Project report

SSY130 Applied signal processing
Project 2

Adaptive Noise Cancellation

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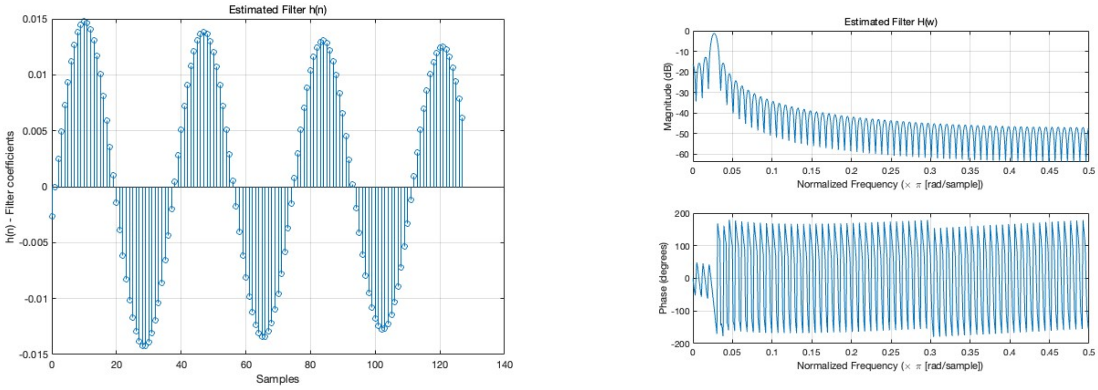
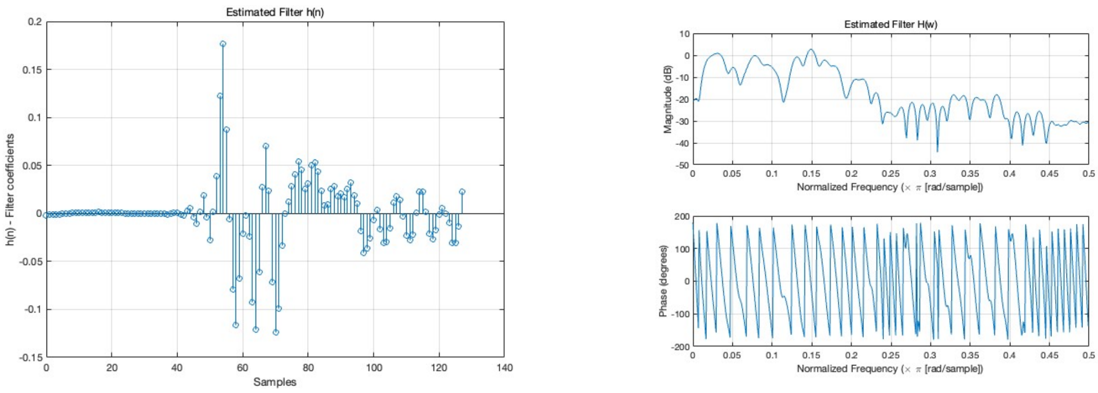
1 Empirical section

1.1 Question 1



(a) h_{sin} **(b)** Magnitude and Phase of H_{sin}

Figure 1: Filter Generated in Sine Noise



(a) h_{bb} **(b)** Magnitude and Phase of H_{bb}

Figure 2: Filter Generated in Broadband Noise

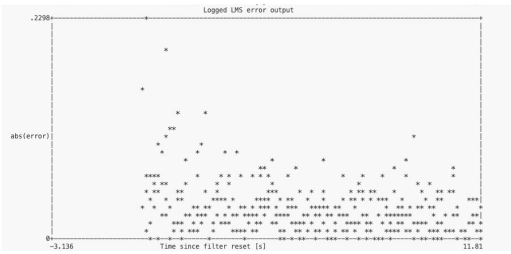
1.2 Question 2

After the filter finished training, set the $\mu = 0$ and test the performance of the filter in the following three cases.

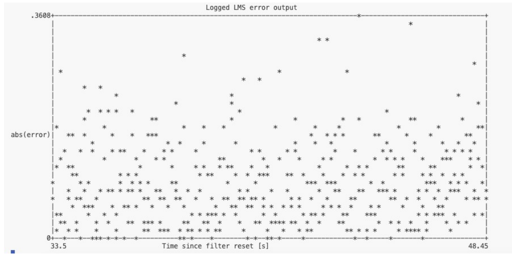
1.2.1 Change Channel Characteristics

Phenomenon Figure 3 shows the performance(i.e., LMS error) with or without a book. We can see an obvious increase in error from about 0.11(after the filter is stable) to 0.36 if we put a book in the middle after the filter is trained.

Reason The filter coefficients h are obtained through error minimization under fixed channel conditions. When the channel characteristics change, the actual channel impulse response h no longer matches the estimated value $\hat{h}$ of the filter. This mismatch prevents the filter from accurately canceling the noise.



(a) LMS Error without Book



(b) LMS Error with Book

Figure 3: LMS Error with/without Book

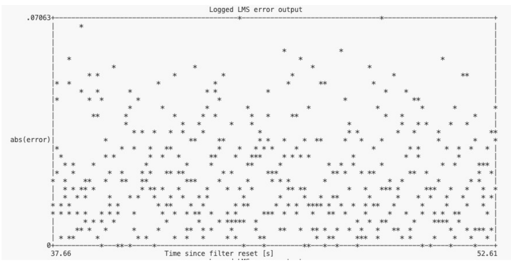
1.2.2 Adjust Speaker Volume

Phenomenon When the volume (noise amplitude) is increased, the LMS error noticeably increases. Conversely, when the volume is reduced, the LMS error decreases.

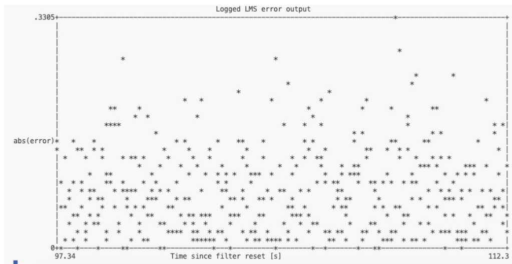
Reason It can be explained with the definition of $e(n)$:

$$e(n) = x(n) - \hat{x}(n)$$

When the amplitude of the noise signal increases, the amplitude of the actual signal $x(n)$ also increases, which in turn causes the estimation error $e(n)$ to grow accordingly. Although the filter still attempts to minimize the error, the overall energy of the input signal increases. As a result, even if the filter model maintains the same relative accuracy, the absolute error will increase. That is also why low volume will lead to less error.



(a) LMS Error in Low Volume



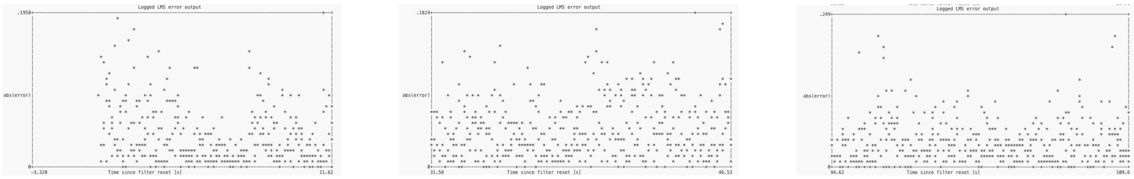
(b) LMS Error in High Volume

Figure 4: LMS Error in Low/High Volume

1.2.3 Using Cell Phone as Signal Source

Phenomenon Train the filter with speaker music and broadband noise, then change the signal source to cell phone and moving around. From Figure 5, we can get the error in these 3 cases are similar, which means that a moving signal source will not affect the performance of the filter.

Reason This is because when the signal source is changed, the noise source remains unchanged, and the noise signal path remains same. Therefore, the estimated channel characteristics of the filter, $\hat{h}$, are still applicable, and the filter can still effectively cancel the noise.

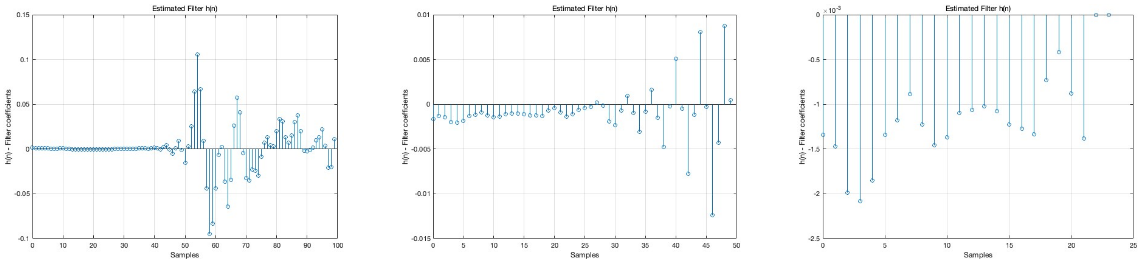


(a) Error with Speaker **(b)** Error with Fixed Phone **(c)** Error with Moving Phone

Figure 5: Using Cell Phone as Signal Source

1.3 Question 3

Coefficients in length 100, 50, 25 Without reset, the filter coefficients for lengths 50 and 25 appear to be truncated versions of the coefficients for a filter length of 100 (Figure 6). Without resetting, the coefficient distributions change more significantly and become less regular, reflecting that the filter is attempting to adapt to the channel under the new length (Figure 8). However, as the filter length decreases, the amplitude of the coefficients decreases regardless of whether the filter is reset or not. This indicates that the filter's ability to affect the noise is significantly reduced as the length decreases.



(a) h_{100} **(b)** h_{50} **(c)** h_{25}

Figure 6: h_{bb} in tap range from 100-50-25 without reset

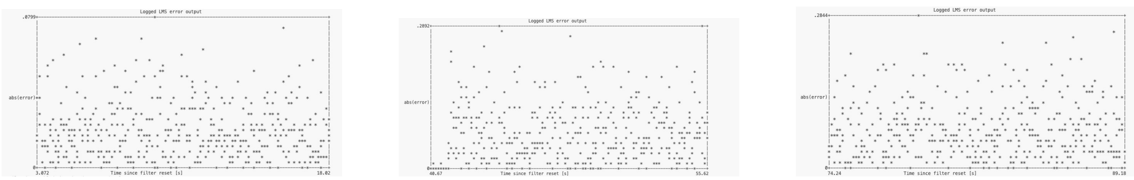
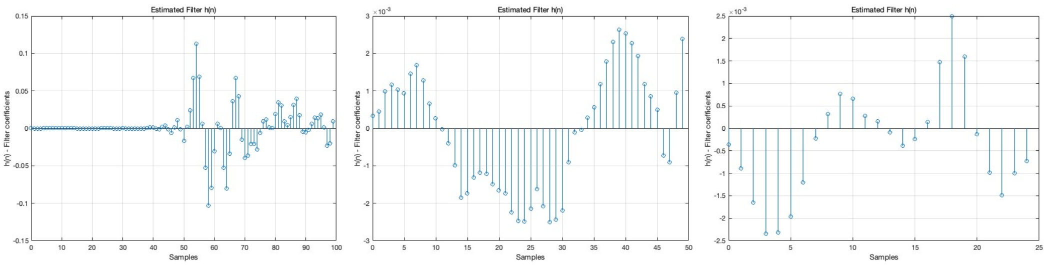


Figure 7: Error using h_{bb} ranging from 100-50-25 without reset

Performance in length 100, 50, 25 Based on Figure 7 and Figure 9, it can be observed that the overall filter performance deteriorates as the filter length decreases, regardless of whether the coefficients are reset or not. However, when the coefficients are reset, the error is relatively lower compared to the case without resetting. This indicates that even with limited filter length, resetting the coefficients allows the filter to adapt to the new channel more effectively, thereby reducing the noise to a certain extent.



(a) h_{100} **(b)** h_{50} **(c)** h_{25}

Figure 8: h_{bb} in tap range from 100-50-25 with reset

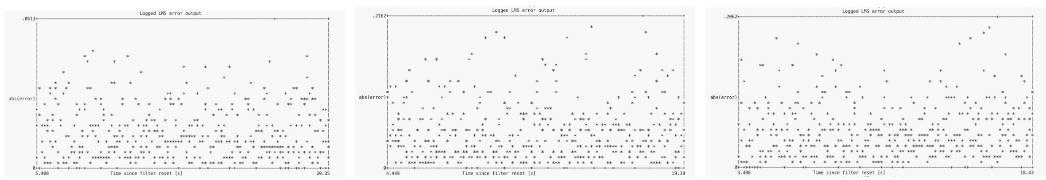


Figure 9: Error using h_{bb} ranging from 100-50-25 with reset

Critical length From Figure 7 and Figure 9, it can be observed that the filter performance significantly degrades when the filter length is reduced from 100 to 50, regardless of whether the coefficients are reset. However, further reducing the length from 50 to 25 does not result in substantial changes. Therefore, we hypothesize that the critical length for performance degradation lies between 50 and 100. After testing, the critical length is found to be around 60.

Referring to Figure 2(a) in Question 1, the significant filter coefficients are concentrated between 50 and 75. This indicates that at least approximately 60 taps are required for the filter to fully capture the characteristics of the channel.

1.4 Question 4

Decrease h_{sin} length from 100 to 10 According to Figure 10 and Figure 11, without resetting the coefficients, when the filter length is reduced, the later coefficients are removed while the earlier ones remain unchanged. As a result, the overall shape and magnitude of the coefficients does not change. With resetting, compared to filter of length $L = 100$, the magnitude of the coefficients for a filter of length $L = 10$ is much larger to compensate the shorter length.

Increase h_{sin} length from 10 to 100 According to Figure 12, when the filter length is increased without resetting the coefficients, the first 10 coefficients remain unchanged, while the newly added coefficients are initialized to zero.1

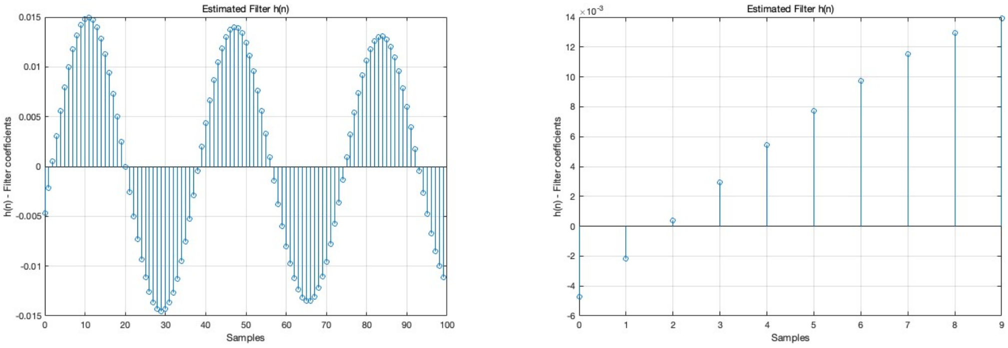


Figure 10: h_{sin} length from 100 to 10 without Reset

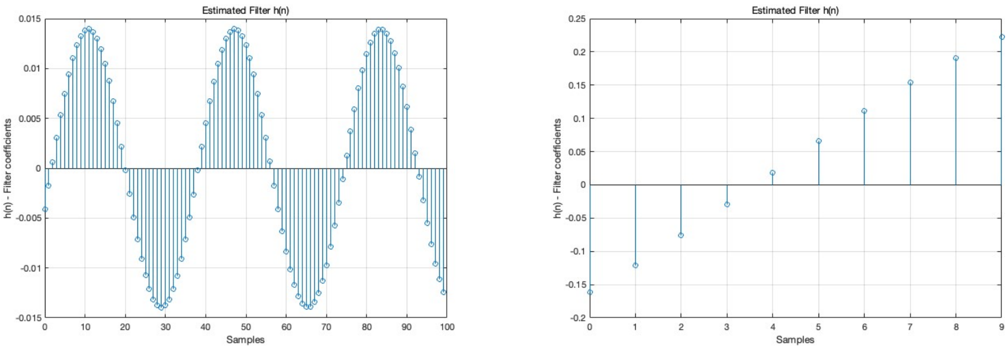


Figure 11: h_{sin} length from 100 to 10 with Reset

Impact of reset or not When the filter length increases from 10 to 100, resetting the filter coefficients has minimal impact, as the sinusoidal noise signal has a single frequency and theoretically requires only a length of 2 to capture its characteristics effectively.
However, when the filter length decreases from 100 to 10, the performance of the filter deteriorates if the coefficients are not reset. This is because the filter retains only the first few coefficients from the length-100 filter (Figure 10), which are often small in magnitude and insufficient to effectively filter the noise.

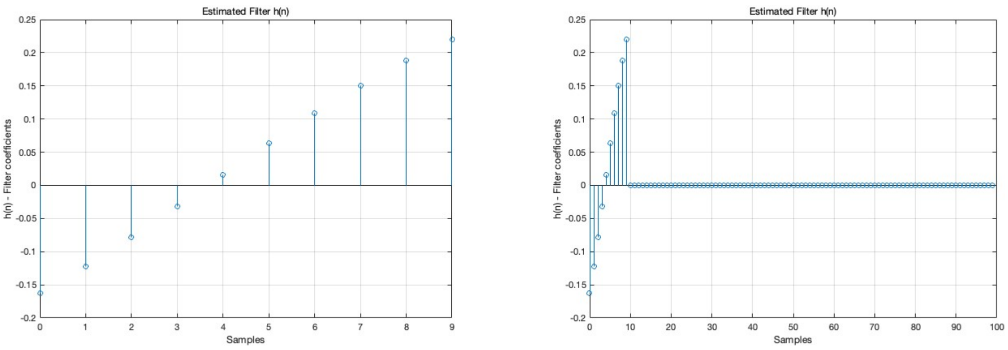


Figure 12: h_{sin} length from 10 to 100 without Reset

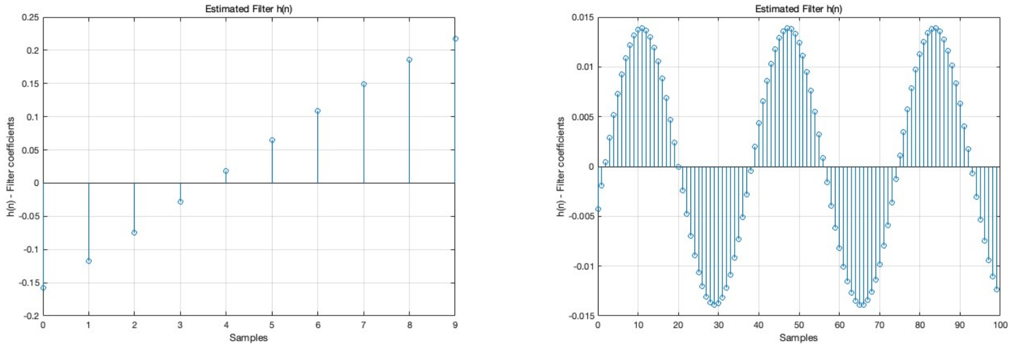
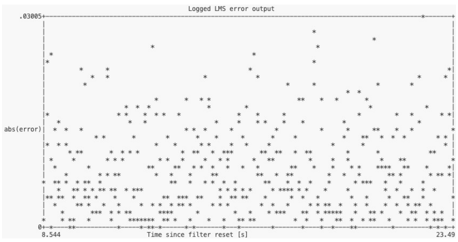


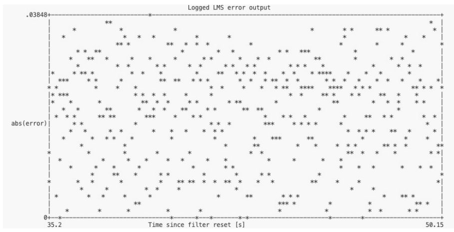
Figure 13: h_{sin} length from 10 to 100 with Reset

1.5 Question 5

Apply h_{bb} to sinusoidal noise According to Figure 14, the filter h_{BB} demonstrates good adaptability to sinusoidal noise and can effectively attenuate it. This is because the broadband noise-trained filter captures multiple frequency components, allowing it to accommodate the single frequency characteristic of sinusoidal noise.



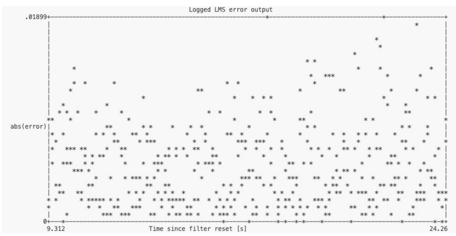
(a) h_{bb} in Broadband Noise



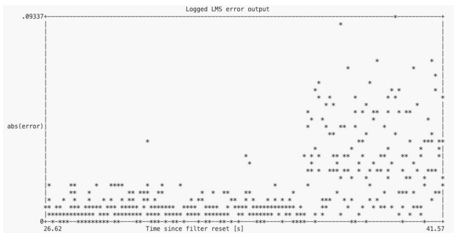
(b) h_{bb} in Sinusoidal Noise

Figure 14: Error from Broadband Noise to Sinusoidal Noise

Apply h_{sin} to broadband noise Conversely, h_{sin} fails to effectively filter broadband noise. As shown in Figure 15(b), when the noise source is switched from sinusoidal noise to broadband noise, the LMS error increases significantly. This is because sinusoidal noise is a single-frequency signal, and the trained filter coefficients of h_{sin} are concentrated around that frequency. Consequently, it lacks the modeling capability for broadband noise, which requires a more distributed set of weights across multiple frequency components.



(a) h_{sin} in Sinusoidal Noise



(b) h_{sin} in Broadband Noise

Figure 15: Error from Sinusoidal Noise to Broadband Noise

1.6 Question 6

Based on Figure 15, when the volume reaches saturation due to excessive levels, the filter coefficients $h_{\text{bb,sat}}$ exhibit a significantly abnormal distribution compared to h_{bb}. This discrepancy arises because the LMS algorithm is designed for linear channels, while the saturation-induced clipping introduces a nonlinear operation. The output under clipping no longer satisfies the linear superposition property, i.e., $f(x+y) \neq f(x) + f(y)$. This nonlinearity disrupts the fundamental assumptions of the LMS algorithm, making it challenging for the filter to accurately estimate the true characteristics of the channel.

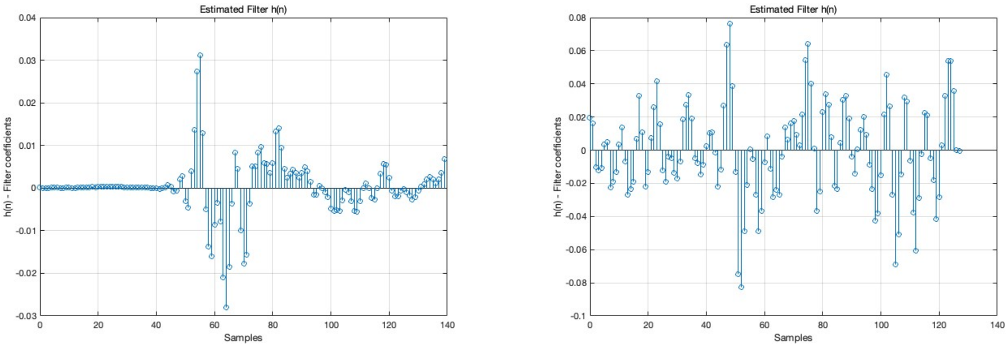


Figure 16: h_{bb} and $h_{bb,sat}$

2 Analytical section

2.1 Question 1

a) The step size μ is directly related to the performance of the filter. If the step size is large enough, the filter will rapidly adapt and exceed the optimal point causing an increase in error, i.e. LMS is diverged. In contrast, the lower the step size, the lower the error. But this also leads to a very slow adaption.

b) Yes, a large enough step size causes the estimated channel to diverge. The bounds on μ guaranteeing convergence are shown in (14.16) from the lecture note as:

$$0 < \mu < \frac{1}{\lambda_1}$$

Where λ_1 is the max value of eigenvalue λ_M. A large step size can cause the overshoot to the optimal point adapting and resulting in oscillation and divergence.

c) Yes, the choice of the step size affects how well the filter works once it has converged. Given that the equation (13.5) from the lecture note:

$$\hat{h}(N+1) = \hat{h}(N) + 2\mu y(N)e(N)$$

On the premise of convergence, the lower step size leads to a lower error and slower convergence and vice versa.

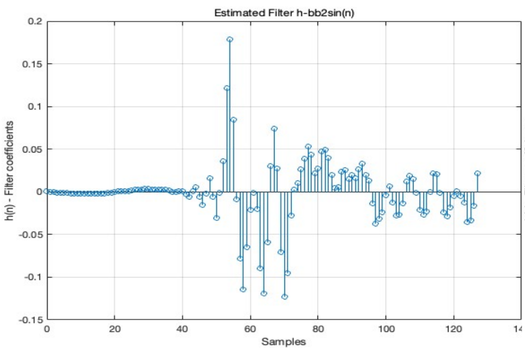
2.2 Question 2

For broadband noise The reduction of filter length causes a significant deterioration. Through reset the filter coefficient can partly reduce that, but those errors are still not bearable. The reason is that broadband noise distribution in a rather wide frequency range and a longer filter. At this point, the objective function becomes stricter, requiring the filter to match the true channel $H(\omega)$ as closely as possible across all frequency points, thereby increasing the uniqueness of the solution. In this project, at least 50 to 70 coefficients is needed to have a good performance.

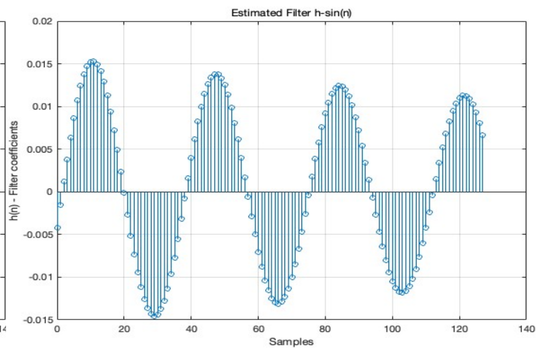
For sinusoidal noise Decreasing filter length does not affect the error performance from 100 to 10 taps. This is because the sinusoidal noise has a concentration in a narrow frequency. In other words, the objective function does not require the filter's response at other frequencies, resulting in a certain degree of freedom in the solution and a non-uniqueness. Consequently, the filter only needs a small number of coefficients to estimate.

2.3 Question 3

It is clear that sinusoidal noise is concentrated in a narrow frequency which overlaps with the original broadband noise. Therefore when the filter is trained using broadband noise can easily adapt to the narrowband sinusoidal noise and does not need to change coefficients.



(a) h_{bb2sin}



(b) h_{sin}

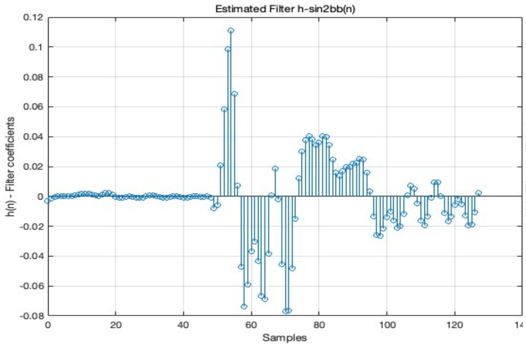
Figure 17: Sinusoidal noise

In contrast, adapting broadband noise requires a longer filter to cover a larger range of noise frequency. In this case, the filter coefficient change is necessary.

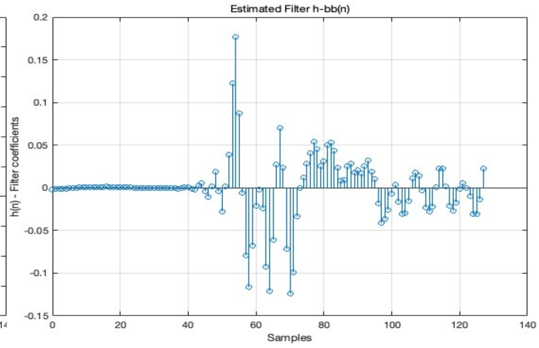
2.4 Question 4

The magnitude and phase of $H_{\sin}(f_0)$ and $H_{\mathrm{BB}}(f_0)$ are as in Fig.19 and Fig.20.

The results are consistent with our expectations They behave similarly at f_0, but $H_{\sin}(f)$ performs better. $H_{\sin}$ exhibits a lower magnitude at this frequency, which indicates that it attenuates the noise at f_0 more effectively. In contrast,



(a) h_{sin2bb}



(b) h_{bb}

Figure 18: Broadband noise

```
For H_sin(f)
At f = 440 Hz:
Magnitude = 0.86803
Phase (degrees) = -103.9211
```

Figure 19: The magnitude and phase of $H_{\text{sin}}(f_0)$

```
For H_bb(f)
At f = 440 Hz:
Magnitude = 1.0576
Phase (degrees) = -109.1978
```

Figure 20: The magnitude and phase of $H_{\text{BB}}(f_0)$

H_{BB} has a slightly higher magnitude, showing weaker attenuation capability at this frequency compared to H_{sin}.

Reason: H_{sin} is specifically trained for sinusoidal noise (single-frequency noise), so it achieves better performance at $f_0 = 440\,\text{Hz}$. H_{BB}, on the other hand, is trained for broadband noise (covering multiple frequencies), and it must balance attenuation across multiple frequencies, leading to slightly weaker performance at a single frequency f_0.

2.5 Question 5

The minimal theoretical length is 2 (N=1, corresponding to h0 and h1.) for the sinusoidal disturbance. The reason is because that to achieve zero error at the frequency, we only need the filter's DTFT to be correct at that point in the frequency domain. As we know, the complex-valued signal contains two real numbers, one for the real part and one for the imaginary part.

$$H_{\text{desired}}(f_0) = Ae^{j\phi},$$

where, A is the amplitude and ϕ is the phase. With a finite impulse response(FIR) filter, each tap contributes to the DTFT as a complex exponential combination. The minimal FIR filter that can produce a complex response at a single frequency requires at least 2 coefficients. For this two-tap filter:

$$\hat{H}(e^{j\omega_0}) = h_0 + h_1 e^{-j\omega_0}.$$

If only one coefficient(h0), it can only scale the input which only adjusts amplitude. The phase shift near the input is lost. When there are two coefficients, they can form a linear combination of the input and it can produce both amplitude and phase adjustments at that frequency. In this case, we can generate a complex response that matches the needed complex gain at the frequency.

Do we need H= for all frequencies to get zero error?

No, we only need to approximate the value at the disturbance frequency. It is unnecessary to match the entire frequency response.

2.6 Question 6

The goal of the LMS algorithm is to make the filter coefficients $\hat{h}$ minimize the Mean Square Error (MSE) function:

$$\hat{h}_{\text{opt}} = \arg\min_{h} E[e(n)^2].$$

From the frequency domain perspective, this optimization problem is equivalent to minimizing:

$$E\left[|Y(H - \hat{H})|^2\right].$$

h_{sin} **has a distinctive form** To ensure that the output accurately matches the sinusoidal wave (i.e., minimizing the error $e(n) = x(n) - \hat{x}(n)$), the filter coefficients $\hat{h}$ must be capable of synthesizing a sinusoidal wave with the same frequency as the input sinusoidal wave.

Mathematically, this means: The form of the filter coefficients $\hat{h}$ is a discrete sinusoidal or cosine wave, with its frequency consistent with the input sinusoidal wave.

Increased length leads to different forms When the filter length increases, the degrees of freedom (number of parameters) of the filter coefficients increase accordingly. More parameters mean that, under the condition of matching the key frequency points $\hat{H}(\omega)$ and $H(\omega)$, there are still multiple ways to allocate the remaining coefficients. At the unconstrained frequency ranges, the filter coefficients can be distributed in various forms, resulting in multiple sets of different $\hat{h}$ that achieve the same minimum MSE.

3 Appendix

```
for (int n = 0; n < block_size; n++) {
    float *y_book = &lms_state[n];
    arm_dot_prod_f32(lms_coeffs, y_book, lms_taps, &xhat[n]);
    e[n] = x[n] - xhat[n];

    for (int i = 0; i < lms_taps; i++) {
        lms_coeffs[i] += 2 * lms_mu * y_book[i] * e[n];
    }
}
```