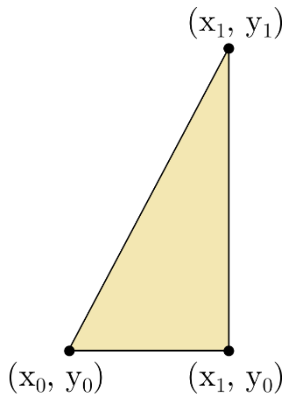
Analytical solution for triangles of various orientation

In Jansen et al. (2019) and Cornelissen et al. (2023), the analytical solution for the strain field for a triangular inclusion with specific geometry. To model the strains and stresses in a faulted reservoir, we also need the analytical solution for triangular inclusions with different orientation. It is not necessary to re-integrate the Green's functions, but instead these solutions can simply be obtained from the given analytical solution for a triangle. Here, we present how these solutions can be obtained.

1. 'Regular' triangle

This is the original triangle for which the analytical solution in Jansen et al. (2019) and Cornelissen et al. (2023) was derived. Here, it is assumed that the hypotenuse of the triangle passes through the point (0,0), which is the origin of the coordinate system. Defining x_0 as the minimum x-coordinate, x_1 as the maximum x-coordinate, y_0 as the minimum y-coordinate, and y_1 as the maximum y-coordinate of the triangle, the integral limits are then given by:



$$G_{xx} = \int_{y_0}^{y_1} \int_{\frac{\xi}{\tan\theta}}^{x_1} g_{xx}$$

$$G_{yy} = \int_{x_0}^{x_1} \int_{y_0}^{\zeta \tan\theta} g_{yy}$$

The analytical solution for G_{xx} and G_{yy} is then given by Jansen et al. (2019) and Cornelissen et al. (2023) as:

$$\begin{aligned} G_{xx}&(x, y, o, p, r, s) \\ &= \operatorname{atan}\left(\frac{y-s}{x-p}\right) - \operatorname{atan}\left(\frac{y-r}{x-p}\right) + \frac{\sin\theta\cos\theta}{2}\ln\left(\frac{(x-o)^2+(y-r)^2}{(x-p)^2+(y-s)^2}\right) \\ &- \sin^2\theta\left(\operatorname{atan}\left(\frac{(x-p)\cot\theta+(y-s)}{x-y\cot\theta}\right) - \operatorname{atan}\left(\frac{(x-o)\cot\theta+(y-r)}{x-y\cot\theta}\right)\right) \end{aligned}$$

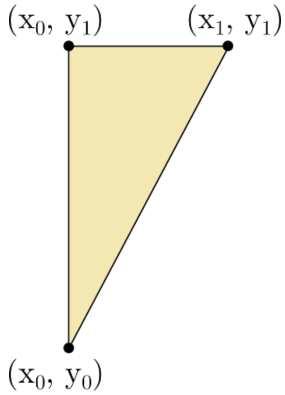
$$\begin{aligned} G_{yy}&(x, y, o, p, r, s) \\ &= \operatorname{atan}\left(\frac{x-o}{y-r}\right) - \operatorname{atan}\left(\frac{x-p}{y-r}\right) - \frac{\sin\theta\cos\theta}{2}\ln\left(\frac{(x-o)^2+(y-r)^2}{(x-p)^2+(y-s)^2}\right) \\ &+ \cos^2\theta\left(\operatorname{atan}\left(\frac{(x-p)+(y-s)\tan\theta}{y-x\tan\theta}\right) - \operatorname{atan}\left(\frac{(x-o)+(y-r)\tan\theta}{y-x\tan\theta}\right)\right) \end{aligned}$$

where the input parameters o, p, r, and s are given by

$o = x_0$	$p = x_1$
$r = y_0$	$s = y_1$

2. Mirrored upside-down triangle

Next, we consider a triangle that is flipped both horizontally and vertically compared to the 'regular' triangle. Again, we define x_0 as the minimum x-coordinate, x_1 as the maximum x-coordinate, y_0 as the minimum y-coordinate, and y_1 as the maximum y-coordinate of the triangle. The integrals can then be written as:



$$G_{xx,2} = \int_{y_0}^{y_1} \int_{x_0}^{\frac{\xi}{\tan\theta}} g_{xx} = -\int_{y_0}^{y_1} \int_{\frac{\xi}{\tan\theta}}^{x_0} g_{xx} = \int_{y_1}^{y_0} \int_{\frac{\xi}{\tan\theta}}^{x_0} g_{xx}$$

$$G_{yy,2} = \int_{x_0}^{x_1} \int_{\zeta\tan\theta}^{y_1} g_{yy} = -\int_{x_0}^{x_1} \int_{y_1}^{\zeta\tan\theta} g_{yy} = \int_{x_1}^{x_0} \int_{y_1}^{\zeta\tan\theta} g_{yy}$$

These integrals have a similar form as for the 'regular' triangle, but now x_0 and x_1 and y_0 and y_1 are switched around. Hence, we have

$$G_{xx,2} = G_{xx}(x, y, o, p, r, s)$$

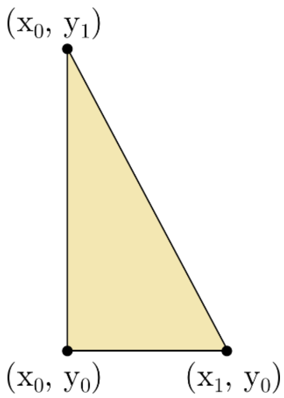
$$G_{yy,2} = G_{yy}(x, y, o, p, r, s)$$

with

$o = x_1$	$p = x_0$
$r = y_1$	$s = y_0$

3. Mirrored triangle

Now we consider a triangle which is mirrored. Again, we define x_0 as the minimum x-coordinate, x_1 as the maximum x-coordinate, y_0 as the minimum y-coordinate, and y_1 as the maximum y-coordinate of the triangle. The integration limits can be written as



$$G_{xx,3} = \int_{y_0}^{y_1} \int_{x_0}^{\frac{\xi}{\tan\theta}} g_{xx} = -\int_{y_0}^{y_1} \int_{\frac{\xi}{\tan\theta}}^{x_0} g_{xx}$$

$$G_{yy,3} = \int_{x_0}^{x_1} \int_{y_0}^{\zeta\tan\theta} g_{yy} = -\int_{x_1}^{x_0} \int_{y_0}^{\zeta\tan\theta} g_{yy}$$

This again has a similar form to the integral of the 'regular' triangle

$$G_{xx,3} = -G_{xx}(x, y, o, p, r, s)$$

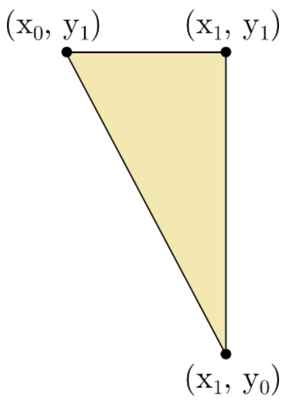
$$G_{yy,3} = -G_{yy}(x, y, o, p, r, s)$$

with

$o = x_1$	$p = x_0$
$r = y_0$	$s = y_1$

4. Upside-down triangle

Finally, we consider an upside-down triangle. Again, we define x_0 as the minimum x-coordinate, x_1 as the maximum x-coordinate, y_0 as the minimum y-coordinate, and y_1 as the maximum y-coordinate of the triangle. The integration limits can then be written as



$$G_{xx,4} = \int_{y_0}^{y_1} \int_{\frac{\xi}{\tan\theta}}^{x_1} g_{xx} = -\int_{y_1}^{y_0} \int_{\frac{\xi}{\tan\theta}}^{x_1} g_{xx}$$

$$G_{yy,4} = \int_{x_0}^{x_1} \int_{\zeta\tan\theta}^{y_1} g_{yy} = -\int_{x_0}^{x_1} \int_{y_1}^{\zeta\tan\theta} g_{yy}$$

This once again has a similar form to the integral of the 'regular' triangle, and hence we have

$$G_{xx,4} = -G_{xx}(x, y, o, p, r, s)$$

$$G_{yy,4} = -G_{yy}(x, y, o, p, r, s)$$

with

$o = x_0$	$p = x_1$
$r = y_1$	$s = y_0$

References

Cornelissen, P., Meulenbroek, B.J., Jansen, J.D., (2023). On the derivation of closed-form expressions for displacements, strains and stresses inside a poroelastic inclusion. Delft University of Technology. https://doi.org/10.4233/uuid:e2f21e31-3221-4d61-bb97-c74bdd3567a8

Jansen, J.D., Singhal, P., Vossepoel, F.C., (2019). Insights from closed-form expressions for injection- and production-induced stresses in displaced faults. Journal of Geophysical Research: Solid Earth, 124, 7193–7212. https://doi.org/10.1029/2019JB017932