

# Fluid Dynamics modelling in Vascular Research

## A journey beyond Hagen and Poiseuille

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Northern Vascular Biology Forum 2019

# Outline

## 1 Introduction

- Fluid Dynamics - Basics
- Importance of Wall Shear Stresses (WSS)
- Non-Newtonian Blood Rheology

## 2 Some Results

- Wall Shear Stress Metrics for Atherosclerosis Risk Prediction
- Influence of Rheology Model and Particle Migration
- What Happens at the *Meso Scale*?

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# Poiseuille Flow

... and why it's a bad way to think about vascular flows

Poiseuille equation for laminar flow in pipes:

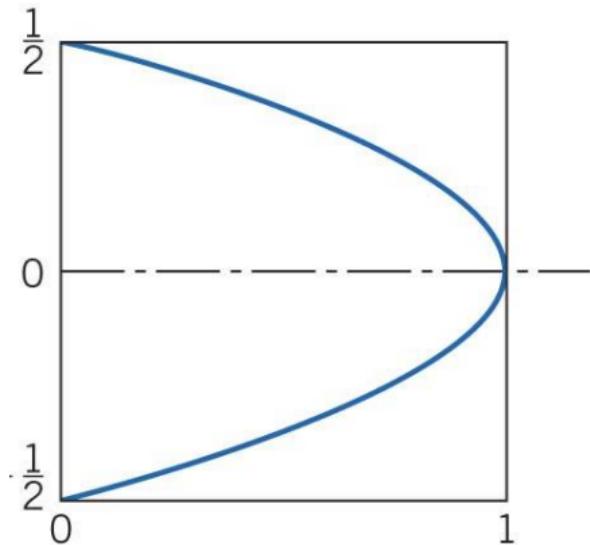
$$v(r) = -\frac{1}{4\mu} \frac{dp}{dx} R^2 \left(1 - \left(\frac{r}{R}\right)^2\right)$$

or:

$$v(r) = 2v_{avg} \left(1 - \left(\frac{r}{R}\right)^2\right)$$

and the WSS is:

$$\tau_w = -\mu \left. \frac{dv}{dr} \right|_{r=R} = 8 \frac{\mu v_{avg}}{D}$$



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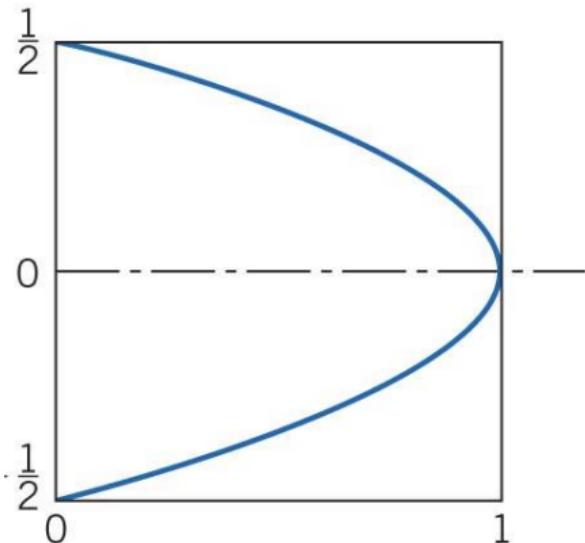
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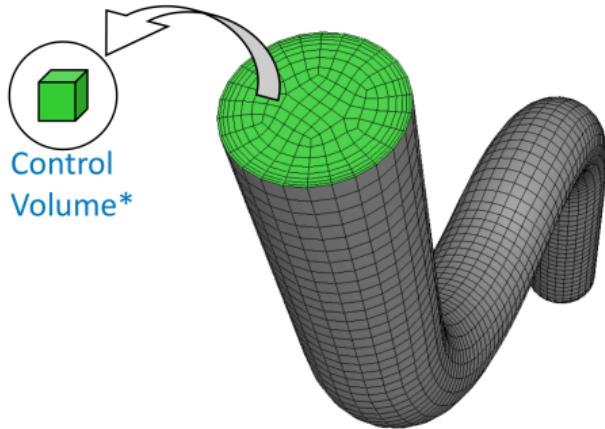
$$\tau_w = -\mu \left. \frac{dv}{dr} \right|_{r=R} = 8 \frac{\mu v_{avg}}{D}$$



Assumptions:

steady state, laminar, Newtonian fluid,  
straight pipe

# How does fluid dynamics modelling work?



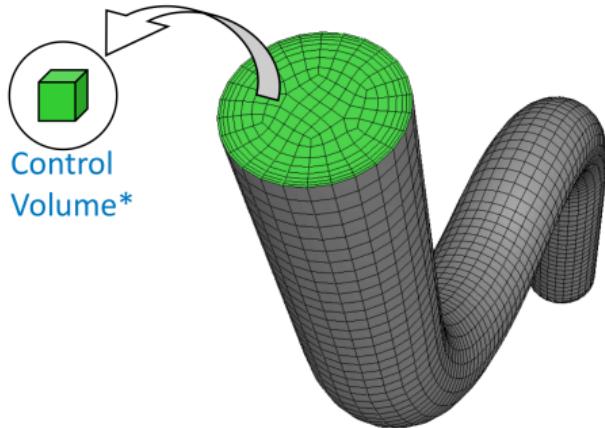
Control  
Volume\*

*Courtesy of ANSYS Inc.*

## Principle:

- Domain is split into small, "finite" volumes
- Equations are balanced over the individual volumes

# How does fluid dynamics modelling work?



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## Principle:

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Courtesy of ANSYS Inc.

Then for me as a modeller all problems look like this:

$$\frac{\partial}{\partial t} \int_V \rho \phi dV + \int_A \rho \phi \vec{v} \cdot dA = \int_A \Gamma_\phi \nabla \phi \cdot dA + \int_V S_\phi dV$$

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# Importance of Wall Shear Stresses (WSS)

$$TAWSS = \frac{1}{T} \int_0^T |\tau| dt$$

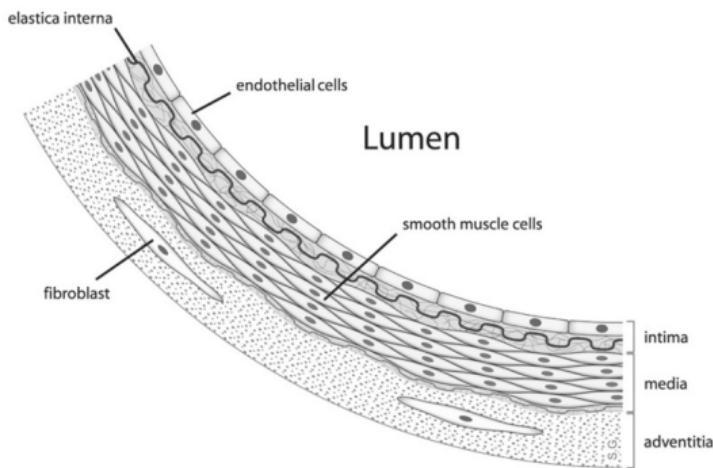
Time Averaged Wall Shear Stress

$$transWSS = \frac{1}{T} \int_0^T \left| \vec{\tau} \times \left( \vec{n} \cdot \frac{\int_0^T \tau dt}{\left| \int_0^T \tau dt \right|} \right) \right|$$

Transverse Wall Shear Stress

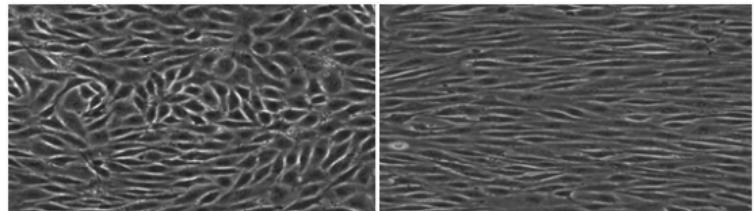
$$OSI = 0.5 \left[ 1 - \left( \frac{\left| \int_0^T \tau \right|}{\int_0^T |\tau| dt} \right) \right]$$

Oscillatory Shear Index



$$RRT \sim [(1 - 2OSI)TAWSS]^{-1}$$

Relative Residence Time



*Remodelling of Endothelial Cells in Culture. Left Panel static culture (no flow). Right Panel Cultured cells exposed to 10 dyn/cm<sup>2</sup> for 24 hrs (flow is from left to right). (Wallace H. Coulter Laboratory for Cardiovascular Dynamics and Biomolecular Transport, CUNY)*

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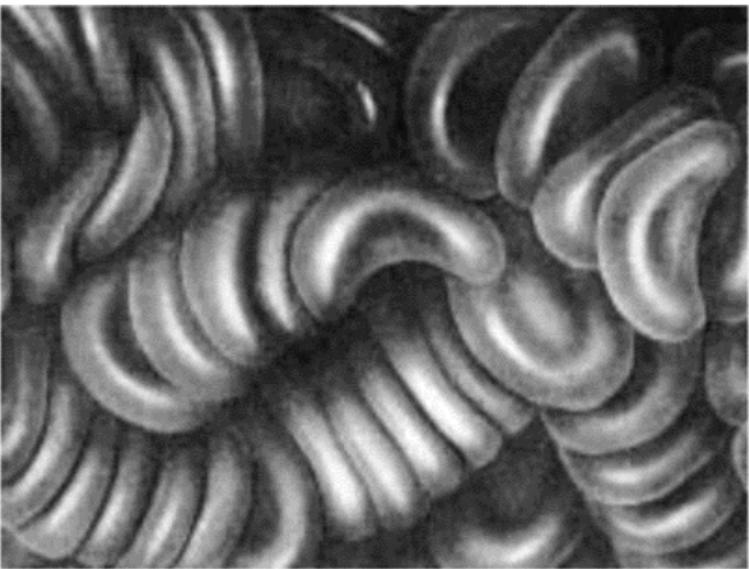
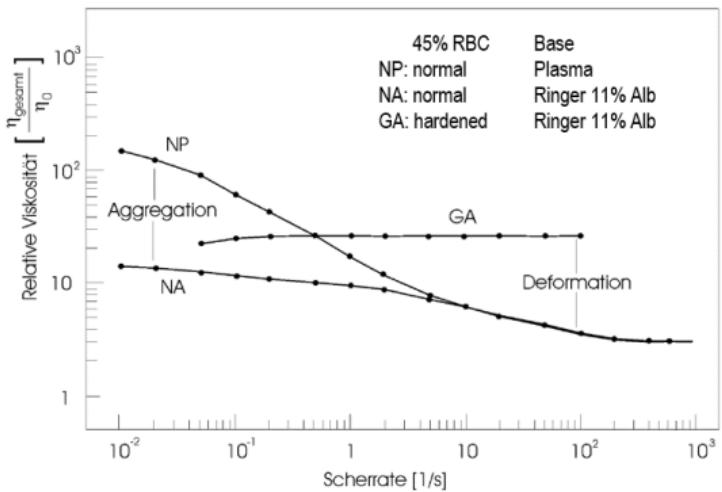
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# Blood as a Non-Newtonian Suspension



Blood viscosity depends on

- shear rate
- haematocrit concentration
- and condition of blood

Data (1), RBC aggregates reproduced from Schmid-Schönbein et al. Exempla hämorheologica. 1980

# Blood Rheology, the Data<sup>1</sup>

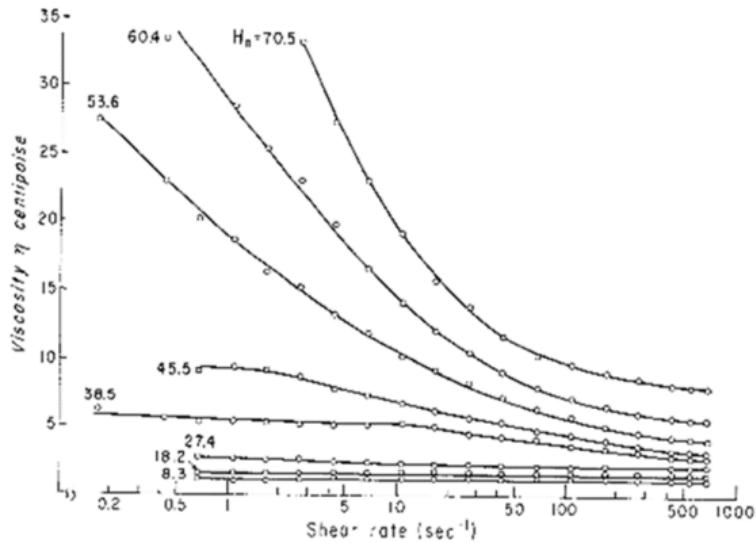


FIG. 2. Relationship between viscosity and rate of shear for human red blood cell suspensions in standard saline at 25 C for various volume concentrations of erythrocytes,  $H_n$  (hematocrit  $\times 0.96$ ).

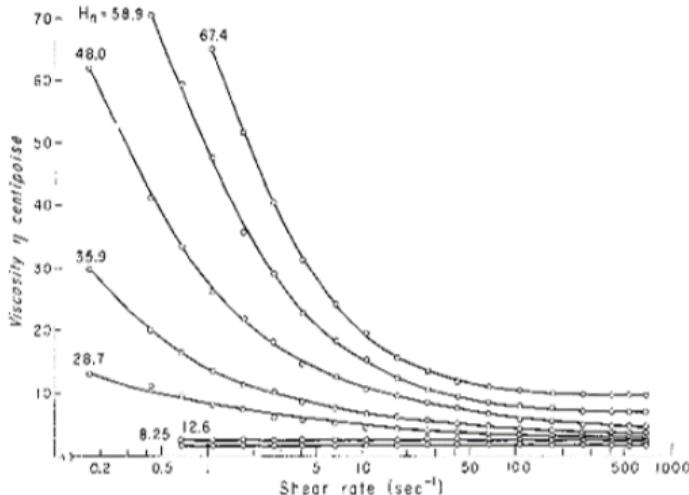
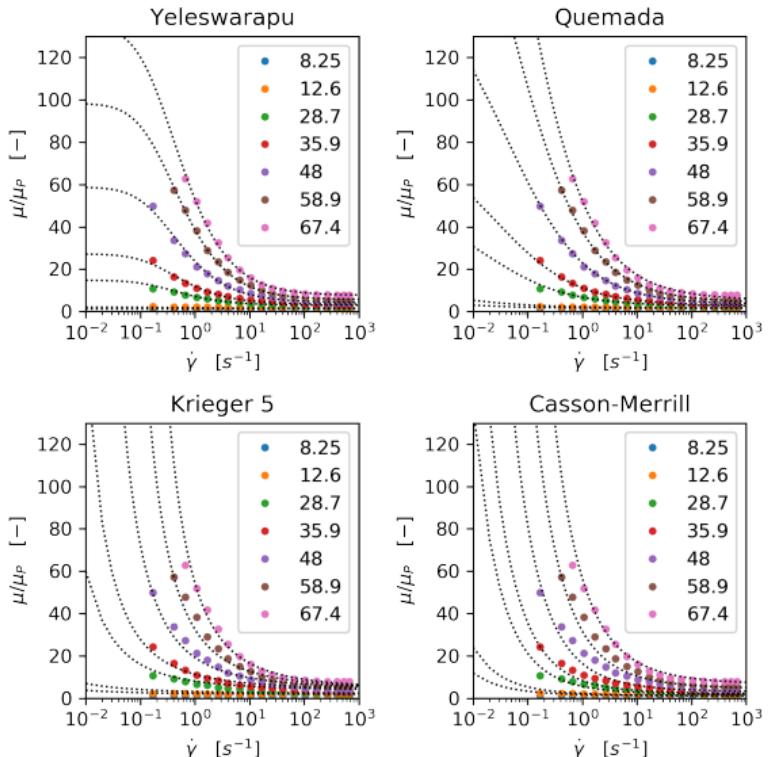


FIG. 3. Relationship between viscosity and rate of shear for human red blood cell suspensions in homologous ACD-plasma at 25 C for various volume concentrations of the erythrocytes,  $H_n$  (hematocrit  $\times 0.96$ ). Note that viscosity range is double that of Fig. 2.

<sup>1</sup>D. E. Brooks et al., *Journal of Applied Physiology* 28, 172–177 (1970).

# Blood Rheology, the Models

- *Carreau* type models: plateau at low shear, typically no haematocrit dependency, has been included in newer variations, e.g., Yeleswarapu model (3).
- *Casson* type models: infinite viscosity at zero shear, can include yield stress.
- *Krieger* type models: particle collision based models, widely used for high particle load suspensions, but typically no shear thinning, included in some derived models (4).
- *Quemada* model: mechanistic model, incorporating haematocrit and shear dependency based on collision considerations (5–7).



*Models matched to Brooks data: dots: experiment data,  
dotted line: model*

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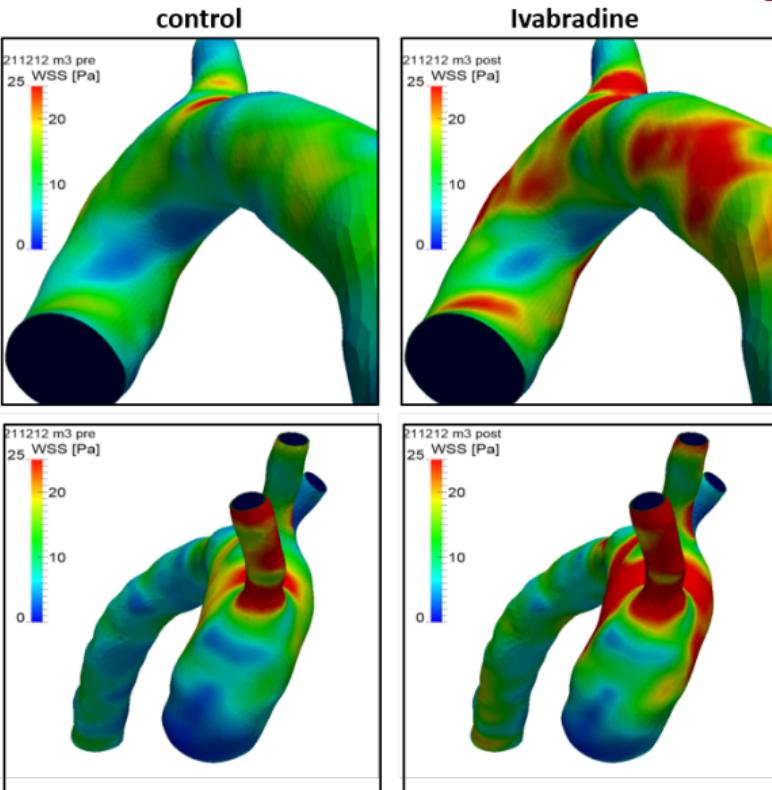
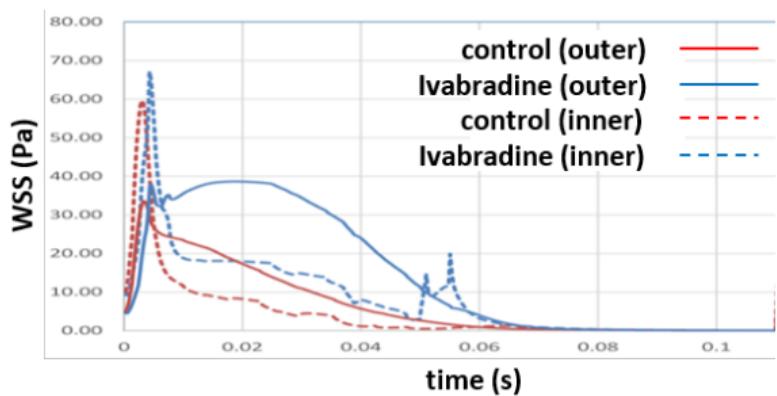
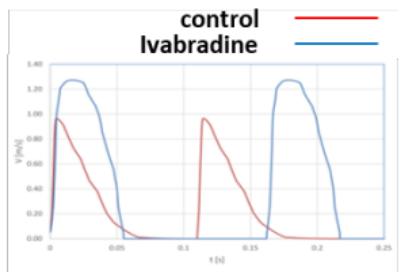
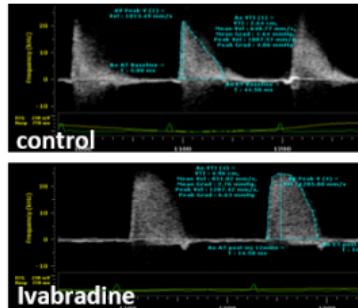
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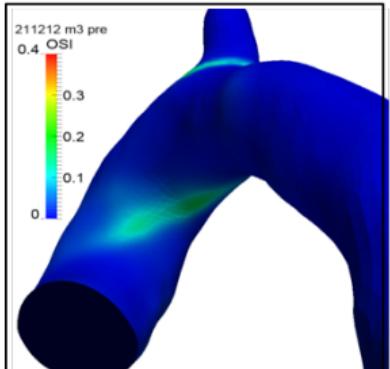
## Murine Aorta - Ivabradine Study<sup>2</sup>



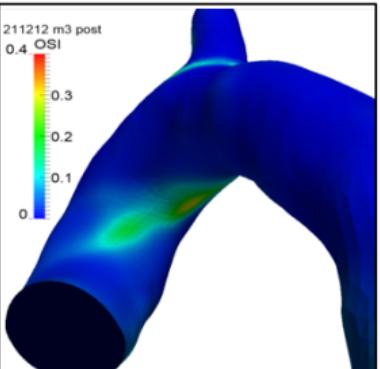
<sup>2</sup>L. Luong et al., *Thrombosis and Haemostasis* **116**, 00000, 181–190 (2016).

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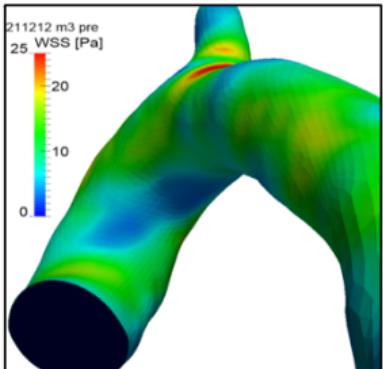
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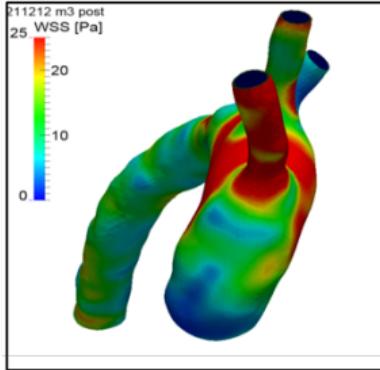
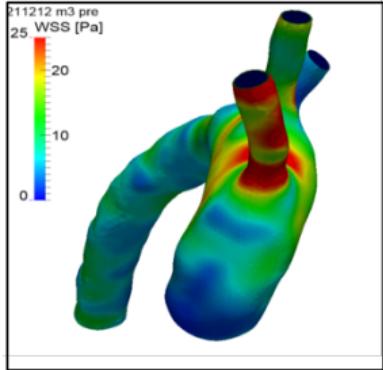
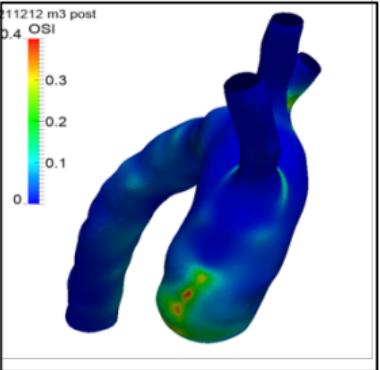
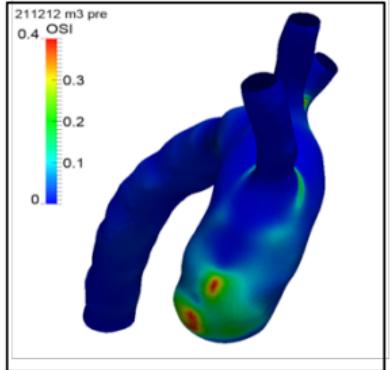
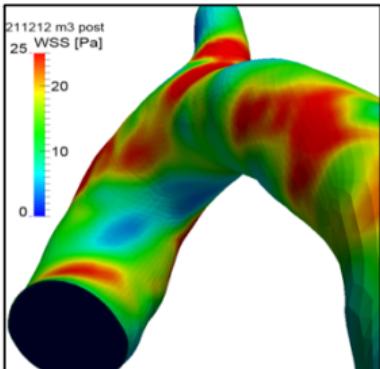
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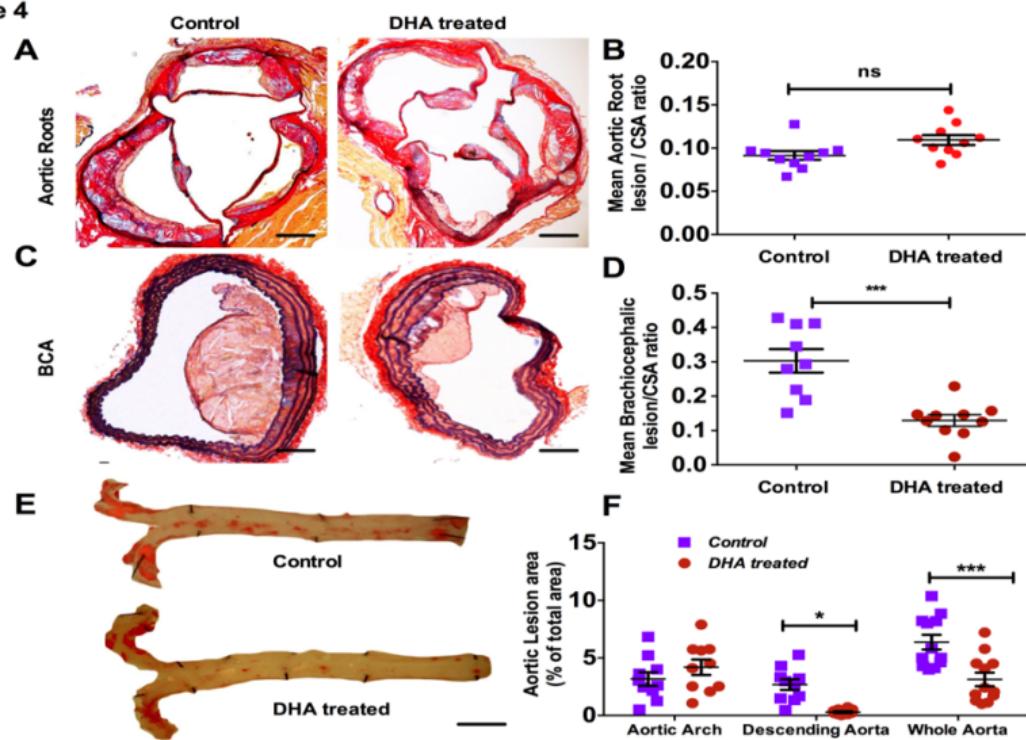
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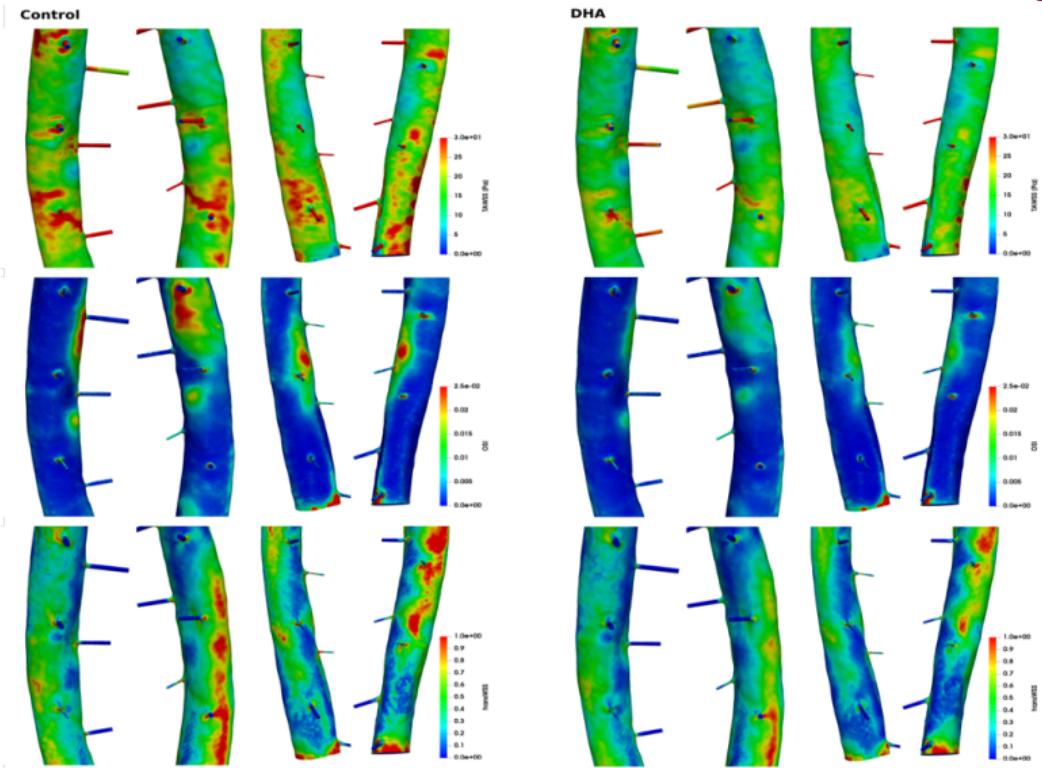
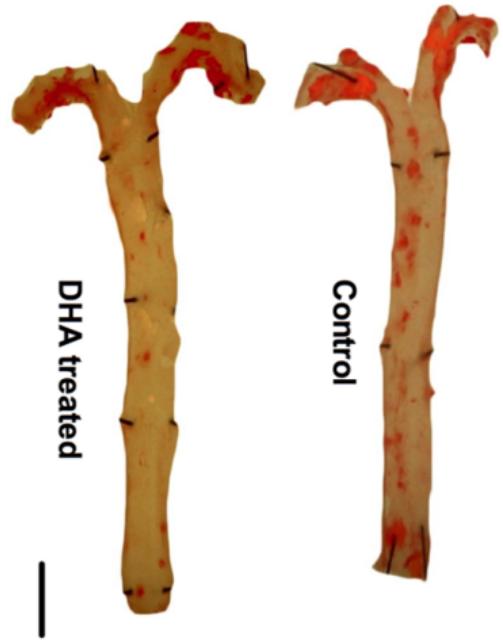
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Murine Aorta - Fish Oil Study<sup>3</sup>

Figure 4

<sup>3</sup>M. A. Alfaidi et al., *Journal of the American Heart Association* 7, e008757 (July 3, 2018).

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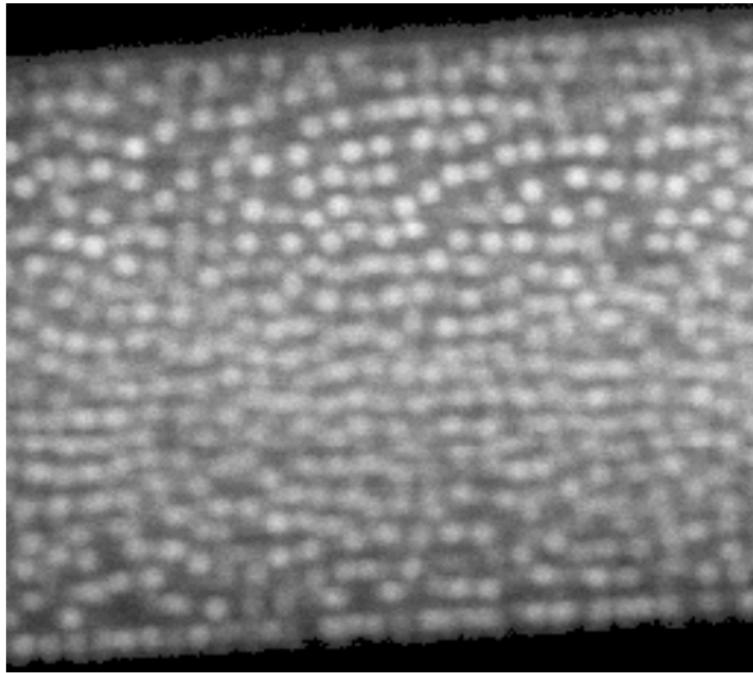
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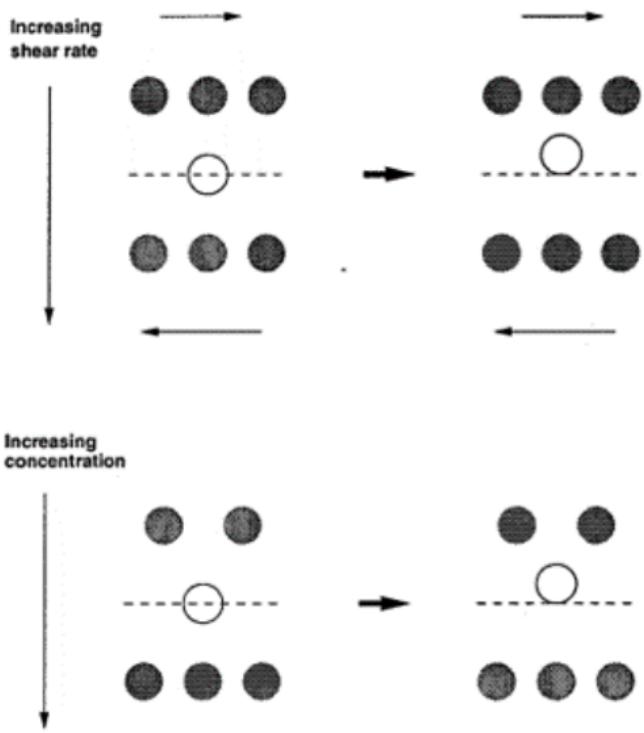
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# Particle Migration Model

Blood is a suspension of particles that move against and collide with each other. This gives rise to bulk migration of particles against the shear gradient:

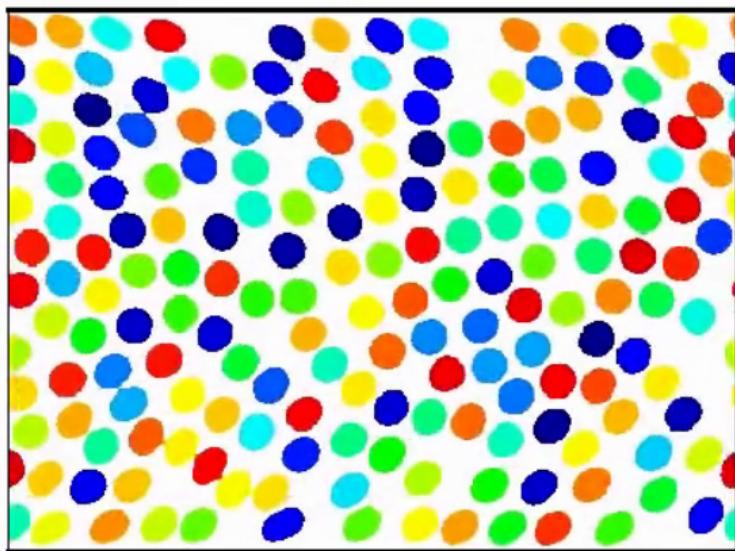


*Rigid, spherical particles. NMR imaging (unknown).*

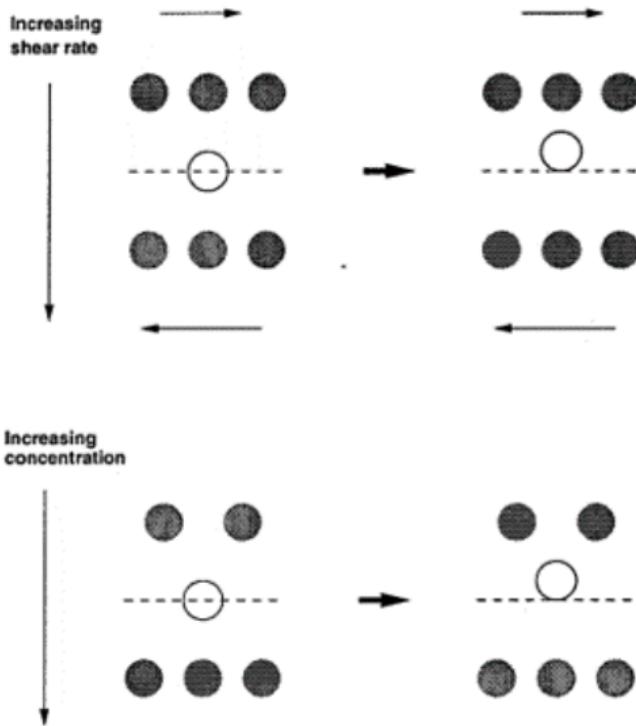


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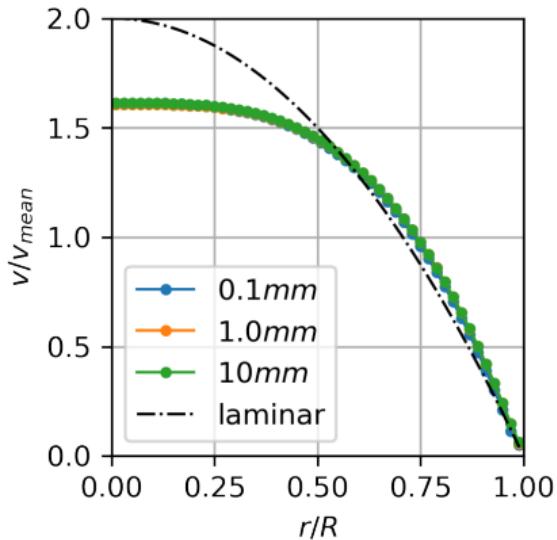
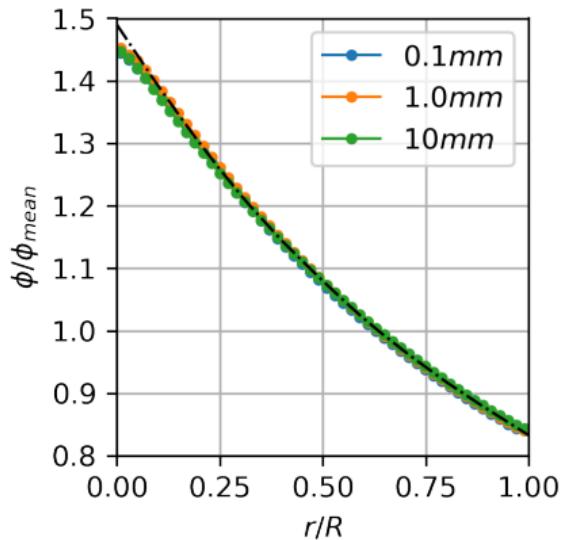
Rigid, spherical particles. Lattice Boltzmann Simulation<sup>4</sup>.



<sup>4</sup>Simulation: Spencer, Halliday (MERI) 2010

# Particle Migration in steady state, laminar, pipe flow

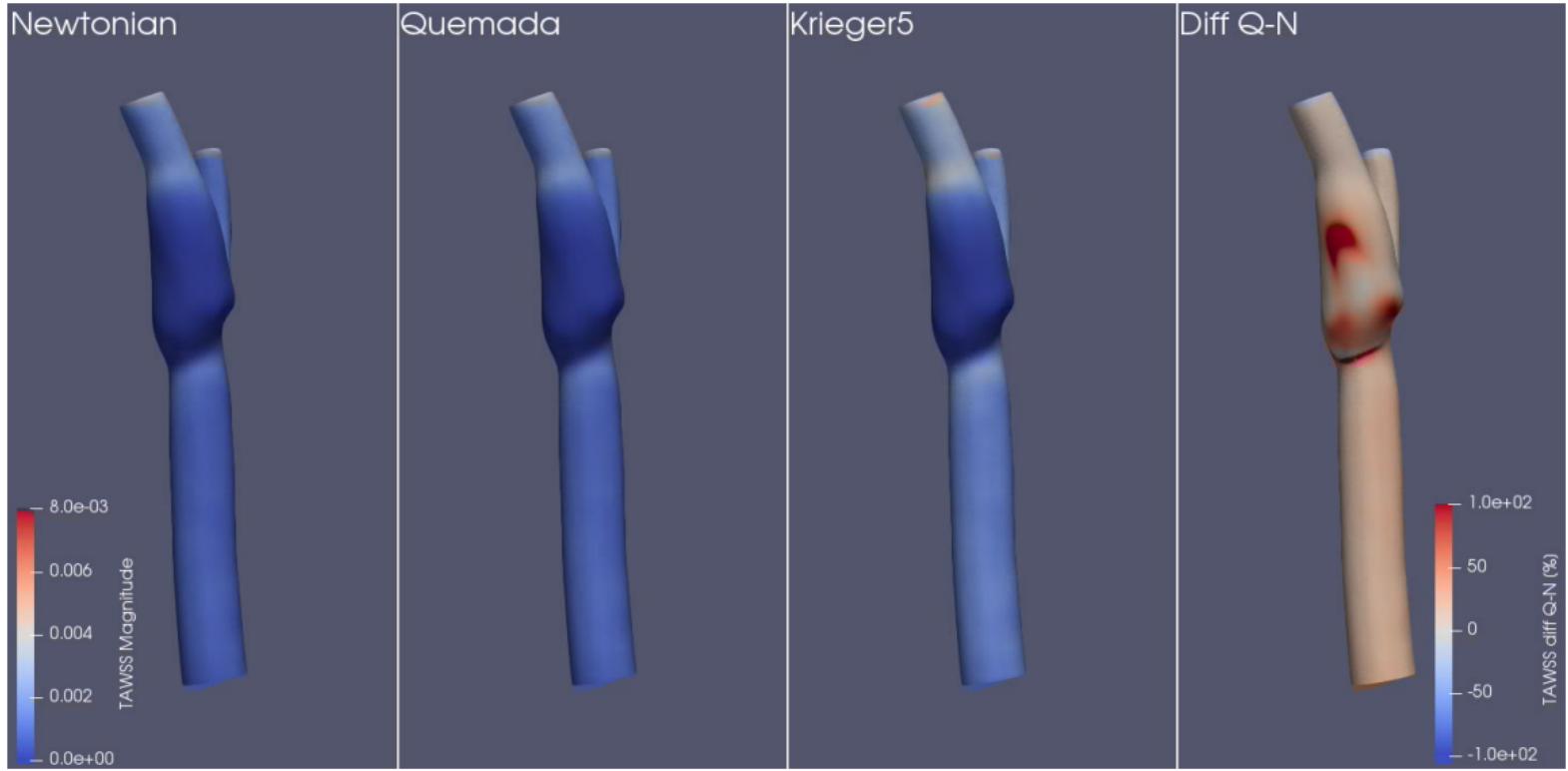
- Shear gradient gives rise to migration of red blood cells to the centre of the pipe.
- This increases Non-Newtonian behaviour: further thinning near wall, thickening at centre.



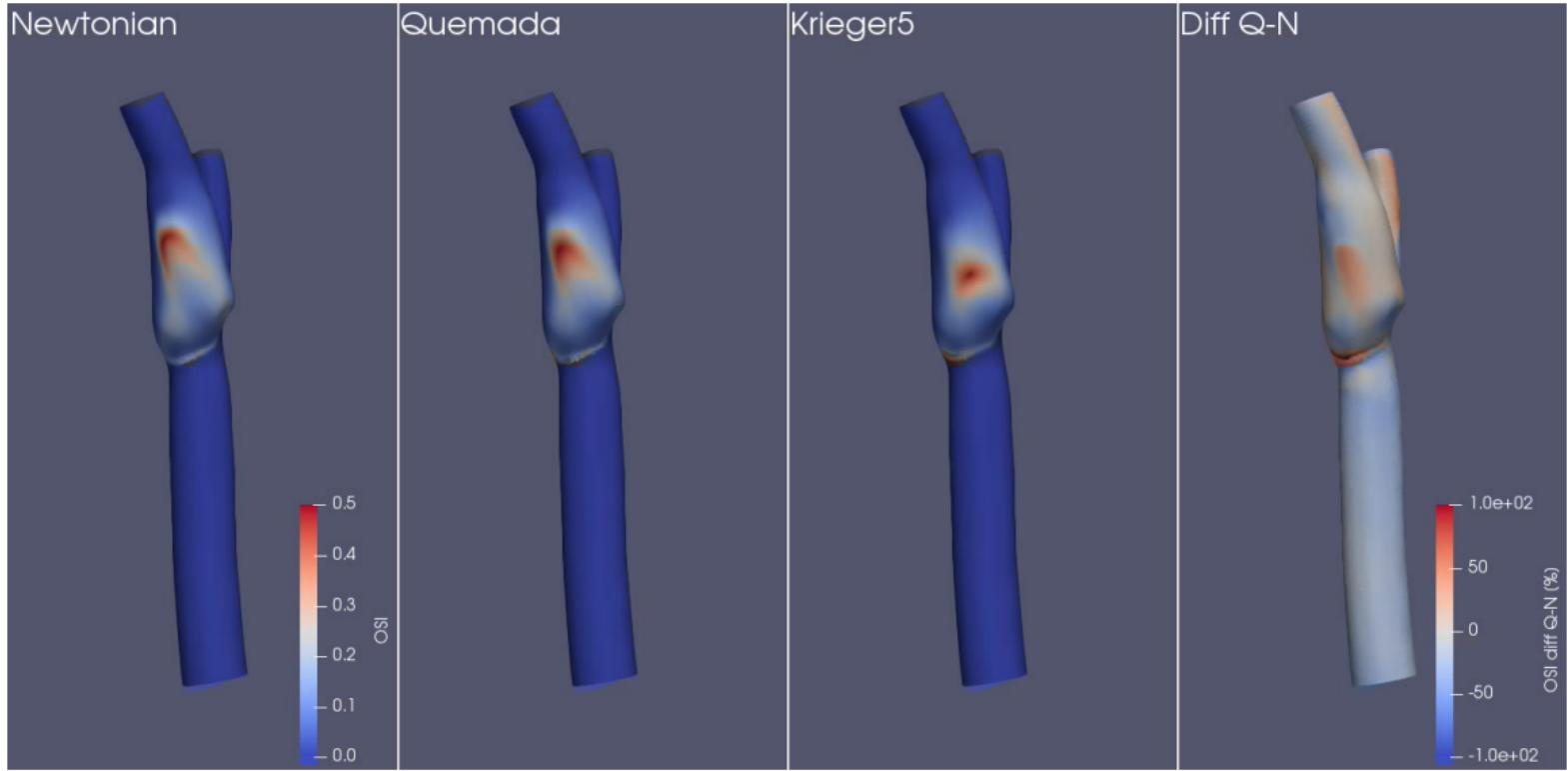
*Steady state particle distribution and velocity profiles for different diameters.*

*Fully developed pipe flow,  $R = 0.05, 0.5, 5\text{mm}$ ,  $V = 0.0065, 0.065, 0.65\text{ms}^{-1}$ ,  $K_c = 0.41$ ,  $K_\mu = 0.62$ ,  $n = 1.82$ ,  $\phi^* = 0.68$ , Standard Krieger-Dougherty Model.*

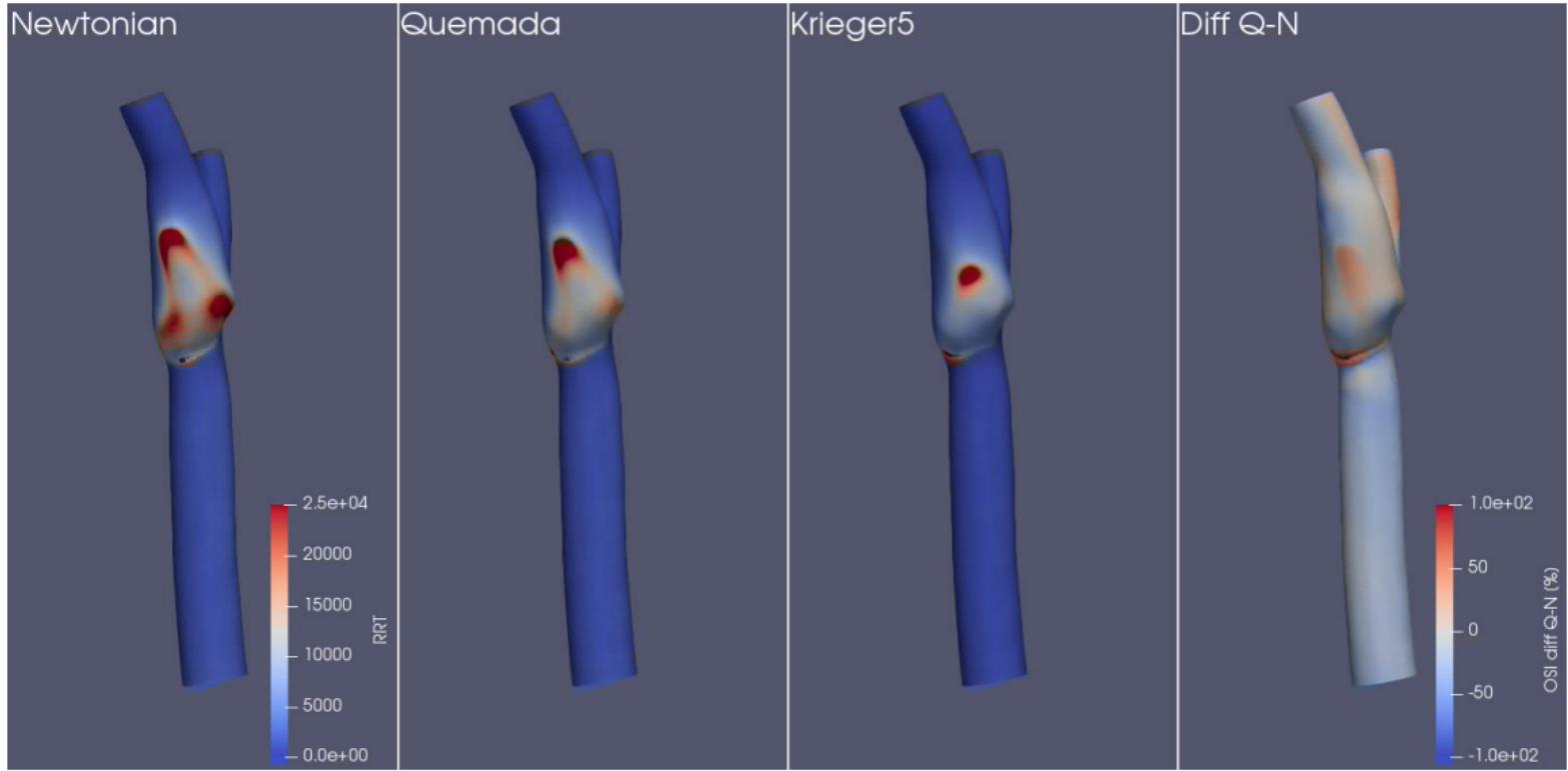
# Rheology Model Comparison - TAWSS



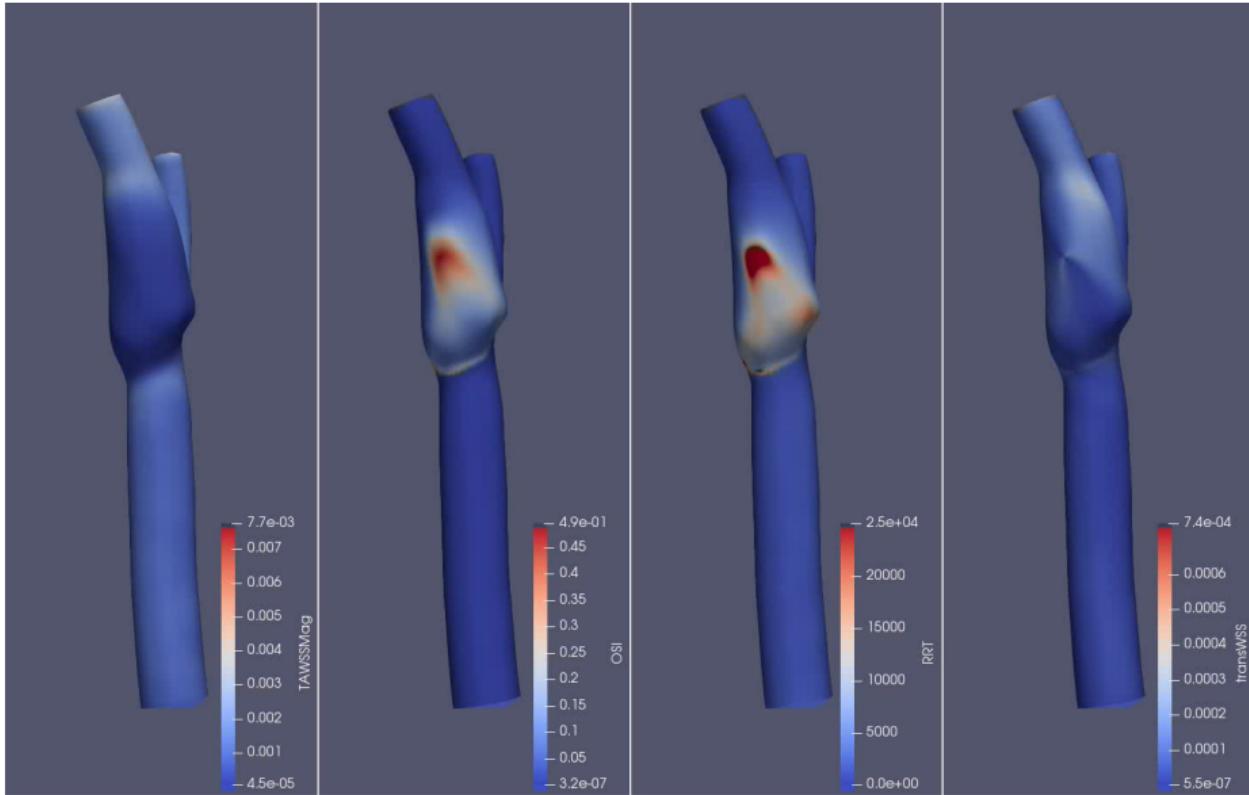
# Rheology Model Comparison - OSI



# Rheology Model Comparison - RRT



# Quemada Model all Results



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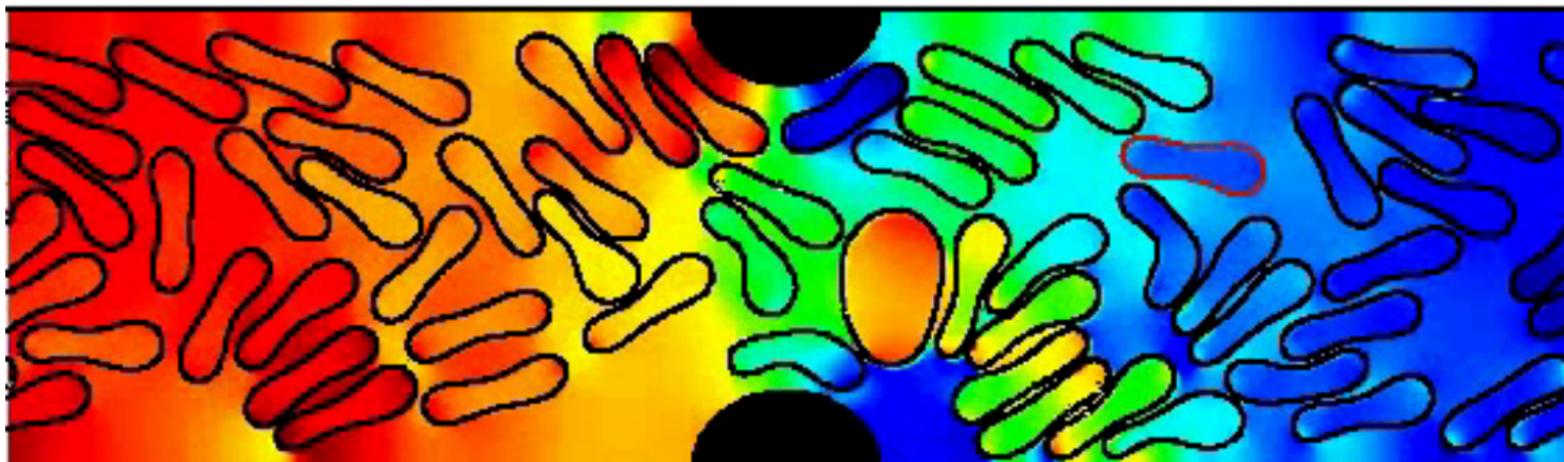
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# On the Scale of Cells



- For small vessels, of up to 10 cell diameters, the cell-scale effects take over,
- best modelled using meso-scale approaches, like the Lattice Boltzmann Method<sup>5</sup>.
- These methods are however too expensive for larger vessels of mm-sizes.
- Results from these meso-scale models can be used to inform continuum-mechanical (large scale) models.

<sup>5</sup>Simulation: Spencer, Halliday (MERI) 2013

# Summary

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- **BUT** simulations can only be as good as their boundary conditions and model parameters.
  
- Blood rheology can be modelled in detail.
  
- Outlook
  - ▶ Advances in computer technology and our understanding of vascular flow and blood will open up new horizons.
  - ▶ Some biochemical processes are already on the cards (thrombus formation, haemolysis, inflammation).

# The Code

## haemoFoam

A modular code for the simulation of blood flow implemented in C++, using the FOAM library:

Implemented so far:

- Haematocrit transport model
- Blood specific non-Newtonian models including haematocrit and shear dependency
- established Wall Shear Stress derived parameters:
  - ▶ TAWSS, TAWSSMag
  - ▶ OSI
  - ▶ transverse WSS
  - ▶ Relative Residence Time (RRT)
  - ▶ temporal and spatial WSS gradients

To be implemented:

- Platelet transport model
- LDL transport model
- viscoelastic rheology model
- volume flow dependent pressure boundary condition (Windkessel)
- Fluid Structure Interaction (FSI) for flexible vessel walls

Code to be published as Open Source.

# Bibliography

1. S. Chien, *Science (New York, N.Y.)* **168**, 977–979 (1970).
2. D. E. Brooks, J. W. Goodwin, G. V. Seaman, *Journal of Applied Physiology* **28**, 172–177 (1970).
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