The gas transmission problem solved by an extension of the Simplex algorithm.

Daniel De Wolf,
GREMARS, Université de Lille 3, France
and SMG, Université libre de Bruxelles, Belgique

Yves Smeers,
CORE, Université Catholique de Louvain, Belgium.

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Abstract

The problem of distributing gas through a network of pipelines is formulated as a cost minimization subject to nonlinear flow-pressure relations, material balances and pressure bounds. The solution method is based on piecewise linear approximations of the nonlinear flow-pressure relations. The approximated problem is solved by an extension of the Simplex method. The solution method is tested on real world data and compared with alternative solution methods.

1 Introduction

The problem considered in this paper is to minimize the total supply cost of a gas transmission company which must satisfy demand in different nodes at a minimal guaranteed pressure. The model consists of a linear objective function subject to linear and nonlinear constraints. The linear constraints express the flow conservation at each node of the network. The arcs correspond to pipelines. The non linear equations give the relation between the flow in each pipeline and the pressure decrease. Arcs can be passive (simple pipelines) or active (pipelines with a compressor).

This problem was considered earlier, among others by O'Neill et al. [17], and by Wilson, Wallace and Furey [19], who also use integer variables to describe the state of the compressors. In these two papers, the solution method is based on Successive Linear Programming (SLP). In this method, a linear approximation of the problem is constructed at some initial point x^0. This linearized problem provides a correct representation of the original constraints and objective function near the point x^0. It is optimized under the condition that the value of each variable x_j can depart from x_j^0 by no more than a constant δ^0, which defines the size of the trust region. The solution of the approximate problem within this trust region, let it be x^1, is then taken as the basis of a new linearization and the procedure is repeated. Researchers differ on the choice of a rule for updating the size of the trust region, noted δ^k at iteration number k. Generally, one compares the predicted decrease of the objective function between two iterations to the effectively observed decrease of the objective function in order to update the trust region. If the approximate model gives a good prediction of the effective decrease of the objective function, one enlarges the trust region. Otherwise, one decreases δ_k, the size of the trust region.

We consider an alternative method which avoids both the selection of an original trust region and the choice of a rule for updating the size of the trust region. A piecewise linear approximation of the nonlinear relations is constructed, the resulting problem is solved by an extension of the Simplex algorithm (See De Wolf and Smeers [4]), and the discretization is refined as long as the tolerance error is exceeded. We refer to this piecewise linear programming approach as PLP.

A variety of quite different algorithmic approaches might also be applied to this problem. The piecewise linear approximations could be represented as "special ordered sets of type 2" so that the piecewise linear problem could be globally solved by a mixed-integer programming code. A nonlinear programming code could also be applied directly to the nonlinear exact formulation to determine a local optimum. These two alternatives are also compared to PLP and SLP. They appear, when converging, much slower than the PLP or SLP methods (See Section 5).

PLP has been implemented on the basis of XMP [14]. Although a rather old code, XMP offers the advantage of a full access to the source code, and of the availability of both an implementation of SLP and Fourer's method [8, 9, 10] for separable programming. This insures that the comparison between the solution methods is not affected by different implementation of the Simplex algorithm. Tests were executed on real world models but with 1989 data in order to avoid problems of confidentiality.

The paper is organized as follows. The formulation of the problem is presented in section 2, the solution method being discussed next. Section 4 introduces a first test problem based on the Belgian transmission system. Section 5 provides a comparison between the method proposed here and alternatives approaches. Conclusions terminate the paper.

2 Formulation of the problem

The formulation of the problem presented in this section applies to a situation where the gas merchant and transmission functions are integrated in a single company. Straightforward adaptations allow one to reformulate the model for a pure gas transmission company. For the sake of simplification, we hereafter refer to the integrated gas merchant and transmission company as the gas company.

Consider a gas company operating a transmission network. The company must decide the quantities of gas to buy from several sources in order to satisfy the demand distributed over different nodes at some minimal guaranteed pressure.

The network of a gas company consists of several supply points where the gas is injected into the system, several demand points where gas flows out of the system and other intermediate nodes where the gas is simply rerouted. Pipelines are represented by arcs linking the nodes.

The following mathematical notation is used. The network is defined as the pair (N, A) where N is the set of nodes and $A \subseteq N \times N$ is the set of arcs connecting these nodes. The network can be represented as in Figure 1.

Two variables are associated to each **node** i of the network: p_i represents the gas pressure at this node and s_i the net gas supply in node i. A positive s_i corresponds to a supply of gas at node i. A negative s_i implies a gas demand $d_i = -s_i$ at node i.

A gas flow f_{ij} is associated with each **arc** (i, j) from i to j. There are two types of arcs: **passive arcs** (whose set is noted A_p) correspond to pipelines and **active arcs** correspond to pipelines with a compressor (whose set is noted A_a).

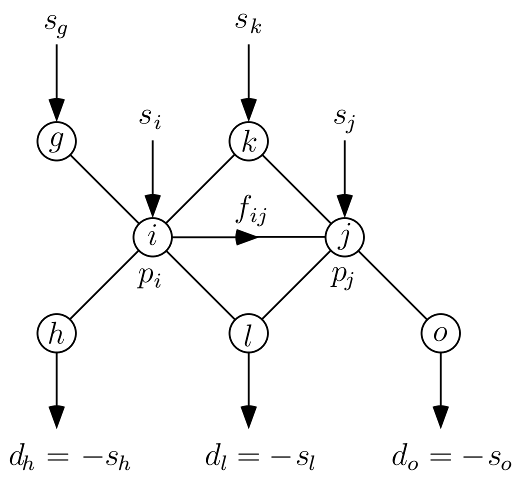

Figure 1: Network representation

The constraints of the model are as follows. At a **supply node** i, the gas inflow s_i must remain within take limitations specified in the contracts. A gas contract specifies an average daily quantity to be taken by the transmission company from the producer. Depending on the flexibility of the contract, the transmission company has the possibility of lifting a quantity ranging between a lower and an upper fraction (e.g. between 0.85 and 1.15) of the average contracted quantity. Mathematically:

$$\underline{s_i} \leq s_i \leq \overline{s_i}$$

At a demand node, the gas outflow $-s_i$ must be greater or equal to d_i, the demand at this node.

The gas transmission company cannot take gas at a pressure higher than the one insured by the supplier at the entry point. Conversely, at each exit point, the demand must be satisfied at a minimal pressure guaranteed to the industrial user or to the local distribution company. Mathematically:

$$\underline{p_i} \leq p_i \leq \overline{p_i}$$

The flow conservation equation at node i (see Figure 2 for the flow conservation at a supply node) insures the gas balance at node i. Mathematically:

$$\sum_{j|(i,j)\in A} f_{ij} = \sum_{j|(j,i)\in A} f_{ji} + s_i$$

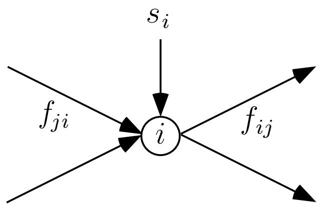

Figure 2: Supply node i

Now, consider the constraints on the arcs. We distinguish between the passive and active arcs. For a **passive arc**, the relation between the flow f_{ij} in the arc (i, j) and the pressures p_i and p_j is of the following form (see O'Neill and al.[17]):

$$sign(f_{ij})f_{ij}^2 = C_{ij}^2(p_i^2 - p_j^2),\ \forall (i, j) \in A_p$$

where C_{ij} is a constant which depends on the length, the diameter and the absolute rugosity of pipe and on the gas composition. Note that the f_{ij} are unrestricted in sign. If $f_{ij} < 0$, the flow $-f_{ij}$ goes from node j to node i.

For an **active arc** corresponding to a pipeline with a compressor, there exists one apparatus which increases the pressure along the pipe allowing a flow larger than the one corresponding to the pressure decrease in the pipe:

$$sign(f_{ij})f_{ij}^2 \geq C_{ij}^2(p_i^2 - p_j^2),\ \forall (i, j) \in A_a$$

For active arcs, the direction of the flow is fixed:

$$f_{ij} \geq 0,\ \forall (i, j) \in A_a$$

The **objective function** of the gas company is to minimize the total cost of the supplies. This can be written:

$$\min z = \sum_{j \in N_s} c_j s_j$$

where c_j is the purchase price of the gas delivered at node j and N_s denotes the set of supply nodes.

The gas transmission problem can thus be formulated as follows:

$$\min z(f,s,p) = \sum_{j \in N_s} c_j s_j$$

$$\text{s.t.} \begin{cases} \sum_{j|(i,j)\in A} f_{ij} = \sum_{j|(j,i)\in A} f_{ji} + s_i & \forall i \in N & (1.1) \\ sign(f_{ij}) f_{ij}^2 = C_{ij}^2 (p_i^2 - p_j^2) & \forall (i,j) \in A_p & (1.2) \\ sign(f_{ij}) f_{ij}^2 \geq C_{ij}^2 (p_i^2 - p_j^2) & \forall (i,j) \in A_a & (1.3) \\ \underline{s_i} \leq s_i \leq \overline{s_i} & \forall i \in N & (1.4) \\ \underline{p_i} \leq p_i \leq \overline{p_i} & \forall i \in N & (1.5) \\ f_{ij} \geq 0 & \forall (i,j) \in A_a & (1.6) \end{cases} \quad (1)$$

A simple change of variables allows one to eliminate the nonlinearities due to the pressure variables. Indeed, defining:

$$\pi_i = p_i^2, \ \forall i \in N$$

the constraints (1.5) are replaced by:

$$\underline{\pi_i} \leq \pi_i \leq \overline{\pi_i}$$

where $\underline{\pi_i}$ and $\overline{\pi_i}$ are respectively the square of $\underline{p_i}$ and $\overline{p_i}$. There only remaining nonlinearities appear in the expressions $sign(f_{ij})f_{ij}^2$ in constraints (1.2) and (1.3).

3 Solution Procedure

We now consider the solution of the problem (1). The formulation of the gas flow-pressure relation in pipes (1.2) is clearly nonconvex. De Wolf and Smeers [6] examine special conditions that render this problem convex. Specifically, they show that if for each demand and supply node, the pressure p_i or the net gas outlet s_i is fixed, then the feasible solution set for (1) is convex. Unfortunately, this condition seems too restrictive for practical purposes, at least if the objective is to optimize a supply mix. Indeed, with such a condition, the feasible region reduces to a singleton and the problem amounts to finding the feasible supply mix.

Some procedure is thus required in general to tackle the nonconvexity of the problem, if only to find a local solution. The approach proposed here is to proceed by successively solving two problems, the first one being expected to produce a good initial point for the second one. Convergence in nonlinear programming may indeed crucially depend on a good choice of the starting point and this is especially true when the problem is nonconvex. Our first problem is obtained by relaxing the pressure constraints and eliminating all compressors in the full model. The solution of this problem is conjectured to provide a good starting point for the **second problem** which is the complete model with pressure bounds and compressors.

3.1 First problem : find a good initial point.

Consider the following convex problem which only accounts for pressure losses along the pipelines:

$$\begin{array}{lll} \min h(f,s) = & \displaystyle\sum_{(i,j)\in A} \frac{|f_{ij}|f_{ij}^2}{3C_{ij}^2} & \\ \text{s.t.} & \displaystyle\sum_{j|(i,j)\in A} f_{ij} - \sum_{j|(j,i)\in A} f_{ji} = s_i & \forall i \in N \\ & \underline{s_i} \leq s_i \leq \overline{s_i} & \forall i \in N \end{array} \tag{2}$$

Since the problem is strictly convex in the flow variables, its optimal solution is unique. The first constraints of (2) then insure that the solution is also unique in the supply variables.

We now prove that the unique optimal solution of the problem (2) satisfies the nonlinear flow pressure relation (1.2). Let π_i be the dual variable associated to the gas balance constraint at node i. The Karush-Kuhn-Tucker necessary conditions (See Bertsekas [1, page 284]) satisfied at the optimum solution of the problem (2) can be written as:

$$sign(f_{ij})\frac{f_{ij}^2}{C_{ij}^2} = \pi_i - \pi_j \quad \forall (i,j) \in A$$

There is no sign constraint on the π variables since π_i is the Lagrange multiplier associated to an equality constraint, namely the gas balance equation at node i. Thus directly replacing $\pi = p_i^2$ is not allowed. Let $\underline{\pi}$ be the value of the lowest dual variable:

$$\underline{\pi} = \min_{i \in N} \{\pi_i\}$$

Replace now

$$p_i^2 = \pi_i - \underline{\pi}.$$

We obtain exactly the flow pressure relations (1.2). The optimal solution of (2) satisfies thus all the constraints of (1) except the pressure bounds constraints (1.5).

It can be shown (See De Wolf and Smeers [6]) that problem (2) has a physical interpretation, in the sense that its objective function is the mechanical energy dissipated per unit of time in the pipes. Because the gas system is designed to be operable during periods of peak demand, it is expected that most of the compressors will only need to be operated a fraction of the time. This implies that the point obtained by minimizing the mechanical energy dissipated in the pipes, that is the natural flow when compressors are not operated, should constitute a good starting point for the complete problem.

Problem (2) can be solved by the recursive use of CPLP, the module of the XMP library. This code is based on the methodology developed by Fourer [8, 9, 10] for minimizing piecewise linear separable convex functions. The approach proceeds by constructing and solving the following sequence of piecewise linear programs. One starts by discretizing each nonlinear term of the objective function of (2) using a certain number of breakpoints (See Figure 3). See also section 4.3 for a discussion of the impact of the initial number of breakpoints.

The resulting problem is solved by CPLP leading to a solution (f^*, s^*) with associated Lagrange multipliers π^*. This solution does not necessarily satisfy the original nonlinear pressure flow relations (See Figure 4). The discretization is then refined through the following procedure. The flow $\overline{f}_{ij}$ that would exactly account for the difference of squares of pressure $\pi_i^* - \pi_j^*$ is computed as

$$\overline{f}_{ij} = sign(\pi_i^* - \pi_j^*) C_{ij} |\pi_i^* - \pi_j^*|^{\frac{1}{2}}$$

If the error $e_{ij} = \overline{f}_{ij} - f_{ij}^*$ is greater than a given tolerance (taken here as 10^{-6}), $\overline{f}_{ij}$ is added as as a new discretization point (see Figure 4) and the procedure is repeated.

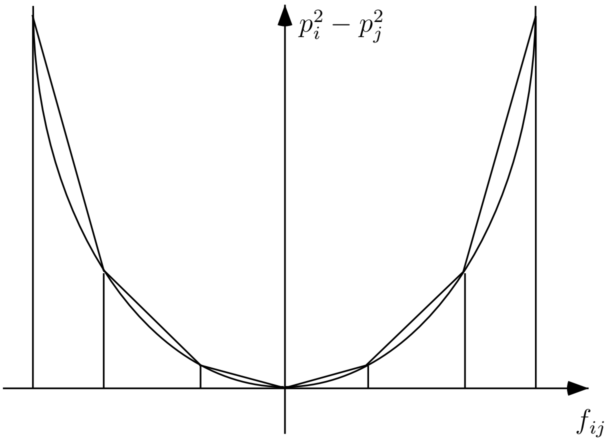

Figure 3: Objective discretization

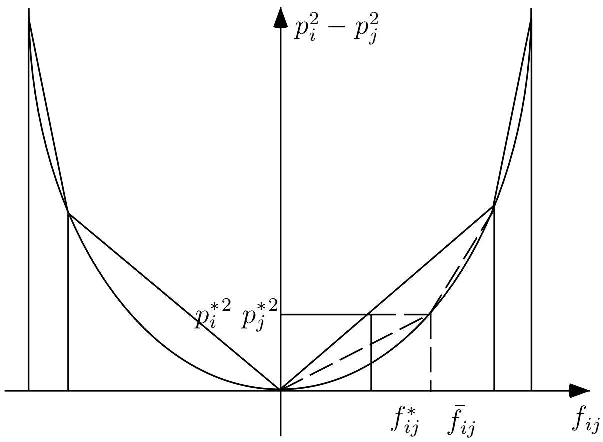

Figure 4: Refining the discretization

This discretization process has the following computational advantages. First, the Simplex algorithm can be restarted from the same basis since the refining process only requires a change in an objective coefficient. This does not create infeasibilities and only results in the loss of optimality. There is thus no need to return to phase I, phase II can be initiated directly and a new optimum solution found in a few iterations.

Second, the discretization process takes advantage of Fourer's treatment of separability. Recall that in the Fourer's methodology [8, page 209] the variables appearing in piecewise linear functions are not explicitly divided in sums of variables. Instead the piecewise linear functions are directly handled without adding any new variable or constraint.

The discretization procedure is repeated until the nonlinear flow pressure relations are satisfied. The behavior of this refining process is illustrated in subsection 4.1.

3.2 Second problem : the complete problem.

The solution to the first problem satisfies all the constraints of problem (1) except for the pressure bounds. We now return to problem (1) and solve it using an approach similar to the one applied to the first problem. This is described in the following algorithm:

Algorithm 1 o) Initialization. *Let* (f^0, p^0, s^0) *be a vector of flows, pressures and net supplies that satisfies (1.1) , (1.2), (1.3), (1.4) and (1.6).*

Replace in (1.2) and (1.3), the function $sign(f_{ij})f_{ij}^2$ *by a piecewise linear approximation including* f_{ij}^0 *as breakpoint. Use* f_{ij}^0 *as starting point for the PLP algorithm (See De Wolf and Smeers [4]). Let* $k = 1$.

i) Iteration k**.** *Solve the approximate problem by the PLP algorithm.*

Let (f^k, p^k, s^k) *be the solution.*

ii) Stopping rule. *Compute* $\overline{f}_{ij}^k$*, the flow corresponding to the differences of pressures as follows:*

$$\overline{f}_{ij}^k = sign(p_i^k - p_j^k)C_{ij}|(p_i^k)^2 - (p_j^k)^2|^{\frac{1}{2}}$$

If *the error* $e_{ij}^k = \overline{f}_{ij}^k - f_{ij}^k$ *is greater than a given tolerance* (10^{-6}), then *add* $\overline{f}_{ij}^k$ *as a new discretization point and return to* **i**)*;*

else *stop: the current solution is optimal.*

This addition of new discretization points is more involved than in the solution of the first problem. Because the piecewise linear pressure flow relations now appear in the constraints, the addition of a new discretization point may make the current point infeasible. It may therefore be necessary to return to phase I in order to get back to feasibility. In practice, only a few phase I iterations are needed to obtain a feasible basic solution.

The PLP algorithm (See De Wolf and Smeers [4]) takes advantage of separability. Similarly to Fourer's treatment of separability, the variables appearing in piecewise linear functions are not explicitly divided in sums of variables in PLP. Instead the piecewise linear functions are directly handled without adding any new variable or constraint.

4 A first problem: the Belgium gas network

The operation of the algorithm is first illustrated on a small size problem constructed on a schematic description of the Belgium gas network. The algorithm has also been tested on larger problems (See below) and for which numerical results are also reported.

Belgium has no domestic gas resources and imports all its natural gas from the Netherlands, Algeria and Norway (See Figure 5). Algerian gas is delivered in LNG form at the Zeebrugge terminal. In the test problem considered here (which no longer corresponds to the current situation), the gas from Norway is piped through the Netherlands and crosses the Belgian border at s'Gravenvoeren. The Dutch gas is also piped and enters the Belgium system at Poppel.

The Belgium gas transmission network carries two types of gas and is therefore divided in two parts. The high calorific gas (10 000 kilocalories per cubic meter), comes from Algeria and Norway. The gas coming from the Netherlands is a low calorific gas (8 400 kilocalories per cubic meter). The test problem presented here refers to the high calorific network. The reader is referred to Appendix A for a more detailed description of the Belgium network.

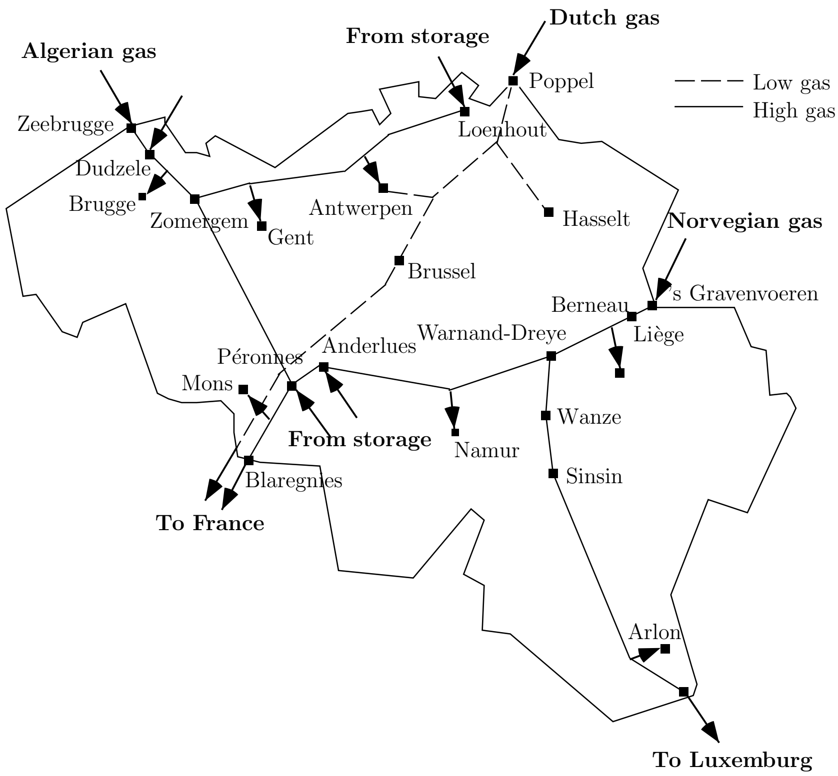

Figure 5: Schematic Belgium gas network

The size of this first test problem is quite small (24 non linear relations, 2 compressors located at Berneau and Sinsin) but the solution method has been used successfully with a more significant test problem corresponding to a larger country of the size of France.

4.1 Behavior of the refining process

The following presents some results related to the discretization process used in the first problem (2). For this example, starting with 50 discretization intervals for each flow variable, an optimal solution to the first problem is obtained in 23 main cycles. One main cycle corresponds to the solution of a piecewise linear approximation of the problem (2) by CPLP. Figure 6 reports the number of new breakpoints added at each cycle.

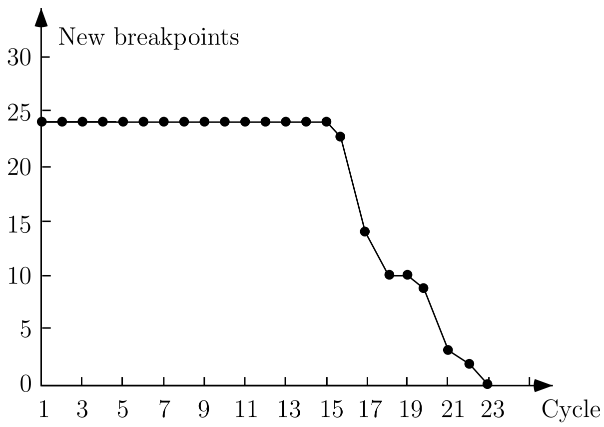

Figure 6: number of new breakpoints added for first problem

As can be seen, a breakpoint is added during the first fifteen cycles for each flow variable (there are 24 flow variables in this example). One can also observe that the number of new breakpoints rapidly decreases to zero so that the refining process quickly comes to an end.

Consider now the second problem (1). The evolution of the number of breakpoints added in each cycle is given in figure 7.

Only a few cycles are needed and the number of new breakpoints again rapidly decreases to zero. But, as seen in Table 1, the time spent on each cycle in problem 2 is greater than the time taken by a cycle in the first problem.

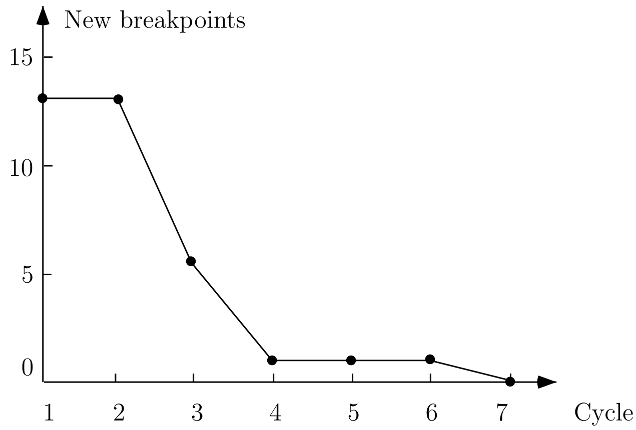

Figure 7: number of new breakpoints added for second problem

	Iteration	Time (sec.)	Time/iteration
Problem 1	24	12	0.5
Problem 2	7	19	2.7

Table 1: execution time per iteration

This can be explained as follows. As indicated above, the impact of a variable crossing a breakpoint in the first problem is limited to a change of slope in the objective function and hence to a loss of optimality. Restoring this latter may, but that is not necessary, require a pivot. In the second problem, the crossing of a breakpoint by a variable implies a change in a constraint and hence a lost of feasibility. This always requires Simplex pivots (See De Wolf and Smeers [4]).

4.2 Role of the first problem

Consider now the gain in processing time achieved by resorting to the first problem. Results are reported on test problems of three different sizes (data relating to these problems can be requested from: `dewolf@univ-lille3.fr`).

As can be seen in Table 2, the gain of efficiency depends on the size of the problem. For small examples, the reduction of the computational time achieved in the second problem is completely lost due to the time spent in the first problem. In contrast, for larger problems the cost of processing the first problem is largely compensated by the savings achieved in the complete problem. Specifically, the global time necessary to successively solve the two problems is about half the time needed for solving directly problem 2 from scratch.

	Size	Time with problem 1 (seconds)	Time without problem 1 (seconds)
Example 1	24	28	26
Example 2	34	56	49
Example 3	60	65	129

Table 2: Computational time gain with first problem

4.3 The initial discretization

Computational efficiency can be expected to depend on both the initial discretization and on the refinement process. Table 3 reports the computation time taken by the solution of the first and second problems for the small example as a function of the original number of breakpoints.

Number of breakpoints (initially)	Time for problem 1 (seconds)	Time for problem 2 (seconds)	Total time
3	15	28	43
5	15	29	44
13	15	37	52
27	16	44	60
53	20	45	65

Table 3: execution time as a function of the original discretization

As can be seen, the number of breakpoints used in the original discretization of the first problem does not really influence the computation time. In contrast, the computation time increases with the initial number of breakpoints in the second problem. It is thus optimal to start with a minimal number of discretization points (3 points) and only add new breakpoints as necessary. This difference in the influence of the original discretization between the two problems can be traced to the difference between the loss of optimality and the loss of feasibility (See end of section 4.1).

5 Other solution procedures

We now compare the execution time for PLP (Piecewise Linear Programming) and for SLP (Successive Linear Programming). We rely,in this paper, on the implementation of SLP due to Lasdon and Plummer [13] even though other implementations exist (Kao et Meyer [12], Meyer [16], Palacios-Gomez et al. [18] and Zhang et al.) Lasdon and Plummer's implementation of SLP offers the advantage of being constructed on the basis of XMP, which makes the comparison with our implementation of PLP independent of the underlying Simplex code.

The computation times for SLP and PLP, starting from the solution of the first problem, are reported in table 4.

	Size of problem	Time for SLP (seconds)	Time for PLP (seconds)	Gain
Example 1	24	22	16	27 %
Example 2	35	47	32	32 %
Example 3	60	69	35	49 %

Table 4: execution times for SLP and PLP

As can be seen, the use of PLP reduces the computational time and this gain is increasing with the size of the problem. Also, for the default choice of specification parameters, SLP concludes to the infeasibility for example 3. This results from too small changes of the objective function while the infeasibility tolerance is not yet reached. These two parameters (fractional change in the objective function and infeasibility tolerance) must therefore be carefully chosen. In particular, they must be kept of the same order of magnitude so that the solution process can stop with both the Karush-Kuhn-Tucker conditions and feasibility satisfied.

Our experience indicates that the SLP method quickly finds a rough solution but requires a lot of iterations to satisfy the nonlinear flow pressure relations with a sufficient accuracy. PLP appears more promising in this respect. The method simply adds a new breakpoint for each nonlinear relation and only requires very few Simplex iterations to reach a solution satisfying the exact flow pressure relations.

We now compare PLP with the two following general purpose approaches:

- The piecewise linear approximations can be cast into "special ordered sets of type 2" so that the successive approximations to (1) can be globally optimized by a mixed-integer programming code.

- A nonlinear programming code can also be applied directly to the exact nonlinear formulation (1) to determine a local optimum.

These two alternatives were tested as follows.

The special ordered sets were implemented using MP-MODEL and the mixed integer solver XPRESS-MP [20] of Dash Associates which is generally reported to be one of the best mixed integer solvers. Tests were constructed using an initial discretization with 21 breakpoints for both PLP and SOS type 2 methods. Table 5 gives the execution times for XPRESS-MP and PLP for the same test problem already used to illustrate the behavior of the refining process. The solution given by XPRESS-MP was in this case not better than the solution obtained by the PLP method (the PLP solution is given in Appendix). But it was proven to be optimal. From this table, it can be concluded that this alternative method is not competitive in time with PLP as the execution time is about 42 times the PLP execution time.

	Size of problem	XPRESS-MP (seconds)	PLP (seconds)	Gain (%)
Example 1	24	2 330	56	97.7

Table 5: Execution times for XPRESS-MP and for PLP.

The second approach was tested using Lancelot, an up-to-date nonlinear solver [2]. Table 6 gives the execution times obtained by using Lancelot and PLP on the same last problem. Both codes found solutions with the same objective function value, but it took Lancelot 37 times the time required by PLP.

	Size of problem	LANCELOT (seconds)	PLP (seconds)	Gain (%)
Example 1	24	75	2	97.3

Table 6: Execution times for Lancelot and PLP.

6 Conclusions

In this paper an alternative method to the Successive Linear Programming method has been presented to solve the gas transmission problem. It relies on the solution of successive piecewise linear problems. The proposed method has been implemented by an original extension of the XMP library in order to facilitate the comparison with existing implementation of SLP. This approach gives computational times smaller than those of the SLP method, and the computational gain appears to increase with the size of the problem.

As the model is in general non convex, the choice of the starting point is crucial if one limits oneself to find only local solutions or upper bounds on the solution in global procedures. Another problem has been defined for producing an initial solution. This first problem has a natural physical interpretation, namely the minimization of the mechanical power dissipated in the pipes. The use of this starting point can reduce the computational times by 50 %. It is conjectured that it can help avoid being trapped in bad local solutions.

The use of the model has been illustrated on three different test problems constructed on the basis of real world situations. The tool has been included in a more general model for investment evaluation (See De Wolf and Smeers [5]).

A Description of the Belgium gas network

A.1 The demand

Demand estimates have been taken from the 1989 statistical yearbook of the federation of the gas industry [7]. Total demand of each Belgian province has been assigned to the main town of the province. Eliminating the demand satisfied by the low gas (Brussels, Hasselt province and 2/3 of the Antwerp province), one obtains the demand for high gas in Belgium for the coolest day of 1989, namely, November 30th. They are given in Table 7 in million of normal cubic meters. This table also includes the gas export to Luxemburg.

The annual quantity of the gas piped from s'Gravenvoeren to Blaregnies for France in 1989 was 5.7 10^9 m^3. This corresponds to a daily quantity of 5.7 $10^9/365 = 15.616$ 10^6 m^3/day.

A.2 The supply

Consider now the supply side of the Belgium gas market. An estimation of the annual contracted quantities and prices can be found in Cedigaz [3]. The daily average contracted quantity is obtained by dividing the annual contracted quantity by 365 days. Gas contracts contain flexibility clauses. In the absence of any precise information, it is assumed that the daily takes can range from 0.85 up to 1.15 of the average contracted quantity. The

Province	Demand 10^6 m^3/day
Antwerpen	4.034
Arlon	0.222
Brugge	3.918
Gent	5.256
Liège	6.365
Mons	6.848
Namur	2.120
Luxemburg	1.919
Total	30.682

Table 7: Belgian daily maximum high gas demand by province in 1989

minimal and maximal daily takes can therefore be stated as in Table 8.

Producer	daily quantity 10^6 m^3/day	minimal 10^6 m^3/day	maximal 10^6 m^3/day	price \$/MBTU
Norway	5.562	4.728	6.396	1.68
Algeria	10.082	8.870	11.594	2.28
Total	15.644	13.297	17.99	

Table 8: Minimal and maximal daily quantities

Available storage capacities are given in Table 9. Because the gas taken from storage has an implicit price, we apply the Algerian price for the gas taken from Dudzele and Loenhout and the Norwegian price for the gas taken from Anderlues and Péronnes.

To link with the notation used in (1), we summarize all the relevant information concerning the nodes in Table 10.

Storage	Maximal outflow	Maximal outflow	price
	m^3/hour	$10^6 m^3$/day	\$/MBTU
Anderlues	50 000	1.2	1.68
Dudzele	350 000	8.4	2.28
Loenhout	200 000	4.8	2.28
Péronnes	40 000	0.96	1.68
Total	290 000	15.36	

Table 9: Storage capacities

node	town	$\underline{s_i}$	$\overline{s_i}$	$\underline{p_i}$	$\overline{p_i}$	c_i
		10^6 scm	10^6 scm	bar	bar	\$/MBTU
1	Zeebrugge	8.870	11.594	0.0	77.0	2.28
2	Dudzele	0.0	8.4	0.0	77.0	2.28
3	Brugge	$-\infty$	-3.918	30.0	80.0	0.00
4	Zomergem	0.0	0.0	0.0	80.0	0.00
5	Loenhout	0.0	4.8	0.0	77.0	2.28
6	Antwerpen	$-\infty$	-4.034	30.0	80.0	0.00
7	Gent	$-\infty$	-5.256	30.0	80.0	0.00
8	Voeren	20.344	22.012	50.0	66.2	1.68
9	Berneau	0.0	0.0	0.0	66.2	0.00
10	Liège	$-\infty$	-6.365	30.0	66.2	0.00
11	Warnand	0.0	0.0	0.0	66.2	0.00
12	Namur	$-\infty$	-2.120	0.0	66.2	0.00
13	Anderlues	0.0	1.2	0.0	66.2	1.68
14	Péronnes	0.0	0.96	0.0	66.2	1.68
15	Mons	$-\infty$	-6.848	0.0	66.2	0.00
16	Blaregnies	$-\infty$	-15.616	50.0	66.2	0.00
17	Wanze	0.0	0.0	0.0	66.2	0.00
18	Sinsin	0.0	0.0	0.0	63.0	0.00
19	Arlon	$-\infty$	-0.222	0.0	66.2	0.00
20	Pétange	$-\infty$	-1.919	25.0	66.2	0.00

Table 10: Nodes description

A.3 Technical description of the network

For each pipeline in the network, we compute the term C_{ij}^2 by the following formula:

$$C_{ij}^2 = 96.074\ 830\ \ 10^{-15} \frac{D_{ij}^5}{\lambda_{ij} z T L \delta}$$

where

$$\frac{1}{\lambda_{ij}} = \left[2log(\frac{3.7 D_{ij}}{\epsilon}) \right]^2,$$

with	L_{ij}	=	length of the pipe $[km]$		
	D_{ij}	=	interior diameter of the pipe $[mm]$		
	T	=	gas temperature $[K]$	=	281.15 K
	ϵ	=	absolute rugosity of pipe $[mm]$	=	0.05 mm
	δ	=	density of the gas relative to air $[-]$	=	0.6106
	z	=	gas compressibility factor $[-]$	=	0.8

The length, interior diameter and corresponding C_{ij}^2 are given in Table 11 for each arc. The last column refers to the type of arc: p is set for *passive arc*, a is set for an *active arc*, i.e. a pipeline with a compressor.

A.4 Optimal solution for PLP

The minimal gas purchase cost founded by PLP is:

$$z^* = 91.056240$$

The optimal flow pattern is given in table 12.
The corresponding pressure and supply patterns are given in table 13.

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arc	from	to	diameter [mm]	length [km]	C_{ij}^2	Pipe type
1	Zeebrugge	Dudzele	890.0	4.0	9.07027	p
2	Zeebrugge	Dudzele	890.0	4.0	9.07027	p
3	Dudzele	Brugge	890.0	6.0	6.04685	p
4	Dudzele	Brugge	890.0	6.0	6.04685	p
5	Brugge	Zomergem	890.0	26.0	1.39543	p
6	Loenhout	Antwerpen	590.1	43.0	0.100256	p
7	Antwerpen	Gent	590.1	29.0	0.148655	p
8	Gent	Zomergem	590.1	19.0	0.226895	p
9	Zomergem	Péronnes	890.0	55.0	0.659656	p
10	Voeren	Berneau	890.0	5.0	7.25622	a
11	Voeren	Berneau	395.5	5.0	0.108033	a
12	Berneau	Liège	890.0	20.0	1.81405	p
13	Berneau	Liège	395.5	20.0	0.0270084	p
14	Liège	Warnand	890.0	25.0	1.45124	p
15	Liège	Warnand	395.5	25.0	0.0216067	p
16	Warnand	Namur	890.0	42.0	0.863836	p
17	Namur	Anderlues	890.0	40.0	0.907027	p
18	Anderlues	Péronnes	890.0	5.0	7.25622	p
19	Péronnes	Mons	890.0	10.0	3.62811	p
20	Mons	Blaregnies	890.0	25.0	1.45124	p
21	Warnand	Wanze	395.5	10.5	0.0514445	p
22	Wanze	Sinsin	315.5	26.0	0.00641977	a
23	Sinsin	Arlon	315.5	98.0	0.00170320	p
24	Arlon	Pétange	315.5	6.0	0.0278190	p

Table 11: Pipe-line description

Arc	from	to	Flow (10^6 SCM)
1	Zeebrugge	Dudzele	5.455644
2	Zeebrugge	Dudzele	5.455644
3	Dudzele	Brugge	9.655644
4	Dudzele	Brugge	9.655644
5	Brugge	Zomergem	15.393288
6	Loenhout	Antwerpen	2.814712
7	Antwerpen	Gent	-1.219288
8	Gent	Zomergem	-6.475288
9	Zomergem	Péronnes	8.918000
10	Voeren	Berneau	19.618224
11	Voeren	Berneau	2.393776
12	Berneau	Liège	19.618224
13	Berneau	Liège	2.393776
14	Liège	Warnand	13.945409
15	Liège	Warnand	1.701591
16	Warnand	Namur	13.506000
17	Namur	Anderlues	11.386000
18	Anderlues	Péronnes	12.586000
19	Péronnes	Mons	22.464000
20	Mons	Blaregnies	15.616000
21	Warnand	Wanze	2.141000
22	Wanze	Sinsin	2.141000
23	Sinsin	Arlon	2.141000
24	Arlon	Pétange	1.919000

Table 12: Optimal flows

Node	Supply (10^6 SCM)	Demand (10^6 SCM)	Pressure (Bars)
1	10.911288		55.822887
2	8.400000		55.793487
3		3.918000	55.655143
4			54.108114
5	2.814712		53.027490
6		4.034000	52.277058
7		5.256000	52.372622
8	22.012000		59.851968
9			59.407217
10		6.365000	57.593877
11			56.418520
12		2.120000	54.514990
13	1.200000		53.187918
14	0.960000		52.982300
15		6.848000	51.653023
16		15.616000	50.000000
17			55.623250
18			63.000000
19		0.222000	35.744537
20		1.919000	33.842225

Table 13: Optimal supplies and pressures

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