A Novel Acceleration Strategy for N-1 Contingency Screening in Distribution System

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***Abstract*—The concern for the security of distribution network renders the necessity of developing an advanced N-1 contingency screening method. However, the complexity of network reconfiguration and islanding operation brings tremendous computation burden for the traditional screening approach. To cope with the problem, a novel acceleration strategy is proposed based on an idea of integrating the optimal power flow calculations under every N-1 contingency into a single search tree. To dynamically check the N-1 security criterion during the solving procedure, the model is formulated in a branch-and-cut framework. At each incumbent solution node, if the load shedding can be resolved by network reconfiguration or islanding operation, a specialized lazy constraint callback approach is implemented to eliminate the current N-1 contingency from the contingency set. Thus, all the calculations for the N-1 contingency are incorporated as a whole and performed only once. The superiority and efficiency of the strategy are verified on the IEEE-33 and 69 bus systems. Numerical results indicate that the proposed approach is an order of magnitude faster than the traditional ones.**

***Index Terms*—N-1 contingency screening, distribution system, network reconfiguration, callback, branch-and-cut.**

I. Introduction

CONTINGENCY screening is an important approach to evaluate the security and reliability of the power system during planning and operation [1]. Although the contingency screening technology has been successfully applied to power transmission systems [2], it was rarely discussed in distribution systems because of the complexity of network reconfiguration and islanding operation. Recently, with the increasing attention to the security of distribution systems [3] [4], it is imperative to develop an efficient method to perform N-1 contingency screening in the distribution system.

The main challenge of N-1 contingency screening is the huge number of power flow calculations. To cope with the problem, numerous scholars have been devoting themselves to alleviating the computational burden. For example, a convolutional neural network was introduced in [5] to automatically generate power flow results without any iterative calculation, so that the computation time spent on each power flow is greatly reduced. In [6], graphics processing unit was utilized to perform parallel generation of Jacobian matrics for accelerating the power flow calculation. Based on the principal component analysis, [7] proposed a new method to simplify the power flow calculations from the perspective of seeking similarity to an existing contingency database. Although these studies claimed that their strategies were hundreds of times faster than the traditional ones, such methods are only applicable to the transmission system whose configuration under N-1 contingency is determined. As to the distribution system with a changeable configuration, the network reconfiguration and the islanding operation turn the power flow calculation under any given N-1 contingency into a mixed-integer optimal power flow (OPF) problem. In other words, the acceleration for a single power flow calculation is not suitable for the N-1 contingency screening in distribution systems.

Due to the absence of acceleration approach, traversing has long been deployed to perform N-1 contingency screening in solving the security-constrained distribution system planning [8] and operation [9] problems. Because the system scale is relatively small, the computational complexity of traversing in these papers is still in the scope of acceptance. Although there have been attempts using the distribution system security region [10] to evaluate the security of operating points, the mathematical deduction of the region is still on the basis of a traversing-based N-1 contingency screening method [11]. The drawback of traversing is obvious. For a distribution system with n branches, the traversing-based contingency screening method requires $n \cdot O(n)$ variables, where $O(n)$ is the number of variables used in a single OPF model. The computational burden will grow exponentially if n keeps rising.

In the community of contingency screening, a large amount of attention is paid to the acceleration strategy inside of the power flow model, taking for granted that OPF calculation in each contingency scenario should be performed independently. On the contrary, the performance of contingency screening is highly dependent on the coordination of all OPF calculations. However, to the best of authors' knowledge, such coordinated technology has never been studied before.

To fill this gap, a novel acceleration strategy is proposed in this paper to improve the performance of N-1 contingency screening in the distribution system. Since the OPF model in the contingency scenario is a mixed integer programming (MIP) problem, the core idea behind the strategy is to work on a single branch-and-cut search tree, so that all the OPF calculations can be performed coordinately and dynamically by applying the technology of incumbent solution update and lazy constraint callback [12] to modify the search path.

The contributions of this paper are as follows.

1) A fast N-1 contingency screening approach is developed for distribution systems where the network reconfiguration and the islanding operation are jointly considered.

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2) A novel acceleration strategy is proposed to perform all OPF calculations within a single branch-and-cut tree.

The remainder of the paper is organized as follows. Section II provides the detailed contingency screening model. Section III presents the novel acceleration strategy. Case studies are conducted in Section IV, and some conclusions are drawn in Section V.

II. Model Formulation

A unified contingency screening model combined with the variables of power flow, feeder utilization and N-1 contingency is proposed here.

A. Objective Function

The objective of the model is given in (1) to minimize the total costs of operation, power loss and load shedding.

$$\min \sum_{i\in\Omega_S} C_i^s p_i^s + \sum_{i\in\Omega_G} C_i^g p_i^g + \sum_{i\in\Omega_B} C_i^c s_i^c \tag{1}$$

where i denotes the index of buses, Ω_S, Ω_G and Ω_B indicate the bus sets of distributed generators (DGs), substations and load, C_i^g, C_i^s and C_i^c stand for the cost coefficients of power generation, power purchasing and load shedding. Variables p_i^s and p_i^g are the power injections from substations and DGs, s_i^c is the curtailment at load buses.

B. Reconfiguration and Islanding

In this paper, the indices of feeder outages are incorporated as free variables rather than loop counters, which formulates the biggest difference compared with the traditional methods in [5]–[7]. The advantages of the incorporation will be given in Section III. Due to the uncertainty of contingency, a new variable λ is introduced to represent the number of islands. If $\lambda = 0$, it means that the load shedding can be restored from substation by network reconfiguration. Otherwise, the system will be divided into λ DG islands [13]. As to the N-1 contingency, the maximum value of λ is set to 1.

Then, the radiality of the distribution system can be determined based on the fictitious load flow constraints [14].

$$|f_{ij}| \le \bar{f}_{ij}\cdot(y_{ij}+c_{ij}\lambda) \quad \forall ij\in\Omega_L \tag{2}$$

$$0 \le s_i \le \bar{s}_i \quad \forall i\in\Omega_S \tag{3}$$

$$\sum_{ji\in\Omega_L} f_{ji} - \sum_{ij\in\Omega_L} f_{ij} - g_i + s_i = d_i \quad \forall i\in\Omega_B \tag{4}$$

$$\sum_{ij\in\Omega_L} y_{ij} = N_B - \lambda - 1 \tag{5}$$

where ij represents the index of branches, Ω_L denotes the set of branches. Binary utilization variable y_{ij} stands for whether the corresponding feeder is in operation. Binary contingency variable c_{ij} indicates the outages status. f_{ij}, g_i, s_i and d_i are fictitious power flow variables, where f_{ij} represents the line flow on the branch, g_i and s_i denote the power injections from DGs and substations, d_i stands for the load demand at bus i. To simplify the optimization, d_i is set to 1 for all $i\in\Omega_B$ and 0 otherwise. Similarly, g_i is set to 1 for all $i\in\Omega_G$ and 0 otherwise. Parameters $\bar{f}_{ij}$ and $\bar{s}_i$ are the upper bounds of f_{ij} and s_i. The total number of bus is denoted by N_B.

Constraint (2) marks one of the innovations in this paper. Due to the possibility of islanding, the original constraints for connected network [14] should be upgraded. Since the topology will remain radial if the fault lines between islands are restored, a new concept called fictitious reconfiguration is introduced, i.e. $y_{ij}+c_{ij}\lambda$. If $\lambda=0$, constraints (2)-(5) are the same with the ones in [14] because the network is connected. If $\lambda=1$, it is assumed that all lines in outage are put into operation fictitiously. Note that $c_{ij}\lambda$ is a bilinear term which should be linearized by the Big-M method.

C. N-1 Contingency

The constraints for contingency are given as follows.

$$y_{ij}+c_{ij} \le 1 \quad \forall ij\in\Omega_L \tag{6}$$

$$\sum_{ij\in\Omega_L} c_{ij} = 1 \tag{7}$$

Constraint (6) denotes that the faulted lines are unavailable for utilization, and (7) illustrates the definition of N-1 contingency, i.e. single contingency.

D. Power Flow

The DistFlow model is applied to formulate the power flow constraints as follows.

$$\begin{aligned}&\sum_{ij\in\alpha_j}(p_{ij}^l - l_{ij}r_{ij}) - \sum_{jk\in\beta_j} p_{jk}^l + \sum_{i\in\gamma_j} p_i^s + \sum_{i\in\delta_j} p_i^g \\ &\quad = (S_j^d - s_j^c)\cos\varphi_i^d; \qquad \forall j\in\Omega_B\end{aligned} \tag{8}$$

$$\begin{aligned}&\sum_{ij\in\alpha_j}(q_{ij}^l - l_{ij}x_{ij}) - \sum_{jk\in\beta_j} q_{jk}^l + \sum_{i\in\gamma_j} q_i^s + \sum_{i\in\delta_j} q_i^g \\ &\quad = (S_j^d - s_j^c)\sin\varphi_i^d; \qquad \forall j\in\Omega_B\end{aligned} \tag{9}$$

$$\begin{aligned}&\left|v_i - v_j - 2\left(p_{ij}^l r_{ij} + q_{ij}^l x_{ij}\right) + l_{ij}(r_{ij}^2 + x_{ij}^2)\right| \\ &\quad \le M(1-y_{ij}) \qquad \forall ij\in\Omega_L\end{aligned} \tag{10}$$

$$v_i^{min} \le v_i \le v_i^{max} \quad \forall i\in\Omega_B \tag{11}$$

$$0 \le l_{ij} \le y_{ij}\cdot I_{ij}^{max} \quad \forall ij\in\Omega_L \tag{12}$$

$$-y_{ij}S_L^{max} \le p_{ij}^l \le y_{ij}S_L^{max} \quad \forall ij\in\Omega_L \tag{13}$$

$$-y_{ij}S_L^{max} \le q_{ij}^l \le y_{ij}S_L^{max} \quad \forall ij\in\Omega_L \tag{14}$$

$$0 \le p_i^s \le S_{S,i}^{max} \quad \forall i\in\Omega_S \tag{15}$$

$$0 \le q_i^s \le S_{S,i}^{max} \quad \forall i\in\Omega_S \tag{16}$$

$$0 \le s_i^g \le S_{G,i}^{max} \quad \forall i\in\Omega_G \tag{17}$$

$$0 \le s_i^c \le S_i^d\cdot\eta \quad \forall i\in\Omega_B \tag{18}$$

$$\left\| 2p_{ij}^l \quad 2q_{ij}^l \quad l_{ij}-v_i \right\|_2 \le l_{ij}+v_i \quad \forall ij\in\Omega_L \tag{19}$$

$$\left\| p_i^g \quad q_i^g \right\|_2 \le S_{G,i}^{max} \quad \forall i\in\Omega_G \tag{20}$$

where variables p_{ij}^l, p_i^s and p_i^g are active power flow through feeders and power injections from substations and DGs, q_{ij}^l, q_i^s and q_i^g are the corresponding reactive power, s_j^c is the load curtailment, l_{ij} is the square of line current, v_i is the square of bus voltage. Parameters r_{ij} and x_{ij} are the line resistance and reactance, S_j^d is the load demand, φ_i^d is the power factor, M is a sufficiently large number, v_i^{min} and v_i^{max} are lower and upper bounds of bus voltage, I_{ij}^{max} is the limit of line current, S_L^{max} is the capacity of feeder, $S_{G,i}^{max}$ is the limit of DG output, η is a proportional coefficient. Moreover, α_j and β_j are sets of lines whose head-end or tail-end is bus j, γ_j and δ_j are sets of substations and DGs connected to bus j.

Constraints (8) and (9) are nodal power balance equations, (10) is the branch flow constraint, (11)-(18) are lower and upper limits of variables, (19) and (20) are second order cone constraints of branches and DGs.

III. Solution Methodology

As can be observed, the proposed model is designed for a single N-1 contingency. If it is directly solved by commercial softwares, the results will be the "mildest" N-1 contingency and the corresponding repair scheme, which is clearly wrong. The motivation for such modeling is to include the N-1 contingency in the solution space, so that the novel acceleration strategy in this paper can be conducted.

A. Generic Version of the Branch-and-Cut Algorithm

The Branch-and-Cut (B&C) algorithm has been widely used to solve MIP models. It consists of a systematic enumeration of candidate solutions by means of state space search and a cutting-plane strategy to tighten the solution space. For a distribution system with n branches, if using the traversing-based strategy, the B&C algorithm will be performed n times to find the optimal repair schemes for each N-1 contingency, which is quite time-consuming.

To accelerate the procedure, this paper focuses on the deep modification of the B&C algorithm. Fig. 1 shows the diagram of the B&C search tree for the traditional method, where paths 1 to 4 represent four different searching paths corresponding to four N-1 contingencies. As can be observed, there are a large number of repeated visits to the same node during the sequential search along paths 1 to 4. Therefore, based on the analysis above, the key idea of the acceleration strategy is to perform all screenings on a single B&C search tree, so that the number of optimization can be reduced from n to 1. Since the model in Section II ensures that all N-1 contingencies and the repair schemes are in the solution space, the following part will focus on the modification of the searching path.

B. Cutting-Plane Based Modification

When monitoring the progress of the MIP optimization, an important intermediate point is the incumbent solution, which is defined as the best integer solution found at any point in the B&C search tree so far. As is often the case, the incumbent solution is a sub-optimal solution. However, it satisfies all the integer constraints. As to the proposed model, any incumbent solution contains a group of feasible candidates $(y_{ij}^*, c_{ij}^*, \lambda^*)$, which indicates that the risk of load shedding under the N-1 contingency (c_{ij}^*) is under control if taking the repair scheme (y_{ij}^*, λ^*). In other words, every time an incumbent solution is

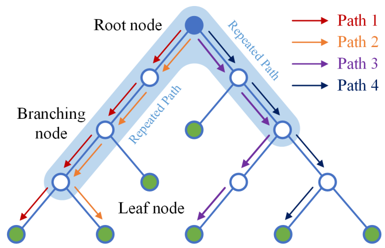


Fig. 1. Diagram of B&C search tree for the traditional traversing-based N-1 contingency screening strategy.

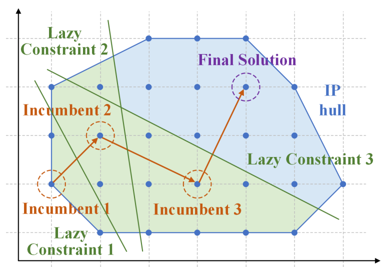


Fig. 2. Diagram of B&C algorithm combined with incumbent solution and lazy constraint callback.

found, the screening for the corresponding N-1 contingency is completed. Although this feature is of great value, the results will still be the "mildest" contingency if no action is taken at each incumbent solution node, not to mention that the B&C search path cannot cover all N-1 contingencies.

To control the searching direction, the cutting-plane technology is introduced in this paper to pare away some specific integer-feasible solutions at the incumbent solution node. In the B&C search tree, every time an incumbent solution is found, a cutting plane will be added to all sub MIPs in the branching queue to prevent the update of incumbent solution, so that the B&C algorithm can change its searching direction to the rest of the solution space until the next incumbent solution is located. Since the cutting planes are added during the search rather than at the start, this process is called lazy constraint callback. The diagram of the strategy above is given in Fig. 2. Following this framework, it is easy to customize the searching path to perform the N-1 contingency screening on a single B&C search tree. Then, the strategy of cutting-plane generation is given as follows.

Let $(y_{ij}^*, c_{ij}^*, \lambda^*)$ be any detected incumbent solution. Since the repair scheme (y_{ij}^*, λ^*) is able to address any contingency on branch ij if the feeder is not in operation, these low-risk contingencies c_{ij}^* can be eliminated from the solution space by appending the constraint in (21). Moreover, the repair scheme itself can also be removed from the search tree by adding the combinatorial cut in (22).

$$c_{ij} = 0 \quad \forall ij \in \{\mathbb{U} \cup \mathbb{V}\} \tag{21}$$

$$\sum_{ij\in\mathbb{V}} y_{ij} + \sum_{ij\in\mathbb{W}} (1 - y_{ij}) \geq 1 \tag{22}$$

where $\mathbb{U}$ is the index set of lines where $c_{ij}^* = 1$, $\mathbb{V}$ is the index set of lines that satisfy $y_{ij}^* = 0$, and $\mathbb{W}$ is the index set of lines that satisfy $y_{ij}^* = 1$.

C. Deep Integration With Modern Solvers

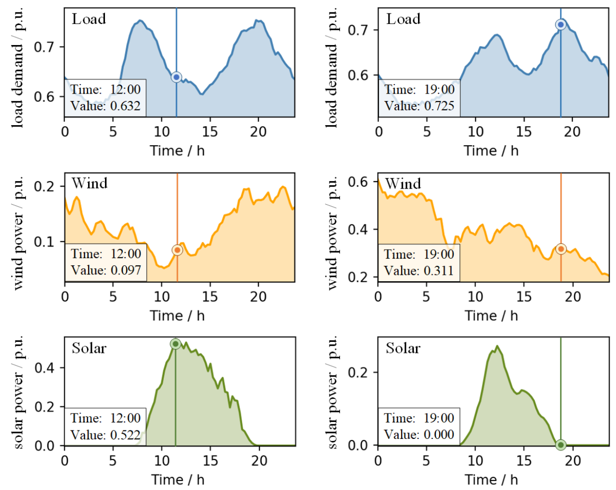
Considering the complexity of rewriting the source code of the B&C algorithm, this paper applies an advanced technique called callback functions [15] to integrate the modifications above into the modern solvers such as Cplex and Gurobi. The core idea is to detect the incumbent solutions by creating monitor modules and to prevent the incumbent updates by adding lazy constraints. Therefore, all the cutting planes are added during the optimization process, which means that the solver only needs to be called once.

Based on the analysis above, the solving procedure for the proposed N-1 contingency screening model using a callback-based B&C algorithm is given as follows.

Algorithm 1 Contingency Screening in Modern Solvers

Input: Data of system configuration, load and DGs
Output: High-risk contingency set Ω_H

Initialize the low-risk contingency set $\Omega_C = \varnothing$, build the model in (1) to (20) and send it to the solver.
while performing the B&C algorithm **do**
  if a new incumbent solution is found **then**
    Add (21) and (22) to the search tree by callback,
    Update $\Omega_C = \Omega_C \cup \{c_{ij}^*\}$.
    if model is infeasible **then**
      Break, and **return** $\Omega_H = \Omega_L - \Omega_C$.
    end if
  end if
end while



(a) IEEE-33 System (b) IEEE-69 System

Fig. 3. Curve of load demand and renewables generation.

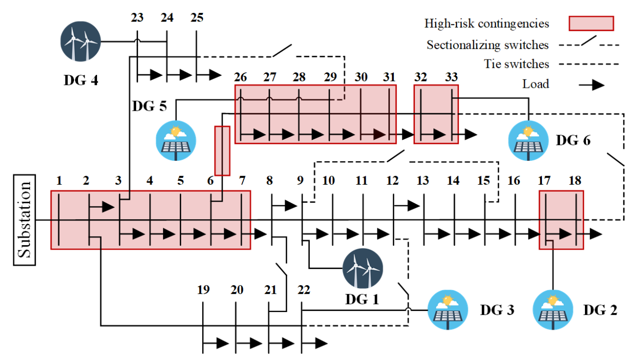


Fig. 4. Result of N-1 contingency screening in IEEE-33 bus system.

TABLE I
DATA OF DGS IN THE TESTING SYSTEMS

No.	IEEE-33 Bus System			IEEE-69 Bus System		
	Bus No.	Capacity	Type	Bus No.	Capacity	Type
1	8	0.20 MVA	Wind	4	0.75 MVA	Wind
2	16	0.25 MVA	PV	18	0.90 MVA	PV
3	21	0.25 MVA	PV	37	0.85 MVA	Wind
4	23	0.20 MVA	Wind	43	1.00 MVA	PV
5	28	0.30 MVA	PV	57	0.75 MVA	Wind
6	31	0.30 MVA	PV	31	0.70 MVA	PV

IV. NUMERICAL RESULT

The performance of the proposed approach is evaluated on the IEEE-33 and 69 bus distribution systems. All algorithms are coded in Python 3.7 and tested on a PC with AMD Ryzen 9 3900X CPU @ 3.80 GHz and 32GB of RAM. The model is solved by Gurobi 8.1.1.

The detailed parameters of the testing systems can be found and downloaded from [16]. The data of DGs in both systems are listed in Table I. To illustrate the uncertainty of load and renewables, all case studies were implemented based on the generation schedule which is depicted in Fig. 3. The hourly load demand is given as a percentage of its peak value, while the generation of wind farms and photovoltaic (PV) stations are normalized between 0 and 1.

A. Case Study

To verify the efficiency and superiority of the acceleration strategy for different systems and scenarios, the contingency screening approaches are conducted on IEEE-33 and 69 bus systems at 12:00 a.m. and 19:00 p.m. respectively.

The results of the N-1 contingency screening on IEEE-33 bus system are shown in Fig. 4. The branches in the dark red square constitute the high-risk contingency set. In other words, any unplanned outage in the set will cause the load shedding rate to exceed its threshold. As can be observed, the outages near root and leaf buses of radial distribution system take higher risk of large-scale load shedding. The total number of severe contingency is 14, which accounts for 42.4% of the total branches. The same analysis can be applied to IEEE-69 bus system, where 35 out of 73 branches are marked as the location of severe contingencies. The detailed screening results are depicted in Fig. 5. The worst N-1 contingency scenario in both systems is branch 1-2, because it is directly connected to the substation bus. The outage of branch 1-2 will cause the system to lose the power supply from the grid. As to the rest of the contingency scenarios, the main reasons for load shedding are overloading of lines and overvoltage of buses.

B. Performance Analysis

To demonstrate the superiority of the proposed acceleration strategy, the CPU time for N-1 contingency screening with the

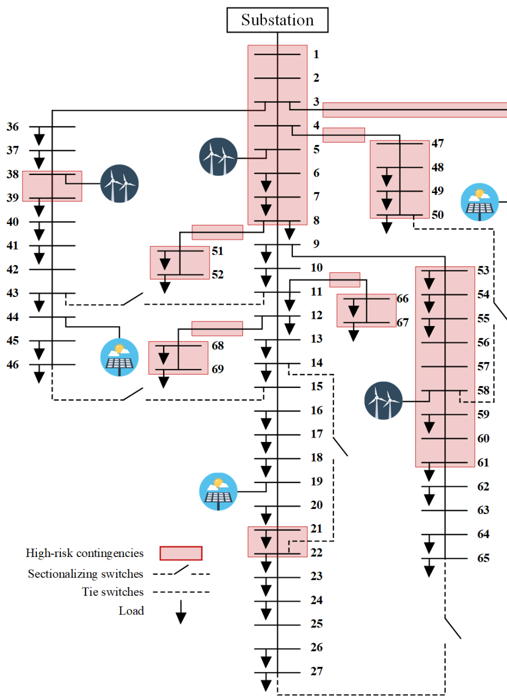


Fig. 5. Result of N-1 contingency screening in IEEE-69 bus system.

TABLE II
CPU TIME FOR THE N-1 CONTINGENCY SCREENING (S)

Hour	IEEE-33 Bus System			IEEE-69 Bus System		
	Callback	Traversal	Ratio	Callback	Traversal	Ratio
00:00	0.28	3.45	8.16%	3.85	32.81	11.74%
03:00	0.29	4.40	6.57%	6.53	41.61	15.69%
06:00	0.25	3.17	8.01%	4.43	27.14	16.33%
09:00	0.26	3.50	7.37%	3.78	21.76	17.37%
12:00	0.93	6.54	14.3%	3.89	21.85	17.79%
15:00	0.44	5.22	8.49%	4.57	24.71	18.50%
18:00	0.26	3.63	7.26%	4.29	20.01	21.46%
21:00	0.17	2.45	6.78%	3.34	20.35	16.42%

callback-accelerated strategy is compared with the traditional traversing-based approach in 8 time sections. The results are listed in Table II. For the IEEE-33 bus system, the average CPU time for performing the contingency screening by the use of the callback-based approach is 8.4% of the traversing-based one, while for the IEEE-69 bus system, the ratio is 16.9%.

The main reason behind such a great improvement is that the calculations for all N-1 contingencies are accomplished at the incumbent solution nodes, so that the solving procedure can be conducted within a single B&C search tree. For one thing, the number of variables needed in the callback-based approach is much less than the traditional one, For another, the optimization only needs to be performed once regardless of the scale of the system.

It is worth mentioning that the incumbent solutions only exist in MIP models, which means that the proposed strategy is not suitable for power transmission systems. However, it is precisely because the traditional algorithm cannot address the problem with integer variables that an order of magnitude improvement achieved in this paper is remarkable.

V. CONCLUSION

The changeability of network configuration brings tremendous computational burden to the N-1 contingency screening in distribution systems. To deal with the problem, this paper proposed a novel acceleration strategy to achieve a fast and reliable screening of all N-1 contingencies. The model was built on a modified DistFlow model where N-1 contingency was incorporated as independent variables. Then, the Branch-and-cut algorithm was customized based on the technology of lazy constraint callback, so that all N-1 contingencies can be identified at the incumbent solution nodes of the search tree. Case studies on IEEE-33 and 69 bus systems show that the proposed strategy is an order of magnitude faster than the traditional traversing-based method.

APPENDIX A

The source code for this article is available online [16] for academic use only.

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