Applied Calculus for Public Health Epidemiology

Problem Set for Health Data Scenarios

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Overview

- Problem 1: Limits and Continuity (conceptual, no code)
- Problem 2: Derivatives – Exponential Growth
- Problem 2.2: Logistic Vaccination Uptake
- Problem 3: Transcendental Functions – Dose Response
- Problem 4: Integration – Cumulative Incidence
- Problem 5: Techniques of Integration (scenario building)
- Problem 6: Differential Equations – Screening Uptake
- Problem 7: Multivariable Calculus – Prevalence Surface
- Problem 8: Optimization – Resource Allocation
- Problem 9: Extrema of Multivariate Functions

Step 1: Shiny Framework

We begin with the Shiny app structure:

```r
library(shiny)
library(ggplot2)

ui <- fluidPage(
titlePanel("Applied Calculus for Public Health Epidemiology"),
sidebarLayout(
sidebarPanel(...),
mainPanel(...)
)
)
server <- function(input, output) {
  output$plot <- renderPlot({ ... })
  output$text <- renderPrint({ ... })
}
shinyApp(ui, server)

```

Step 2: Exponential Growth (Problem 2)

```r
    t <- 0:input$tmax
    k <- log(2)/input$doubling
    I <- input$I0 * exp(k*t)
    df <- data.frame(t,I)
    ggplot(df, aes(t,I)) +
    geom_line(color="blue") +
    labs(title="Exponential Incidence Growth")
```

Step 3: Logistic Vaccination Uptake (Problem 2.2)

```r
t <- 0:input$tmax_log
V <- 1/(1+exp(-input$k*(t-input$t0)))
Vprime <- (input$k*exp(-input$k*(t-input$t0))) /
((1+exp(-input$k*(t-input$t0)))^2)

ggplot(data.frame(t,V,Vprime), aes(t)) +
geom_line(aes(y=V, color="Coverage")) +
geom_line(aes(y=Vprime, color="Daily uptake rate"))
```

Step 4: Dose-Response (Problem 3)

```r
    dose <- 1:input$dosemax
    risk <- 1 - exp(-input$beta*dose)

    ggplot(data.frame(dose,risk), aes(dose,risk)) +
    geom_line(color="blue") +
    labs(title="Dose-Response: Infection Risk")
```

Step 5: Cumulative Incidence (Problem 4)

```r
t <- 0:input$tmax_cum
I <- input$I0_cum * exp(-input$decay*t)
C <- cumsum(I)

df <- data.frame(t,I,C)
ggplot(df, aes(t)) +
geom_line(aes(y=I, color="Daily incidence")) +
geom_line(aes(y=C, color="Cumulative cases"))

```

Step 6: Screening Uptake ODE (Problem 6)

```r
    t <- 0:input$tmax_screen
    ystar <- input$kappa/(input$kappa+input$mu)
    y <- ystar + (input$y0 - ystar) *
    exp(-(input$kappa+input$mu)*t)

    ggplot(data.frame(t,y), aes(t,y)) +
    geom_line(color="blue") +
    labs(title="Screening Uptake Dynamics")

```

Step 7: Resource Allocation (Problem 8)

```r
    x <- 0:input$B
    y <- input$B - x
    Tval <- input$a1*sqrt(x) + input$a2*sqrt(y)
    df <- data.frame(x,y,Tval)

    ggplot(df, aes(x,Tval)) +
    geom_line(color="purple") +
    labs(title="Total Tests vs Staff Allocation")
```

Step 8: Multivariable Surface (Problem 7)

```r
x <- seq(-5,5,0.2); y <- seq(-5,5,0.2)
grid <- expand.grid(x=x, y=y)
grid$z <- input$alpha*grid$x^2 +
input$beta2*grid$y^2 +
input$gamma*grid$x*grid$y

ggplot(grid, aes(x,y,z=z)) +
geom_contour_filled() +
labs(title="Prevalence Surface")
```

Step 9: Extrema Classification (Problem 9)

```r
    H <- matrix(c(2*input$a, input$c,
    input$c, 2*input$b), 2, 2)
    eig <- eigen(H)$values

    if (all(eig > 0)) result <- "Local minimum"
    else if (all(eig < 0)) result <- "Local maximum"
    else result <- "Saddle point"
```

Conclusion

- Each calculus concept is mapped to a public health scenario.
- Shiny enables interactive exploration by adjusting parameters.
- Problems 7 and 9 introduce multivariate thinking:
  - Surfaces (visualizing prevalence).
  - Hessians for extrema classification.

Thank You !!!

- **For More Information, Please Contact:**

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