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Channel Estimation in OFDM Systems

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Abstract: Orthogonal frequency division multiplexing (OFDM) provides an effective and low complexity means of eliminating inter symbol interference for transmission over frequency selective fading channels. This technique has eceived a lot of interest in mobile communication research as the radio channel is usually frequency selective and time variant. In OFDM system, modulation may be coherent or differential. Channel state information (CSI) is required for the OFDM receiver to perform coherent detection or diversity combining, if multiple transmit and receive antennas are deployed. In practice, CSI can be reliably estimated at the receiver by transmitting pilots along with data symbols. Pilot symbol assisted channel estimation is especially attractive for wireless links, where the channel is time-varying. When sing differential modulation there is no need for a channel estimate but its performance is inferior to coherent system. In this paper we investigate and compare various efficient pilot based channel estimation schemes for OFDM systems. In this present study, two major types of pilot arrangement such as block type and comb-type pilot have been focused employing Least Square Error (LSE) and Minimum Mean Square Error (MMSE) channel estimators. Block type pilot sub-carriers is especially suitable for slow-fading radio channels whereas comb type pilots provide better resistance to fast fading channels. Also comb type pilot arrangement is sensitive to frequency selectivity when comparing to block type arrangement. The channel estimation algorithm based on comb type pilots is divided into pilot signal estimation and channel interpolation. The symbol error rate (SER) performances of OFDM system for both block type and comb type pilot subcarriers are presented in this paper.

Keywords: OFDM; channel estimation; pilot based channel estimation; Rayleigh Fading.

1. Introduction

Wireless systems are expected to require high data rates with low delay and low bit-error-rate (BER).In such situations, the performance of wireless communication systems is mainly governed by the wireless channel environment. In addition, high data rate transmission and high mobility of transmitters and/or receivers usually result in frequency-selective and time-selective, i.e., doubly selective, fading channels for future mobile broadband wireless systems. Therefore, mitigating such doubly selective fading effects is critical for efficient data transmission. Moreover, perfect channel state information (CSI) is not available at the receiver. Thus in practise, accurate estimate of the CSI has a major impact on the whole system performance [1]. It is also because, in contrast to the typically static and predictable characteristics of a wired channel, the wireless channel is rather dynamic and unpredictable, which makes an exact analysis of the wireless communication system often difficult.

For a typical wireless system, RF signal transmission between two antennas commonly suffers from power loss, which affects its performance. This power loss between transmitter and receiver is a result of three different phenomena: 1) distance-dependent decrease of the power density called path loss or free space attenuation, 2) absorption due to the molecules in the atmosphere and 3) signal fading caused by terrain and weather conditions in the propagation path. Atmospheric absorption is due to the electrons, uncondensed water vapor and molecules of various gases. Path loss is a theoretical attenuation which occurs under free-line-of-sight conditions and which increases with the distance between base station and mobile.

Fading refers to the variation of the signal amplitude over time and frequency. In contrast with the additive noise as the most common source of signal degradation, fading is another source of signal degradation that is characterized as a non-additive signal disturbance in the wireless channel. Fading may be either due to multipath propagation, referred to as multi-path (induced) fading, or to shadowing from obstacles that affect the propagation of a radio wave, referred to as shadow fading. Fading channel models are often used to model electromagnetic transmission of information over wireless media such as cellular phone and broadcast communication.

The shadow fading occurs when terminals move through areas with obstacles of various sizes, such as mountains, buildings and tunnels. Occasionally, these obstacles will shadow or completely cut off the signal. Although the consequences of such shadowing effects will depend on the size of an obstacle and on the distance to it, the received signal strength will inevitably vary. The effect of shadow fading can be decreased with some awareness in network planning. For example, by placing base stations as high as possible or close to each other it is possible to avoid some obstacles in transmission. However, it is not only the shadow fading that is the most unpredictable power loss. Multi-path fading, Rayleigh fading (Figure 1) or short term fading are another types of fading, involving irregular signal strength variations and are usually problematic to overcome.

Rayleigh fading is a result of a reception of several signals at the receiver incoming and reflected from many different objects and directions in the area. Due to their different traveling distances, the signals are usually not in phase, reinforcing or extinguishing each other. The movement of the terminal causes continuous and unpredictable variations of the signal phases over time, making the attenuation vary variable and extremely high at some points (fading dips). Rayleigh fading is most perceptible in urban areas. Dips will occur more frequently at higher frequencies and more rapid mobile movement. To avoid dips it is necessary to attain a sufficient fading margin. The average value of the signal must be at least as many decibels above the receiver sensitivity level as the strongest expected dip. To overcome these problems special reception techniques are normally used, namely the multiple receiver combining techniques known as diversity.

Broadly, the fading phenomenon can be broadly classified into two different types: 1) large-scale fading and 2) small-scale fading. This is shown in Figure 2

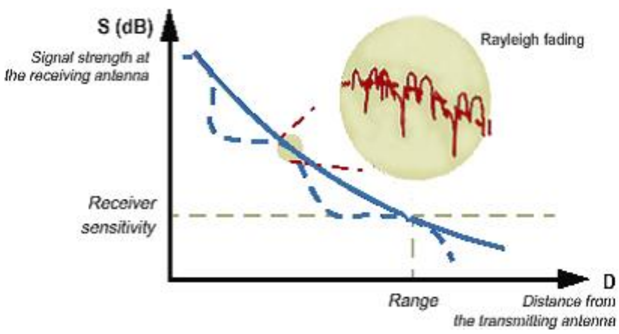


Figure 1: Rayleigh fading [2].

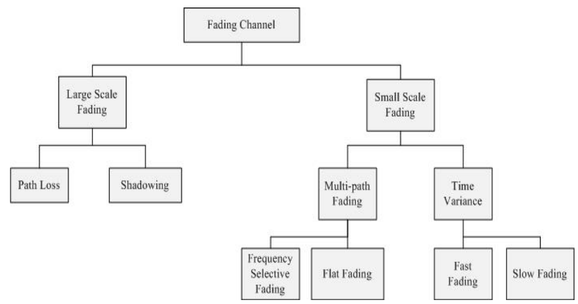


Figure 2: Classification of fading channels.

Large-scale fading occurs as the mobile moves through a large distance, for example, a distance of the order of cell size. It is caused by path loss of signal as a function of distance and shadowing by large objects such as buildings, intervening terrains, and vegetation. Shadowing is a slow fading process characterized by variation of median path loss between the transmitter and receiver in fixed locations. In other words, large-scale fading is characterized by average path loss and shadowing. On the other hand, small-scale fading refers to rapid variation of signal levels due to the constructive and destructive interference of multiple signal paths (multi-paths) when the mobile station moves short distances. Depending on the relative extent of a multipath, frequency selectivity of a channel is characterized (e.g., by frequency-selective or frequency flat) for small-scaling fading. Meanwhile, depending on the time variation in a channel due to mobile speed (characterized by the Doppler spread), short-term fading can be classified as either fast fading or slow fading.

This paper focuses on investigating the effect of fading in modern digital communication techniques such as orthogonal frequency division multiplexing (OFDM). It is because OFDM is most commonly used in modern mobile broadband wireless communication systems such as mobile WiMAX and long-term extension (LTE). Therefore, channel estimation techniques for OFDM systems in doubly selective channels are the topic of interest in this paper. Due to its high bandwidth efficiency, it's simple implementation and its robustness over frequency-selective channels, OFDM has been widely applied in wireless communication systems. For conventional coherent detection, accurate CSI is needed for the receiver processing. Although channel estimation can be avoided by using differential modulation techniques, these techniques will fail catastrophically in the fast fading channel, where the channel impulse response (CIR) varies significantly within the symbol duration. In fact, differential modulation techniques assume that the channel is stationary over the period of two OFDM symbols which is not true for the fast fading channels. The orthogonality among the subcarriers is destroyed and intercarrier interference (ICI) is created, which, if left uncompensated can cause high bit error rates (BERs). Generally, the compensation for the ICI due to the fast fading channel is based on more complex equalizers such as minimum mean-square error (MMSE) equalizers, which need not only the individual subcarrier frequency responses but also the interference among subcarriers in each OFDM symbol. Hence, channel estimation is more challenging for OFDM systems in fast fading channels than in slow fading systems. In other words, the channel estimation is an integral part of the receiver for fast fading channels and the receiver needs to perform channel estimation for each OFDM symbol.

1.1 OFDM systems :

The key elements of OFDM systems are described below.

- **Orthogonality:** In OFDM systems, the two periodic signals are orthogonal when the integral of their product over a period is equal to zero. This can be represented in continuous time as:

$$\int_0^t \cos(2\pi f_0 nt)\cos(2\pi f_0 mt)dt = 0 \tag{1}$$

For the case of discrete time, it can be represented as:

$$\sum_{k=0}^{N-1}\cos\left(\frac{2\pi kn}{N}\right)\cos\left(\frac{2\pi km}{N}\right)dt = 0 \tag{2}$$

where, $m \neq n$ in both cases.

- **Sub-carriers in OFDM systems:** Each subcarrier in an OFDM system is a sinusoid with a frequency that is an integer multiple of fundamental frequency. Each subcarrier can be expressed as a Fourier series component of the composite signal, i.e. an OFDM symbol. The subcarriers waveform can be mathematically expressed as:

$$s(t) = \cos(2\pi f_c t + \theta_k) = a_n \cos(2\pi n f_0 t) + b_n \sin(2\pi n f_0 t) \tag{3}$$

where, $\varphi_n = \tan^{-1}(b_n / a_n)$. The sum of these subcarriers is then referred to baseband OFDM signal as:

$$s_b(t) = \sum_{n=0}^{N-1}\{a_n \cos(2\pi n f_0 t) - b_n \sin(2\pi n f_0 t)\} \tag{4}$$

- **Inter Symbol Interference:** Inter Symbol Interference (ISI) is a form of distortion of a signal in which one symbol interferes with subsequent symbols. This is an unwanted phenomenon as the previous symbols have similar effect as noise; thus, making the communication less reliable. ISI is usually caused by multipath propagation or the inherent non–linear frequency response of a channel causing successive symbols to "blur" together. The presence of ISI in the system introduces error in decision device at the receiver output. Therefore, in the design of transmitting and receiving filters, the objective is to minimize the effects of ISI and; thereby, deliver the digital data to its destination with the smallest error rate possible.
- **Inter-carrier Interference:** Presence of Doppler shifts and frequency and phase offsets in an OFDM system causes loss in orthogonality of the subcarriers. As a result, interference is observed between subcarriers. This phenomenon is known as inter-carrier interference (ICI)**.**
- **Cyclic Prefix:** The Cyclic Prefix (CP) or Guard Interval is a periodic extension of the last part of an OFDM symbol that is added to the front of symbol in a transmitter, and is removed at the receiver before demodulation. Cyclic prefix acts as a guard interval. It eliminates the inter-symbol interference from the previous symbol. It acts as a repetition of the end of the symbol, thus allowing the linear convolution of a frequency–selective multipath channel to be modeled as circular convolution which in turn may be transformed to the frequency domain using a discrete Fourier transform. This approach allows for simple frequency–domain processing such as channel estimation and equalization.
- **Inverse Discrete Fourier Transform:** In frequency domain in OFDM, the modulated data symbols are fed onto the orthogonal sub-carriers. However, transfer of signal over a channel is only possible in its time-domain. Therefore, IDFT of signal is usually taken before, which converts the OFDM signal from frequency domain to time domain. IDFT being a linear transformation can be easily applied to the system and DFT can be applied at the receiver end to regain the original data in frequency domain at the receiver end. As, the basis of Fourier transform is orthogonal in nature, the time domain equivalent of OFDM signal can be implemented from its frequency components.
- **Modulation:** In an OFDM system, the high data rate information is divided into small packets of data which are placed orthogonal to each other. This is achieved by modulating the data by a modulation technique such as QPSK and QAM. After this, IFFT is performed on the modulated signal which is further processed by passing through a parallel–to–serial converter. In order to avoid ISI, a cyclic prefix is added to the signal as discussed above.
- **Demodulation:** In this case, the received data is first passed through a low pass filter to remove the cyclic prefix. After this, FFT is performed and the serial data obtained is converted into the parallel signal. A demodulator is used to get back the original signal.
- **Communication channel:** This is the medium/channel through which the data is transferred. Presence of noise in this medium affects the signal and causes distortion in its data content.

2. System Model

The block diagram of discrete-time baseband OFDM system is depicted in Figure 3. In Figure 3, an OFDM signal consists of N subcarriers that are modulated by N complex symbols selected from a particular QAM constellation. These baseband modulated symbols are then passed through serial to parallel converter which generates complex vector of size N. This complex vector of size N can be expressed as

$$\mathrm{X} = [\mathrm{X}_0, \mathrm{X}_1, \mathrm{X}_2, \mathrm{X}_3 \ldots \mathrm{X}_{\mathrm{N}-1}] \tag{5}$$

X is then passed through the IFFT block to give

$$x = WX \tag{6}$$

Where, W is the $N \times N$ IFFT matrix. Thus, the complex baseband OFDM signal with subcarriers into time domain samples can be written as

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{\frac{j2\pi kn}{N}} \quad n = 0, 1, \ldots, N-1. \tag{7}$$

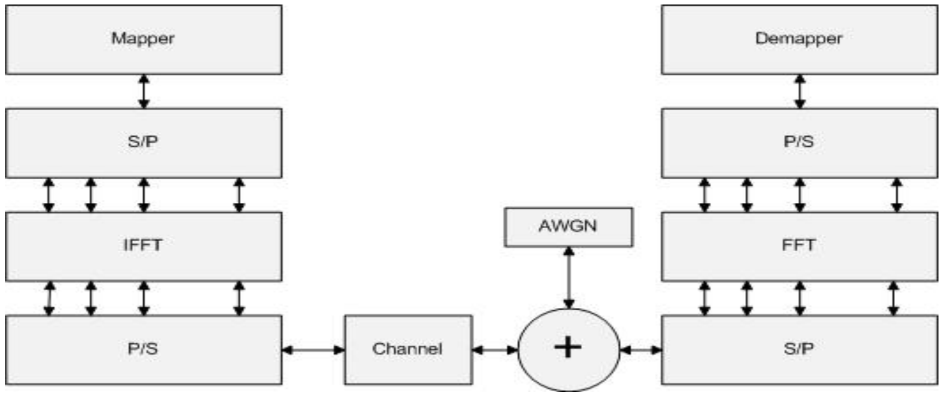


Figure 3: A Discrete-Time Baseband OFDM System.

After parallel-to-serial conversion, a cyclic prefix with a length of N_g samples is appended before the IFFT output to form the time-domain OFDM symbol, $s = [s_0, \ldots, s_{N+Ng-1}]$, where, $s_i = x_{(i-N_g)_N}$ and $\langle i \rangle_N \triangleq i \bmod N$. The useful part of OFDM symbol does not include the N_g prefix samples and has duration of T_u seconds. The samples (s) are then amplified, with the amplifier characteristics is given by function F. The output of amplifier produces a set of samples given by:

$$y = [y_0, y_1, \ldots, y_{N+Ng-1}] \tag{8}$$

This signal is then serially transmitted through a multipath radio propagation channel which is subject to additive white Gaussian noise (AWGN) with variance $\sigma^2 = N_0/2$, where N_0power spectral density is. At the receiver front end, the received signal is applied to a matched filter and then sampled at a rate $T_s = T_u/N$. After dropping the CP samples (N_g), the received sequence z, assuming an additive white Gaussian noise (AWGN) channel, can be expressed as

$$z = F(Wd) + \eta \tag{9}$$

Where, the noise vector η consists of N independent and normally distributed complex random variables with zero mean and variance $\sigma_n^2 = E\{|\eta_n|^2\}$. Subsequently, the sequence z is fed to the fast Fourier transform (FFT), which produces the frequency-domain sequence r as

$$r = W^H z \tag{10}$$

where, k_{th} element of r is given by

$$r_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} z_n e^{\frac{-j2\pi kn}{N}} \quad k=0, 1, 2, \ldots, N-1 \tag{11}$$

Finally, the estimated symbols vector $\hat{d}$ can be obtained from r. It is to be noted that the demodulation is performed based on the assumption of perfect symbol timing, carrier frequency, and phase synchronization. It is also to be noted that due to the use of CP, the inter-block interference between contiguous OFDM blocks in frequency domain is eliminated so each OFDM block can be processed independently, provided that the length of CP is equal to or larger than the delay spread of the channel. The main drawbacks of OFDM systems are high peak-to-average-power ratio (PAPR), bit error rate (BER) and high sensitivity to carrier frequency offset (CFO). Moreover, OFDM does not obtain frequency diversity. If a deep fade occurs close to the frequency of a subcarrier, reliable data detection carried by these faded subcarriers becomes difficult [2, 3]. This problem can be solved by using error-control codes in conjunction with interleaving, which helps reducing the diversity loss. Typical examples of error control codes are block codes (e.g., Reed-Solomon (RS) or Bose-Chaudhuri-Hocquenghem (BCH)), convolutional codes, trellis codes, turbo codes, and low-density parity-check (LDPC) codes.

The major advantage of OFDM lies in processing frequency-selective channels as multiple flat-fading sub-channels. If the channel is time invariant (slow fading) over the period of an OFDM symbol block, the orthogonality property is maintained between the subcarriers. In such a case, channel estimation or data detection is simple as each subcarrier is equalized with a single-tap equalizer. However, when the channel is time-varying over one OFDM symbol period, the orthogonality among subcarriers is destroyed, resulting in ICI, which degrades the bit error rate performance compared to the slow fading channels. The ICI may occur due to the presence of the fast fading channel or the presence of a carrier frequency offset (CFO) between the transmitter and receiver caused by imperfect synchronization. CFO can be estimated by using various algorithms such as a maximum likelihood (ML) estimation algorithm [4, 5, 6]. The potential performance degradation of OFDM caused by fading channels is a function of the fading rate, with faster fading channels requiring more significant mitigation methods to achieve the same error performance as slow fading channels. Furthermore, in the presence of ICI due to fast fading, the channel estimation is more challenging since both the individual subcarrier and the interference created by each subcarrier to its neighbouring subcarriers need to be estimated. Therefore, this paper focuses on channel estimation techniques for both slow fading and fast fading channels.

2.1) Communication system :

As discussed above, wireless signals usually subject to fading and dispersion. This offers implications for the design of wireless communication systems. In such cases, a transmitted signal will be composed of a sequence of shift-orthogonal and overlapping pulses, distorted by the channel. OFDM systems usually experience frequency selectivity in a channel, so the same require equalisation in the receiver. The term *equalisation* means the removal of distortion or the de-convolution of channel response from the received signal. Mathematically, a communication system can be described in terms of the complex baseband transmitted signal as:

$$s(t) = \sum_{l \epsilon Z} a_l\, \delta(t - lT) \tag{12}$$

A pulse shaping filter is also usually introduced, which makes the above equation as:

$$s_T(t) = \sum_{l \epsilon Z} a_l\, c_T(t - lT) \tag{13}$$

where Z is the set of integers, a_l is an element from the complex symbol sequence, $c_T(t)$ is the transmitter pulse shape and T is the symbol spacing. The bandwidth of this transmitted pulse shape $c_T(t)$ is denoted by W. Any digital modulation techniques such as PAM, PSK or QAM can be used in OFDM systems; however in this paper, QAM is considered as modulation technique. The channel state information is usually expressed mathematically in terms of the channel impulse response (CIR); which is a function of time and delay. It is denoted as $h(t, \xi)$.This impulse response incorporates both fading, time selective effects and dispersive, frequency selective effects. To determine the CIR at a given time (say *t*), the channel has the impulse response described as a function of delay (ξ). In OFDM systems, the receiver usually have a fixed front-end filter with impulse response $c_R(t)$.The impulse response of the overall pulse shape is given by

$$c(t) = c_T(t) \otimes c_R(t) \tag{14}$$

As discussed above, the overall CIR in terms of pulse shape is defined as

$$u(t, \xi) = c(\xi) \otimes h(t, \xi) \tag{15}$$

where, $\otimes$ denotes convolution. The overall CIR with $\xi = t - lT$ and the convolution explicitly shown is expressed as

$$u(t, t - lT) = \int_{-\infty}^{\infty} c(t - lT - \tau)\, h(t, \tau) d\tau \tag{16}$$

At the output of the front-end filter, the received signal is given by

$$s_R(t) = \sum_{l \epsilon Z} a_l\, u(t, t - lT) + \eta(t) \tag{17}$$

where, $\eta(t)$ is the filtered output due to additive white Gaussian noise (AWGN) with two-sided power spectral density (PSD) $N_0/2$.

2.2) Channel estimation techniques :

In wireless communication, the channel is usually unknown *a priori* to the receiver. Therefore to do the channel estimation, a pilot symbol aided modulation is used, where known pilot signals are periodically sent during the transmission. In general, the performance of channel estimation depends on the number, the location, and the power of pilot symbols inserted into OFDM blocks. To mathematically analyse this, consider a fading multipath channel with the multipath delay spread τ_{max} and the maximum Doppler frequency(f_d). To recover the channel state information (CSI), the spaces between pilot symbols in the time and frequency domain must satisfy two-dimensional (2-D) sampling theorem [7], that is,

$$f_d T d_t \leq 1/2 \tag{18}$$

$$\tau_{max}\, \Delta f\, d_f \leq 1 \tag{19}$$

Where T is the OFDM block duration, Δf is the subcarrier spacing; $d_t$and d_f are the numbers of samples between pilot symbols in the time domain and frequency domain, respectively [8].Within the OFDM symbol duration, the number of pilot symbols in frequency domain is related to the delay spread; on the other hand, the number of pilot symbols in time domain is related to the normalized Doppler frequency($f_d T$). Based on 2-D arrangement of pilot symbols, 2-D channel estimators are too complex in practice [9]. Therefore, channel estimation is exploited in one-dimension (1-D) for OFDM systems in general.

2.3) Channel estimation in slow fading channels:

In slow fading channels, the channel slowly changes over a number of OFDM symbol blocks. In such channels, channel estimation is based on pilot symbols, which are inserted into all subcarriers of OFDM symbol blocks within a specific period. These pilot symbols are usually called training symbols and a batch of OFDM symbols follows the training symbols. For channel estimation based on training symbols, the first step is to estimate CSI corresponding to training symbols. Using this information, the CSI corresponding to subsequent data symbols can be tracked and further improved by decision directed channel estimation [10]. If the channel varies slowly over OFDM blocks, the estimated CSI based on previous training symbols are generally reliable so such estimated channel state may be used in data detection. However in situations, the channel varies fast over time, the training symbols will be sent more frequently to get reliable channel estimates, and thus, the overall system efficiency is reduced [11]. In such a case, channel estimation can be based on pilot symbols, which are periodically inserted into different subcarriers for each OFDM symbol block. This helps improving the bandwidth efficiency. In addition, to improve further bandwidth efficiency, the superimposed pilot scheme was proposed for flat-selective fading channels [12], frequency selective fading channels [13, 14, 15], and for doubly selective fading channels [16, 17]. In this approach, a pseudo-noise (PN) sequence is synchronously added to the information symbols at the transmitter prior to modulation transmission. Although the use of superimposed pilots can improve the bandwidth efficiency, the performance based on superimposed pilot aided channel estimation is worse than that of traditional PSAM. Therefore, this paper focuses on channel estimation based on traditional PSAM.

For slow fading channels, different pilot designs have been discussed in [18, 19, 20]. For these channels, several types of estimation techniques are exploited by using least squares (LS), ML, minimum mean square error (MMSE) or linear minimum mean square error (LMMSE) methods. For slow fading channels, depending on the pilot arrangement, these estimation schemes are performed in the frequency domain using either training symbols or pilot symbols.

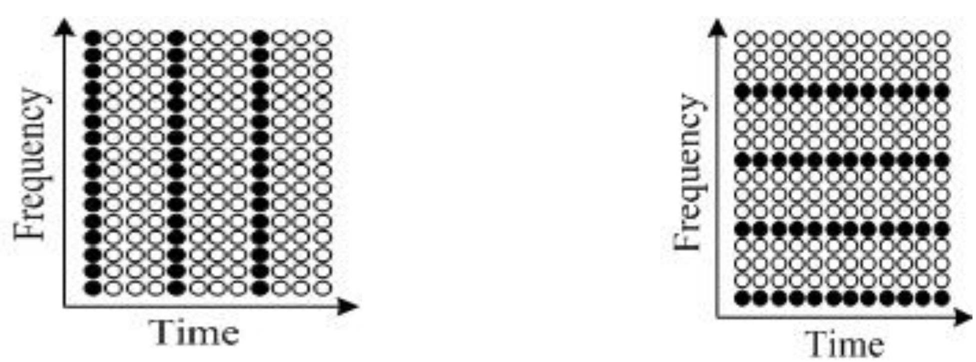
2.4) Channel estimation in fast fading channels :

In fast fading channels, the channel varies significantly over one OFDM symbol block. In such cases, the orthogonality among the OFDM subcarriers is usually lost and the ICI is created. The severity of ICI depends on the normalized Doppler frequency ($f_d T$). In the presence of ICI, the amount of channel states that need to be estimated for reliable data detection increases. Not only the individual subcarrier frequency responses but also the interference among subcarriers in each OFDM symbol are to be estimated. In such a case, an underdetermined system occurs if standard channel modelling is employed, as the number of unknowns are more than the number of measurements (pilot symbols). In order to reduce the number of unknown channel parameters, simplification approaches are exploited for channel estimation. One approach is that the channel is approximated to a piece-wise linear model over one or two adjacent OFDM blocks [21, 22]. However, this modelling approach degrades the performance of the channel estimation at high normalized Doppler frequencies such as 10% mentioned in [23]. Another approach is to model the channel by a basis expansion model (BEM), where the samples of the channel state are characterized as a linear combination of a finite number of known basis functions weighted by unknown basis coefficients.

In addition, in order to minimize the MSE of the channel estimator, the optimal number of pilot (data) blocks should be equal to the number of basis coefficients for a given transmission block. From a performance viewpoint, to minimize the total MSE of the estimator which includes the BEM modelling error, the number of pilot (data) blocks may be larger than the number of basis coefficients [24]. Similar to the time-domain pilot scheme, the pilot block in the form of $[\mathbf{0}_{1xQ}\ 1\ \mathbf{0}_{1xQ}]$ is equally placed between data blocks in the frequency domain, where Q is the number of basis coefficients, defined as $Q \geq 2\lceil f_d N T_s \rceil$ and $\lceil\ \ \rceil$ denotes the integer ceiling.

3. CHANNEL ESTIMATION OF OFDM SYSTEMS :

For an OFDM mobile communication system, the channel transfer function at different subcarriers appears unequal in both frequency and time domains. Therefore, a dynamic estimation of the channel is always required. Pilot-based approaches are widely used to estimate the channel properties and correct the received signal. In this paper, two types of pilot arrangements, as shown in Figure 4 are investigated.



(a) (b)

Figure 4: (a) Block type pilot arrangement and (b) comb type pilot arrangement.

The first kind of pilot arrangement, shown in Figure 4, is denoted as block-type pilot arrangement. This is sent periodically in time-domain and is particularly suitable for slow-fading radio channels. Because the training block contains all pilots, channel interpolation in frequency domain is not required. Therefore, this type of pilot arrangement is relatively insensitive to frequency selectivity. The second kind of pilot arrangement, shown in Figure 4, is denoted as comb-type pilot arrangement. In this case, the pilot arrangements are uniformly distributed within each OFDM block. Assuming that the payloads of pilot arrangements are the same, the comb-type pilot arrangement has a higher re-transmission rate. Thus, the comb-type pilot arrangement system provides better resistance to fast-fading channels. Since only some sub-carriers contain the pilot signal, the channel response of non-pilot sub-carriers will be estimated by interpolating neighbouring pilot sub-channels. Thus, the comb-type pilot arrangement is sensitive to frequency selectivity when comparing to the block-type pilot arrangement system.

3.1) Description of pilot channel estimation based OFDM systems :

The block diagram of pilot channel estimation based OFDM system is shown in Figure 5

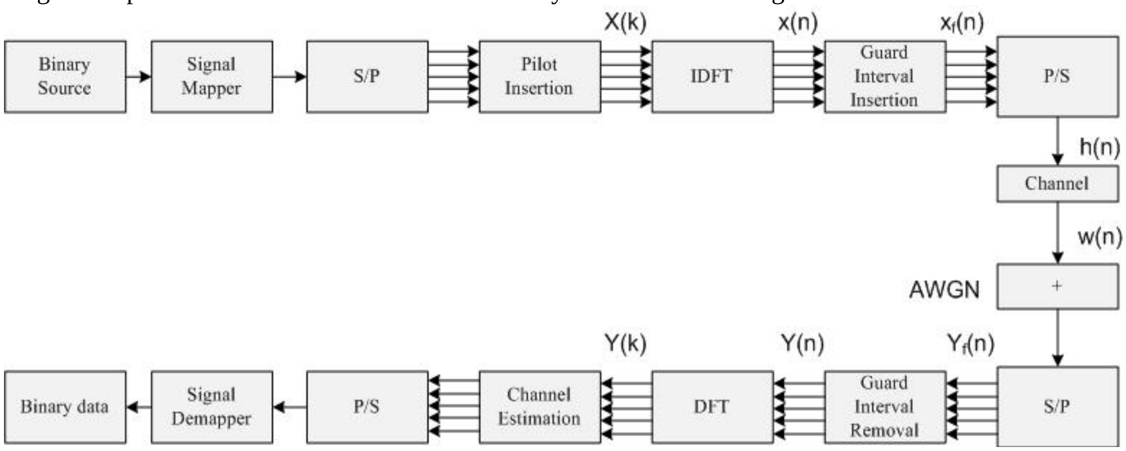


Figure 5: Baseband OFDM system.

In Figure 5, the binary information is first grouped and mapped according to the modulation in signal mapper. In modern OFDM systems, usually QAM is used as the modulation technique. After inserting pilots either to all sub-carriers with a specific period or uniformly between the information data sequence, IDFT block is used to transform the data sequence of length into time domain signal using the following equation:

$$x(n) = IDFT\{X(k)\} = \sum_{k=0}^{N-1} X(k) e^{j(2\pi kn/N)} \qquad n = 0,1,2...N-1 \tag{20}$$

Where N is the DFT length. Following IDFT block, guard time, which is chosen to be larger than the expected delay spread, is inserted to prevent inter-symbol interference. This guard time helps eliminating the inter-carrier interfernce. Therefore, the resultant OFDM symbol is given as:

$$\begin{aligned} x_f(n) &= x(N+n) \qquad & n &= -N_g, -N_g+1, ..., -1 \\ &= x(n) & n &= 0,1,....N-1 \end{aligned} \tag{21}$$

Where N_g is the length of guard interval. The transmitted signal $x_f(n)$ will pass through the frequency selective time varying fading channel with additive noise. The received signal is given by:

$$y_f(n) = x_f(n) \otimes h(n) + w(n) \tag{22}$$

Where $w(n)$ is AWGN and $h(n)$ is the channel impulse response. Thus, the overall channel response can be represented as:

$$h(n) = \sum_{i=0}^{r-1} h_i e^{j(2\pi/N) f_{di} Tn} \delta(\lambda - \tau_i) \qquad n = 0,1,2...N-1 \tag{23}$$

Where r is the total number of propagation paths, h_i is the complex impulse response of the i^{th} path, f_{di} is the i^{th} path Doppler frequency shift, λ is delay spread index, T is the sample period and τ_i is the i^{th} path delay normalized by the sampling time. At the receiver, the analog signal received is converted to discrete domain and the guard time is removed to give the received signal as:

$$\begin{aligned} &y_f(n) \qquad && \text{for } (-N_g \le n \le N-1) \\ &y(n) = y_f(n+N_g) && n = 0,1,....N-1 \end{aligned} \tag{24}$$

This received signal $y(n)$ is then sent to DFT block to yield:

$$Y(k) = DFT\{y(n)\} = \frac{1}{N} \sum_{n=0}^{N-1} y(n) e^{-j(2\pi kn/N)} \qquad k = 0,1,2...N-1 \tag{25}$$

Assuming there is no ISI, the relation of the resulting $Y(k)$ to $H(k)$=DFT{$h(n)$}, and $W(k)$=DFT{$w(n)$}, is given by:

$$Y(k) = X(k)H(k) + W(k) \tag{26}$$

Following DFT block, the pilot signals are extracted and the estimated channel $\hat{H}(k)$ for the data sub-channels is obtained in channel estimation block. Then the transmitted data is estimated by:

$$\hat{X} = \frac{Y(k)}{\hat{H}(k)} \qquad k = 0,1,...., N-1 \tag{27}$$

Then the binary information data is obtained back in signal demapper block. Based on principle of OFDM transmission scheme, it is easy to assign the pilot both in time domain and in frequency domain.

3.2) <u>Block-type pilot based channel estimation</u>

In block-type pilot based channel estimation, OFDM channel estimation symbols are transmitted periodically, in which all sub-carriers are used as pilots. If the channel is constant during the block, there will be no channel estimation error as the pilots are sent at all carriers. The estimation can be performed by using either LSE or MMSE. If inter symbol interference is eliminated by the guard interval, then:

$$Y = XFh + W = XH + W \tag{28}$$

Where

$$X = diag\{X(0), X(1)......X(N-1)\} \tag{29}$$
$$Y = [Y(0), Y(1)......Y(N-1)]^T$$
$$W = [W(0), W(1)......W(N-1)]^T$$
$$H = [H(0), H(1)......H(N-1)]^T$$
$$F = \begin{bmatrix} W_N^{00} & \cdots & W_N^{0(N-1)} \\ \vdots & \ddots & \vdots \\ W_N^{(N-1)0} & \cdots & W_N^{(N-1)(N-1)} \end{bmatrix}$$
$$W_N^{nk} = \frac{1}{N} e^{-j2\pi(nk/N)}$$

3.2.1) <u>Minimum mean square error estimation</u>

The mean square error (MSE) is given by

$$J(e) = E[(H - \hat{H})^2] = E[(H - \hat{H})^H (H - \hat{H})] \tag{30}$$

Here $\hat{H} = MY$, where M is a linear estimator. Invoking the well-known orthogonality principle in order to minimize the mean square error vector $e = (H - \hat{H})$ has to be set orthogonal by the MMSE equalizer to the estimators input vector Y. That is:

$$E[(H - \hat{H})Y^{\mathscr{H}}] = 0 \tag{31}$$
$$\Rightarrow E[FhY^{\mathscr{H}}] - ME[YY^{\mathscr{H}}] = 0$$

If the time domain channel vector h is Gaussian and uncorrelated with the channel noise W, then

$$FR_{hY} = MR_{YY} \tag{32}$$

Where $R_{hY} = E[hY^{\mathscr{H}}]$ and $R_{YY} = E[YY^{\mathscr{H}}]$, $R_{hY} = E[hY^{\mathscr{H}}] = E[h(XFh + w)^{\mathscr{H}}] = R_{hh}F^{\mathscr{H}}X^{\mathscr{H}}$. Because of $E[hw^{\mathscr{H}}] = 0$, i.e., h is uncorrelated with w. And,

$$R_{YY} = E[YY^{\mathscr{H}}] = E[(XFh + w)(XFh + w)^{\mathscr{H}}] = XFR_{hh}F^{\mathscr{H}}X^{\mathscr{H}} + \sigma^2 I_N \tag{33}$$

Where σ^2 is the variance of noise.

$$M = FR_{hY}R_{YY}^{-1} \quad \text{and} \quad \hat{H} = FR_{hY}R_{YY}^{-1}Y \tag{34}$$

The time domain MMSE estimate of h is given by

$$\hat{h}_{MMSE} = R_{hY} R_{YY}^{-1} \mathrm{Y} \tag{35}$$

3.2.2) Least square error estimation

For least square error estimation:

$$\begin{aligned} J &= (Y - XH)^{\mathcal{H}} (Y - XH) = (Y^T - H^{\mathcal{H}} X^{\mathcal{H}})(Y - XH) \\ &= Y^{\mathcal{H}} Y - Y^{\mathcal{H}} XH - H^{\mathcal{H}} X^{\mathcal{H}} Y + H^{\mathcal{H}} X^{\mathcal{H}} XH \end{aligned} \tag{36}$$

For minimization of J, it is required to differentiate J with respect to H as

$$\left.\frac{\partial J}{\partial H}\right|_{\hat{H}} = 0 \tag{37}$$

That is

$$-2Y^{\mathcal{H}} X - 2\hat{H}^{\mathcal{H}} X^{\mathcal{H}} X = 0 \tag{38}$$

$$\begin{aligned} &\Rightarrow Y^{\mathcal{H}} X = \hat{H}^{\mathcal{H}} X^{\mathcal{H}} X \\ &\Rightarrow (Y^{\mathcal{H}} X)(X^{\mathcal{H}} X)^{-1} = \hat{H}^{\mathcal{H}} (X^{\mathcal{H}} X)(X^{\mathcal{H}} X^{-1}) \\ &\Rightarrow Y^{\mathcal{H}} X X^{-1} (X^{\mathcal{H}})^{-1} = \hat{H}^{\mathcal{H}} \\ &\Rightarrow \hat{H} = [(X^{\mathcal{H}})^{-1}]^{\mathcal{H}} Y = X^{-1} Y \end{aligned} \tag{39}$$

The time domain LS estimate of h is given by

$$\hat{h} = F^{\mathcal{H}} X^{-1} Y \tag{40}$$

3.3) Channel estimation based on comb type pilot arrangement

In comb-type based channel estimation, the n_p pilot signal are uniformly inserted into X(k) according to the following equation

$$X(k) = X(mL + l) \tag{41}$$

where L=number of carriers/n_p. Suppose that the frequency-selective channels remain invariant over an OFDM block, and length of the cyclic prefix exceeds the channel order. After demodulation, the received signal on the nth subcarrier corresponding to pilot symbols can be written as:

$$Y[k] = \sqrt{\varepsilon_p} H(k) X(n) + w(k), \qquad k \in \Im_P \tag{42}$$

Where $\Im_P$ denotes the set of subcarriers on which the pilot symbols are transmitted, ε_p is the transmitted power per pilot symbol, $H(k)$ is the channel frequency response on k^th^ carrier $X(k),\ k \in \Im_P$, is the pilot symbol and $w(k)$ is the complex Additive White Gaussian Noise (AWGN) with zero mean and variance $N_0/2$. The received samples corresponding to information symbols can be expressed as:

$$Y[k] = \sqrt{\varepsilon_s} H(k) X(k) + w(k), \qquad k \in \Im_s \tag{43}$$

Where, ε_s is the transmitted power per information in symbol, and $\Im_s$ denotes the set of subcarriers on which the information symbols are transmitted. Suppose that the total number of subcarrier is N, and set of $\Im_P$ is $|\Im_P| = P$. For simplicity, it is assumed that the size of $\Im_s$ is $|\Im_s| = N - P$, although it is possible that $|\Im_s| < N - P$, when null subcarriers are inserted for spectrum shaping. Selecting information symbols from M-PSK constellation, it is also that $|X(k)| = 1, \forall k \in \Im_s$. The frequency selective channel is assumed to be Rayleigh-fading with channel impulse response $h = [h(0), \ldots\ldots, h(L-1)]^T$, where L denoting the number of taps; i.e. $h(l), \forall l \in [0.L-1]$ are uncorrelated complex Gaussian random variables with zero-mean. Channels are normalized so that $\sum_{l=0}^{L-1} \sigma_h^2(l) = 1$. Defining the LxN matrix by $[F]_{l,n} = \exp(j2\pi(l-1)(k-1)/N)$, and let f_n be a function of F. Then $H(k) = f_k^N h$, is a complex Gaussian variable with zero-mean and unit variance. The average signal-to-noise ratio (SNR) per pilot (information) symbol is $\varepsilon_P / N_0 (\varepsilon_s / N_0)$.

The AWGN variables $w(k)$ are assumed to be uncorrelated, $\forall k$. Suppose that the set of pilot subcarrier is given by $\Im_P = \{k_l\}_{l=1}^{P}$. Letting $H_P = [H(k_1),.....,H(k_P)]^T$ contains frequency response on pilot subcrriers and defining $F_P = [f_{k_1},....., f_{k_P}]$, the fast Fourier Trasform (FFT) pair can be related via: $H_P = F_P^N h$. Let the Px1 vector $Y = [Y(k_1),.....,Y(k_P)]^T$ consist of the received pilot samples per block, and define $X_P = [X(k_1),.....,X(k_P)]^T$ and $w = [w(k_1),.....,w(k_P)]^T$. Thus,

$$Y = \sqrt{\varepsilon_P} D(X_P) H_P + w = \sqrt{\varepsilon_P} D(X_P) F_P^N h + w \tag{44}$$

Given X_P and Y, the parameter h can be estimated from the above equation. While it may be possible to use pilot samples from different OFDM blocks to estimate the channel. However, this paper will rely on pilots from only one block to estimate the channel on a per block basis. This is particularly suitable for packet data transmission, where the receiver may receive different blocks with unknown delays.

3.3.1) Minimum mean square error estimation

With knowledge of channel statistics, channel estimation in MMSE may can be written as

$$\begin{aligned}
\hat{h} &= R_{yh}^{\mathcal{H}} R_{yy}^{-1} y \\
R_{yy} &= E[yy^{\mathcal{H}}] = \varepsilon_P D(X_P) F_P^{\mathcal{H}} R_{hh} F_P D^{\mathcal{H}}(X_P) + N_0 I_P \\
R_{yh} &= E[yh^{\mathcal{H}}] = \sqrt{\varepsilon_P} D(X_P) F_P^{\mathcal{H}} R_{hh} \\
R_{hh} &= E[hh^{\mathcal{H}}] = diag(\sigma_h^2(0),....,\sigma_h^2(L-1))
\end{aligned} \tag{45}$$

The channel estimator is given by $\in = h - \hat{h}$, which is Gaussian distributed with zero-mean, and covariance $R_{\in} = E[\in\in^{\mathcal{H}}] = (R_{hh}^{-1} + \varepsilon_p F_p F_P^{\mathcal{H}} / N_0)^{-1}$ where $\sigma_h^2(l) \neq 0, \forall l$ so that R_{hh} is invertible. The estimated channel frequency response of nth carrier can be obtained as: $\hat{H}(k) = f_k^{\mathcal{H}} \hat{h} = H(k) - \in(k)$, where $\in(k) = f_k^{\mathcal{H}} \in$ with $\in(k) \sim CN(0, \sigma_{e(k)}^2)$ and $\sigma_{e(k)}^2 = f_k^{\mathcal{H}} R_{\in} f_k$. The estimator $\hat{H}(k)$ is Gaussian distributed with zero mean. Since the orthogonality principle renders $\in$ uncorrelated h, $\in(k)$ and $\hat{H}(k)$ are uncorrelated.

3.3.3) Least square error estimation

In this case,

$$G = (\varepsilon_P F_P D^{\mathcal{H}}(X_P) D(X_P) F_P^{\mathcal{H}})^{-1} (\sqrt{\varepsilon_P} D(X_P) F_P^{\mathcal{H}})^{\mathcal{H}} \tag{46}$$

then the least square error (LSE) estimate of channel impulse response is given by

$$\hat{h} = Gy = h + \eta \tag{47}$$

Where, $\eta = Gw$. Using the fact that $D^{\mathcal{H}}(X_P) D(X_P) = I_P$, it follows readily that $\eta \sim CN(0, (F_P F_P^{\mathcal{H}})^{-1} N_0 / \varepsilon_P)$. The estimated channel frequency response on the k^{th} subcarrier can be obtained as:

$$\hat{H}(k) = f_k^H \hat{h} = H(k) + v(k) \tag{48}$$

Where $v(n) \sim CN(0, \sigma_{v(k)}^2)$ with $\sigma_{v(k)}^2 = f_k^{\mathcal{H}} (F_P F_P^{\mathcal{H}})^{-1} f_k N_0 / \varepsilon_P$.

4. IMPLEMENTATION

In this paper, an OFDM system is implemented using Matlab as shown in Figure 5. The aim is to measure the performance of simulated OFDM system under different channel conditions, and to allow for different OFDM configurations to be tested.

The system, is designed using the following commands and functions in Matlab.

- **Random data generation:** The input random data is generated by *randn*() function in Matlab.
- **Serial to parallel conversion:** The input serial data stream is formatted into the word size required for transmission, e.g. 2bit/word for QPSK, and shifted into a parallel format using the command *reshape*(). The data is then transmitted in parallel by assigning each data word to one carrier in the transmission.
- **Modulating data:** The data to be transmitted on each carrier is modulated into a QAM and M-ary PSK format.

- **Inverse Fourier Transform:** The purpose of Inverse Fourier Transform is to find the corresponding time waveform. This is done using the command *IFFT* in Matlab. The guard period is then added to the start of each symbol.
- **Channel model:** A channel model is then applied to the transmitted signal. In this channel the signal-to-noise is varied and multipath path is then introduced. The signal to noise ratio is set by adding a known amount of white noise to the transmitted signal. The channels used are described below:

(1) ***Block type pilot arrangement:*** In this scheme, 16-QAM modulation scheme is used for a 64-subcarrier OFDM system, with a two ray multipath channel. The channel impulse response $h(t)$ is a time limited pulse train in the form of

$$h(t) = \sum_m \alpha_m \delta(t - \tau_m T_s) \tag{49}$$

Where, the amplitudes α_m are complex valued, τ_m is m_{th} path delay and T_s is sampling time. Guard time T_G is taken such that $0 \le \tau_m T_s \le T_G$.The above continuous time relationship can be represented as a discrete time version having discrete channel impulse response $h(n)$ as:

$$h(n) = \sum_m \alpha_m e^{-j\frac{\pi}{N}(n+(N-1)\tau_m)} \frac{\sin(\pi\tau_m)}{\sin(\frac{\pi}{N}(\tau_m - n))} \tag{50}$$

In the simulation for block type pilot arrangement, two ray multipath channels have been taken as:

$$h(t) = \delta(t - 0.5T_s) + \delta(t - 3.5T_s) \tag{51}$$

(2) ***Comb type pilot arrangement:*** In comb type pilot arrangement, Rayleigh-fading channel is considered, with channel impulse response $h(l) = [h(0),....,h(L-1)]^T$. Here, L=40 is the number of taps and are uncorrelated complex Gaussian random variables with zero mean.

5. RESULT & DISCUSSION

In the simulations performed in this paper, a system operating with a bandwidth of 500 kHz, divided into 64 tones with total symbol period of 138µs, of which 10µs is a cyclic prefix is considered. Sampling is performed with a 500 kHz rate. A symbol thus consists of 69 samples, five of which are contained in the cyclic prefix. 10,000 channels are randomized per average SNR. The channel is two ray channel: $h(t) = \delta(t - 0.5T_s) + \delta(t - 3.5T_s)$. Figure 6 demonstrates the mean square error of channel estimation at different SNRs in dB. As SNR increases mean square error decreases for both LSE and MMSE. Figure 7 shows SNR versus Symbol Error Rate (SER). As SNR increases Symbol Error Rate decreases for both cases. For a given SNR, MMSE estimator shows better performance than LSE estimator. The complexity of MMSE estimators will be larger than LSE estimators but give better performance in comparison to LSE. It should be noticed that MMSE estimators have been derived under assumption of known channel correlation and noise variance. In practice these quantities R_{hh} and σ_n^2 □ , are either taken as fixed or estimated, possibly in an adaptive way. This will increase the estimator complexity, but also improve its performance over LSE estimators.

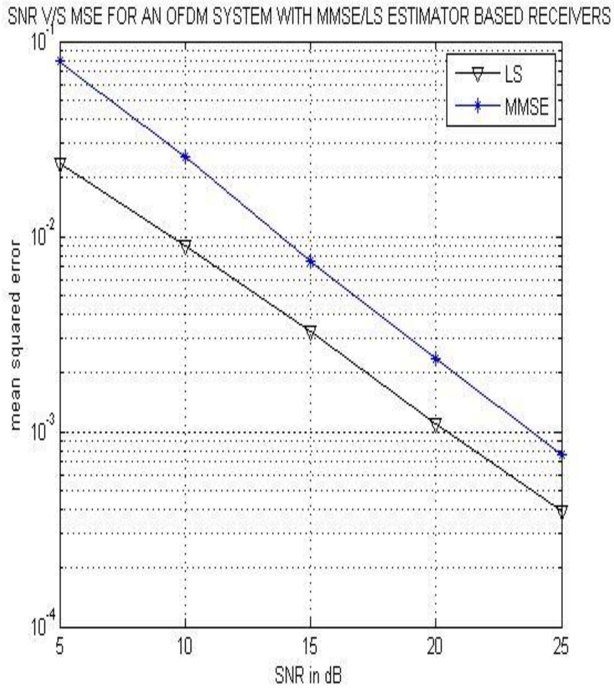


Figure 6: Mean Square Error for LSE and MMSE estimators at different SNRs

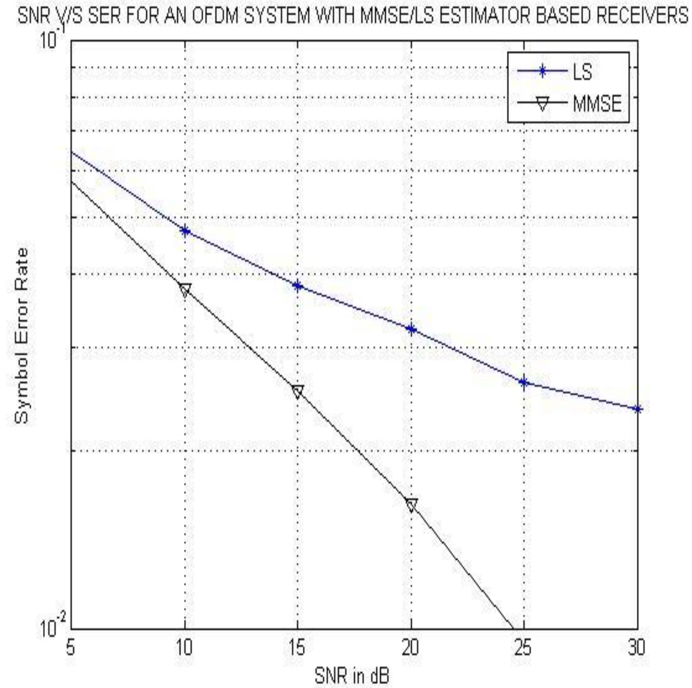


Figure 7: SER for LSE and MMSE estimators at different SNRs

5.1) <u>Simulation results for comb type pilot arrangement</u>

For comb type pilot arrangement, an OFDM system with N=1024 subcarriers is considered. As mentioned above, the frequency selective Rayleigh channel has L=40 zero-mean uncorrelated complex Gaussian random taps. The spacing between pilots are taken as 4. So the number of pilots are 256 and number of information symbols are 768. In the simulation, PSK modulation is considered. Figure 8 and Figure 9 demonstrate Symbol Error Rate (SER) performance (SNR versus SER) for different modulations in MMSE and LSE estimators respectively. It shows that as SNR increases the symbol error rate decreases, as theoretically expected.

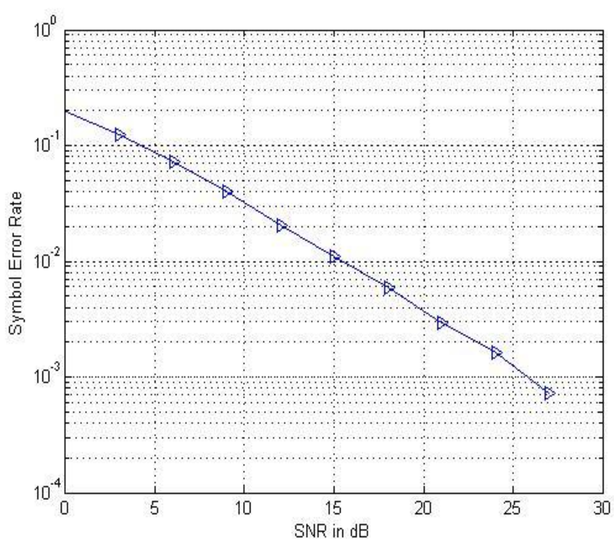


Figure 8: SER (MMSE channel estimation) for M-PSK modulation for different SNRs.

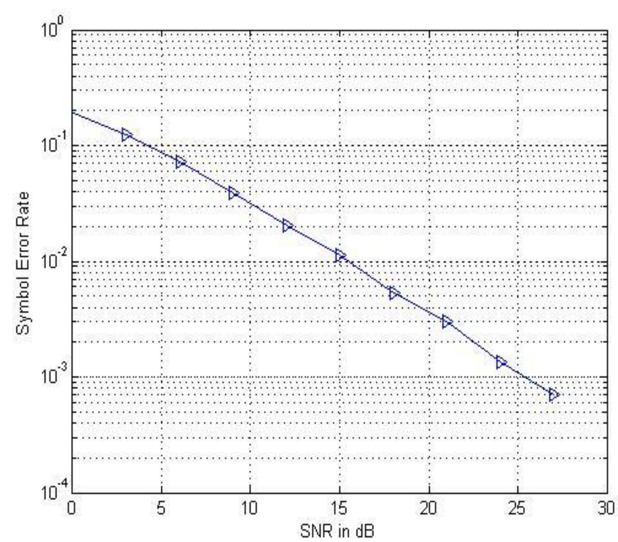
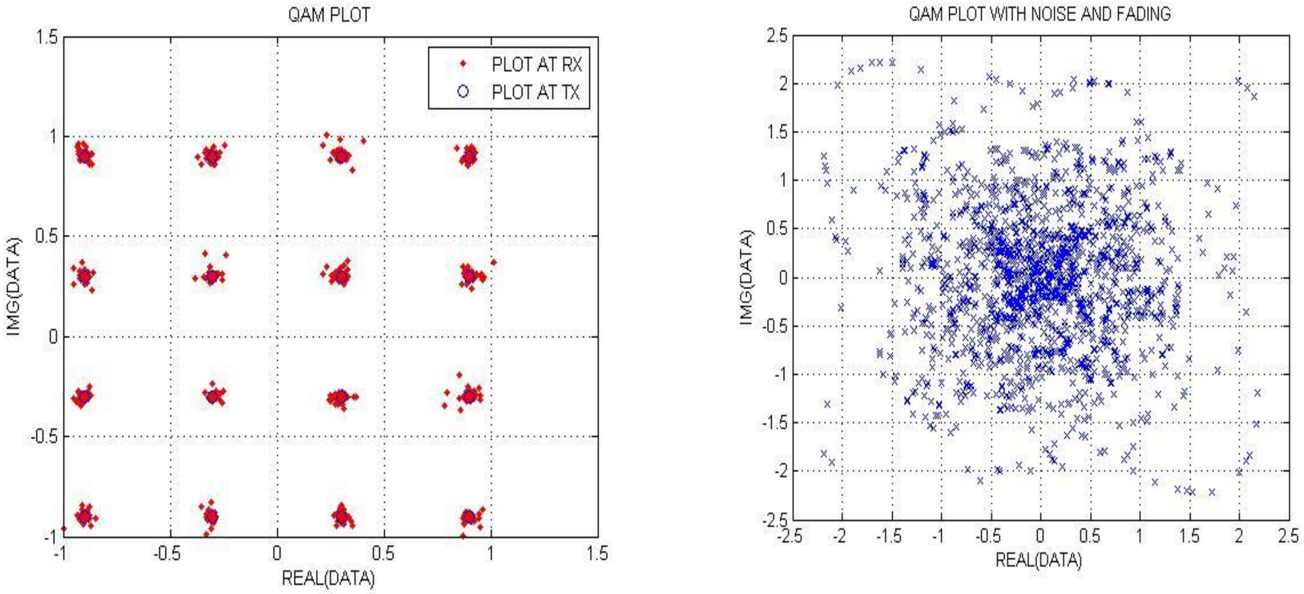


Figure 9: Bit Error Rate versus SNR (LSE channel estimation) for M-PSK modulation

From Figures 8 and 9, the MMSE estimators have less Symbol Error Rate than LSE estimators in low SNRs. But at higher SNRs, both will have nearly equal performance. At lower SNRs noise is the prominent factor. In this case, MMSE estimator works better. But at higher SNRs, it is better to go for LSE estimator because of its simplicity where noise is less effective

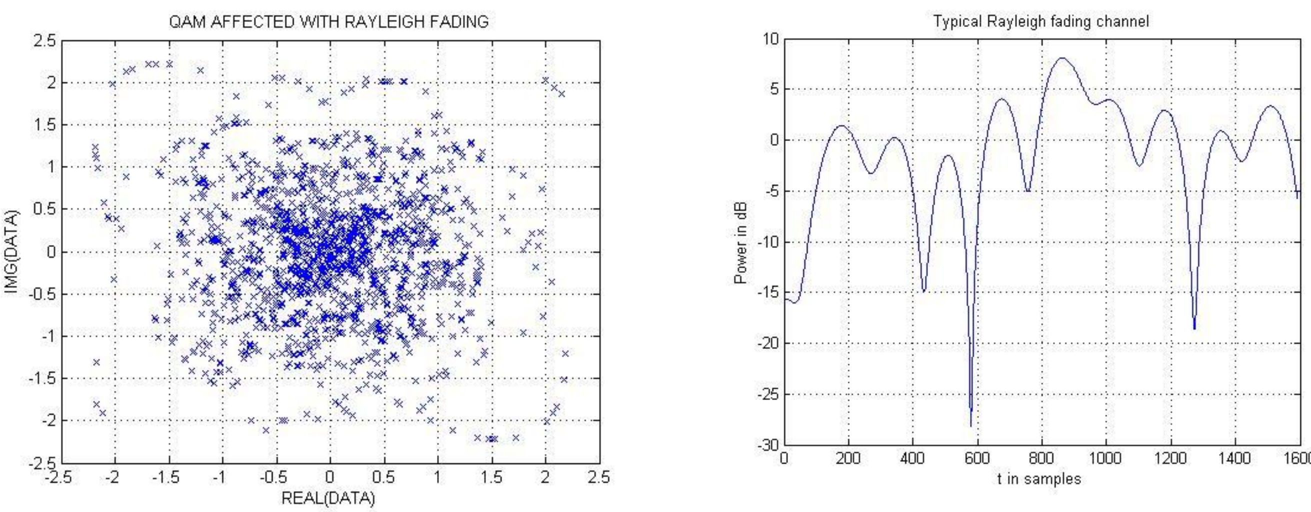
5.2) Simulation results for interpolation techniques

Figures 10 and 11 shows the results for various interpolation techniques used in the simulation. Figure 10 shows the simulated QAM signal and the effect of noise and Rayleigh Fading channel. The FFT, spline, linear and cubic interpolation interpolation techniques are applied using the LSE estimation. It is found that spline interpolation having better performance than linear and other interpolation techniques (Figure 12).



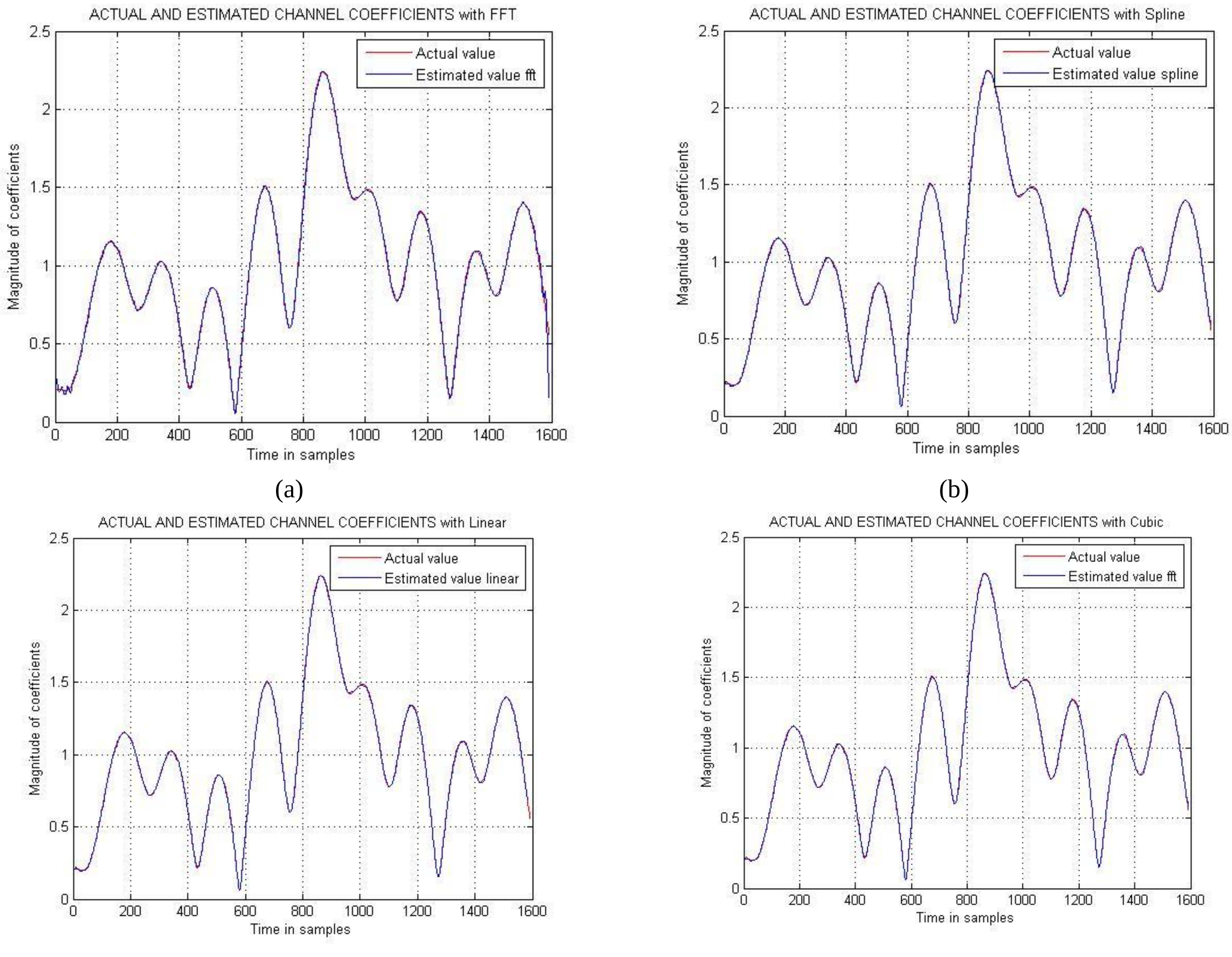
(a) (b)

Figure 10: (a) Simulated QAM signal, (b) QAM plot with noise and fading,



(c) (d)

Figure 10: (c) QAM affected with Rayleigh Fading, and (d) a typical example of Rayleigh Fading channel.



(a) (b) (c) (d)

Figure 11: (a) Results for FFT interpolation, (b) results for spline interpolation, (c) results for linear interpolation, and (d) results for cubic interpolation.

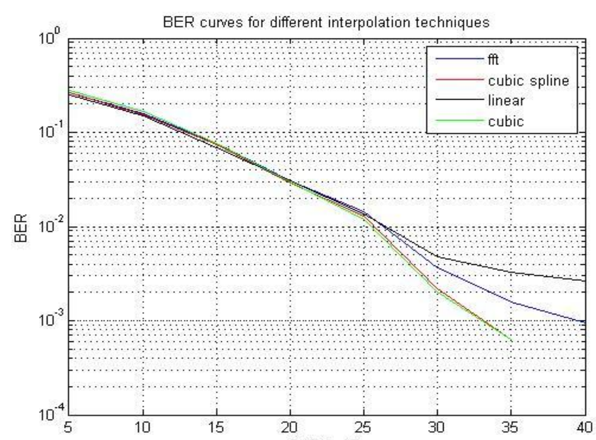


Figure 12: BER curves for different FFT, spine, linear and cubic interpolation techniques.

6. CONCLUSIONS

In this paper, the performance of two types of estimators (LSE and MMSE estimators) has been theoretically and experimentally evaluated for both block type and comb type pilot arrangements. The estimators in this study can be used to efficiently estimate the channel in an OFDM system, given certain knowledge about channel statistics. The MMSE estimators assume a priori knowledge of noise variance and channel covariance. Moreover, its complexity is large compare to the LSE estimator. For high SNRs, the LSE estimator is both simple and adequate. The MMSE estimator has good performance but high complexity. The LSE estimator has low complexity, but its performance is not as good as that MMSE estimator basically at low SNRs. In comparison between block and comb type pilot arrangement, block type of pilot arrangement is suitable to use for slow fading channel, where channel impulse response is not changing very fast. So the channel estimated in one block of OFDM symbols through pilot carriers can be used in next block for recovery the data which are degraded by the channel.

Comb type pilot arrangement is suitable to use for fast fading channel where the channel impulse response is changing very fast, even if one OFDM block is present. So, comb type of pilot arrangement is not suitable in this case. Both data and pilot carriers in one block of OFDM symbols are used. Pilot carriers are used to estimate the channel impulse response. The estimated channel can be used to get back the data sent by transmitter certainly with some error. In the simulation, 1024 number of carriers in one OFDM block is used, in which one fourth are used for pilot carriers and rest are of data carriers. BER for different SNR conditions for M-PSK signalling is calculated. The performance of LSE with MMSE estimator is also investigated. MMSE estimation is better that LSE estimator in low SNRs; whereas at high SNRs, performance of LSE estimator approaches to MMSE estimator. Various interpolation techniques for channel estimation are also used. It is found that higher order interpolation technique (spline) is giving better performance than lower order interpolation technique (linear).

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