

OFDM Notes

EE 442 – Spring 2017

Lecture 16

Orthogonal Frequency Division Multiplexing (OFDM) is a digital multi-carrier modulation technique extending the concept of single subcarrier modulation by using multiple sub-carriers over the channel.

Rather than transmit a high-rate stream of data with a single carrier, OFDM makes use of a large number of closely spaced orthogonal sub-carriers that are transmitted in parallel.

Each sub-carrier is modulated with a conventional digital modulation scheme (such as QPSK, 16QAM, etc.) at a lower symbol rate. However, the combination of many sub-carriers enables data rates similar to conventional single-carrier modulation schemes using equivalent bandwidths.

OFDM Benefits

Q: What are the benefits of using OFDM?

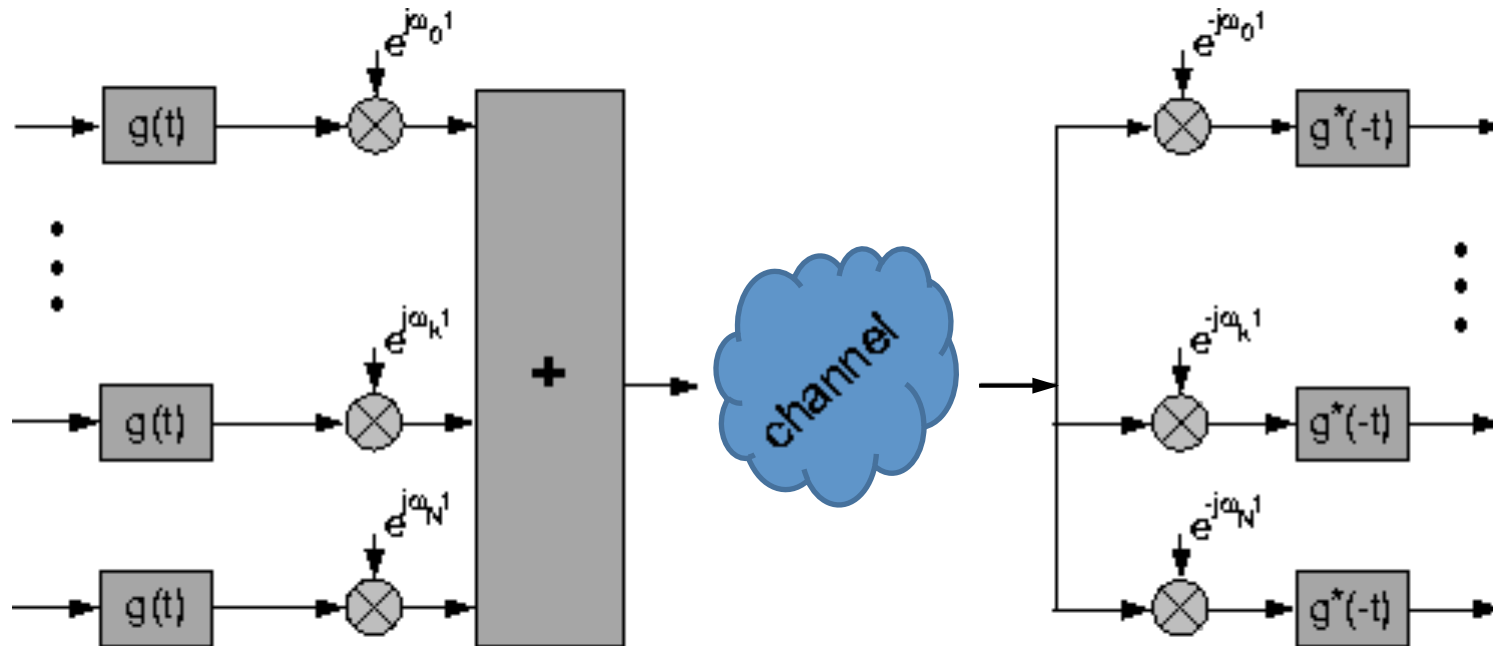
A: The first reason is **spectral efficiency**, also called **bandwidth efficiency**. What that term really means is that you can transmit more data faster in a given bandwidth in the presence of noise. The measure of spectral efficiency is bits per second per Hertz, or bps/Hz.

For a given chunk of spectrum space, different modulation methods will give you widely varying maximum data rates for a given bit error rate (BER) and noise level. Simple digital modulation methods like amplitude shift keying (**ASK**) and frequency shift keying (**FSK**) are simple but don't give the best BER performance. **BPSK** and **QPSK** do much better. **QAM** is very good but more susceptible to noise and low signal levels issues. Code division multiple access (**CDMA**) methods are even better performance.

But none is better than **OFDM** with respect to attaining the maximum data capacity out of a given channel bandwidth. It approaches the Shannon limit defining channel capacity C in bits per second (bps).

Multicarrier Communication Systems (Uses Multiple Oscillators)

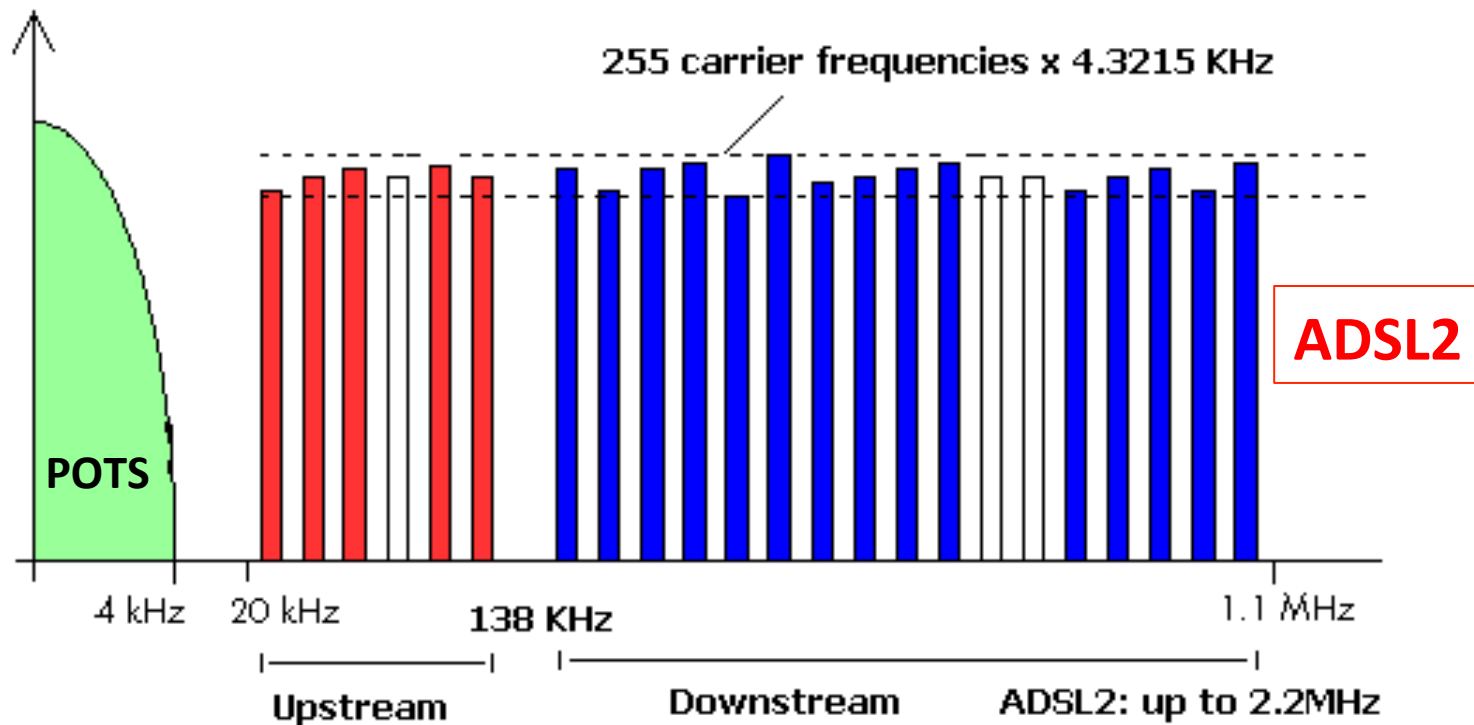
What is the problem with this architecture?



Answer: It is nearly impossible to keep multiple oscillators in synchronization with each other. And it is difficult to have hundreds of oscillators in a practical system.

Discrete Multitone (*aka* Frequency Division Multiplexing)

The basic idea of Discrete Multitone (DMT) is to split the available bandwidth into a large number of sub-channels. ADSL2 is an example application of DMT.

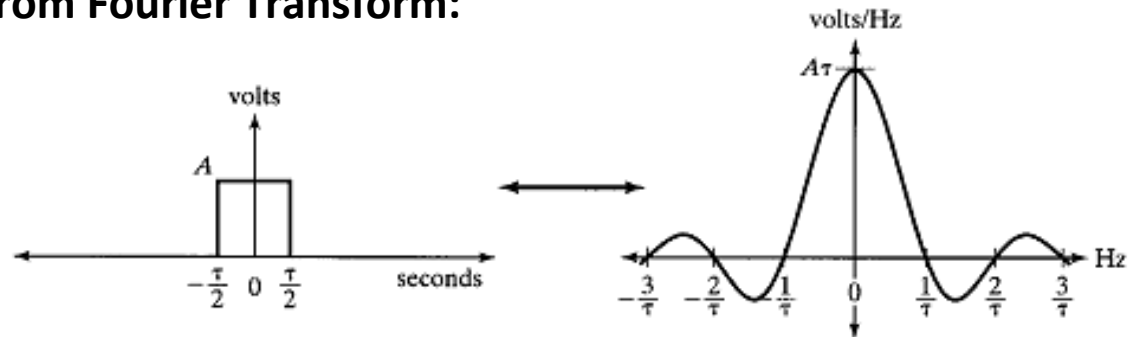


DMT uses available frequencies on the telephone line and splits them into 256/512 equal sized frequency bins of 4.3125 kHz each.

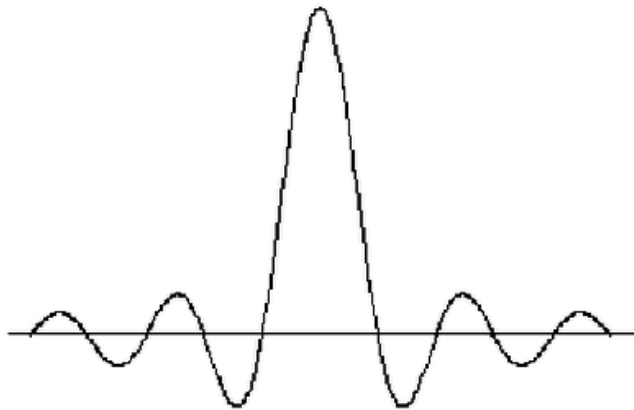
<http://educypedia.karadimov.info/computer/modemadsl.htm>

Can we get rid of the guard-bands in DMT?

Recall from Fourier Transform:

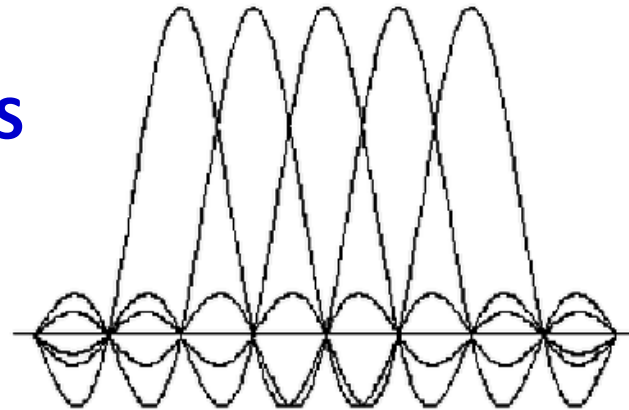


Spectrum of one OFDM Sub-channel

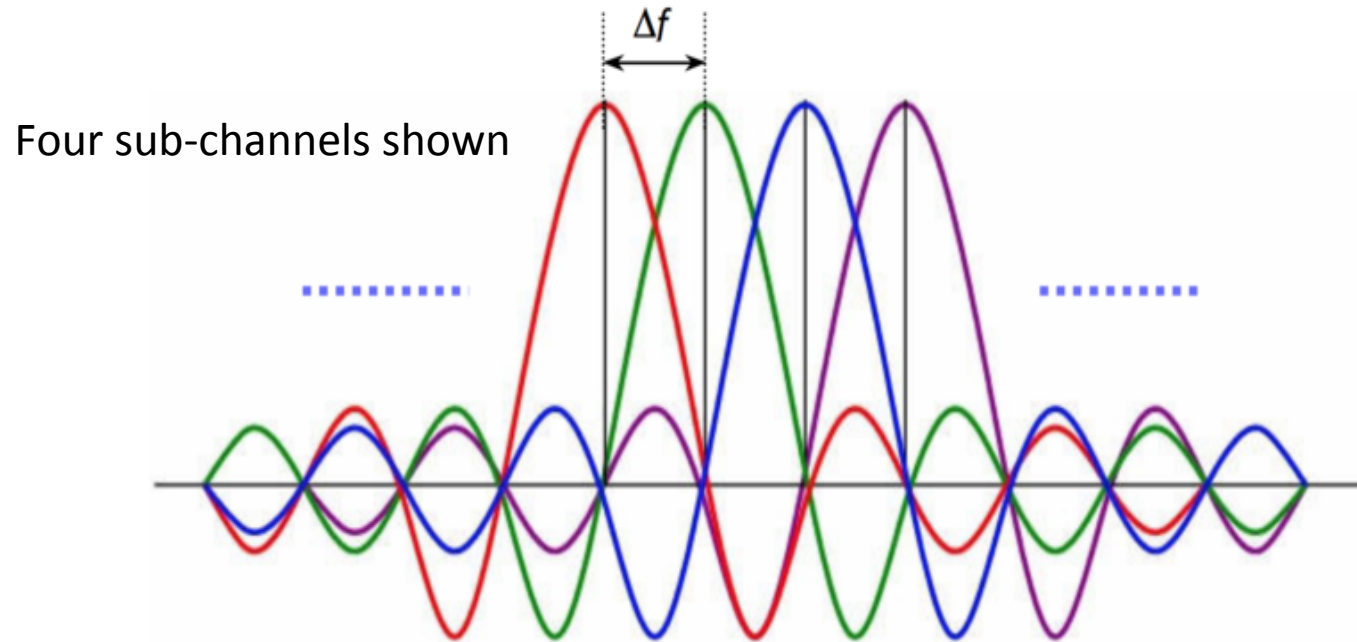


YES

OFDM Spectrum

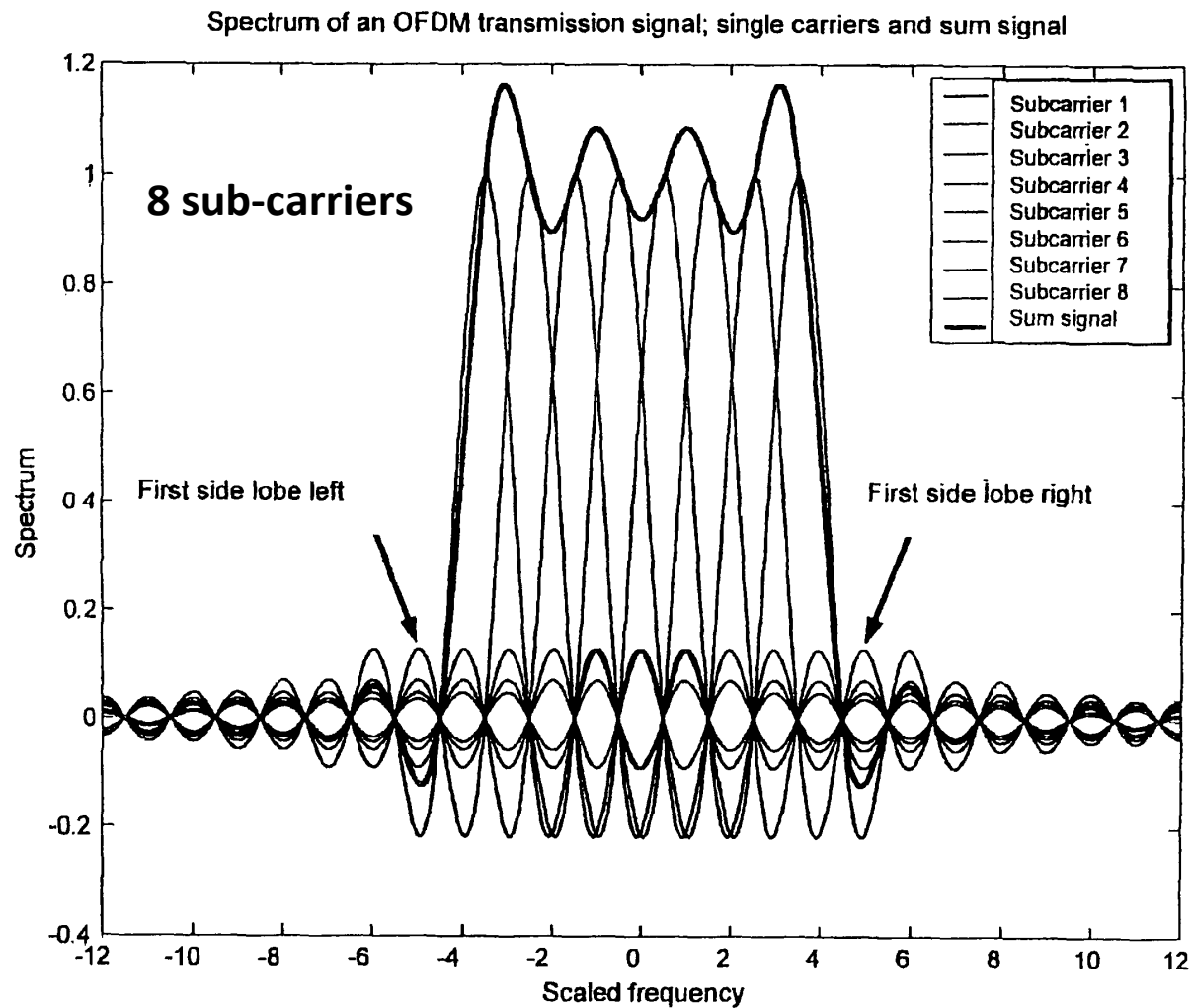


OFDM Packs Sub-channels Closer Together



*OFDM divides each channel into many narrower subcarriers. The spacing is such that these subcarriers are **orthogonal** so they don't interfere with one another in spite of the lack of guard bands between them. This results from the subcarrier spacing equal to the reciprocal of symbol time. All subcarriers have a complete number of sine wave cycles that sum to zero upon demodulation.*

Spectrum of OFDM Signal Transmission



In OFDM the Sub-Carriers are Orthogonal

ORTHOGONALITY

- **Two conditions must be considered for the orthogonality between the subcarriers.**
 - Each subcarrier has exactly an integer number of cycles in the FFT interval.
 - The number of cycles between adjacent subcarriers differs by exactly one.

OFDM Summary

Parallel data streams (all at lower data rates)

⇒ large number of subcarriers without guard bands

Data coding based upon amplitude modulation

Multiple subcarriers are multiplexed into one symbol

Subcarriers are computed using IFFT (*i.e.*, Inverse Fast Fourier Transform)

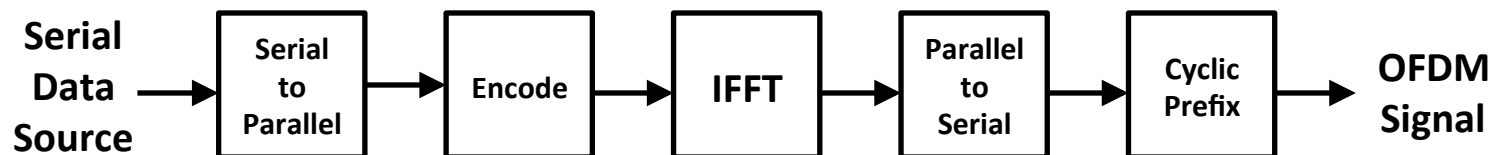
They are spaced at precise frequencies and phases

This spacing selection provides orthogonality

Orthogonality Principle results in close packing of subcarriers

Provides for **multi-user diversity**

Synchronization using pilot signals



What Makes OFDM Possible?

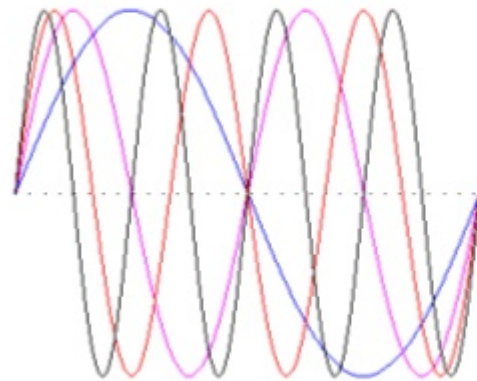
Q: How is OFDM implemented in the real world?

A: OFDM is accomplished using digital signal processing (DSP). You can program IFFT and FFT math functions on any fast PC, but it is usually done using a special DSP IC, or an appropriately programmed FPGA, or hardwired digital logic. With today's super-fast chips, even complex math routines like the FFT are relatively easy to implement. In brief, you can put it all on a single IC chip.

What Does Orthogonality Mean?

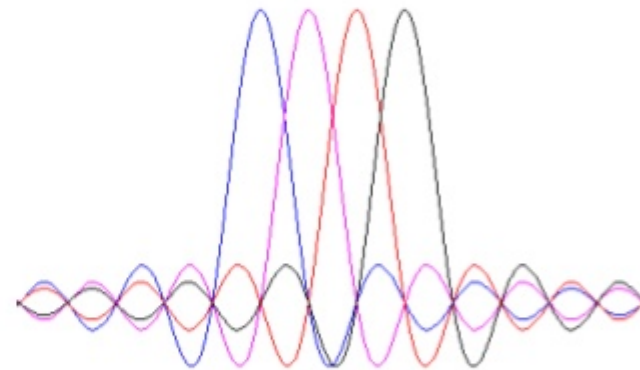
ORTHOGONALITY

Time domain



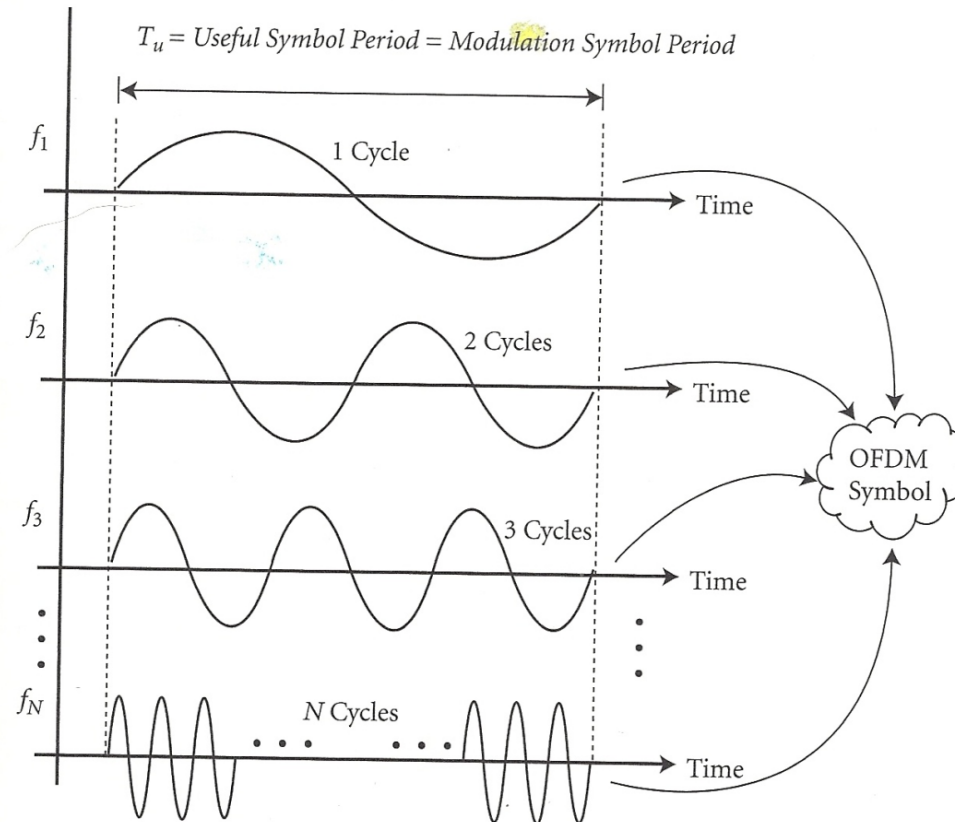
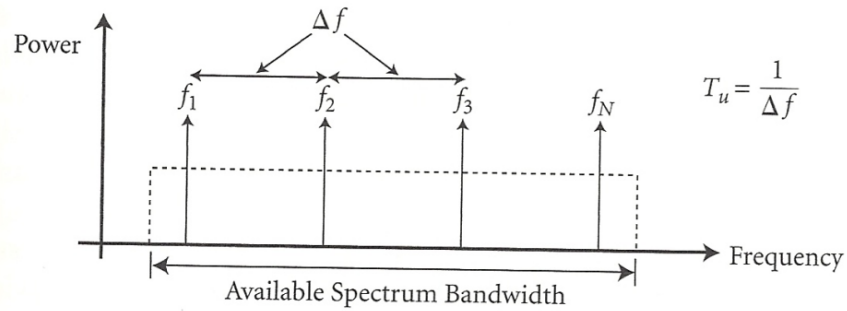
Example of four subcarriers within one OFDM symbol

Frequency domain



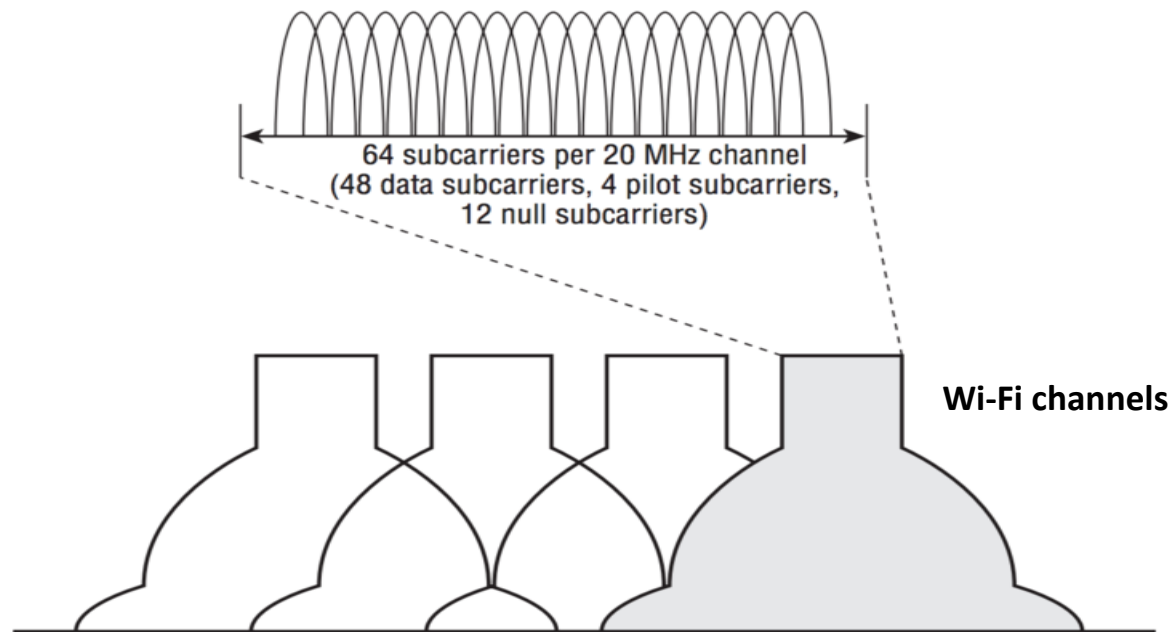
Spectra of individual subcarriers

Subcarrier Orthogonality



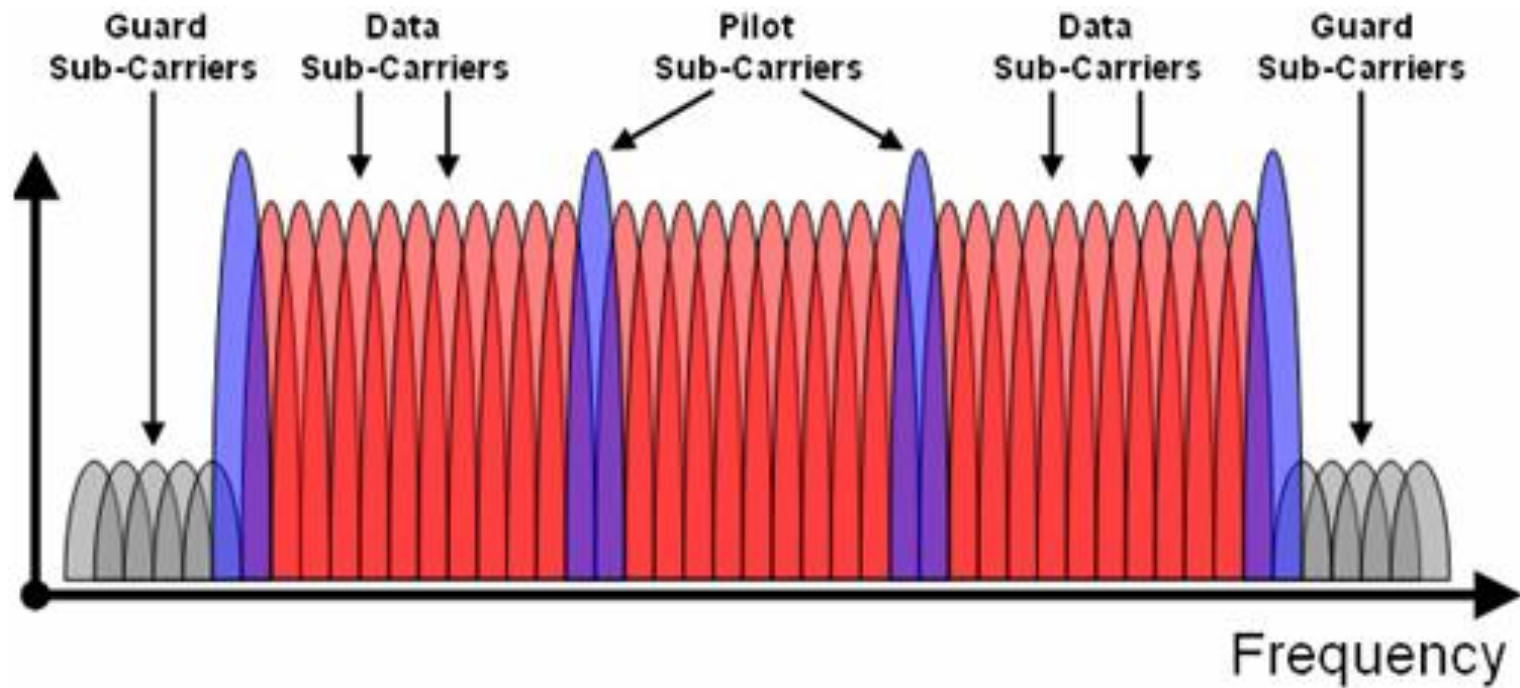
Example: OFDM Subcarriers in 802.11a/g Wi-Fi

Each 20 MHz channel, whether it's 802.11a/g/n/ac, is composed of 64 subcarriers spaced **312.5 KHz** apart. This spacing is chosen because we use 64-point FFT sampling. 802.11a/g use 48 subcarriers for data, 4 for pilot, and 12 as null subcarriers. 802.11n/ac use 52 subcarriers for data, 4 for pilot, and 8 as null.

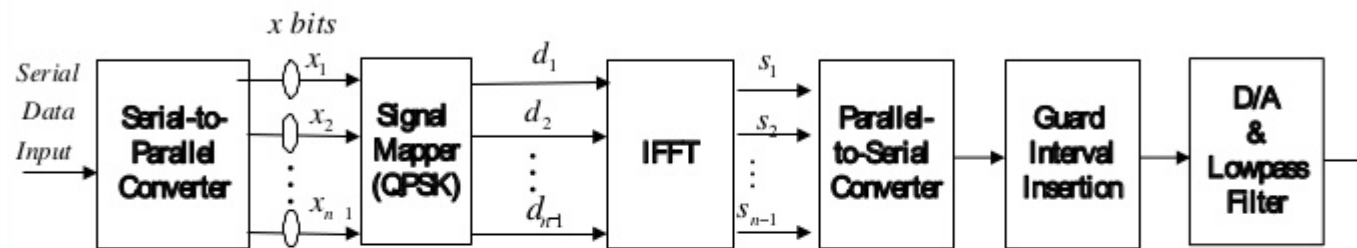


A standard Wi-Fi symbol is $4\mu\text{s}$, composed of a $3.2\mu\text{s}$ IFFT useful symbol duration plus $0.8\mu\text{s}$ guard interval. (When using a shorter guard interval of $0.4\mu\text{s}$ then the total symbol time is reduced to $3.6\mu\text{s}$).

OFDM Sub-Carriers with Guard-bands & Pilot Signals



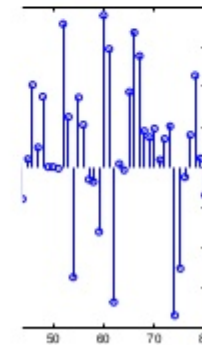
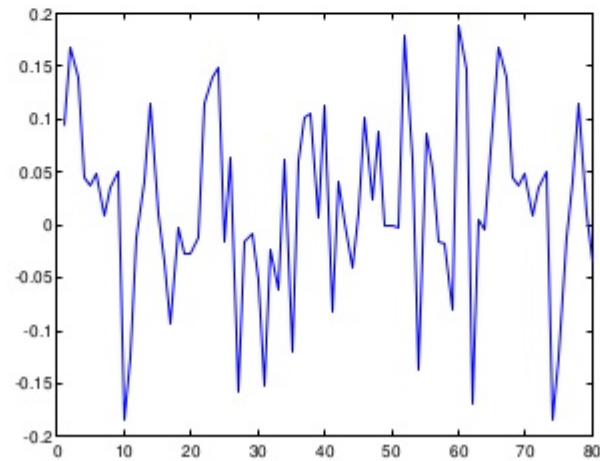
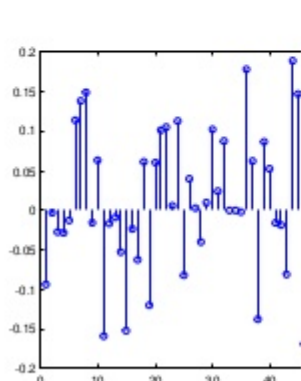
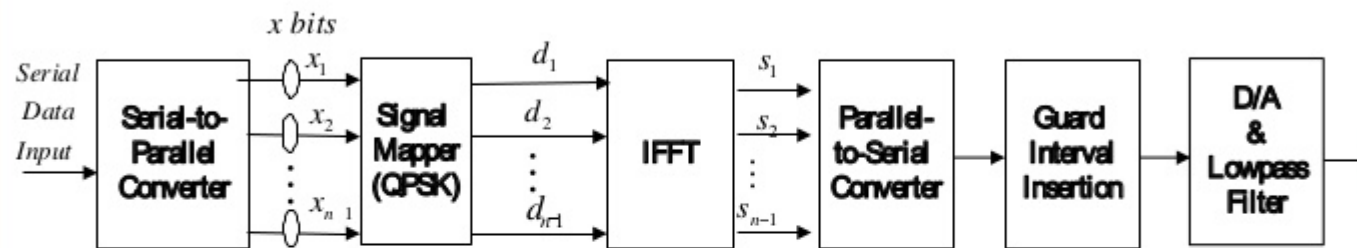
OQDM IMPLEMENTATION



$x = [1, 0, 1, 1, 0, 0, \dots]$

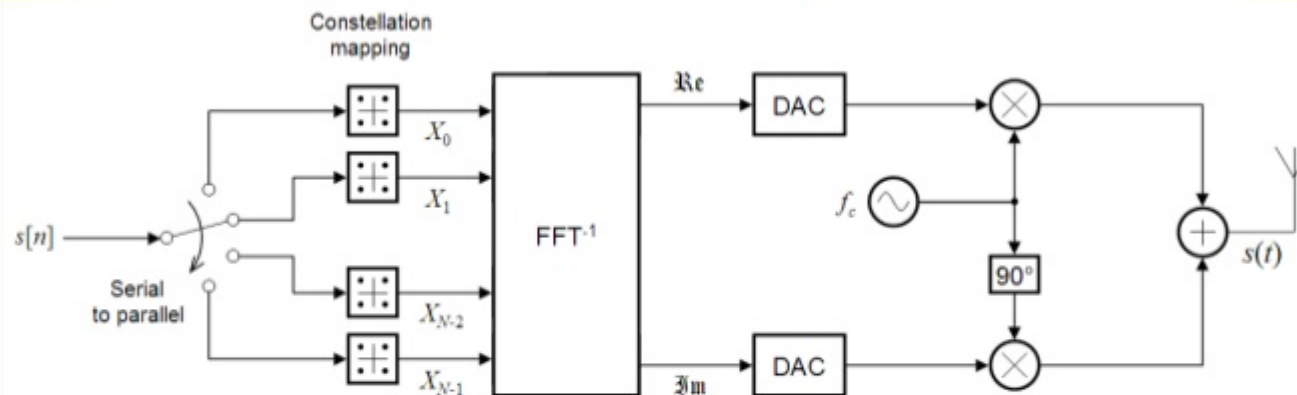
$\longrightarrow x_1 = [1, 0]$	$\longrightarrow d_1 = [-1]$
$\longrightarrow x_2 = [1, 1]$	$\longrightarrow d_2 = [-i]$
$\longrightarrow x_3 = [0, 0]$	$\longrightarrow d_3 = [1]$
$\longrightarrow \dots$	$\longrightarrow \dots$

OFDM IMPLEMENTATION



OFDM System Model

Transmitter
Receiver



Serial to parallel conversion

- Series of bits divided into N parallel frames of size F
- N - # of subchannels, F - # bits per symbol = 4

$$\sum_{l=1}^F b_l = B_f$$

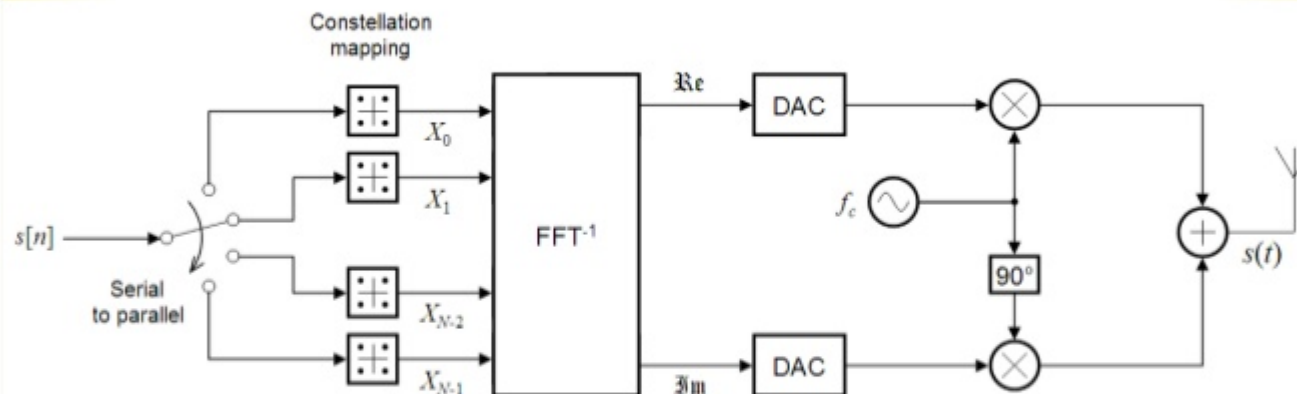
QPSK modulation

- Differ signal by phase to transfer information
- X_n - Modulated symbol

$$s[n] = \cos \left[2\pi f n + \frac{\pi}{4} (2m - 1) \right], m = [1 \ 4]$$

OFDM System Model

Transmitter
Receiver



Inverse Fast Fourier Transform

- Creates N carriers that are all **orthogonal**

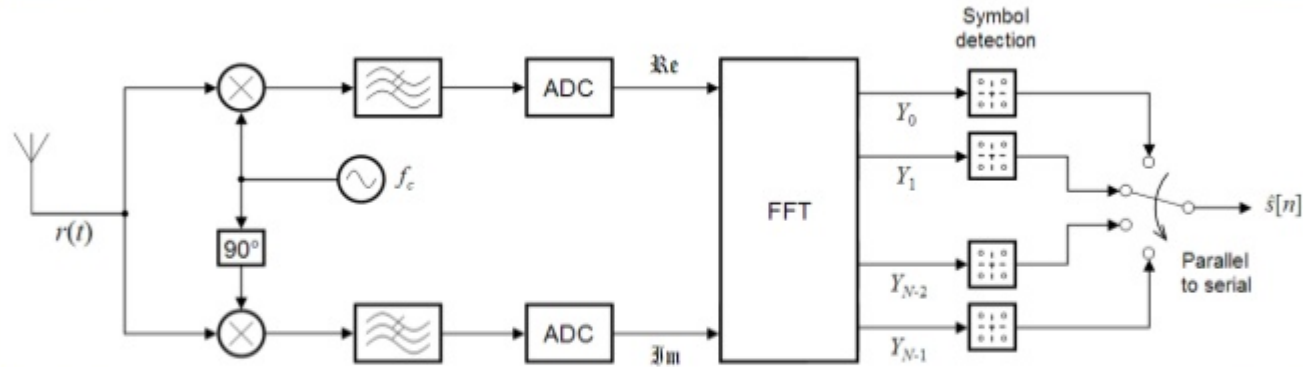
$$x_p[n] = \frac{1}{N} \sum_{k=0}^{N-1} X_p[k] e^{j(2\pi/N)kn}$$

DAC

- Integrated circuit that converts the digital signal to analog signal

OFDM System Model

Transmitter
Receiver



Low Pass Filter

- Eliminate unwanted signals

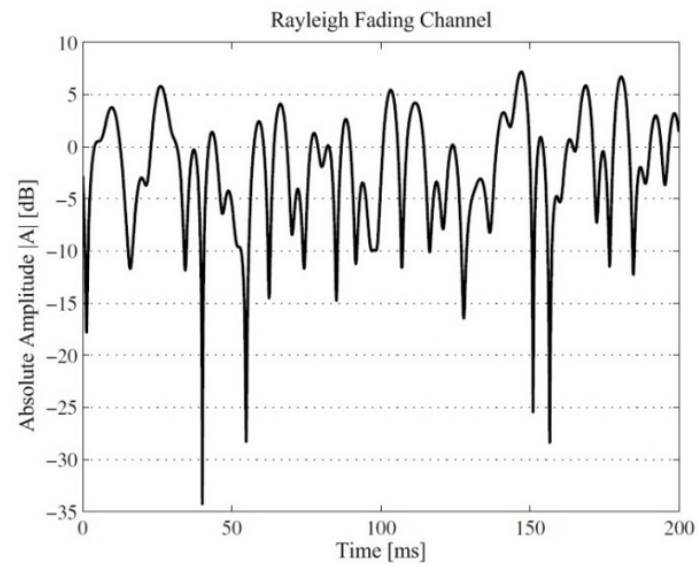
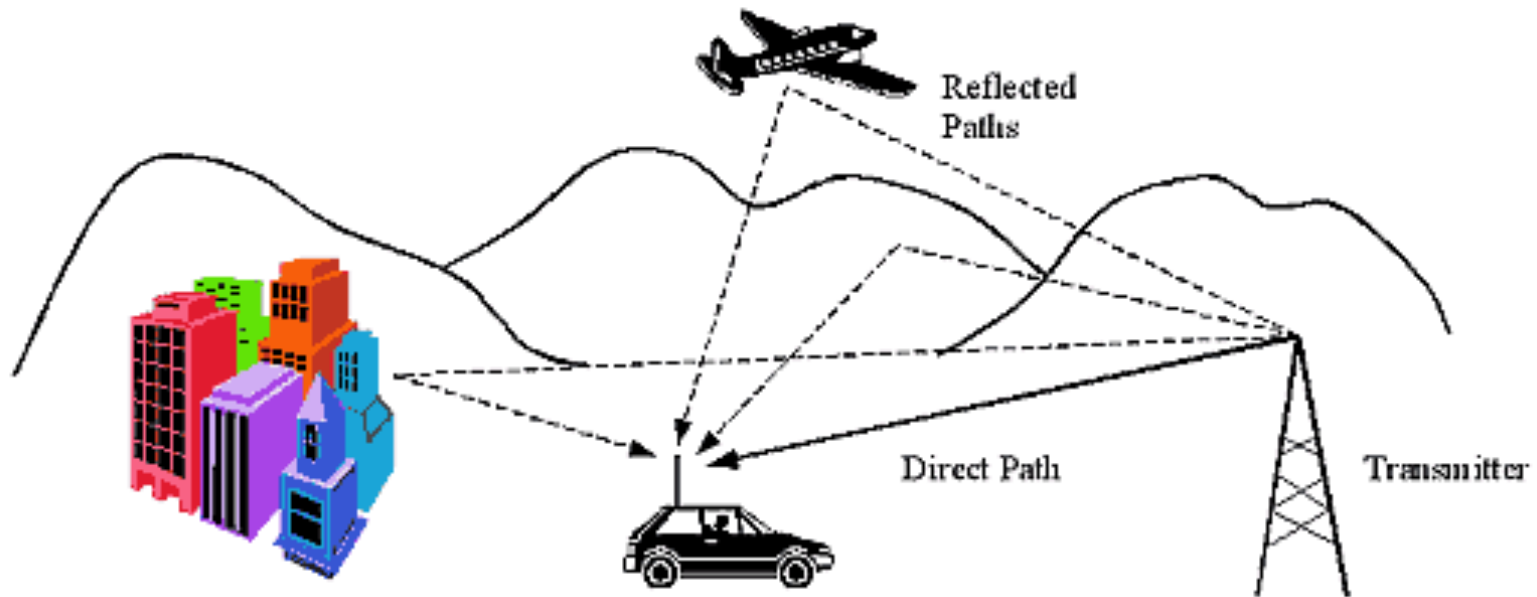
ADC

- Integrated circuit that converts the analog signal to digital signal

Fast Fourier Transform

$$X_p[k] = \sum_{n=0}^{N-1} x_p[n] e^{-j(2\pi/N)kn}$$

The Multipath Problem in Wireless

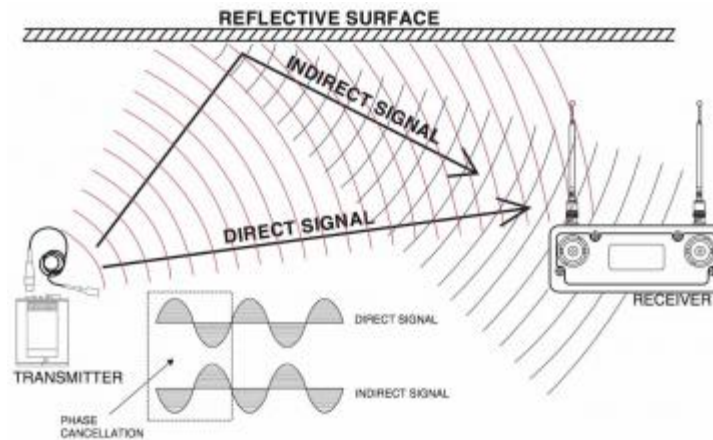


Path Losses and Fading

In wireless telecommunications, **multipath** is the propagation phenomenon that results in radio signals reaching the receiving antenna by two or more paths.

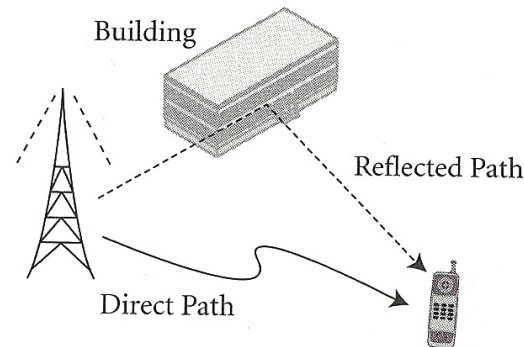
Causes of multipath include atmospheric ducting, ionospheric reflection and refraction, and reflection from water bodies and terrestrial objects such as mountains and buildings. Path losses generally vary inversely with distance, namely, $\sim 1/d^n$, where n is approximately 2, but can be as high as $n = 4$.

Multipath causes multipath interference including constructive and destructive interference, and phase shifting of the signal. Destructive interference causes fading.

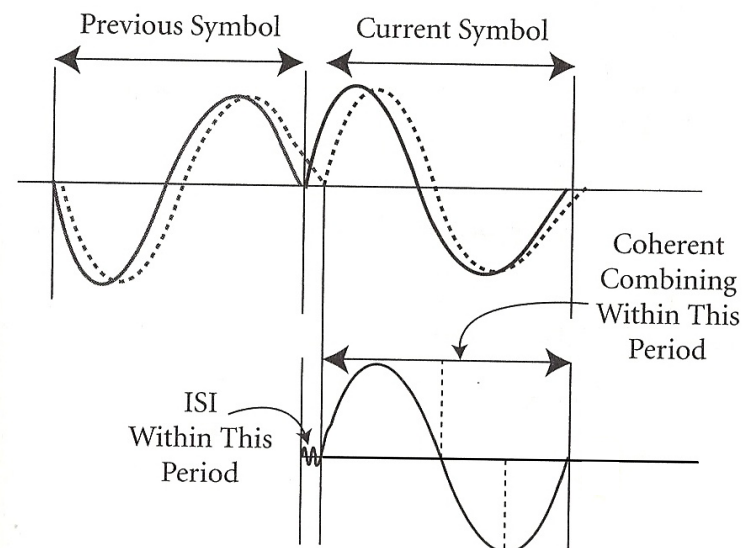


OFDM Performs To Reduce the Problem of Fading

Multipath destroys orthogonality. The figure below shows two copies of the signal (direct path and reflected path with time difference).



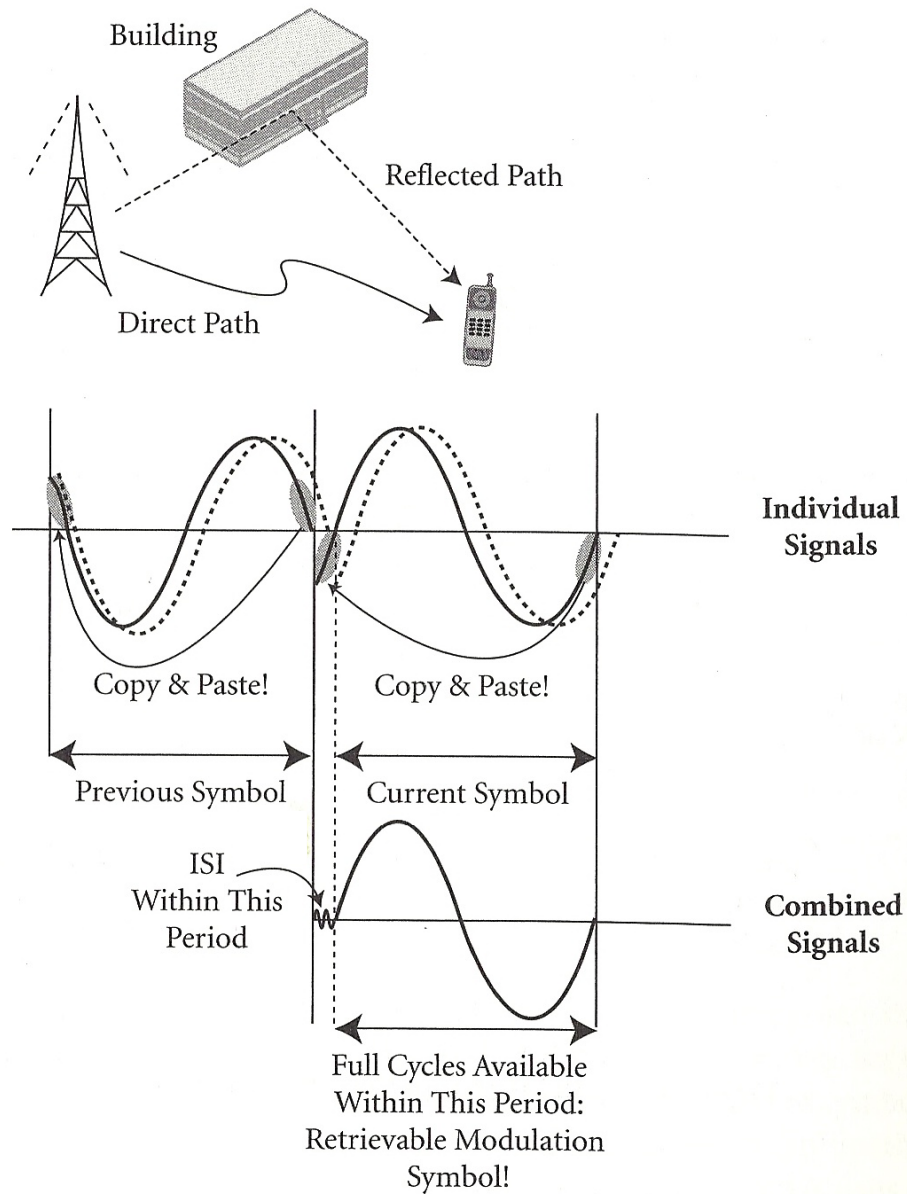
Time delay is evident



**Individual
Signals**

**Combined
Signals**

Adding a Cyclic Prefix



GUARD INTERVAL AND CYCLIC EXTENSION

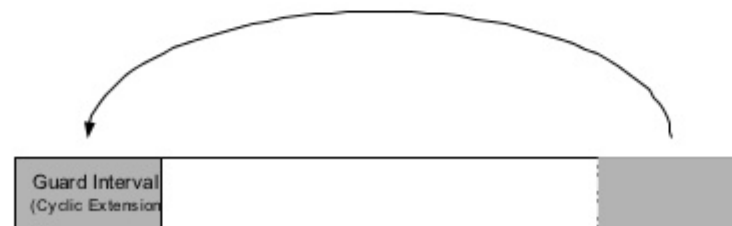
- For the purpose to eliminate the effect of ISI, the guard interval could consist of no signals at all.
- Guard interval (or cyclic extension) is used in OFDM systems to combat against multipath fading.

$$\begin{aligned} T_g &: \text{guard interval} \\ T_{\text{delay-spread}} &: \text{multi path delay spread} \\ T_g &> T_{\text{delay-spread}} \end{aligned}$$

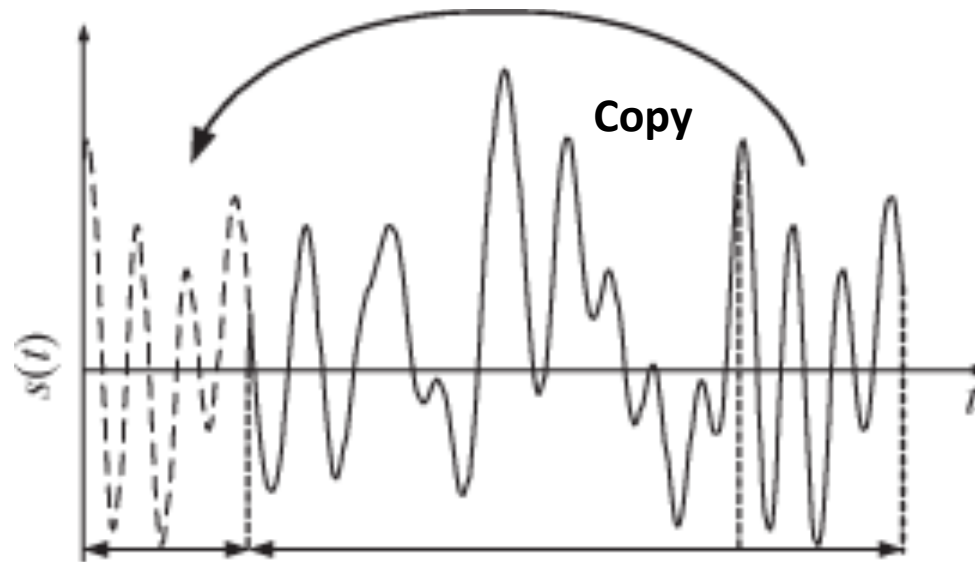
- In that case, however, the problem of inter-carrier interference (ICI) would arise.
- The reason is that there is no integer number of cycles difference between subcarriers within the FFT interval.

GUARD INTERVAL AND CYCLIC EXTENSION

- To eliminate ICI, the OFDM symbol is cyclically extended in the guard interval.
- This ensures that delayed replicas of the OFDM symbol always have an integer number of cycles within the FFT interval, as long as the delay is smaller than the guard interval.

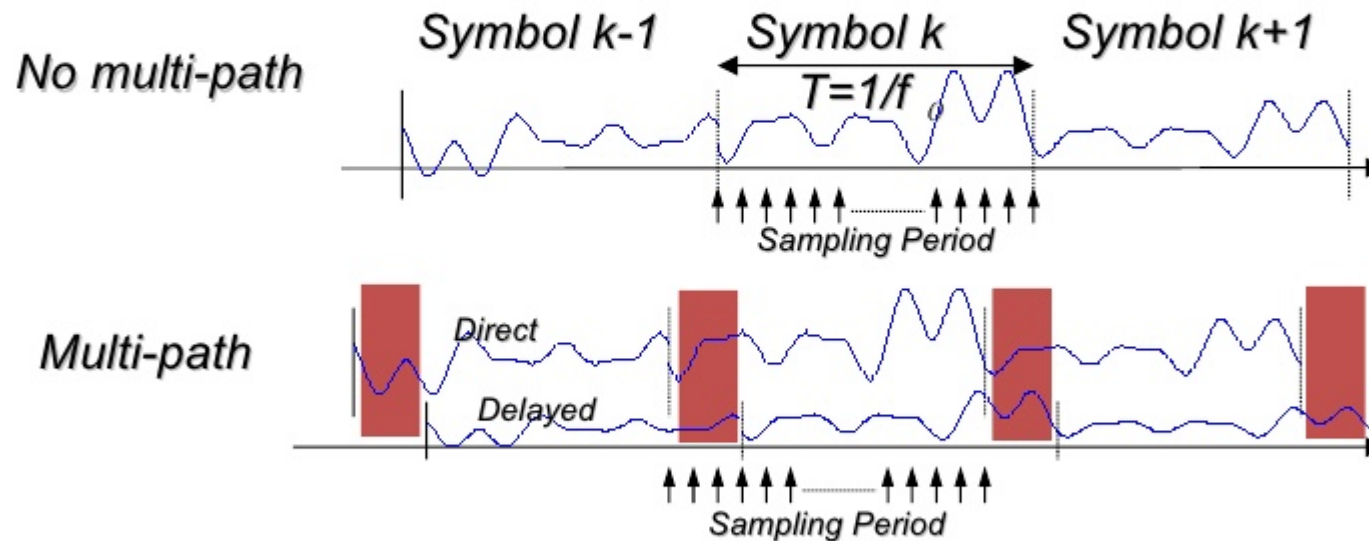


Cyclic Prefix



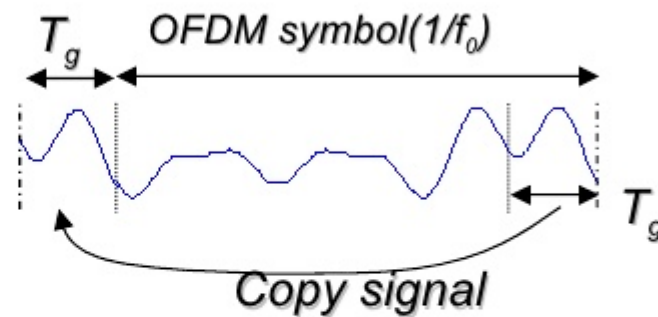
A guard-time interval is used to increase the robustness of OFDM. A cyclic prefix is formed by part of the OFDM symbol which is copied to the beginning of the symbol period.

Multi-pass effect

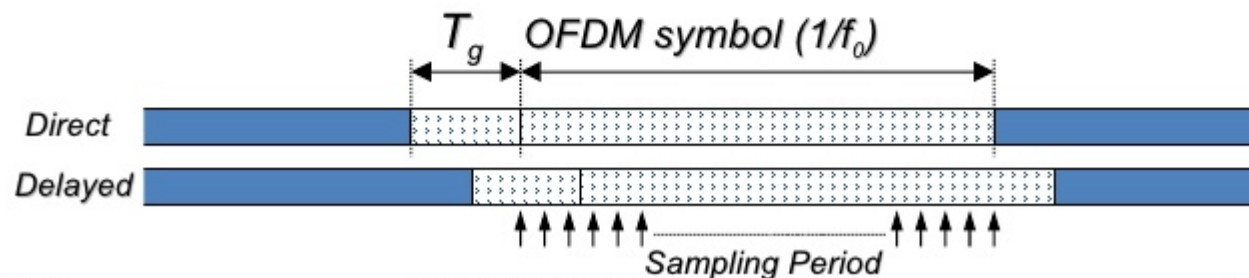


- Inter symbol interference (ISI) happens in Multi-path condition

Guard Interval T_g



- By adding the Guard Interval Period, ISI can be avoided



Thus, OFDM is Resistant to Multipath Signal Fading

Q: What else is good about OFDM?

A: OFDM is resistant to the multipath problem in high-frequency wireless. Very short-wavelength signals normally travel in a straight line (line of sight) from the transmit antenna to the receive antenna. Yet trees, buildings, cars, planes, hills, water towers, and even people will reflect some of the radiated signal. These reflections are copies of the original signal that also go to the receive antenna. If the time delays of the reflections are in the same range as the bit or symbol periods of the data signal, then the reflected signals will add to the most direct signal and create cancellations or other anomalies. Multipath fading is also called **Raleigh fading**.

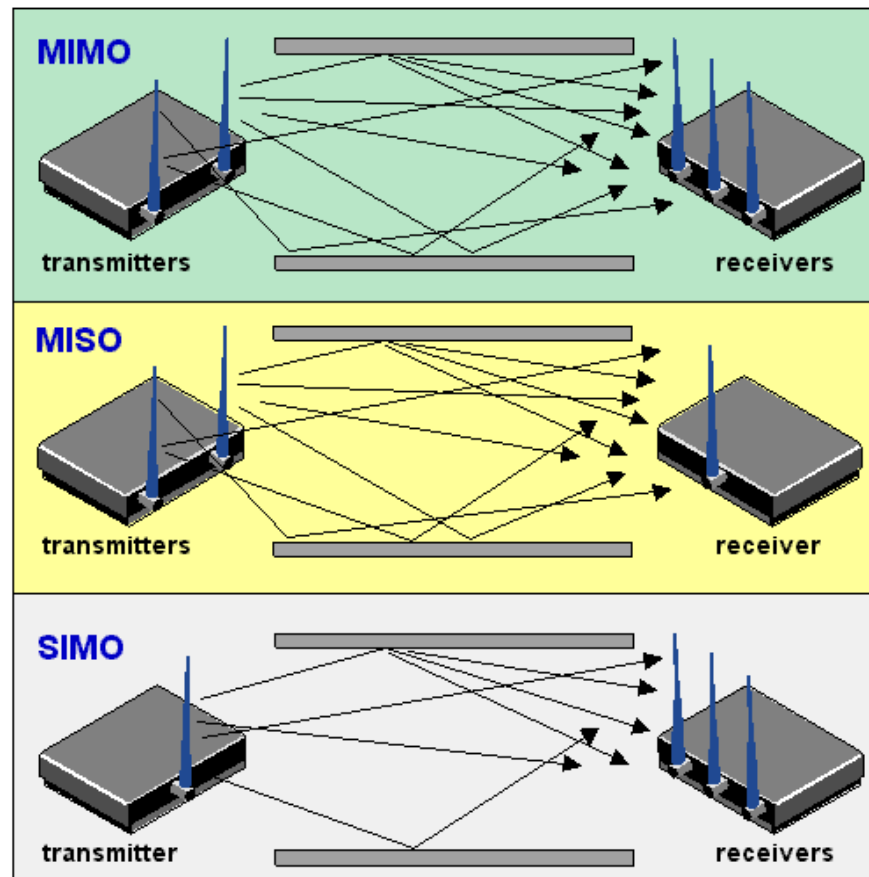
Antenna Diversity

Antenna diversity, also known as **space diversity** or **spatial diversity**, is any one of several wireless diversity schemes that uses two or more antennas to improve the quality and reliability of a wireless link.

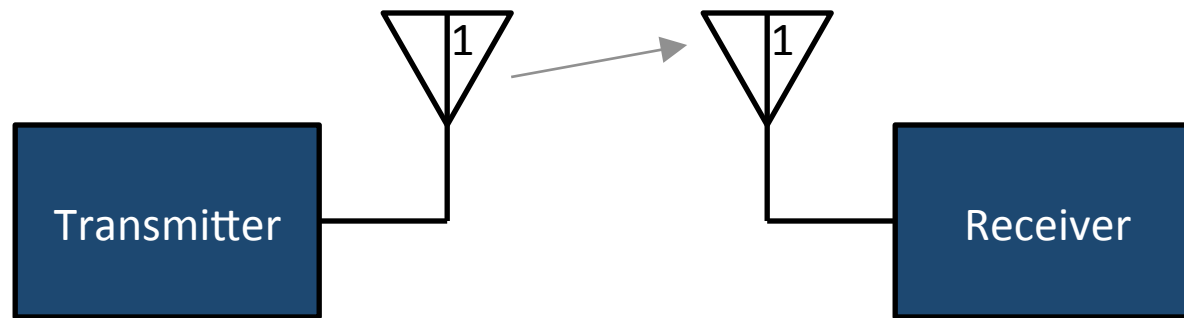
Antenna diversity is especially effective at mitigating these multipath situations. This is because multiple antennas offer a receiver several observations of the same signal. Each antenna will experience a different interference environment. Thus, if one antenna is experiencing a deep fade, it is likely that another has a sufficient signal. Collectively such a system can provide a robust link.

Antenna Diversity

Multiple Input Multiple Output (MIMO)



Single-Input Single-Output (SISO)



Conventional communication systems use one transmit antenna and one receive antenna.

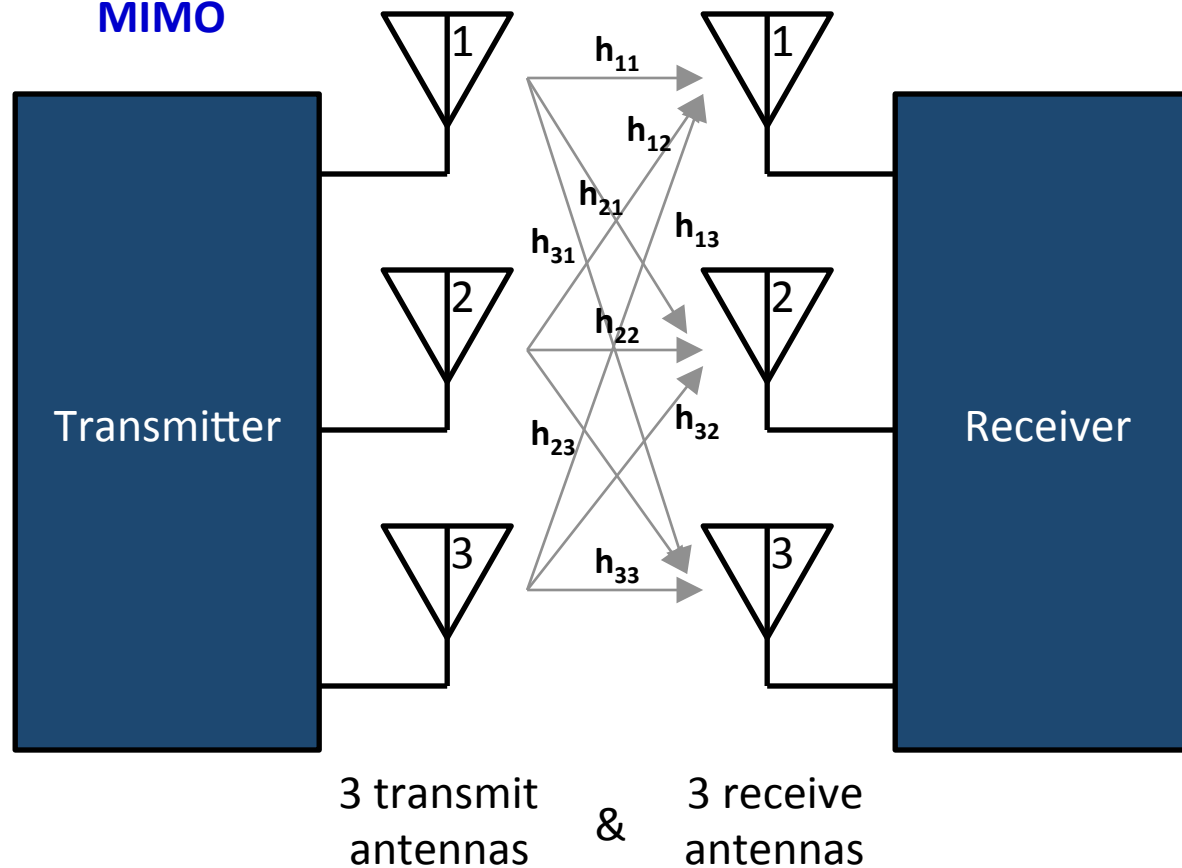
This is **Single Input Single Output (SISO)**.

Shannon-Hartley:

$$C = B \log_2 \left(\left[1 + \frac{S}{N} \right] \right)$$

Multiple Input Multiple Output (MIMO)

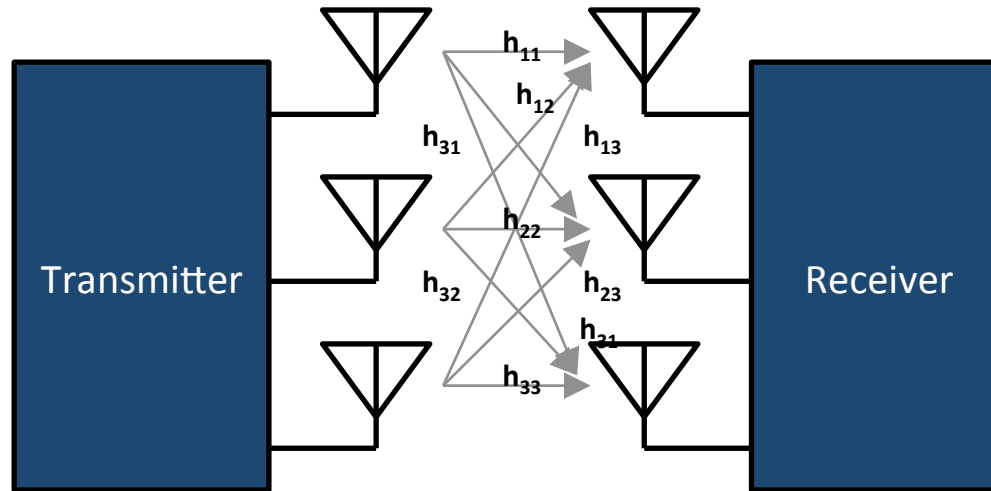
3 x 3
MIMO



We have a nine-element transmission matrix $[H]$

Note: Antenna spacing is important in using MIMO.

Multiple Input Multiple Output (MIMO)



Data to be transmitted is divided into individual data streams. Three transmit antennas mean three data streams.

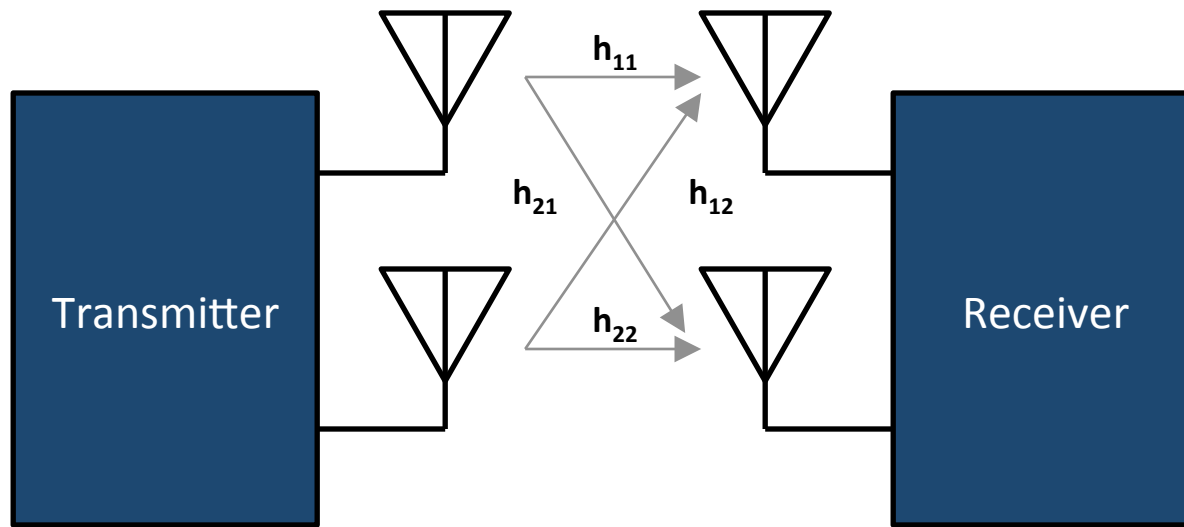
If m transmitters is not equal to n receiver antennas, then the number of data streams is the smaller of integers m and n .

The capacity can increase directly with the number of data streams.

$$C = N_D \cdot B \log_2 \left(\left[1 + \frac{S}{N} \right] \right)$$

N_D = number of data streams

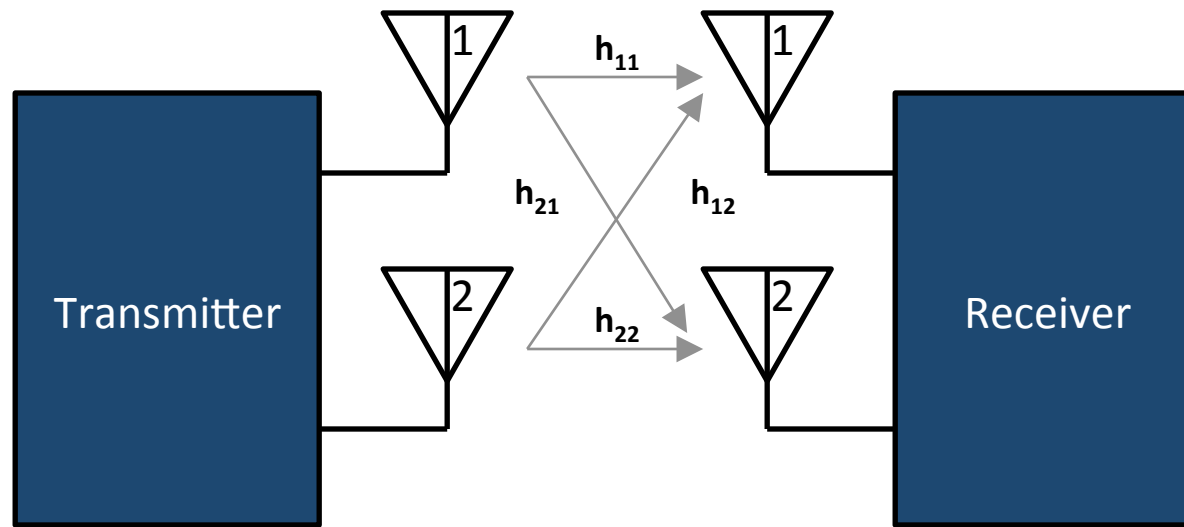
Single User MIMO (SU-MIMO)



This configuration doubles the data rate under ideal conditions.

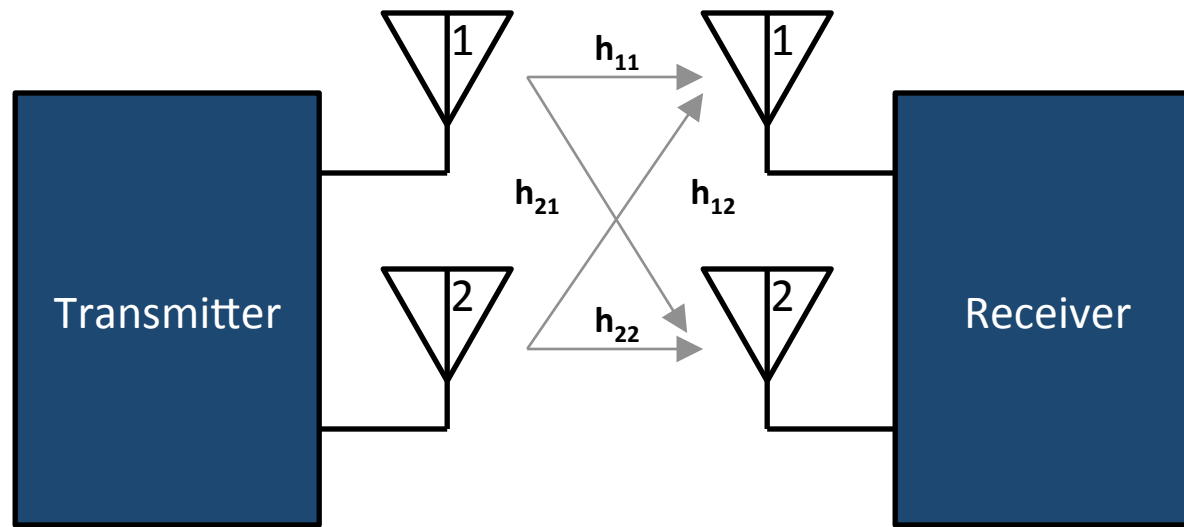
Why?

Multi-User MIMO (MU-MIMO)



When individual data streams are assigned to individual users we have multi-user MIMO. This mode is very useful for cellular uplinks because of the power limitations of the UE (cell phone) – it must be kept to a minimum in complexity. The base station has multiple antennas to enhance reception of the UE's signal.

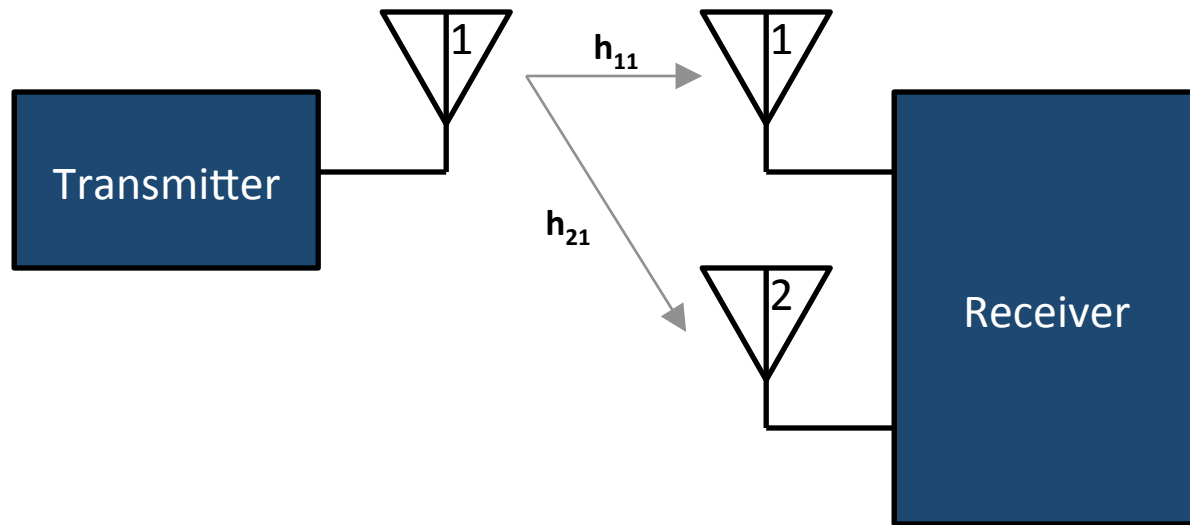
Spatial Diversity



The purpose of **spatial diversity** is to make the transmission more robust. In the case shown there is no increase in the data rate.

When does it make transmission more robust?

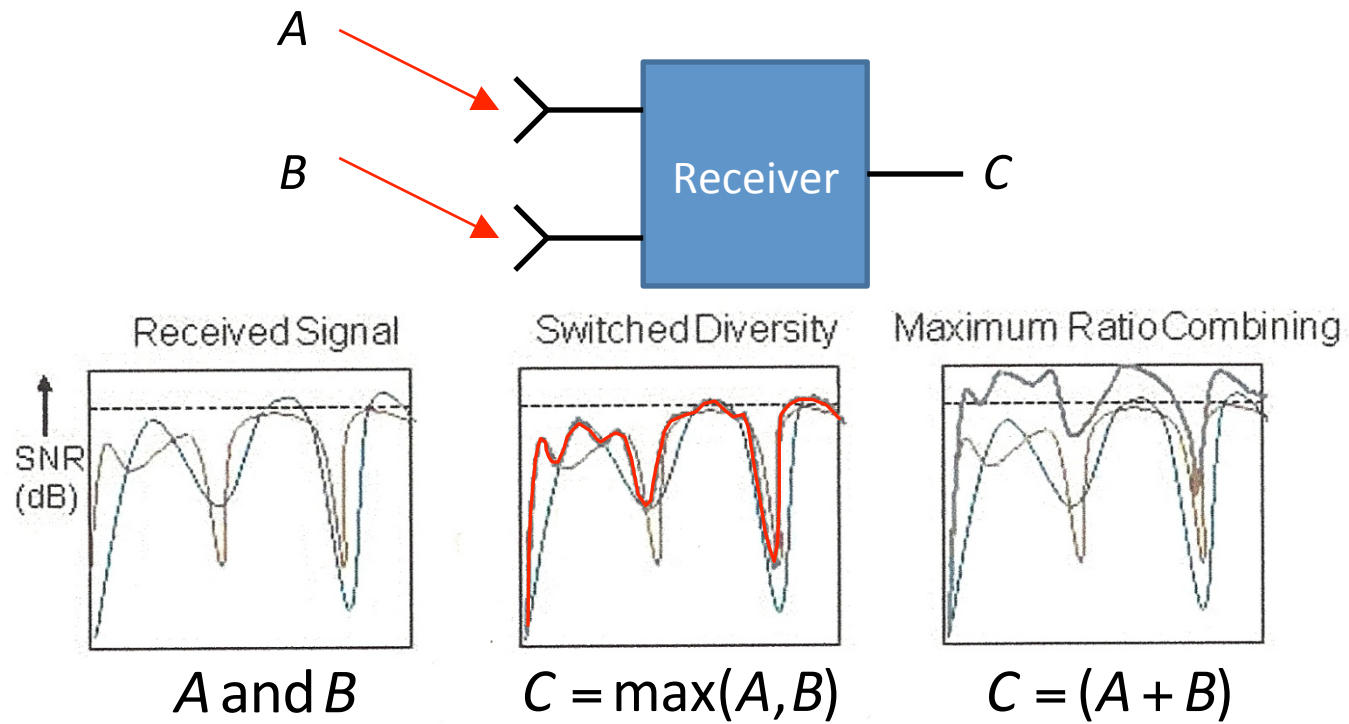
Receiver Diversity



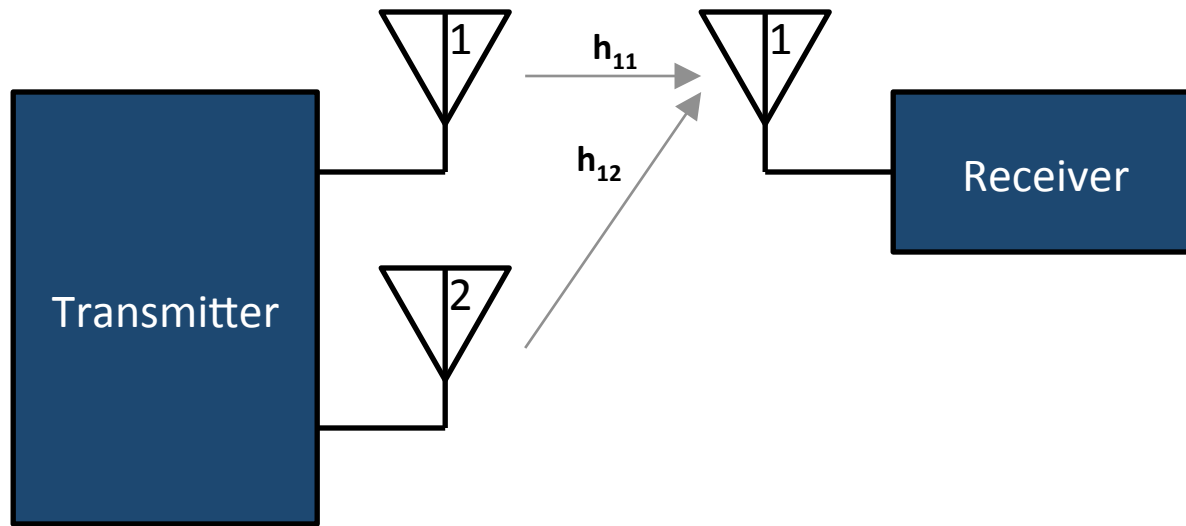
Receiver diversity uses more antennas on the receiver than the transmitter. A 1 x 2 MIMO configuration is the simplest version of receiver diversity. Redundant data is transmitted to the receiver.

Receiver 1 x 2 diversity is simple to implement and requires no special coding.

Example of Receiver Diversity



Transmitter Diversity



The simplest transmitter diversity configuration is shown here. The same data is redundantly transmitted to the receiver.

Space-time codes are used in transmitter diversity.

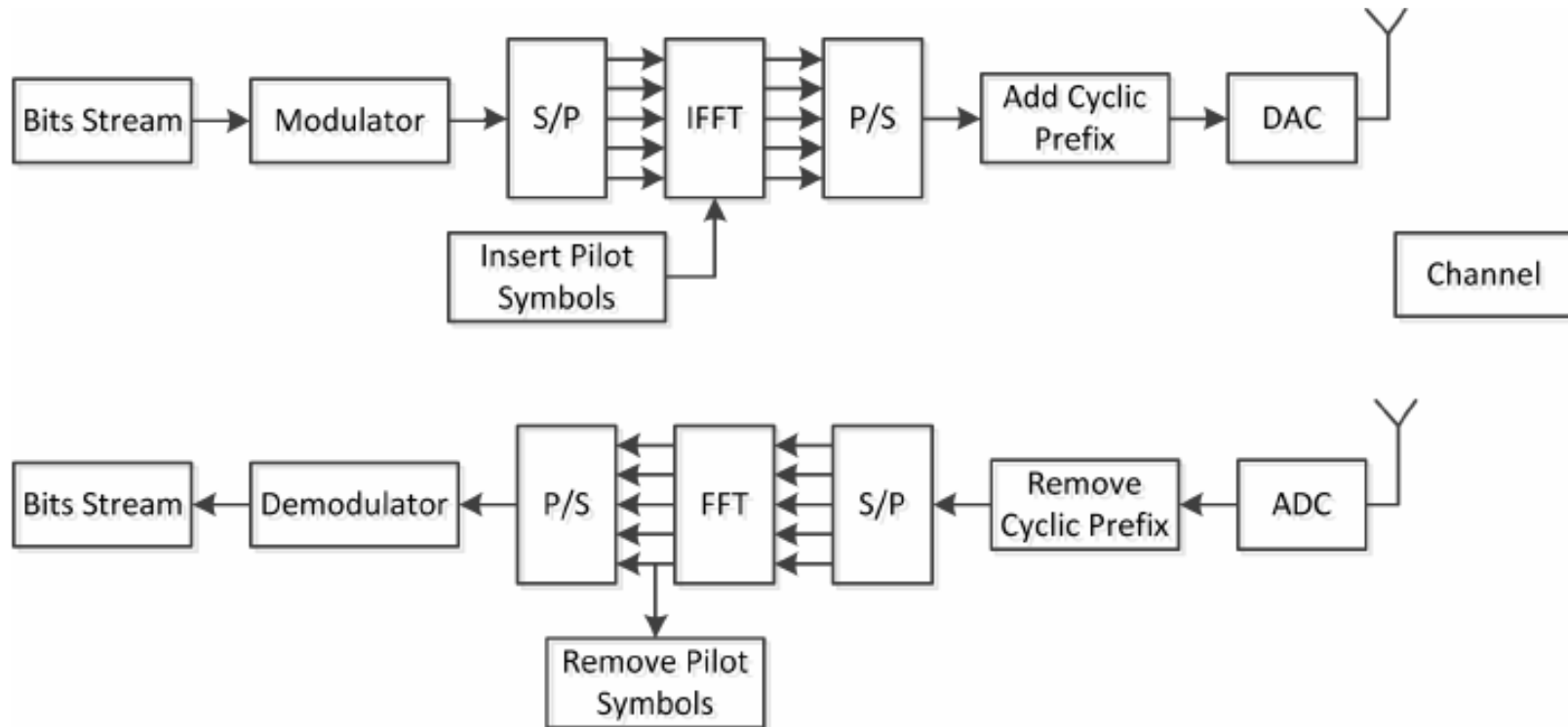
Some Applications Using MIMO

- ✓ IEEE 802.11n standard has adopted MIMO
 - Antenna Diversity upto 4 x 4
 - Tx Beamforming
 - Space Division Multiplexing (SDM)
- ✓ 2.4/5 GHz ISM band
 - 20/40 MHz Bandwidth
- ✓ PHY Data rates upto 600 Mbps
 - Throughput > 200 Mbps
- ✓ Extended Range
 - Indoor 70 m
 - Outdoor 250 m

MIMO in 4G LTE-Advanced

- ✓ 3GPP Rel 10 (LTE-A) has full featured MIMO
 - Antenna Diversity
 - Beamforming
 - Space Division Multiplexing (SDM)
- ✓ 22 Freq. bands covering 698-3600 MHz
 - Scalable Bandwidth (20-100 MHz)
- ✓ Enhanced Throughput
 - 1 Gbps Downlink
 - 500 Mbps Uplink
- ✓ Single or Multi User MIMO
 - SU-MIMO
 - MU-MIMO

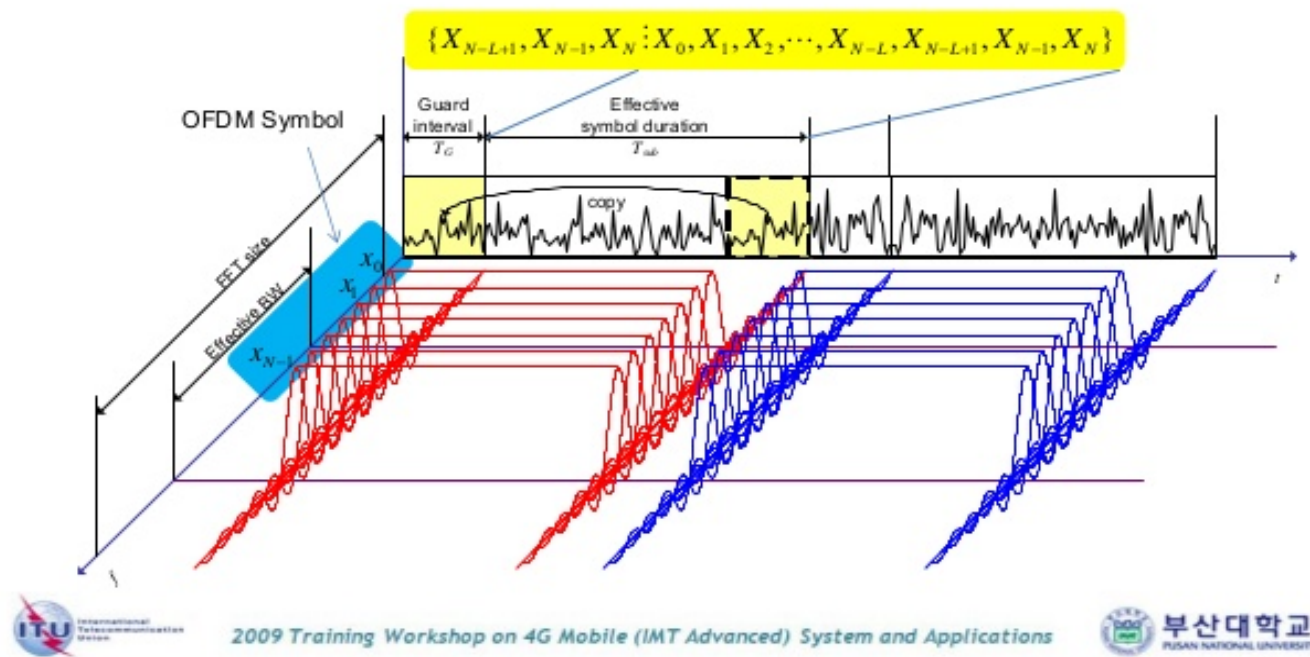
Digital Processing in OFDM Transmission System



https://www.researchgate.net/figure/272623859_fig1_Figure-1-Block-Diagram-of-OFDM-Transmission-System

OFDM: Overall Picture

- OFDM Symbol in 3D



Discrete Fourier Transform (DFT)

