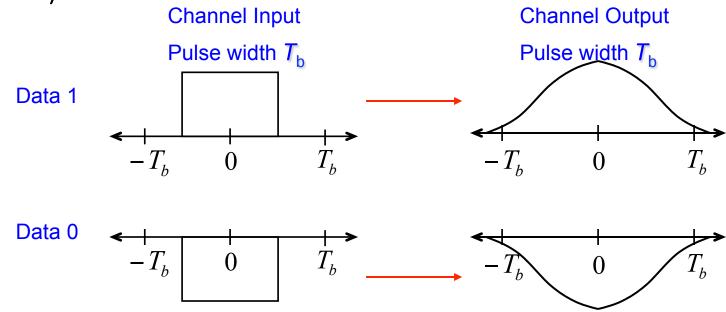
Intersymbol interference (ISI) occurs when a pulse spreads out in such a way that it interferes with adjacent at the sample instant.

Example: assume polar NRZ line code. The channel outputs are shown as "smeared" (width  $T_b$  becomes  $2T_b$ ) pulses (Spreading due to bandlimited channel)



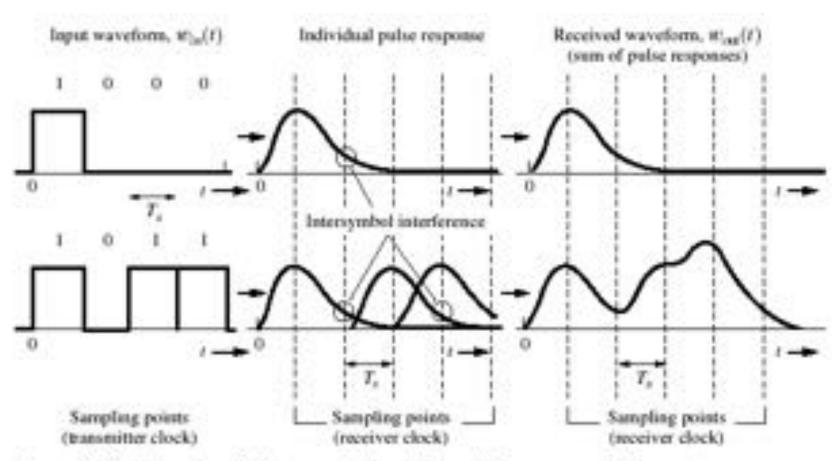
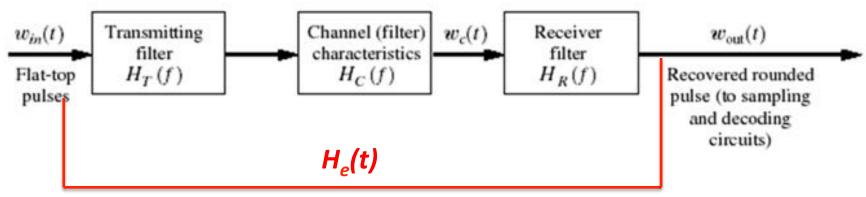


Figure 3-23 Examples of ISI on received pulses in a binary communication system.



$$w_{in}(t) = \pi \sum_{n} a_{n}h(t - nT_{s})$$

$$w_{out}(t) = \left[\sum_{n} a_{n}\delta(t - nT_{s})\right] * h_{e}(t)$$

$$h_{e}(t) = h(t) * h_{T}(t) * h_{C}(t) * h_{R}(t)$$

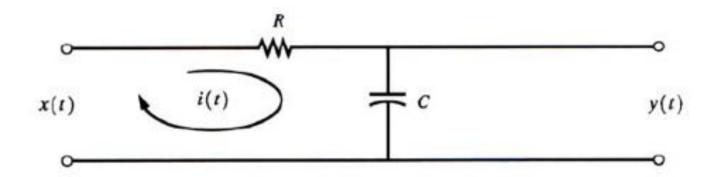
$$w_{out}(t) = \sum_{n} a_{n}h_{e}(t - nT_{s})$$

$$\Re$$

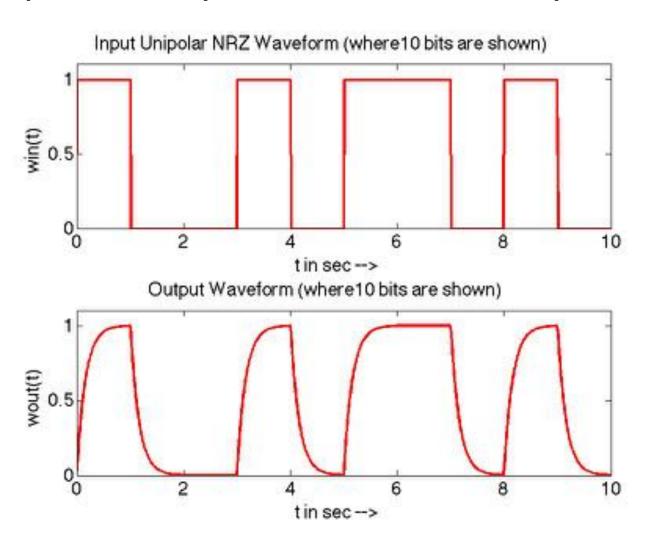
$$H_{e}(f) = H(f)H_{T}(f)H_{C}(f)H_{R}(f)$$
where
$$H(f) = \Re\left[\Pi\left(\frac{t}{T_{s}}\right)\right] = T_{s}\left(\frac{\sin \pi T_{s}f}{\pi T_{s}f}\right)$$

#### **Example 3-13 Intersymbol Interference Caused by RC Filtering**

Plot the output waveform when a channel filters a unipolar NRZ signal. Assume that the overall filtering effect of the transmitter, channel, and the receiver is that of an RC-low pass filter where the 3 dB bandwidth is 1 HZ. Assume that the unipolar NRZ input signal has a bit rate of  $R_b = 1$  HZ and that the data on the unipolar NRZ signal is  $[1\ 0\ 0\ 1\ 0\ 1\ 0\ 1\ 0\ 1\ 0]$ . Plot the waveform at the receiver output and observe the intersymbol interference.



#### **Example 3-13 Intersymbol Interference Caused by RC Filtering**



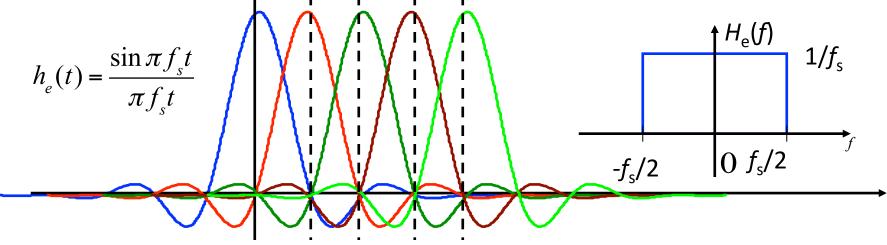
- Nyquist three criteria
  - Pulse amplitudes can be detected correctly despite pulse spreading or overlapping, if there is no ISI at the decisionmaking instants
    - 1: At sampling points, no ISI
    - 2: At threshold, no ISI
    - 3: Areas within symbol period is zero, then no ISI
  - At least 14 points in the finals
    - 4 point for questions
    - 10 point like the homework



#### Nyquist's First Method (Zero ISI)

Nyquist's first method for eliminating ISI is to use an equivalent transfer function  $H_{\rho}(f)$ :

$$H_{e}(f) = \frac{1}{f_{s}} \Pi\left(\frac{f}{f_{s}}\right) \qquad h_{e}(kT_{s} + \tau) = \begin{cases} C, & k = 0 \\ 0, & k \neq 0 \end{cases} \qquad \text{where} \qquad f_{s} = \frac{1}{T_{s}}$$



Nyquist's First Method (Zero ISI)

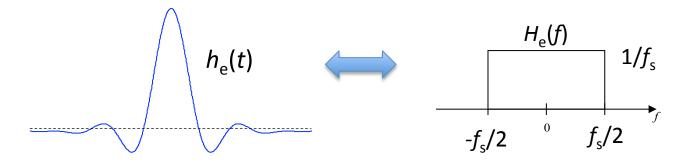
This type of pulse will allow signaling at a baud rate of  $D=1/T_s=2B$  (for Binary R=D)

Absolute bandwidth is:  $B = \frac{f_s}{2}$  MINIMUM BANDWIDTH

Signaling Rate is:  $D=1/T_s = 2B$  Pulses/sec

#### Nyquist's First Method (Zero ISI)

This type of pulse will allow signaling at a baud rate of  $D=1/T_s=2B$  (for Binary R=D)



- ♦ Since pulses are not possible to create due to:
  - Infinite time duration.
  - Sharp transition band in the frequency domain.
- ♦ The Sinc pulse shape can cause significant ISI in the presence of timing errors.
  - If the received signal is not sampled at *exactly* the bit instant (Synchronization Errors), then ISI will occur.

#### Nyquist's First Method (Zero ISI)

**THEOREM:** A filter is said to be a **Nyquist filter** if the effective transfer function is:

$$H_{e}(f) = \begin{cases} \Pi\left(\frac{f}{2f_{0}}\right) + Y(f), |f| < 2f_{0} \\ 0, elsewhere \end{cases}$$

Y(f) is a real function and **even symmetric** about f = 0;

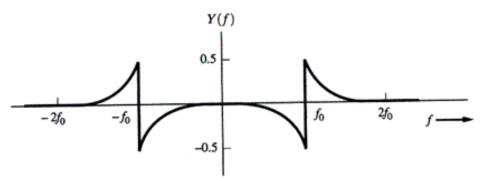
$$Y(f) = Y(-f), |f| < 2f_0$$

Y is **odd symmetric** about  $f = f_0$ ;

$$Y(-f + f_0) = Y(-f + f_0), |f| < 2f_0$$

There will be no intersymbol interference at the system output if the symbol rate is:

$$D = f_s = 2f_0$$

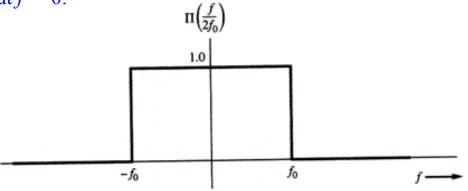


Y(f) is a real function and even symmetric about f = 0:

$$Y(-f) = Y(f), |f| < 2f_0$$

Y is odd symmetric about  $f = f_0$ :

$$Y(-f + f_0) = -Y(f + f_0),$$
  $|f| < f_0$ 



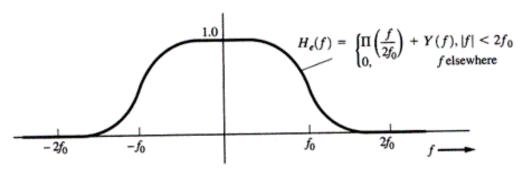


Figure 3-27 Nyquist filter characteristic.

A more general class of filter: Raised Cosine-Rolloff Filter

$$H_{e}(f) = \begin{cases} 1, & |f| < f_{1} \\ 1/2 \left\{ 1 + \cos \left[ \frac{\pi \left( |f| - f_{1} \right)}{2f_{\Delta}} \right] \right\}, & f_{1} < |f| < B \end{cases}$$

$$0, & |f| > B$$

Where **B** is the *absolute bandwidth* and the parameters

$$f_{\Delta} = B - f_0$$

and 
$$f_1 = f_0 - f_{\Delta}$$

 $f_0$  is the 6-dB bandwidth of the filter. The **rolloff factor** is defined to be

$$r = \frac{f_{\Delta}}{f_0}$$

A more general class of filter: Raised Cosine-Rolloff Filter

$$h_{e}(t) = \Im^{-1} \left[ H_{e}(f) \right] = 2f_{0} \left( \frac{\sin 2\pi f_{0}t}{2\pi f_{0}t} \right) \left[ \frac{\cos 2\pi f_{\Delta}t}{1 - \left(4f_{\Delta}t\right)^{2}} \right]$$

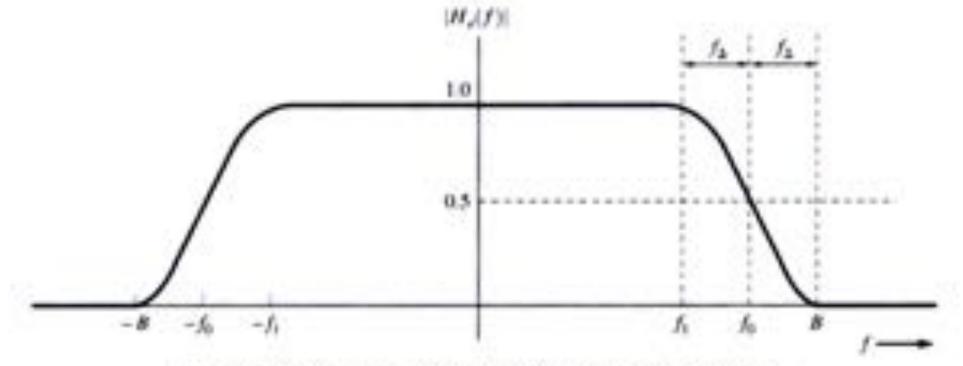
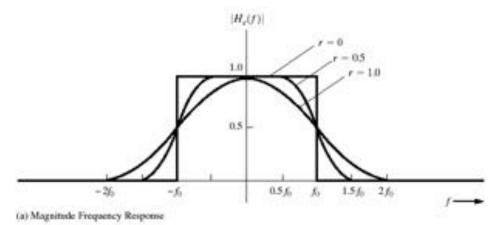


Figure 3-25 Raised cosine-rolloff Nyquist filter characteristics.

A more general class of filter: Raised Cosine-Rolloff Filter

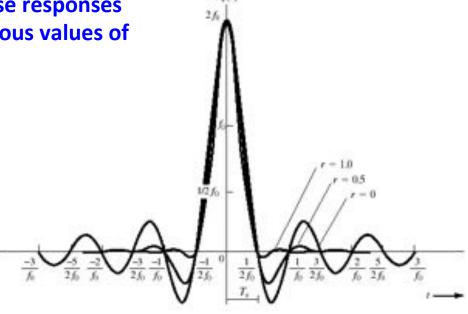


Frequency response and impulse responses of Raised Cosine pulses for various values of the roll off parameter.

$$r \uparrow \rightarrow B \uparrow$$

$$r \uparrow \rightarrow ISI \downarrow$$

$$r \uparrow \rightarrow ISI \downarrow$$



A more general class of filter: Raised Cosine-Rolloff Filter

$$H_{e}(f) = \begin{cases} 1, & |f| < f_{1} \\ 1/2 \left\{ 1 + \cos \left[ \frac{\pi \left( |f| - f_{1} \right)}{2f_{\Delta}} \right] \right\}, & f_{1} < |f| < B \end{cases}$$

$$0, & |f| > B$$

Baud rate: 
$$D = \frac{2B}{1+r}$$

Where *B* is the absolute bandwidth of the system and *r* is the system rolloff factor.

#### Problem 3.49

Assume that a pulse transmission system has the overall raised cosine-rolloff Nyquist filter characteristic described by

$$H_{e}(f) = \begin{cases} 1, & |f| < f_{1} \\ 1/2 \left\{ 1 + \cos \left[ \frac{\pi (|f| - f_{1})}{2f_{\Delta}} \right] \right\}, & f_{1} < |f| < B \end{cases}$$

$$0, & |f| > B$$

Find the Y(f) Nyquist function of 
$$H_{e}(f) = \begin{cases} \Pi\left(\frac{f}{2f_{0}}\right) + Y(f), & |f| < 2f_{0} \\ 0, & elsewhere \end{cases}$$

#### Problem 3.50

An analog signal is to be converted into a PCM signal that is a binary polar NRZ line code. The signal is transmitted over a channel that is absolutely bandlimited to 4 kHz. Assume that the PCM quantizer has 16 steps and that the overall equivalent system transfer function is of the raised cosine-rolloff type with r = 0.5.

- a.) Find the maximum PCM bit rate that can be supported by this system without introducing ISI.
- b.) Find the maximum bandwidth that can be permitted for the analog signal.