

# **The application of signal detection and channel estimation to OFDM systems**

**A course project on Signal detection and Estimation**

**By:**

**Tesfay Gidey Hailu**

**Student IDNo: 201914010103**

**Dec 20, 2019**

## **Abstract**

This course project aims to survey and compare the performances of channel estimation techniques for OFDM systems based on pilot arrangements. A number of recent articles on emerging topics of signal detection have been surveyed, to better understand each model for channel estimation and detection in OFDM systems in general and make decision criterion based on their model efficiency in particular. Those papers that I have surveyed basically focuses on investigation and comparing various efficient pilot-based channel estimation schemes for OFDM systems. In this survey, two major types of pilot arrangement such as block type and comb-type pilot have been focused employing Least Square Error (LSE) and Minimum Mean Square Error (MMSE) channel estimators. It has been also indicated that, block type pilot sub-carriers is especially suitable for slow-fading radio channels whereas comb type pilots provide better resistance to fast fading channels.

To address channel distortion was also their interest of problem, and the researchers have applied a deep learning model. Besides, the model was first trained offline using the data generated from simulation based on channel statistics and then used for recovering the online transmitted data directly. From their simulation results, the deep learning-based approach can address channel distortion and detect the transmitted symbols with better performance comparable to the minimum mean square error (MMSE) estimator.

The deep learning-based approach was also found to be more robust than conventional methods when fewer training pilots are used, the cyclic prefix (CP) is omitted, and nonlinear clipping noise exists. In summary, it has been pointed out that, deep learning is a promising tool for channel estimation and signal detection in wireless communications with complicated channel distortion and interference.

## **1. Introduction**

Orthogonal Frequency Division Multiplexing (OFDM) is a digital multi-carrier modulation scheme that extends the concept of single subcarrier modulation by using multiple subcarriers within the same single channel. OFDM makes use of a large number of closely spaced orthogonal subcarriers that are transmitted in parallel rather than transmit a high-rate stream of data with a single subcarrier. It has recently been applied widely in wireless communication systems due to its high data rate transmission capability with high bandwidth efficiency and its robustness to multipath delay. Wireless systems are expected to require high data rates with low delay and low bit error-rate (BER). In such situations, the performance of wireless communication systems is mainly governed by the wireless channel environment.

In addition, high data rate transmission and high mobility of transmitters and/or receivers usually result in frequency-selective and time selective, i.e., doubly selective, fading channels for future mobile broadband wireless systems. Therefore, mitigating such doubly selective fading effects is critical for efficient data transmission. Moreover, perfect channel state information (CSI) is not available at the receiver. Thus, in practice, accurate estimate of the CSI has a major impact on the whole system performance [1]. It is also because, in contrast to the typically static and predictable characteristics of a wired channel, the wireless channel is rather dynamic and unpredictable, which makes an exact analysis of the wireless communication system often difficult.

A dynamic estimation of channel is necessary before the demodulation of OFDM signals since the radio channel is frequency selective and time-varying for wideband mobile communication systems [2]. The channel estimation can be performed by either inserting pilot tones into all of the subcarriers of OFDM symbols with a specific period or inserting pilot tones into each OFDM symbol.

This project aims to assess and compare the performances of channel estimation techniques for OFDM systems based on pilot arrangements. The estimation of the channel for this block-type pilot arrangement has been analyzed using Least Square (LS) or Minimum Mean-Square (MMSE). It has been noted that the MMSE estimate has been shown to give 10-15 dB gain in signal-to-noise ratio (SNR) for the same mean square error of channel estimation over LS estimate [2]. A low-rank approximation was applied to linear MMSE by using the frequency correlation of the channel in order to eliminate the major drawback of MMSE, which is complexity [3].

On the other hand, the comb-type pilot channel estimation, has been introduced to satisfy the need for equalizing when the channel changes even in one OFDM block. The comb-type pilot channel estimation consists of algorithms to estimate the channel at pilot frequencies and to interpolate the channel. In line with this, the estimation of the channel at the pilot frequencies for comb-type based channel estimation can be based on LS, MMSE or Least Mean-Square (LMS). The MMSE has been shown to perform much better than Least Square. It has been also indicated that the complexity of MMSE is reduced by deriving an optimal low rank estimator with singular-value decomposition [4]. Moreover, the interpolation of the channel for comb-type based channel estimation can depend on linear interpolation, second order interpolation, low-pass interpolation, spline cubic interpolation, and time domain interpolation. In [4], second-order interpolation has been shown to perform better than the linear interpolation. In [5], time-domain interpolation has been proven to give lower bit-error rate (BER) compared to linear interpolation.

Furthermore, for the first time, researchers have been exploited deep learning to handle wireless OFDM channels in an end-to-end manner. It is noted that the approach they used as being different from existing OFDM receivers that first estimate channel state information (CSI) explicitly and then detect/recover the transmitted symbols using the estimated CSI, the proposed deep learning-based approach estimates CSI implicitly and recovers the transmitted symbols directly.

Their simulation result has revealed that, the deep learning-based approach can address channel distortion and detect the transmitted symbols with performance comparable to the minimum mean square error (MMSE) estimator. Furthermore, the deep learning-based approach is more robust than conventional methods when fewer training pilots are used, the cyclic prefix (CP) is omitted, and nonlinear clipping noise exists. Deep learning is therefore found as a promising tool for channel estimation and signal detection in wireless communications with complicated channel distortion and interference.

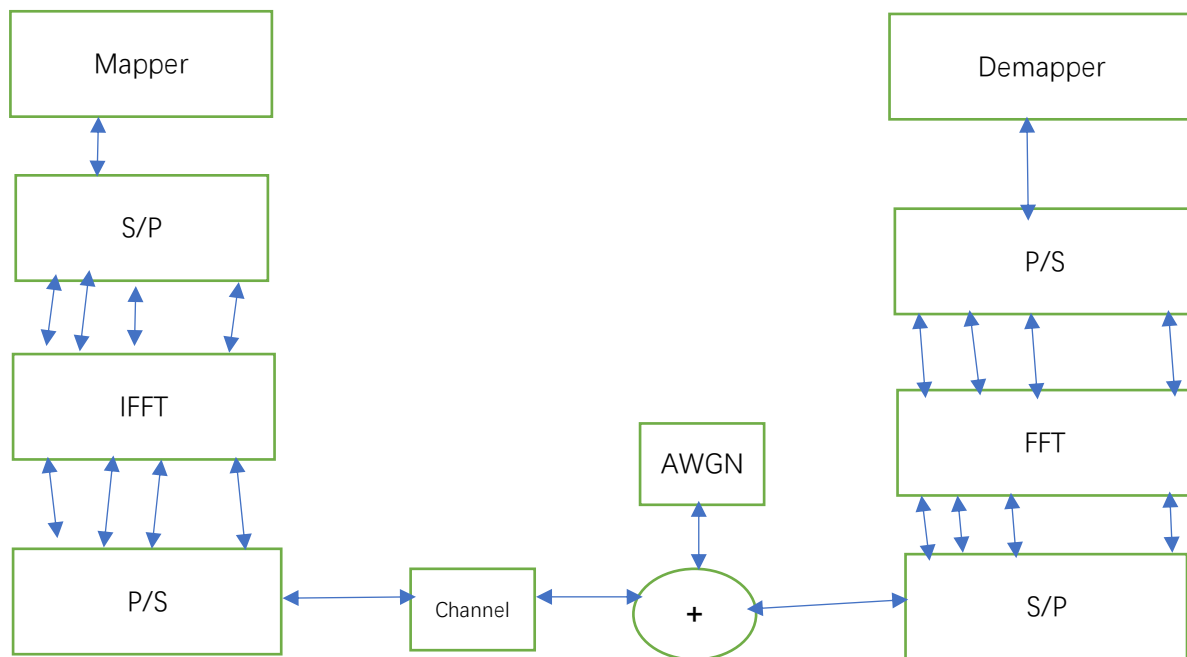
This project therefore aims to assess and compare the performances of channel estimation techniques for OFDM systems based on pilot arrangements. Furthermore, I surveyed a number of recent articles on emerging topics of signal detection, to better understand each model for channel estimation and detection in OFDM systems in general and make decision criterion based on their model efficiency in particular. In section II, the description of the OFDM system based on pilot channel estimation is given. In section III, the estimation of the channel based on block-type pilot arrangement is discussed. In section IV, the different channel estimation techniques are introduced. In section V, the simulation environment and results are discussed. Conclusions and discussions will be presented in section VI.

## 2. System model

The block diagram of discrete-time baseband OFDM system is depicted in Figure 1. In Figure 3, an OFDM signal consists of  $N$  subcarriers that are modulated by  $N$  complex symbols selected from a particular QAM constellation. These basebands modulated symbols are then passed through serial to parallel converter which generates complex vector of size  $N$ . This complex vector of size  $N$  can be expressed as  $X = [X_0, X_1, \dots, X_{N-1}]$ .  $X$  is then passed through the IFFT block to give  $x = WX$ . Where,  $W$

is the  $N \times N$  IFFT matrix. Thus, the complex baseband OFDM signal with subcarriers into time domain samples can be written as:

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{\frac{j2\pi kn}{N}} \quad n = 0, 1, 2, \dots, N-1.$$



**Figure 1: A Discrete-Time Baseband OFDM System**

After parallel-to-serial conversion, a cyclic prefix with a length of  $N_g$  samples is appended before the IFFT output to form the time-domain OFDM symbol,  $s = [s_0, \dots, s_{N+N_g-1}]$ , where,  $s_i = x_{i-N_g}$  and  $i \bmod N$ . The useful part of OFDM symbol does not include the  $N_g$  prefix samples and has duration of  $T_u$  seconds. The samples ( $s$ ) are then amplified, with the amplifier characteristics is given by function  $F$ . The output of amplifier produces a set of samples given by:

$$y = [y_0, y_1, \dots, y_{N+N_g-1}]$$

This signal is then serially transmitted through a multipath radio propagation channel which is subject to additive white Gaussian noise (AWGN) with variance  $\sigma^2 = N_0/2$ , where  $N_0$  power spectral density is. At the receiver front end, the received signal is applied to a matched filter and then sampled at a rate  $T_s = T_u/N$ . After dropping the CP

samples ( $N_g$ ), the received sequence  $z$ , assuming an additive white Gaussian noise (AWGN) channel, can be expressed as

$$z = F(Wd) + \eta$$

Where, the noise vector  $\eta$  consists of  $N$  independent and normally distributed complex random variables with zero mean and variance  $\sigma_n^2 = E\{|\eta|^2\}$ . Subsequently, the sequence  $z$  is fed to the fast Fourier transform (FFT), which produces the frequency-domain sequence  $r$  as

$$r = W^H Z$$

where,  $k$ th element of  $r$  is given by

$$r_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} z_n e^{\frac{j2\pi kn}{N}} \quad n = 0, 1, 2, \dots, N-1.$$

Finally, the estimated symbols vector  $d$  can be obtained from  $r$ . It is to be noted that the demodulation is performed based on the assumption of perfect symbol timing, carrier frequency, and phase synchronization. It is also to be noted that due to the use of CP, the inter-block interference between contiguous OFDM blocks in frequency domain is eliminated so each OFDM block can be processed independently, provided that the length of CP is equal to or larger than the delay spread of the channel.

The main drawbacks of OFDM systems are high peak-to-average-power ratio (PAPR), bit error rate (BER) and high sensitivity to carrier frequency offset (CFO). Moreover, OFDM does not obtain frequency diversity. If a deep fade occurs close to the frequency of a subcarrier, reliable data detection carried by these faded subcarriers becomes difficult [7, 8]. This problem can be solved by using error-control codes in conjunction with interleaving, which helps reducing the diversity loss. Typical examples of error control codes are block codes (e.g., Reed-Solomon (RS) or Bose-Chaudhuri-Hocquenghem (BCH)), convolutional codes, trellis codes, turbo codes, and low-density parity-check (LDPC) codes.

The major advantage of OFDM lies in processing frequency-selective channels as multiple flat-fading sub-channels. If the channel is time invariant (slow fading) over the

period of an OFDM symbol block, the orthogonality property is maintained between the subcarriers. In such a case, channel estimation or data detection is simple as each subcarrier is equalized with a single-tap equalizer. However, when the channel is time-varying over one OFDM symbol period, the orthogonality among subcarriers is destroyed, resulting in ICI, which degrades the bit error rate performance compared to the slow fading channels. The ICI may occur due to the presence of the fast fading channel or the presence of a carrier frequency offset (CFO) between the transmitter and receiver caused by imperfect synchronization. CFO can be estimated by using various algorithms such as a maximum likelihood (ML) estimation algorithm [9, 10, 11].

The potential performance degradation of OFDM caused by fading channels is a function of the fading rate, with faster fading channels requiring more significant mitigation methods to achieve the same error performance as slow fading channels. Furthermore, in the presence of ICI due to fast fading, the channel estimation is more challenging since both the individual subcarrier and the interference created by each subcarrier to its neighboring subcarriers need to be estimated. Therefore, this paper focuses on channel estimation techniques for both slow fading and fast fading channels.

a) **Communication system:**

As discussed above, wireless signals usually subject to fading and dispersion. This offers implications for the design of wireless communication systems. In such cases, a transmitted signal will be composed of a sequence of shift-orthogonal and overlapping pulses, distorted by the channel. OFDM systems usually experience frequency selectivity in a channel, so the same require equalization in the receiver. The term *equalization* means the removal of distortion or the de-convolution of channel response from the received signal. Mathematically, a communication system can be described in terms of the complex baseband transmitted signal as:

$$S(t) = \sum_{l \in \mathbb{Z}} a_l \delta(t - lT)$$

A pulse shaping filter is also usually introduced, which makes the above equation as:



$$S_T(t) = \sum_{l \in \mathbb{Z}} a_l C_T \delta(t - lT)$$

where  $\mathbb{Z}$  is the set of integers,  $a_l$  is an element from the complex symbol sequence,  $C_T(t)$  is the transmitter pulse shape and  $T$  is the symbol spacing. The bandwidth of this transmitted pulse shape  $C_T(t)$  is denoted by  $W$ . The channel state information is usually expressed mathematically in terms of the channel impulse response (CIR); which is a function of time and delay. It is denoted as  $h(t, \xi)$ . This impulse response incorporates both fading, time selective effects and dispersive, frequency selective effects. To determine the CIR at a given time (say  $t$ ), the channel has the impulse response described as a function of delay ( $\xi$ ). In OFDM systems, the receiver usually has a fixed front-end filter with impulse response  $C_R(t)$ . The impulse response of the overall pulse shape is given by:

$$\mathbf{c}(t) = C_T(t) \otimes C_R(t)$$

As discussed above, the overall CIR in terms of pulse shape is defined as

$$\mathbf{u}(t) = \mathcal{C}(\xi) \otimes h(t, \xi)$$

where,  $\otimes$  denotes convolution. The overall CIR with  $\xi = t - lT$  and the convolution explicitly shown is expressed as:

$$\mathbf{u}(t, t - lT) = \int_{\tau=-\infty}^{\infty} \mathbf{c}(t - lT - \tau) h(t, \tau) d\tau.$$

At the output of the front-end filter, the received signal is given by

$$\mathbf{S}_R(t) = \sum_{l \in \mathbb{Z}} \mathbf{a}_l \mathbf{u}(t, t - lT) + \boldsymbol{\eta}(t).$$

Where,  $\boldsymbol{\eta}(t)$  is the filtered output due to additive white Gaussian noise (AWGN) with two-sided power spectral density (PSD)  $N_0/2$ .

### 3. Channel Estimation Techniques

In wireless communication, the channel is usually unknown *a priori* to the receiver. Therefore, to do the channel estimation, a pilot symbol aided modulation is used, where known pilot signals are periodically sent during the transmission. In general, the performance of channel estimation depends on the number, the location, and the power of pilot symbols inserted into OFDM blocks. To mathematically analyze this, consider

a fading multipath channel with the multipath delay spread  $\tau_{max}$  and the maximum Doppler frequency ( $fd$ ). To recover the channel state information (CSI), the spaces between pilot symbols in the time and frequency domain must satisfy two-dimensional (2-D) sampling theorem [12], that is,

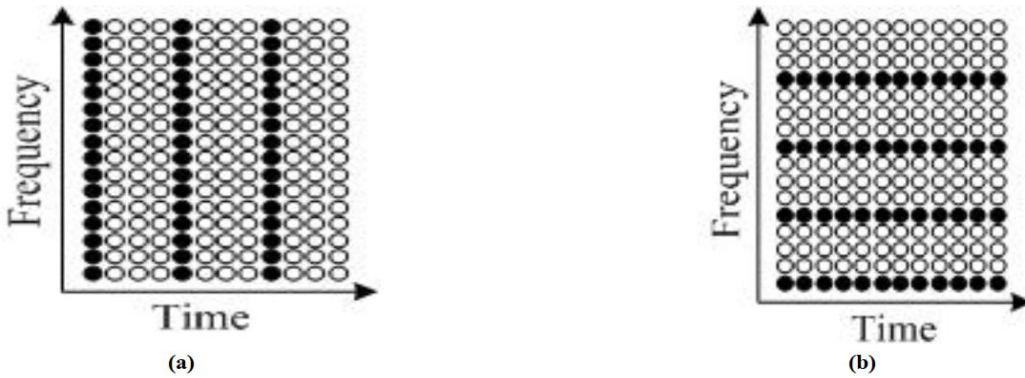
$$f_d T d_t \leq 1/2$$

$$\tau_{max} \Delta f d_f \leq 1$$

Where  $T$  is the OFDM block duration,  $\Delta f$  is the subcarrier spacing;  $d_t$  and  $d_f$  are the numbers of samples between pilot symbols in the time domain and frequency domain, respectively [13]. Within the OFDM symbol duration, the number of pilot symbols in frequency domain is related to the delay spread; on the other hand, the number of pilot symbols in time domain is related to the normalized Doppler frequency ( $f_d T$ ). Based on 2-D arrangement of pilot symbols, 2-D channel estimators are too complex in practice [14]. Therefore, channel estimation is exploited in one-dimension (1-D) for OFDM systems in general.

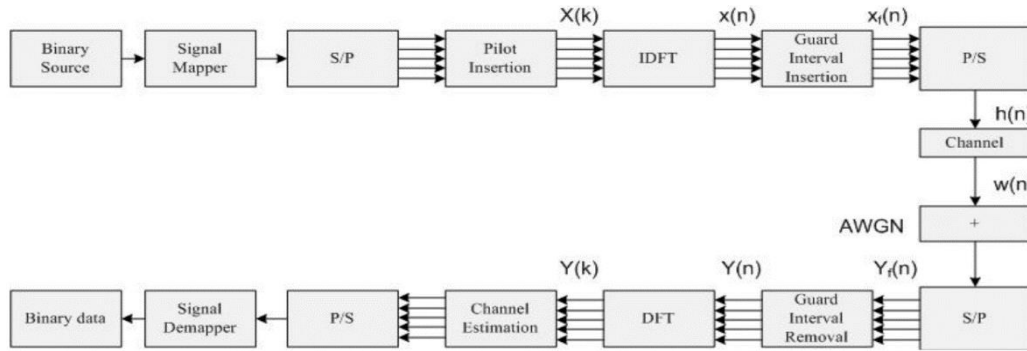
### 3.1 OFDM Systems

For an OFDM mobile communication system, the channel transfer function at different subcarriers appears unequal in both frequency and time domains. Therefore, a dynamic estimation of the channel is always required. Pilot-based approaches are widely used to estimate the channel properties and correct the received signal. In this project, two types of pilot arrangements are considered, as shown in Figure 2.



**Figure 2: (a) Block type pilot arrangement and (b) comb type pilot arrangement.**

The first kind of pilot arrangement, shown in Figure 2, is denoted as block-type pilot arrangement. This is sent periodically in time-domain and is particularly suitable for slow-fading radio channels. Because the training block contains all pilots, channel interpolation in frequency domain is not required. Therefore, this type of pilot arrangement is relatively insensitive to frequency selectivity. The second kind of pilot arrangement, shown in Figure 3, is denoted as comb-type pilot arrangement. In this case, the pilot arrangements are uniformly distributed within each OFDM block. Assuming that the payloads of pilot arrangements are the same, the comb-type pilot arrangement has a higher re-transmission rate. Thus, the comb-type pilot arrangement system provides better resistance to fast-fading channels. Since only some sub-carriers contain the pilot signal, the channel response of non-pilot sub-carriers will be estimated by interpolating neighbouring pilot sub-channels. Thus, the comb-type pilot arrangement is sensitive to frequency selectivity when comparing to the block-type pilot arrangement system.



**Figure 3: Baseband OFDM system**

In Figure 3, the binary information is first grouped and mapped according to the modulation in signal mapper. In modern OFDM systems, usually QAM is used as the modulation technique. After inserting pilots either to all sub-carriers with a specific period or uniformly between the information data sequence, IDFT block is used to transform the data sequence of length into time domain signal using the following equation:

$$x(n) = IDFT\{X(k)\} = \sum_{K=0}^{N-1} X(k) e^{\frac{j(2\pi kn)}{N}} \quad n = 0, 1, 2 \dots \dots N - 1.$$

Where  $N$  is the DFT length. Following IDFT block, guard time, which is chosen to be larger than the expected delay spread, is inserted to prevent inter-symbol interference. This guard time helps eliminating the inter-carrier interference. Therefore, the resultant OFDM symbol is given as:

$$\begin{aligned} x_f(n) &= x(N + n), \quad n = -N_g, -N_g + 1, \dots, -1 \\ &= x(n) \quad n = 1, 2, \dots, N - 1 \end{aligned}$$

Where,  $N_g$  is the length of guard interval. The transmitted signal  $x_f(n)$  will pass through the frequency selective time varying fading channel with additive noise. The received signal is given by:

$$y_f(n) = x_f(n) \otimes h(n) + w(n)$$

Where  $w(n)$  is AWGN and  $h(n)$  is the channel impulse response. Thus, the overall channel response can be represented as:

$$h(n) = \sum_{i=1}^{r-1} h_i e^{j\left(\frac{2\pi}{N}\right) f_{di} T n} \delta(\lambda - \tau_i) \quad n = 0, 1, 2, \dots, N - 1$$

Where  $r$  is the total number of propagation paths,  $h_i$  is the complex impulse response of the  $i^{\text{th}}$  path,  $f_{di}$  is the  $i^{\text{th}}$  path Doppler frequency shift,  $\lambda$  is delay spread index,  $T$  is the sample period and  $\tau_i$  is the  $i^{\text{th}}$  path delay normalized by the sampling time. At the receiver, the analog signal received is converted to discrete domain and the guard time is removed to give the received signal as:

$$\begin{aligned} y_f(n) & \quad \text{for } -N_g \leq n \leq N - 1 \\ y(n) &= y_f(n + N_g), \quad n = 1, 2, \dots, N - 1 \end{aligned}$$

This received signal  $y(n)$  is then sent to DFT block to yield:

$$Y(k) = DFT\{y(k)\} = \frac{1}{N} \sum_{K=0}^{N-1} y(n) e^{\frac{-j(2\pi kn)}{N}} \quad k = 0, 1, 2, \dots, N - 1.$$

Assuming there is no ISI, the relation of the resulting  $Y(k)$  to  $H(k)=DFT\{h(n)\}$ , and  $W(k)=DFT\{w(n)\}$ , is given by:

$$Y(k) = X(k)H(k) + W(k)$$

Following DFT block, the pilot signals are extracted and the estimated channel  $\hat{H}(k)$  for the data sub-channels is obtained in channel estimation block. Then the transmitted data is estimated by:

$$\hat{X} = \frac{Y(k)}{\hat{H}(k)} \quad k = 0, 1, \dots, N-1.$$

Then the binary information data is obtained back in signal demapper block. Based on principle of OFDM transmission scheme, it is easy to assign the pilot both in time domain and in frequency domain.

### 3.2) Block-type pilot-based channel estimation

In block-type pilot-based channel estimation, OFDM channel estimation symbols are transmitted periodically, in which all sub-carriers are used as pilots. If the channel is constant during the block, there will be no channel estimation error as the pilots are sent at all carriers. The estimation can be performed by using either LSE or MMSE. If inter symbol interference is eliminated by the guard interval, then:

$$Y = XF_h + W = X_H + W$$

Where

$$X = \text{diag}\{X(0), X(1), \dots, X(N-1)\}$$

$$Y = [Y(0), Y(1), \dots, Y(N-1)]^T$$

$$W = [W(0), W(1), \dots, W(N-1)]^T$$

$$H = [H(0), H(1), \dots, H(N-1)]^T$$

$$\begin{bmatrix} W_N^{00} & \dots & W_N^{0(N-1)} \\ \vdots & \ddots & \vdots \\ W_N^{(N-1)0} & \dots & W_N^{(N-1)(N-1)} \end{bmatrix}$$

$$W_N^{nk} = \frac{1}{N} e^{-j2\pi(nk/N)}$$

#### 3.2.1 Minimum mean square error estimation

The mean square error is given by

$$J(e) = E \left[ (H - \hat{H})^2 \right] = E \left[ (H - \hat{H})^H (H - \hat{H}) \right]$$

Here,  $\hat{H} = MY$ , where  $M$  is a linear estimator. Invoking the well-known orthogonality principle in order to minimize the mean square error vector  $e = (H - \hat{H})$ , has to be set orthogonal by the MMSE equalizer to the estimators input vector  $Y$ .

### 3.2.2) Least square error estimation

For least square error estimation:

$$J = (Y - XH)^T(Y - XH) = (Y^T - H^T X^T)(Y - XH)$$

For minimization of  $J$ , it is required to differentiate  $J$  with respect to  $H$  as

$$\left. \frac{\partial J}{\partial H} \right|_{\hat{H}} = 0 \quad \text{and the time domain LS estimate of } h \text{ is given by: } \hat{h} = F^T X^{-1} Y$$

### 3.3 Channel estimation based on comb type pilot arrangement

In comb-type based channel estimation, the  $np$  pilot signal are uniformly inserted into  $X(k)$  according to the following equation

$$X(K) = X(mL + l)$$

where  $L$ =number of carriers/ $np$ . Suppose that the frequency-selective channels remain invariant over an OFDM block, and length of the cyclic prefix exceeds the channel order. After demodulation, the received signal on the  $n$ th subcarrier corresponding to pilot symbols can be written as:

$$Y[k] = \sqrt{\varepsilon_p} H(k) X(n) + w(k), \quad k \in \mathfrak{F}_p$$

Where  $\mathfrak{F}_p$  denotes the set of subcarriers on which the pilot symbols are transmitted,  $\varepsilon_p$  is the transmitted power per pilot symbol,  $H(k)$  is the channel frequency response on  $k$ th carrier  $X(k)$ ,  $k \in \mathfrak{F}_p$ , is the pilot symbol and  $w(k)$  is the complex Additive White Gaussian Noise (AWGN) with zero mean and variance  $N_0/2$ . The received samples corresponding to information symbols can be expressed as:

$$Y[k] = \sqrt{\varepsilon_s} H(k) X(k) + w(k), \quad k \in \mathfrak{F}_s$$

Where,  $\varepsilon_s$  is the transmitted power per information in symbol, and  $\mathfrak{F}_s$  denotes the set of subcarriers on which the information symbols are transmitted.

### 3.3.1 Minimum mean square error estimation

With knowledge of channel statistics, channel estimation in MMSE may can be written as:

$$\hat{h} = R_{yh}^T R_{yy}^{-1} y$$

$$R_{yy} = E[yy^T] = \varepsilon_p D(X_p) F_p^T R_{hh} F_p D^T(X_p) + N_0 I_P$$

$$R_{yh} = E[yh^T] = \sqrt{\varepsilon_p} D(X_p) F_p^T R_{hh}$$

$$R_{hh} = E[hh^T] = \text{diag}(\sigma_h^2(0), \dots, \sigma_h^2(N-1))$$

The channel estimator is given by  $\epsilon = h - \hat{h}$ , which is Gaussian distributed with zero mean, and covariance  $R_\epsilon = E[\epsilon\epsilon^T] = (R_{hh}^{-1} + \varepsilon_p F_p F_p^T / N_0)^{-1}$  where  $\sigma_h^2(l) \neq 0, \forall l$  so that  $R_{hh}$  is invertible. The estimated channel frequency response of the  $n^{\text{th}}$  carrier can be obtained as:  $\hat{H}(k) = f_k^T \hat{h} = H(k) - \epsilon(k)$ , where  $\epsilon(k) = f_k^T \epsilon$  with  $\epsilon(k) \sim \text{CN}(0, \sigma_{\epsilon(k)}^2)$  and  $\sigma_{\epsilon(k)}^2 = f_k^T R_\epsilon f_k$ . The estimator  $\hat{H}(k)$  is Gaussian distributed with zero mean. Since the orthogonality principle renders  $\epsilon$  uncorrelated with  $\hat{h}$ ,  $\epsilon(k)$  and  $\hat{H}(k)$  are uncorrelated.

### 3.3.2) Least square error estimation

In this case,

$$G = (\varepsilon_p F_p D^T(X_p) D(X_p) F_p^T)^{-1} (\sqrt{\varepsilon_p} D(X_p) F_p^T)^T$$

Then the least square error (LSE) estimate of channel impulse response is given by

$$\hat{h} = Gy = h + \eta$$

Where  $\eta = Gw$ . Using the fact that  $D^T(X_p) D(X_p) = I_P$ , it follows readily that  $\eta \sim \text{CN}(0, F_p F_p^T)^{-1} N_0 / \varepsilon_p$ . The estimated channel frequency response on the  $k^{\text{th}}$  subcarrier

can be obtained as:  $\hat{H}(k) = f_k^T \hat{h} = H(k) + v(k)$  where  $v(k) \sim \text{CN}(0, \sigma_{v(k)}^2)$  with

$$\sigma_{v(k)}^2 = f_k^T (F_p F_p^T)^{-1} f_k N_0 / \varepsilon_p.$$

### 3.4 Implementation and Simulations Result

It has been mentioned that an OFDM system is implemented using Matlab and the aim is to measure the performance of simulated OFDM system under different channel conditions, and to allow for different OFDM configurations to be tested. The system, is designed using the following commands and functions in Matlab.

- **Random data generation:** The input random data is generated by *randn()* function in Matlab.
- **Serial to parallel conversion:** The input serial data stream is formatted into the word size required for transmission, e.g. 2bit/word for QPSK, and shifted into a parallel format using the command *reshape()*. The data is then transmitted in parallel by assigning each data word to one carrier in the transmission.
- **Modulating data:** The data to be transmitted on each carrier is modulated into a QAM and M-ary PSK format.
- **Inverse Fourier Transform:** The purpose of Inverse Fourier Transform is to find the corresponding time waveform. This is done using the command *IFFT* in Matlab. The guard period is then added to the start of each symbol.
- **Channel model:** A channel model is then applied to the transmitted signal. In this channel the signal-to-noise is varied and multipath path is then introduced. The signal to noise ratio is set by adding a known amount of white noise to the transmitted signal. The channels used are described below:

#### (1) **Block type pilot arrangement:**

The researchers for this paper indicated that, 16-QAM modulation scheme is used for a 64-subcarrier OFDM system, with a two-ray multipath channel. The channel impulse response  $h(t)$  is a time limited pulse train in the form of:

$$h(t) = \sum_m \alpha_m \delta(t - \tau_m T_s)$$

Where, the amplitudes  $\alpha_m$  are complex valued,  $\tau_m$  is  $m_{th}$  path delay and  $T_s$  is sampling time. Guard time  $T_G$  is taken such that  $0 \leq \tau_m T_s \leq T_G$ . The above continuous time



relationship can be represented as a discrete time version having discrete channel impulse response  $h(n)$  as:

$$h(n) = \sum_m \alpha_m e^{-j\frac{\pi(n+(N-1)\tau_m)}{N}} \frac{\sin(\pi\tau_m)}{\sin(\frac{\pi}{N}(\tau_m - n))}$$

In the simulation for the block type pilot arrangement, two ray multipath channels have been taken as:

$$h(t) = \delta(t - 0.5T_s) + \delta(t - 3.5T_s)$$

**2) comb type pilot arrangement:** In comb type pilot arrangement, Rayleigh-fading is considered, with channel impulse response  $h(t) = [h(0), \dots, h(L-1)]^T$ . Here,  $L=40$  is the number of taps and are uncorrelated complex Gaussian random variables with zero mean.

### 3.4.1 Results and discussions

In this paper, the researchers have used the performance of two types of estimators (LSE and MMSE estimators) which are theoretically and experimentally evaluated for both block type and comb type pilot arrangements. The estimators in this study can be used to efficiently estimate the channel in an OFDM system, given certain knowledge about channel statistics. The MMSE estimators assume a priori knowledge of noise variance and channel covariance. Moreover, its complexity is large compare to the LSE estimator. For high SNRs, the LSE estimator is both simple and adequate. The MMSE estimator has good performance but high complexity. They have revealed that, the LSE estimator has low complexity, but its performance is not as good as that MMSE estimator basically at low SNRs.

In comparison between block and comb type pilot arrangement, block type of pilot arrangement is suitable to use for slow fading channel, where channel impulse response is not changing very fast. Therefore, the channel estimated in one block of OFDM symbols through pilot carriers can be used in next block for recovery the data which are degraded by the channel. Comb type pilot arrangement is suitable to use for fast

fading channel where the channel impulse response is changing very fast, even if one OFDM block is present. Hence, comb type of pilot arrangement is not suitable in this case. Both data and pilot carriers in one block of OFDM symbols are used. Pilot carriers are used to estimate the channel impulse response. The estimated channel can be used to get back the data sent by transmitter certainly with some error. In the simulation, 1024 number of carriers in one OFDM block is used, in which one fourth are used for pilot carriers and rest are of data carriers. BER for different SNR conditions for M-PSK signaling is calculated. The researchers have also investigated the performances of LSE with MMSE estimator for channel estimation. They have concluded that MMSE estimation is better than LSE estimator in low SNRs; whereas at high SNRs, performance of LSE estimator approaches to MMSE estimator. Various interpolation techniques for channel estimation had also used. They have found that higher order interpolation technique (spline) has given better performance than lower order interpolation technique (linear).

#### **4. Deep learning approach to Channel Estimation and Signal Detection**

This section would present a method where deep learning is exploited as an end-to-end approach for channel estimation and symbol detection. The DNN model is trained based on simulated data offline, which views OFDM and the wireless channel as complete black boxes. Orthogonal frequency-division multiplexing (OFDM) is a popular modulation scheme that has been widely adopted in wireless broadband systems to combat frequency-selective fading in wireless channels. Channel state information (CSI) is vital to coherent detection and decoding in OFDM systems. Usually, the CSI can be estimated by means of pilots prior to the detection of the transmitted data. With the estimated CSI, transmitted symbols can be recovered at the receiver.

Historically, channel estimation in OFDM systems has been thoroughly studied. The traditional estimation methods, i.e., least square (LS) and minimum mean-square error

(MMSE), have been utilized and optimized in various conditions [15]. The method of LS estimation requires no prior channel statistics, but its performance may be inadequate. The MMSE estimation in general leads to much better detection performance by utilizing the second order statistics of channels. In this article, the authors have introduced a deep learning approach to channel estimation and symbol detection in an OFDM system. Deep learning and artificial neural networks (ANNs) have numerous applications. In particular, it has been successfully applied in localization based on CSI [16], channel equalization [17], and channel decoding [18] in communication systems. With the improving computational resources on devices and the availability of data in large quantity, we expect deep learning to find more applications in communication systems. ANNs have been demonstrated for channel equalization with online training, which is to adjust the parameters according to the online pilot data.

However, such methods cannot be applied directly since, with deep neural networks (DNNs), the number of parameters becomes much increased, which requires a large number of training data together with the burden of a long training period. To address the issue, the researchers have trained a DNN model that predicts the transmitted data in diverse channel conditions. Then the model is used in online deployment to recover the transmitted data. The researchers' have presented their initial results in deep learning for channel estimation and symbol detection in an end-to-end manner. It demonstrates that DNNs have the ability to learn and analyze the characteristics of wireless channels that may suffer from nonlinear distortion and interference in addition to frequency selectivity.

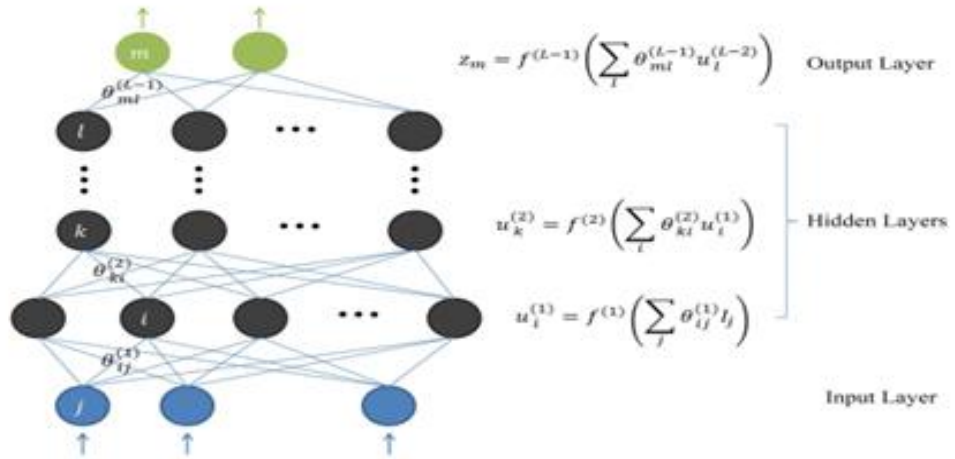
To the best of my review I did, this is the first attempt to use learning methods to deal with wireless channels without online training. The simulation results show that deep learning models achieve performance comparable to traditional methods if there are enough pilots in OFDM systems, and it can work better with limited pilots, channel

interference, and nonlinear noise. the initial research results indicate that deep learning can be potentially applied in many directions in signal processing and communications.

#### 4.1 Deep Learning Methods

Deep learning has been successfully applied in a wide range of areas with significant performance improvement, including computer vision [19], natural language processing [20], speech recognition [21], and so on. A comprehensive introduction to deep learning and machine learning can be found in [22]. The structure of a DNN model is shown in Fig. 4. Generally speaking, DNNs are deeper versions of ANNs by increasing the number of hidden layers in order to improve the ability in representation or recognition. Each layer of the network consists of multiple neurons, each of which has an output that is a nonlinear function of a weighted sum of neurons of its preceding layer, as shown in Fig. 4. The nonlinear function may be the Sigmoid function, or the Relu function, defined as

$$f_s(a) = \frac{1}{1+e^{-a}}, \text{ and } f_R(a) = \max(0, a), \text{ respectively.}$$



**Figure 4:** an example of deep learning models

Hence, the output of the network  $z$  is a cascade of nonlinear transformation of input data  $I$ , mathematically expressed as

$$Z = f(I, \theta) = f^{(L-1)} \left( f^{(L-2)} (\dots f^{(1)}(I)) \right)$$

where  $L$  stands for the number of layers, and  $\theta$  denotes the weights of the neural network. The parameters of the model are the weights for the neurons, which need to be optimized before the online deployment. The optimal weights are usually learned on a training set, with known desired outputs.

## 4.2 System architecture

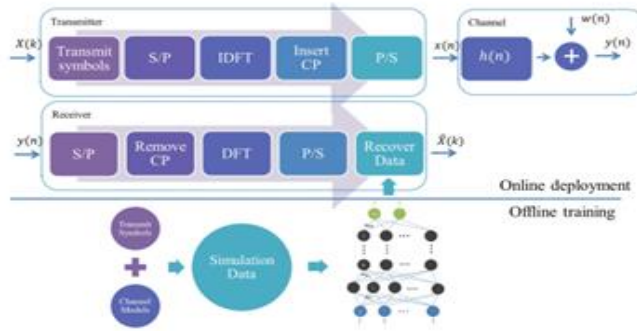


Figure 5: System model

The architecture of the OFDM system with deep learning-based channel estimation and signal detection is illustrated in Fig. 5. The baseband OFDM system is the same as the conventional ones. On the transmitter side, the transmitted symbols inserted with pilots are first converted to a paralleled data stream, then the inverse discrete Fourier transform (IDFT) is used to convert the signal from the frequency domain to the time domain. After that, a cyclic prefix (CP) is inserted to mitigate the inter-symbol interference (ISI). The length of the CP should be no shorter than the max delay spread of the channel.

We consider a sample-spaced multi-path channel described by complex random variable  $\{h(n)\}_{n=0}^{N-1}$ . Thus, the received signal,  $y(n)$ , can be expressed as  $y(n) = x(n) \otimes h(n) + w(n)$ , where  $\otimes$  denotes the circular convolution while  $x(n)$  and  $w(n)$  represent the transmitted signal and the additive white Gaussian noise (AWGN), respectively. After removing the CP and performing DFT, the received frequency domain signal is  $Y$

$Y(k) = X(k)H(k) + W(k)$ , where  $Y(k)$ ,  $X(k)$ ,  $H(k)$ , and  $W(k)$  are the DFT of  $y(n)$ ,  $x(n)$ ,  $h(n)$  and  $w(n)$ , respectively. We assume that the pilot symbols are in the first OFDM block while the following OFDM blocks consist of the transmitted data. Together they form a *frame*. The channel can be treated as constant spanning over the pilot block and the data blocks, but change from one frame to another.

The DNN model takes as input the received data consisting of one pilot block and one data block in our initial study, and recovers the transmitted data in an end-to-end manner. As shown in Fig. 2, to obtain an effective DNN model for joint channel estimation and symbol detection, two stages are included. In the offline training stage, the model is trained with the received OFDM samples that are generated with variant information sequences and under diverse channel conditions with certain statistical properties, such as typical urban or hilly terrain delay profile. In the online deployment stage, the DNN model generates the output that recovers the transmitted data without explicitly estimating the wireless channel.

### 4.3 Model Training

The researchers' have trained the models by viewing OFDM modulation and the wireless channels as black boxes. Historically, researchers have developed many channel models for CSI that well describe the real channels in terms of channel statistics. With these channel models, the training data can be obtained by simulation. In each simulation, a random data sequence is first generated as the transmitted symbols and the correspondent OFDM frame is formed with pilot symbols. The current random channel state is simulated based on the channel models. The received OFDM signal is obtained based on the OFDM frames undergoing the current channel distortion, including the channel noise. The received signal and the original transmitted data are collected as the training data. The model is trained to minimize the difference between the output of the neural network and the transmitted data. The difference can be portrayed in several ways. In our experiment settings, we choose the L2 loss,

$$L_2 = \frac{1}{N} \sum_k (\hat{X}(k) - X(k))^2,$$

where  $\hat{X}(k)$  is the prediction and  $X(k)$  is the supervision message, which is the transmitted symbols in this situation. The DNN model we use consists of five layers, three of which are hidden layers. The numbers of neurons in each layer are 256, 500, 250, 120, 16. The input number corresponds to the number of real parts and imaginary parts of 2 OFDM blocks that contain the pilots and transmitted symbols, respectively. Every 16 bits of the transmitted data are grouped and predicted based on a single model trained independently, which is then concatenated for the final output. The Relu-function, is used as the activation function in most layers except in the last layer where the Sigmoid function is applied to map the output to the interval  $[0, 1]$ .

#### 4.4 Analysis of simulation results

The researchers have demonstrated their initial efforts to apply the power of deep learning approach to channel estimation and symbol detection in an OFDM system. They have trained their model offline based on the simulated data that view OFDM and the wireless channels as black boxes. The simulation results have shown that the deep learning method have advantages when wireless channels are complicated by serious distortion and interference, which proves that DNNs have the ability to remember and analyze the complicated characteristics of the wireless channels.

For real world applications, it is important for the DNN model to have a good generalization ability so that it can still work effectively when the conditions of online deployment do not exactly agree with the channel models used in the training stage. An initial experiment has been conducted in this article to illustrate the generalization ability of DNN model with respect to some parameters of the channel model. More rigorous analysis and more comprehensive experiments are left for the future work. In addition, for practical use, samples generated from the real wireless channels could be collected to retrain or fine-tune the model for better performance.

## **5. Conclusions and discussions**

The papers I have surveyed basically focuses on investigation and comparing various efficient pilot-based channel estimation schemes for OFDM systems. In this survey, two major types of pilot arrangement such as block type and comb-type pilot have been focused employing Least Square Error (LSE) and Minimum Mean Square Error (MMSE) channel estimators. Block type pilot sub-carriers is especially suitable for slow-fading radio channels whereas comb type pilots provide better resistance to fast fading channels.

Moreover, to address channel distortion, the researchers have applied a deep learning model and was first trained offline using the data generated from simulation based on channel statistics and then used for recovering the online transmitted data directly. From their simulation results, the deep learning-based approach can address channel distortion and detect the transmitted symbols with performance comparable to the minimum mean square error (MMSE) estimator.

Furthermore, the deep learning-based approach is more robust than conventional methods when fewer training pilots are used, the cyclic prefix (CP) is omitted, and nonlinear clipping noise exists. In summary, it has been pointed out that, deep learning is a promising tool for channel estimation and signal detection in wireless communications with complicated channel distortion and interference.



## 6. References

1. H. Meyr, M. Moeneclaey, and S. A. Fechtel, "Digital Communication Receivers", John Wiley and Sons, 1998.
2. A.R.S. Bahai, B. R. Saltzberg Multi-carrier digital communications: theory and applications of OFDM Kluwer Academic/Plenum, 1999.
3. J.-J van de Beek, O. Edfors, M. Sandell, S.K. Wilson and P.O. Borjesson, On channel estimation in OFDM systems in Proc. IEEE 45th Vehicular Technology Conference, Chicago, IL, Jul. 1995, pp. 815-819
4. O. Edfors, M. Sandell, J.-J. van de Beek, S.K. Wilson, and P.O. Brjesson. OFDM channel estimation by singular value decomposition. IEEE Transactions on Communications, vol. 46, no. 7, pp. 931-939, July 1998.
5. M. Hsieh and C. Wei, Channel estimation for OFDM systems based on comb-type pilot arrangement in frequency selective fading channels in IEEE Transactions on Consumer Electronics, vol. 44, no.1, February 1998
6. R. Steele, Mobile Radio Communications, London, England, Pentech Press Limited, 1992
7. Z. Wang and G. B. Giannakis, "Complex-field coding for OFDM over fading wireless channels," IEEE Trans. Inform. Theory, vol. 49, no. 3, pp. 707-720, Mar. 2003.
8. X. Cai, S. Zhou, and G. B. Giannakis, "Group-orthogonal multicarrier CDMA," IEEE Trans. Commun., vol. 52, no. 1, pp. 90-99, Jan. 2004.
9. P. H. Moose, "A technique for orthogonal frequency division multiplexing frequency offset correction," IEEE Trans. Commun., vol. 42, no. 10, pp. 2908-2914, Oct. 1994.
10. T. M. Schmidl and D. C. Cox, "Robust frequency and timing synchronization for OFDM," IEEE Trans. Commun., vol. 45, no. 12, pp. 1613-1621, Dec. 1997.

11. Y. Li, H. Minn, N. Al-Dhahir, and A. R. Calderbank, "Pilot designs for consistent frequency-offset estimation in OFDM systems," *IEEE Trans. Commun.*, vol. 55, no. 5, pp. 864-877, May 2007.
12. Moon, JAe Kyoung and Choi, Song In., "Performance of channel estimation methods for OFDM systems in multipath fading channels." *IEEE Transaction on Communication Electronics*. vol. 46, (February 2000): pp. 161-170.
13. P. Hoeher, S. Kaiser, and P. Robertson, "Two-dimensional pilot-symbol-aided channel estimation by Wiener filtering," in *Proc. ICASSP97*, Munich, German, vol. 3, Apr. 1997, pp. 1845-1848.
14. Y. Shen and E. Martinez, "Channel estimation in OFDM systems," *Freescall Semiconductor*, pp. 1-15, 2006.
15. Y. G. Li, L. J. Cimini, and N. R. Sollenberger, "Robust channel estimation for OFDM systems with rapid dispersive fading channels," *IEEE Trans. Commun.*, vol. 46, no. 7, pp. 902-915, Jul. 1998.
16. X. Wang, L. Gao, S. Mao and S. Pandey, 2017. "CSI-based fingerprinting for indoor localization: A deep learning approach," *IEEE Trans. Veh. Technol.*, vol. 66, no. 1, pp. 763-776, Jan. 2017.
17. E. Nachmani, Y. Beery, and D. Burshtein, "Learning to decode linear codes using deep learning," *54'th Annual Allerton Conf. On Commun., Control and Computing*, Mouticello, IL, Sept. 2016
18. S. Chen, G. Gibson, C. Cown, and P. Grant, "Adaptive equalization of finite nonlinear channels using multilayer perceptrons," *IEEE Trans. Signal Process.*, vol. 20, no. 2, pp. 107-119, Jun. 1990.
19. A. Krizhevsky, I. Sutskever, and G. E. Hinton, "Imagenet classification with deep convolutional neural networks," in *Proc. Adv. Neural Inf. Process. Syst.*, 2012, pp. 1097-1105.

20. K. Cho *et al.* “Learning phrase representations using RNN encoder- decoder for statistical machine translation.” [Online]. <http://arxiv.org/abs/1406.1078>, 2014.  
Available
21. C. Weng, D. Yu, S. Watanabe, and B. H. F. Juang, “Recurrent deep neural networks for robust speech recognition,” in *Proc. ICASSP*, May 2014, pp. 5532–5536
22. J. Schmidhuber, “Deep learning in neural networks: An overview,” *Neural Networks*, vol. 61, pp. 85–117, Jan. 2015.