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A statistical CSI model for indoor positioning using fingerprinting

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Abstract: Indoor positioning methods are still inaccurate, as they use Wi-Fi signals which are distorted by multipath propagation. A promising approach consists in using channel state information (CSI) for fingerprinting. We propose a new method for this approach. To do so, we start by building a statistical model of the CSI information of the room where positioning needs to be resolved. The key of our method lies on how we build this model. We do so by considering the room model as a realization of a random model, and building the posterior distribution of the room model given the measurements acquired during initialization. This permits obtaining an accurate interpolation of the CSI behavior of the room at points where CSI was not initially measured. Once the statistical room model is built, the position is estimated using the maximum likelihood criterion. We present experimental results showing the higher positioning accuracy of the proposed method, in comparison with other available alternatives.

Key Words: Indoor Positioning, Fingerprint Positioning, Channel State Information

1 Introduction

Reliable outdoor positioning can be achieved using the global positioning system (GPS). Indoor positioning systems are typically based on Wi-Fi signals. This is because of its widespread compared with other signals such as Bluetooth, ultrasound, Zigbee, etc. However, these positioning systems are not highly reliable due to the inaccuracies caused by the multipath propagation of Wi-Fi signals. Obtaining a reliable indoor positioning system is currently an active research problem due the increasing demand for a solution.

There are two main approaches for indoor localization, namely, geometric mapping and fingerprinting [1]. Geometric mapping relies on the estimation of geometric parameters, e.g., angles or distance with respect to certain reference points. A drawback of this approach is that, as mentioned above, the multipath effects caused by indoor obstacles cause serious estimation errors. To go around this, the fingerprinting approach builds a database of certain features of the received Wi-Fi signals, representing the multipath effect at different locations. The localization problem then becomes a problem of matching the characteristics of the signal received at an unknown location, with the information available within the database.

A key aspect of designing an fingerprint-based indoor positioning system is how to choose Wi-Fi signal features, so that positions can be uniquely determined from them. In [2], this feature was chosen to be a vector of received signal strength indications (RSSIs) from a number of Wi-Fi access points (APs). The position is estimated as the one from the database whose feature is the closest from the measured one. A drawback of this method is that all positions are assigned to one from the database. To solve this, Youssef [3] assumes that signals obtained from different APs are statistically independent and builds a posterior probability of the unknown position. The position is then estimated using the maximum a posteriori criterion.

The advantage of using RSSI for fingerprinting lies in its ease of computation. However, while this approach yields accurate localization estimates at positions close to the points used to build the database, it often significantly looses accuracy at other points. This is because RSSI neglects useful information available at Wi-Fi signal, which could be used to improve the localization accuracy. This extra information is used by designs based on channel state information (CSI), i.e., the phase and amplitude of the OFDM channel gain at each sub-carrier [1]. Within this line, a method called fine-grained indoor fingerprinting system (FIFS) was proposed in [4]. In this method, the square sum of channel gain amplitudes is used as fingerprint. Then, Pearson correlation coefficients between the measured fingerprint and those in the database are used to estimate the position as a weighted sum of the corresponding positions within the database. Chapre [5] developed an indoor positioning system using the amplitude and phase difference of adjacent subcarriers as their fingerprints. Also, Wang Xuyu [6, 7] used a linear transformation to eliminate the phase bias from CSIs, and estimate the position using a deep autoencoder. A drawback of these methods, however, is that their positioning accuracy, at points outside the database grid, is very sensitive to errors in the database.

In this paper we propose a new method for CSI-based indoor localization using fingerprinting. Our method consists in two stages. In the first one, we build a statistical model for CSI fingerprints within the room, i.e., a function mapping positions to CSIs. To this end, we assume that the room's model is a realization of a random mapping function. In the second stage, using this model, we

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estimate the position. To this end, we build the posterior probability of the CSI mapping function, using the information available in the database, and then estimate the position using the maximum likelihood criterion. The key advantage of our approach is that the use of a statistical model for the room's CSIs permits a smooth interpolation of CSI patterns at positions outside the grid used to build the database. This leads to more accurate position estimates at those points. We present numerical experiments confirming this claim.

The rest of the paper is organized as follows: In Section 2 we describe the problem. In Section 3 we introduce the statistical model used for CSI fingerprints. In Section 4 we describe the two stages of our proposed positioning method. In Section 5 we present experimental results and we give concluding remarks in Section 6.

2 Problem description

We have a number A of WiFi access points (APs) located in a room ($\mathbb{R}^2$). There is also a receptor, located at $x \in \mathbb{R}^2$, having B antennas. The communication from AP a to antenna b is done through an OFDM channel with sub-carrier frequencies $f_m \in \mathbb{R}$, $m = 1, \cdots, M$. We use $h^{(a,b)} \in \mathbb{R}^M$ to denote the set of frequency gains of this channel. We also use $h^{(a)\top} = \left[h^{(a,1)\top}, \cdots, h^{(a,B)\top}\right]$ to denote all frequency gains from AP a to all receptor antennas and $h^\top = \left[h^{(1)\top}, \cdots, h^{(A)\top}\right]$ to denote the set of all frequency gains from all APs to all receptor antennas. We refer to $h \in \mathbb{R}^N$, with $N = MAB$, as the CSI at the receptor's position x. Fig. 1 shows a number of measurements of a CSI with one AP and one receiving antenna.

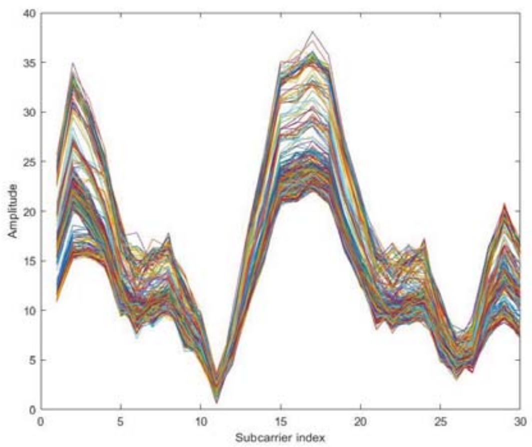

Fig. 1: Different measurements of a CSI with $M = 30$ frequencies, $A = 1$ AP, and $B = 1$ receiving antenna.

We assume that CSI values are relatively stable at a given position, and vary continuously with the receptor's position and the sub-carrier frequency. Based on this assumption, our goal is to estimate the receptor's location x. The approach for doing so is composed of two stages. In the first (initialization) stage, we build a model for the room's CSI values. To this end we have a set of points $p_i \in \mathbb{R}^2$, $i = 1, \cdots, I$, and for each point $i = 1, \cdots, I$, we have K measurements $g_{i,k} \in \mathbb{R}$, $k = 1, \cdots, K$ of the CSI value at p_i. In the second (positioning) stage, we use the model built in the initialization stage to estimate the receptor's location x, given the CSI value h measured at x. Fig. 2 summarizes this two-stage procedure.

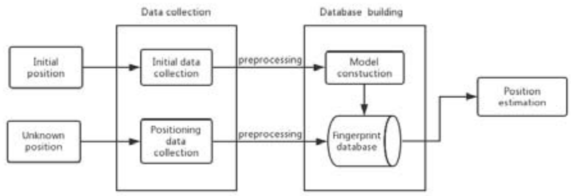

Fig. 2: Positioning strategy including model building and position estimation.

Notation 1. We use $\mathbf{p}^\top = \left[p_1^\top, \cdots, p_I^\top\right]$ to denote the vector of all measuring positions. We also use $\mathbf{g}_k^\top = \left[g_{1,k}^\top, \cdots, g_{I,k}^\top\right]$ to denote the vector of all measurements at all positions $\mathbf{p}$, and $\bar{\mathbf{g}} = \frac{1}{K}\sum_{k=1}^{K} \mathbf{g}_k$ to denote its average.

3 Proposed model for CSI

We assume that the measurements $h, g_{i,k} \in \mathbb{R}^N$, $i = 1, \cdots, I$, $k = 1, \cdots, K$, are normal random vectors given by

$$h = m(x) + w,$$
$$g_{i,k} = m(p_i) + w_{i,k},$$

with $w, w_{i,k} \sim \mathcal{N}(0, Q)$ being jointly independent. We also assume that the function $m : \mathbb{R}^2 \to \mathbb{R}^N$ is a realization of a normal random process indexed by $\mathbb{R}^2$ (i.e., for each $x \in \mathbb{R}^2$, $m(x) \in \mathbb{R}^N$ is a normal random vector), satisfying that, for $x, y \in \mathbb{R}^2$,

$$\begin{bmatrix} m(x) \\ m(y) \end{bmatrix} \sim \mathcal{N}\left(0, \begin{bmatrix} C(x,x) & C(x,y) \\ C(y,x) & C(y,y) \end{bmatrix}\right).$$

To define the covariance map $C : \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}^N$ we use

$$[C(x,y)]_{n,m} = \sigma^2 \Pi(x,y) [\Phi]_{n,m},$$

where $\Phi \in \mathbb{R}^{N \times N}$ and the map $\Pi_{a,\alpha} : \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$ is given by a rational quadratic covariance function, i.e.,

$$\Pi_{a,\alpha}(x,y) = \left(1 + \frac{\|x-y\|^2}{2\alpha a^2}\right)^{-\alpha}.$$

4 Positioning method

The model described in Section 3 depends on the unknown parameters $\eta = \left\{\sigma^2, a, \alpha, Q, \Phi\right\}$. Since also the receptor's position x is unknown, a natural choice for solving the positioning problem is to use the maximum likelihood (ML) criterion, i.e.,

$$(\hat{x}, \hat{\eta}) = \arg\max_{x,\eta} \mathcal{P}(\mathbf{h}, \bar{\mathbf{g}}; x, \eta). \quad (1)$$

The problem with the above approach is that solving (1) is numerically very expensive. In order to go around this, we propose below a numerically tractable approximation of (1). As already explained, this solution is composed of two stages. In the first (initialization) stage, we use $\mathbf{p}$ and $\bar{\mathbf{g}}$ to estimate the model parameters η. In the second (positioning) stage, we use η and h, to estimate x.

4.1 Initialization stage: Estimation of the CSI model parameters η

In this section we describe how to obtain an estimation $\hat{\eta} = \left\{ \hat{\sigma}^2, \hat{a}, \hat{\alpha}, \hat{Q}, \hat{\Phi} \right\}$ of η.

4.1.1 Estimation of Q

We obtain an estimation $\hat{Q}$ of Q by averaging all available CSI measurements, i.e.,

$$\left[\hat{Q}\right]_{n,m} = \frac{1}{I} \sum_{i=1}^{I} \frac{1}{K-1} \sum_{k=1}^{K} \left([g_{i,k}]_n - [\bar{\mathbf{g}}_i]_n \right) \left([g_{i,k}]_m - [\bar{\mathbf{g}}_i]_m \right),$$

where

$$\bar{\mathbf{g}}_i = \frac{1}{K} \sum_{k=1}^{K} g_{i,k}.$$

4.1.2 Estimation of Φ

We estimate Φ using

$$\left[\hat{\Phi}\right]_{n,m} = \frac{1}{I-1} \sum_{i=1}^{I} \left([\bar{\mathbf{g}}_i]_n - [\bar{\bar{\mathbf{g}}}]_n \right) \left([\bar{\mathbf{g}}_i]_m - [\bar{\bar{\mathbf{g}}}]_m \right),$$

where

$$\bar{\bar{\mathbf{g}}} = \frac{1}{I} \sum_{i=1}^{I} \bar{\mathbf{g}}_i. \tag{2}$$

4.1.3 Estimation of σ^2, a and α

We start by building a sample approximation of Π evaluated at the points p_i, $i = 1, \cdots, I$,

$$\hat{\Pi}(p_i, p_j) = \frac{1}{N-1} \sum_{n=1}^{N} \left([\bar{\mathbf{g}}_i]_n - \bar{\bar{\mathbf{g}}}_i \right) \left([\bar{\mathbf{g}}_j]_n - \bar{\bar{\mathbf{g}}}_j \right),$$

where

$$\bar{\bar{\mathbf{g}}}_i = \frac{1}{N} \sum_{n=1}^{N} [\bar{\mathbf{g}}_i]_n .$$

We then compute

$$(\hat{\sigma}, \hat{a}, \hat{\alpha}) = \arg\max_{\sigma,a,\alpha} \sum_{i,j=1}^{I} \left(\sigma^2 \Pi_{a,\alpha}(p_i, p_j) - \hat{\Pi}(p_i, p_j) \right)^2 . \tag{3}$$

4.2 Positioning stage: Estimation of the receptor's position x

Since we know an estimation $\hat{\eta}$ of the CSI model parameters η, we can estimate the receptor's position by solving

$$\begin{aligned} \hat{x} &= \arg\max_x \mathcal{P}(h, \bar{\mathbf{g}}; x, \hat{\eta}) \\ &= \arg\max_x \mathcal{P}(h|\bar{\mathbf{g}}; x, \hat{\eta}) \\ &= \arg\max_x \mathcal{N}(h; \mu(x), \Sigma(x)), \end{aligned}$$

where

$$\mu(x) = \mathcal{E}\left\{ h\bar{\mathbf{g}}^\top \right\} \mathcal{E}^{-1}\left\{ \bar{\mathbf{g}}\bar{\mathbf{g}}^\top \right\} \bar{\mathbf{g}}, \tag{4}$$

$$\Sigma(x) = \mathcal{E}\left\{ h\mathbf{h}^\top \right\} - \mathcal{E}\left\{ h\bar{\mathbf{g}}^\top \right\} \mathcal{E}^{-1}\left\{ \bar{\mathbf{g}}\bar{\mathbf{g}}^\top \right\} \mathcal{E}\left\{ \bar{\mathbf{g}}h^\top \right\}. \tag{5}$$

Now,

$$\mathcal{E}\left\{ h\bar{\mathbf{g}}^\top \right\} = \hat{\sigma}^2 \mathbf{\Pi}_{\hat{a},\hat{\alpha}}(x, \mathbf{p}) \otimes \mathbf{\Phi}, \tag{6}$$

$$\mathcal{E}\left\{ \bar{\mathbf{g}}\bar{\mathbf{g}}^\top \right\} = \hat{\sigma}^2 \mathbf{\Pi}_{\hat{a},\hat{\alpha}}(\mathbf{p}, \mathbf{p}) \otimes \mathbf{\Phi} + \frac{1}{K} \mathbf{I}_I \otimes Q, \tag{7}$$

$$\mathcal{E}\left\{ hh^\top \right\} = \hat{\sigma}^2 \Pi_{\hat{a},\hat{\alpha}}(x, x) \otimes \mathbf{\Phi} + Q, \tag{8}$$

where

$$\mathbf{\Pi}_{\hat{a},\hat{\alpha}}(x, \mathbf{p}) = \left[\Pi_{\hat{a},\hat{\alpha}}(x, p_1), \cdots, \Pi_{\hat{a},\hat{\alpha}}(x, p_I) \right],$$

$$\mathbf{\Pi}_{\hat{a},\hat{\alpha}}(\mathbf{p}, \mathbf{p}) = \begin{bmatrix} \Pi_{\hat{a},\hat{\alpha}}(p_1, p_1) & \cdots & \Pi_{\hat{a},\hat{\alpha}}(p_1, p_I) \\ \vdots & \ddots & \vdots \\ \Pi_{\hat{a},\hat{\alpha}}(p_I, p_1) & \cdots & \Pi_{\hat{a},\hat{\alpha}}(p_I, p_I) \end{bmatrix}.$$

If the number K used to average measurements is very large, we can neglect the second summand in (7). Then, putting (6)-(8) into (4) we obtain

$$\begin{aligned} \mu(x) &\simeq \left[\mathbf{\Pi}_{\hat{a},\hat{\alpha}}(x, \mathbf{p}) \otimes \mathbf{\Phi} \right] \left[\mathbf{\Pi}_{\hat{a},\hat{\alpha}}(\mathbf{p}, \mathbf{p}) \otimes \mathbf{\Phi} \right]^{-1} \bar{\mathbf{g}} \\ &= \left[\mathbf{\Pi}_{\hat{a},\hat{\alpha}}(x, \mathbf{p}) \otimes \mathbf{\Phi} \right] \left[\mathbf{\Pi}^{-1}_{\hat{a},\hat{\alpha}}(\mathbf{p}, \mathbf{p}) \otimes \mathbf{\Phi}^{-1} \right] \bar{\mathbf{g}} \\ &= \left[\mathbf{\Pi}_{\hat{a},\hat{\alpha}}(x, \mathbf{p}) \mathbf{\Pi}^{-1}_{\hat{a},\hat{\alpha}}(\mathbf{p}, \mathbf{p}) \otimes \mathbf{I}_N \right] \bar{\mathbf{g}} \\ &= \left[\mathbf{\Pi}_{\hat{a},\hat{\alpha}}(x, \mathbf{p}) \otimes \mathbf{I}_N \right] \theta, \end{aligned}$$

with

$$\theta = \left[\mathbf{\Pi}^{-1}_{\hat{a},\hat{\alpha}}(\mathbf{p}, \mathbf{p}) \otimes \mathbf{I}_N \right] \bar{\mathbf{g}}.$$

Also, putting the same equations into (5) we get

$$\begin{aligned} \Sigma(x) \simeq& \hat{\sigma}^2 \Pi_{\hat{a},\hat{\alpha}}(x, x) \otimes \mathbf{\Phi} + Q \qquad (9) \\ &- \hat{\sigma}^2 \left[\mathbf{\Pi}_{\hat{a},\hat{\alpha}}(x, \mathbf{p}) \mathbf{\Pi}^{-1}_{\hat{a},\hat{\alpha}}(\mathbf{p}, \mathbf{p}) \mathbf{\Pi}^\top_{\hat{a},\hat{\alpha}}(x, \mathbf{p}) \otimes \mathbf{\Phi} \right] \\ =& Q + R(x), \qquad (10) \end{aligned}$$

with

$$\begin{aligned} R(x) =& \hat{\sigma}^2 \left[\Pi_{\hat{a},\hat{\alpha}}(x, x) \right., \\ &\left. - \mathbf{\Pi}_{\hat{a},\hat{\alpha}}(x, \mathbf{p}) \mathbf{\Pi}^{-1}_{\hat{a},\hat{\alpha}}(\mathbf{p}, \mathbf{p}) \mathbf{\Pi}^\top_{\hat{a},\hat{\alpha}}(x, \mathbf{p}) \right] \otimes \mathbf{\Phi}, \end{aligned}$$

Equation (10), indicates that the covariance of the CSI measurement h at x has two component. The first one, Q, depends on the measurement noise w. The second one, $R(x)$, depends on the uncertainty of the CSI at x. In other words, if x is close to one of the measuring points p_i used to build the model, $R(x)$ is very small. Hence, if the measuring grid is dense enough, we can neglect $R(x)$. We then obtain $\Sigma(x) = Q$ and

$$\hat{x} = \arg\max_x \mathcal{N}(h; \mu(x), Q).$$

5 Experiments

In this section we compare the positioning accuracy of our proposed CSI-based fingerprinting method, with those of the FIFS method [4] and the RSSI method [2]. To this end, we use the public CSI Tool [8]. This tool gives, for each package received from each AP at each receiving antenna, a vector of 30 complex number representing the average amplitude and phase of 30 subcarrier groups from the the 56 subcarriers of a 20 MHz channel or the 114 carriers in a 40 MHz channel of the IEEE 802.11n standard. We then build the CSI by forming, for each received package, a vector with all amplitudes from all subcarrier groups, all APs and all receiving antennas. We used a room with two round tables and placed the Wi-Fi transmitter in a corner, as depicted in Fig, 3. The figure also shows the 61 positions (labeled as initialization points) we used to build the CSI fingerprint database.

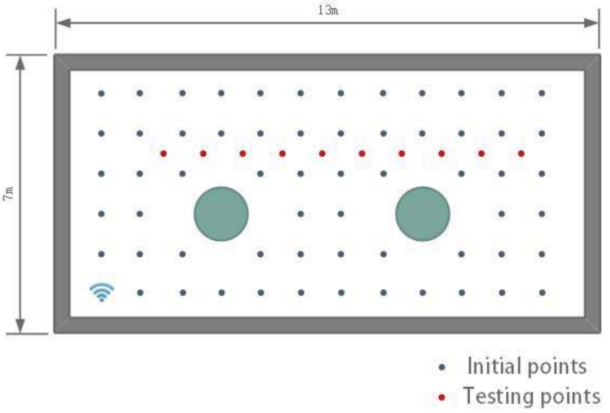

Fig. 3: Room setup for method comparison. The two big circles in the middle are tables, and the Wi-Fi transmitter appears in the lower-left corner.

In the first experiment we used ten testing points to compare the positioning accuracy of the three methods. Fig.4 shows, for each error (in meters) the proportion of testing points whose positioning accuracy is smaller that this error. We see that our method makes substantial improvement over the FIFS and RSSI methods.

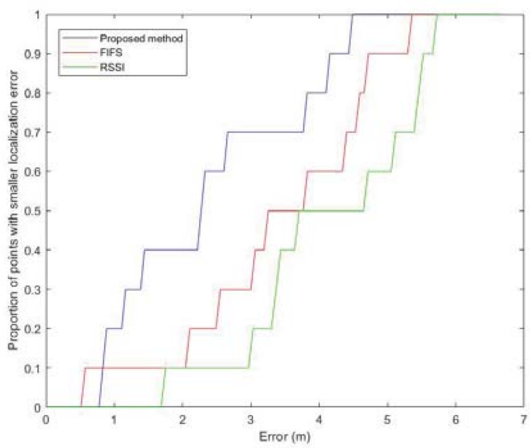

Fig. 4: Localization error vs. proportion of testing points with smaller positioning error.

In the second experiment we show how the positioning accuracy of our proposed method improves with the number of Wi-Fi APs. To this end, we use a simple room model, based on two-dimensional trigonometric expansions, to synthesize signals from an arbitrary number of APs. Fig. 5 shows how the mean squared positioning error decreases with an increase in the number of APs.

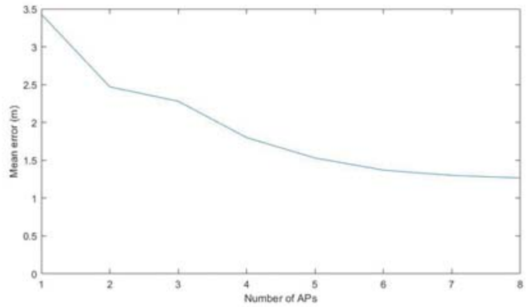

Fig. 5: Mean squared positioning error vs. number of APs

6 Conclusion

We proposed a new method for indoor positioning based on fingerprinting and using CSI. To this end, we build a statistical model of the CSI patterns of the whole room. The key component of our method consists in considering this model as a realization of a random model, and building the posterior distribution of the room model given the information available in the database. This yields an accurate interpolation of CSI patterns at points outside the grid used to build the fingerprinting database. Once this statistical model is built, the position is estimated using the maximum likelihood criterion. We present experimental results showing the higher positioning accuracy of our method, when compared with other available alternatives.

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