Chapter 5

Digital transmission through the AWGN channel

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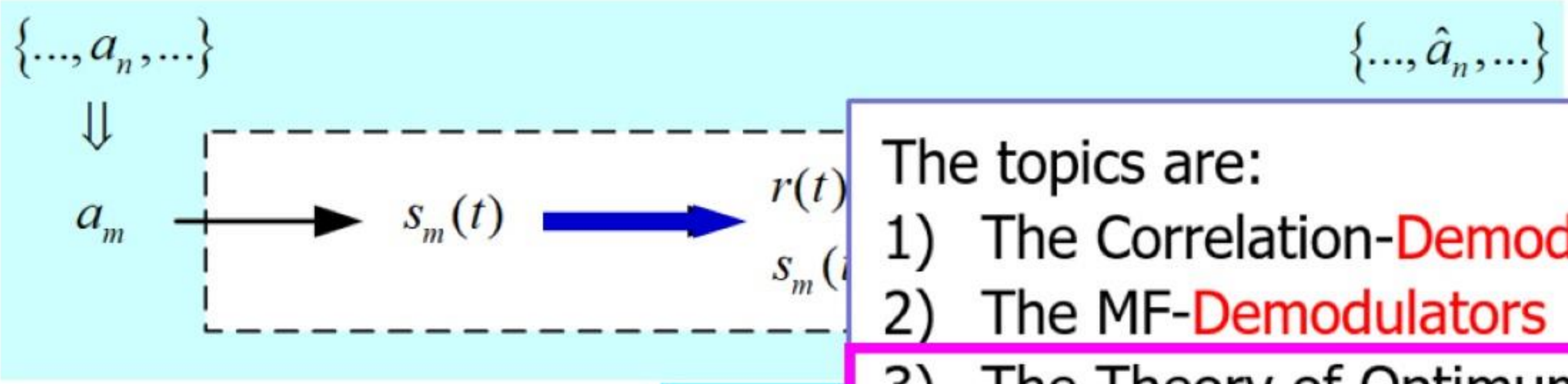
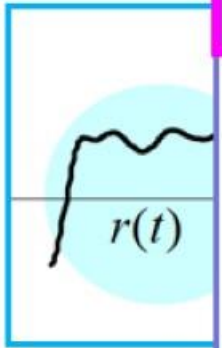
Contents 16 Hrs

- Introduction
- Geometric rep. of the sig waveforms
- Pulse amplitude modulation
- 2-d signal waveforms
- M-d signal waveforms
- Opt. reception for the sig. In AWGN
- Optimal receivers and probs of err

5.5 Opt. reception for the sig. In AWGN

In the nth interval, the process is as follows,





The topics are:

1) The Correlation-Demodulators
2) The MF-Demodulators
3) The Theory of Optimum Detector

5.6 Four Typical Optimal receivers

- Carrier-amp (1-D signals)
- Carrier-phs (2-D signals)
- QAM (2-D signals)
- FSK (M-D orthogonal signals)

It is convenient to divide the receiv

1)**Demodulator**: produce an obse

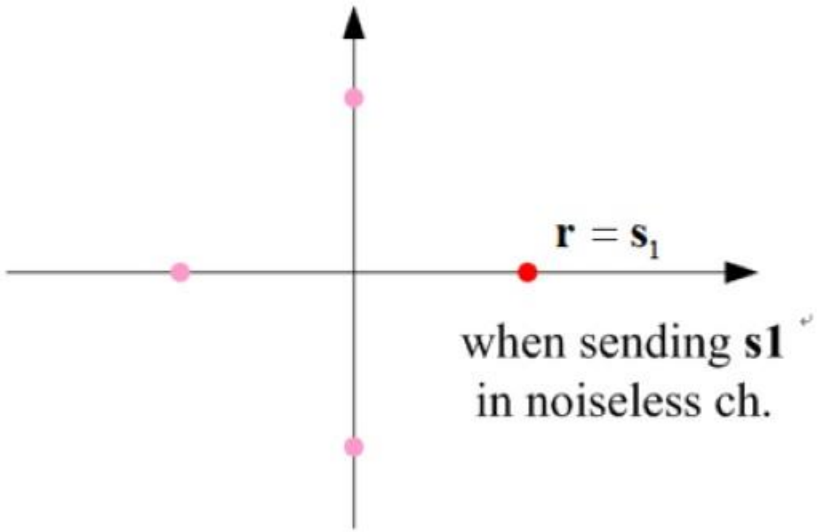
2)**Detector**: estimate the sym from the observation

5.5.2 The optimum detector

Detector: estimate the sym from the observation

What is the problem in the detector ?

With observation $\mathbf{r} = (r_1, \ldots, r_N)$ in hand, we recall that $\mathbf{r} = \mathbf{s}_m + \mathbf{n}$ and picture it.



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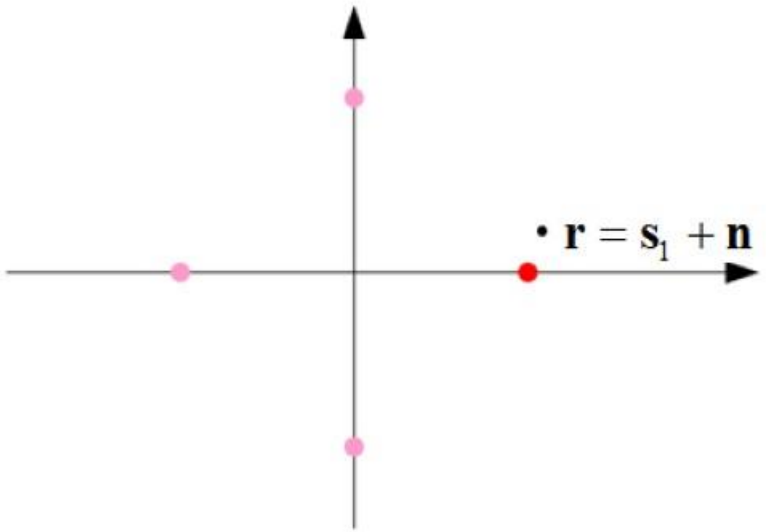

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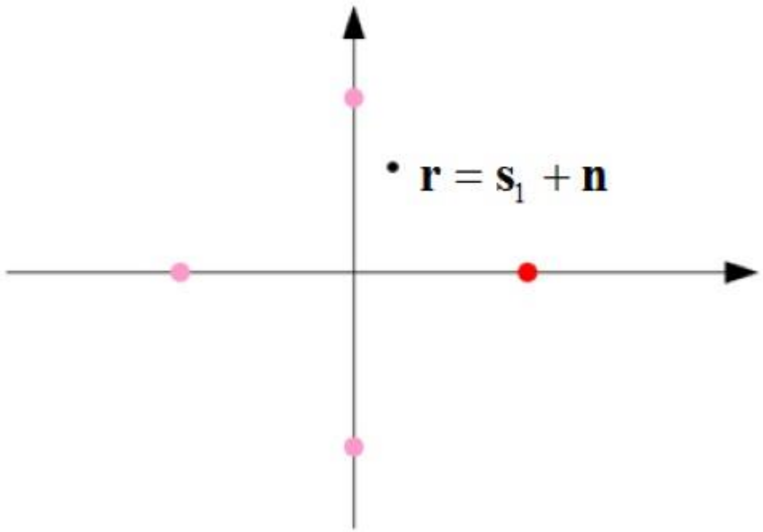

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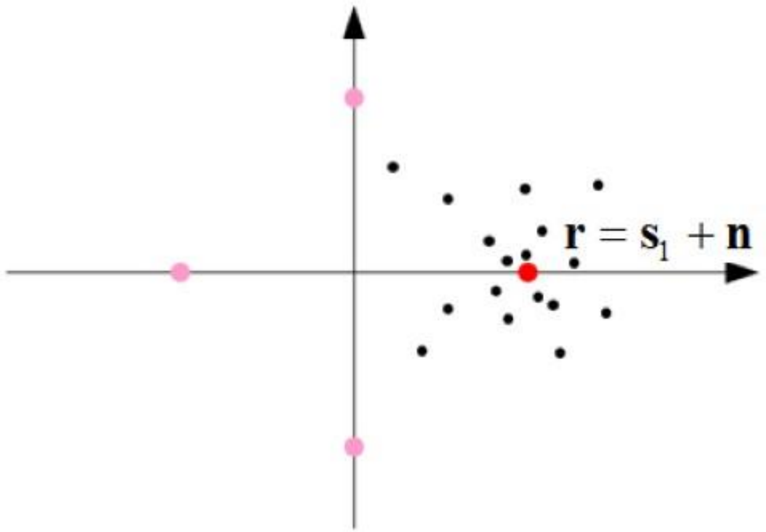
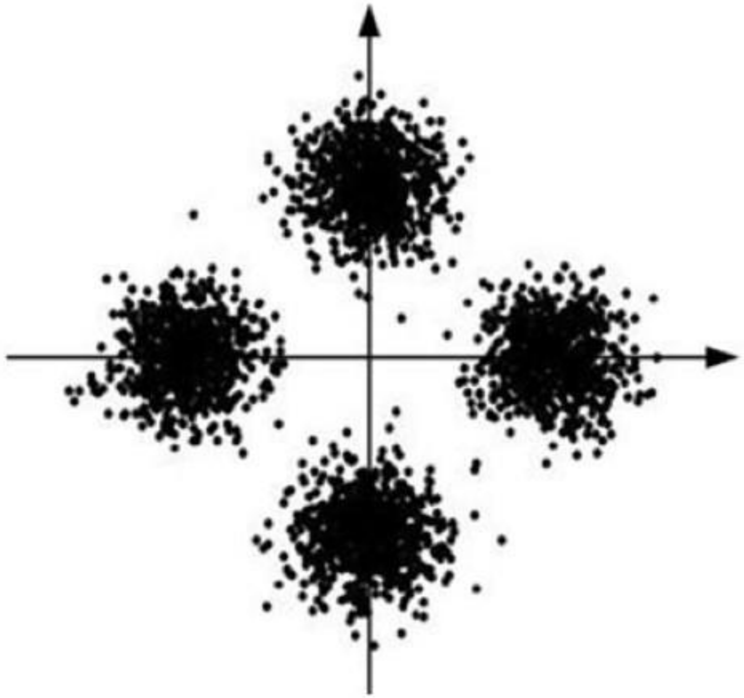


5.5.2 The optimum detector

Detector: estimate the sym from the observation

All the possibilities form a cloud centered at **s1.**
The dense of the cloud is higher at the center, and becomes less as departing from the center. The dense indicates basically the probability density or **r**.



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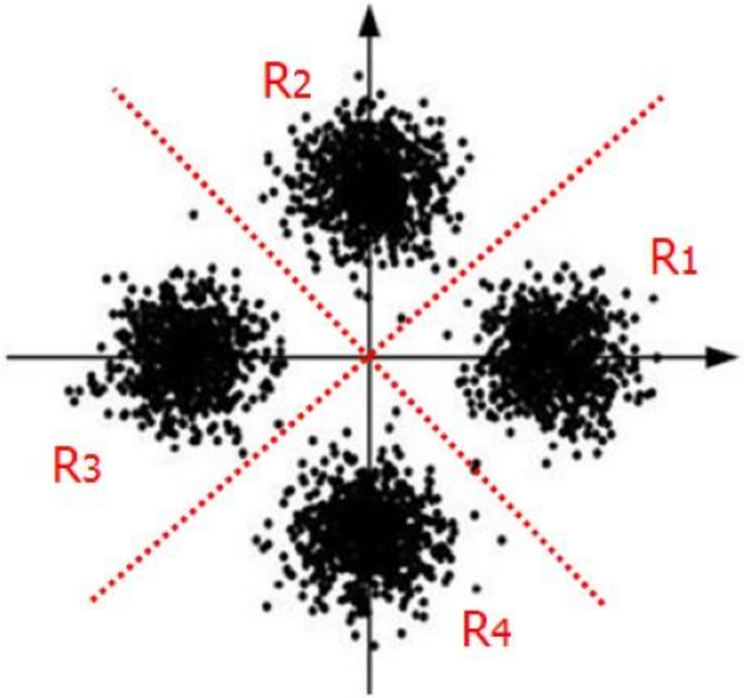

5.5.2 The optimum detector

What is a good detector ? ---- Make error as less as posible

How to measure error ? ---- **Probability of error**

Let R_m be the region in the space for which we select $\mathbf{s}_m$, R_m^c be the complement of R_m. They are decided by the criterion.

The average probability of errors is,

$$\boxed{P_e} = P(e) = 1 - \sum_{m=1}^{M} \int_{R_m} P(\mathbf{s}_m \mid \mathbf{r}) f(\mathbf{r}) d\mathbf{r}$$

where, $P(\mathbf{s}_m \mid \mathbf{r})$ is the prob that $\mathbf{s}_m$ has been transmitted on the reception of **r**

$f(\mathbf{r})$ is the unconditional pdf of **r**.

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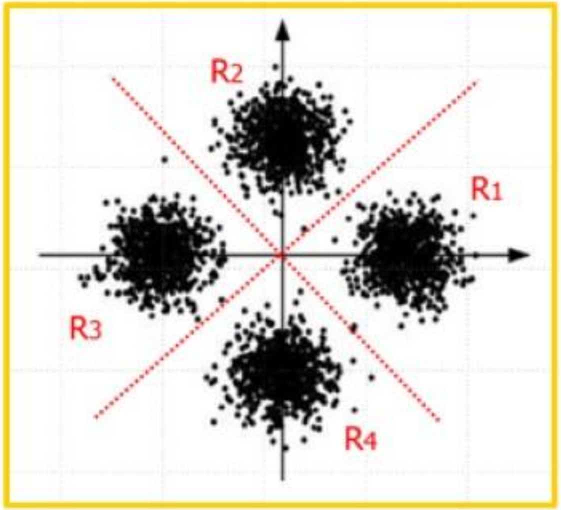

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$f(\mathbf{r})$ is the unconditional pdf of $\mathbf{r}$.

5.5.2 The optimum detector

What is the best detector ?

P_e is minimum when R_m corresponds to max $P(\mathbf{s}_m \mid \mathbf{r})$.

That lead to MAP criterion as follows.

$$P_e = P(e) = 1 - \sum_{m=1}^{M} \int_{R_m} P(\mathbf{s}_m \mid \mathbf{r}) f(\mathbf{r}) d\mathbf{r}$$

The Optimum Detector:

MAP: Maximum a posterior probability

ML : Maximum-likelihood

Min-Dist : Minimum-distance

Max-Corr: Maximum Correlation

Maximum a posterior probability (MAP

$\mathbf{s}_m$ that max the $P(\mathbf{s}_m \mid \mathbf{r})$. Denoted by,

$$\hat{\mathbf{s}}_m = \arg\max_{\mathbf{s}_m} \{P(\mathbf{s}_m \mid \mathbf{r})\}$$

Note that $P(\mathbf{s}_m \mid \mathbf{r})$ is often called a posterior probability.

5.5.3 The optimum detector

Using Bayes' rule,

$$P(\mathbf{s}_m \mid \mathbf{r}) = \frac{f(\mathbf{r} \mid \mathbf{s}_m) P(\mathbf{s}_m)}{f(\mathbf{r})}$$

where, $f(\mathbf{r} \mid \mathbf{s}_m)$ is the condi pdf of **r** given $\mathbf{s}_m$;

$P(\mathbf{s}_m)$ is prob of transmitting $\mathbf{s}_m$, called priori probability

Let $PM(\mathbf{r}, \mathbf{s}_m) = f(\mathbf{r} \mid \mathbf{s}_m) P(\mathbf{s}_m)$

MAP is equivalent to: $\hat{\mathbf{s}}_m = \arg\max_{\mathbf{s}_m} PM(\mathbf{r}, \mathbf{s}_m)$

Maximum a posterior probability (MAP): to select the $\mathbf{s}_m$ that max the $P(\mathbf{s}_m \mid \mathbf{r})$. Denoted by,

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5.5.3 The optimum detector

Note that $f(\mathbf{r}\,|\,\mathbf{s}_m)$ is usually called likelihood function.

Maximum-likelihood (ML): to select the $\mathbf{s}_m$ that max the $f(\mathbf{r}\,|\,\mathbf{s}_m)$.

Denoted by, $$\hat{\mathbf{s}}_m = \arg\max_{\mathbf{s}_m} \{f(\mathbf{r}\,|\,\mathbf{s}_m)\}$$

Obviously, when the symbols are equally probable, we have, **ML=MAP**
Equally-probable case is very common, thus ML is widely used in practical.

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Denoted by, $\hat{\mathbf{s}}_m = \arg\max_{\mathbf{s}_m}\{f(\mathbf{r}\mid\mathbf{s}_m)\}$

Minimum-distance (Min-Dist): to select the $\mathbf{s}_m$ that is nearest to **r**,

that is $\hat{\mathbf{s}}_m = \arg\min_{\mathbf{s}_m} D(\mathbf{r},\mathbf{s}_m)$ and, $D(\mathbf{r},\mathbf{s}_m) = \|\mathbf{r}-\mathbf{s}_m\|^2$

Recall that, $f(\mathbf{r}\mid\mathbf{s}_m) = \left(\frac{1}{\sqrt{\pi N_0}}\right)^N \exp\left[-\frac{\|\mathbf{r}-\mathbf{s}_m\|^2}{N_0}\right]$

It is more convenient to work with, $\ln f(\mathbf{r}|\mathbf{s}_m) = -\frac{N}{2}\ln(\pi N_0) - \frac{1}{N_0}\|\mathbf{r}-\mathbf{s}_m\|^2$

Clearly, max of $f(\mathbf{r}\mid\mathbf{s}_m)$ is same as min of $\|\mathbf{r}-\mathbf{s}_m\|^2$, which is the distance bwtween **r** and $\mathbf{s}_m$

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We can say more, $D(\mathbf{r},\mathbf{s}_m) = \|\mathbf{r}\|^2 - 2\mathbf{r}\bullet\mathbf{s}_m + \|\mathbf{s}_m\|^2$

Note $\|\mathbf{r}\|^2$ is common to all $\mathbf{s}_m$,

$\mathbf{r}\bullet\mathbf{s}_m$ is the correlation

and, $\|\mathbf{s}_m\|^2$ is the energy of the symbol.

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Let $E_m = \|\mathbf{s}_m\|^2$ and $C(\mathbf{r},\mathbf{s}_m) = \mathbf{r}\bullet\mathbf{s}_m - E_m/2$

Maximum Correlation (Max-Corr): to select the $\mathbf{s}_m$ that is most correlated with **r**, possibly compensated by some bias due to unequal energy, that is,

$$\hat{\mathbf{s}}_m = \arg\max_{\mathbf{s}_m} C(\mathbf{r},\mathbf{s}_m)$$

5.5.3 The optimum detector

ML is often very simple and common. It is nice to see that ,

ML=MinDist=MaxCorr.

Example 5.5.3 on page 285.

1) The observation vector;

2) The pdfs

3) MAP, ML(Min-Dist), or Mac-Corr

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