

Principles of Signal Detection and Parameter Estimation

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In these matters the only certainty is that nothing is certain.

Pliny the Elder

Preface

As a discipline, signal detection has evolved significantly over the last 40 years. Some changes have been caused by technical advances, like the development of robust detection methods, or the use of the theory of large deviations to characterize the asymptotic performance of tests, but most changes have been caused by transformations in the engineering systems to which detection techniques are applied. While early applications of signal detection focused on radar and sonar signal processing or the design of digital communication receivers, newer areas of application include image analysis and interpretation, document authentication, biometrics, and sensor or actuator failure detection. This expanded scope of application has required some adjustment in standard ways of formulating detection problems. For example, image processing applications typically combine parameter estimation and detection tasks, so the separation of parameter estimation and detection in distinct operations typical of early communication systems, where parameter estimation was accomplished through the use of training signals, needs to be abandoned. Other changes have occurred in the design of communication systems which make it increasingly difficult to treat the detection of communications signals and of radar/sonar signals in a unified manner. This common framework assumes implicitly that intersymbol interference is not present and that channel coding and modulation are implemented separately, since in this case modulated signals can be detected one symbol at a time. But modern communication systems are typically designed to operate over bandlimited channels where intersymbol interference is present, and starting with the introduction of trellis coded modulation, modulation and coding have become intertwined. In this context, the detection of modulated signals can no longer be treated on a symbol-by-symbol basis but needs to be viewed as a sequence detection problem, where the sequence is generated by a Markov chain. Another feature of modern radar and communication systems, in particular wireless systems, is that they often need to operate in a rapidly changing environment. So even if training or calibration signals are available to estimate the system parameters, because parameters may change quickly, it is desirable to constantly

refresh estimates while at the same time performing detection tasks on received signals. In other words, detection and estimation need to be performed simultaneously and can no longer be viewed as separate tasks. Finally, another feature of modern engineering systems to which detection algorithms are applied is that due to modelling errors, imperfect calibration, changes in the environment, as well as the presence of interfering signals, it is not entirely realistic to assume that accurate models are available, and thus robust detection techniques need to be applied.

The objective of this book is to give a modern presentation of signal detection which incorporates new technical advances, while at the same time addressing issues that reflect the evolution of contemporary detection systems. Recent advances which are covered include the use of the theory of large deviations to characterize the asymptotic performance of detectors, not only for the case of independent identically distributed observations, but also for detection problems involving Gaussian processes or Markov chains. In addition, a chapter discusses robust signal detection, and another the application of the EM algorithm to parameter estimation problems where ML estimates cannot be evaluated in closed form. At the same time, changes in modern communications technology are addressed by examining the detection of partially observed Markov chains, both for the case when the Markov chain model is known, or when the model includes unknown parameters that need to be estimated. To accommodate the need for joint estimation and detection in modern communication systems, particular attention is given to the generalized likelihood ratio test (GLRT), since it explicitly implements detection and estimation as a combined task, and because of its attractive invariance and asymptotic properties.

This book is primarily intended for use in signal detection courses directed at first or second year graduate electrical engineering students. Thus, even though the material presented has been abstracted from actual engineering systems, the emphasis is on fundamental detection principles, rather than on implementation details targeted at specific applications. It is expected that after mastering the concepts discussed here, a student or practicing engineer will be able to analyze a specific detection problem, read the available literature, and design a detector meeting applicable specifications. Since the book is addressed at engineering students, certain compromises have been made concerning the level of precision applied to mathematical arguments. In particular, no formal exposure to measure theory, modern real analysis, and the theory of operators in Hilbert spaces is assumed. As a consequence, even though derivations are conceptually accurate, they often leave some technical details out. This relatively casual presentation style has for objective to ensure that most students will be able to benefit from the material presented, regardless of preparation. On the other hand, it is expected that readers will have a solid background in the areas of random processes, linear algebra, and convex optimization, which in the aggregate form the common frame of reference of the statistical signal processing community.

Another aspect of this book that may be controversial is that it does not follow a theorem/proof format. To explain this choice, I would like to point out that whereas the hypothesis testing and parameter estimation techniques used in signal detection lend themselves naturally to a formal presentation style, because of its applied nature, signal detection consists primarily of a methodology for converting an observed signal model and some specifications for the detector to be constructed into first a formulation of the problem in hypothesis testing format, followed by a solution meeting the given specifications. In this context, the most important skills needed are first the ability to think geometrically in higher dimensional spaces, and second the capacity to reason in a manner consistent with the assumed observation model. For example, if the parameters appearing in the signal model admit probability distributions, a Bayesian framework needs to be employed to construct a detector, whereas when parameters are unknown but nonrandom, the parameters need to be estimated as part of the detector construction. Slightly different modelling assumptions for the same problem may lead to different detector structures. Accordingly, signal detection cannot really be reduced to a collection of mathematical results. Instead, it is primarily a methodology that can be best explained by employing a continuous presentation flow, without attempting to slice the material into elementary pieces. The continuous flow approach has also the advantage that it makes it easier to connect ideas presented in different parts of the book without having to wait until each analytical derivation is complete.

Obviously, since the field of signal detection covers a vast range of subjects, it has been necessary to leave out certain topics that are either covered elsewhere or that are too advanced or complex to be presented concisely in an introductory text. Accordingly, although Kalman and Wiener filters are employed in the discussion of Gaussian signal detection in Chapter 10, it is assumed that optimal filtering is covered elsewhere as part of a stand-alone course, possibly in combination with adaptive filtering, as is the case at UC Davis. In any case, several excellent presentations of optimal and adaptive filtering are currently available in textbook form, so it makes little sense to duplicate these efforts. Two other topics that have been left out, but for entirely different reasons, are change detection/failure detection, and iterative detection. To explain this choice, let me indicate first that change detection and failure detection represent one of the most interesting and challenging fields of application of the methods presented in this book, since in addition to detecting whether a change occurs, it is necessary to detect when the change occurred, and for safety critical applications, to do so as quickly as possible. However, important advances have occurred in this area over the last 20 years, and it does not appear possible to give a concise presentation of these results in a manner that would do justice to this topic. As for the iterative detection techniques introduced recently for iterative decoding and equalization, it was felt that these results are probably best presented in the context of the communications applications for which they were developed.

The scope of the material presented in this book is sufficiently broad to allow different course organizations depending on length (quarter or semester) and on the intended audience. At UC Davis, within the context of a one quarter course, I usually cover Chapter 2 (hypothesis testing), followed by Chapter 4 (parameter estimation), and the first half of Chapter 5 (composite hypothesis testing). Then I move on to Chapter 7 presenting the Karhunen-Loève decomposition of Gaussian processes, followed by Chapters 8 and 9 discussing the detection of known signals, possibly with unknown parameters, in white and colored Gaussian noise. A semester length presentation directed at a statistical signal processing audience would allow coverage of the second half of Chapter 3 on sequential hypothesis testing, as well as Chapters 10 and 11 on the detection of Gaussian signals, possibly with unknown parameters. On the other hand, a semester course focusing on communications applications would probably add Chapters 12, 13 and parts of Chapter 11 to the one-quarter version of the course outlined above.

The idea of writing this book originated with a lunch conversation I had with a UC Davis colleague, Prof. Zhi Ding about four years ago. I was complaining that available textbooks on signal detection did not include several topics that I thought were essential for a modern presentation of the material, and after listening politely, Zhi pointed out that since I had all these bright ideas, maybe I should write my own book. Against my better judgement, I decided to follow Zhi's suggestion when I became eligible for a sabbatical year in 2004–2005. In spite of the hard work involved, this has been a rewarding experience, since it gave me an opportunity to express my views on signal detection and parameter estimation in a coherent manner. Along the way, I realized how much my understanding of this field had been impacted by teachers, mentors, friends, collaborators, and students. Among the many individuals to whom I am indebted, I would like to start with my teachers Pierre Faurre and Pierre Bernhard at the Ecole des Mines in Paris, who got me interested in optimal filtering and encouraged me to go to Stanford to pursue graduate studies. As soon as I arrived at Stanford, I knew this was the right choice, since in addition to the expert guidance and scientific insights provided by my advisors, Tom Kailath and Martin Morf, I was very fortunate to interact with an unusually talented and lively group of classmates including Sun-Yuan Kung, George Verghese, and Erik Verriest. Later, during my professional life at MIT and UC Davis, I benefited greatly from the mentorship and advice provided by Alan Willsky, Sanjoy Mitter, and Art Krener. I am particularly grateful to Art for showing me through example that good research and fun are not mutually exclusive. In addition, I would like to thank Albert Benveniste and Ramine Nikoukhah for fruitful research collaborations during and after sabbatical visits at INRIA in France. Like most professors, I have learnt a lot from my students, and among those whose research was directly related to the topic of this book, I would like to acknowledge Ahmed Tewfik, Mutlu Koca, Hoang Nguyen and Yongfang Guo. A number of volunteers have helped me in the preparation of this book. Yongfang Guo helped

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A Note to Instructors

A password protected, solutions manual is available online for qualified instructors utilizing this book in their courses. Please see www.Springer.com for more details or contact Dr Bernard C. Levy directly.