Chapter 5

Digital transmission through the AWGN channel

—— by Prof. XIAOFENG LI
SICE, UESTC

Problems

Ch5 Digital transmission through the AWGN channels

Section 5.1-5.4: 5.3, 5.7

Section 5.5: 5.8

Section 5.6: 5.9, 5.10, 5.18, 5.34, 5.43, 5.47, 5.54

Introduction

In this chapter, we focus on how to transmit digital info signals with waveforms.

The topics include:

1. Geometric representation of sig waveforms;
2. Diff types of waveforms for digital transmissions;
3. Optimal reception;
4. Performance evaluation on the AWGN channel
5. Comparison of the methods.

Contents 16 Hrs

- Introduction
- Geometric rep. of the sig waveforms
- Pulse amplitude modulation
- 2-d signal waveforms
- M-d signal waveforms
- Opt. reception for the sig. In AWGN
- Optimal receivers and probs of err

5.1 Geometric rep. of the sig waveforms

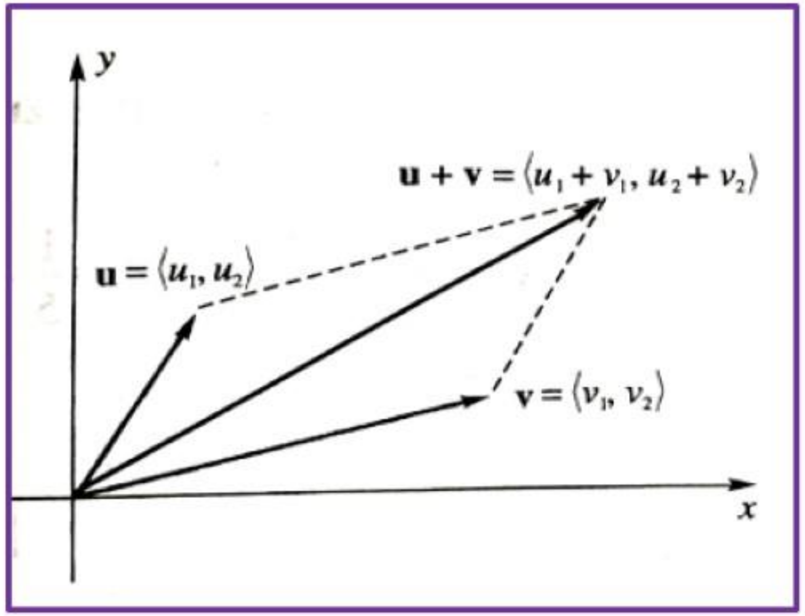
1. Review of the Vectors and Vector Space

Concepts:

- length and direction
- Coordinates

$$\mathbf{v} = (v_x, v_y)$$

- vector addition,
- distance calculation
- inner products



- axes or basis vectors
- Geometric representation
- Algebraic representation, $\mathbf{v} = v_x \times \mathbf{i} + v_y \times \mathbf{j}$

Dimensions: 2-space, 3-space, N-space

5.1 Geometric rep. of the sig waveforms

1. Review of the Vectors and Vector Space

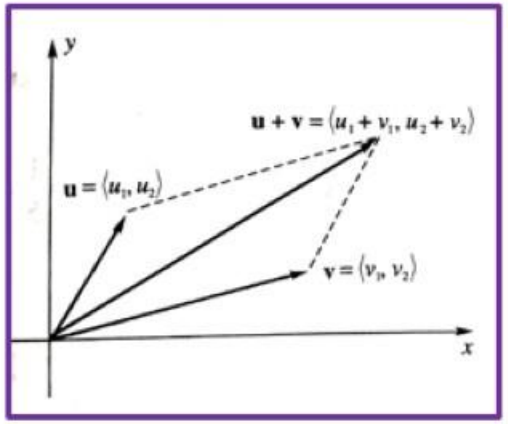
Let $B = \{\mathbf{e}_1, \dots, \mathbf{e}_N\}$ be an orthonormal basis for a N dimensional **vector space** R^N.

If **v** is any vector in the space, then it can be written uniquely in the form

$$\mathbf{v} = a_1\mathbf{e}_1 + a_2\mathbf{e}_2 + \dots + a_N\mathbf{e}_N$$

$a_1, \dots, a_N$ are called the coordinates of **v**.

$$\mathbf{v} = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \end{bmatrix}$$

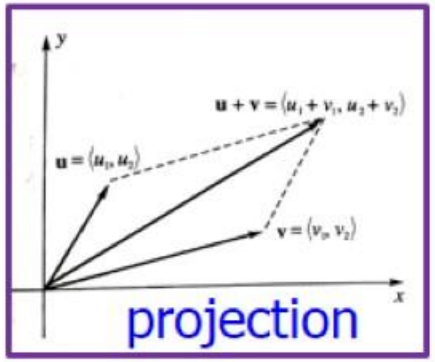


5.1 Geometric rep. of the sig waveforms

1. Review of the Vectors and Vector Space

With the dot product (or inner product), denotes as '·', a coordinate is calculated by following,

$$a_i = \mathbf{v} \cdot \mathbf{e_i}$$



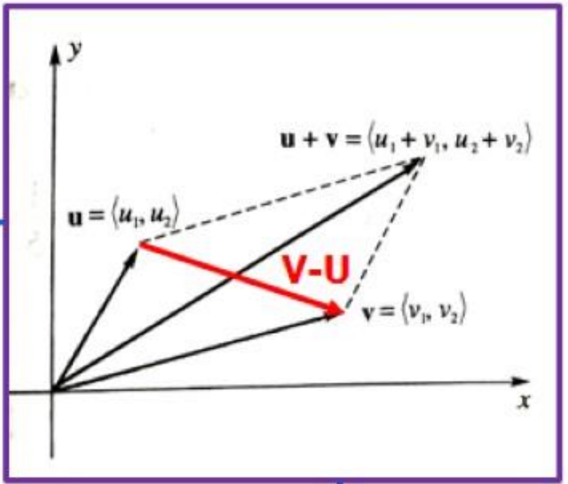
Because,

$$\begin{aligned}
\mathbf{v} \cdot \mathbf{e_i} &= (a_1\mathbf{e}_1 + a_2\mathbf{e}_2 + \ldots + a_N\mathbf{e}_N) \cdot \mathbf{e_i} \\
&= a_1(\mathbf{e}_1 \cdot \mathbf{e_i}) + a_2(\mathbf{e}_2 \cdot \mathbf{e_i}) + \ldots + a_i(\mathbf{e_i} \cdot \mathbf{e_i}) \cdots + a_N(\mathbf{e}_N \cdot \mathbf{e_i}) \\
&= a_1 \times 0 + a_2 \times 0 + \ldots + a_i \times 1 \cdots + a_N \times 0 \\
&= a_i
\end{aligned}$$

orthonormal basis

5.1 Geometric rep. of the sig waveforms

1. Review of the Vectors and Vector Space

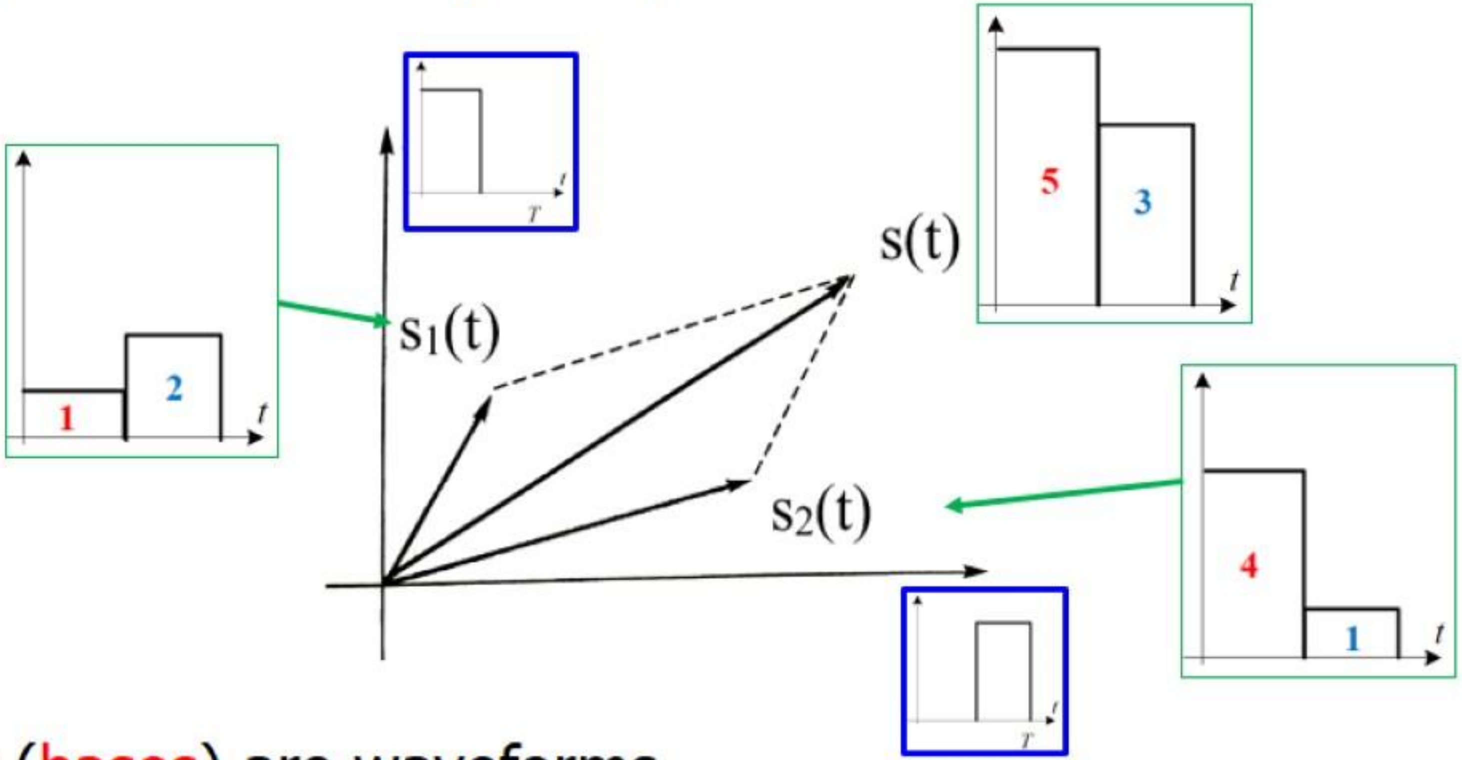


With coordinates, we have,

1. The inner product $$\mathbf{v}\cdot\mathbf{u}=\sum_{i=1}^{N} v_i u_i$$
2. The norm $$\|\mathbf{v}\|=\sqrt{\sum_{i=1}^{N} v_i^{\,2}}$$
3. The distance $$d=\|\mathbf{v}-\mathbf{u}\|=\sqrt{\sum_{i=1}^{N} (v_i-u_i)^2}$$

5.1 Geometric rep. of the sig waveforms

2. Now think of an signal waveform as a vector (or element) in a space. The space is called **signal space.**



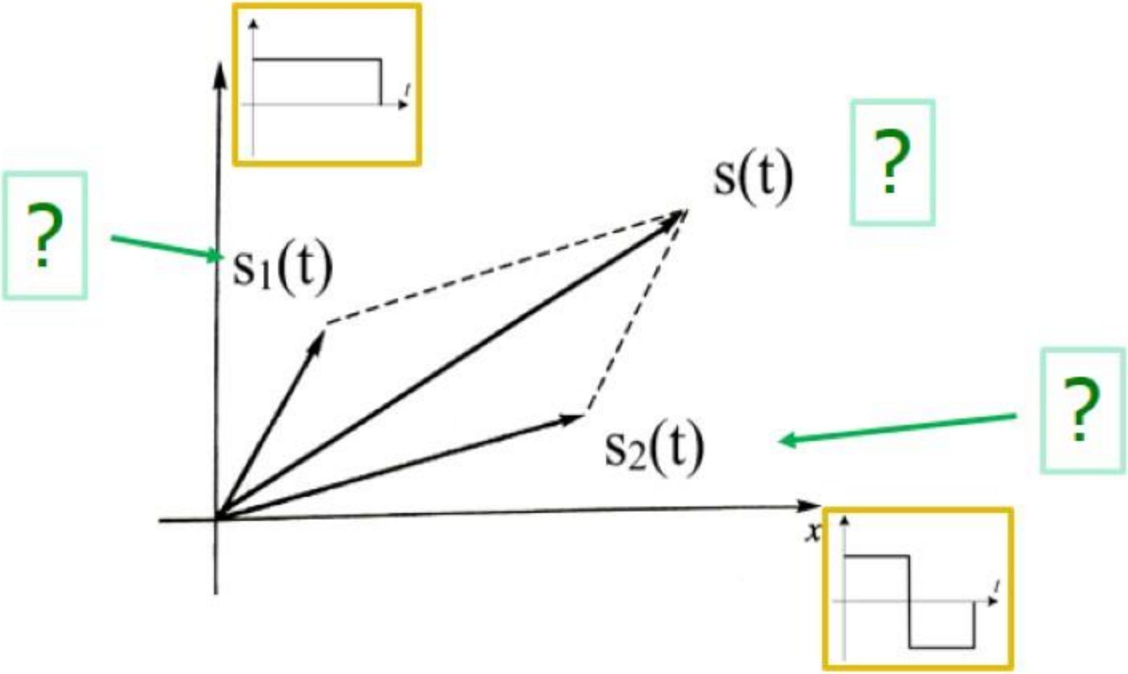
- Axes (bases) are waveforms,

  $\Psi_1(t)$, $\Psi_2(t)$

  Say, two rect-pulses

5.1 Geometric rep. of the sig waveforms

2. Now think of an signal waveform as a vector (or element) in a space.
   The space is called **signal space.**



Questions:

- For new pulses, what are s1(t) , s2(t) and s(t)?
- For $\sin 2\pi f_c t$ and $\cos 2\pi f_c t$, what are the signals?
- Are the basis signals orthonormal?

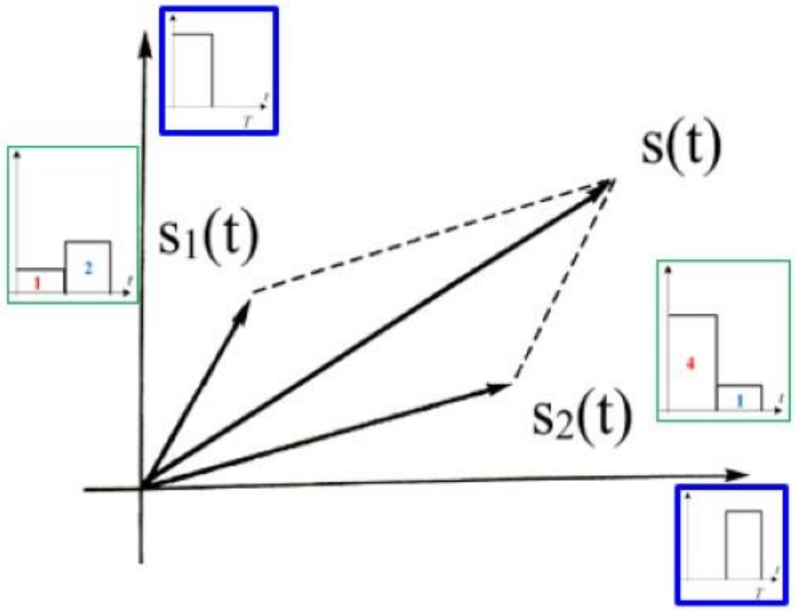
5.1 Geometric rep. of the sig waveforms

2. Now think of an signal waveform as a vector (or element) in a space. The space is called **signal space.**

Let $\mathbf{s}, \mathbf{s}_1, \mathbf{s}_2$ denote $s(t), s_1(t), s_2(t)$, and

define the inner product of $\mathbf{s}_1$ and $\mathbf{s}_2$ as

$$\mathbf{s}_1 \cdot \mathbf{s}_2 = \int_{-\infty}^{\infty} s_1(t) s_2(t) dt$$



5.1 Geometric rep. of the sig waveforms

2. Now think of an signal waveform as a vector (or element) in a space.
   The space is called **signal space.**

Let $\mathbf{s}, \mathbf{s}_1, \mathbf{s}_2$ denote $s(t), s_1(t), s_2(t)$, and

define the inner product of $\mathbf{s}_1$ and $\mathbf{s}_2$ as

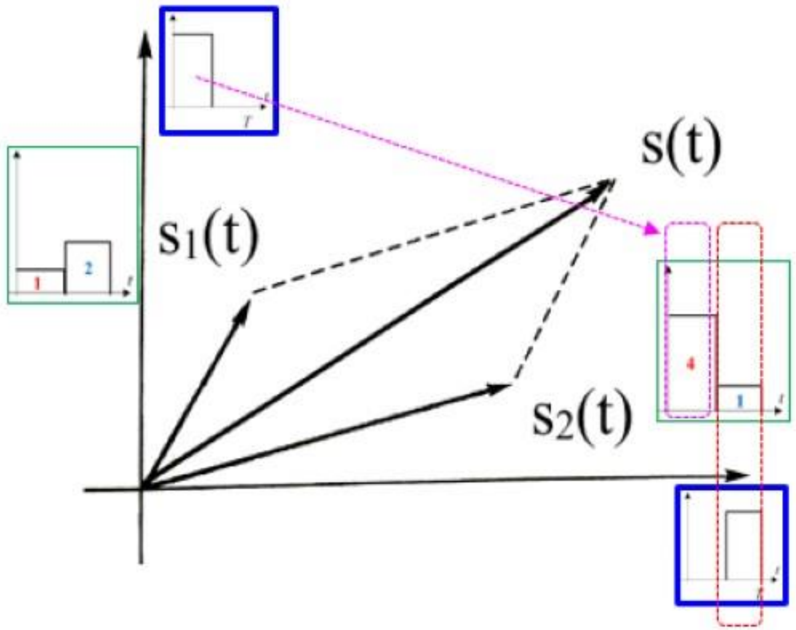
$$\mathbf{s}_1 \cdot \mathbf{s}_2 = \int_{-\infty}^{\infty} s_1(t) s_2(t) dt$$

The coordinates of signal $s_m(t)$

can be calculated by,

$$s_{mi} = \mathbf{s}_m \cdot \boldsymbol{\psi}_\mathbf{i} = \int_{-\infty}^{\infty} s_m(t) \psi_i(t) dt$$

for all $i = 1, 2, \ldots N$.



5.1 Geometric rep. of the sig waveforms

2. Now think of an signal waveform as a vector (or element) in a space. The space is called **signal space.**

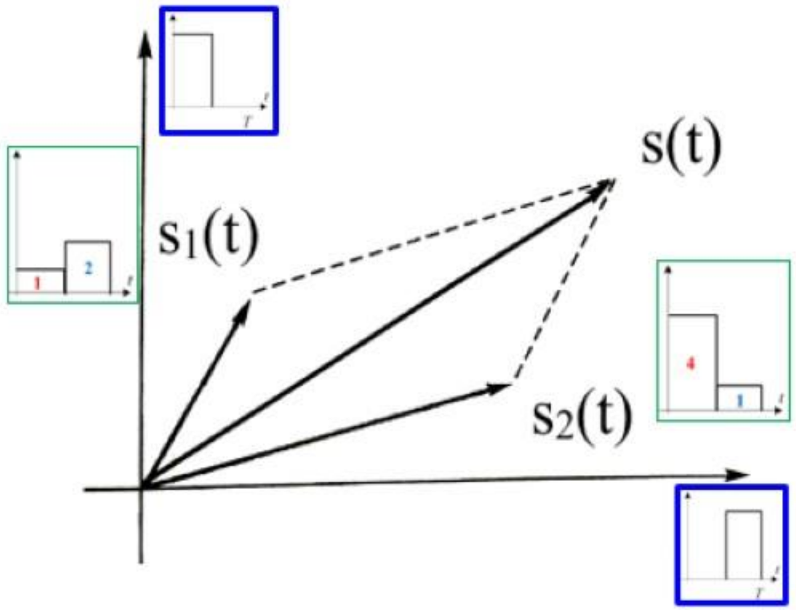
Let $\mathbf{s}, \mathbf{s}_1, \mathbf{s}_2$ denote $s(t), s_1(t), s_2(t)$, and

define the inner product of $\mathbf{s}_1$ and $\mathbf{s}_2$ as

$$\mathbf{s}_1 \cdot \mathbf{s}_2 = \int_{-\infty}^{\infty} s_1(t) s_2(t) dt$$

Physical meaning by,

$$\mathbf{s}_1 \cdot \mathbf{s}_1 = \int_{-\infty}^{\infty} s_1(t) s_1(t) dt$$
$$= Energy$$



5.1 Geometric rep. of the sig waveforms

2. Now think of an signal waveform as a vector (or element) in a space. The space is called **signal space.**

Suppose the orthonormal basis is $\boldsymbol{\Psi}_1, \dots, \boldsymbol{\Psi}_N$, which is equivalent to $\psi_1(t), \dots, \psi_N(t)$.

With coordinates, we have,

1) The signal $\mathbf{s}_m = \sum_{i=1}^{N} s_{mi} \boldsymbol{\Psi}_\mathbf{i}$

2) The energy $E_s = \|\mathbf{s}_m\|^2 = \sum_{i=1}^{N} s_{mi}^2$

3) The rms $\|\mathbf{s}_m\| = \sqrt{\sum_{i=1}^{N} s_{mi}^2}$

4) The distance $d = \|\mathbf{s}_m - \mathbf{s}_k\| = \sqrt{\sum_{i=1}^{N} (s_{mi} - s_{ki})^2}$

$$\mathbf{v} = a_1 \mathbf{e}_1 + a_2 \mathbf{e}_2 + \dots + a_N \mathbf{e}_N$$

$$\mathbf{v} \cdot \mathbf{u} = \sum_{i=1}^{N} v_i u_i$$

$$\|\mathbf{v}\| = \sqrt{\sum_{i=1}^{N} v_i^2}$$

$$d = \|\mathbf{v} - \mathbf{u}\| = \sqrt{\sum_{i=1}^{N} (v_i - u_i)^2}$$

5.1 Geometric rep. of the sig waveforms

3. Given M signals $s_1(t), \ldots, s_M(t)$, what is the SPACE for them?

   Note SPACE is specified by basis signals.

Q: how to find the basis $\psi_1, \ldots, \psi_N$ based on $\mathbf{s}_1, \ldots, \mathbf{s}_M$?

What is N and M?

A: We employ the **Gram-Schmidt Procedure**.

See p283

4. Examples of Gram-Schmidt Procedure

   Example 5.1.1

   Example 5.1.2