Chapter 5

Digital transmission through the AWGN channel

—— by Prof. XIAOFENG LI

SICE, UESTC

Problems

Ch5 Digital transmission through the AWGN channels

Section 5.1-5.4: 5.3, 5.7

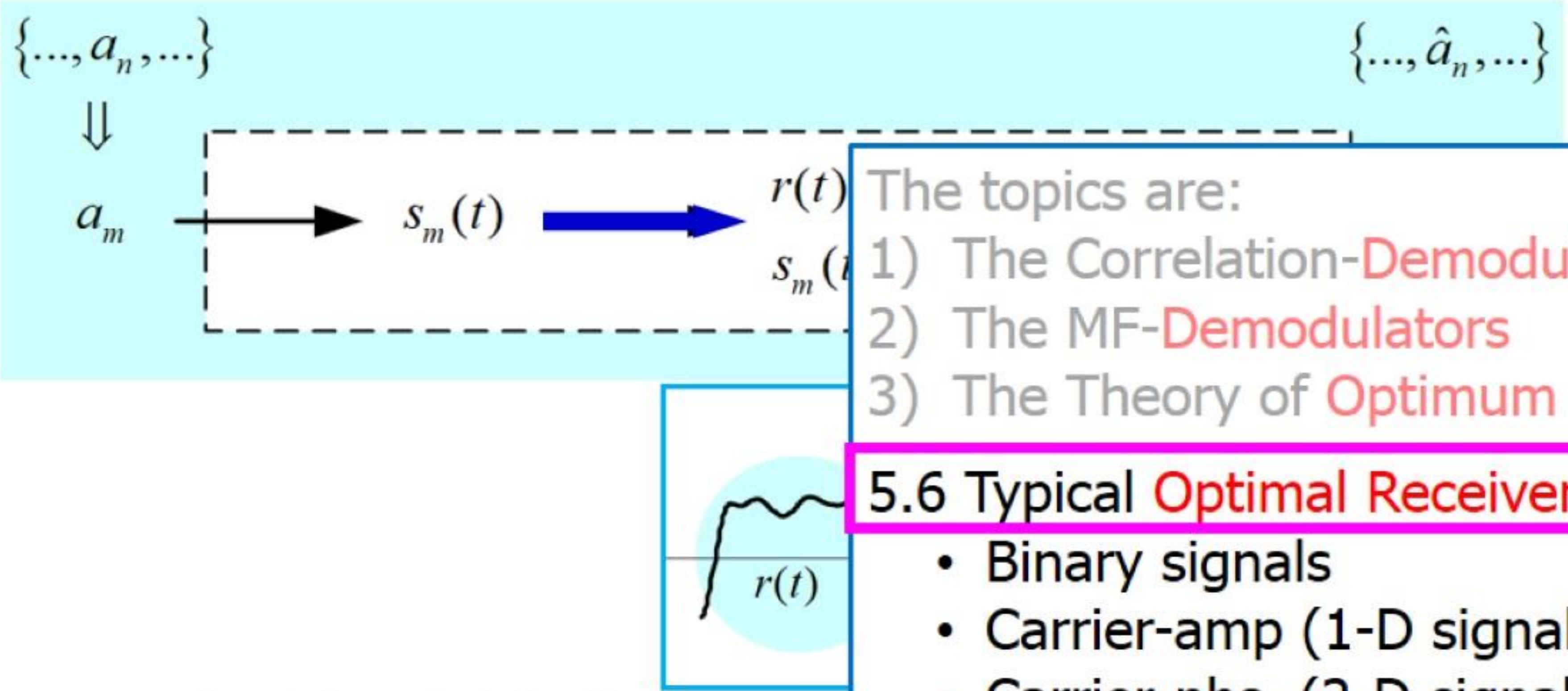
Section 5.5: 5.8

Section 5.6: 5.9, 5.10, 5.18, 5.34, 5.43, 5.47, 5.54

Contents 16 Hrs

5.5 Opt. reception for the sig. In AWGN

In the nth interval, the process is as follows,



The topics are:

1) The Correlation-Demodulators
2) The MF-Demodulators
3) The Theory of Optimum Detectors

5.6 Typical Optimal Receivers

- Binary signals
- Carrier-amp (1-D signals)
- Carrier-phs (2-D signals)
- QAM (2-D signals)
- FSK (M-D orthogonal signals)

It is convenient to divide the receiv
1)**Demodulator**: produce an obse
2)**Detector**: estimate the sym fron

5.6.1 Binary modulations

Baseband

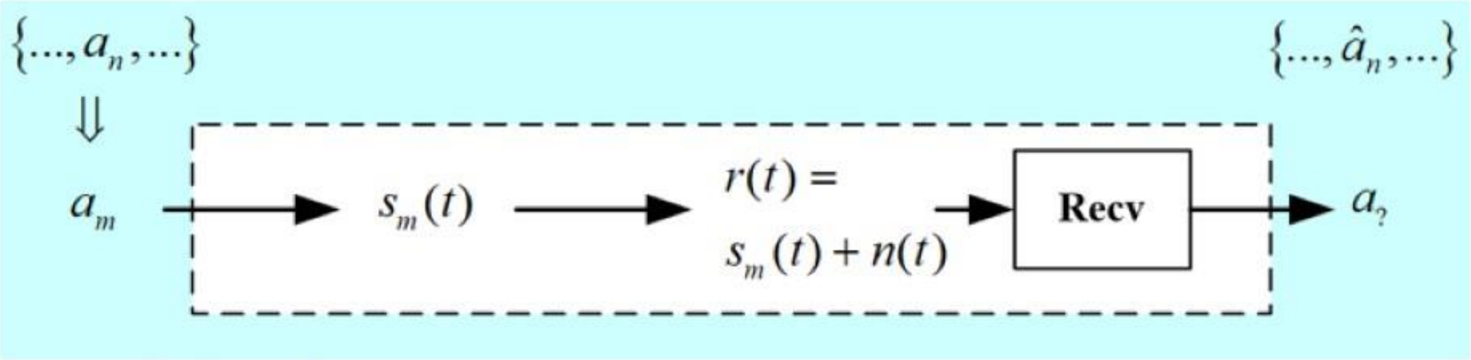
- Binary PAM (the antipodal, unipolar)
- Orthogonal signaling with M=2

Passband

- BPSK (the antipodal)
- OOK or BASK (the unipolar)
- BFSK (Orthogonal signaling with M=2)

5.6.1 Binary modulations

Antipodal

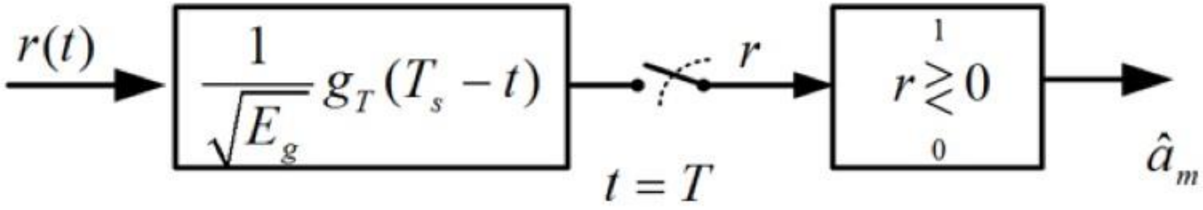


The signals: $s_1(t) = A g_T(t)$,

$$s_0(t) = -A g_T(t)$$

The 1-D basis: $\psi(t) = \frac{1}{\sqrt{E_g}} g_T(t)$

A MF-ML receiver is given by,

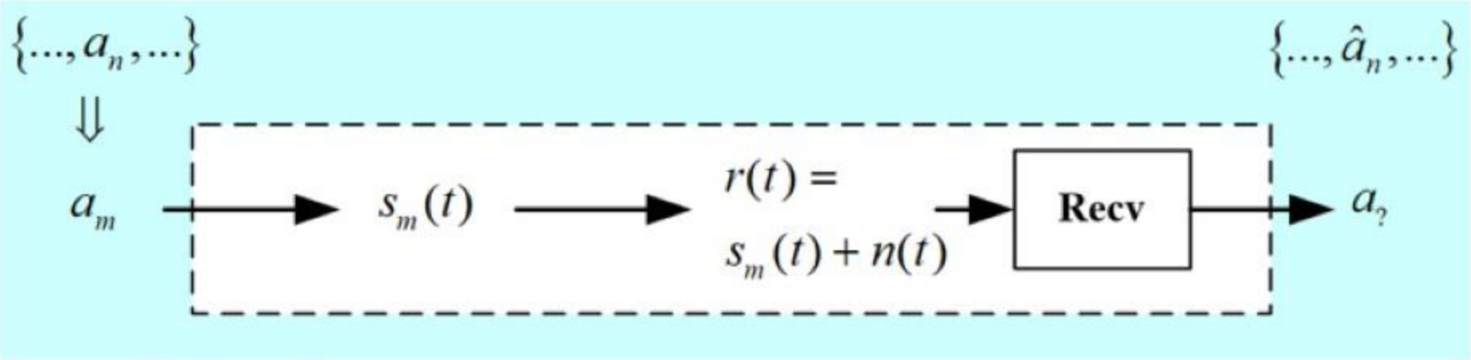


Prob of err is computed by,

$$P_e = P(e) = 1 - \sum_{m=1}^{M} \int_{R_m} P(\mathbf{s}_m \mid \mathbf{r}) f(\mathbf{r}) d\mathbf{r}$$

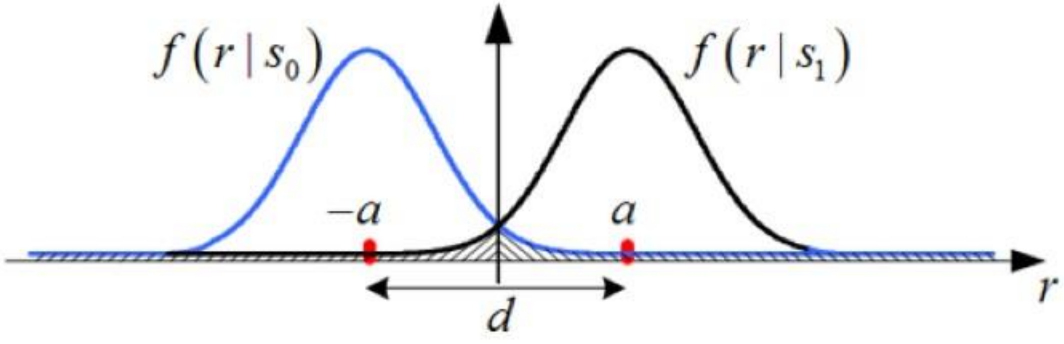
5.6.1 Binary modulations

Antipodal



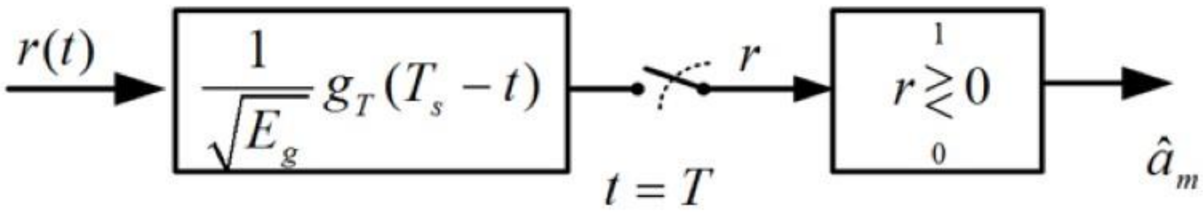
The signals: $s_1(t) = Ag_T(t)$,
$s_0(t) = -Ag_T(t)$

The constellation and likelihood functions:



ML leads to a decision rule of : $r \underset{0}{\overset{1}{\gtrless}} 0$

A MF-ML receiver is given by,



Prob of err is computed by,

$$P_e = 1 - \frac{1}{2}\int_{R_0} f(r|s_0)dr - \frac{1}{2}\int_{R_1} f(r|s_1)dr$$

where, $R_0 = (-\infty, 0]$ and $R_1 = [0, +\infty)$.

5.6.1 Binary modulations

$$P_e = 1 - \int_{R_0} f(r|s_0)dr = \int_{[0,+\infty)} \frac{1}{\sqrt{\pi N_0}} e^{-\frac{(r+d/2)^2}{N_0}} dr$$

$$= \int_{\sqrt{\frac{d^2}{2N_0}}}^{+\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx \qquad \left(\text{Let} \quad x = \frac{r+d/2}{\sqrt{N_0/2}}\right)$$

$$= Q\left(\sqrt{\frac{d^2}{2N_0}}\right)$$

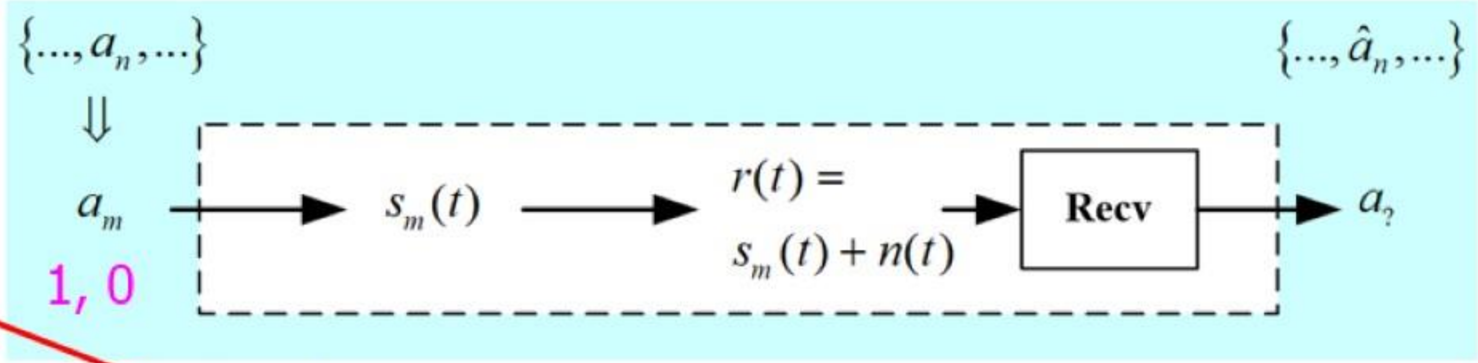
where, $Q(x) = \int_x^{+\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx$

By convention, engineers evaluate performance in terms of $P_e \sim \frac{E_b}{N_0}$,

where E_b is the average bit-energy. And $\frac{E_b}{N_0}$ is often called SNR.

For binary antipodal system, $E_b = E_s = a^2 = d^2/4$, thus, $\boxed{P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)}$

5.6.1 Binary modulations



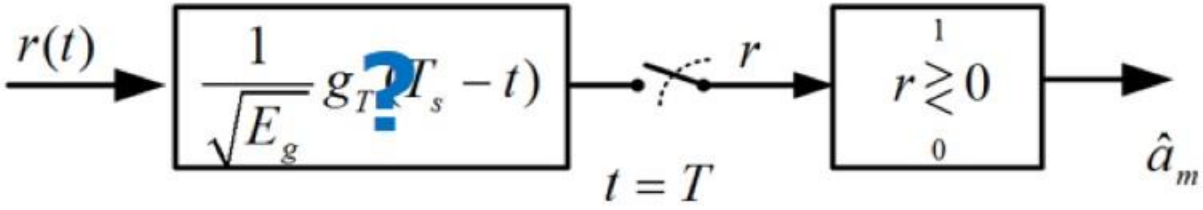
~~Antipodal~~
Unipolar

The signals: $s_1(t) = Ag_T(t),$

$s_0(t) = 0$

The 1-D basis: $\psi(t) = \frac{1}{\sqrt{E_g}} g_T(t)$

A MF-ML receiver is given by,



Prob of err is computed by,

$$P_e = P(e) = 1 - \sum_{m=1}^{M} \int_{R_m} P(\mathbf{s}_m \mid \mathbf{r}) f(\mathbf{r}) d\mathbf{r}$$

$$\boxed{P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)}$$

?

Leave the details to you, after class

5.6.1 Binary modulations

Baseband

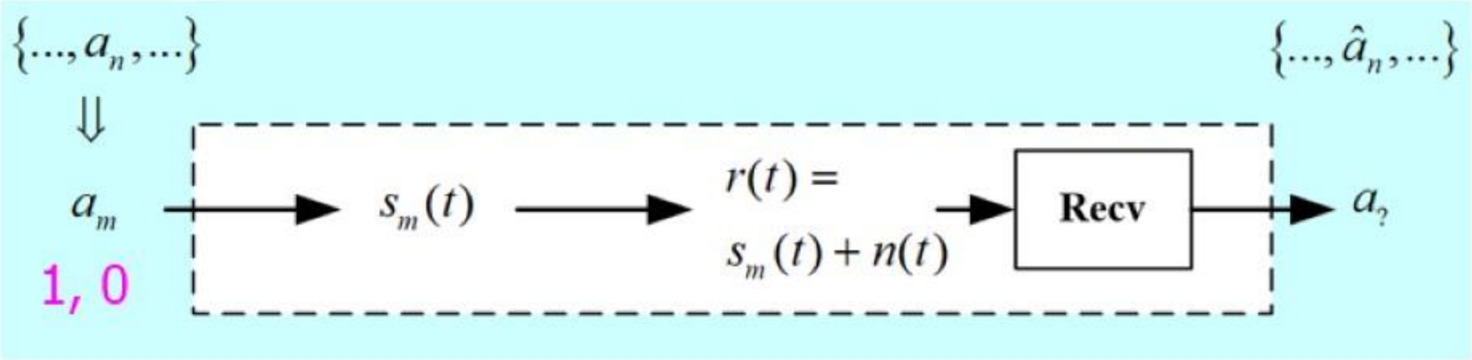
- Binary PAM (the antipodal, unipolar)
- Orthogonal signaling with M=2

Passband

- BPSK (the antipodal)
- OOK or BASK (the unipolar)
- BFSK (Orthogonal signaling with M=2)

5.6.1 Binary modulations

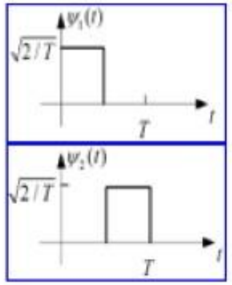
Orthogonal



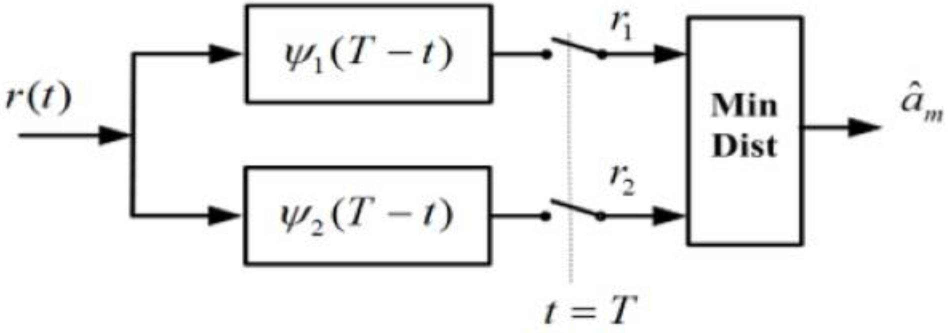
The 2-D basis: $\psi_1(t), \psi_2(t)$

The signals: $s_1(t) = a\psi_1(t)$,

$s_0(t) = a\psi_2(t)$



A MF-ML receiver is given by,



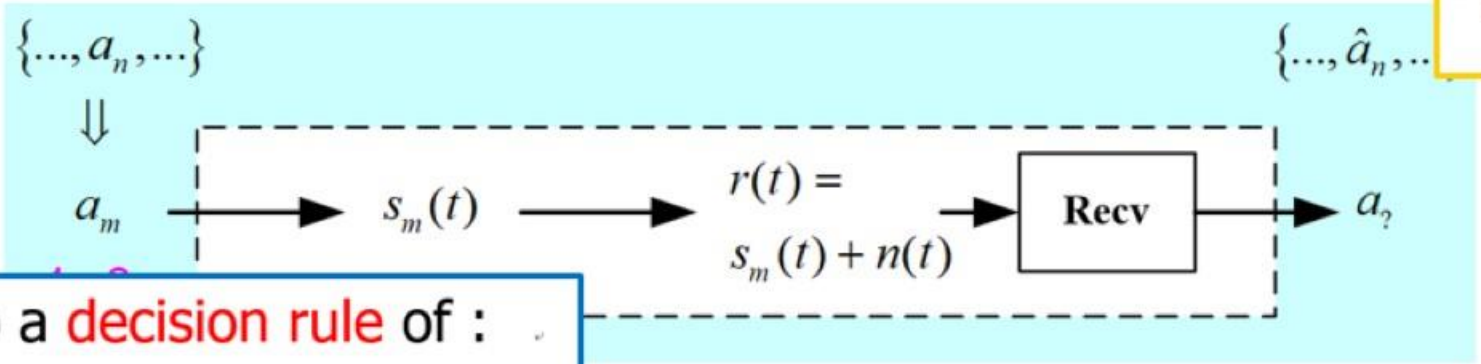
Prob of err is computed by,

$$P_e = P(e) = 1 - \sum_{m=1}^{M} \int_{R_m} P(\mathbf{s}_m \mid \mathbf{r}) f(\mathbf{r}) d\mathbf{r}$$

5.6.1 Binary modulations

$$P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

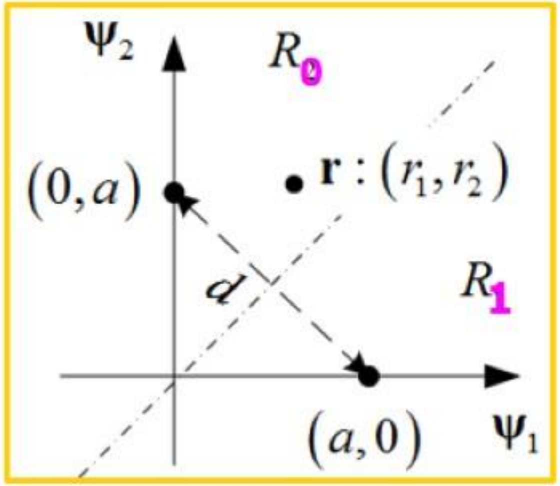
Orthogonal



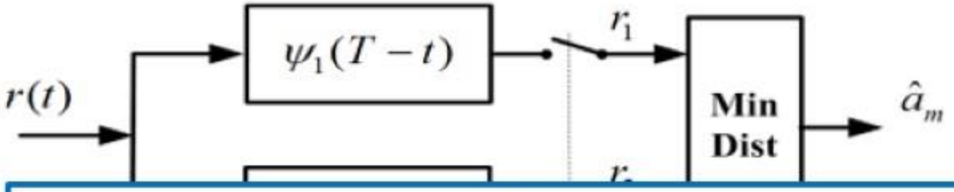
MinDist leads to a decision rule of :

$$\left[(r_1 - a)^2 + r_2^2\right] \underset{1}{\overset{0}{\gtrless}} \left[r_1^2 + (r_2 - a)^2\right]$$

or $r_1 \underset{0}{\overset{1}{\gtrless}} r_2$



A MF-ML receiver is given by,



In terms of $P_e \sim \frac{E_b}{N_0}$,

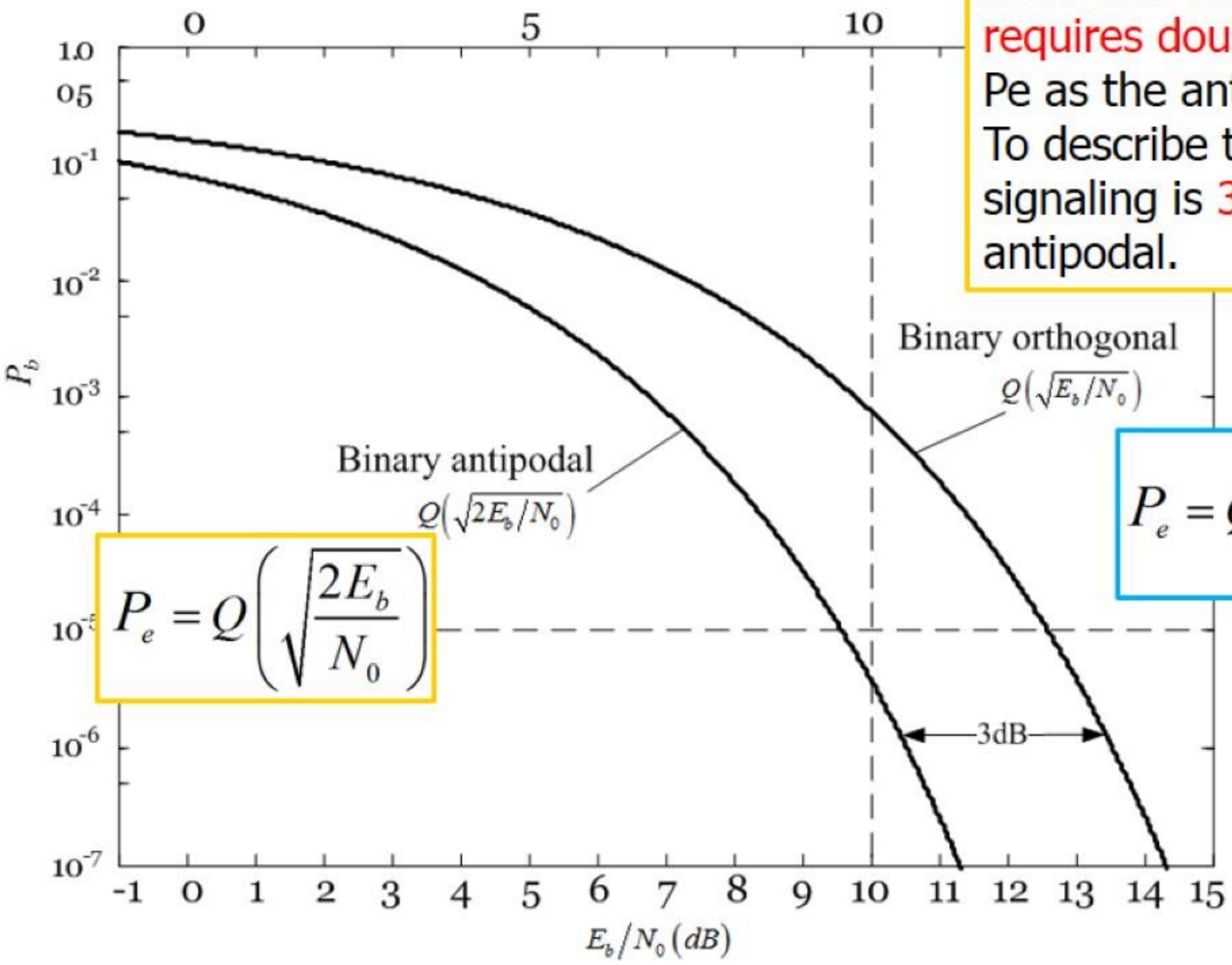
$$P_e = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

$$E_b = E_s = a^2 = d^2 / 2$$

Prob

$$= 1 - \int_{R_0} f(\mathbf{r}|\mathbf{s}_0)dr = \ldots = Q\left(\sqrt{\frac{d^2}{2N_0}}\right)$$

5.6.1 Binary modulations



$$P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

$$P_e = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

Note that the Bin-orthog.signaling requires doubled SNR to achieve same Pe as the antipodal.
To describe this we say that Bin-orthog. signaling is 3dB worse than the antipodal.

5.6.1 Binary modulations

Baseband

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Passband

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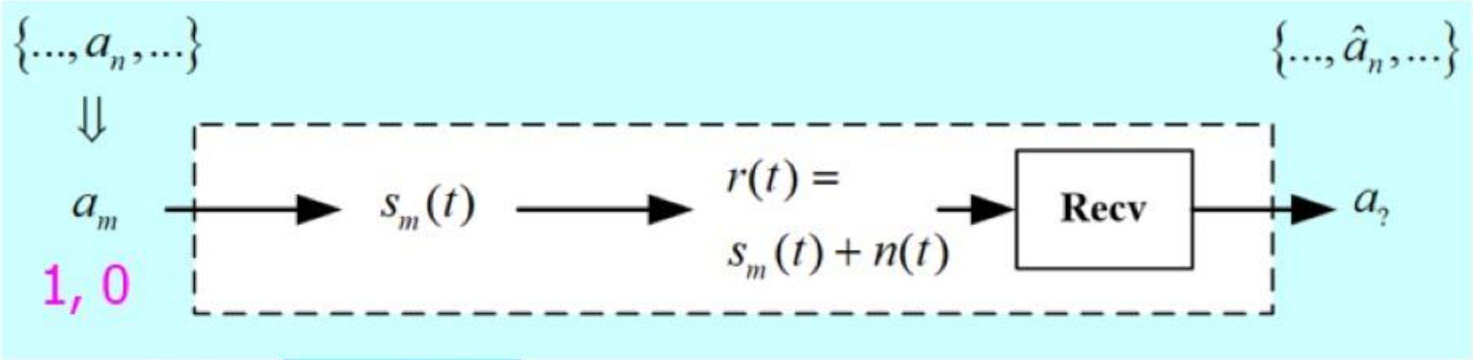
5.6.1 Binary modulations

PSK

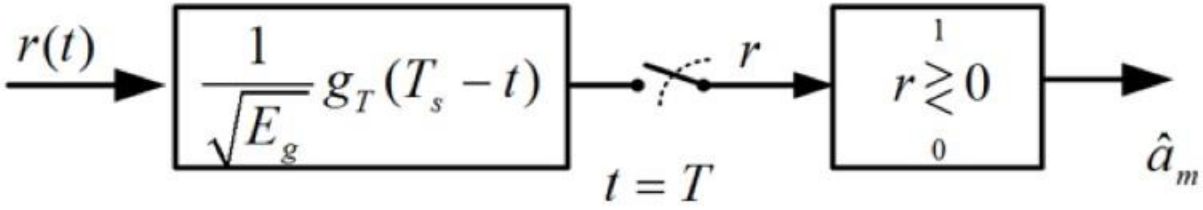


The signals: $s_1(t) = \boxed{a\psi(t)},$

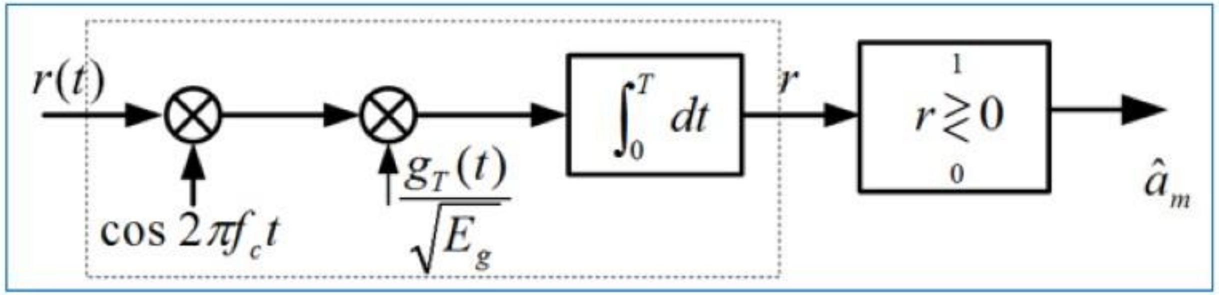
$s_0(t) = \boxed{-a\psi(t)}$

The 1-D basis: $\boxed{\psi(t) = \sqrt{\frac{2}{E_g}} g_T(t) \cos 2\pi f_c t}$

A MF-ML receiver is given by,

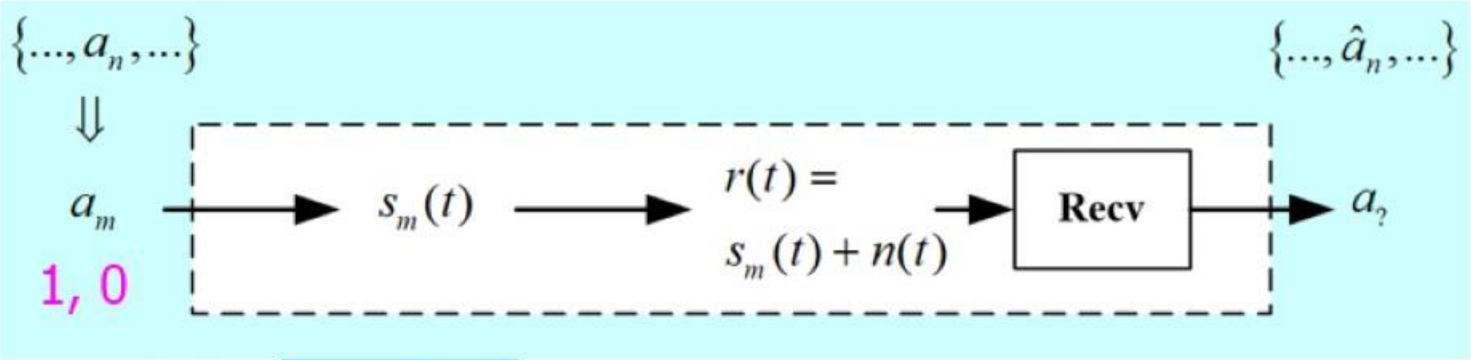


A Corr-ML receiver is given by,



5.6.1 Binary modulations

PSK



The signals: $s_1(t) = \boxed{a\psi(t),}$
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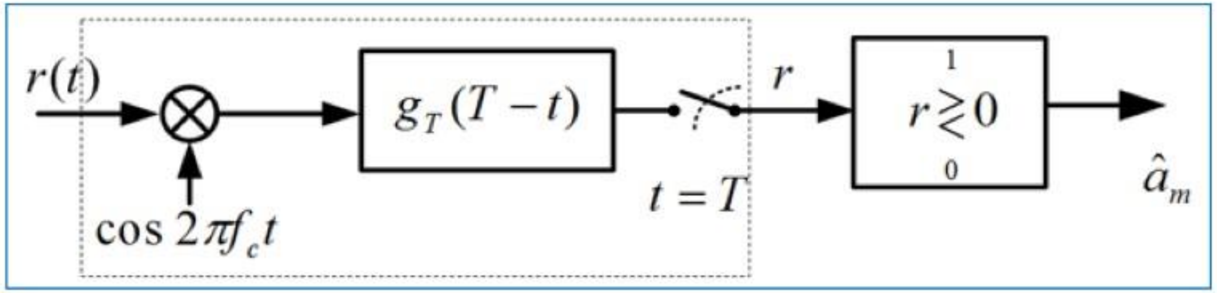
The 1-D basis: $\boxed{\psi(t) = \sqrt{\frac{2}{E_g}} g_T(t) \cos 2\pi f_c t}$

A mixer plus a baseband MF, then ML,



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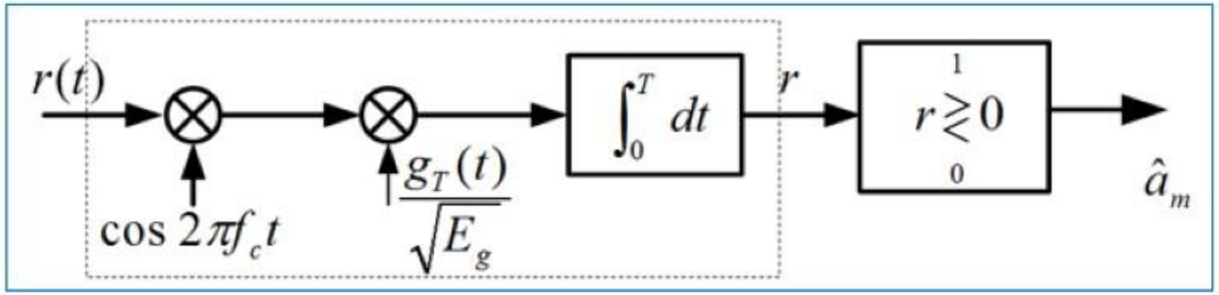

5.6.1 Binary modulations

PSK

$\{\ldots, a_n, \ldots\}$ $\Downarrow$ a_m (1, 0) $\longrightarrow s_m(t) \longrightarrow r(t) = s_m(t) + n(t) \longrightarrow$ **Recv** $\longrightarrow a_?$ $\{\ldots, \hat{a}_n, \ldots\}$

The signals: $s_1(t) = \boxed{a\psi(t)}$, $s_0(t) = \boxed{-a\psi(t)}$

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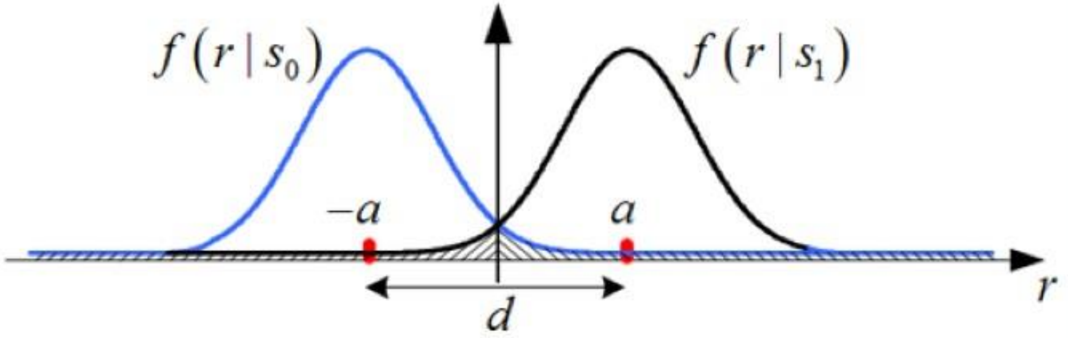

ML leads to a decision rule of : $r \overset{1}{\underset{0}{\gtrless}} 0$

Prob of err is computed by,

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- BFSK (Orthogonal signaling with M=2)

For OOK, leave the details to you, after class

For BFSK, discuss later