Chapter 5

Digital transmission through the AWGN channel

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Problems

Ch5 Digital transmission through the AWGN channels

Section 5.1-5.4: 5.3, 5.7

Section 5.5: 5.8

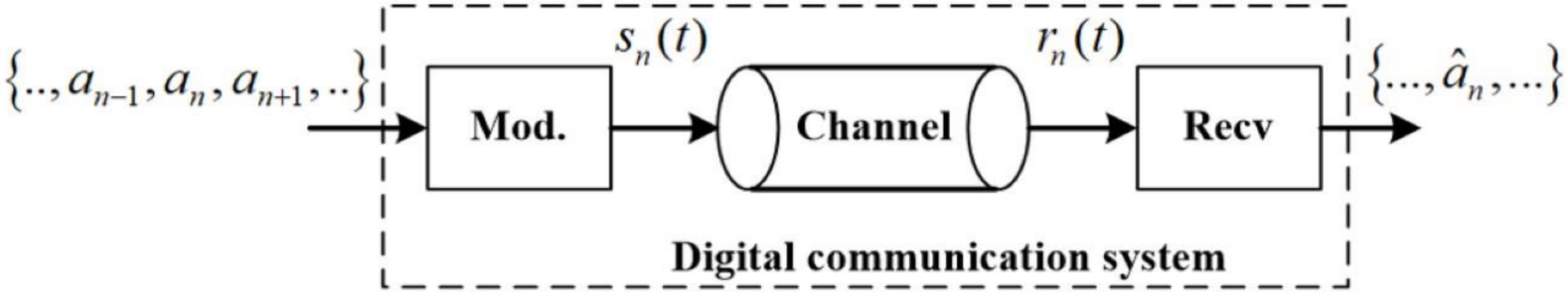
Section 5.6: 5.9, 5.10, 5.18, 5.34, 5.43, 5.47, 5.54

Contents 16 Hrs

- Introduction
- Geometric rep. of the sig waveforms
- Pulse amplitude modulation
- 2-d signal waveforms
- M-d signal waveforms
- Opt. reception for the sig. In AWGN
- Optimal receivers and probs of err

5.5 Opt. reception for the sig. In AWGN

A general block diagram of a Digital Comm Sys is shown as,



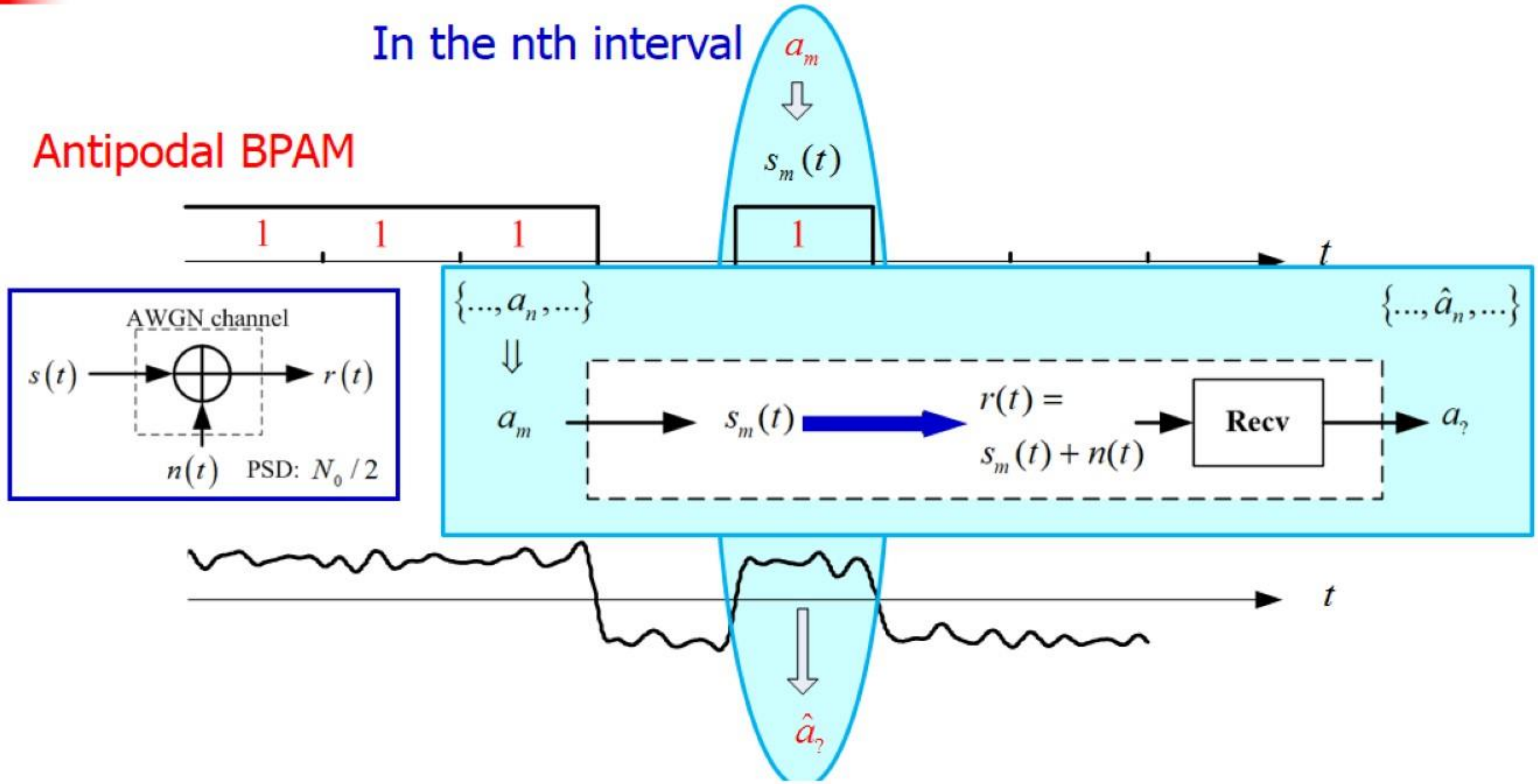
The symbols are independent and are represented by specified waveforms. The received signals is often a little different from the sent and we recover the symbols from them.

Take antipodal BPAM for msg 1110100…

5.5 Opt. reception for the sig. In AWGN

In the nth interval

Antipodal BPAM



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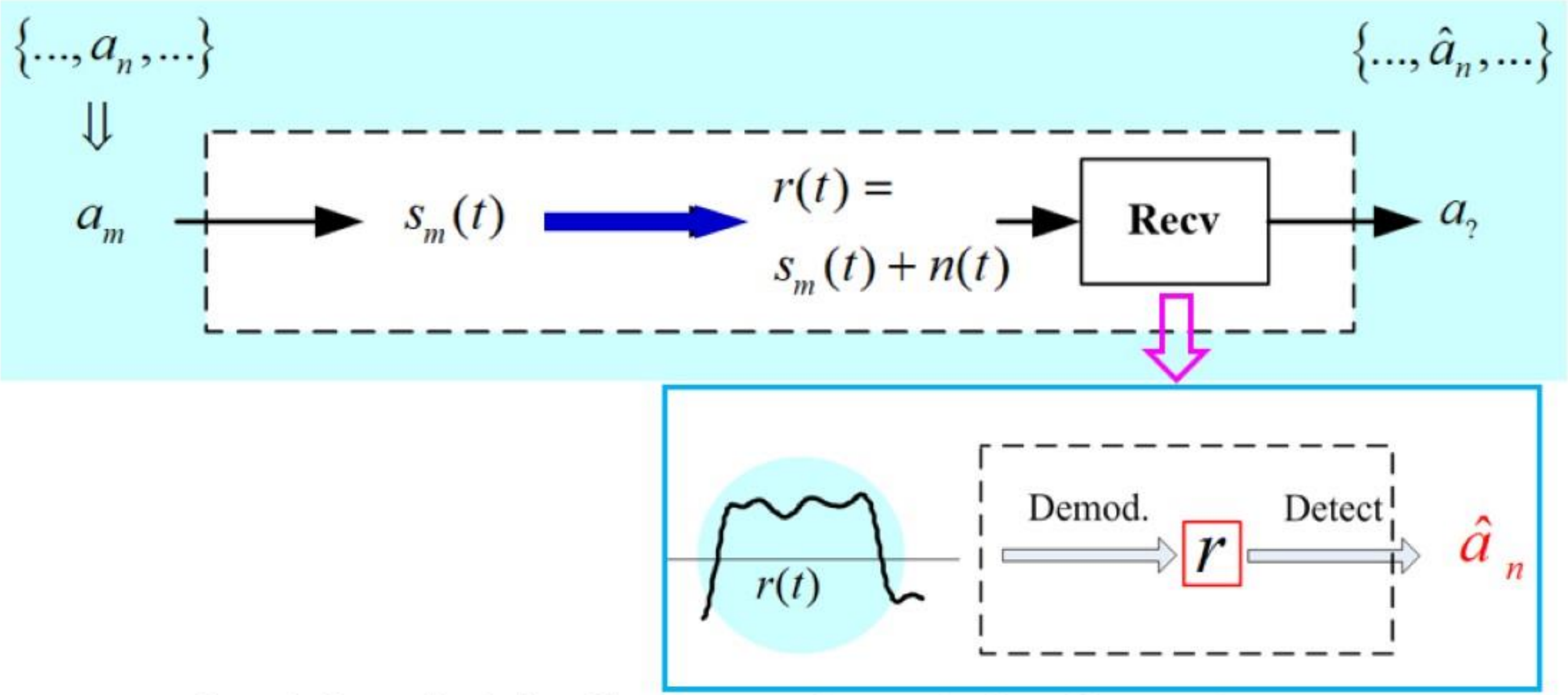
5.5 Opt. reception for the sig. In AWGN

In the nth interval, the process is as follows,



It is convenient to divide the receiver into 2 parts:

1)**Demodulator**: produce an observation from the waveform

2)**Detector**: estimate the sym from the observation

5.5 Opt. reception for the sig. In AWGN

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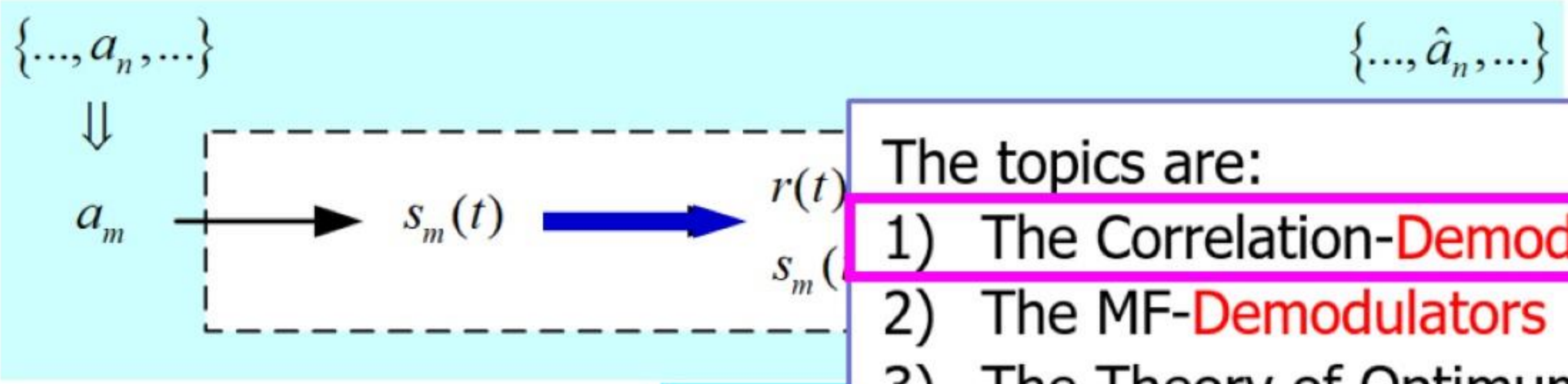
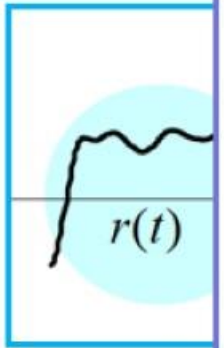




The topics are:

1) The Correlation-Demodulators
2) The MF-Demodulators
3) The Theory of Optimum Detector

5.6 Four Typical Optimal receivers

- Carrier-amp (1-D signals)
- Carrier-phs (2-D signals)
- QAM (2-D signals)
- FSK (M-D orthogonal signals)

It is convenient to divide the receiv

1)**Demodulator**: produce an obse

2)**Detector**: estimate the sym from the observation

5.5.1 Correlation-type demodulator

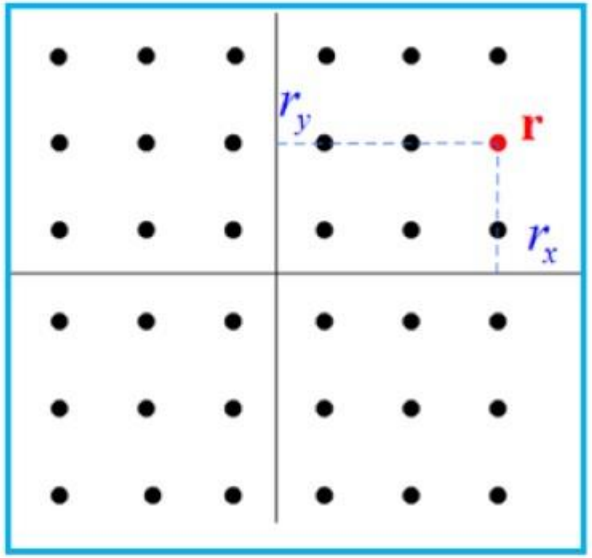
Demodulator: produce an observation from the waveform

An **observation** is a set of measurements that characterize the received waveform.

How to measure a waveform?

As a spot (vector), its coordinates specify the waveform.

It is straightforward to compute the coordinates of the waveform. So, an observation is a vector containing N coordinates



5.5.1 Correlation-type demodulator

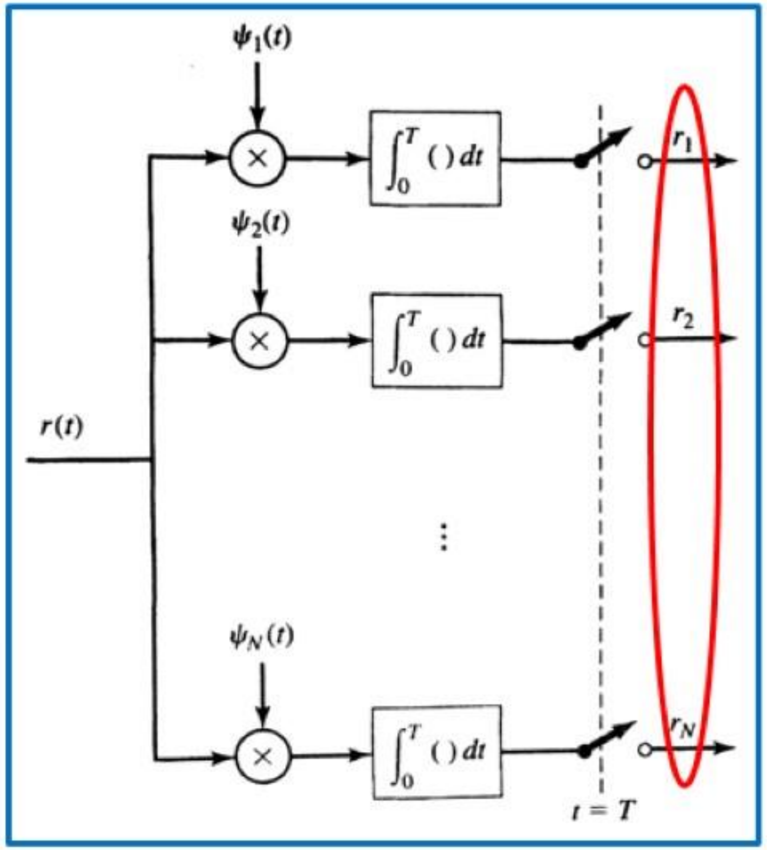
How to compute the coordinates ?

Given an N-d transmission waveform $s_m(t)$, $m = 1, 2, ..., M$,
the received signal is

$$r(t) = s_m(t) + n(t)$$

Let the orthonormal basis be $\psi_1(t), \ldots, \psi_N(t)$.

$$r_k = \int_{-\infty}^{\infty} r(t)\psi_k(t)dt$$
$$= \int_{-\infty}^{\infty} s_m(t)\psi_k(t)dt + \int_{-\infty}^{\infty} n(t)\psi_k(t)dt$$



In vector language, that is

$$\mathbf{r} = \mathbf{s}_m + \mathbf{n}$$

$$(r_1, \ldots, r_N) = (s_{m1}, \ldots, s_{mN}) + (n_1, \ldots, n_N)$$

5.5.1 Correlation-type demodulator

What is the **Characteristics** of $(r_1,\ldots,r_N)$?

$$\boxed{\begin{aligned} r_k &= \int_{-\infty}^{\infty} s_m(t)\psi_k(t)dt + \int_{-\infty}^{\infty} n(t)\psi_k(t)dt \\ &= s_{mk} + n_k \end{aligned}}$$

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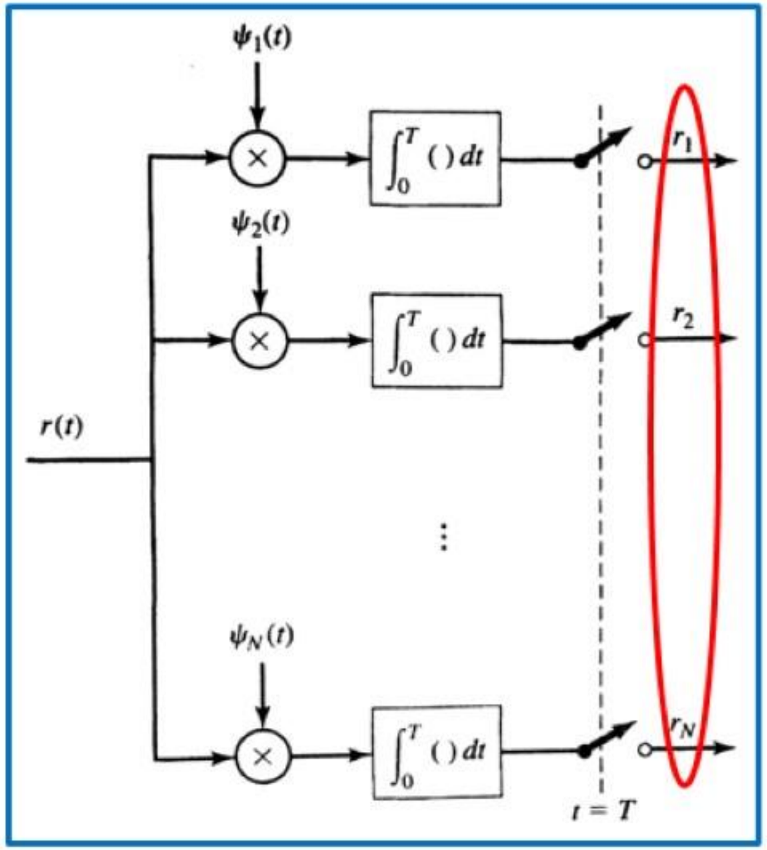
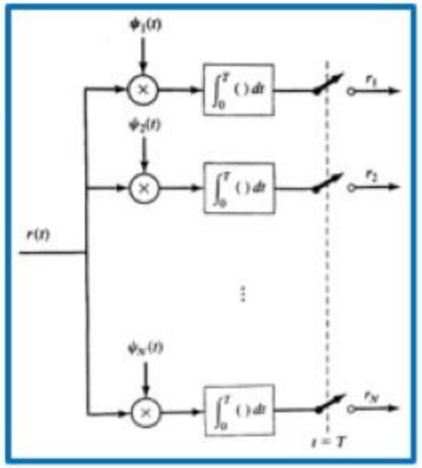


5.5.1 Correlation-type demodulator



What is the **Characteristics** of $(r_1, \ldots, r_N)$?

$$r_k = \int_{-\infty}^{\infty} s_m(t)\psi_k(t)dt + \int_{-\infty}^{\infty} n(t)\psi_k(t)dt$$
$$= s_{mk} + n_k$$

Note that $n(t)$ is AWGN with PSD of $N_0 / 2$. We have,

1) $\boxed{s_{mk}}$'s are constants determined by the trans. signal.

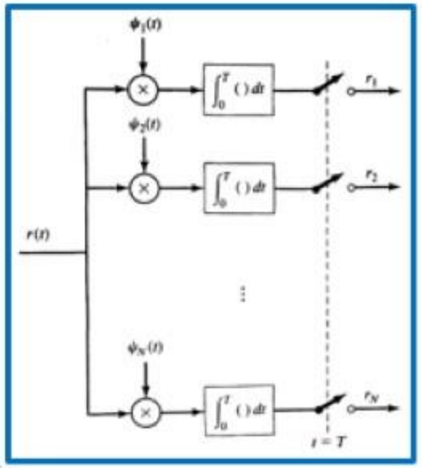
2) $\boxed{n_k}$'s are i.i.d Gaussian RVs with distribution of $N(0, N_0 / 2)$.

Therefore,

1) r_k's are independent Gaussian RVs with distribution of $N(s_{mk}, N_0 / 2)$;

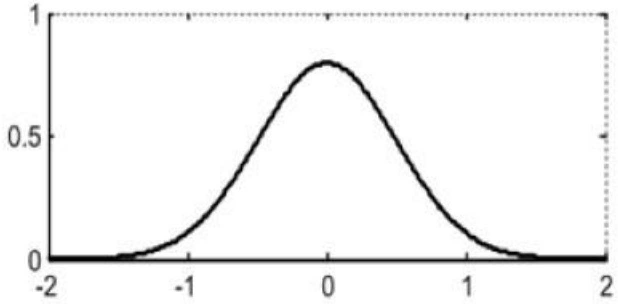
2) $\mathbf{r}$ is an N-d Gaussian Vector.

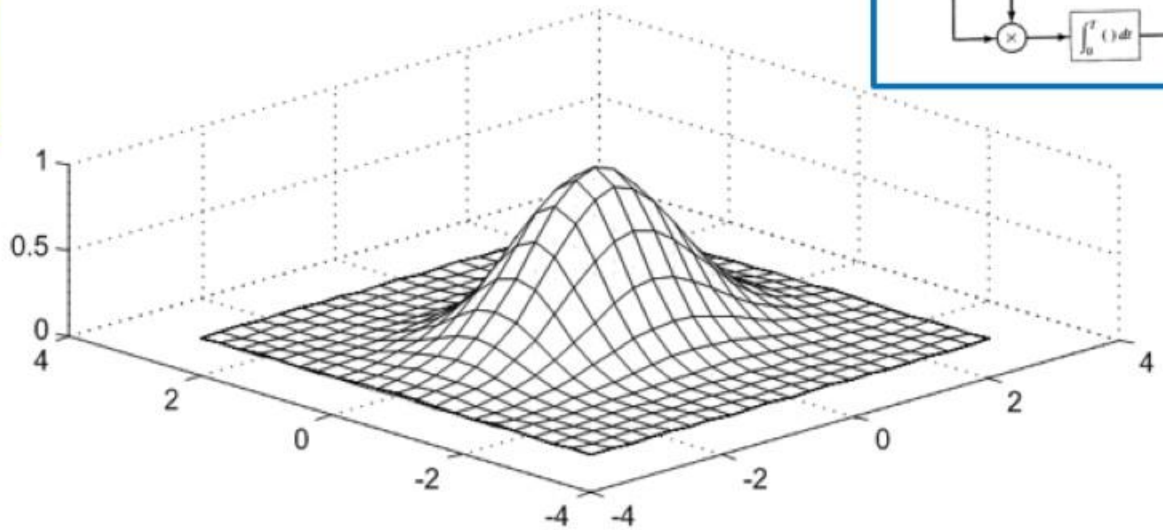
5.5.1 Correlation-type demodulator



What is the **Characteristics** of $(r_1, \ldots, r_N)$?

$$f(r_k \mid \mathbf{s}_m) = \frac{1}{\sqrt{2\pi N_0}} \exp\left[-\frac{(r_k - s_{mk})^2}{N_0} \right]$$





Therefore,

1) r_k's are independent Gaussian RVs with distribution of $N(s_{mk}, N_0/2)$;

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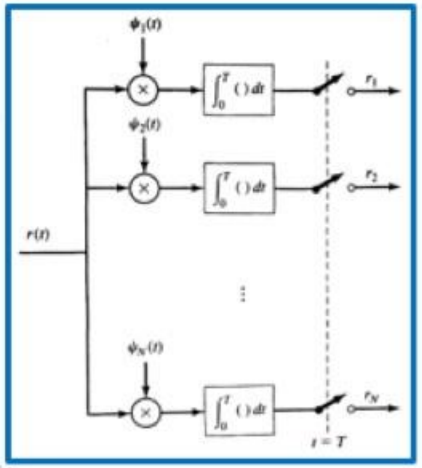
5.5.1 Correlation-type demodulator



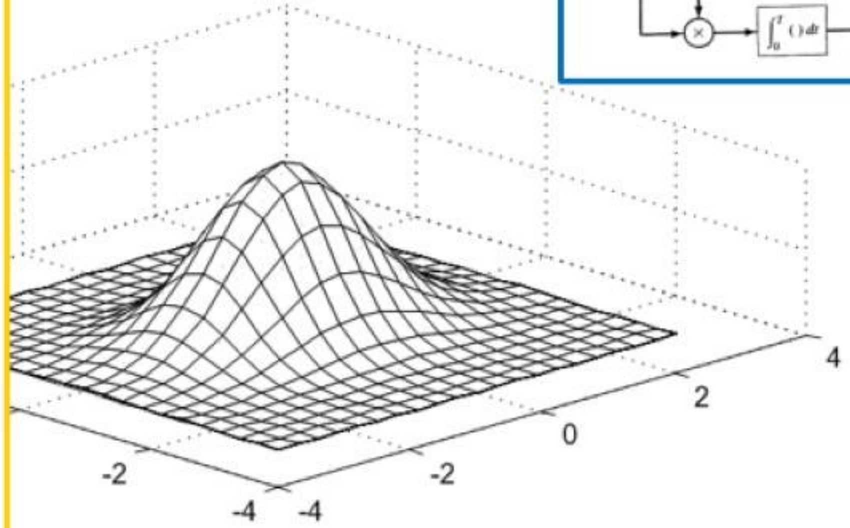
What is the **Characteristics** of $(r_1, \ldots, r_N)$? $_N)$?

Generally,

$$f(\mathbf{r} \mid \mathbf{s}_m) = \prod_{k=1}^{N} f(r_k \mid s_{mk}) = \prod_{k=1}^{N} \frac{1}{\sqrt{\pi N_0}} \exp\left[-\frac{(r_k - s_{mk})^2}{N_0} \right]$$

$$= \left(\frac{1}{\sqrt{\pi N_0}} \right)^N \exp\left[-\frac{1}{N_0} \sum_{k=1}^{N} (r_k - s_{mk})^2 \right]$$

$$= \left(\frac{1}{\sqrt{\pi N_0}} \right)^N \exp\left[-\frac{\left\| \mathbf{r} - \mathbf{s}_m \right\|^2}{N_0} \right]$$



Therefore,

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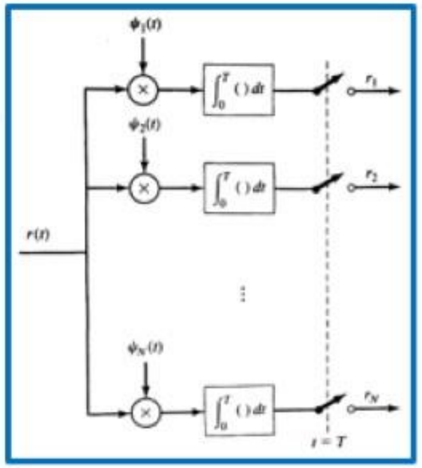


Geometric interpretation of demodulation:

$$\left(r_1,\ldots,r_N\right)=\left(s_{m1},\ldots,s_{mN}\right)+\left(n_1,\ldots,n_N\right)$$

$$\boxed{\sum_{k=1}^{N} r_k\psi_k(t)}=\sum_{k=1}^{N} s_{mk}\psi_k(t)+\sum_{k=1}^{N} n_k\psi_k(t)$$

$$=s_m(t)+?$$

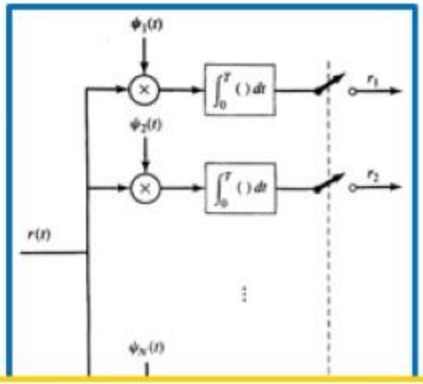
Q: Does $n(t)=\sum_{k=1}^{N} n_k\psi_k(t)$?

A: No! The noise can take any waveform that is much more complex than the signals in the N-d space.

Let $n(t)=\sum_{k=1}^{N} n_k\psi_k(t)+\boxed{n'(t)}$, where $n'(t)$ the "lost" part.

So, $r(t)=s_m(t)+n(t)$

$$=\boxed{\sum_{k=1}^{N} s_{mk}\psi_k(t)+\sum_{k=1}^{N} n_k\psi_k(t)}+\boxed{n'(t)}$$



Geometric interpretation of demodulation:

$$(r_1, \ldots, r_N) = (s_{m1}, \ldots, s_{mN}) + (n_1, \ldots, n_N)$$

$$\boxed{\sum_{k=1}^{N} r_k \psi_k(t)} = \sum_{k=1}^{N} s$$

$$= s_m(t) +$$

$s_m(t)$ is the signal in the N-d signal space.

$\sum_{k=1}^{N} n_k \psi_k(t)$ is the projection of $n(t)$ that is in the space.

$\sum_{k=1}^{N} r_k \psi_k(t)$ is the projection of $r(t)$ that is in the space.

$n'(t)$ is the part of $n(t)$, also part of $r(t)$, that is perpendicular to the space.

Let $n(t) = \sum_{k=1}^{N} n_k \psi_k($

So, $r(t) = s_m(t) + n($

$$= \boxed{\sum_{k=1}^{N} s_{mk} \psi_k(t) + \sum_{k=1}^{N} n_k \psi_k(t)} + \boxed{n'(t)}$$