# Chapter 5

# Digital transmission through the AWGN channel

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# Contents 16 Hrs

- Introduction
- Geometric rep. of the sig waveforms
- Pulse amplitude modulation
- 2-d signal waveforms
- M-d signal waveforms
- Opt. reception for the sig. in AWGN
- Optimal receivers and probs of err

# 5.4 Multi-d signal waveforms

Sets of multiple dimensional signals are used to transmit M-ary symbols.

Common types are

1. Orthogonal signals;
2. *Bi-orthogonal signals;
3. Simplex signals;
4. Binary coded signals;

## 5.4.1 Orthogonal signal waveforms

Consider a space with orthonormal basis of $\{\psi_1, \psi_2, \cdots, \psi_M\}$, for M-ary transmission

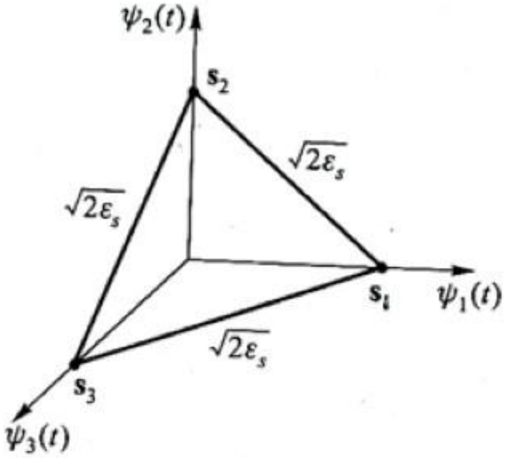
And, construct signal set $\{\mathbf{s}_1, \mathbf{s}_2, \cdots, \mathbf{s}_M\}$ such that,

$$\mathbf{s}_1 = \sqrt{E_s}\psi_1, \ \mathbf{s}_2 = \sqrt{E_s}\psi_2 \ ,\ldots, \ \mathbf{s}_M = \sqrt{E_s}\psi_M$$

They are, 1) **orthogonal**, 2) M-dimensional, 3) having equal energies .

For convenience of graphing, take M=3 in our discussion.
The coordinates and constellations are,

$$\mathbf{s}_1 = \left(\sqrt{E_s}, 0, 0\right)$$
$$\mathbf{s}_2 = \left(0, \sqrt{E_s}, 0\right)$$
$$\mathbf{s}_3 = \left(0, 0, \sqrt{E_s}\right)$$




$$\begin{aligned} d_{mn} &= \|\mathbf{s}_m - \mathbf{s}_n\| \\ &= \sqrt{\left(\sqrt{E_s}\right)^2 + \left(-\sqrt{E_s}\right)^2} \\ &= \sqrt{2E_s} = \sqrt{2}\,\|\mathbf{s}_m\| \end{aligned}$$

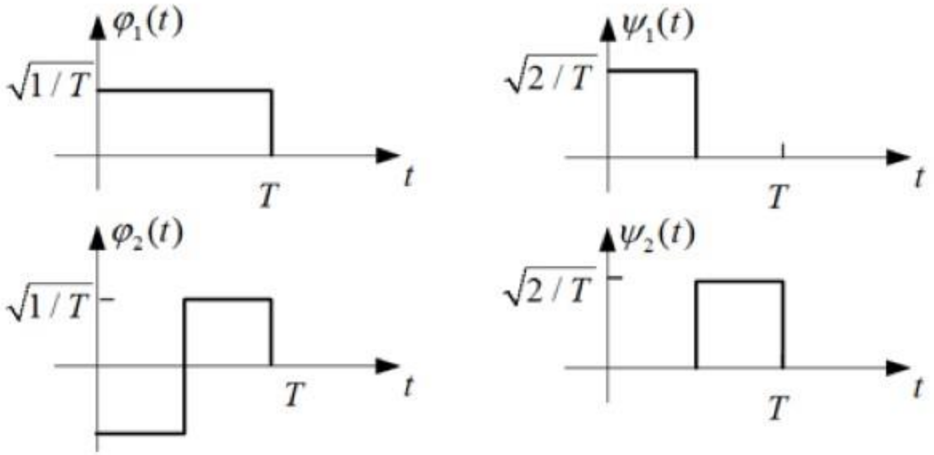
# 5.4.1 Orthogonal signal waveforms

$$\boxed{\mathbf{s}_1 = \sqrt{E_s}\boldsymbol{\psi}_1, \quad \mathbf{s}_2 = \sqrt{E_s}\boldsymbol{\psi}_2 \quad ,\ldots, \quad \mathbf{s}_M = \sqrt{E_s}\boldsymbol{\psi}_M}$$

**Baseband examples**

- "Left" =signals which are completely overlapped
- "Right" =signals which are completely **non-**overlapped

For Binary (M=2)



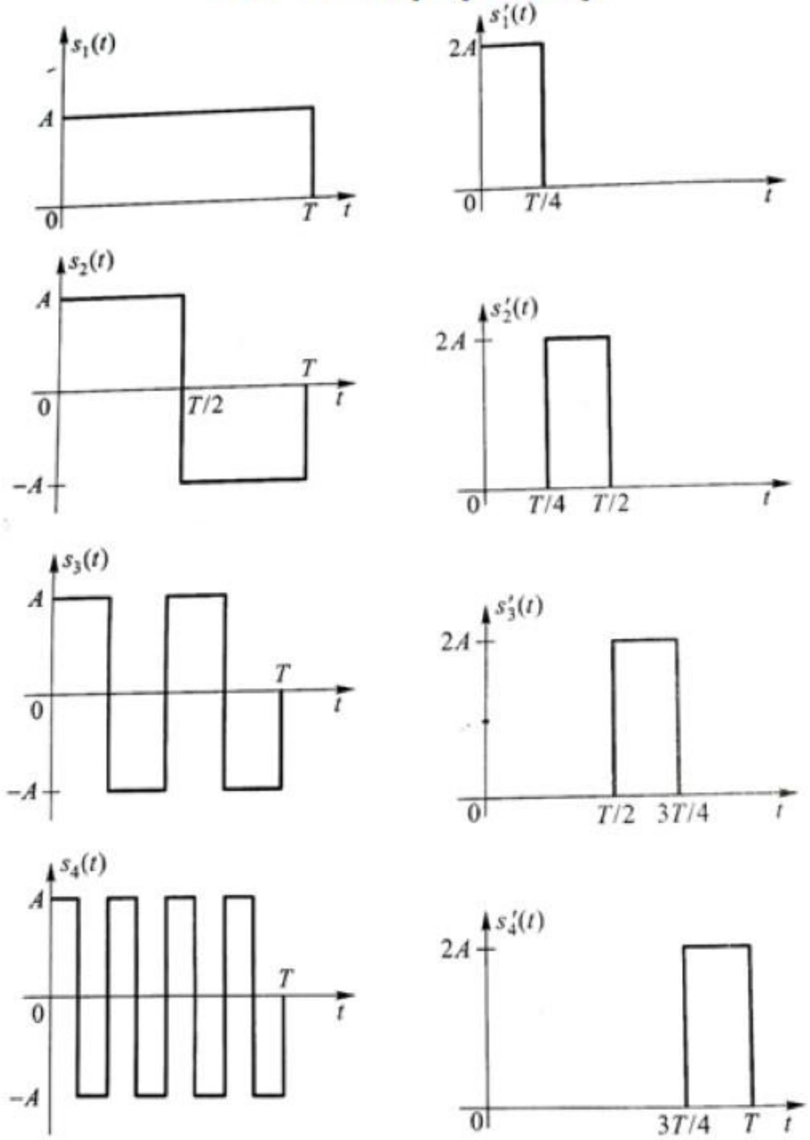
# 5.4.1 Orthogonal signal waveforms

$$\mathbf{s}_1 = \sqrt{E_s}\boldsymbol{\psi}_1, \quad \mathbf{s}_2 = \sqrt{E_s}\boldsymbol{\psi}_2, \quad \ldots, \quad \mathbf{s}_M = \sqrt{E_s}\boldsymbol{\psi}_M$$

**Baseband examples**

- "Left" =signals which are completely overlapped
- "Right" =signals which are completely **non-**overlapped

For M-ary (M=4)




In practical,

- The "Left" is usually generated by a famous procedure called Hardamard/ Walsh-Hardamard Transform (WHT).
- The "Right" is often called PPM (Pulse position modulation), which are pulses with diff. position.

## 5.4.1 Orthogonal signal waveforms

$$\boxed{\mathbf{s}_1 = \sqrt{E_s}\,\boldsymbol{\psi}_1,\ \ \mathbf{s}_2 = \sqrt{E_s}\,\boldsymbol{\psi}_2\ \ ,\ldots,\ \ \mathbf{s}_M = \sqrt{E_s}\,\boldsymbol{\psi}_M}$$

**Passband examples**

A very common type is call **FSK** (**Freq shift keying**).

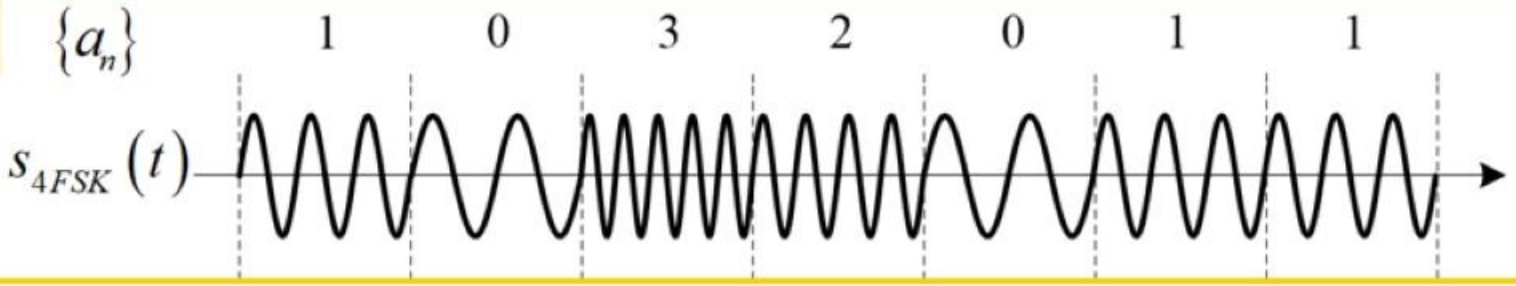
The signals are given by,

$$u_m(t) = A\cos\left[2\pi(f_0 + m\times\Delta f)t\right] \qquad t\in[0,T], m = 0,1,\ldots,M-1$$

Note that:

1. $\Delta f$ is the freq separation which is some value making the signals orthogonal.
2. With same amplitude the signals are all of equal energy.

Take M=4 (4FSK)



# 5.4.1 Orthogonal signal waveforms

$$\mathbf{s}_1 = \sqrt{E_s}\boldsymbol{\psi}_1, \quad \mathbf{s}_2 = \sqrt{E_s}\boldsymbol{\psi}_2, \ \ldots, \ \mathbf{s}_M = \sqrt{E_s}\boldsymbol{\psi}_M$$

FSK: $u_m(t) = A\cos\left[2\pi(f_0 + m\times\Delta f)t\right]$ $\quad t$

What is $\Delta f$? To make the signals mu

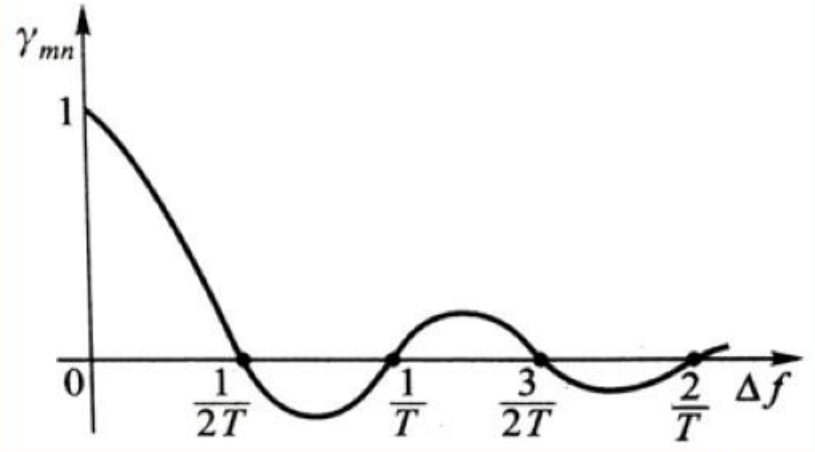
Compute the correlation coeff. of any two sign

$$\gamma_{mn} = \frac{\mathbf{u}_m\cdot\mathbf{u}_n}{\|\mathbf{u}_m\|\|\mathbf{u}_n\|} = \frac{A^2}{E_s}\int_0^T \cos[2\pi(f_c+m\Delta f)t]\cos[2\pi(f_c+n\Delta f)t]dt$$

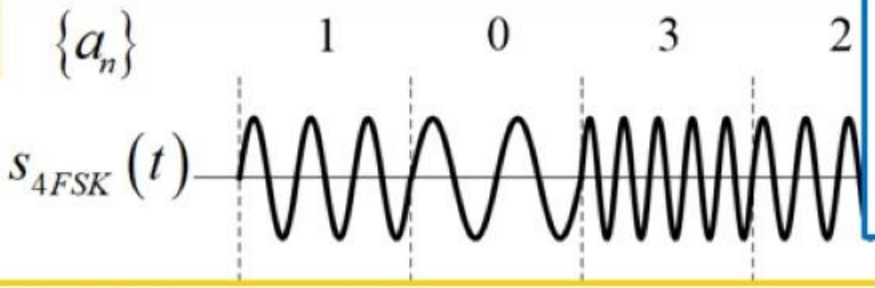
$$= \ldots = \frac{\sin 2\pi(m-n)\Delta f T}{2\pi(m-n)\Delta f T}$$

For orthogonal, we require that $\gamma_{mn} = 0$ and the orthogonal condi is $\Delta f = 0.5kR_s$, where $R_s = 1/T$ and k is any posi. interger.

The smallest freq separation is $\Delta f = 0.5R_s$




Take M=4 (4FSK)



# 5.4.1 Orthogonal signal waveforms

FSK: $u_m(t) = A\cos\left[2\pi(f_0 + m\times\Delta f)t\right] \qquad t\in[0,T], m = 0,1,\ldots,M-1$

The Binary example is called **BFSK(**or **2FSK).**

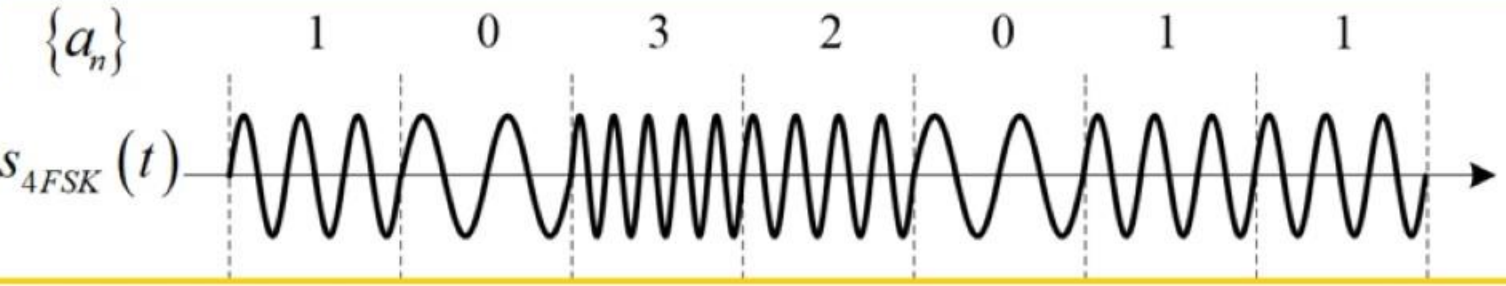
$$\begin{cases} u_1(t) = A\cos 2\pi f_1 t \\ u_2(t) = A\cos 2\pi f_2 t \end{cases} \quad \text{where,} \begin{cases} f_1 = f_c + \Delta f/2 \\ f_0 = f_c - \Delta f/2 \end{cases}$$

And, $f_c = [f_0 + f_1]/2$

The **MSK** is the BFSK with minimum freq separation and continuous phase. Thus,

$$f_1 = f_c + R_s/4, \qquad f_0 = f_c - R_s/4$$

Take M=4 (4FSK)



## 5.4.2 Bi-Orthogonal signal waveforms

A set of M-ary bi-orthogonal signals is constructed from a set of M/2-ary orthogonal signals.

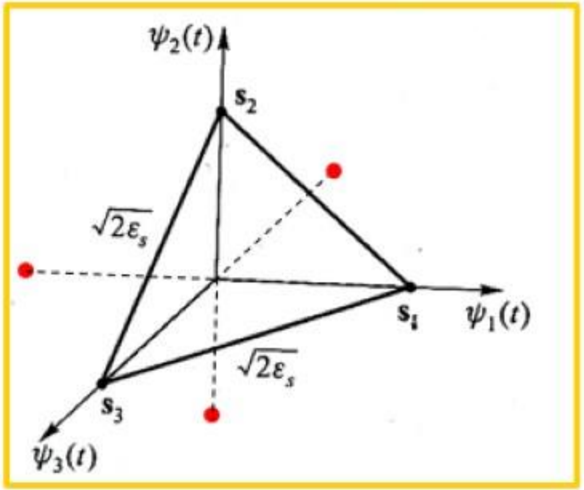
Given M=6 and 3 orthogonal signals $\{\mathbf{s}_1, \mathbf{s}_2, \mathbf{s}_3\}$, the set of bi-orthogonal signals is simply given by,

$$\boxed{\{\mathbf{s}_1, \mathbf{s}_2, \mathbf{s}_3, -\mathbf{s}_1, -\mathbf{s}_2, -\mathbf{s}_3\}}$$

Or expressed as,

$$\mathbf{s}_1 = \left(\sqrt{E_s}, 0, 0\right) \quad \mathbf{s}_2 = \left(0, \sqrt{E_s}, 0\right) \quad \mathbf{s}_3 = \left(0, 0, \sqrt{E_s}\right)$$

$$\mathbf{s}_4 = \left(-\sqrt{E_s}, 0, 0\right) \quad \mathbf{s}_5 = \left(0, -\sqrt{E_s}, 0\right) \quad \mathbf{s}_6 = \left(0, 0, -\sqrt{E_s}\right)$$




We have the followings:

- M-ary bi-orthogonal signals are M/2 dimensional.
- The energy of signals are equal. Two distances between signals are, $\sqrt{2E_s}$ and $2\sqrt{E_s}$.
- Baseband bi-orthogonal signals are from baseband orthogonal signals, Bandpass signals from bandpass ones.