Chapter 5

Digital transmission through the AWGN channel

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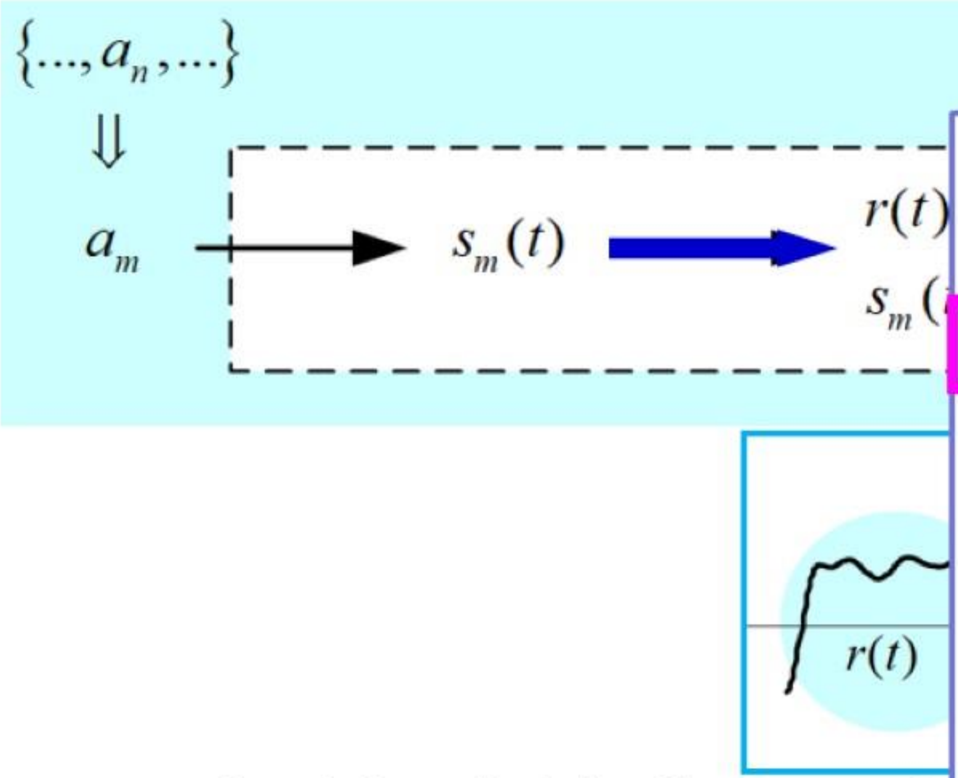
SICE, UESTC

Contents 16 Hrs

- Introduction
- Geometric rep. of the sig waveforms
- Pulse amplitude modulation
- 2-d signal waveforms
- M-d signal waveforms
- Opt. reception for the sig. In AWGN
- Optimal receivers and probs of err

5.5 Opt. reception for the sig. In AWGN

In the nth interval, the process is as follows,



The topics are:

1) The Correlation-Demodulators
2) The MF-Demodulators
3) The Theory of Optimum Detector

5.6 Four Typical Optimal receivers

- Carrier-amp (1-D signals)
- Carrier-phs (2-D signals)
- QAM (2-D signals)
- FSK (M-D orthogonal signals)

It is convenient to divide the receiv

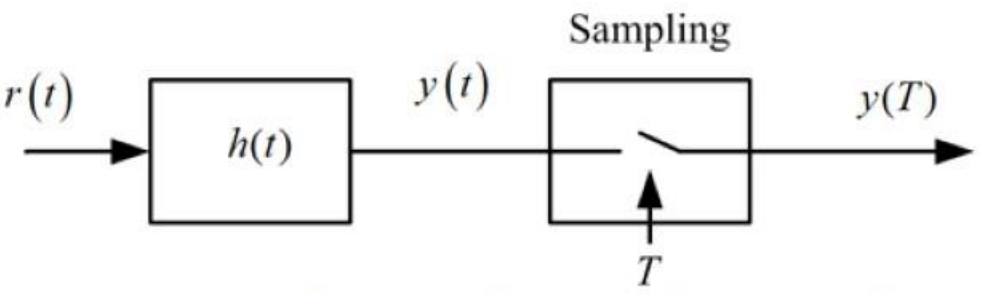
1)**Demodulator**: produce an obse

2)**Detector**: estimate the sym from the observation

5.5.2 Matched-filter type demodulator

A **Matched Filter (MF)** to s(t) is defined as a LTI filter whose impulse response is as $h(t) = Cs(T-t)$, where C is any non-zero constant and T is a given sample-time.

Consider a signal s(t) being corrupted as r(t)=s(t)+n(t), where n(t) is AWGN. Suppose that r(t) is measured as follows.



The filter h(t) is to reduce the noise and sampling is to obtain a measurement.

We design the filter h(t) to obtain max-SNR in y(T), so that a GOOD measurement is obtained.

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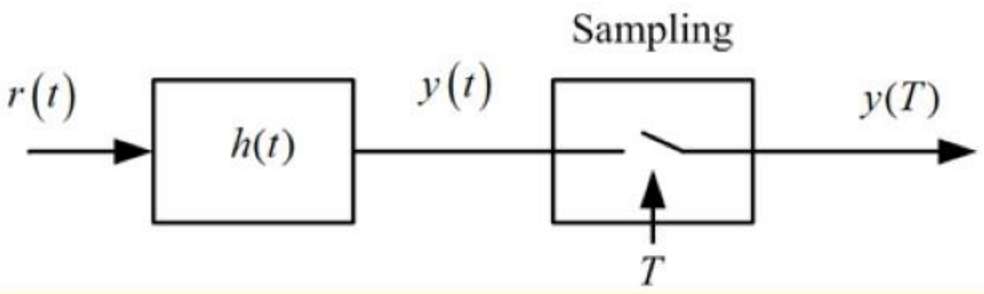

Properties of MF:

If a given signal is corrupted by AWGN, a matched filter to the signal maximizes the output SNR.

Simply, MF is the optimal filter for this case

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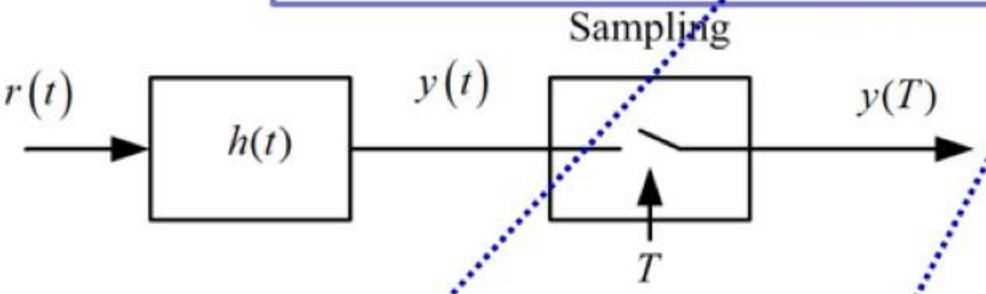
The SNR is defined as,

$$\left(\frac{S}{N}\right)_0 = \frac{\left(\int_{-\infty}^{\infty} h(\tau)s(T-\tau)d\tau\right)^2}{E\left(\int_{-\infty}^{\infty} h(\tau)n(T-\tau)d\tau\right)^2}$$

Consider a signal s(t) b n(t) is AWGN. Suppose that r(t)



Compute,

$$y(T) = h(t) * r(t)\big|_{t=T} = \int_{-\infty}^{\infty} h(\tau)r(t-\tau)d\tau\Big|_{t=T} = \int_{-\infty}^{\infty} h(\tau)r(T-\tau)d\tau$$

$$= \int_{-\infty}^{\infty} h(\tau)s(T-\tau)d\tau + \int_{-\infty}^{\infty} h(\tau)n(T-\tau)d\tau$$

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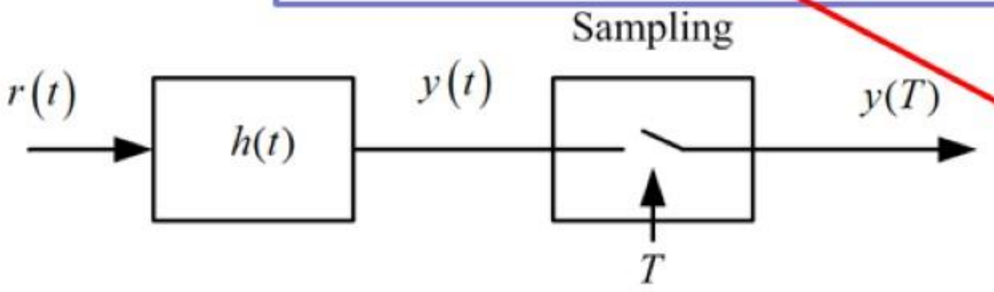
The SNR is defined as,

$$\left(\frac{S}{N}\right)_{0\_max} = \frac{2G_E E_s}{N_0 G_E} = \frac{2E_s}{N_0}$$



MF is the optimal filter !

when $h(t) = Cs(T-t)$

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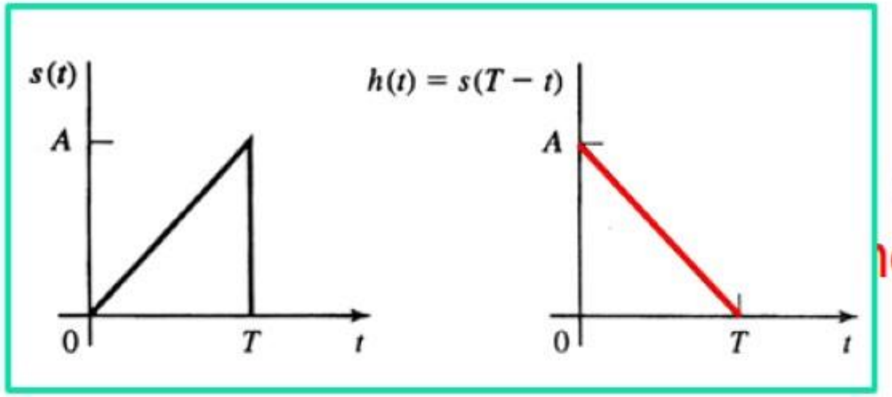

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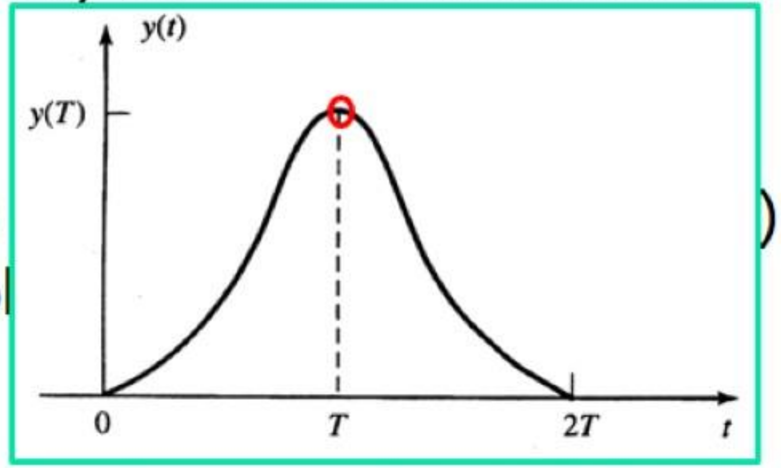

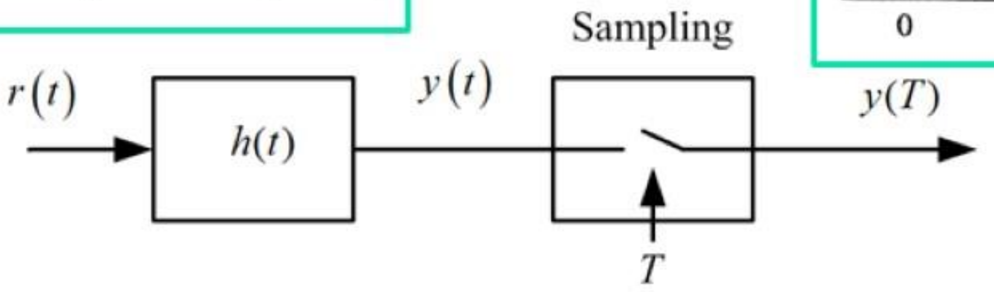

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Compute,

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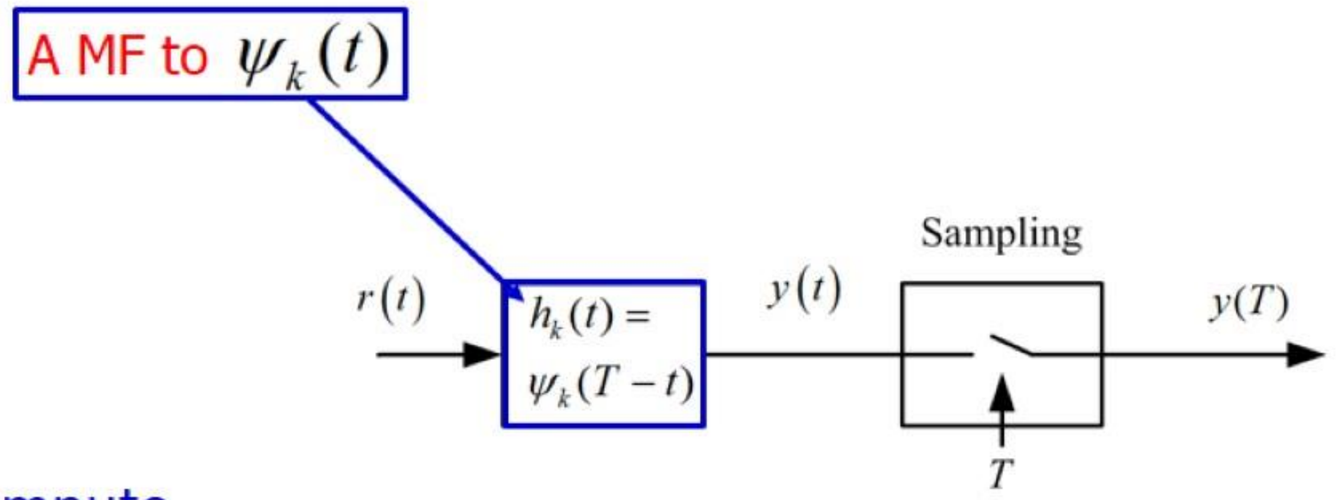

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$$y(T) = \boxed{h(t) * r(t)\Big|_{t=T} = \int_{-\infty}^{\infty} h(\tau) r(t-\tau) d\tau \Big|_{t=T}} = \int_{-\infty}^{\infty} h(\tau) r(T-\tau) d\tau$$

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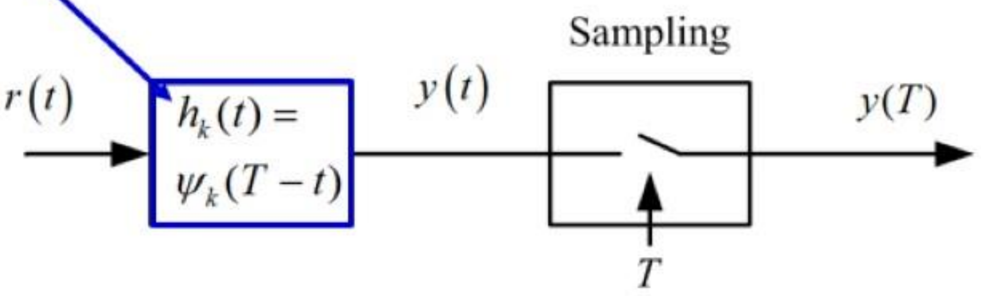
A MF to $\psi_k(t)$



MF is the optimal filter !

Compute,

$$y_k(t) = \boxed{\int_{-\infty}^{\infty} r(\tau)h_k(t-\tau)d\tau} = \int_{-\infty}^{\infty} r(\tau)\psi_k[T -$$

So,

$$y_k(T) = \int_{-\infty}^{\infty} r(\tau)\psi_k(\tau)d\tau = \boxed{r_k}$$

A matched filter is equivalent to a correlator. An alternate demodulator is as Fig 5.35 on page 276.

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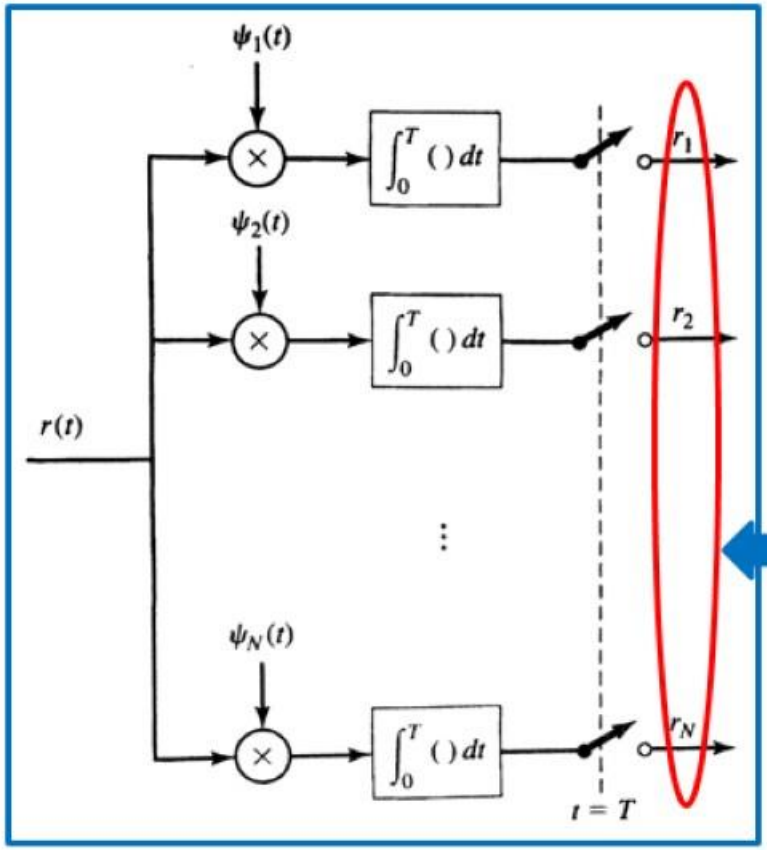

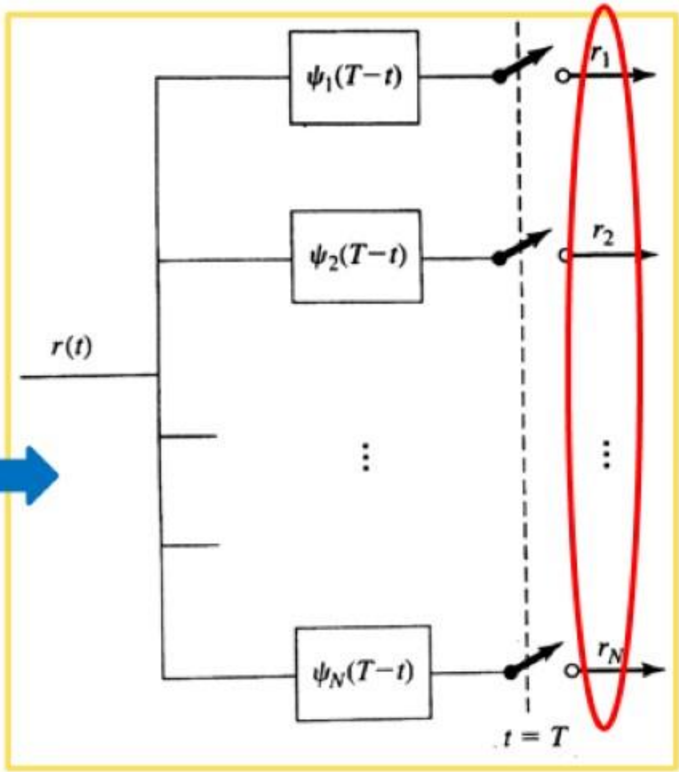

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