# Chapter 5

# Digital transmission through the AWGN channel

—— by Prof. XIAOFENG LI
SICE, UESTC

# 5.6

**Binary**

Baseband: BPAM (antipodal, unipolar), Orthogonal signaling;

Passband: BPSK, OOK or BASK, BFSK (Orthogonal)

**M-ary, 1-D signaling**

Baseband: MPAM

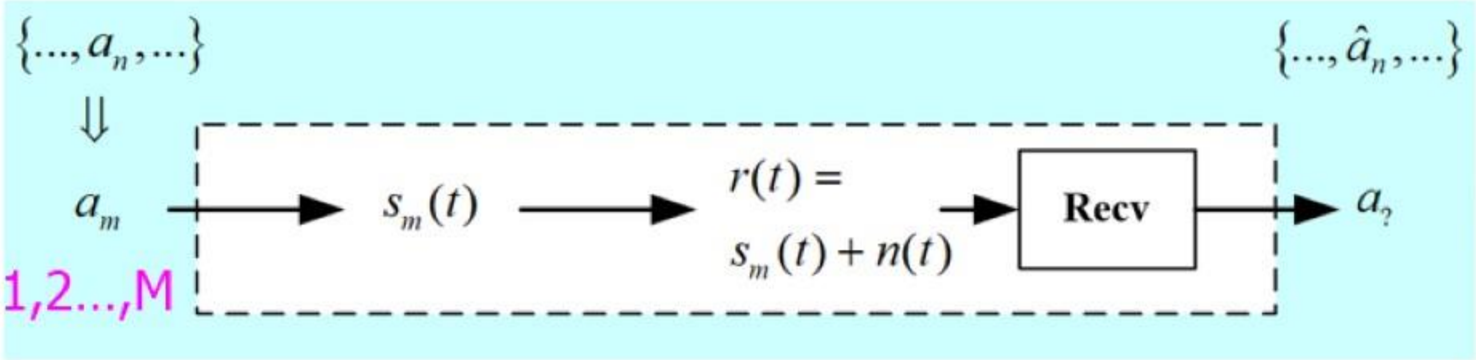
Passband: ~~MASK~~

**M-ary, 2-D signaling (Passband)**

MPSK, QAM

# 5.6.3 MPSK

MPSK
(Carrier-phs)




In many simple case the $g_T(t)$ is a rect pulse, and the MF is equivalent to an integrator.

The signals: $s_m(t) = g_T(t)\cos\left(2\pi f_c t + \frac{2\pi m}{M}\right)$

$$= A_{mc} g_T(t)\cos 2\pi f_c t - A_{ms} g_T(t)\sin 2\pi f_c t$$

The basis: $\psi_1(t) = \sqrt{\frac{2}{E_g}} g_T(t)\cos 2\pi f_c t,$

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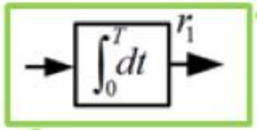
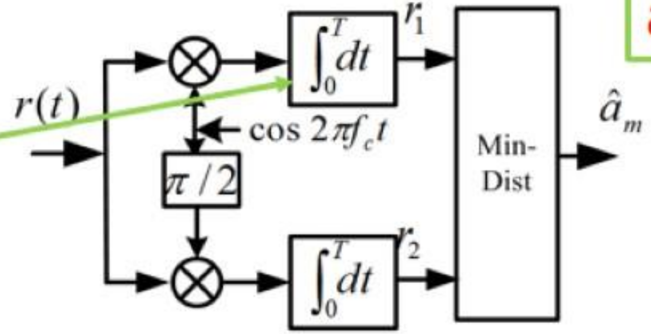
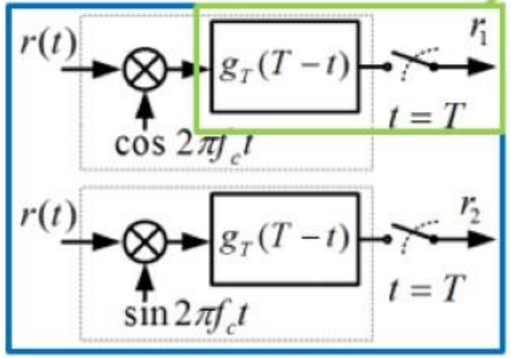
A MF-ML receiver is given by,




Prob of err is computed by,

Two **sync demod**. that requires:

- The local cos/sin are coherent with the carriers in recv sig.
- The sampler/integrator is synchronous with the symbols in recv sig.

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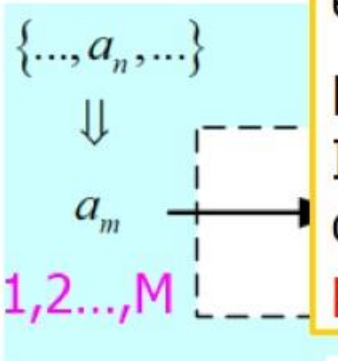
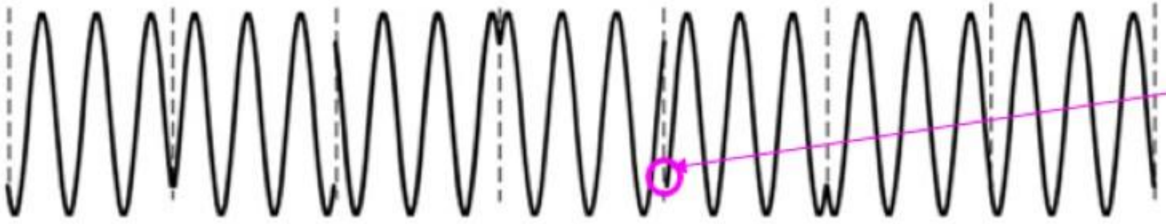


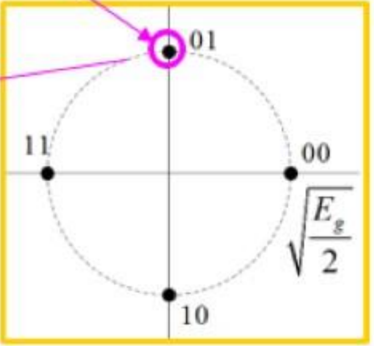

Unfortunately, the LO has phase ambiguities and DPSK is often employed for solution.

**DPSK**

Instead of absolute phase-mapping of info sym, PSK with differential phase-mapping is called DPSK.

| Sym | Abs-mapping: $\theta$ | Relative mapping: $\Delta\theta$ |
|---|---|---|
| 0 | $0$ | $0$ |
| 1 | $\pi/2$ | $\pi/2$ |
| 2 | $\pi$ | $\pi$ |
| 3 | $3\pi/2$ | $3\pi/2$ |



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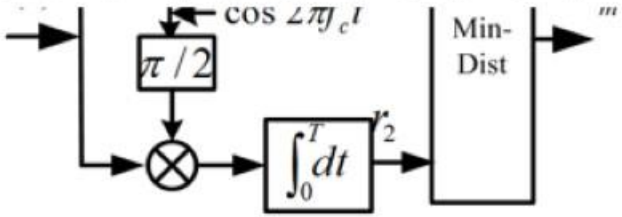
$= A_{mc} g_T(t)\cos 2\pi f_c t -$

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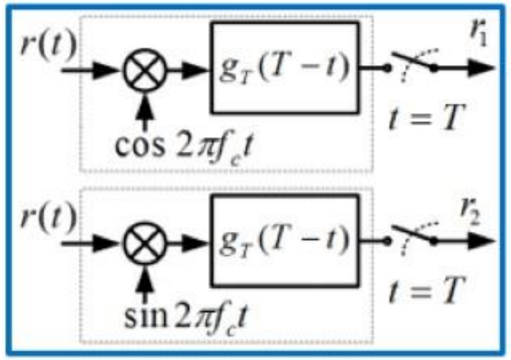


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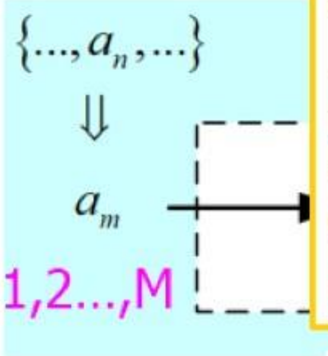


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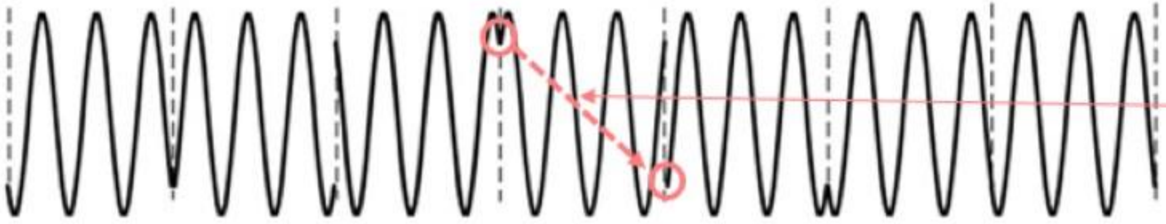

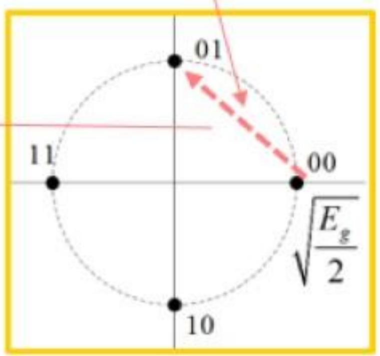


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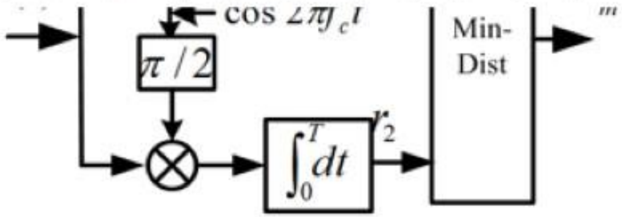


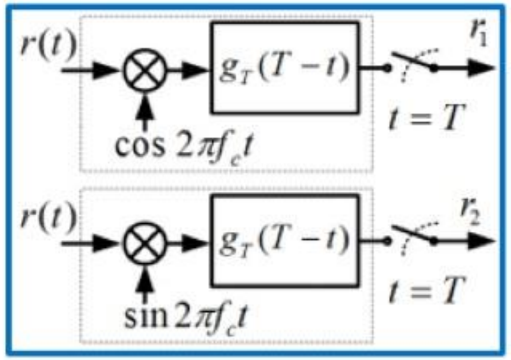


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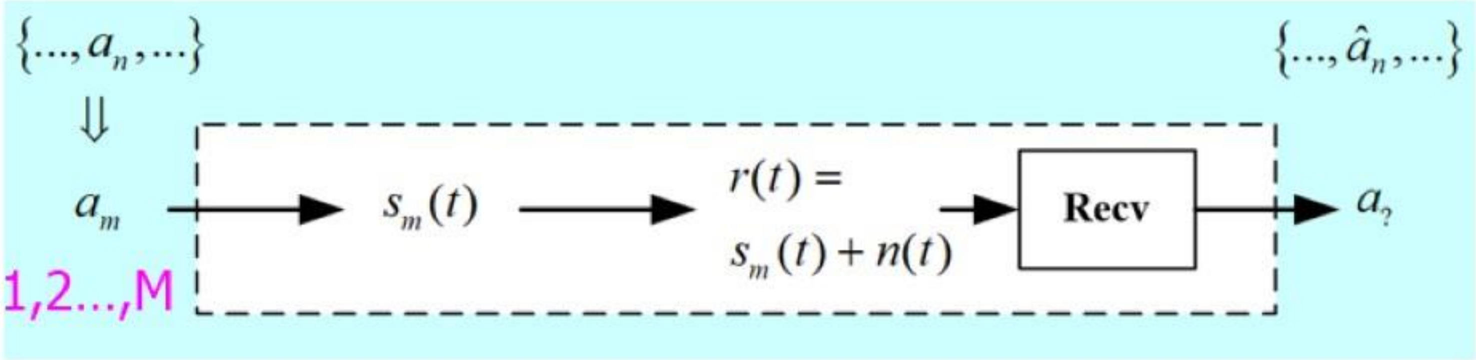


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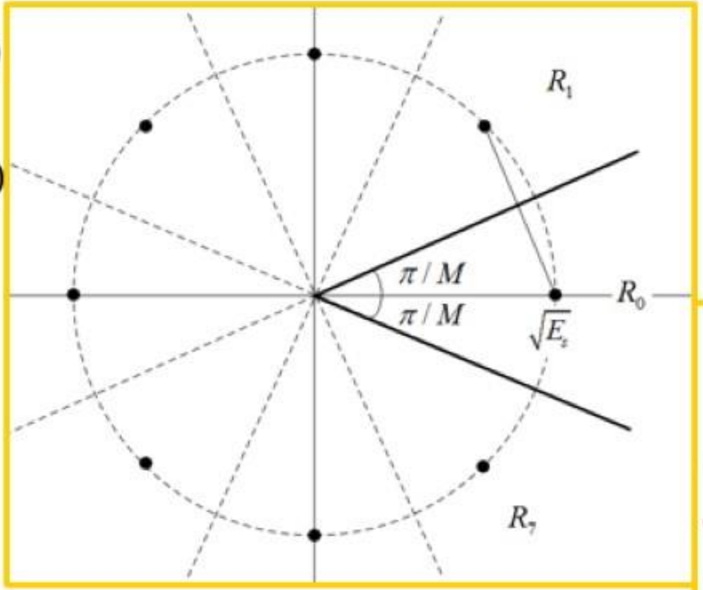


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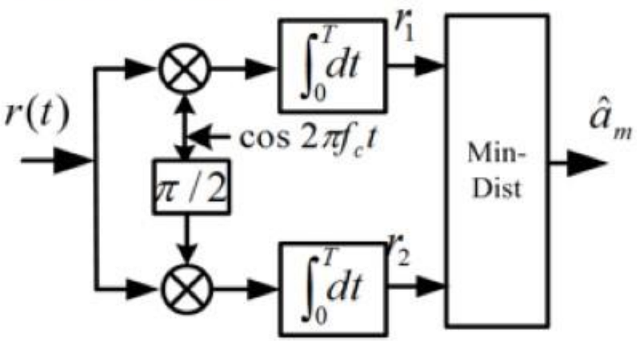



Prob of err is computed by,

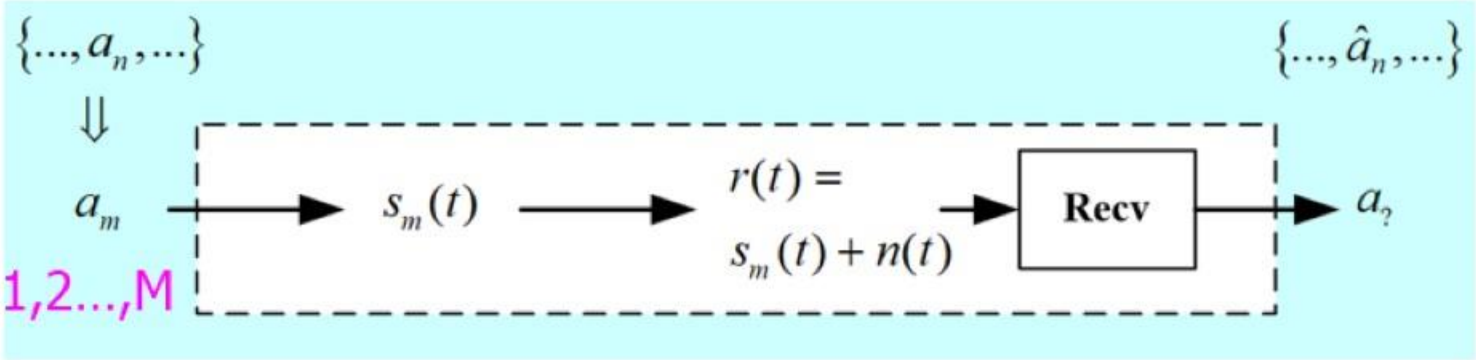
Let $\Theta_m = 2\pi m / M$ be the angle. of $\mathbf{s}_m$.

Compute $\Theta_r = \arctan(r_2 / r_1)$.

The decision rule: select the $\mathbf{s}_m$ whose angle is closest to $\Theta_r$.

# 5.6.3 MPSK

MPSK
(Carrier-phs)

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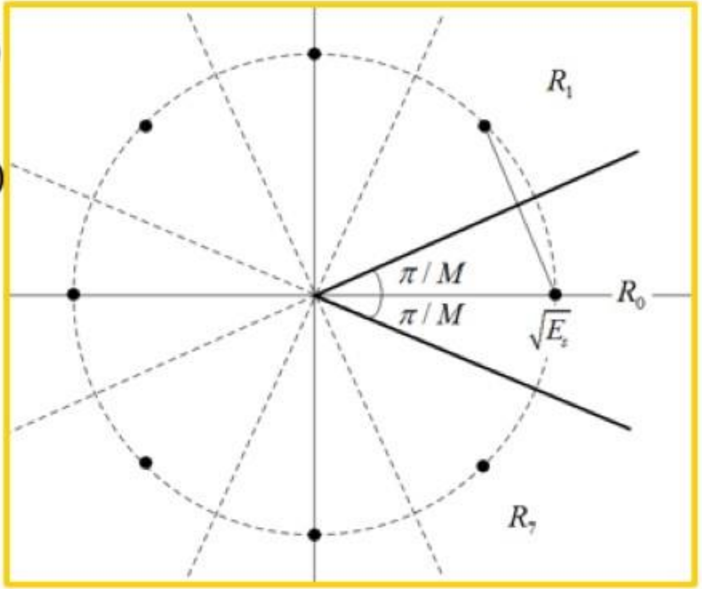



Prob of err is computed by,

$$P_e = 1 - \frac{1}{M}\sum_{k=1}^{M}\int_{R_k} f(\mathbf{r}|\mathbf{s}_k)dr = 1 - \int_{R_0} f(\mathbf{r}|\mathbf{s}_0)dr$$

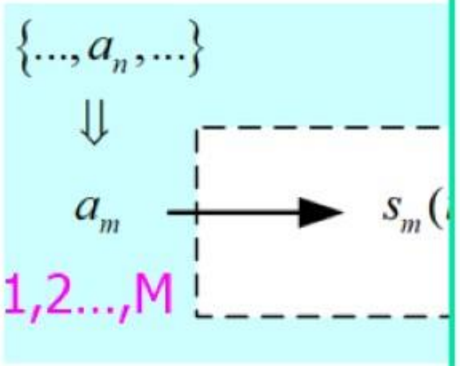
$$= 1 - \int_{-\pi/M}^{+\pi/M} f(\theta_r|\mathbf{s}_0)d\theta_r$$

For simple cases of M=2 or 4， BPSK and QPSK

$$\boxed{P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)}$$

# 5.6.3 MPSK

MPSK
(Carrier-phs)

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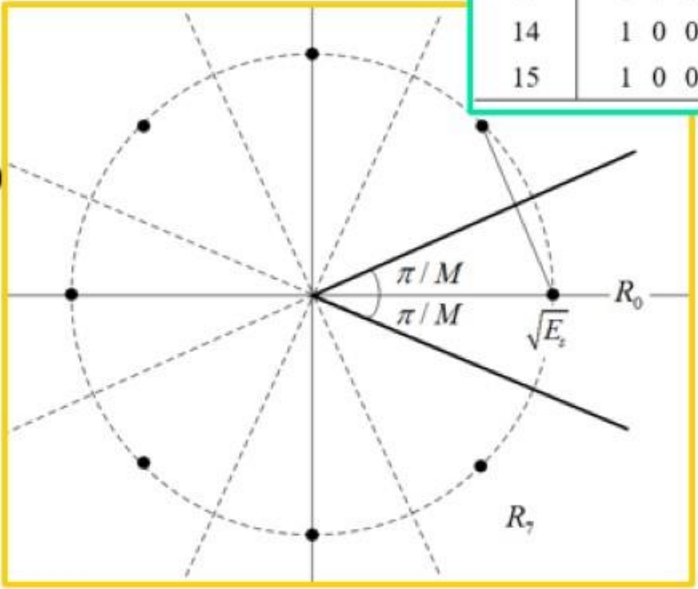



| Sym | Gray | Natural |
|---|---|---|
| 0 | 0 0 0 0 | 0 0 0 0 |
| 1 | 0 0 0 1 | 0 0 0 1 |
| 2 | 0 0 1 1 | 0 0 1 0 |
| 3 | 0 0 1 0 | 0 0 1 1 |
| 4 | 0 1 1 0 | [illegible] |
| 5 | 0 1 1 1 | 0 1 0 1 |
| 6 | 0 1 0 1 | 0 1 1 0 |
| 7 | 0 1 0 0 | 0 1 1 1 |
| 8 | 1 1 0 0 | 1 0 0 0 |
| 9 | 1 1 0 1 | 1 0 0 1 |
| 10 | 1 1 1 1 | 1 0 1 0 |
| 11 | 1 1 1 0 | 1 0 1 1 |
| 12 | 1 0 1 0 | 1 1 0 0 |
| 13 | 1 0 1 1 | 1 1 0 1 |
| 14 | 1 0 0 1 | 1 1 1 0 |
| 15 | 1 0 0 0 | 1 1 1 1 |

Reflection

**The equivalent bit error probability,**
In M-ary systems, an error is an incorrect symbol which may cause more than 1 bit errors.
Gray code is a special designed bit-mapping code which make two adjacent symbols differ in only one bit.
When there is an error, a sym is naturally mistaken by an adjacent sym with very high prob. So, with Gray code, an error causes only one bit error and,

$$\mathbf{P_b \sim= P_e/\log_2 M}$$

In general, we do not have a simple form of the Pe, however a good approximation of Pe for many case is given by,

$$\approx 2Q\left(\sqrt{2\frac{E_s}{N_0}\sin^2\frac{\pi}{M}}\right) = 2Q\left(\sqrt{2\log_2 M\frac{E_b}{N_0}\sin^2\frac{\pi}{M}}\right)$$

# 5.6

**Binary**

<u>Baseband</u>: BPAM (antipodal, unipolar), Orthogonal signaling;

<u>Passband</u>: BPSK, OOK or BASK, BFSK (Orthogonal)

**M-ary, 1-D signaling**

<u>Baseband</u>: MPAM

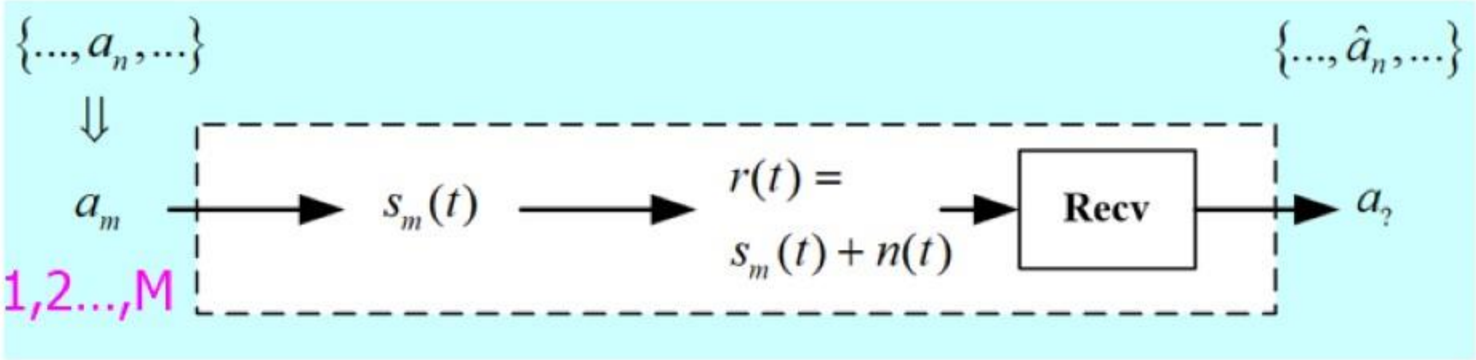
<u>Passband</u>: ~~MASK~~

**M-ary, 2-D signaling (Passband)**

MPSK, QAM

# 5.6.4 QAM

QAM
(Amp/phs)

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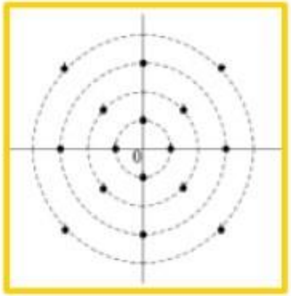

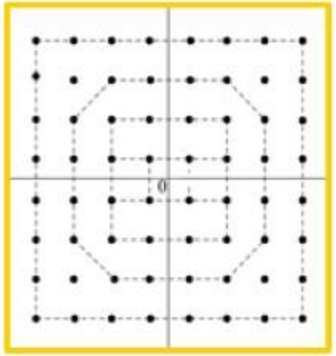

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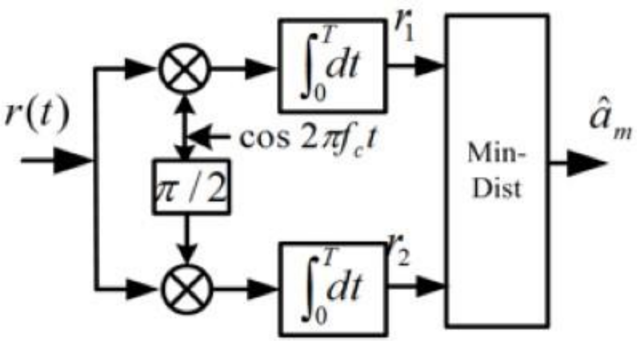
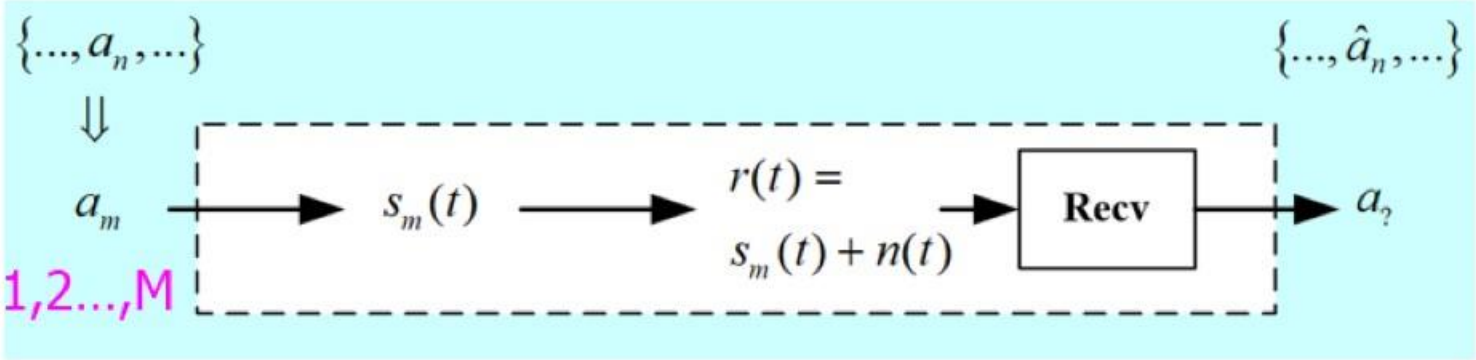



Prob of err is computed by,

# 5.6.4 QAM

QAM (Amp/phs)

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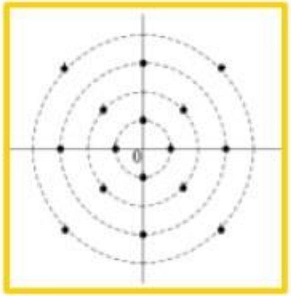

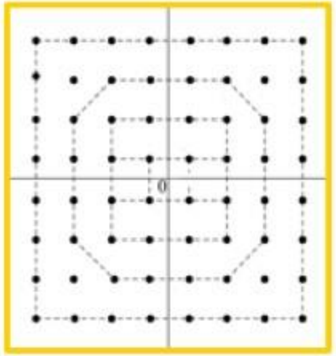

Prob of err is computed by,

The $P_e$ of QAM depends on its specific constellation. The distance between pairs of points and the average energy are two key paras.

Rectangular QAM constellations have distinct advantages of being equivalent to 2 PAMs. Though not the best, it is only slightly poorer than the best, so it is frequently used in practical.

$$P_{M\_QAM} \approx 2P_{\sqrt{M}\_PAM} = 2\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3E_{av}}{(M-1)N_0}}\right)$$

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**M-ary, 1-D signaling**

Baseband: MPAM

Passband: MASK

**M-ary, 2-D signaling (Passband)**

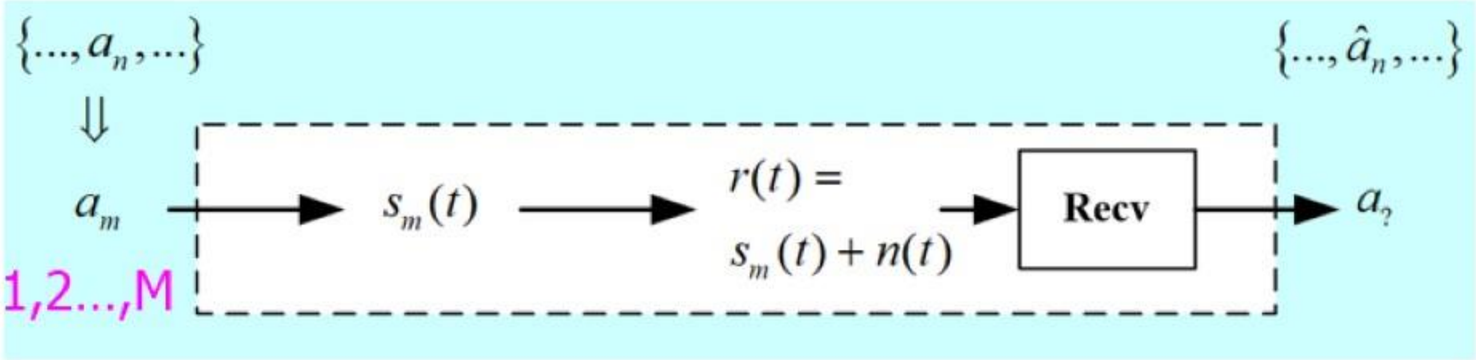
MPSK, QAM

**M-ary, M-D signaling (Passband)**

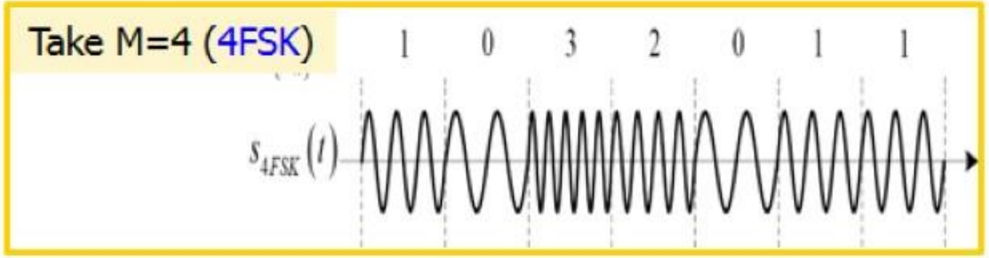
MFSK

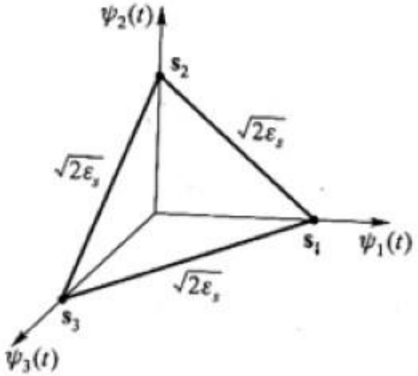
# 5.6.5 MFSK

MFSK
(M-d)




The signals: $s_m(t) = A\cos\left[2\pi(f_0 + m\times\Delta f)t\right]$




A MF-ML receiver is given by,

The receiver consists of a demod. of M correlation-branches and a detector with MaxCorr rule.
See block diag of Fig 5.46 on p302.

Prob of err is computed by,

$$P_e = \dots = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\left\{1-\left[1-Q(x)\right]^{M-1}\right\}e^{-\frac{\left(x-\sqrt{2E_s/N_0}\right)^2}{2}}dx$$

In contrast to QAM and PSK, the $P_b$ of MFSK decreases as M increases.
Note that MFSK is M-dimensional while QAM and PSK are 2-dimensional.