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MOOC: Spread Spectrum Communications & Jamming

Assignment 1: Solutions to Problems on Direct Sequence Spread Spectrum

Due date: Max. marks: 20

1. Consider a DS-BPSK spread spectrum transmitter in the Fig. 1. Let d(t) be a binary sequence 1101 arriving at a rate of 100 bps, where left most bit is the earliest bit. Let c(t) be the pseudorandom binary sequence 100110111000 with a clock rate rate of 300 Hz. Assuming a bipolar signaling scheme with a binary '0' and binary '1' represented by a signal levels '-1' and '+1', respectively:

   (a) The final transmitted binary sequence corresponding to the bipolar signal sequence, p(t), is:

   Solution:

   d(t): 1101
   x(t): +1 +1 -1 +1
   c(t): 100110111000
   g(t): +1 -1 -1 +1 +1 -1 +1 +1 +1 -1 -1 -1
   Spread sequence, p(t) = x(t)g(t):
   x(t): +1 +1 +1 +1 +1 +1 -1 -1 -1 +1 +1 +1
   g(t): +1 -1 -1 +1 +1 -1 +1 +1 +1 -1 -1 -1
   p(t): +1 -1 -1 +1 +1 -1 -1 -1 -1 -1 -1 -1
   Binary sequence corresponding to p(t): 100110000000
   (**correct option iv.**)

   (b) The bandwidth of the transmitted spread signal is:

   Solution:

   Bandwidth of the transmitted spread signal (DSSS-BPSK) is same as clock rate of the pseudorandom binary sequence (= 300 Hz).
   (**correct option i.**)

(c) The processing gain in dB is:

Solution:

Processing gain = (Pseudorandom chip rate)/(data rate)=300/100 = 3 = 10log(3) dB = 4.77 dB (**correct option ii.**)

(d) Assuming that the spread signal is not corrupted by noise, suppose the estimated delay at a spread spectrum receiver is too large by one chip time, the despread and decoded signal sequence (assuming majority logic decoding) is:

Solution:

Despread sequence, (p(t))(delayed g(t)) =

p(t): +1 -1 -1 +1 +1 -1 -1 -1 -1 -1 -1 -1

g'(t): -1 +1 -1 -1 +1 +1 -1 +1 +1 +1 -1 -1

r(t): -1 -1 +1 -1 +1 -1 +1 -1 -1 -1 +1 +1

Majority logic decoding (taking 3 bits at a time) yields: 0001
(**correct option i.**)

(e) The number of bit errors due to the estimation delay is:

Solution: In comparison with d(t), we see that the decoded bit sequence differs in bit positions 1 and 2. Thus, the no. of bit errors in the decoded sequence is 2. (**correct option iii.**)

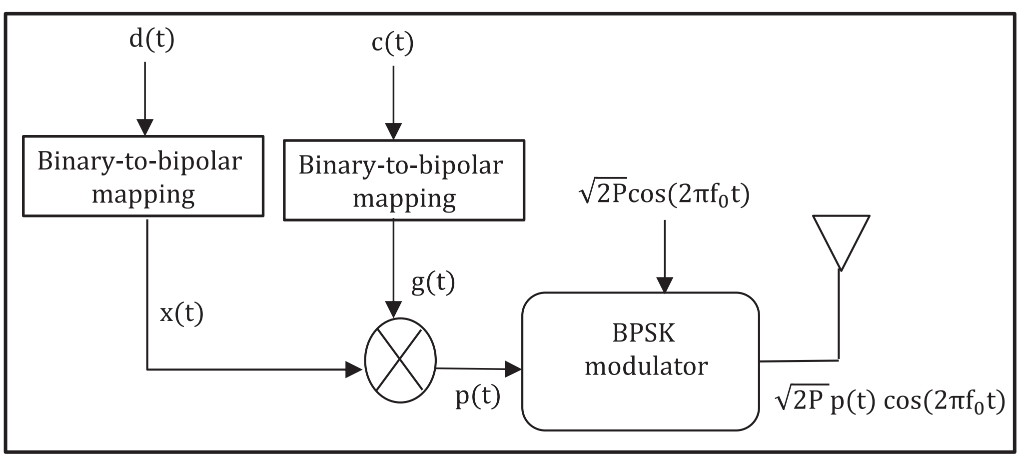

Figure 1: DSSS-BPSK System

2. A DSSS system transmits at a rate of 1000 bits/sec in the presence of a tone jammer. The average jammer power is 20 dB greater than the average desired signal power and the required E_b/J_o to achieve satisfactory performance is 10 dB. Note: E_b denotes average bit energy and J_o denotes jamming power spectral density.

   (a) The ratio of spreading bandwidth, W_s, to the transmission rate, R, is: (Hint: Express E_b in terms of corresponding average power, and transmission rate.)

   Solution:

$$\frac{E_b}{J_o} = \frac{P_{av}T_b}{\left(\frac{J_{av}}{W_s}\right)} = \frac{P_{av}}{J_{av}}.\frac{W_s}{R}$$

   where P_{av} denotes average transmit power, J_{av} denotes the average jamming power and $T_b = \frac{1}{R}$ denotes bit duration with R being the transmission rate.

   Thus,

$$\frac{W_s}{R}(dB) = \frac{E_b}{J_o}(dB) + \frac{J_{av}}{P_{av}}(dB) = 10 + 20 = 30\ dB$$

   (**correct option iv.**)

   (b) The bandwidth of the spread signal is:

   Solution:

$$\frac{W_s}{R}(dB) = 30\ dB = 10^3$$

   and $R = 10^3$ (given)

   Thus,

   $W_s = 10^3.10^3 = 10^6 = 1$ MHz

   (**correct option i.**)

3. A ground-to-synchronous satellite link is to operate at a data rate is 10 kbps with a ground station antenna of 80 feet and a transmit power of 10 kW. It employs a 10 Mbps DSSS code. The receiver E_b/J_0 required for reliable communication is 20 dB. A jammer with 100 feet antenna intends to disrupt the communication link. Assume equal space and propagation losses, and that receiver noise is negligible. Note: E_b denotes average bit energy and J_o denotes jamming power spectral density.

   (a) The processing gain of the spread spectrum system is:
   Solution:
   Processing gain, G_p = (Chip Rate)/(Data rate) = $(10 \times 10^6)/(10 \times 10^3) = 10^3 =$ 30 dB
   (**correct option ii.**)

   (b) The jammer power required to disrupt the communication system is:
   Solution:

$$P_T = \left(\frac{E_b}{J_o}\right)_r \frac{P_J}{G_p} \frac{A_{eJ}}{A_{eT}}$$

   where P_T denotes transmitter power, P_J denotes jammer power, A_{eJ} and A_{eT} denote effective aperture areas of jammer and transmitter antennas, respectively.

   Given, $\left(\frac{E_b}{J_o}\right)_r = 20 \text{ dB} = 10^2$

   Therefore,

$$P_J = \frac{P_T}{\left(\frac{E_b}{J_o}\right)_r} G_p \frac{A_{eT}}{A_{eJ}} = \frac{(10^4)}{(10^2)}(10^3)\frac{(80)^2}{(100)^2} = 64 \textbf{ kW}$$

   (**correct option iv.**)

4. A DSSS system uses BPSK modulation for transmitting data. It is required that the bit error probability be 10^{-5}, and that $E_{ch}/I_o \leq -20dB$. Assume perfect synchronization and negligible noise at the receiver. Note: E_{ch} denotes average chip energy and I_o denotes Gaussian interference power spectral density (Refer Fig.2 and Fig.3 for typical BER plot and/or Q-function table if necessary.)

   (a) The E_b/I_o ratio for the specified probability of error is(choose the nearest value from the options below:
   Solution:
   Probability of error for BPSK modulation:

$$P_e = Q\left(\sqrt{\frac{2E_b}{N_o}}\right)$$

Given that probability of error (bit error rate) = 10^{-5}, and referring to Fig. 2 (or alternatively to the table in Fig. 3), we see that the required E_b/N_o (or E_b/I_o in this case since noise is negligible) is approximately 9.6 dB.

(**correct option ii.**)

(b) The processing gain, G, calculated in terms of the E_{ch}/I_o and E_b/I_o, is):

Solution:

Processing gain,

$$G_p = \frac{R_c}{R_b} = \frac{T_b}{T_c} = \frac{E_b}{E_c}$$

where, R_c and R_b are the chip rate and bit rate, respectively, and T_c and T_b are the chip duration and bit duration, respectively.

$$G_p = \frac{\frac{E_b}{I_o}}{\frac{E_{ch}}{I_o}} = \frac{E_b}{I_o}(dB) - \frac{E_{ch}}{I_o}(dB) = 9.6 - (-20) = 29.6 \text{ dB}$$

(**correct option iv.**)

(c) The minimum number of chips/bit required is:

Solution:

Since G_p = 29.6 dB, which is approximately equal to 912, and we need the chip rate to be greater than G_p, the number of chips/bit should be greater than 912 (nearest option is i, i.e 10^3) (**correct option i.**)

5. In a DS/BPSK system delivers a processing gain of 20 dB. The system is required to have a probability of error due to externally generated interfering signals that does not exceed 10^{-6}. (Refer Fig. 2 and Fig. 3 for typical BER plot and/or Q-function table if necessary)

(a) The E_b/I_o ratio for the specified probability of error is (choose the nearest value from the options below):

Solution:

Probability of error for BPSK modulation:

$$P_e = Q\left(\sqrt{\frac{2E_b}{N_o}}\right)$$

Given that probability of error (bit error rate) = 10^{-6}, and referring to Fig. 2 (or alternatively to the table in Fig. 3), we see that the required E_b/N_o (or E_b/I_o in

this case since noise is negligible) is approximately 10.5 dB.
(**correct option iii.**)

(b) The jamming margin is:
Solution:
Jamming margin (dB) = G_p - (E_b/I_o) = 20 - 10.5 = 9.5 dB
(**correct option iv.**)

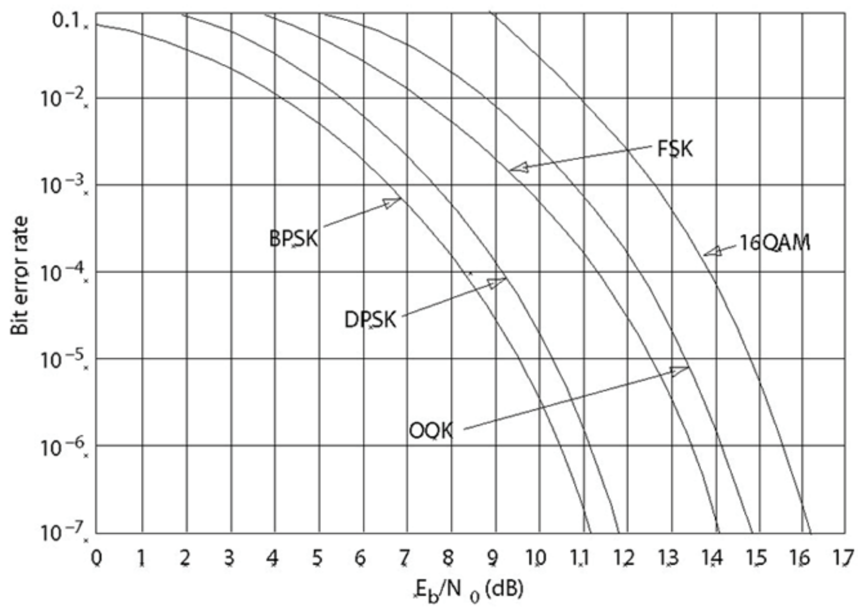

Figure 2: BER plot for typical modulation schemes.

Values of $Q(x)$ for $0 \leq x \leq 9$

x	$Q(x)$	x	$Q(x)$	x	$Q(x)$	x	$Q(x)$
0.00	0.5	2.30	0.010724	4.55	2.6823×10^{-6}	6.80	5.231×10^{-12}
0.05	0.48006	2.35	0.0093867	4.60	2.1125×10^{-6}	6.85	3.6925×10^{-12}
0.10	0.46017	2.40	0.0081975	4.65	1.6597×10^{-6}	6.90	2.6001×10^{-12}
0.15	0.44038	2.45	0.0071428	4.70	1.3008×10^{-6}	6.95	1.8264×10^{-12}
0.20	0.42074	2.50	0.0062097	4.75	1.0171×10^{-6}	7.00	1.2798×10^{-12}
0.25	0.40129	2.55	0.0053861	4.80	7.9333×10^{-7}	7.05	8.9459×10^{-13}
0.30	0.38209	2.60	0.0046612	4.85	6.1731×10^{-7}	7.10	6.2378×10^{-13}
0.35	0.36317	2.65	0.0040246	4.90	4.7918×10^{-7}	7.15	4.3389×10^{-13}
0.40	0.34458	2.70	0.003467	4.95	3.7107×10^{-7}	7.20	3.0106×10^{-13}
0.45	0.32636	2.75	0.0029798	5.00	2.8665×10^{-7}	7.25	2.0839×10^{-13}
0.50	0.30854	2.80	0.0025551	5.05	2.2091×10^{-7}	7.30	1.4388×10^{-13}
0.55	0.29116	2.85	0.002186	5.10	1.6983×10^{-7}	7.35	9.9103×10^{-14}
0.60	0.27425	2.90	0.0018658	5.15	1.3024×10^{-7}	7.40	6.8092×10^{-14}
0.65	0.25785	2.95	0.0015889	5.20	9.9644×10^{-8}	7.45	4.667×10^{-14}
0.70	0.24196	3.00	0.0013499	5.25	7.605×10^{-8}	7.50	3.1909×10^{-14}
0.75	0.22663	3.05	0.0011442	5.30	5.7901×10^{-8}	7.55	2.1763×10^{-14}
0.80	0.21186	3.10	0.0009676	5.35	4.3977×10^{-8}	7.60	1.4807×10^{-14}
0.85	0.19766	3.15	0.00081635	5.40	3.332×10^{-8}	7.65	1.0049×10^{-14}
0.90	0.18406	3.20	0.00068714	5.45	2.5185×10^{-8}	7.70	6.8033×10^{-15}
0.95	0.17106	3.25	0.00057703	5.50	1.899×10^{-8}	7.75	4.5946×10^{-15}
1.00	0.15866	3.30	0.00048342	5.55	1.4283×10^{-8}	7.80	3.0954×10^{-15}
1.05	0.14686	3.35	0.00040406	5.60	1.0718×10^{-8}	7.85	2.0802×10^{-15}
1.10	0.13567	3.40	0.00033693	5.65	8.0224×10^{-9}	7.90	1.3945×10^{-15}
1.15	0.12507	3.45	0.00028029	5.70	5.9904×10^{-9}	7.95	9.3256×10^{-16}
1.20	0.11507	3.50	0.00023263	5.75	4.4622×10^{-9}	8.00	6.221×10^{-16}
1.25	0.10565	3.55	0.00019262	5.80	3.3157×10^{-9}	8.05	4.1397×10^{-16}
1.30	0.0968	3.60	0.00015911	5.85	2.4579×10^{-9}	8.10	2.748×10^{-16}
1.35	0.088508	3.65	0.00013112	5.90	1.8175×10^{-9}	8.15	1.8196×10^{-16}
1.40	0.080757	3.70	0.0001078	5.95	1.3407×10^{-9}	8.20	1.2019×10^{-16}
1.45	0.073529	3.75	8.8417×10^{-5}	6.00	9.8659×10^{-10}	8.25	7.9197×10^{-17}
1.50	0.066807	3.80	7.2348×10^{-5}	6.05	7.2423×10^{-10}	8.30	5.2056×10^{-17}
1.55	0.060571	3.85	5.9059×10^{-5}	6.10	5.3034×10^{-10}	8.35	3.4131×10^{-17}
1.60	0.054799	3.90	4.8096×10^{-5}	6.15	3.8741×10^{-10}	8.40	2.2324×10^{-17}
1.65	0.049471	3.95	3.9076×10^{-5}	6.20	2.8232×10^{-10}	8.45	1.4565×10^{-17}
1.70	0.044565	4.00	3.1671×10^{-5}	6.25	2.0523×10^{-10}	8.50	9.4795×10^{-18}
1.75	0.040059	4.05	2.5609×10^{-5}	6.30	1.4882×10^{-10}	8.55	6.1544×10^{-18}
1.80	0.03593	4.10	2.0658×10^{-5}	6.35	1.0766×10^{-10}	8.60	3.9858×10^{-18}
1.85	0.032157	4.15	1.6624×10^{-5}	6.40	7.7688×10^{-11}	8.65	2.575×10^{-18}
1.90	0.028717	4.20	1.3346×10^{-5}	6.45	5.5925×10^{-11}	8.70	1.6594×10^{-18}
1.95	0.025588	4.25	1.0689×10^{-5}	6.50	4.016×10^{-11}	8.75	1.0668×10^{-18}
2.00	0.02275	4.30	8.5399×10^{-6}	6.55	2.8769×10^{-11}	8.80	6.8408×10^{-19}
2.05	0.020182	4.35	6.8069×10^{-6}	6.60	2.0558×10^{-11}	8.85	4.376×10^{-19}
2.10	0.017864	4.40	5.4125×10^{-6}	6.65	1.4655×10^{-11}	8.90	2.7923×10^{-19}
2.15	0.015778	4.45	4.2935×10^{-6}	6.70	1.0421×10^{-11}	8.95	1.7774×10^{-19}
2.20	0.013903	4.50	3.3977×10^{-6}	6.75	7.3923×10^{-12}	9.00	1.1286×10^{-19}
2.25	0.012224						

Figure 3: Q-function table