Chapter 5

Digital transmission through the AWGN channel

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SICE, UESTC

Contents 16 Hrs

- Introduction
- Geometric rep. of the sig waveforms
- Pulse amplitude modulation
- 2-d signal waveforms
- M-d signal waveforms
- Opt. reception for the sig. in AWGN
- Optimal receivers and probs of err

5.2 Pulse amplitude modulation

5.2.1. PAM： Binary= info bits are 1 and 0

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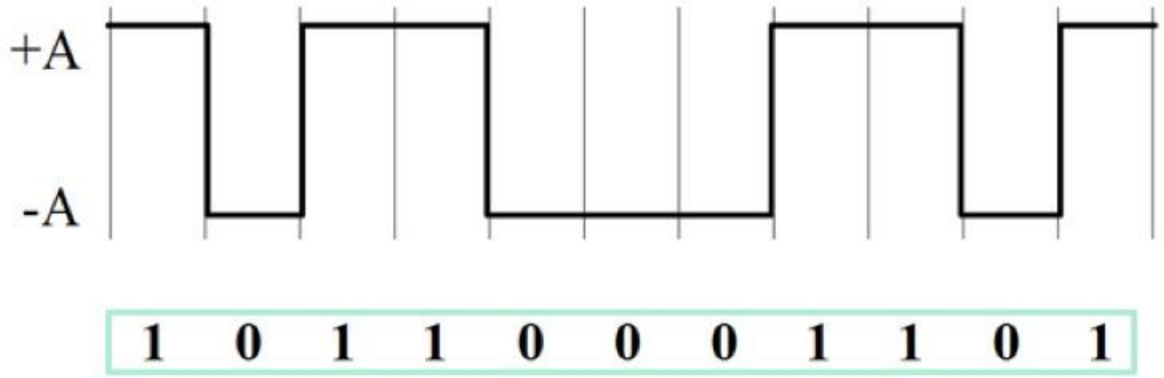
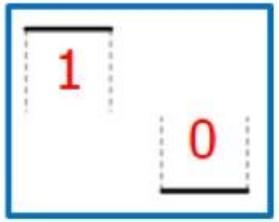
Pulses of amp A and –A is used, called antipodal (or polar) signaling.





If one bit takes an interval of T_b seconds, the rate of signaling is $R_b = 1/T_b$ bps (bit-per-sec).

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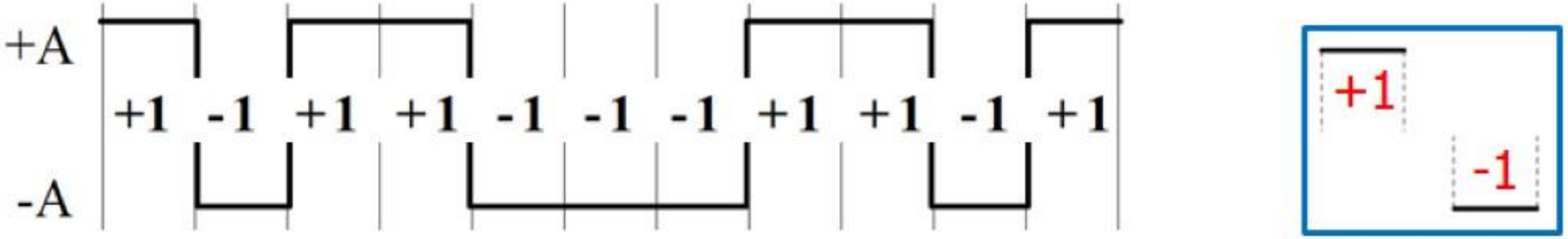
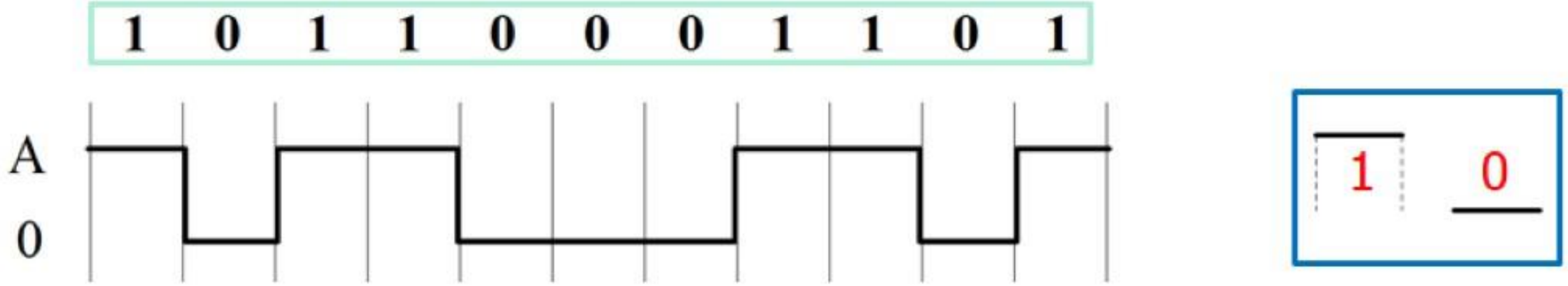


Alternately, A pulse of amp A and 0 may be used, called unipolar signaling.



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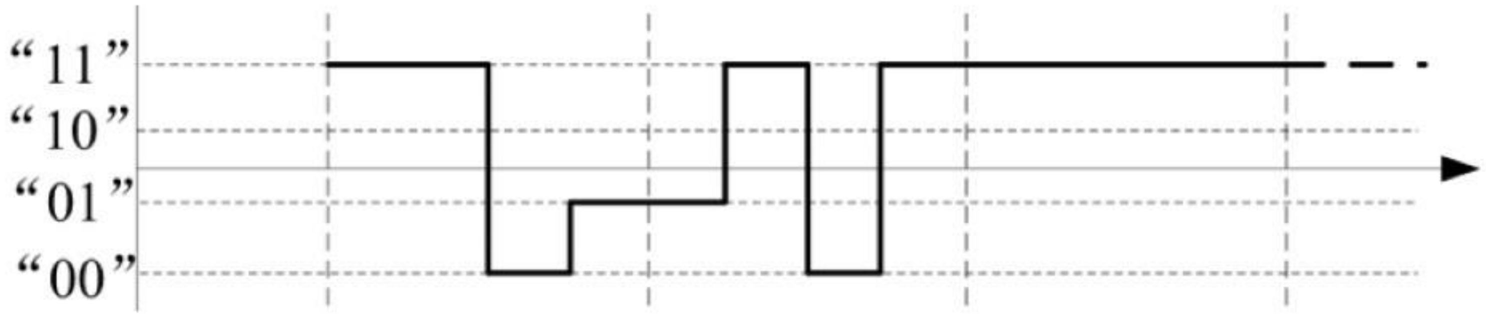
- Take k=2, then M=4, and there are 4 symbols: 00, 01, 10, 11.
- We need 4 signals, pulses of 4 different amps, to send them.



If one symbol takes an interval of T_s seconds, the rate of signaling is $R_s = 1/T_s$ symbols/s (also called Bauds).

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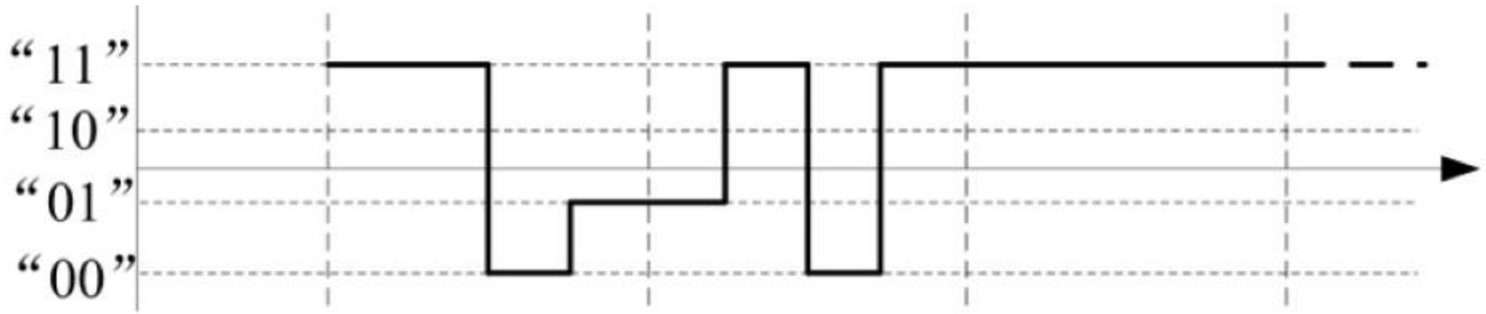

More, 1) R_s is called the symbol rate, or baud rate.

2) The bit rate is $R_b = kR_s$ bps.

Equivalently, the bit interval is $T_b = T_s / k$.

5.2 Pulse amplitude modulation

5.2.1. PAM： signal expression.

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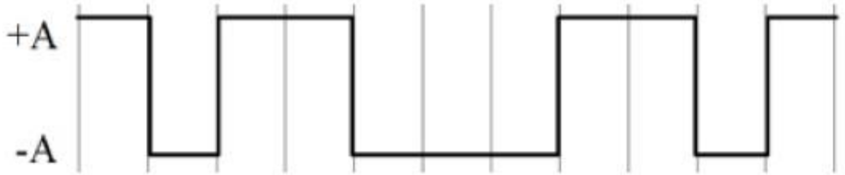
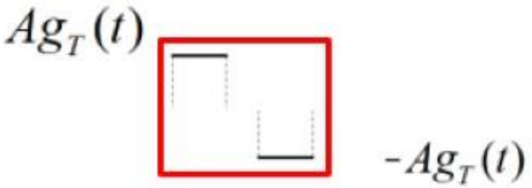
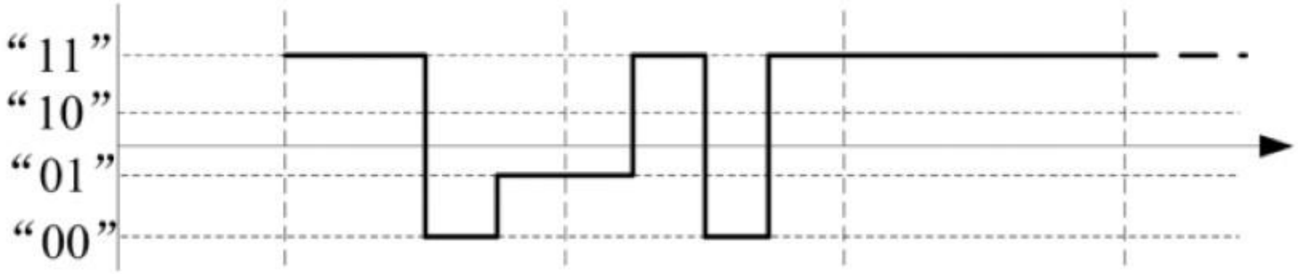
5.2.1. PAM: signal expression.

$$\boxed{s_m(t) = A_m g_T(t)}$$

The signal 'element' is a rect pulse, and generally, can be of other shapes.







Pulse Amp Modulation

- For polar 2-PAM, A_m are A and –A.
- For polar 4-PAM, A_m are 3A, A, –A and –3A.

Generally, let $g_T(t)$ be the pulse, the signals for M symbols is,

$$s_m(t) = A_m g_T(t) \qquad m = 1, 2, \ldots, M$$

where A_m are amps.

5.2 Pulse amplitude modulation

5.2.1. PAM: geometric representation

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$$\boxed{s_m(t) = A_m g_T(t)}$$

1) Basis function: $\psi(t) = k_0 g_T(t)$

where $k_0 = 1\big/\sqrt{E_g}$, $E_g = \int_{-\infty}^{\infty} g_T^2(t)dt$ = the energy of the $g_T(t)$.

The space is of 1-d and N=1.

2) The signal vectors (points): $\mathbf{s}_m = (a_m)$ $\quad m = 1, 2, \ldots, M$

where, $a_m = \mathbf{s}_m \cdot \boldsymbol{\psi}_\mathbf{i} = \int_{-\infty}^{\infty} A_m g_T(t) \frac{g_T(t)}{\sqrt{E_g}} dt$

$= A_m \sqrt{E_g}$

5.2.1. PAM: geometric representation

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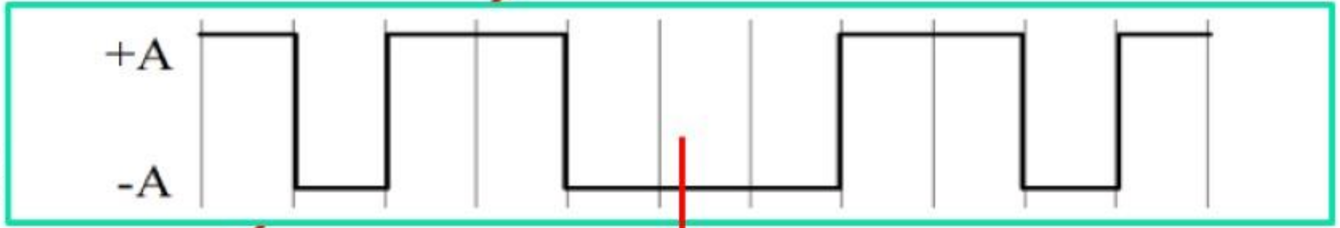

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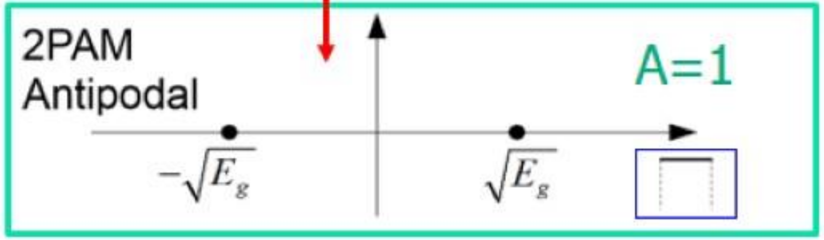

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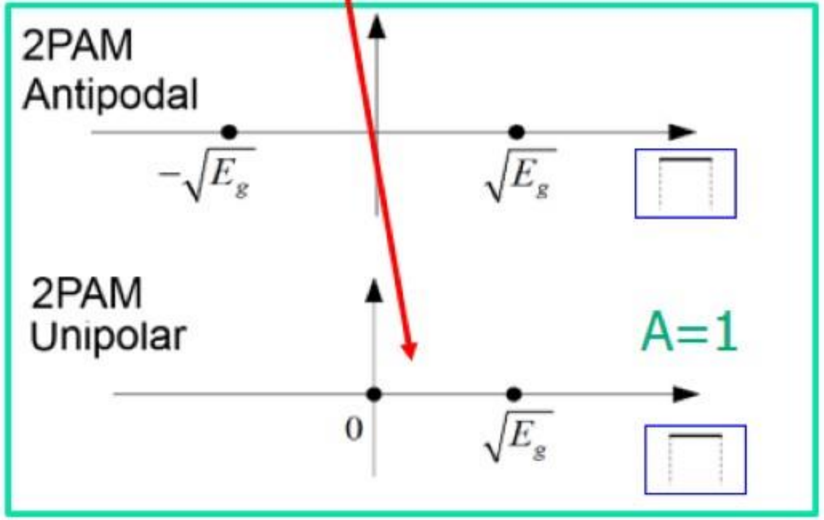

The energy of signals

$$E_m = \|\mathbf{s}_m\|^2 = \left(A_m \sqrt{E_g}\right)^2 = A_m^2 E_g$$

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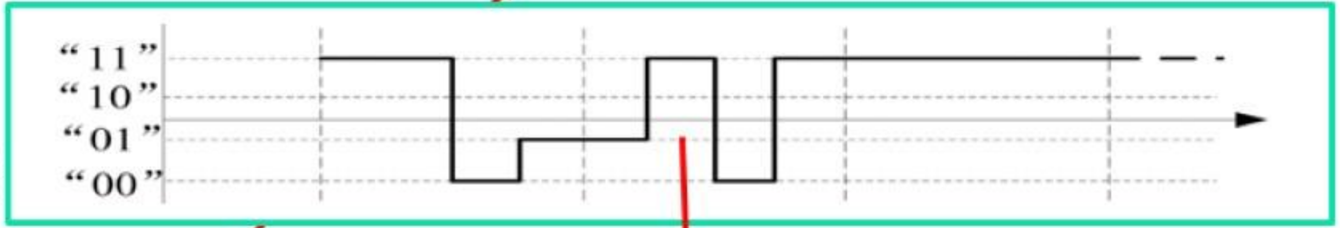

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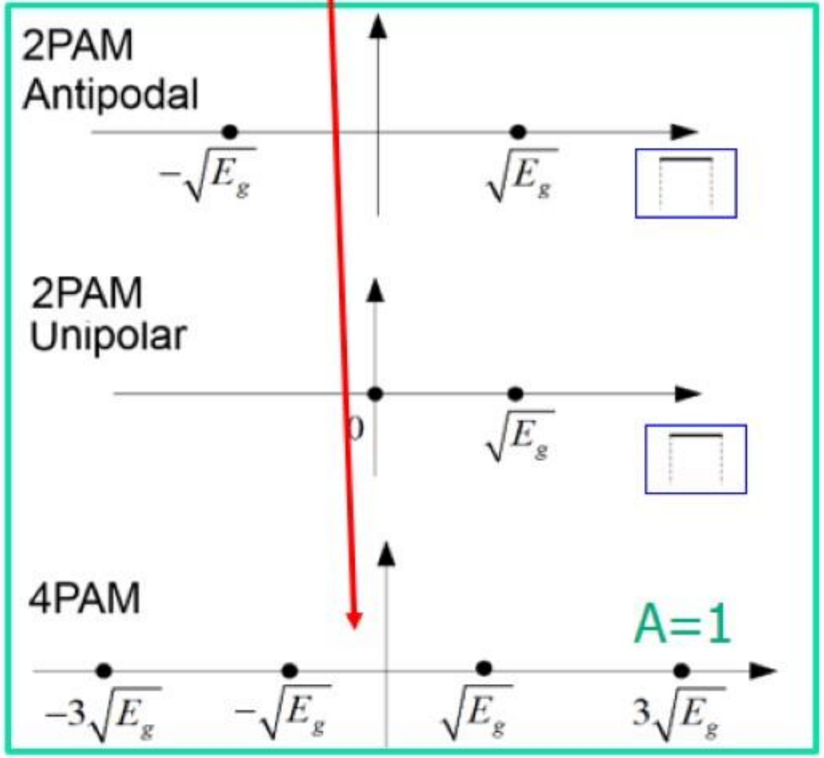

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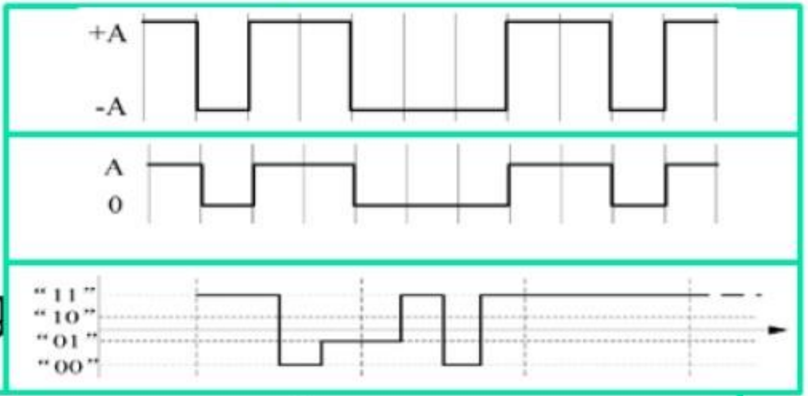

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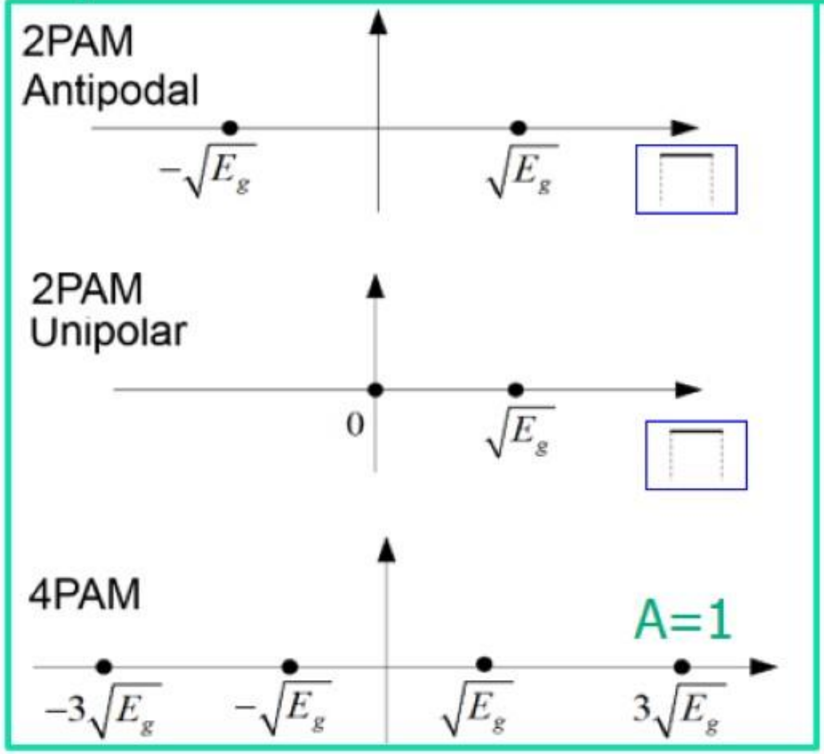

5.2 Pulse amplitude modulation

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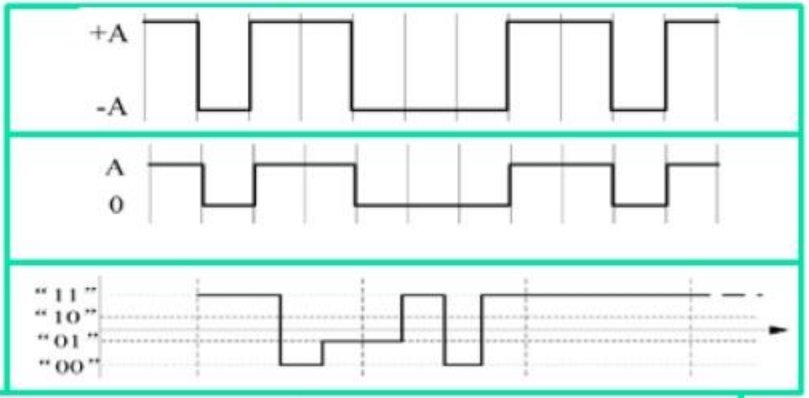
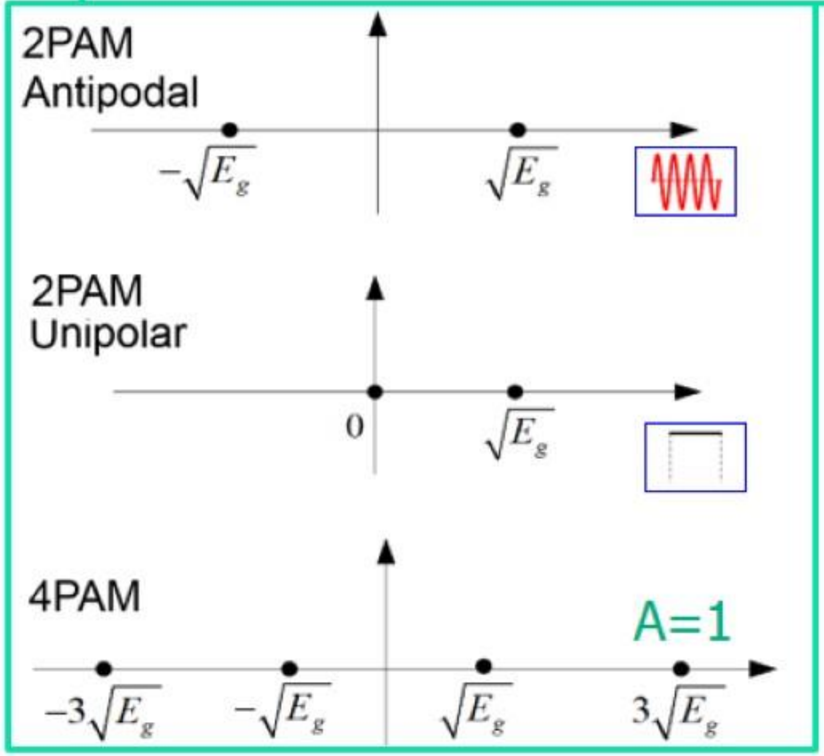
5.2.1. PAM: passband signals



- ✓ Baseband signal: its freq band is close to zero, often used in wire-line transmission.
- ✓ Passband signal: its freq band is away from zero, a cos-like waveform, and widely used in wireless transmission.



5.2 Pulse amplitude modulation

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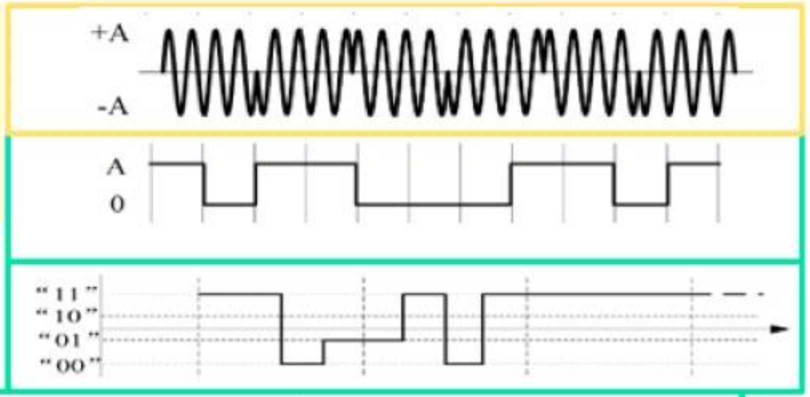
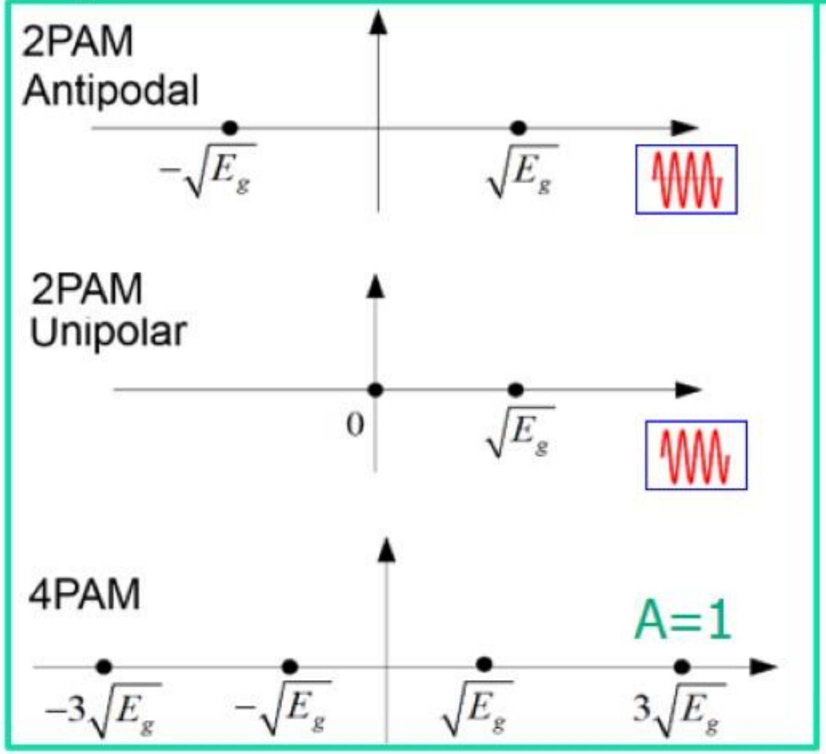
5.2.1. PAM: passband signals

BPSK=Binary Phase-Shift-Keying





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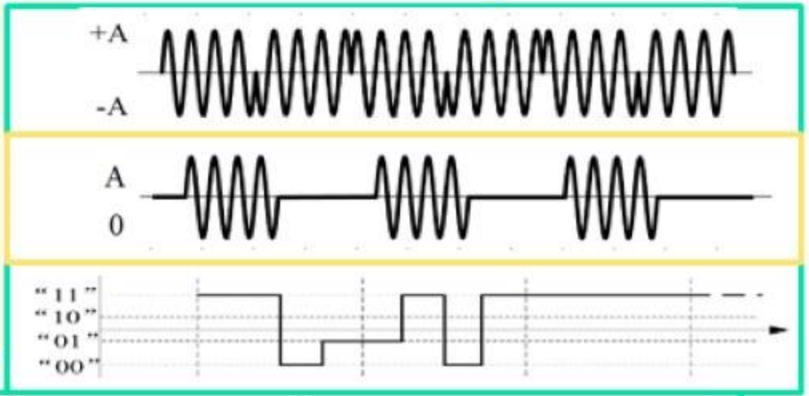
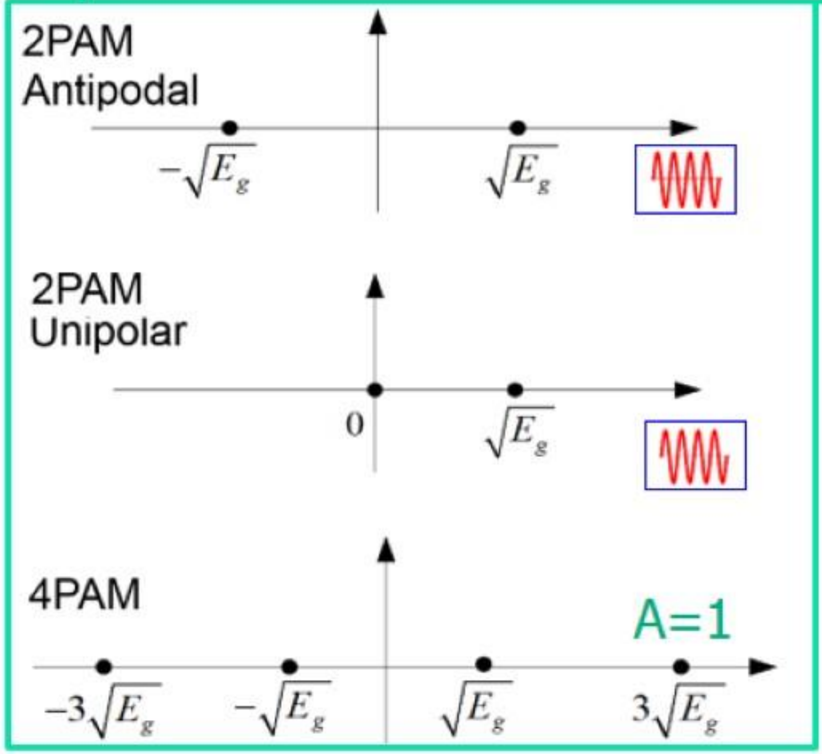
BASK=Binary Amp-Shift-Keying
OOK=On-Off-Keying





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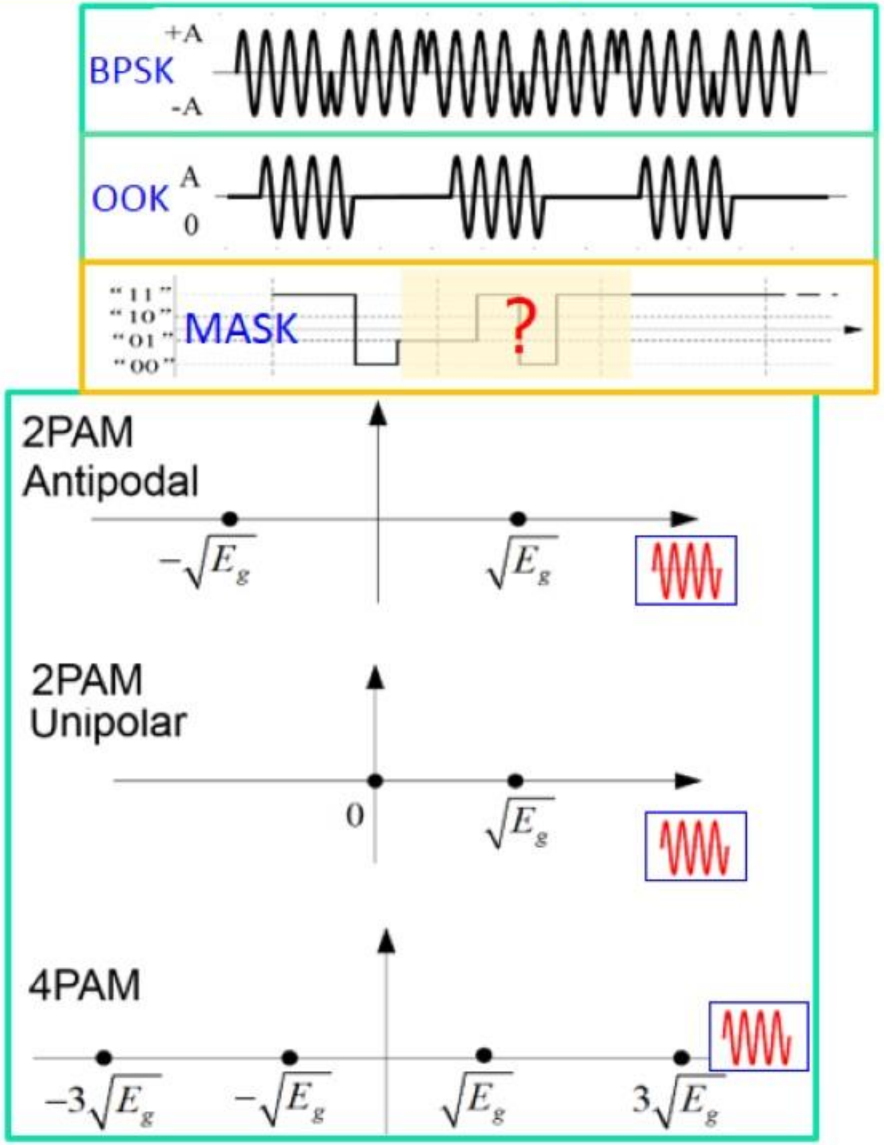

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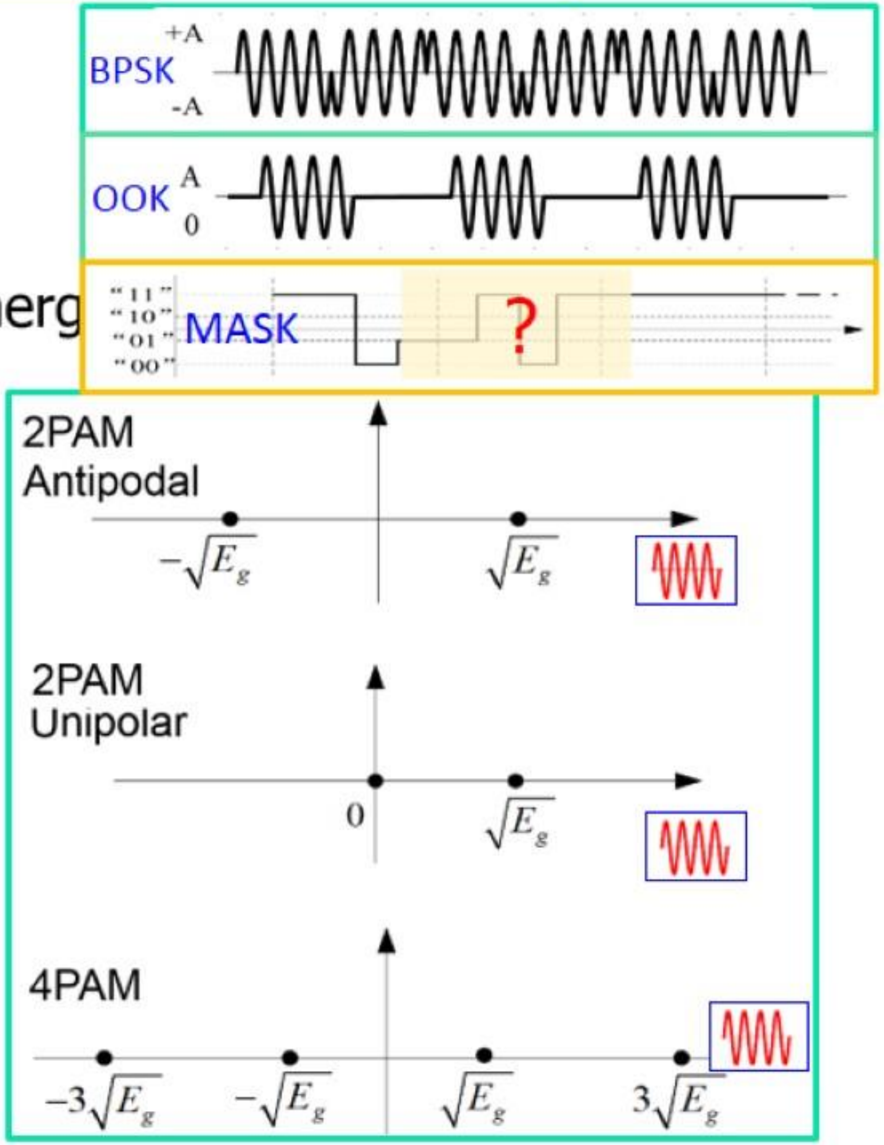

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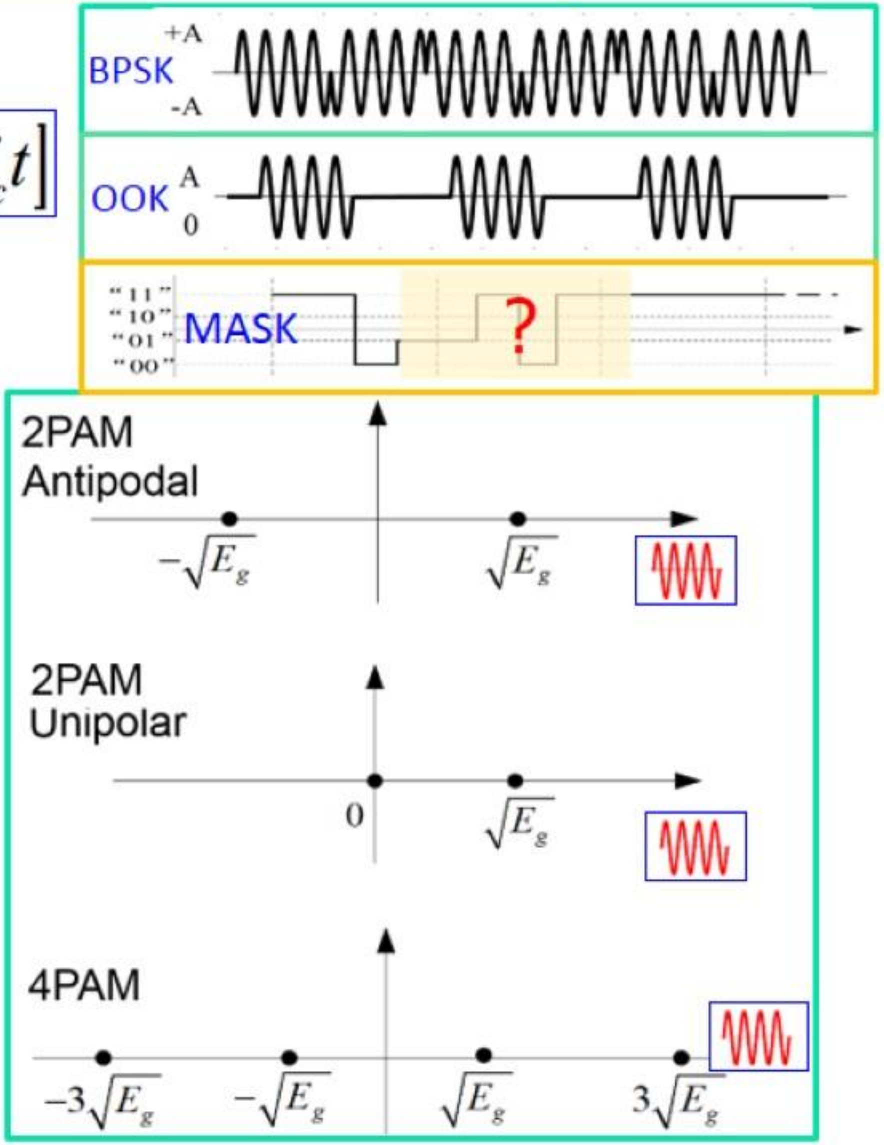

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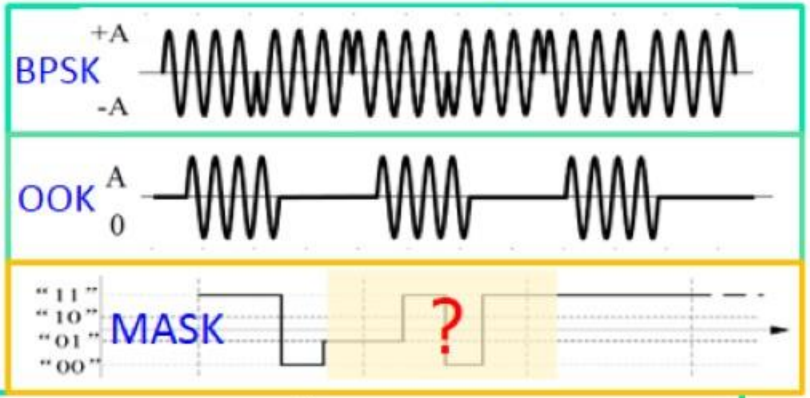
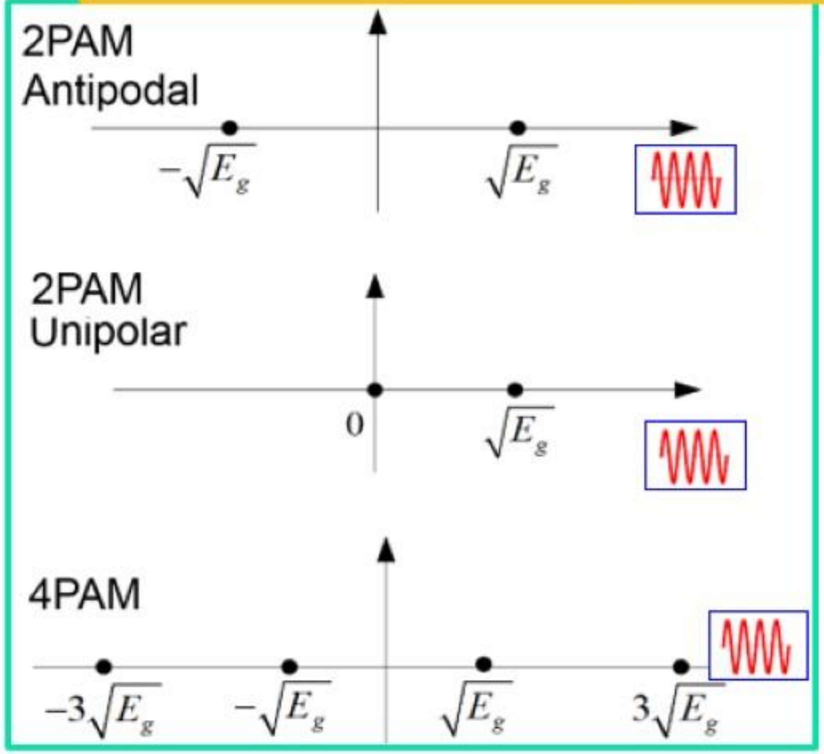
$$\boxed{\begin{aligned} \text{Note that } E_{gc} &= \int_{-\infty}^{\infty} g_T^2(t)\cos^2(2\pi f_c t)dt \\ &= \int_{-\infty}^{\infty} \frac{g_T^2(t)(1-\cos 4\pi f_c t)}{2}dt = \frac{E_g}{2} \end{aligned}}$$





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The graph is called the constellation

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