



# Chapter 6

## Digital transmission through band-limited AWGN channels

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- ISI and zero-ISI condition  
(Ref p380-381, p393-394)
- Design of BL signals for zero-ISI  
(Ref p396-399)
- OFDM

## 6.2 Design of BL signals for zero-ISI

Most channels are practically **BAND-LIMITED**. Examples such as,

- **Subscribe-line** in a standard telephone system is limited to 4kHz.
- **Uplink channel in GSM** is for data from mobile user to the base and is a bandpass channel limited to 200kHz.

The **freq response** of channel,  $C(f)$

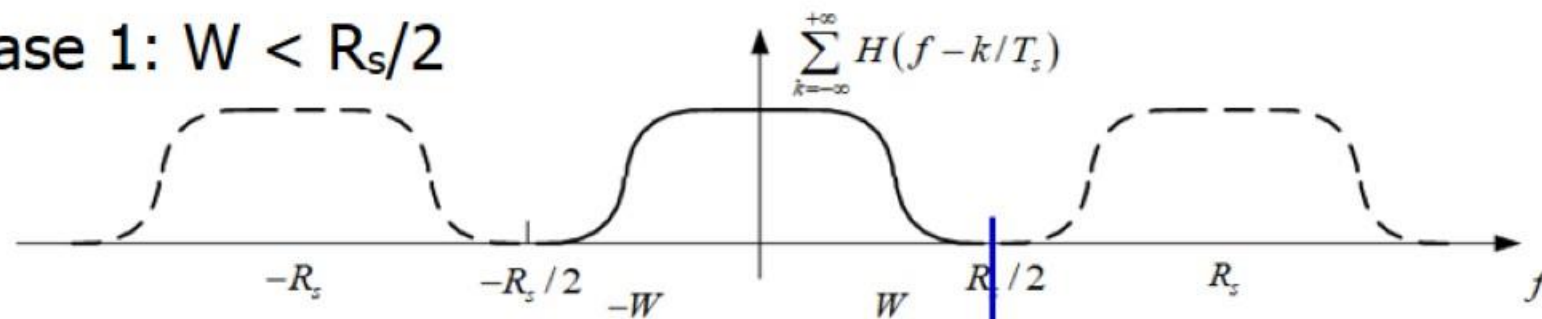
1. For BB, **limited** to some  $w$ , and  $C(f) = 0, |f| > W$
2. For PB, **limited** to some  $w$  about its central freq  $f_c$ , and

$$C(f) = 0, |f - f_c| \leq W / 2$$

## 6.2 Design of BL signals for zero-ISI

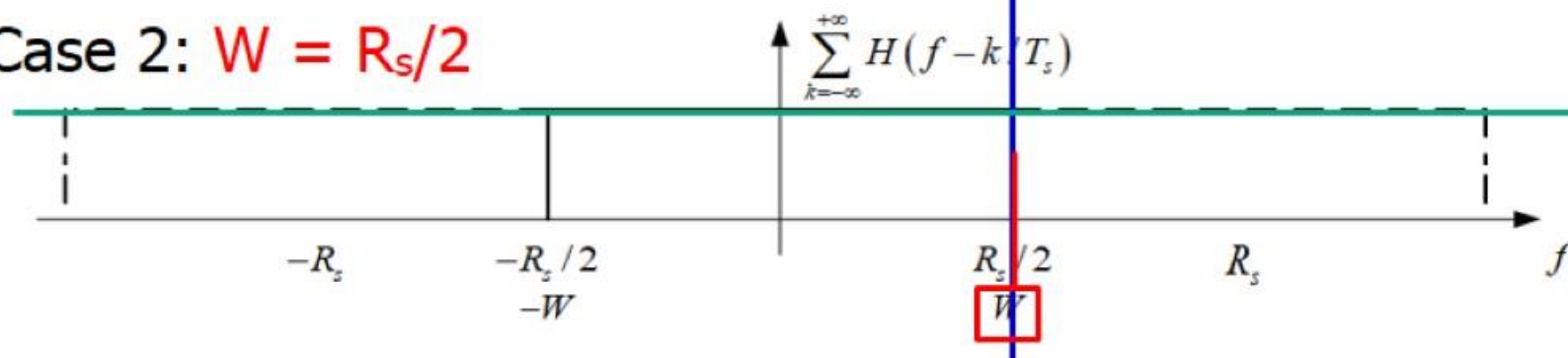
Think about the channel with **BW of W Hz**.

Case 1:  $W < R_s/2$



Flat? **Impossible!**

Case 2:  $W = R_s/2$



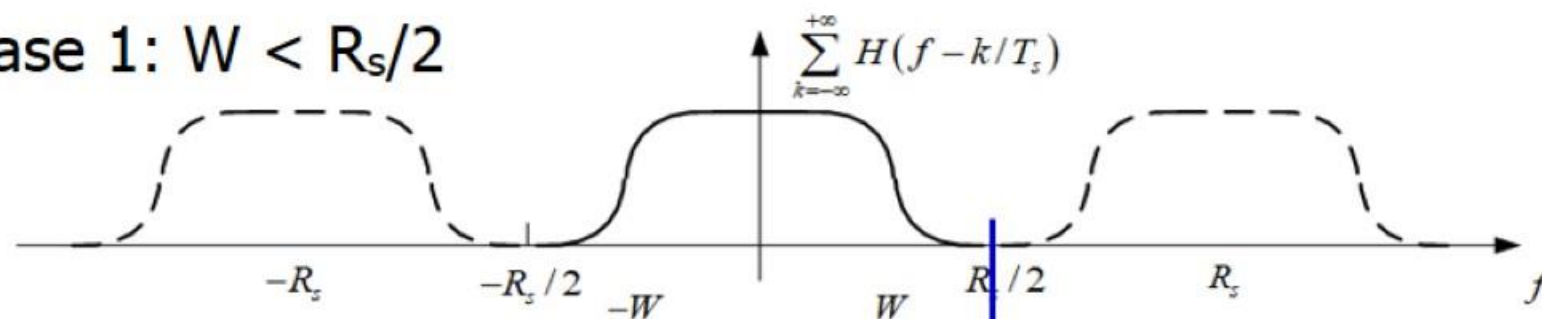
Flat? If and only if  $H(t)$  is **rect**.



## 6.2 Design of BL signals for zero-ISI

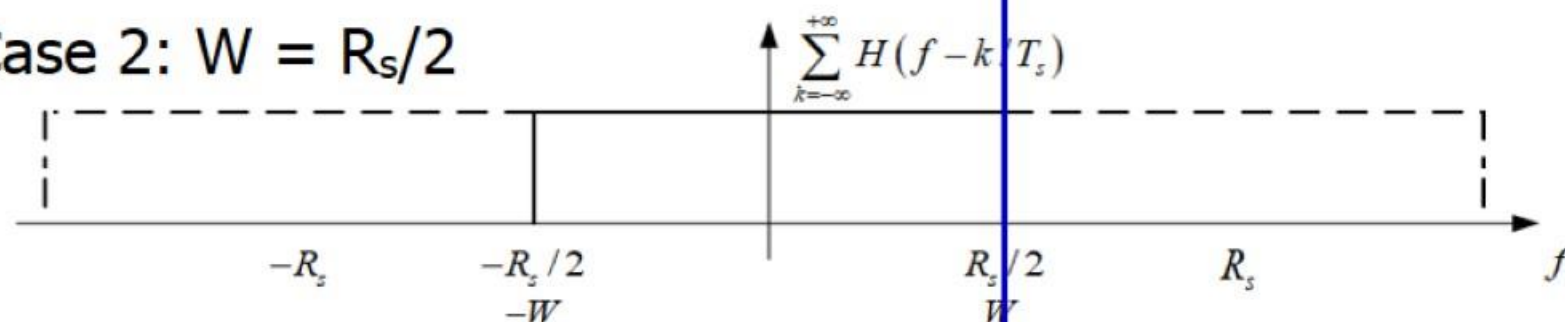
Think about the channel with **BW of WHz**.

Case 1:  $W < R_s/2$



Flat? **Impossible!**

Case 2:  $W = R_s/2$



Flat? If and only if  $H(t)$  is **rect**.

Case 3:  $W > R_s/2$



Flat? **Many chances**, once  $H(t)$  is **complement about  $R_s/2$** .

## 6.2 Design of BL signals for zero-ISI

Think about the channel with **BW of WHz**.

Case 1:  $W < R_s/2$

$$\uparrow \sum_{k=-\infty}^{+\infty} H(f - k/T_s)$$

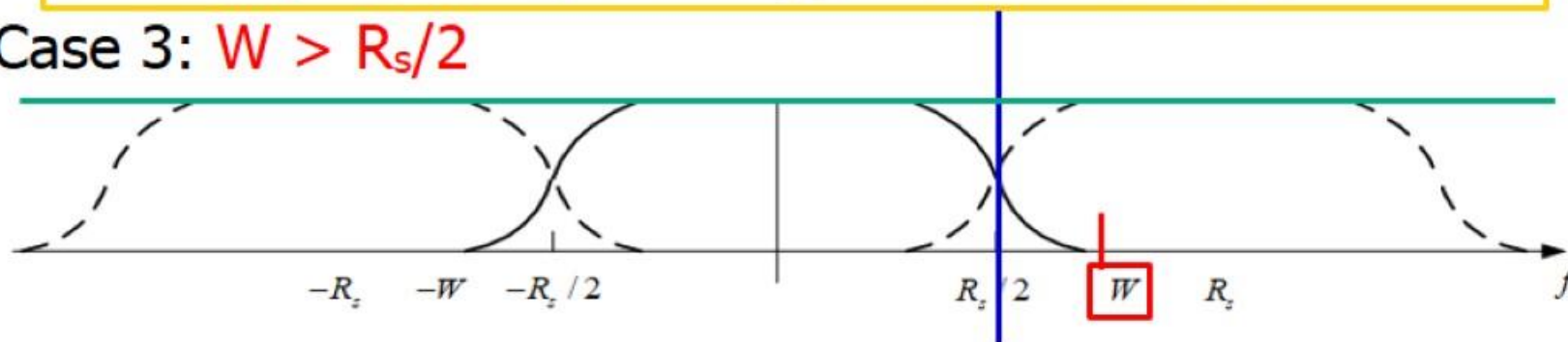
### Conclusion for BL transmission:

A digital communication system for zero-ISI must meet  $R_s < 2W$

Case 2 **Nyquist Rate:** The **maximum symbol rate** for zero-ISI transmission over a WHz-channel is  **$2W$  baud**.

The **highest rate** can only be reached when the overall system response  $h(t)$  is a **LPF of WHz**.

Case 3:  $W > R_s/2$



Flat? **Impossible!**

Flat? If and only if  $H(t)$  is **rect**.

Flat? Many chances, once  $H(t)$  is **complement about  $R_s/2$** .

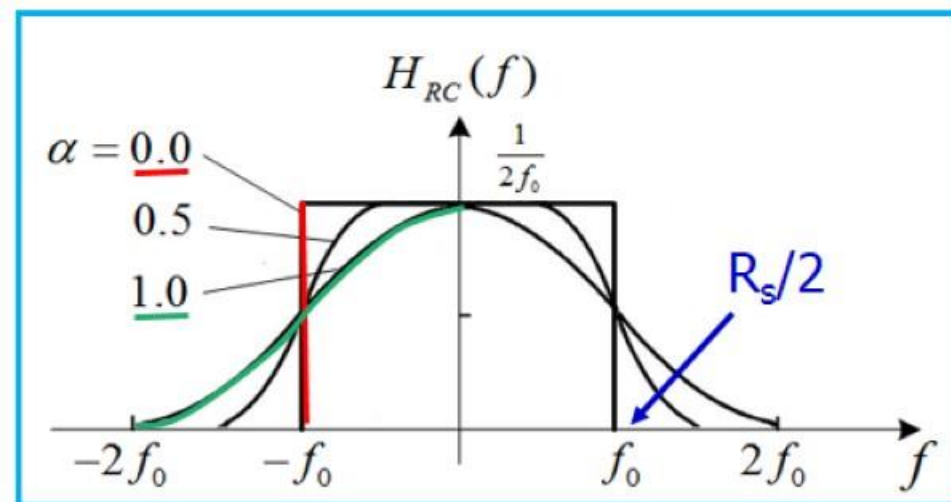
## 6.2 Design of BL signals for zero-ISI

In the design of BL systems for zero-ISI, **Raised-Cosine (RC) spectrum** is widely used for  $H(f)$ .

A RC **spectrum** and its **time response** are defined by,

$$H_{RC}(f) = \begin{cases} \frac{1}{2f_0} & 0 \leq |f| \leq (1-\alpha)f_0 \\ \frac{1}{4f_0} \left\{ 1 + \cos \frac{\pi [|f| - (1-\alpha)f_0]}{2\alpha f_0} \right\} & (1-\alpha)f_0 < |f| \leq (1+\alpha)f_0 \\ 0 & |f| > (1+\alpha)f_0 \end{cases}$$

respectively, where  $\alpha$  is called the **roll-off** factor and  $f_0 = R_s/2$  called **6dB-BW**.





## 6.2 Design of BL signals for zero-ISI

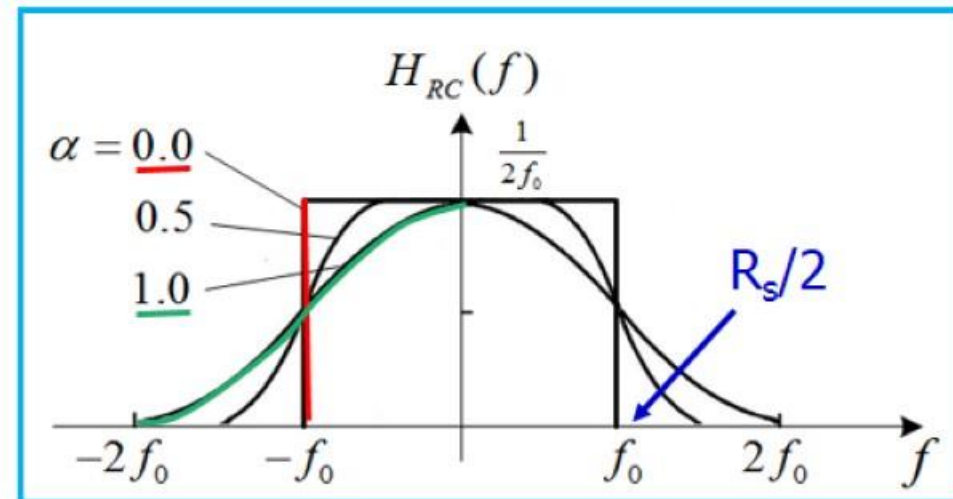
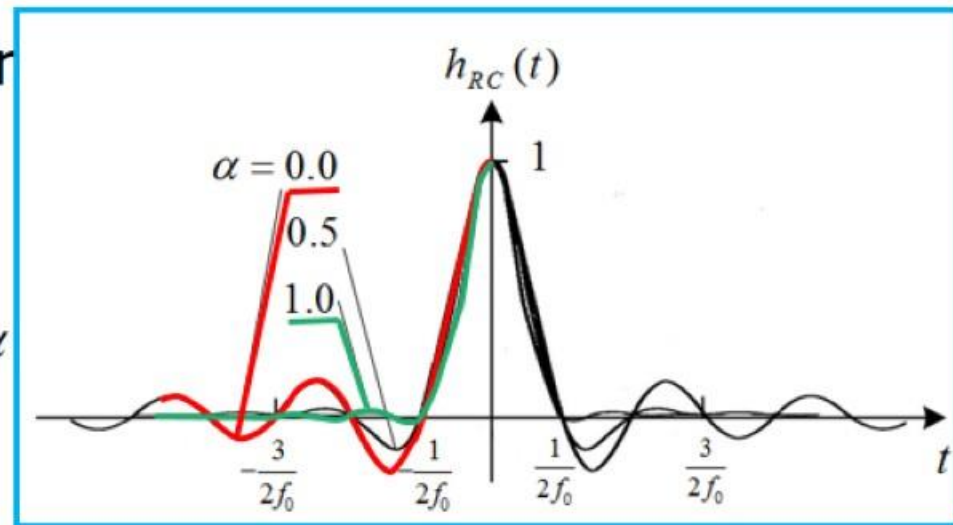
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respectively, where  $\alpha$  is called the **roll-off factor** and  $f_0 = R_s/2$  called **6dB-BW**.  
and

$$h_{RC}(t) = \frac{\sin(2\pi f_0 t)}{2\pi f_0 t} \cdot \frac{\cos(2\pi \alpha f_0 t)}{1 - (4\alpha f_0 t)^2}$$



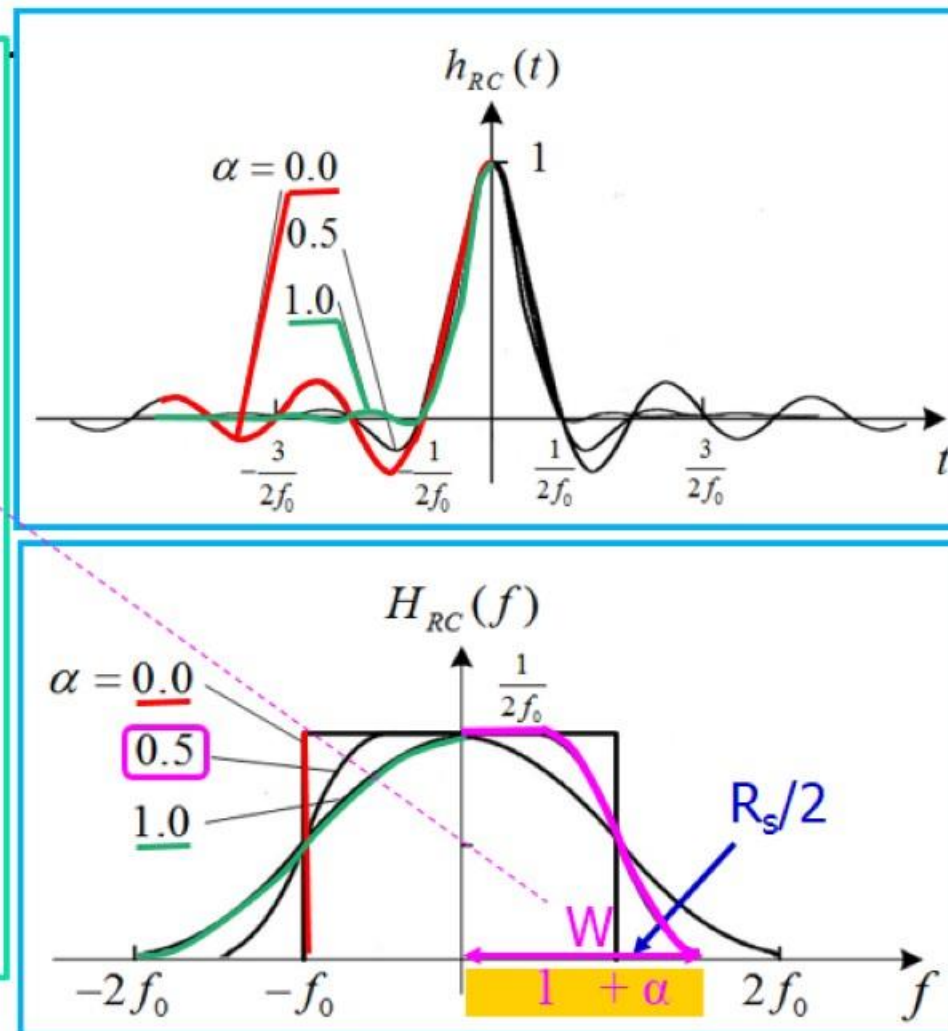


## 6.2 Design of BL signals for zero-ISI

In the design of BL systems for zero-ISI, **Raised-Cosine (RC) spectrum** is widely used for  $H(f)$ .

From the figure, we read  $W=(1+\alpha)R_s/2$ .  $\alpha$  controls the **transition of the band**. We note that:

- 1) **Large  $\alpha$**  gives a smooth transition and requires large BW, while **small  $\alpha$**  sharp transition and small BW.
- 2) The RC spectrum for  $\alpha=0$  is exactly **the LPF** and BW reaches the minimum.
- 3) The smoothness of transition band with **non-zero  $\alpha$**  is very important in practical implementation of filters.



Example: a **passband** channel of  $600 < f < 3600 \text{ Hz}$ , for  $7200 \text{ kbps}$  transmission.

Analysis:

1.  $f_c = (3600 + 600)/2 = 2100 \text{ (Hz)}$

2.  $W/2 = 3600 - 2100 = 1500$

Make  $W/2 \geq R_s/2 = [7200/\log_2(M)]/2$

$$\log_2(M) \geq 7200/(2 \times 1500) = 2.4,$$

Take  $M=8$ , and **8PSK**.  $R_s = 2400 \text{ Baud}$

3. Take  $W/2 \geq (1 + \alpha)R_s/2$ , that is  $1500 \geq 1200(1 + \alpha)$ ,

$$\alpha \leq 1500/1200 - 1 = 5/4 - 1 = 1/4, \quad \text{Take a RC filter with } \alpha = 0.25$$

4. The **spectral efficiency** is:  $R_b/W = 7200 \text{ bps}/3000 \text{ Hz} = 2.4 \text{ bps/Hz}$

