Chapter 5

Digital transmission through the AWGN channel

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Contents 16 Hrs

- Introduction
- Geometric rep. of the sig waveforms
- Pulse amplitude modulation
- 2-d signal waveforms
- M-d signal waveforms
- Opt. reception for the sig. in AWGN
- Optimal receivers and probs of err

5.3.1 Baseband

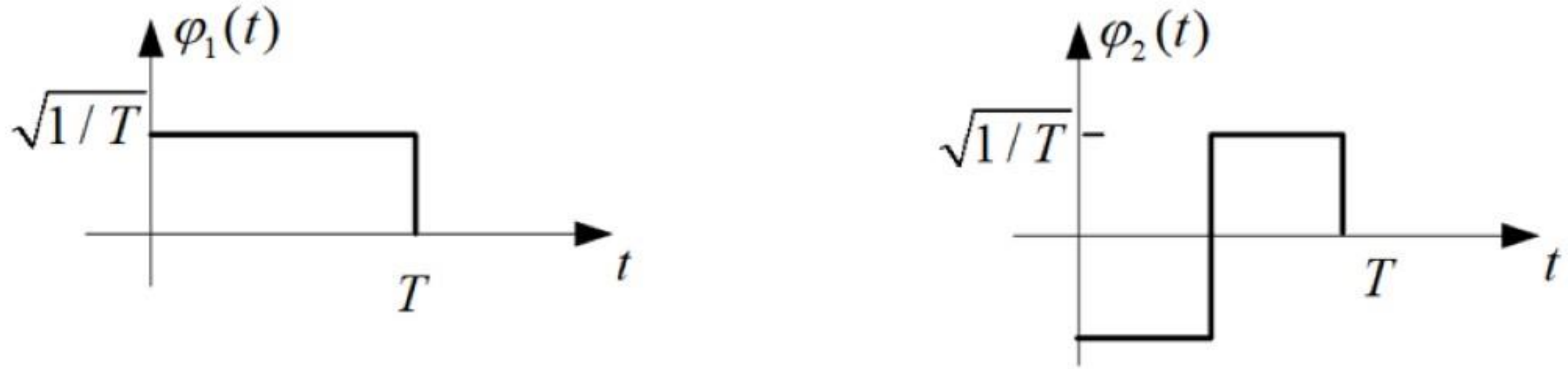
Previous signals are all one dimensional.

5.3.1 Baseband

Previous signals are all one dimensional. We need 2-d basis:

1) two overlapping signals;

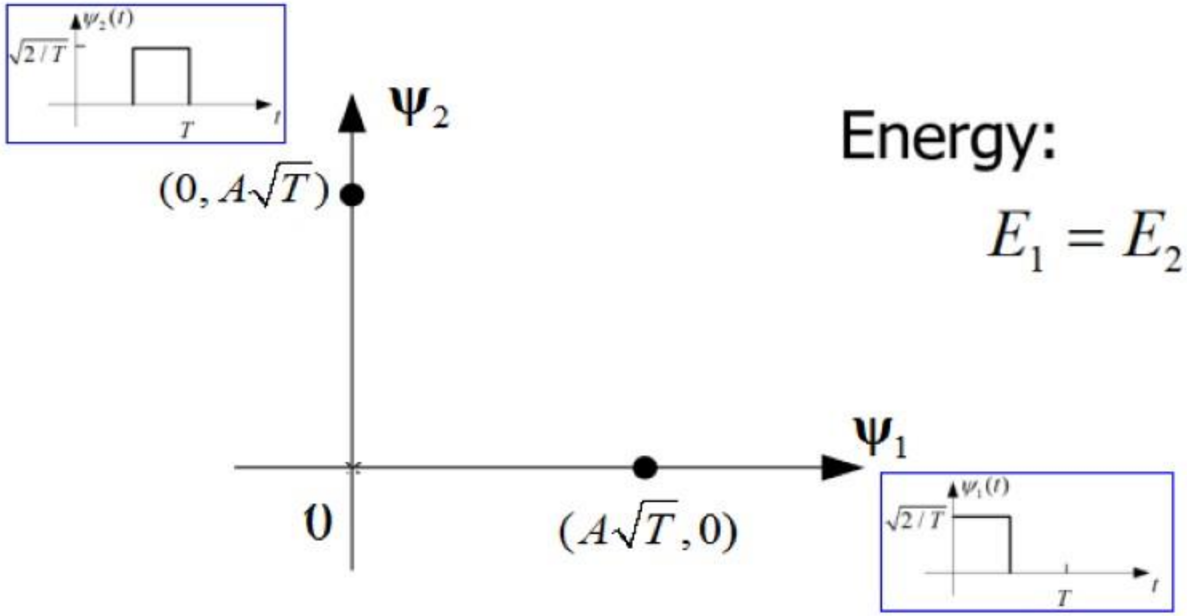
2) two non-overlapping signals



5.3.1 Baseband

Binary signals:

Scheme 1: $s_1(t) = A\sqrt{T} \times \psi_1(t)$ $\qquad s_2(t) = A\sqrt{T} \times \psi_2(t)$



Energy:

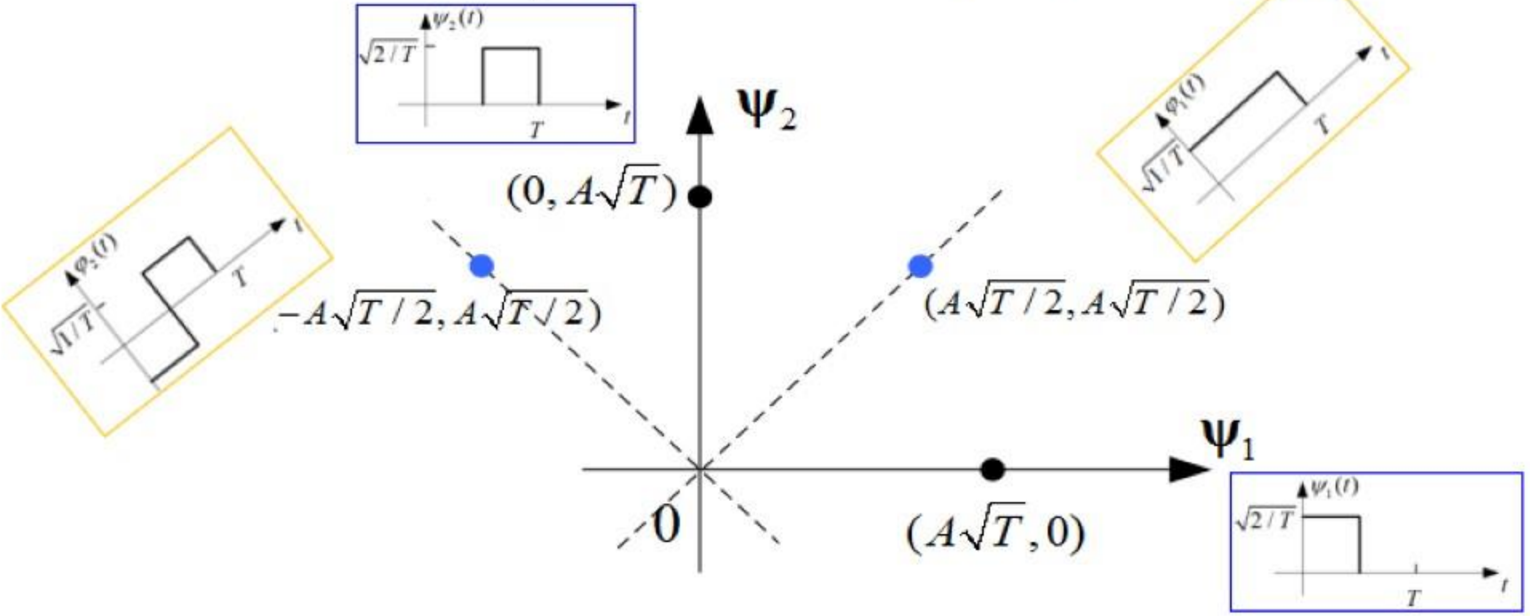
$$E_1 = E_2 = A^2 T$$

Q:What is the waveform for msg of 10110001101...?

Binary signals:

Scheme 1: $s_1(t) = A\sqrt{T} \times \psi_1(t) \qquad s_2(t) = A\sqrt{T} \times \psi_2(t)$

Scheme 2: $$\begin{cases} u_1(t) = A\sqrt{T/2}\left[\psi_1(t) + \psi_2(t)\right] = A\sqrt{T} \times \varphi_1(t) \\ u_2(t) = A\sqrt{T/2}\left[-\psi_1(t) + \psi_2(t)\right] = A\sqrt{T} \times \varphi_2(t) \end{cases}$$

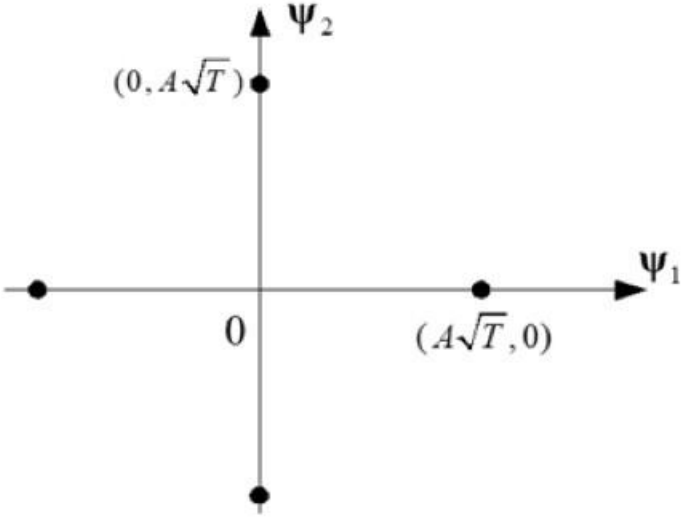


Q: What is the waveform for msg of 10110001101...?

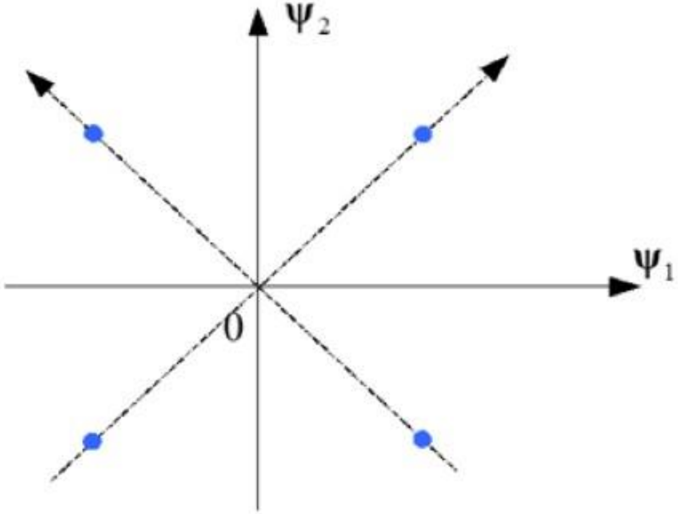
5.3.1 Baseband

4-ary signals

Scheme 1:
$$\begin{cases} s_1(t) = a_1\psi_1(t) \\ s_2(t) = a_2\psi_2(t) \\ s_3(t) = -a_1\psi_1(t) \\ s_4(t) = -a_2\psi_2(t) \end{cases}$$



Scheme 2:
$$\begin{cases} s_1(t) = a_{11}\psi_1(t) + a_{12}\psi_2(t) \\ s_2(t) = a_{21}\psi_1(t) + a_{22}\psi_2(t) \\ s_3(t) = -[a_{11}\psi_1(t) + a_{12}\psi_2(t)] \\ s_4(t) = -[a_{21}\psi_1(t) + a_{22}\psi_2(t)] \end{cases}$$



M-ary signals: more to See Fig5.15 and Fig5.16

5.3.2 Passband

Passband signals are cos-like

5.3.2 Passband

Passband signals are cos-like, and are generally in the form of,

$$\begin{aligned} s(t) &= \underline{r(t)\cos[2\pi f_c t + \theta(t)]} \\ &= r(t)\cos\theta(t)\cos\left(2\pi f_c t\right) - r(t)\sin\theta(t)\sin\left(2\pi f_c t\right) \\ &= \underline{a_c(t)\cos\left(2\pi f_c t\right) + a_s(t)\sin\left(2\pi f_c t\right)} \end{aligned}$$

With $\psi_1(t) = k_{c0}\boxed{\cos\left(2\pi f_c t\right)}$, $\psi_2(t) = k_{s0}\boxed{\sin\left(2\pi f_c t\right)}$, They are 2D signals.

Note that $\boldsymbol{\psi}_1 \cdot \boldsymbol{\psi}_2 = k_{s0}k_{c0}\int_{-\infty}^{+\infty} g_T^2(t)\sin\left(2\pi f_c t\right)\cos\left(2\pi f_c t\right)dt = 0$

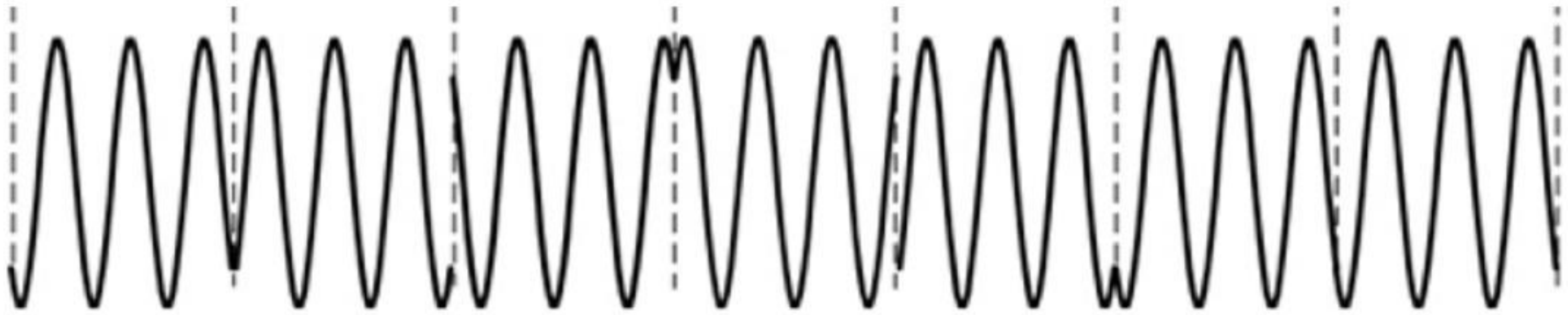
5.3.2 Passband

Carrier-phase modulated signals

$$u_m(t) = g_T(t)\cos\left(2\pi f_c t + \boxed{\frac{2\pi m}{M}}\right) \qquad m = 0, 2, \ldots, M-1$$

It is often called **PSK** (**Phase shift keying**).

Take M=4, QPSK. There are 4 possible phases: $\theta_m = \frac{2\pi m}{M} = 0, \pi/2, , \pi, 3\pi/2$



5.3.2 Passband

In fact, $u_m(t) = g_T(t)\cos\left(2\pi f_c t + \frac{2\pi m}{M}\right)$

$$= A_{mc} g_T(t)\cos(2\pi f_c t) - A_{ms} g_T(t)\sin(2\pi f_c t)$$

where, $A_{mc} = \cos\left(\frac{2\pi m}{M}\right)$, $A_{ms} = \sin\left(\frac{2\pi m}{M}\right)$

Let basis be $\psi_1(t) = k_0 g_T(t)\cos(2\pi f_c t)$, $\psi_2(t) = -k_0 g_T(t)\sin(2\pi f_c t)$

Where $k_0 = \sqrt{2/E_g}$, required by unit basis.

Then, $\boxed{u_m(t) = \left(A_{mc}\sqrt{E_g/2}\right)\times\psi_1(t) - \left(A_{mc}\sqrt{E_g/2}\right)\times\psi_1(t)}$

And the m signal points, $\mathbf{u}_m = \left(A_{mc}\sqrt{E_g/2},\ A_{ms}\sqrt{E_g/2}\right)$

5.3.2 Passband

In fact, $u_m(t) = g_T(t)\cos\left(2\pi f_c t + \frac{2}{\ }\right.$

$= A_{mc} g_T(t)\cos(2\pi f_c t)$

where, $A_{mc} = \cos\left(\frac{2\pi m}{M}\right)$, $A_{ms} = \sin\left(\right.$

Take M=4, QPSK.

$$\theta_m = \frac{2\pi m}{M} = 0, \pi/2,, \pi, 3\pi/2,$$

$$\mathbf{u}_0 = \left(\sqrt{E_g/2},\ 0\right), \mathbf{u}_1 = \left(0,\ \sqrt{E_g/2}\right),$$

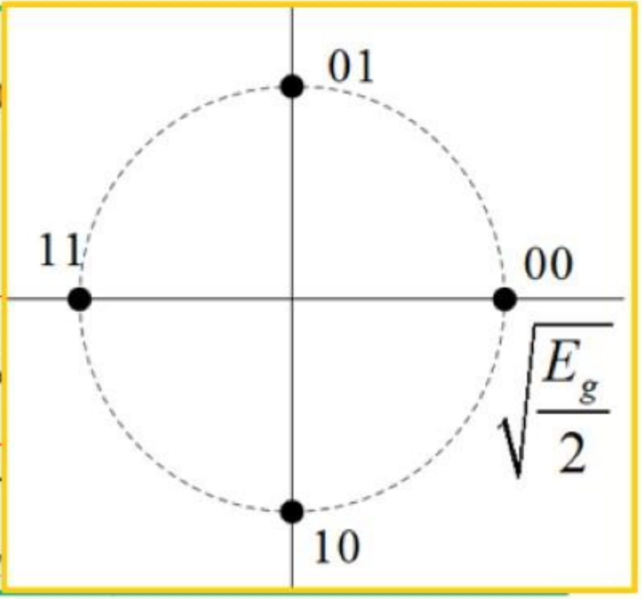
$$\mathbf{u}_2 = \left(-\sqrt{E_g/2},\ 0\right), \mathbf{u}_3 = \left(0,\ -\sqrt{E_g/2}\right)$$

Let basis be $\psi_1(t) = k_0 g_T(t)\cos(2\pi f_c t)$, $\psi_2(t) = -k_0$

Where $k_0 = \sqrt{2/E_g}$, required by unit basis.

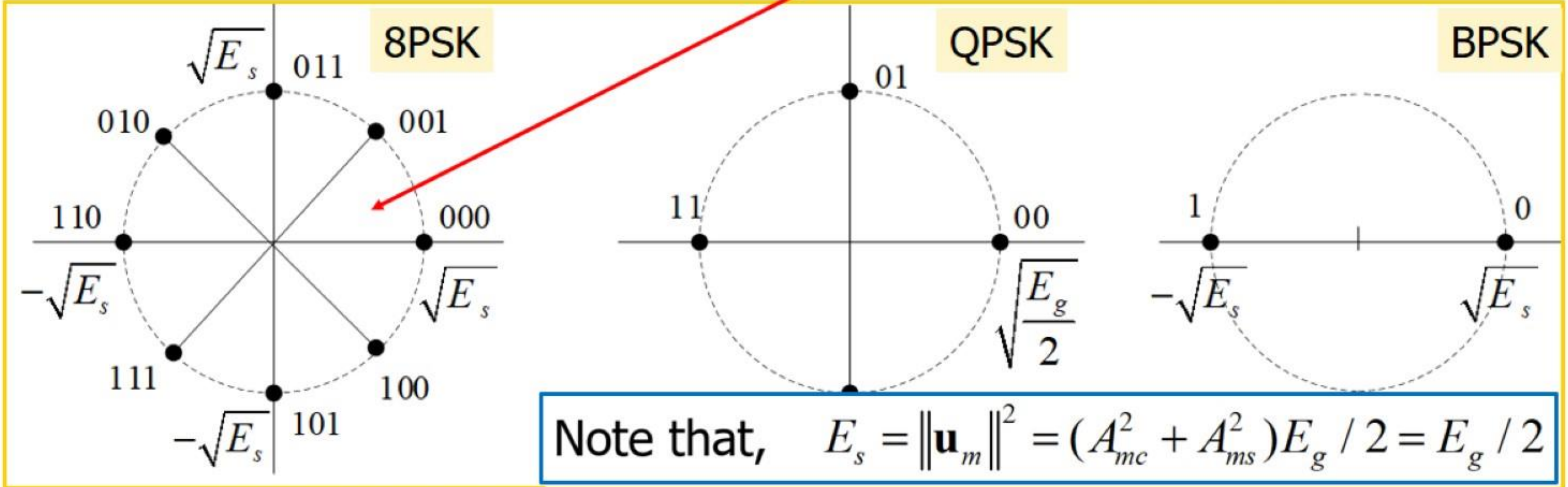
Then, $\boxed{u_m(t) = \left(A_{mc}\sqrt{E_g/2}\right)\times\psi_1(t) - \left(A_{mc}\sqrt{E_g/2}\right)\times}$

And the m signal points, $\mathbf{u}_m = \left(A_{mc}\sqrt{E_g/2},\ A_{ms}\sqrt{E_g}\right.$



5.3.2 Passband

In fact, $$u_m(t) = g_T(t)\cos\left(2\pi f_c t + \boxed{\frac{2\pi m}{M}}\right)$$



Note that, $E_s = \|\mathbf{u}_m\|^2 = (A_{mc}^2 + A_{ms}^2)E_g/2 = E_g/2$

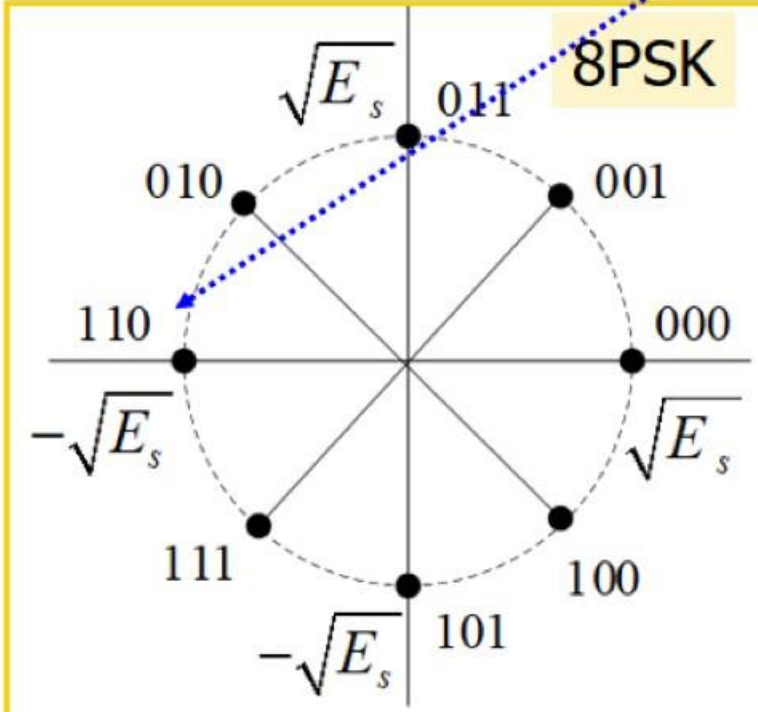
Constellation for M=8, 4, 2

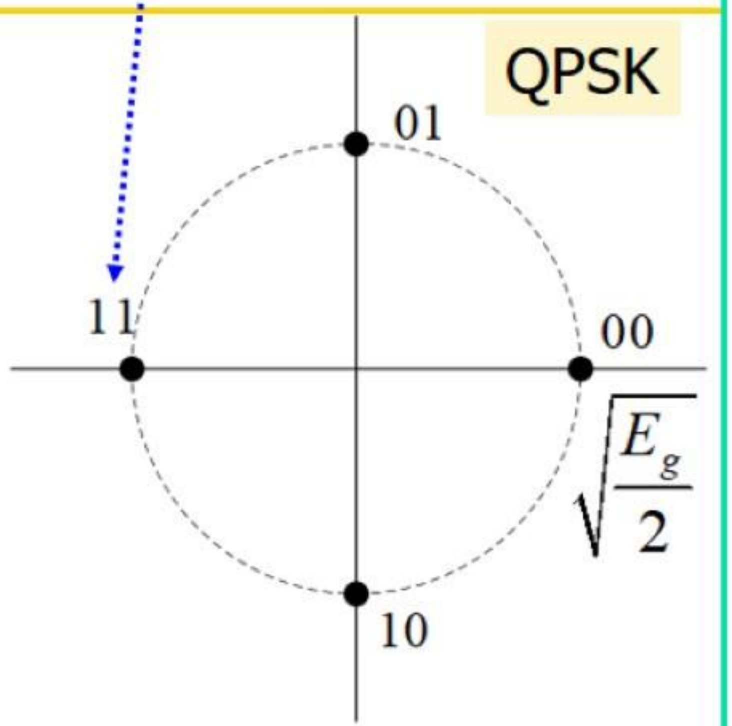
$$\mathbf{u}_m = \left(A_{mc}\sqrt{E_g/2},\ \ A_{ms}\sqrt{E_g/2}\right) \qquad A_{mc} = \cos\left(\frac{2\pi m}{M}\right),\ \ A_{ms} = \sin\left(\frac{2\pi m}{M}\right)$$

5.3.2 Passband

Gray code for M-ary

In fact, $u_m(t) = g_T(t)\cos\left(2\pi f_c t + \frac{2\pi m}{M}\right)$





Constellation for M=8, 4, 2

Sym	Gray	Natural
0	0 0 0 0	0 0 0 0
1	0 0 0 1	0 0 0 1
2	0 0 1 1	0 0 1 0
3	0 0 1 0	0 0 1 1
4	0 1 1 0	0 1 0 0
5	0 1 1 1	0 1 0 1
6	0 1 0 1	0 1 1 0
7	0 1 0 0	0 1 1 1
8	1 1 0 0	1 0 0 0
9	1 1 0 1	1 0 0 1
10	1 1 1 1	1 0 1 0
11	1 1 1 0	1 0 1 1
12	1 0 1 0	1 1 0 0
13	1 0 1 1	1 1 0 1
14	1 0 0 1	1 1 1 0
15	1 0 0 0	1 1 1 1

Reflection

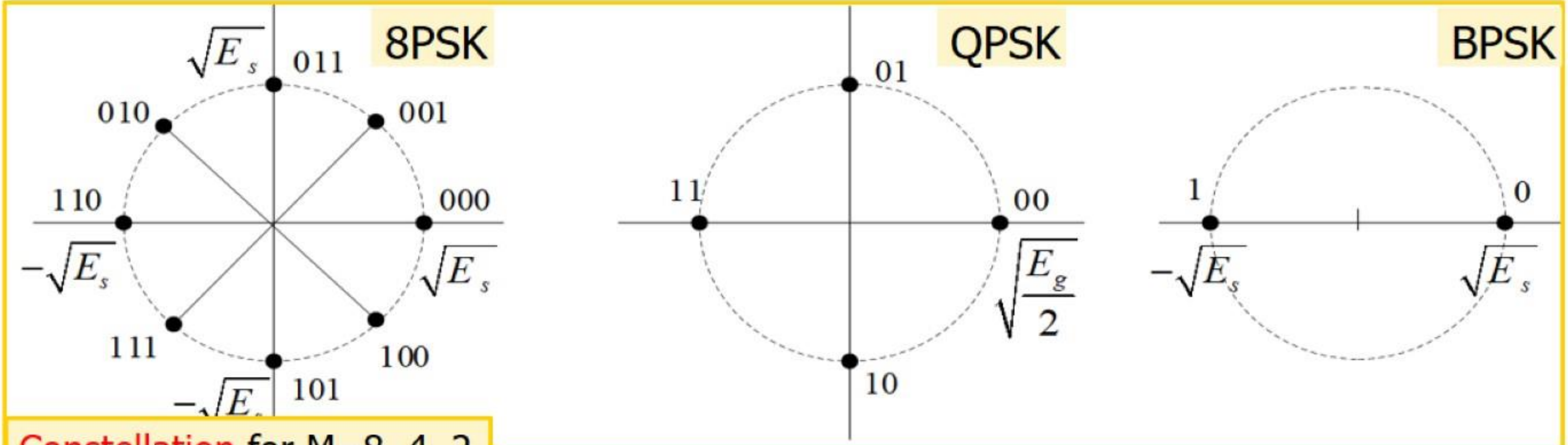
$$\mathbf{u}_m = \left(A_{mc}\sqrt{E_g/2},\ A_{ms}\sqrt{E_g/2}\right) \qquad A_{mc} = \cos\left(\frac{2\ldots}{M}\right),\ A_{ms} \ldots \left(\frac{\ldots}{M}\right)$$

5.3.2 Passband

In PSK, $$u_m(t) = A_{mc} g_T(t)\cos\left(2\pi f_c t\right) - A_{ms} g_T(t)\sin\left(2\pi f_c t\right)$$

where the information symbol corresponds a pair (A_{mc}, A_{ms}). The signals are 2-d points.

Note that the constraint of $A_{mc}^2 + A_{ms}^2 = const.$, i.e. the points are on a circle.



Constellation for M=8, 4, 2

5.3.2 Passband

In PSK, $$u_m(t) = A_{mc} g_T(t)\cos\left(2\pi f_c t\right) - A_{ms} g_T(t)\sin\left(2\pi f_c t\right)$$

where the information symbol corresponds a pair (A_{mc}, A_{ms}). The signals are 2-d points.

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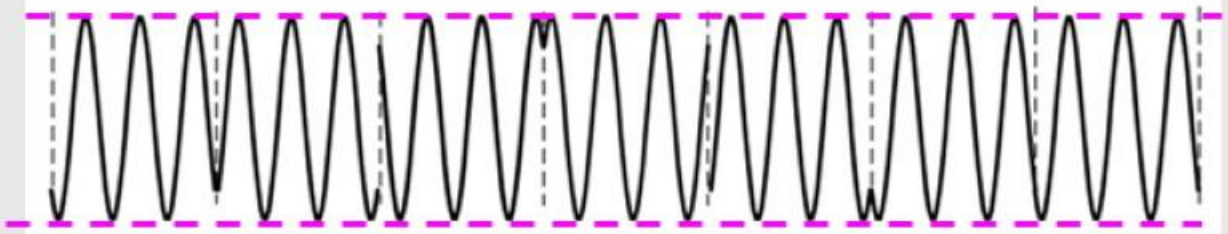
The waveforms of the PSK have constant envelope, a good feature in many applications.



Constellation for M=8, 4, 2

5.3.2 Passband

In PSK, $$u_m(t) = A_{mc} g_T(t) \cos\left(2\pi f_c t\right) - A_{ms} g_T(t) \sin\left(2\pi f_c t\right)$$

where the information symbol corresponds a pair (A_{mc}, A_{ms}). The signals are 2-d points.

Note that the constraint of $A_{mc}^2 + A_{ms}^2 \cancel{=} const.$, i.e. the points are on a circle.

In general, what is the scheme with no constraint on the points !

5.3.3 Quadrature Amplitude Modulation

Generally,

$$u_m(t) = x_m g_T(t)\cos\left(2\pi f_c t\right) - y_m g_T(t)\sin\left(2\pi f_c t\right)$$

where the information symbol corresponds a pair (x_m, y_m), with NO constraint on the (x_m, y_m).

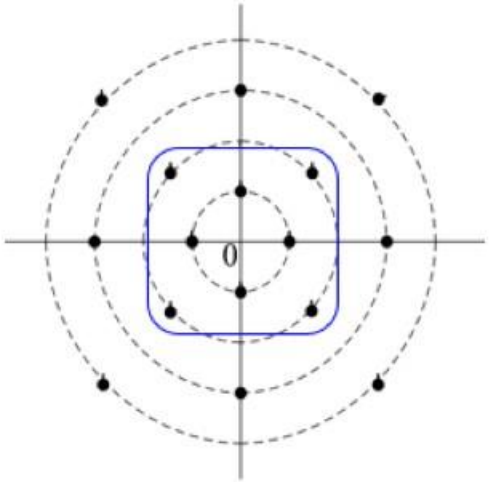
The signal points are, $\mathbf{u}_m = \left(k_1 x_m, k_1 y_m\right)$ where, $k_1 = \sqrt{E_g / 2}$

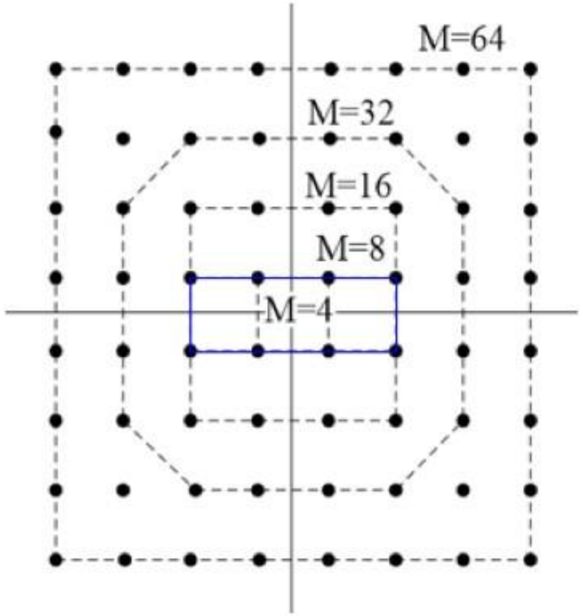
Due to the orthogonality, $\cos 2\pi f_c t$ and $\sin 2\pi f_c t$ are called quadrature carriers. This type of modulation is called **QAM** (**quadrature amplitude modulation**).

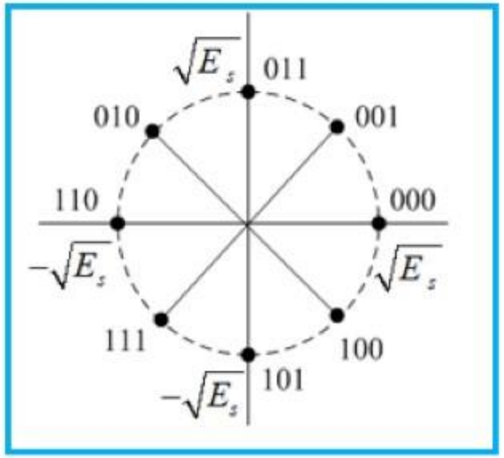
5.3.3 QAM

The signal points are, $\mathbf{u}_m = (k_1 x_m, k_1 y_m)$ where, $k_1 = \sqrt{E_g / 2}$

In general, the followings are constellations for QAM.







The energy of signals are unequal, and we have,

$$E_m = \|\mathbf{u}_m\|^2 = (x_m^2 + y_m^2) k_1^2 = A_m^2 E_g / 2$$

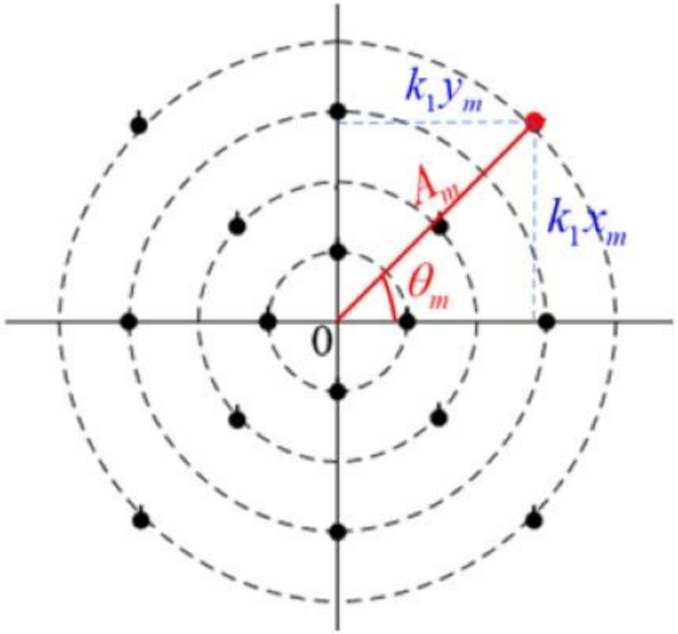
The average energy is, $E_{av} = \dfrac{1}{M} \sum_{m=1}^{M} \|\mathbf{u}_m\|^2 = \dfrac{E_g}{2M} \sum_{m=1}^{M} A_m^2$

5.3.3 QAM

The signal points are, $\mathbf{u}_m = (k_1 x_m, k_1 y_m)$ where, $k_1 = \sqrt{E_g / 2}$

Also,
$$u_m(t) = \boxed{x_m} g_T(t)\cos 2\pi f_c t - \boxed{y_m} g_T(t)\sin 2\pi f_c t$$
$$= \sqrt{x_m^2 + y_m^2}\, g_T(t)\left[\frac{x_m}{\sqrt{x_m^2 + y_m^2}}\cos 2\pi f_c t - \frac{y_m}{\sqrt{x_m^2 + y_m^2}}\sin 2\pi f_c t\right]$$
$$= \boxed{A_m} g_T(t)\cos\left(2\pi f_c t + \boxed{\theta_m}\right)$$

where, $A_m = \sqrt{x_m^2 + y_m^2}$, $\theta_m = arc\tan\dfrac{y_m}{x_m}$



A QAM signal can be given by a pair of radius and angle. So, QAM is also combined amplitude- and phase-modulation.