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# Pilot-based channel estimation in OFDM system

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A Thesis

entitled

Pilot-Based Channel Estimation in OFDM System

by

Fei Wang

Submitted to the Graduate Faculty as partial fulfillment of the requirements  
for the Master of Science Degree in Electrical Engineering

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College of Graduate Studies

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May 2011

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An Abstract of  
Pilot-Based Channel Estimation in OFDM System

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Orthogonal frequency division multiplexing (OFDM) is a multi-carrier transmission technology in wireless environment, and can also be seen as a multi-carrier digital modulation or multi-carrier digital multiplexing technology. A large number of orthogonal sub-carriers are used to transmit information. OFDM system has high utilization of frequency spectrum and satisfactory capability of reducing multi-path inference. So, OFDM has been considered as one of the core technologies of 4<sup>th</sup> generation (4G) wireless communication system in the future.

Channel estimation plays a very important role in OFDM system. As a research hotspot, many related algorithms have been presented these years, which can be generally separated into two methods, pilot-based channel estimation and blind channel estimation. Pilot-based channel estimation estimates the channel information by obtaining the impulse response from all sub-carriers by pilot. Compared with blind channel estimation, which uses statistical information of the received signals, pilot-based channel estimation is a practical and an effective method.

This thesis is on the pilot-based channel estimation of OFDM system. Firstly, it introduces the basic principle and realization of OFDM system, and describes the system

construction and model with summary of some key technologies, such as fast Fourier transform (FFT) and cyclic prefix (CP). We also analyze OFDM modulation in the frequency domain, and discusses some advantages and disadvantages of OFDM system. Next, a summary of multi-path and time varying statistical properties of general wireless channel of OFDM system are presented. This thesis also investigates principles and performances of the channel estimation methods, block type pilot and comb type pilot. In the arrangement of block type pilot, the performance of channel estimation is analyzed with estimators based on three different algorithms, least square (LS) algorithm, linear minimum mean square error (LMMSE) algorithm and singular value decomposition (SVD) algorithm. To use comb-type pilot arrangement, the thesis introduces three methods of interpolation: linear interpolation, second order interpolation and cubic spline interpolation. Finally, simulation results using Matlab are used to compare bit error rate (BER) performance of different modulation schemes and types of pilot.

For my parents and friends

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Firstly, I sincerely thank my advisor Dr. Junghwan Kim for giving me the opportunity to pursue my research under his guidance. I have learned much from him on academic. I would also thank Dr. Dong-Shik Kim and Dr. Mohammed Niamat for being my committee members of my thesis.

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# Contents

<b>Abstrat.....</b>	<b>iii</b>
<b>Acknowledgements .....</b>	<b>v</b>
<b>Contents .....</b>	<b>vi</b>
<b>List of Tables.....</b>	<b>x</b>
<b>List of Figures.....</b>	<b>xi</b>
<b>1. Introduction.....</b>	<b>1</b>
1.1    Background.....	1
1.1.1    Development of Mobile Communication System.....	1
1.1.2    Development and Application of OFDM.....	3
1.2    Thesis Outline.....	4
<b>2. OFDM System Fundamentals.....</b>	<b>6</b>
2.1    Single-carrier and Multi-carrier Communication System.....	6
2.1.1    Single-carrier Transmission System.....	6
2.1.2    Multi-carrier Transmission System.....	7
2.2    Frequency Division Multiplexing and Orthogonal Frequency Division Multiplexing.....	8



2.2.1	Frequency Division Multiplexing.....	8
2.2.2	Orthogonal Frequency Division Multiplexing.....	11
2.3	Principles of OFDM.....	12
2.3.1	M-ary Digital Modulation.....	12
2.3.2	OFDM Principles.....	15
2.3.3	Fast Fourier Transform.....	19
2.3.4	Cyclic Prefix in OFDM System.....	19
2.3.5	Equalization.....	21
2.3.6	OFDM System Components.....	23
2.4	Advantages and Disadvantages of OFDM .....	28
2.4.1	Advantages of OFDM.....	28
2.4.2	Disadvantages of OFDM.....	29
<b>3.</b>	<b>Channel Characteristics of OFDM System.....</b>	<b>30</b>
3.1	Wireless Channel.....	30
3.1.1	Multipath Propagation and Time-Varying.....	32
3.1.1.1	Multipath Propagation .....	32
3.1.1.2	Time Varying.....	33
3.1.2	Parameters of Mobile Multipath Channel.....	33
3.1.3	Fading Channels.....	36
3.1.3.1	Flat Versus Frequency Selective Fading.....	36

3.1.3.2	Slow Versus Fast Fading.....	37
3.1.4	Fading Channel Properties .....	38
3.2	OFDM Channel Model.....	40
3.2.1	AWGN Channel .....	40
3.2.2	Rayleigh Multipath Channel.....	41
3.2.3	Doppler Spread Channel.....	41
<b>4.</b>	<b>Pilot-Based Channel Estimation in OFDM System .....</b>	<b>43</b>
4.1	Types of Pilot.....	44
4.2	Block Type Pilot-Based Channel Estimation.....	46
4.2.1	Least Square (LS) Estimator.....	46
4.2.2	Minimum Mean Square Error (MMSE) Estimator.....	48
4.2.3	Linear Minimum Mean Square Error (LMMSE) Estimator .....	49
4.2.4	Singular Value Decomposition (SVD) Estimator.....	51
4.3	Comb Type Pilot-Based Channel Estimation .....	53
4.3.1	Piecewise Constant Interpolation .....	54
4.3.2	Linear Interpolation .....	54
4.3.3	Second Order Interpolation.....	55
4.3.4	Cubic Spline Interpolation.....	56
<b>5.</b>	<b>Simulation Results.....</b>	<b>57</b>
5.1	Simulation Scenarios .....	57

5.2	Simulation Results.....	58
<b>6.</b>	<b>Conclusion and Future Work.....</b>	<b>76</b>
	<b>References.....</b>	<b>78</b>

# List of Tables

5-1 OFDM system simulation parameter.....58

## List of Figures

2-1 Frame of the single carrier .....	7
2-2 Frame of the multi-carriers.....	8
2-3 Frame of FDM system .....	9
2-4 Spectrum analysis of FDM [6] .....	10
2-5 Spectra of FDM and OFDM.....	12
2-6 Constellation of QPSK.....	14
2-7 Constellation of 16QAM.....	15
2-8 Frame of the basic model of OFDM .....	16
2-9 Example of an OFDM symbol with 4 carriers.....	17
2-10 Spectrum of sub-carriers of OFDM .....	18
2-11 Cyclic Prefix.....	20
2-12 Diagram of ZF equalizer .....	22
2-13 Block diagram of OFDM system.....	23
2-14 OFDM carrier magnitude before IFFT .....	24
2-15 OFDM carrier phase before IFFT .....	25
2-16 OFDM signal in time domain.....	26
2-17 OFDM carrier magnitude after FFT.....	27
2-18 OFDM carrier phase after FFT .....	27
3-1 AWGN channel model .....	41

3-2 Rayleigh channel model.....	42
3-3 Doppler spread channel model.....	42
4-1 Block-type pilot and comb-type pilot.....	46
4-2 Block diagram of SVD estimator.....	53
4-3 Sketch map of second order interpolation.....	55
5-1 Comparison of DFT and FFT operations .....	59
5-2 OFDM performance with or without CP.....	59
5-3 OFDM performance with or without equalizer.....	60
5-4 Eye pattern before ZF equalizer.....	61
5-5 Eye pattern after ZF equalizer.....	61
5-6 Comparison of received signal waveform with and without ZF equalizer.....	62
5-7 OFDM performance under AWGN and multipath.....	63
5-8 Comparison of pilot-based and no channel estimation .....	63
5-9 Comparison of block type and comb type pilot in fast fading channel.....	64
5-10 Comparison of block type and comb type pilot in slow fading channel .....	65
5-11 Comparison of LS, LMMSE and SVD of block type pilot.....	65
5-12 Comparison of SVD with different p .....	67
5-13 Comparison of block type when Doppler frequency is 20 Hz.....	67
5-14 Comparison of block type when Doppler frequency is 40 Hz.....	68
5-15 Comparison of block type when Doppler frequency is 80 Hz.....	68
5-16 Comparison of block type when Doppler frequency is 120 Hz.....	69
5-17 Comparison of different interpolations of comb type pilot.....	70
5-18 Comparison of comb type with 8 pilots.....	71
5-19 Comparison of comb type with 16 pilots.....	71

5-20 Comparison of comb type with 32 pilots.....	72
5-21 Comparison of comb type with 64 pilots.....	72
5-22 Comparison of QPSK and 16QAM under multipath.....	73
5-23 Comparison of block type under QPSK modulation .....	74
5-24 Comparison of block type under 16QAM modulation.....	74
5-25 Comparison of comb type under QPSK modulation.....	75
5-26 Comparison of comb type under 16QAM modulation.....	75

# Chapter 1

## Introduction

Over the past two decades, the rapid development of wireless communication technology has brought great convenience to people's lives and work. In the 21st century, wireless communication technologies, especially mobile communication technology, presents unprecedented development. The goal of next generation of mobile wireless communication system is to achieve ubiquitous, high-quality, high-speed mobile multimedia transmission. To achieve this goal, various new technologies are constantly being applied to mobile communication systems. Academia and industry have reached a consensus that OFDM is one of the most promising core technologies in new generation of wireless mobile communication system.

### 1.1 Background

#### 1.1.1 Development of Mobile Communication System

Since Guglielmo M. Marconi invented wireless telegraph a century ago, the wireless transmission technology allows people to communicate without any physical connection. In recent decades, there is more rapid development of wireless mobile communications, from the initial 1G analog mobile communication system. Currently, 4G mobile



communication system has been started in the world and begun the trial. Development of mobile communication is shown as following steps [1, 2].

The 1G mobile communication system mainly uses analog technology and frequency division multiple access (FDMA) technology. Due to bandwidth constraints, mobile communications cannot do long distance roaming, but only be a regional mobile communications system. The main drawback is low spectrum utilization and signaling voice traffic interference.

The 2G mobile communication alternatives 1G mobile communication system and completed changes from analog to digital technology. It mainly uses the time division multiple access (TDMA) technology and code division multiple access (CDMA) technology. Its main business is voice, and the feature is to provide digital voice services and low-speed data services. However, due to the limited bandwidth, application of data services is limited, and cannot achieve high rates of business such as mobile multimedia services.

The concept of 3G mobile communication is proposed by the International Telecommunication Union (ITU) in 1985 originally, and was officially named as International Mobile telecom System 2000 (IMT2000). In May 2000, it finally passed the 3G mobile communication air interface standards, and was officially named as IMT2000 wireless interface specification. This specification includes two types of CDMA and TDMA. Its main features include support for multimedia services, data transmission rate of at least 384kbit/s, and global roaming. Compared with the 1G and 2G, high-speed data transmission and broadband multimedia service can be achieved in 3G communication system. However, because of different standards of regional communication systems, 3G is still unable to meet the future requirements of higher data transfer rates.

Therefore, 4G mobile communication system, with the OFDM modulation technique, started to enter the horizon and became a research hotspot. For high-volume and high-speed wireless mobile communication systems, OFDM is a promising modulation scheme, and will play an increasingly important role in the future development of wireless mobile communication network.

### **1.1.2 Development and Application of OFDM**

OFDM is a multi-carrier transmission technology in wireless environment, and can also be seen as a multi-carrier digital modulation or multi-carrier digital multiplexing technology. Because of using of orthogonal carrier technology without interference and no guard band between single carriers, OFDM system requires much less bandwidth compared with the conventional frequency division multiplexing (FDM) system, and gets higher bandwidth utilization.

Theoretical formation of OFDM and application starts in the area of wireless mobile communication, which is based on discrete Fourier transform (DFT).

The sub-carriers overlap  $1/2$  between them but remain orthogonal to each other. The signals are separated by demodulation in the receiver. To reduce the complexity of multi-carrier system, in 1977, S.B. Weinstein and P.M. Ebert [3] proposed following theories, “Before passing the filter, the spectral shape of sub-carrier is a sinc function and non-band limited. DFT can do modulation and demodulation of multi-carrier baseband. The empty slot between symbols can be used as a guard interval to eliminate ISI”. In 1980, A. Peled and A. Ruiz replaced the empty slot as cyclic prefix (CP) in order to satisfy the orthogonality of each carrier in dispersive channels. In 1985, Cimini [3] applied the OFDM

to wireless cellular mobile communication system, and established the OFDM wireless mobile communication systems theory.

OFDM is widely applied from theory to practice. In the past decade, the rapid development of large scale integrated circuit technology and realization of a large number points high-speed FFT promote the wide application of OFDM theory.

At present, OFDM has been widely used in broadcast audio and video and civilian communications, including asymmetric digital subscriber line (ADSL), digital audio broadcasting (DAB), digital video broadcasting (DVB), high definition television (HDTV), wireless local area networks (WLAN), and etc.. However, because of the dramatic increasing of mobile users, mobile speed and amount of data transmitted, there are still many problems of OFDM to be resolved in the high-speed mobile environment. And one of the problems is the channel estimation.

## **1.2 Thesis Outline**

Channel estimation is a key technology of OFDM system, and the accuracy of it directly determines the quality of the system transmission. This thesis is on the research of pilot-based channel estimation of OFDM for wireless mobile system. The first two chapters introduce the development of OFDM technology, basic principles, structural features and key technologies. The third chapter describes the wireless channel transmission characteristics and the impact on the OFDM system, and multipath fading channel model used in thesis. The fourth chapter describes the pilot-based channel estimation in OFDM system. The thesis analyzes LS estimator, LMMSE estimator and SVD estimator based on block type pilot, along with linear interpolation, second order interpolation and cubic spline interpolation based on comb type pilot. The fifth chapter

analyzes the simulation results and compares the differences of the algorithms. The last chapter is on the conclusions and future works.

## Chapter 2

# OFDM System Fundamentals

### 2.1 Single-carrier and Multi-carrier Communication System

#### 2.1.1 Single-carrier Transmission System

When the system data rate is not too high, and of interference between symbols caused by multipath signal is not particularly serious, the single-carrier transmission system is normally used as is shown in Figure 2-1, where the  $g(t)$  is matching filter. Then an appropriate equalization algorithm could be used to allow the system to work properly. But for broadband services of the higher rate data transmission, delay spread caused by overlap between the data symbols results in inter-symbol interference (ISI) between the symbols, which requires a higher equalization requirements. From another point of view, when the signal bandwidth is over and close to the channel coherence bandwidth, time dispersion will result in frequency selective fading and making the same signal in different frequency components reflect the different fading characteristics, which is we do not want to see.

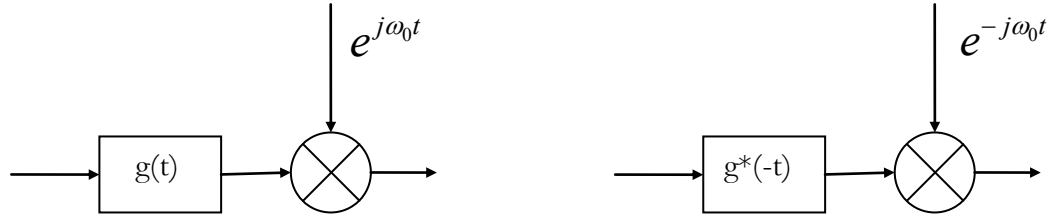


Figure 2-1 Frame of the single carrier

### 2.1.2 Multi-carrier Transmission System

Multi-carrier transmission system is a method of transmitting data by splitting it into several components, and sending each of these components over separate carrier signals. The individual carriers have narrow bandwidth, but the composite signal can have broad bandwidth. The advantages of multi-carrier transmission system include relative immunity to fading caused by transmission over more than one path at a time (multipath fading), less susceptibility than single-carrier systems to interference caused by impulse noise, and enhanced immunity to inter-symbol interference. Limitations include difficulty in synchronizing the carriers under marginal conditions, and a relatively strict requirement that amplification be linear. Figure 2-2 shows the basic structure of multi-carrier system schematic.

There are several kinds of Multi-carrier transmission system, for example, OFDM, discrete multi-tone (DMT) and multi-carrier modulation (MCM). These three kinds of methods are generally same, but in OFDM, sub-carrier remains orthogonal to each other, which is not always the establishment of MCM [3].

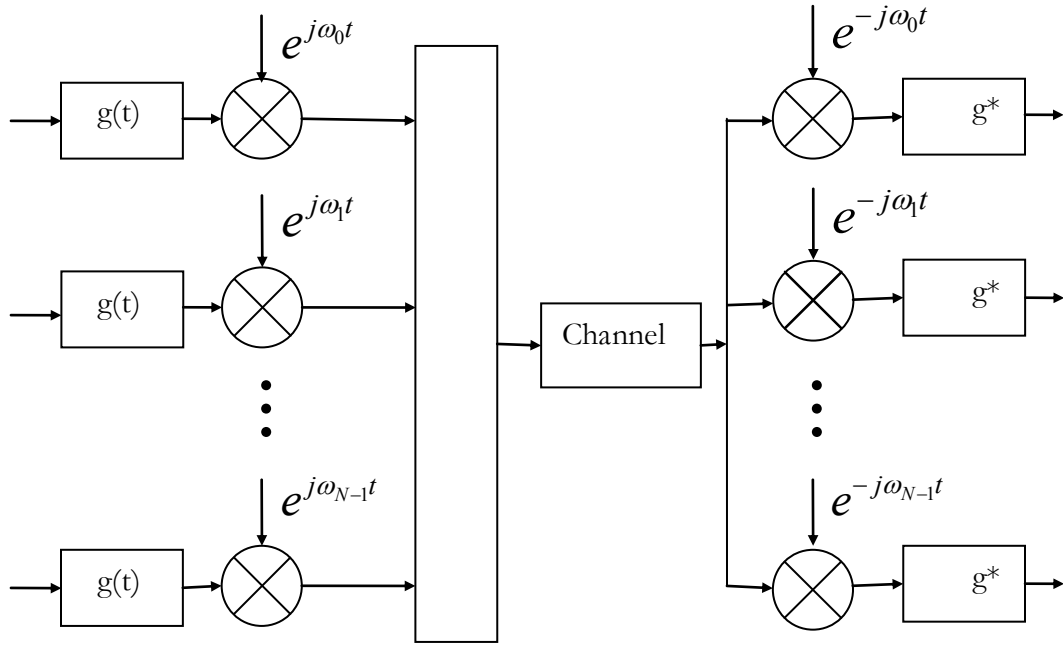


Figure 2-2 Frame of the multi-carriers

## 2.2 Frequency Division Multiplexing and Orthogonal Frequency Division Multiplexing

### 2.2.1 Frequency Division Multiplexing

Frequency-division multiplexing (FDM) is a scheme in which numerous signals are combined for transmission on a single communications line or channel. Each signal is assigned a different frequency as sub-channels within the main channel. The sending part of frequency division multiplexing transmission system block diagram is shown in Figure 2-3, and the receiving part is a reverse process.

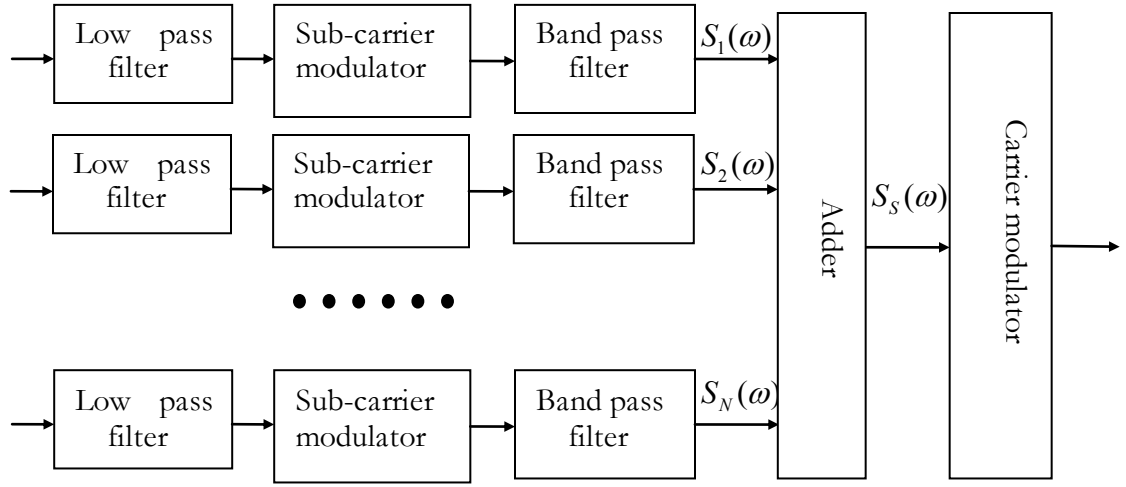


Figure 2-3 Frame of FDM system

We assume there are  $N$  signals  $f_1(t)$ ,  $f_2(t)$ , ...,  $f_N(t)$  with the same bandwidth waiting for transmit. They pass through low pass filter separately to ensure that the bandwidth does not exceed  $2\omega_f$ , where  $\omega_f$  is the highest frequency of each signal. Because signals occupying the same frequency band, the receiver will not be able to distinguish them if they are in the same channel. So we need to change the frequency of signals in order not to overlap in the frequency axis. Therefore, various sub-carrier signals are modulated to achieve the frequency moving. Let us define a set of sine waves with the same frequency as the sub-carrier, and the corresponding frequency is called sub-carrier frequency. To limit the frequency band shared by each sub-carrier, we set up a band pass filter before the adding device in each way. Multi-channel signal is still a base-band signal and can be directly transferred by wire. Signals are non-overlapping on frequency bands at this time, so adding devices can be used to transfer  $N$  signals together. Frequency division multiplex signal can be expressed as below



$$S_s(t) = \sum_{n=1}^N f_n \cos \omega_n(t) \quad (2-1)$$

In order to achieve the wireless transmission, the signal needs to be synthesized on a modulated RF carrier, and it is called the primary or secondary modulated carrier modulation. In the receiver, demodulation process is the opposite transformation. First of all, demodulate the main carrier of the RF signal, and the recovered signal is added to all multi-band pass filters on each way. The center frequency of each band pass filter limits bandwidth and sub-carrier frequency, allowing only the signals on the channel pass, in order to achieve the division of the frequency domain [5]. After demodulating the subcarrier of signals, the spectra information can be obtained from various quarters of FDM as shown in Figure 2-4.

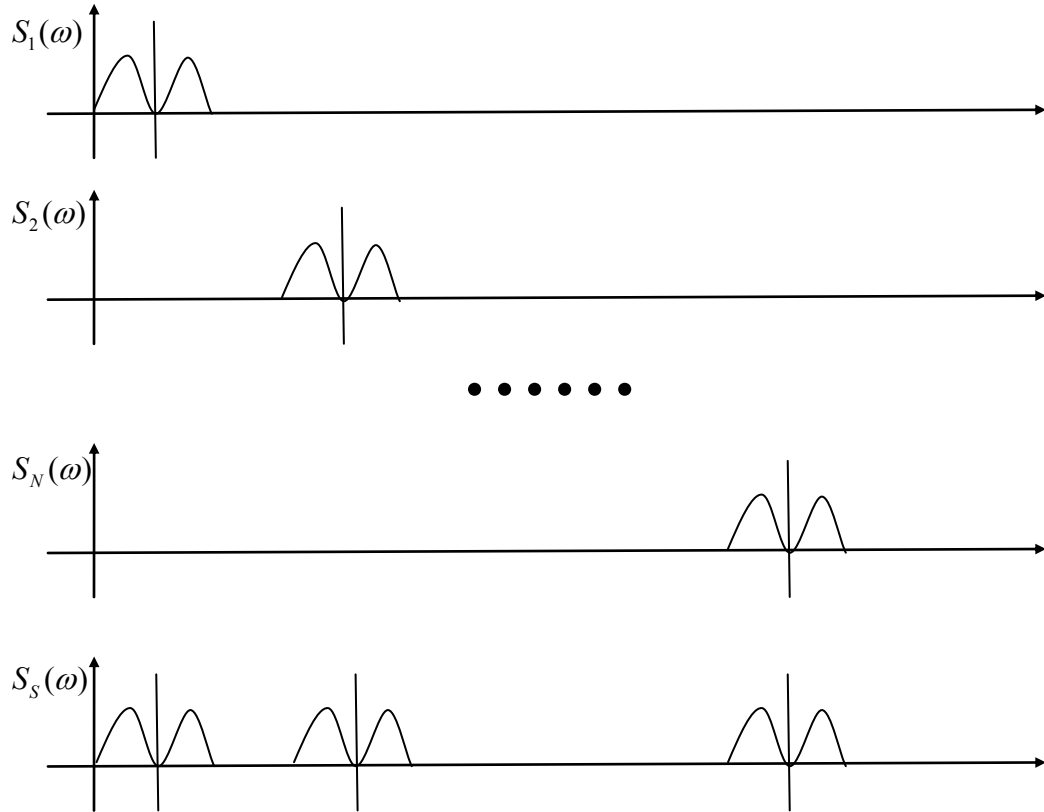


Figure 2-4 Spectrum analysis of FDM [6]

### 2.2.2 Orthogonal Frequency Division Multiplexing

Based on the principle of FDM, subcarrier sets of OFDM uses orthogonal sine or cosine function. The orthogonality of  $\{\cos n\omega t\}$  and  $\{\sin m\omega t\}$  ( $n, m=1, 2, 3, \dots$ ) occurs in  $(t_0, t_0 + T)$  as below

$$\int_{t_0}^{t_0+T} \cos n\omega t \sin m\omega t dt = \begin{cases} 0 & (n \neq m) \\ T/2 & (n = m) \\ T & (n = m = 0) \end{cases} \quad T = \frac{2\pi}{\omega} \quad (2-2)$$

The sin function is similar as cosine function.

According to the theory, let frequencies of  $N$  sub carriers are  $f_1, \dots, f_N$  and  $f_k = f_0 + k/T_N, k=1, \dots, N$ , where  $T_N$  is unit code duration. The single sub-carrier signal is defined as

$$f_k(t) = \begin{cases} \cos(2\pi f_k t) & 0 \leq t < T_N \\ 0 & \text{Others} \end{cases} \quad (2-3)$$

We know from the orthogonality that,

$$\int f_n(t) f_m(t) dt = \begin{cases} T_N & m = n \\ 0 & m \neq n \end{cases} \quad (2-4)$$

So sub-carriers are orthogonal each other. The receiver can demodulate signal modulated by orthogonality if signal is strictly synchronized. The OFDM signal is same as FDM as below

$$S(t) = \sum_{n=1}^N f_n \cos \omega_n(t) \quad (2-5)$$

But, the difference is spectrum shown as in Figure 2-5.

From Figure 2-5, FDM requires the protection of a wide interval because of frequency division required in the receiver. But the received OFDM signal spectrum needs smaller bandwidth, since the spectrum of adjacent sub-channels could be overlapped.

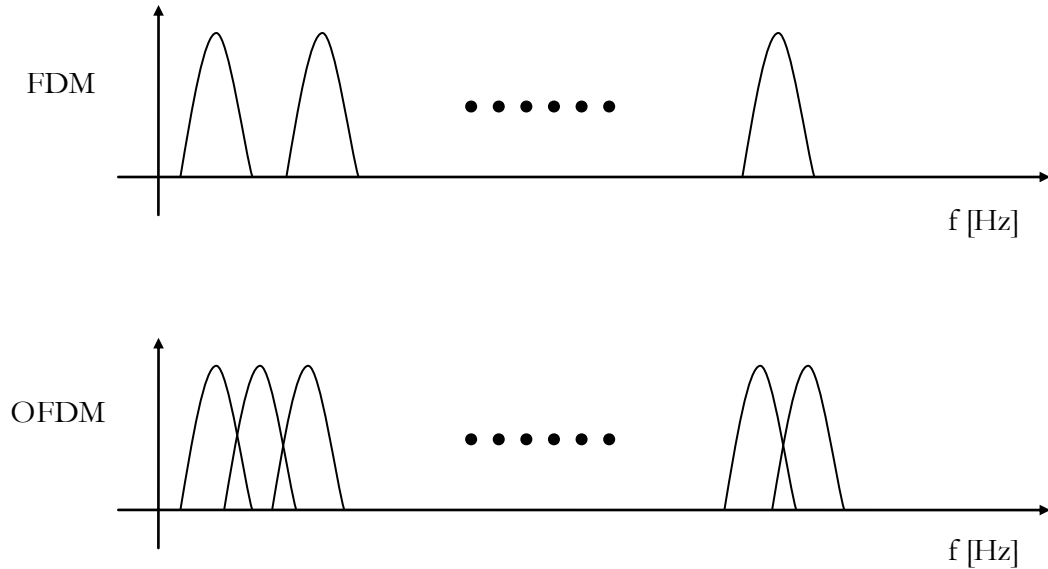


Figure 2-5 Spectra of FDM and OFDM

## 2.3 Principles of OFDM

### 2.3.1 M-ary Digital Modulation

In order to achieve efficient information transmission, M-ary digital modulation could be used to transmit data symbols. Compared with the binary digital modulation, a M-ary symbol can carry  $\log_2 M$  bits of information, whereas a binary symbol can only carry one bit of information. Commonly, M-ary digital modulation methods used in digital communication systems includes constant amplitude modulation and non-constant

amplitude modulation. A typical example of two modulation methods are M-ary phase shift keying (MPSK) and quadrature amplitude modulation (QAM) [1].

In MPSK modulation, carrier phase could be chosen by different  $M$ , then  $\theta_i = 2\pi i / M$ ,  $i = 0, 1, 2 \dots M-1$ . The function of phase after modulation is

$$\begin{aligned} s_i &= \sqrt{E_s} \cos(2\pi f_c t + 2\pi i / M) \\ &= \sqrt{E_s} \cos(2\pi i / M) \cos(2\pi f_c t) - \sqrt{E_s} \sin(2\pi i / M) \sin(2\pi f_c t) \end{aligned} \quad (2-6)$$

$\sqrt{E_s}$  means the energy per symbol.

Let me take quadrature phase shift keying (QPSK) for an example to introduce MPSK.

QPSK uses four points on the constellation diagram, equispaced around a circle. With four phases, QPSK can encode two bits per symbol, as shown in Figure 2-6, with Gray coding to minimize the bit error rate (BER) — sometimes misperceived as twice the BER of binary phase shift keying (BPSK)

The implementation of QPSK is more general than that of BPSK and also indicates the implementation of higher-order PSK. Writing the symbols in the constellation diagram in terms of the sine and cosine waves used to transmit them.

$$s_i(t) = \sqrt{\frac{2E_s}{T}} \cos(2\pi f_c t + (2n-1)\frac{\pi}{4}), \quad n = 1, 2, 3, 4 \quad (2-7)$$

This yields the four phases  $\pi/4$ ,  $3\pi/4$ ,  $5\pi/4$  and  $7\pi/4$  as needed. This results in a two-dimensional signal space with unit basis functions.

$$\phi_1(t) = \sqrt{\frac{2}{T_s}} \cos(2\pi f_c t) \quad (2-8)$$

$$\phi_2(t) = \sqrt{\frac{2}{T_s}} \sin(2\pi f_c t) \quad (2-9)$$

The first basis function is used as the in-phase component of the signal and the second as the quadrature component of the signal. Hence, the signal constellation consists of the signal-space 4 points.  $(\pm\sqrt{E_s}/2, \pm\sqrt{E_s}/2)$  The factors of 1/2 indicate that the total power is split equally between the two carriers.

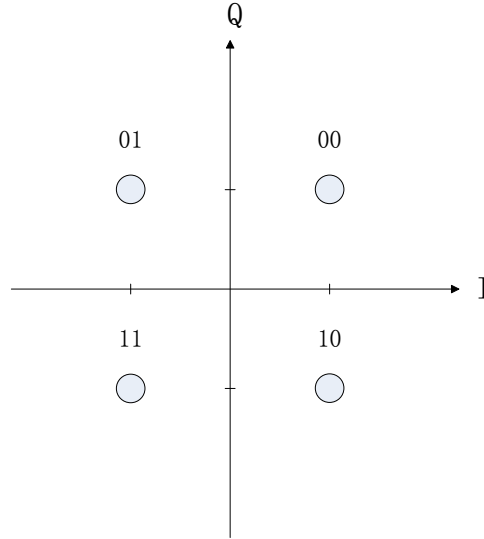


Figure 2-6 Constellation of QPSK

Because amplitude of MPSK modulation is kept constant, so we can get circular constellation map. If the phase and amplitude of signal modulated can be changed, we can get QAM method with non-constant amplitude. The signal function after QAM modulation could be written as below

$$s_i = \sqrt{E_{\min}} a_i \cos(2\pi f_c t) + \sqrt{E_{\min}} b_i \sin(2\pi f_c t) \quad (2-10)$$

$\sqrt{E_{\min}}$  means the energy of symbol with minimum amplitude.  $a_i, b_i$  ( $i=1, 2, 3 \dots M-1$ ) are a pair of independent integer numbers that could be determined by constellation.

Figure 2-7 shows the signal constellation of 16QAM.

The reason we are concerned on 16-QAM is as follows. A brief consideration reveals that 2-QAM and 4-QAM are in fact BPSK and QPSK, respectively. Also, the error-rate

performance of 8-QAM is close to that of 16-QAM (only about 0.5 dB better), but its data rate is only three-quarters of 16-QAM's.

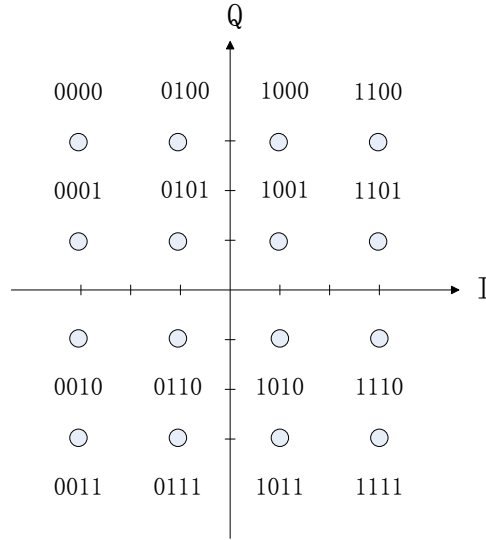


Figure 2-7 Constellation of 16QAM

### 2.3.2 OFDM Principles

One OFDM symbol includes a number of sub-carrier modulated signal synthesis, and each sub-carrier can be modulated by PSK or QAM. We define  $N$  as number of sub-carrier and  $T$  as the symbol duration. As data symbol assigned to each sub-carrier, let  $f_0$  as the carrier frequency of  $0^{\text{th}}$  sub-carrier. Also define  $d_i$  ( $i = 0, 1, \dots, N-1$ ) is the data symbol on each sub-channel, and  $\text{rect}(t) = 1, |t| \leq T/2$ , respectively.

From  $t = t_s$ , where  $t_s$  is any point of time, the symbol of OFDM could be written as

$$s(t) = \text{Re} \left\{ \sum_{i=0}^{N-1} d_i \text{rect}\left(t - t_s - \frac{T}{2}\right) \exp \left[ j2\pi \left(f + \frac{i}{T}\right)(t - t_s) \right] \right\}, \quad t_s \leq t \leq t_s + T \quad (2-11)$$

$$s(t) = 0, \quad t < t_s \quad \text{or} \quad t > t_s + T$$

However, in most of literature, the output signal of OFDM signal always be described by the equivalent complex baseband form as in Equation 2-12.

$$s(t) = \sum_{i=0}^{N-1} d_i \text{rect}(t - t_s - \frac{T}{2}) \exp \left[ j 2\pi \frac{i}{T} (t - t_s) \right], \quad t_s \leq t \leq t_s + T \quad (2-12)$$

$$s(t) = 0, \quad t < t_s \quad \text{or} \quad t > t_s + T$$

Where the real and imaginary parts correspond to the in-phase and quadrature components of OFDM symbol respectively [7]. The basic model of OFDM is shown in Figure 2-8.

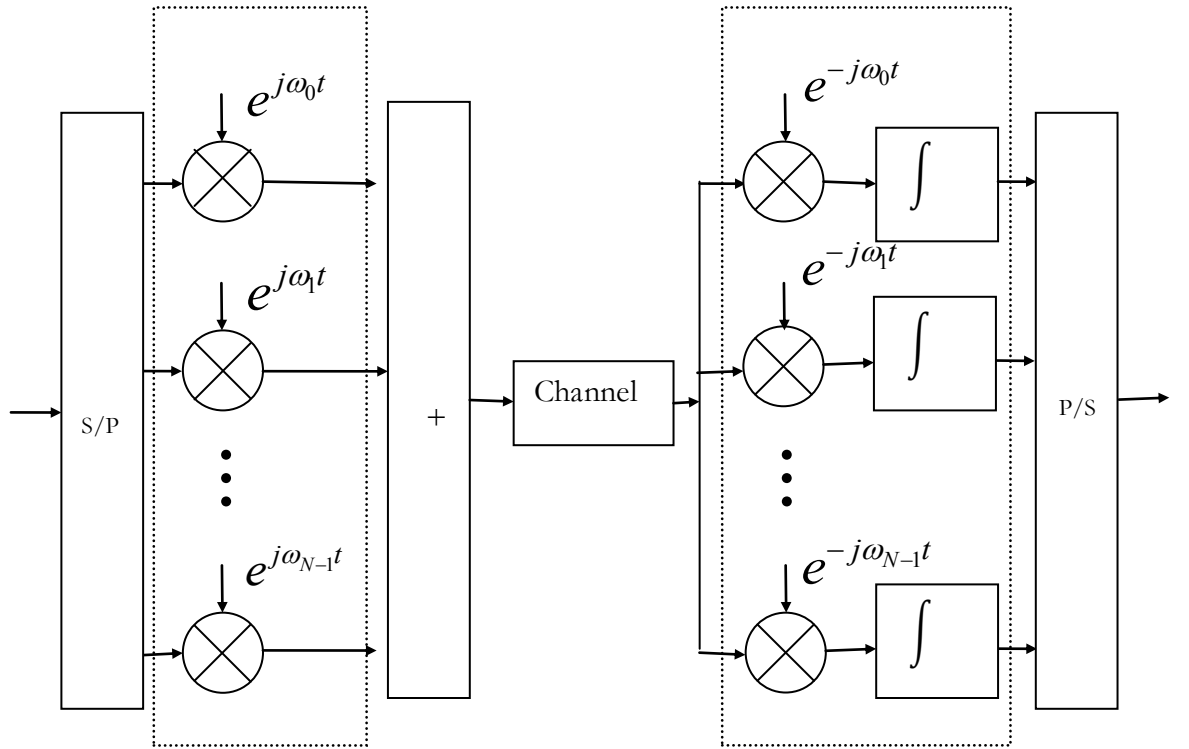


Figure 2-8 The basic model of OFDM

An example of an OFDM symbol with 4 carriers is shown in Figure 2-9.

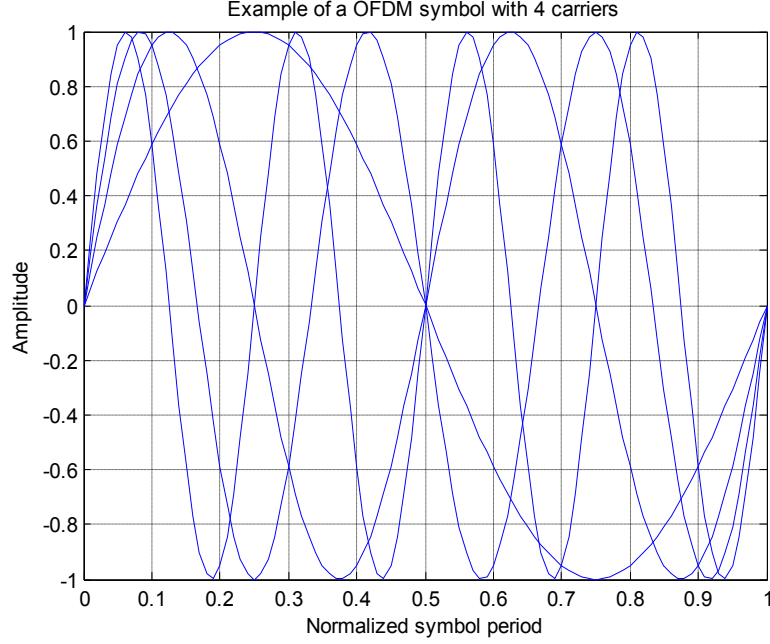


Figure 2-9 Example of an OFDM symbol with 4 carriers

All the subcarriers have the same amplitude and phase, but in practical applications, because of the modulation of data symbols, each sub-carrier amplitude and phase may be different. From Figure 2-9, we can find that each sub-carrier of OFDM symbols contains an integer multiple of a cycle, but the difference between each adjacent sub-carrier is one cycle [8]. This feature can be used to explain the orthogonality between subcarriers.

$$\frac{1}{T} \int_0^T \exp(j\omega_n t) * \exp(-j\omega_m t) dt = \begin{cases} 1 & m = n \\ 0 & m \neq n \end{cases} \quad (2-13)$$

For example, let us demodulate the  $j^{th}$  sub-carrier in (2-12), and integrate it in the length of time  $T$ . We could get,

$$\begin{aligned} \hat{d}_j &= \frac{1}{T} \int_{t_s}^{t_s+T} \exp(-j2\pi \frac{j}{T} (t-t_s)) * \sum_{i=0}^{N-1} d_i \exp(j2\pi \frac{i}{T} (t-t_s)) dt \\ &= \frac{1}{T} \sum_{i=0}^{N-1} d_i \int_{t_s}^{t_s+T} \exp(j2\pi \frac{i-j}{T} (t-t_s)) dt \\ &= d_j \end{aligned} \quad (2-14)$$



According to the above formula, we could get the expected symbols  $d_j$  by demodulate the  $j^{th}$  sub-carrier. For other carriers, because of the frequency difference  $(i-j)/T$ , in the integration interval, it causes a integer multiple of cycle, so the integral is zero.

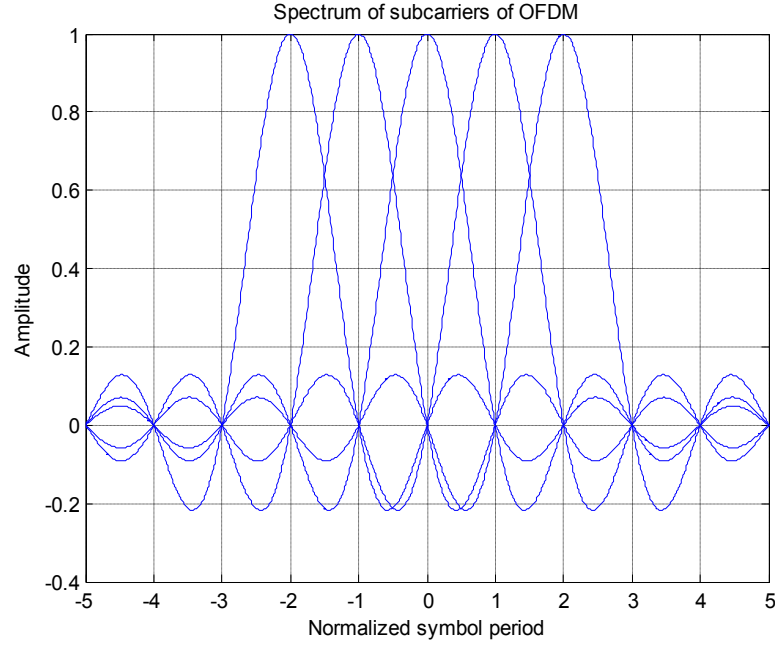


Figure 2-10 Spectrum of sub-carriers of OFDM

Figure 2-10 gives each individual sub-carrier within the coverage through the rectangular wave forming the symbol of the sinc function by the spectrum. At the maximum value of each sub-carrier frequency, the value of all other sub-channel spectrum is zero, exactly. In demodulation process of symbols of the OFDM, we need to calculate these points corresponding to the maximum value for each sub-carrier frequency [8]. So we can extract each sub-channel spectrum symbol from many overlapping sub-channels with no interference by other sub-channels.

### 2.3.3 Fast Fourier Transform

From the Figure 2-3, we could find that many sine wave generators, filters, modulators and demodulators are needed when  $N$  is large. And it costs too much for system.

To solve the complexity and cost problems of OFDM system, we often use inverse fast Fourier transform/fast Fourier transform (IFFT/ FFT) to implement the system modulation and demodulation.

After doing discretization for  $s(t)$  in  $t = mT_s$ , we could get

$$s(mT_s) = \sum [a(n) \cos 2\pi f_n(mT_s) + b(n) \sin 2\pi f_n(mT_s)] \quad 0 \leq m < N \quad (2-15)$$

When  $f_n = f_c + n\Delta f$ ,

$$s(m) = \text{Re} \left\{ \sum_{n=0}^{N-1} [a(n) + jb(n)] \exp\left(\frac{j2\pi nm}{N}\right) \right\} \quad m = 0, 1, \dots, N-1 \quad (2-16)$$

The function in the brackets is the form of discrete Fourier transform (DFT) of  $d(n) = a(n) + jb(n) \quad n = 0, 1, \dots, N$ . So, the part in the dashed box of Figure 2-8 could be achieved by IDFT/DFT.

But, the disadvantage of DFT is the complex operation. However, as the development of FFT, the IFFT/FFT operation can be easily used in OFDM system to achieve modulation and demodulation, and it makes the practical application of OFDM available [18].

### 2.3.4 Cyclic Prefix in OFDM System

When the OFDM signal is used for transmission over a wide frequency, unless there is a large number of sub-carriers, sub-carrier signal is difficult to make a smaller bandwidth

than the coherence bandwidth of channel. So, the residual ISI is too large, damaging orthogonality between subcarriers, causing demodulation errors, and result in bit error rate.

In order to eliminate ISI as much as possible, we could add protection, guard interval (GI), before the information symbol, and make the time length of GI larger than the estimated delay spread of channel. However, if GI is empty, the orthogonality between sub-carriers is no longer available because of ICI caused by multipath. To eliminate the ISI and the ICI, cyclic of OFDM symbol extends into GI. Therefore, cyclic prefix (CP) is usually added in OFDM symbol, as shown in Figure 2-11, and the protection interval is guaranteed to be wider than the channel multipath delay spread, so that ISI and ICI can be eliminated.

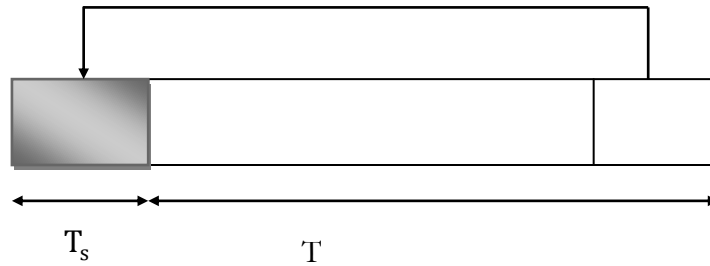


Figure 2-11 Cyclic Prefix (CP)

Where  $T$  is the length of one OFDM symbol and  $T_s$  is the length of CP in time domain respectively in Figure 2-11. The shadowed part in Figure 2-11 is the cyclic prefix. It copies the rear part of OFDM symbol and puts it to the front of the symbol, so the period will increase from  $T$  to  $T + T_s$  and  $T_s$  is the CP. The length of CP must be larger than the maximum delay spread of channel, which makes the multipath copy of the front symbol to appear in the range of cycle expansion of after symbol, thus eliminates the ISI between front and after symbols. Another aspect, the addition of CP cycles to the OFDM signal

symbol in the integration range to must be repeated. So different sub-carriers of the same OFDM symbol are still able to be maintained orthogonal, which also prevents ICI.

### **2.3.5 Equalization**

Equalizer is always used in both time and frequency domains in traditional communication system. In the time domain, for traditional FDM system, equalization is indispensable. Because equalizer is used to balance the channel characteristics. In the receiver, equalizer produces the opposite characteristics of channel to offset ISI by time varying multi-path channel. But equalization is not a satisfactory method for OFDM system. The reason is that CP is used to avoid equalization. Although in highly scattering channel, the memory length of channel is very long, and the length of CP is needed for longer in order to eliminate ISI. The performance of system, with small number of sub-carriers especially, will be worse [9]. In that special case, equalization is used to shorten the length of CP by increasing the complexity of the system for the improvement of the system bandwidth efficiency. However, equalizer of time domain is not considered in OFDM system normally.

But in the frequency domain, equalizer is an important tool used for reducing the ICI in the part of channel estimation. Because of the influence of residual frequency offset and Doppler frequency shift effects, especially in rapidly changing channel, the orthogonality between subcarriers loses, and will introduce significant ICI. At this point, generally, equalizer in frequency domain is set to eliminate the interference. In this thesis, zero forcing (ZF) equalizer is used in channel estimation. ZF equalizer is a very simple equalizer as the criterion to minimize the peak distortion. Figure 2-12 shows the diagram of equalizer.

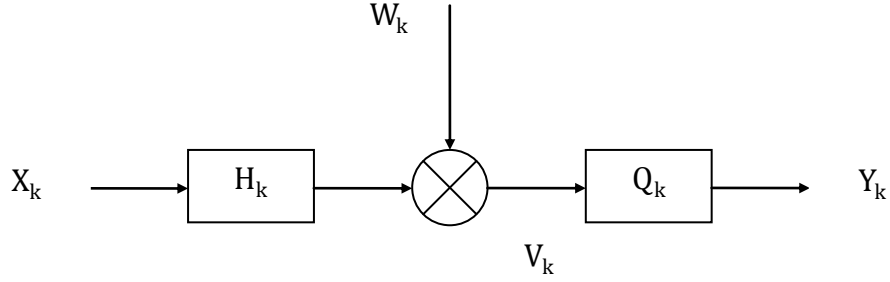


Figure 2-12 Diagram of ZF equalizer

From Figure 2-12,

$$V_k = X_k H_k + W_k \quad (2-17)$$

$$Y_k = V_k Q_k \quad (2-18)$$

Where,  $X_k$  is input signal,  $H_k$  is channel impulse response,  $W_k$  is AWGN,  $Q_k$  is equalizer, respectively.

From Equations 2-17 and 2-18, we could get

$$Y_k = (X_k H_k + W_k) Q_k = X_k H_k Q_k + W_k Q_k \quad (2-19)$$

$$Y_k - X_k = X_k H_k Q_k + W_k Q_k - X_k \quad (2-20)$$

In ZF equalizer, let  $E\{Y_k - X_k\} = 0$ , because AWGN is irrelevant, then

$$E\{X_k\} H_k Q_k - E\{X_k\} = 0 \quad (2-21)$$

In OFDM system, because ISI is eliminated, ZF equalizer could gain satisfactory performance. In the part of channel estimation, ZF equalization is achieved by received signal divided by channel impulse response easily, from Equation 2-21, as below

$$Q_k = \frac{1}{H_k} \quad (2-22)$$

### 2.3.6 OFDM System Components

Figure 2-12 shows a typical block diagram of OFDM system.

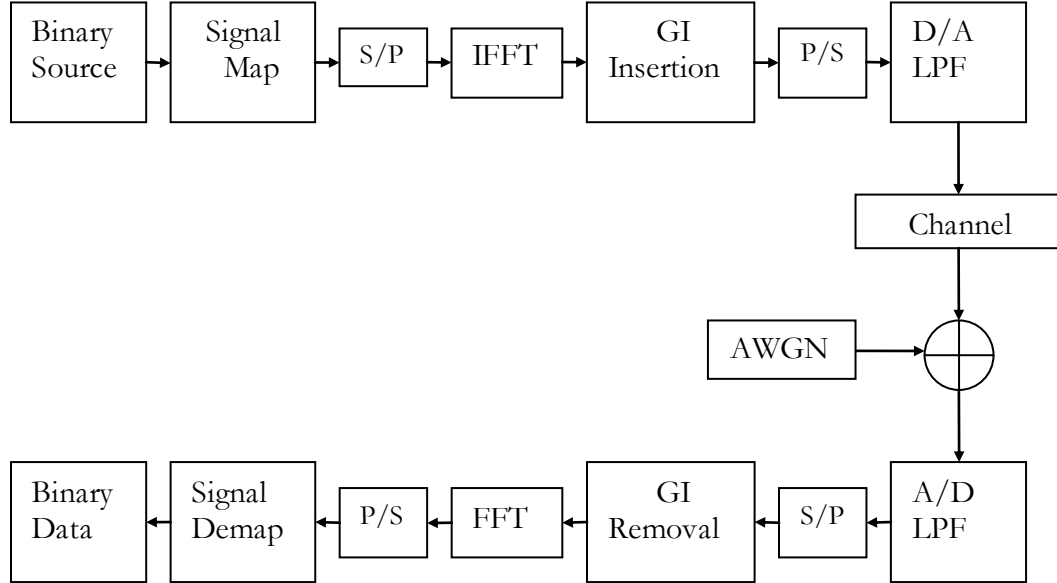


Figure 2-13 Block diagram of OFDM system

Because of the characteristic of OFDM system, only the modulation and demodulation procedure in frequency domain are analyzed in the thesis.

In order to get the distinguishable results, the parameters are set as 8 carriers, 128 FFT bin points, 1 bit/symbol.

Firstly, we combine the binary data, 0 and 1 sequence, into symbols according to the number of bits/symbol. Then, the serial symbol stream is converted into parallel segments by the number of carriers. After that, symbol sequences are formed for next step. The differential coding is used for each carrier symbol sequence in order to convert them into complex phase representations. Therefore, each carrier sequence is assigned to the appropriate IFFT bin.

The OFDM signal after modulation is a group of impulse, shown as delta curved shape, functions. And the phases of them are determined by symbol used in modulation. We could find that the frequency is separated by  $N$  as IFFT bin size.  $N$  is 128 in this example. One OFDM carrier in frequency domain is shown as below

$$S(k) = e^{j\theta_m} \delta(k - m - \frac{N}{2}) + e^{-j\theta_m} \delta(k + m - \frac{N}{2}) \quad (2-23)$$

$N$  is the IFFT bin size,  $m$  is OFDM carrier and  $k$  is frequency from 0 to  $N-1$ , respectively.

From the parameters, Figure 2-14 gives an example of magnitude of OFDM carrier frequency prior to IFFT operation. The magnitude of each carrier is 1.

In the binary case, the value of symbol is 1 or 0, so the phase for each carrier is 0 or 180 degrees. In the example, the 3<sup>rd</sup>, 6<sup>th</sup> and 8<sup>th</sup> bits are 1, as 180 degrees, and the other bits are 0, as 0 degree. Figure 2-15 shows the OFDM carrier phase before IFFT.

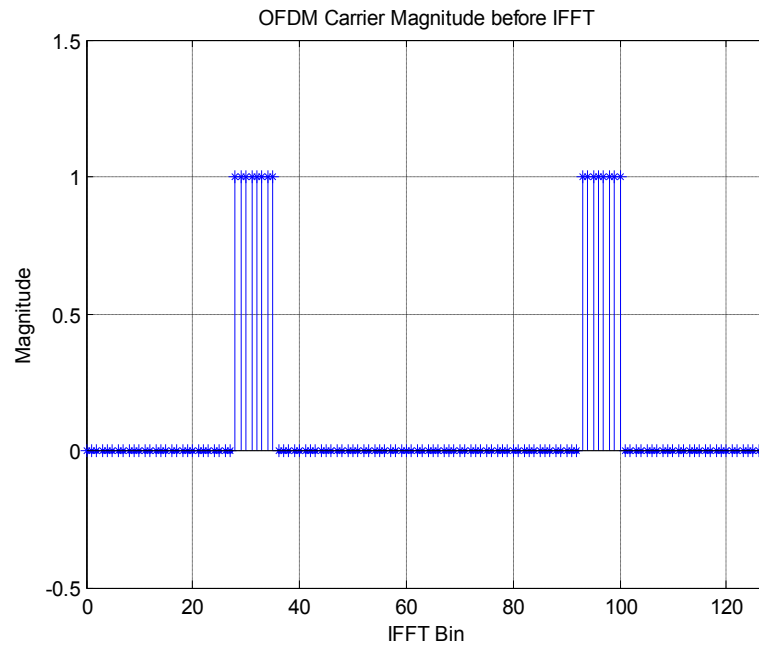


Figure 2-14 OFDM carrier magnitude before IFFT

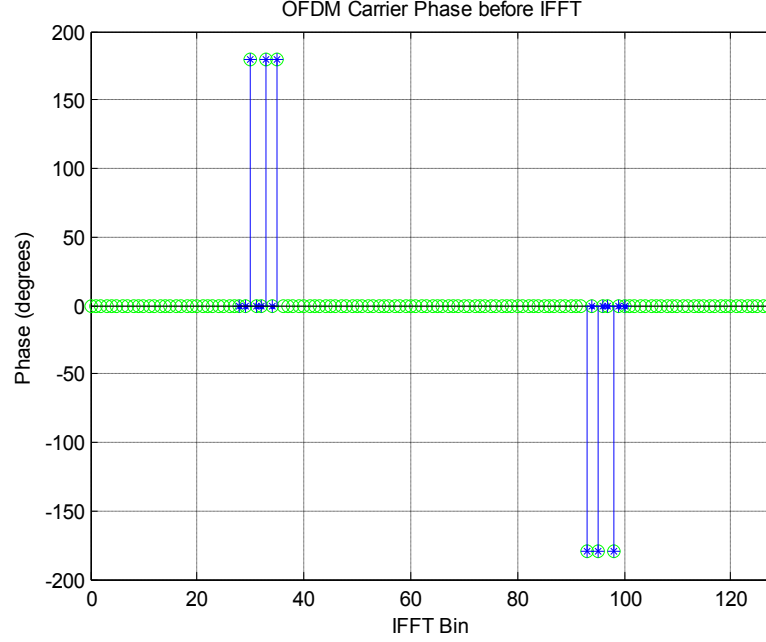


Figure 2-15 OFDM carrier phase before IFFT

After that, IFFT is applied to generate one symbol period in the time domain. In Matlab software, the “ifft” function is used to transfer  $s(k)$  to  $s(n)$ .

$$s(n) = \sum_{m=C_1}^{C_n} \sum_{n=0}^{N-1} \cos\left(\frac{2\pi mn}{N} + \theta_m\right) \quad (2-24)$$

$N$  is IFFT bin size,  $n$  is time sample,  $m$  is OFDM carrier,  $\theta_m$  is phase modulation,  $C_1$  and  $C_n$  are first and last carrier, respectively.

Figure 2-16 shows the IFFT result of one symbol period. It is clear to see the graph is not smooth. But the varying amplitude needs to be kept unchanged. If the amplitude is modified, the signal after the FFT will no longer result in the expected frequency characteristics. On the other hand, this figure shows a drawback of OFDM system, which requires linear amplification. The large amplitude peaks directly results in the high peak-to-average power ratio (PAPR). It means that the amplifier has to have a large enough dynamic range to avoid loss of the peaks.



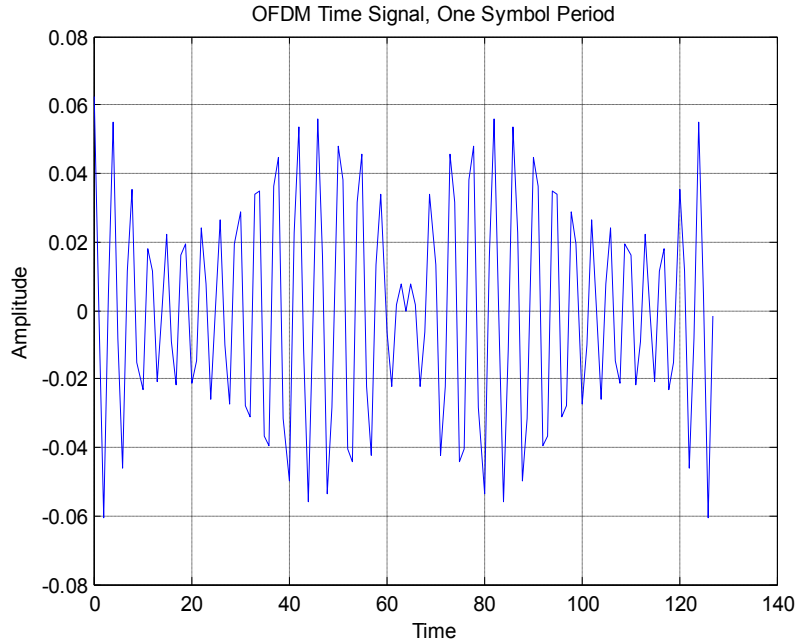


Figure 2-16 OFDM signal in time domain

In the receiver part, the OFDM signal is converted by passing A/D. The demodulation procedure is almost a reverse direction of modulation, and also applied in frequency domain. And FFT plays a role for transferring the OFDM signal from time domain to frequency domain. We could use “fft” function in Matlab to convert. Figure 2-18 and 2-19 show the magnitude and phase of OFDM signal after the FFT. The channel is defined as simple AWGN and signal-to-noise ratio (SNR) is 10 dB.

Compared with Figures 2-17 and 2-18, the OFDM signal is very easy to recover. In Figure 2-18, we could note that the unused frequency bins have varying phase values. But it does not matter because they are not decoded.

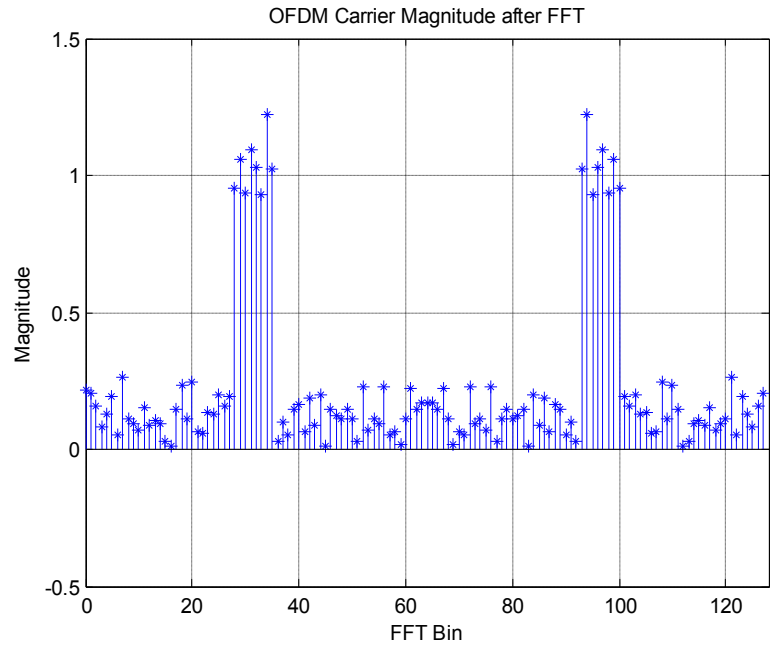


Figure 2-17 OFDM carrier magnitude after FFT

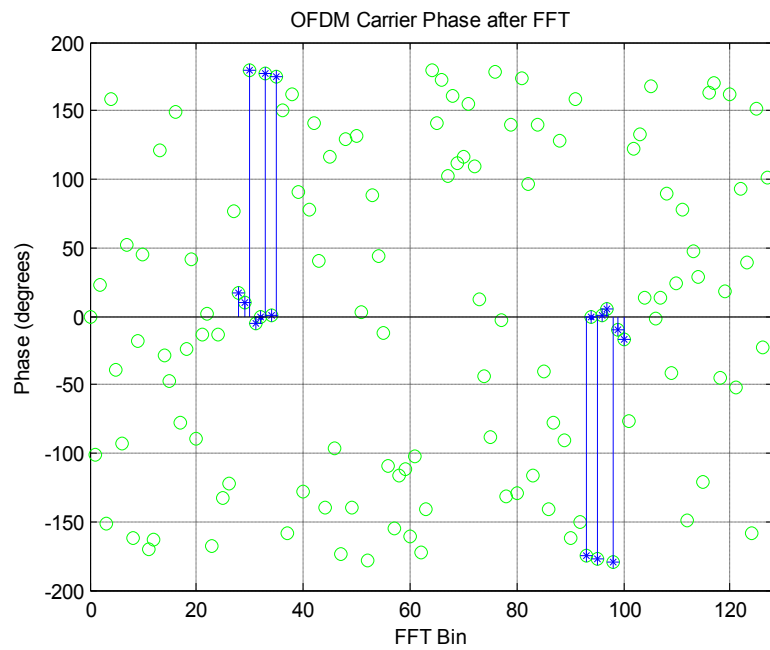


Figure 2-18 OFDM carrier phase after FFT

## **2.4 Advantages and Disadvantages of OFDM**

### **2.4.1 Advantages of OFDM**

The high-speed data streams are converted from serial to parallel, so that the continued length of data symbols on each sub-carrier increases, which can reduce the ISI caused by the radio channel time dispersion effectively. By reducing the balanced complexity of the receiver, equalizer may not be needed, and the adverse effects of ISI can be eliminated by using of cyclic prefix insertion.

In the conventional FDM method, the frequency band is divided into several disjoint sub-frequency band to transmit parallel data streams, and separated to the various sub-channels at the receiver with a set of filters. Compared with this, the orthogonality of sub-carriers of OFDM system allows overlapping spectrum of sub-channels, hence OFDM system can maximize the using of spectrum resources.

In each sub-channel, IDFT and DFT can be used to achieve orthogonal modulation and demodulation. When  $N$  is a large number, we can use FFT to achieve. IFFT and FFT are very easy to implement using DSP technology.

Wireless data services are generally non-symmetrical. In terms of the requirement of data services of user, and consideration of the mobile communication system, non-symmetrical high-speed data transmission is expected to support by physical layer. The OFDM system can be easy to achieve the different uplink and downlink transmission rates by using a different number of sub-channels.

OFDM system can be easily combined with a variety of other access methods to form multi-carrier code division multiple access (MC-CDMA), frequency hopping OFDM. It allows multiple users to transmit information by OFDM technology simultaneously.

Because the narrow-band interference can only affect a small part of the sub-carriers, the OFDM system can resist this narrow-band interference to some extent.

### **2.4.2 Disadvantages of OFDM**

It is vulnerable to the impact of frequency deviation. Because of the time variability of wireless channel, wireless signal frequency offset always occurs during transmission, such as Doppler frequency shift. Then the orthogonality between subcarriers of OFDM system will be destroyed, resulting in ICI between sub-channels. Hence sensitivity to the frequency deviation is the main drawback of OFDM system.

There exists a problem of higher PAPR. Compared with the single-carrier system, because the output of multi-carrier modulation system is a superposition of multiple sub-channel signals, so if the phases of some signals are same, instantaneous signal power of the superposition of signals received will be far greater than the average power, which lead to larger PAPR. It needs higher level linear requirement for amplifier of transmitter. If the dynamic range of amplifier cannot meet the signal changes, it will bring the signal distortion, which could change the spectrum of signal. So, the orthogonality between different sub-channels will be destroyed, making the system performance worse.

## Chapter 3

### Channel Characteristics of OFDM System

Performance of wireless mobile communication system is mainly constrained by the wireless channel, which consists of base station antennas and propagation paths between the user antennas. Communication between the transmitter and receiver path can be more complex, because of variety of complex topography, such as buildings, mountains, forests, etc.. Compared with the predictable channel like cable, radio channel is very random, which results in distortion of amplitude, phase and frequency of received signal. So, it is necessary to have an overall understanding about wireless communication channel.

#### 3.1 Wireless Channel

In the wireless mobile communication systems, electromagnetic wave propagation can be divided into direct wave, ground reflected wave and scattering, reflection and diffraction of the radiation energy in the dissemination of path caused by a variety of obstacles in the path. In the land mobile system, mobile station is built in the city among the buildings or in areas with complex terrain, and the antenna will receive the signals coming from multiple paths. And because of the movement of the mobile station itself, it makes the channel between mobile base station changeful and difficult to control. So, signals are attenuated through the wireless channel [10]. The received power is

$$P(d) = |d|^{-n} S(d) R(d) \quad (3-1)$$

Where  $|d|$  means the distance between the mobile station and base station. From Equation 3-1, effect of wireless signal from channel could be divided into three categories as defined by  $|d|^{-n}$ ,  $S(d)$  and  $R(d)$ .

$|d|^{-n}$  means propagation loss of electromagnetic waves in free space, also known as large-scale fading, where  $n$  is always set from 3 to 4 [3]. It is the function of distance between mobile station and base station. The strength of received signal within the distance between the transmitter and receiver changes in the large-scale space (a few hundred feet or a few feet).

$S(d)$  is shadowed attenuation, which means the fading caused by terrain, buildings and other obstacles. It is mainly about the intermediate scale (hundreds of wavelengths).

$R(d)$  is the multi-path fading which is the propagation phenomenon that results in radio signals reaching the receiving antenna by two or more paths. Causes of multipath include atmospheric ducting, ionospheric reflection and refraction, and reflection from water bodies and terrestrial objects such as mountains and buildings. It is about the small scale (several or tens of wavelengths).

In addition, because of the movement of the mobile station and the other objects in wireless channel environment, the instantaneous change of space is converted to the instantaneous change of signal, when the mobile station moves through the multi-path area. It presents the time-varying of wireless channel, such as Doppler shift.

Large scale fading and shadow effects covers wireless local area primarily, and rational design can eliminate them. Small-scale fading is essential for the choice of transmission

technology and design of digital receiver. Therefore, this section is devoted only for multipath fading of small-scale fading wireless channel, along with time varying characteristics.

### **3.1.1 Multipath Propagation and Time-varying**

#### **3.1.1.1 Multipath Propagation**

The main characteristic of wireless mobile channel is multipath propagation. The signal transmitting via the mobile channel from transmit antenna to mobile station antenna is not from a single path, but a number of different paths of many reflected waves. Because the path of electromagnetic wave, distance of transmission, emission coefficient of the launch point are different, so time and phase of reflected waves received from each path are different. Many signals with different phase are synthesized in receiver, which resulting in strengthening with signals of same phase and weaken if not. Thus, the received signal will change in time domain, frequency domain and spatial domain. And the signal amplitude produces drastic changes, which is fading known as multipath propagation.

Multipath propagation caused by multipath mobile channel can be described from both time and space. In the point of view of space, within the direction of movement of mobile station, the received signal amplitude changes with distance. From time domain, because of different length of the various paths, the arriving time of signal is different. In this way, after base station sends a pulse signal, the signal received not only contains the pulse, but also includes all delayed version of signals. Extension of time could be measured from the signal received first to the last one.

In general, the main consideration in the simulation of mobile systems is the amplitude change of the received signal caused by multipath. But the main consideration in digital

mobile systems is delay spread. Because the delay spread will cause the ISI, which seriously affect the quality of digital signal transmission.

### 3.1.1.2 Time Varying

An important feature of wireless channel is time-varying, which is the transfer function of channel changes within time. So, the transmitter sends the same signal at different time, but the signals received by receiver are not the same. Doppler shift is one concrete manifestation of the time varying in the mobile communication system.

When the mobile station is doing communication in motion, the frequency of the received signal will change. In multipath conditions, each multipath wave has a frequency shift, called Doppler spread. The shift of the mobile received signal frequency caused by the movement is called Doppler frequency shift, and it is proportional to the speed of mobile users.

$$f_d = \frac{v}{\lambda} \cos \theta_i = f_c \frac{v}{c} \cos \theta_i = f_m \cos \theta_i \quad (3-2)$$

Where  $v$  is speed of mobile station,  $\lambda$  is radio wavelength,  $f_c$  is carrier frequency of transmitter,  $c$  is speed of light,  $\theta_i$  is angle between radio and mobile station,  $f_m$  is the maximum Doppler frequency shift, respectively.

### 3.1.2 Parameters of Mobile Multipath Channel

Mobile multipath channel parameters are delay spread, coherence bandwidth, Doppler spread and coherence time. The first two are used to describe multipath mobile channel dispersion in time, and the latter two are used to describe multipath mobile channel dispersion in frequency.

- (1) Parameters of time dispersion and frequency selectivity



Time dispersion and frequency selectivity are effects generated by superposition of the different multipath delay signals, depends on the geometric relationship between the transmitter, receiver and the surrounding physical environment. These two effects occur simultaneously, but in different forms. Time dispersion is reflected in the time domain, and frequency selectivity reflected in the frequency domain. Time dispersion is a signal to the transmitter unfolded along the time axis, so that the duration of the received signal is longer than the signal sent. The frequency selectivity refers to the transmitted signal is filtered, the signal components of different frequencies in the range with different fading. It means when components are very close in frequency, and their decline is also very close, but when far apart in frequency, and their decline varies widely.

In multipath propagation conditions, the received signal will produce delay spread. Parameters used to describe time extended are the average additional delay  $\bar{\tau}$ , root mean square (RMS) delay spread  $\sigma_\tau$  and maximal delay spread. The first two parameters are related to power delay profile  $P(\tau)$ . Power delay profile (PDP) is a function of additional delay based on fixed time delay  $\tau_0$  by getting average of local instantaneous power delay.

Average additional delay is the first order matrix of PDP and written as

$$\bar{\tau} = \frac{\sum_k a_k^2 \tau_k}{\sum_k a_k^2} = \frac{\sum_k P(\tau_k) \tau_k}{\sum_k P(\tau_k)} \quad (3-3)$$

Where  $a_k$  is attenuation factor of  $k^{\text{th}}$  path,  $P(\tau_k)$  is the relative power of multi-path fading at  $\tau_k$ , respectively.

RMS delay spread is square root of second order matrix of PDP.

$$\sigma_\tau = \sqrt{E(\tau^2) - (\bar{\tau})^2} \quad (3-4)$$

$$E(\tau^2) = \frac{\sum_k a_k^2 \tau_k^2}{\sum_k a_k^2} = \frac{\sum_k P(\tau_k) \tau_k^2}{\sum_k P(\tau_k)} \quad (3-5)$$

Coherence bandwidth  $B_c$  is used to describe the parameters of frequency selective fading and it envelopes the signal bandwidth with correlation for a particular value. That is, when the interval of frequency of the two components is less than the coherence bandwidth, they have a strong correlation between the amplitude. On the other hand, when the interval of two components is greater than the coherence bandwidth, their amplitude correlation is very small. Delay spread is phenomena caused by reflection and scattering in the propagation path. And the coherence bandwidth is a determined value from RMS delay spread and could be written as below

$$B_c \approx \frac{1}{\tau_{\max}} \quad (3-6)$$

## (2) Parameters of frequency dispersion and time selectivity

Due to the movement of the mobile station, the phenomenon of Doppler shift occurs, as frequency dispersion, and makes the channel time varying. Doppler spread and coherence time are two important parameters to describe the mobile channel frequency dispersion and time-varying characteristics, and there is the inverse relationship between them.

Doppler spread is measured by the spectrum broadening, which is determined by speed of movement of the mobile station and the environment object. It is a frequency range, defined as the spectrum of the received signal spectrum is not equal to 0, and depending on the  $f_d$ . When a single frequency sine wave signal with frequency of  $f_c$  is sent by transmitter, its spectrum is a spectral line of carrier frequency. But, because of Doppler

frequency shift, the spectral line with the frequency of  $f_c$  will be extended to limited spectrum bandwidth from  $f_c - f_d$  to  $f_c + f_d$ .

Coherence time  $T_c$  is measured by time change rate of channel. Larger coherence time yields smaller Doppler frequency shift, and slower channel changes. On the other hand, smaller coherence time yields larger Doppler frequency shift, and faster channel changes.

### 3.1.3 Fading Channels

When signal transmitted through the mobile wireless channel, signal fading characteristics are determined by the transmitted signal and channel characteristics. Time dispersion and frequency selective fading are caused by multipath time delay spread, and frequency dispersion and time selective fading are caused by Doppler frequency spread. According to the frequency selectivity of channel, the channel can be divided into flat fading channels and frequency selective fading channel. And according to the time selectivity of channel, the channel can be divided into the fast fading channel and slow fading channel [10].

#### 3.1.3.1 Flat Versus Frequency Selective Fading

Because the carrier frequency of a signal is varied, the change of amplitude will vary. The coherence bandwidth measures the separation in frequency after two signals will experience unrelated fading. Therefore, according to frequency selectivity of channel, the channel fading is divided into flat fading channel and frequency selective fading channel.

In flat fading, when signal is transmitted slowly, the coherence bandwidth of the channel is larger than the bandwidth of the signal, and the period of the signal is larger than delay spread. From the frequency domain, components with different frequency have same

fading. From the time domain, signal received passes only one distinguished path fading, so the ISI could be negligible and the signal wave fading is not distorted. Flat fading is name as frequency non-selective fading and meets as below

$$B_s \ll B_c \quad (3-7)$$

$$T_s \gg \sigma_\tau \quad (3-8)$$

Where  $B_s$  is the signal bandwidth,  $B_c$  is the coherence bandwidth,  $T_s$  is the period of signal, and  $\sigma_\tau$  is the delay spread of channel, respectively.

In frequency selective fading, when signal is transmitted fast, the coherence bandwidth of the channel is smaller than the bandwidth of the signal, and the period of the signal is smaller than delay spread. From the frequency domain, components with different frequency have different fading. From the time domain, signal received passes many distinguished path fading, so serious ISI occurs.

$$B_s > B_c \quad (3-9)$$

$$T_s < \sigma_\tau \quad (3-10)$$

Coherence bandwidth is a feature of wireless channels. The signals through the wireless channel affected by frequency selective fading or flat fading depend on the bandwidth of the signal itself.

In this thesis, only flat fading channel is used in the simulation.

### 3.1.3.2 Slow Versus Fast Fading

The terms slow and fast fading refer to the rate of the magnitude and phase change enforced by the channel on the signal changes. The coherence time is a measure of the minimum time required for the magnitude change of the channel to become uncorrelated from its previous value.

Slow fading occurs when the coherence time is larger than the delay constraint of the channel. In this condition, the amplitude and phase change imposed by the channel can be considered roughly constant over the period of use. Slow fading could be caused by some events, for example, shadowing as a large obstruction such as hills, buildings. In slow fading, the parameters of channel could be considered to be steady in one or more periods of signal symbols, hence

$$T_s \ll T_c \quad (3-11)$$

$$B_s \gg B_d \quad (3-12)$$

Where  $B_d$  is Doppler spread.

Fast fading occurs when the coherence time is smaller than the delay constraint of the channel. In this condition, the amplitude and phase change imposed by the channel varies considerably over the period of use, hence

$$T_s > T_c \quad (3-13)$$

$$B_s < B_d \quad (3-14)$$

In short, the mobile communication system channel characteristics are very complex, including the waves of multipath propagation, delay spread, fading characteristics and the Doppler effects. Solving the negative impact of these features is the key technology of mobile communication system.

### 3.1.4 Fading Channel Properties

Because of the multipath channel effects and Doppler effects, after signal sent from the transmitter, the envelop of received signal shows random properties. There are two types of fading in multipath channels, Rayleigh fading and Rician fading. Rayleigh fading is viewed as a reasonable model for tropospheric and ionospheric signal propagation as well

as the effect of heavily built-up urban environments on radio signals. Rayleigh fading is most applicable when there is no dominant propagation path along a line of sight between the transmitter and receiver. If there is a dominant line of sight, Rician fading may be more applicable.

Rician fading is a stochastic model for radio propagation anomaly caused by partial cancellation of a radio signal by itself — the signal arrives at the receiver by several different paths (hence exhibiting multipath interference), and at least one of the paths is changing (lengthening or shortening). Rician fading occurs when one of the paths, typically a line of sight signal, is much stronger than the others.

But, considering a typical urban environment, a straight line as channel path between transmitter and receiver does not exist. Therefore, we can consider the wireless channels in the spread of a single antenna system obey the Rayleigh distribution envelope.

In the mobile communication channel, there are so many refractions and reflections between the base station and mobile station, so the channel impulse response could be expressed as

$$h(\tau) = \sum_{k=1}^L u_k \delta(\tau - \tau_k) e^{j\phi_k} \quad (3-15)$$

Where  $L$  is the number of multipath,  $u_k$  is the signal magnitude of  $k^{th}$  path,  $\tau_k$  is the delay of  $k^{th}$  path compared with the first path,  $\phi_k$  is the signal phase of  $k^{th}$  path, respectively.

In the condition of many communication paths, we could assume there is no stronger straight path in channel, and the channel impulse response could be considered as a complex Gaussian process and shown as

$$p(u) = \frac{u}{\sigma^2} e^{\frac{-u^2}{2\sigma^2}}, \quad 0 \leq u < \infty \quad (3-16)$$

And the average of signal enveloped  $\bar{u}$ , RMS  $u_{rms}$ , variance  $\sigma_r^2$  are

$$\bar{u} = E(u) = \int_0^\infty u p(u) du = \sqrt{\frac{\pi}{2}} \sigma \approx 1.253 \sigma \quad (3-17)$$

$$u_{rms} = \sqrt{E(u^2)} = \sqrt{\int_0^\infty u^2 p(u) du} = \sqrt{2} \sigma \approx 1.414 \sigma \quad (3-18)$$

$$\sigma_r^2 = E(u^2) - E^2(u) = 0.429 \sigma^2 \quad (3-19)$$

The signal is defined as fading when  $u(t)$  is smaller than  $u_{rms}$ .

And the probability density function of phase  $\varphi(t)$  is uniform as is shown below,

$$p(\varphi) = \frac{1}{2\pi}, \quad 0 \leq \varphi < 2\pi \quad (3-20)$$

## 3.2 OFDM Channel Model

All channel models mentioned in this section are used in the simulation.

### 3.2.1 AWGN Channel

AWGN channel is the simplest channel and the only parameter is the SNR (dB).

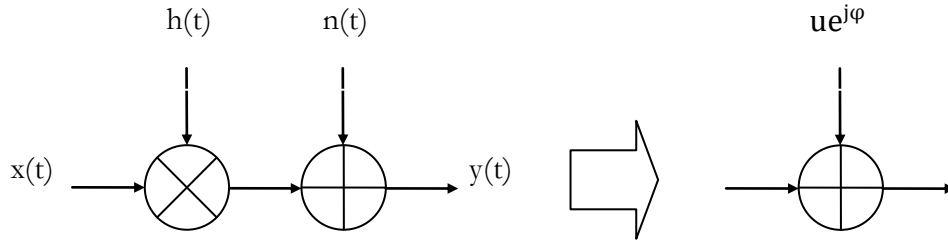


Figure 3-1 AWGN channel model

### 3.2.2 Rayleigh Multipath Channel

The channel impulse response of Rayleigh multipath channel could be expressed as

$$h(t, \tau) = \sum_{K=0}^{L-1} u_k e^{j\phi_k} \delta(t - \tau_k) \quad (3-21)$$

$L$  is the number of multipath,  $\tau_k$  is delay of the  $k^{th}$  path,  $u_k e^{j\phi_k}$  is gain coefficient, respectively. The channel model is shown as Figure 3-2.

### 3.2.3 Doppler Spread Channel

Doppler spread channel could be shown as below

$$h(t) = u e^{j\phi} e^{j2\pi f_d t} \quad (3-22)$$

Where  $f_d$  is Doppler frequency and the model is shown as Figure 3-3.



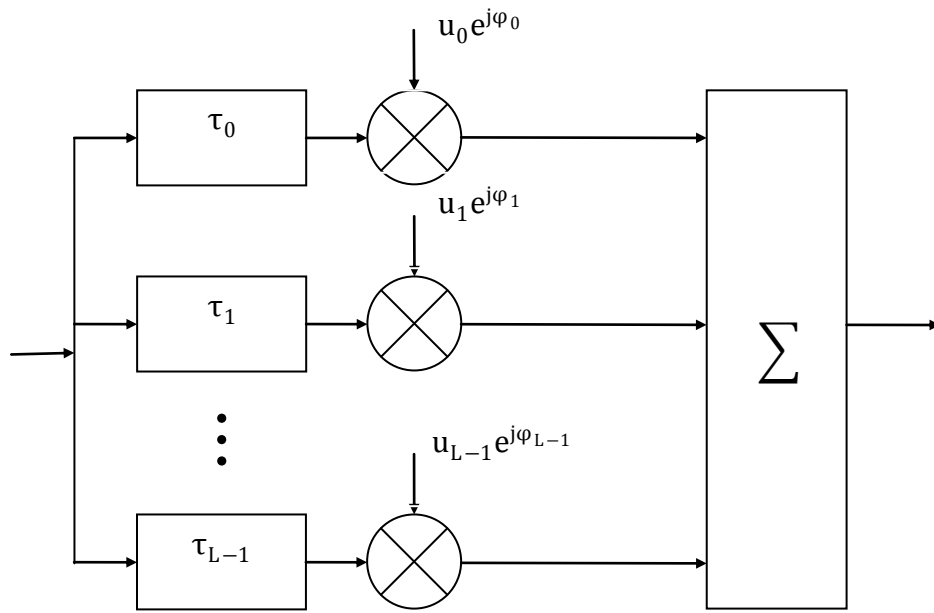


Figure 3-2 Rayleigh channel model

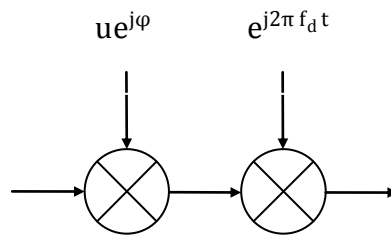


Figure 3-3 Doppler spread channel model

## Chapter 4

# Pilot-based Channel Estimation in OFDM System

OFDM signals can be demodulated either coherently or differentially coherent manners. The most important advantage of differential demodulation is not requiring channel information, and the receiver is relatively simple. However, compared with coherent demodulation, system performance will be degraded from 3 to 4 dB [11]. Moreover, differential demodulation cannot be applied in multi-level modulation. So coherent demodulation is preferred to achieve higher data rates, spectrum efficiency and good performance. Since coherent demodulation depends on the change of phase and amplitude of carrier signal, an accurate estimation of the channel is needed [12]. In OFDM systems, it is very important to know how to get the best channel estimation.

OFDM system channel estimation method can be divided into two ways, pilot-based channel estimation and blind channel estimation. The pilot channel estimation methods are based on the pilot channel and pilot symbol. However, due to two-dimensional time-frequency structure of OFDM system, pilot symbol assisted modulation (PSAM) is more flexible [14]. PSAM is the method doing channel estimation by using pilot sequence and symbol, which are inserted into some fixed positions of signals sent by transmitter. The

pilot symbol sent by transmitter makes spectral efficiency and power utilization lower with the trade-off of quick response to the channel variation. Blind channel estimation is focusing on the correlation between the data sent and received, without knowing the information of the transmitted data. Although it yields higher spectral and power efficiencies by using blind channel estimation, it needs more data to analyze. Hence it is suitable for slow varying channel [19]. This thesis is concentrated on PSAM.

For pilot based channel estimation of OFDM system, following three are required. Firstly, suitable pilot pattern needs to be considered. Secondly, pilot-based channel estimation algorithm with low complexity should be identified. Thirdly, proper demodulation method toward effective channel estimation has to be developed.

## 4.1 Types of Pilot

In order to obtain the channel information, pilot symbols are inserted in the information from transmitter, and the receiver get the channel information by using pilot symbols received. In essence, the problem of pilot pattern design is to determine where to insert the pilot and how closely between pilots. A suitable way of inserting could be calculated according to the known communication environment and estimated speed from the terminal [17].

Therefore, the most two important parameters of pilot, maximum Doppler shift  $f_m$  which determines the minimum coherent time, and maximum multipath time delay  $\tau_{\max}$  that decides the minimum coherent bandwidth, should be discussed [13, 14, 15].

The fading channel of the OFDM system can be viewed as a 2D lattice in a time-frequency plane, because signal is transmitted in the fixed position. And the 2D sampling should satisfy the Nyquist sampling theorem in order to eliminate the distortion. So the

minimum limit of pilot symbols inserted is decided by Nyquist theorem. From Nyquist theorem, the interval of time domain  $N_t$  and frequency domain  $N_f$  should satisfy [16]

$$f_m \cdot T \cdot N_t \leq \frac{1}{2}, \quad \tau_{\max} \cdot \Delta f \cdot N_f \leq \frac{1}{2} \quad (4-1)$$

Where  $\Delta f$  is bandwidth of sub-carrier and  $T$  is period of signal.

For practical application, oversampling with 2 times of the minimum is used. Hence,

$$f_m \cdot T \cdot N_t \approx \frac{1}{4}, \quad \tau_{\max} \cdot \Delta f \cdot N_f \approx \frac{1}{4} \quad (4-2)$$

The two basic channel estimations in OFDM systems, block-type pilot and comb-type pilot, are illustrated in Figure 4-1.

The first one, block-type pilot channel estimation, is performed by inserting pilot tones into all subcarriers of OFDM symbols with a specific period in time. The pilot symbols, because covering all frequencies, could be effective against the selective frequency fading, but more sensitive for the impact of fast fading channel. Therefore, the block-type pilot is developed under the assumption of slow fading channel. In case of same number of pilots, the performance is decided by channel change rate, known as coherent time.

The second one, comb-type pilot channel estimation, is performed by inserting pilot tones into certain subcarriers of each OFDM symbol, where the interpolation is needed to estimate the conditions of data subcarriers. Compared with the block-type pilot, because pilot symbols are inserted into subcarriers with same interval, comb-type pilot channel estimation is introduced to satisfy the need for equalizing when the channel changes even from one OFDM block to the subsequent one. In case of same number of pilots, the performance is decided by channel multipath time delay, known as coherent bandwidth.

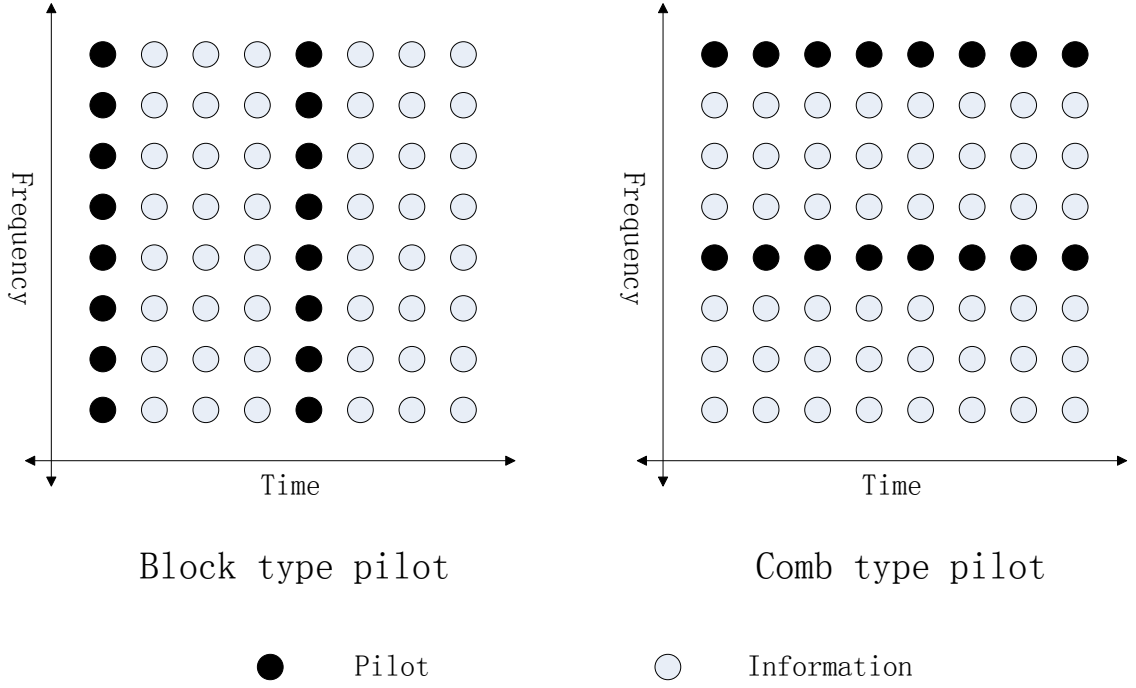


Figure 4-1 Block-type pilot and comb-type pilot

## 4.2 Block Type Pilot-Based Channel Estimation

### 4.2.1 Least Square (LS) Estimator

The impulse response of multipath channel could be written as below

$$h(t, \tau) = \sum_{k=0}^{L-1} \hat{\partial}_k(t) \delta(t - \tau_k) \quad (4-3)$$

Where  $\tau_k$  is the delay of  $k^{\text{th}}$  path,  $\hat{\partial}_k(t)$  is the amplitude of  $k^{\text{th}}$  path, and  $L$  is the number of sub-carriers, respectively. Because  $\hat{\partial}_k(t)$  is a generalized stationary narrowband complex Gaussian random process, it is independent for each other from different paths.

If there is no ISI, the signal received is written as

$$Y = XF\hat{h} + W \quad (4-4)$$

Where  $Y$  is the vector of output signal after OFDM demodulation as  $Y = [Y_0, Y_1, \dots, Y_{N-1}]^T$ ,  $^T$  is transpose,  $X$  is the diagonal matrix of pilots as  $X = \text{diag}\{X_0, X_1, \dots, X_{N-1}\}$ ,  $N$  is the number of pilots in one OFDM symbol,  $\hat{h}$  is the impulse response of the pilots of one OFDM symbol, and  $W$  is the channel noise (always assumed to be AWGN), respectively. Also  $F$  is the Fourier transfer matrix as below,

$$F = \begin{bmatrix} W_N^{00} & \dots & W_N^{0(N-1)} \\ \vdots & \ddots & \vdots \\ W_N^{(N-1)0} & \dots & W_N^{(N-1)(N-1)} \end{bmatrix} \quad (4-5)$$

Where  $W_N^{i,k} = \frac{1}{\sqrt{N}} e^{-j2\pi(\frac{ik}{N})}$

The cost function of LS algorithm is written as,

$$\begin{aligned} J &= \|Y - XF\hat{h}\|^2 \\ &= (Y - XF\hat{h})^H (Y - XF\hat{h}) \\ &= Y^H Y - Y^H XF\hat{h} - X^H Y F^H \hat{h}^H + X^H F^H \hat{h}^H XF\hat{h} \end{aligned} \quad (4-6)$$

Where  $^H$  denotes conjugate transpose.

The purpose of LS algorithm [12] is to minimize the cost function  $J$  without noise. For the minimization of  $J$ , let  $\frac{\partial J}{\partial \hat{h}^H} = 0$ .

Then, from Equation 4-6,

$$\begin{aligned} \frac{\partial J}{\partial \hat{h}^H} &= 0 - 0 - \frac{\partial J}{\partial \hat{h}^H} (X^H Y F^H \hat{h}^H) + \frac{\partial J}{\partial \hat{h}^H} (X^H F^H \hat{h}^H XF\hat{h}) \\ &= -F^H X^H Y + F^H X^H XF\hat{h} \\ &= 0 \end{aligned} \quad (4-7)$$

Then we could get

$$\hat{h}_{LS} = (F^H X^H XF)^{-1} F^H X^H Y = F^{-1} X^{-1} Y \quad (4-8)$$

Because  $\hat{H} = F\hat{h}$ , where  $\hat{H}$  is the impulse response of the channel,

$$\begin{aligned}\hat{H}_{LS} &= X^{-1}Y \\ &= [\frac{Y_0}{X_0} \quad \frac{Y_1}{X_1} \dots \frac{Y_{N-1}}{X_{N-1}}]^T\end{aligned}\quad (4-8)$$

The advantage of LS algorithm is its simplicity, because no consideration of noise and ICI. So, without using any knowledge of the statistics of the channels, the LS estimators are calculated with very low complexity, but obviously it suffers from a high MSE. LS method, in general, is utilized to get initial channel estimates at the pilot subcarriers, which are then further improved via different methods.

#### 4.2.2 Minimum Mean Square Error (MMSE) Estimator

The MMSE estimator employs the second-order statistics of the channel conditions to minimize the MSE [20]. Let us denote the error of channel estimation  $e$  as

$$e = H - \hat{H} \quad (4-9)$$

Where  $H$  is actual channel estimation and  $\hat{H}$  is raw channel estimation, respectively.

And the MSE of channel estimation is

$$\begin{aligned}E\{|e|^2\} &= E\{|H - \hat{H}|^2\} \\ &= E\{(H - \hat{H})(H - \hat{H})^H\}\end{aligned}\quad (4-10)$$

Where  $E\{\}$  is the expectation.

Since the channel and AWGN are not correlated, we can rewrite Equation 4-10 as

$$\hat{H}_{MMSE} = R_{HY} R_{YY}^{-1} Y \quad (4-11)$$

Let us denote the auto-covariance matrixes of  $H$ ,  $Y$  by  $R_{HH}$ ,  $R_{YY}$  respectively, and cross covariance matrix between  $H$  and  $Y$  by  $R_{HY}$ . Let  $\sigma_N^2$  is the noise-variance, since the channel and AWGN are not correlated, we could get

$$\begin{aligned}
R_{HY} &= E\{HY^H\} \\
&= E\{H(HX + W)^H\} \\
&= E\{HH^H X^H + HW^H\} \\
&= E\{HH^H\}X^H + 0 \\
&= R_{HH}X^H
\end{aligned} \tag{4-12}$$

$$\begin{aligned}
R_{YY} &= E\{YY^H\} \\
&= E\{(HX + W)(HX + W)^H\} \\
&= E\{HXX^H + HXW + WH^H X^H + WW^H\} \\
&= E\{HH^H\}XX^H + 0 + 0 + E\{WW^H\} \\
&= XR_{HH}X^H + \sigma_N^2 I_N
\end{aligned} \tag{4-13}$$

If  $R_{HH}$  and  $\sigma_N^2$  are known to the receiver, channel impulse response could be calculated by MMSE estimator as below

$$\begin{aligned}
\hat{H}_{MMSE} &= R_{HY}R_{YY}^{-1}Y \\
&= R_{HH}X^H(XR_{HH}X^H + \sigma_N^2 I_N)^{-1}X\hat{H}_{LS} \\
&= R_{HH}(R_{HH} + \sigma_N^2(X^H X)^{-1})^{-1}\hat{H}_{LS}
\end{aligned} \tag{4-14}$$

The performance of MMSE estimator is much better than LS estimator, especially under the lower  $E_b/N_0$ . And MMSE estimator could gain 10-15 dB more of performance than LS [21]. However, because of the required matrix inversions, the computation is very complex when the number of subcarriers of OFDM system increases. Therefore, an important drawback of the MMSE estimator can be the high computational complexity.

### 4.2.3 Linear Minimum Mean Square Error (LMMSE) Estimator

From the  $\hat{H}_{MMSE}$  in Equation 4-14, we could find that the channel estimator need to get the inverse matrix of  $R_{HH} + \sigma_N^2(X^H X)^{-1}$ . Because  $(X^H X)^{-1}$  are not the same in different OFDM symbols, its inverse matrix should be updated every time for the different



OFDM symbols, which needs much computation. A simplification of MMSE estimator is to replace the  $(X^H X)^{-1}$  by its expectation  $E\{(X^H X)^{-1}\}$ , which means the average power of all subcarriers replace the instantaneous power of each subcarrier in order to reduce the computation, since matrix inversion of  $R_{HH} + \sigma_N^2 (X^H X)^{-1}$  is no longer needed [22, 24]. Assuming the same signal constellation on all tones and equal probability on all constellation points, we get

$$E\{(X^H X)^{-1}\} = E\left\{\frac{1}{|X_k|^2}\right\} I \quad (4-15)$$

Where  $I$  is the identity matrix.

Let the average of SNR is

$$\overline{SNR} = \frac{E\{|X_k|^2\}}{\sigma_N^2} \quad (4-16)$$

The term  $\sigma_N^2 (X^H X)^{-1}$  is then approximated by  $\frac{\beta}{\overline{SNR}} I$ .

Where  $\beta$  is defined as

$$\beta = \frac{E\{|X_k|^2\}}{E\left\{\frac{1}{|X_k|^2}\right\}} \quad (4-17)$$

Note that  $\beta$  is a constant depending only on the signal constellation.

For example, when 16QAM is used, from the constellation, the values of  $X_k$  are

$(\pm 1 \pm i)$ ,  $(\pm 3 \pm i)$ ,  $(\pm 1 \pm 3i)$ ,  $(\pm 3 \pm 3i)$ . From Equation 4-17, the  $\beta$  is  $\frac{17}{9}$ .

Then the modified MMSE can be written as

$$\hat{H}_{LMMSE} = R_{HH} (R_{HH} + \frac{\beta}{\overline{SNR}} I)^{-1} \hat{H}_{LS} \quad (4-18)$$

Therefore, if  $R_{HH}$  and SNR are known or fixed as known values, matrix inversion of  $R_{HH} + \sigma_N^2(X^H X)^{-1}$  is just needed to be calculated only once. But, because of consideration of influence of noise, the MSE of LMMSE is smaller than MMSE.

#### 4.2.4 Singular Value Decomposition (SVD) Estimator

To further reduce the complexity of computation of LMMSE estimator, we can consider the SVD estimator [23]. Because the channel energy is mainly in low frequency domain, the first  $P$  orders could be chosen to be used in estimator. Let us denote the SVD of channel correlation matrix,

$$R_{HH} = U \Lambda U^H \quad (4-19)$$

Where  $U$  is a unitary matrix, and  $\Lambda$  is a diagonal matrix with the singular values of  $R_{HH}$  as  $\lambda_0 \geq \lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_{N-1} \geq 0$ .

Therefore, we could get the estimator as below

$$\begin{aligned} \hat{H}_{LMMSE} &= U \Lambda U^H (U \Lambda U^H + \frac{\beta}{SNR} I)^{-1} \hat{H}_{LS} \\ &= U \Lambda (\Lambda + \frac{\beta}{SNR} I)^{-1} U^H \hat{H}_{LS} \\ &= U \Delta U^H \hat{H}_{LS} \end{aligned} \quad (4-20)$$

Where  $\Delta = \Lambda (\Lambda + \frac{\beta}{SNR} I)^{-1} = \frac{\lambda_k}{\lambda_k + \frac{\beta}{SNR}}$ , and the values on the diagonal of  $\Delta$  could

be written by

$$\delta_k = \begin{cases} \frac{\lambda_k}{\lambda_k + \frac{\beta}{SNR}} & k = 0, 1, \dots, p-1 \\ 0 & k = p, p+1, \dots, N-1 \end{cases} \quad (4-21)$$

Then we could get the channel estimation by SVD algorithm as

$$\hat{H}_{SVD} = U \begin{pmatrix} \Delta_P & 0 \\ 0 & 0 \end{pmatrix} U^H \hat{H}_{LS} \quad (4-22)$$

Where the  $\Delta_P$  is the left up  $P \times P$  corner of  $\Delta$  as

$$\Delta_P = \begin{bmatrix} \frac{\lambda_0}{\lambda_0 + \frac{\beta}{SNR}} & & & 0 \\ & \frac{\lambda_1}{\lambda_1 + \frac{\beta}{SNR}} & & \\ & & \ddots & \\ 0 & & & \frac{\lambda_{p-1}}{\lambda_{p-1} + \frac{\beta}{SNR}} \end{bmatrix} \quad (4-23)$$

Assuming the matrix  $U^H$  as a transform, the singular value  $\lambda_k$  is the channel energy of information received after  $\hat{H}_{LS}$  is transformed by  $U^H$ . We choose the larger P points of all N points and assume the channel energy of them is larger than noise, so the channel energy of other N-P points is smaller. Therefore, the P points are used for estimation but the N-P points are set as 0, in order to reduce the complexity of computation and influence of noise. The block diagram of SVD estimator is shown as below in Figure 4-2.

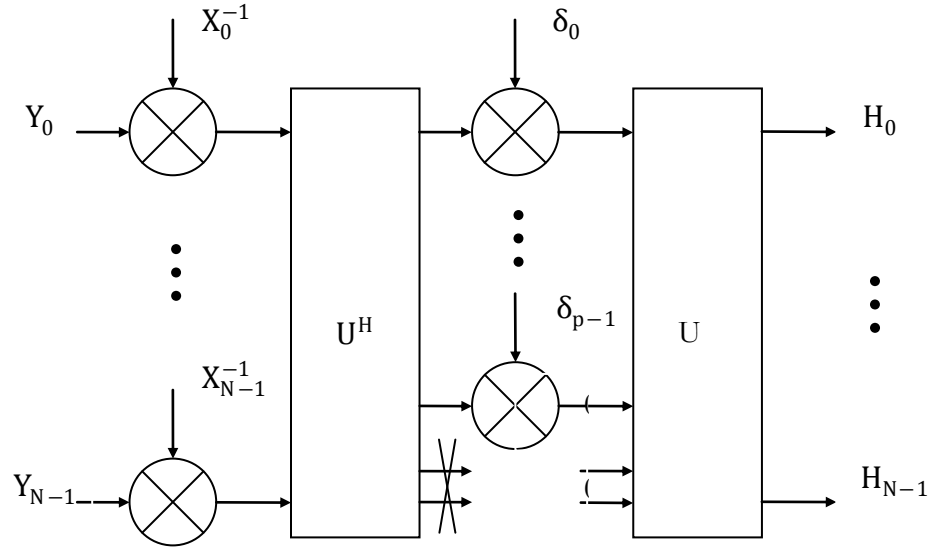


Figure 4-2 Block diagram of SVD estimator

In the Figure 4-2,  $Y = \begin{bmatrix} Y_0 \\ \vdots \\ Y_{N-1} \end{bmatrix}$ ,  $X = \begin{bmatrix} X_0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & X_{N-1} \end{bmatrix}$ .

### 4.3 Comb Type Pilot-Based Channel Estimation

In comb type pilot-based channel estimation, as shown in Figure 4-1, for each transmitted OFDM symbol,  $N_p$  pilot signals are inserted into subcarriers with same interval in frequency from each other [25].

$$X(k) = X(mN_f + l) = \begin{cases} x_p(m) & l = 0 \\ Data & l = 1, 2, \dots, N_f - 1 \end{cases} \quad (4-24)$$

Where  $X(k)$  is the information, including pilot and data, of all sub-carriers,  $x_p(m)$  is the value of  $m^{th}$  subcarrier pilot, and  $N_f$  is the frequency interval of inserted pilot, respectively. If there are  $N$  subcarriers,

$$N_f = \frac{N}{N_p} \quad (4-25)$$

According to the pilot position, frequency response of corresponding sub-channel is calculated in the receiver.

$$\hat{H}_p(m) = \frac{Y_p(mN_f)}{X_p(mN_f)} \quad m = 0, 1, \dots, N_p - 1 \quad (4-26)$$

Where  $Y_p$  and  $X_p$  are output and input of pilot subcarrier, respectively.

When the pilot interval is shorter than coherent bandwidth, after the frequency response of pilot sub-channel is estimated, interpolation is used in frequency domain to get the channel estimation [26]. Different interpolation methods will yield different accuracy.

#### 4.3.1 Piecewise Constant Interpolation

Piecewise constant interpolation is the simplest method. The channel is estimated by previous pilot. And the channel estimation is given by,

$$\hat{H}(k) = \hat{H}(mN_f + l) = \hat{H}_p(m), \quad 0 \leq l \leq N_f, \quad m = 0, 1, \dots, N_p - 1 \quad (4-27)$$

We could note that the channel estimation  $\hat{H}(k)$  of all  $l$  sub-carriers is determined by the estimation of  $m^{th}$  pilot directly. As the simplest method, piecewise constant interpolation provides the worst performance.

#### 4.3.2 Linear Interpolation

Linear interpolation performs better than the piecewise constant interpolation [27]. The channel estimation at the data subcarrier is obtained by estimation of response of two adjacent pilot sub-channels. But the precondition is linearity of transmitted functions of adjacent sub-channels. The linear interpolation is shown as below,

$$\hat{H}(k) = \hat{H}(mN_f + l) = (\hat{H}_p(m+1) - \hat{H}_p(m)) \frac{1}{N_f} + \hat{H}_p(m) \quad m = 0, 1, \dots, N_p - 1 \quad (4-28)$$

In Equation 4-28, note that only two pilots are used in linear interpolation for channel estimation. Hence complexity of computation is simple. But performance is not necessarily satisfactory.

### 4.3.3 Second Order Interpolation

Second order interpolation is better than linear interpolation, where the channel estimation at the data subcarrier is calculated by used linear combination of three adjacent pilots [28]. Figure 4-3 shows the diagram of second order interpolation.

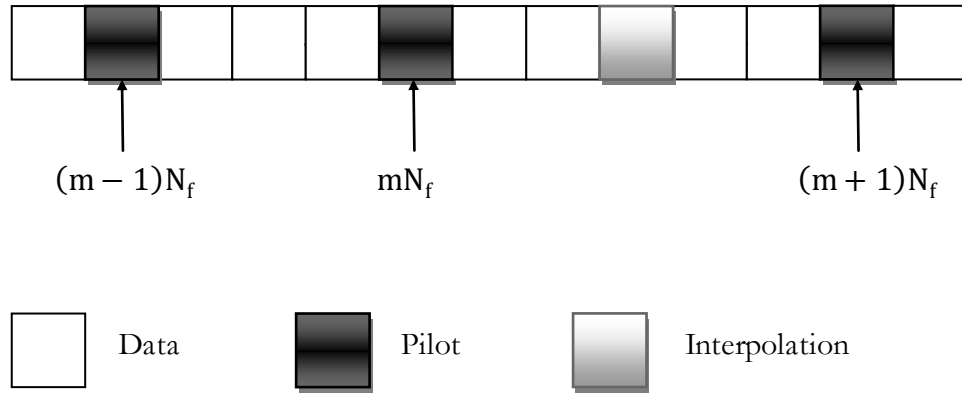


Figure 4-3 Sketch map of second order interpolation

Theoretically, high order interpolation yields better channel estimation because of using more pilots. And the channel estimation will be close to the true channel response. But computation complexity also is increased as increasing of order. The channel estimation of second order interpolation is given by

$$\begin{aligned}\hat{H}(k) = \hat{H}(mN_f + l) &= c_1 \hat{H}_p(m-1) + c_0 \hat{H}_p(m) + c_{-1} \hat{H}_p(m+1) \\ m &= 0, 1, \dots, N_p - 1\end{aligned}\quad (4-29)$$

The coefficients are defined as,

$$\begin{aligned}c_1 &= \frac{\alpha(\alpha-1)}{2} \\ c_0 &= -(\alpha-1)(\alpha+1), \quad \alpha = \frac{l}{N_f} \\ c_{-1} &= \frac{\alpha(\alpha+1)}{2}\end{aligned}\quad (4-30)$$

#### 4.3.4 Cubic Spline Interpolation

The cubic spline interpolation method produces a smooth and continuous polynomial fitted to given data points [29], which is given by

$$\begin{aligned}\hat{H}(k) &= \hat{H}(mN_f + l) \\ &= \alpha_1 \hat{H}_p(m+1) + \alpha_0 \hat{H}_p(m) + N_f \alpha_1 \hat{H}'_p(m+1) - N_f \alpha_0 \hat{H}'_p(m) \\ m &= 0, 1, \dots, N_p - 1\end{aligned}\quad (4-31)$$

$\hat{H}'_p(m)$  is the first order derivative of  $\hat{H}_p(m)$ , and

$$\begin{aligned}\alpha_1 &= \frac{3(N_f - l)^2}{N_f^2} - \frac{2(N_f - l)^3}{N_f^3} \\ \alpha_0 &= \frac{3l^2}{N_f^2} - \frac{2l^3}{N_f^3}\end{aligned}\quad (4-32)$$

Although cubic spline interpolation with higher order interpolation can be used for better interpolation accuracy, the performance improvement is not obviously proven.

# Chapter 5

## Simulation Results

This chapter discusses the simulation setup and results of pilot-based channel estimation in OFDM system. Firstly, the simulation parameters are shown as a scenario of OFDM system. Then some results about the core techniques of OFDM system, mentioned in Chapter 2, are analyzed. The performance under different channel condition of Chapter 3 is discussed. Lastly, all algorithms of pilot-based channel estimation introduced in Chapter 5 are simulated by Matlab. The performance of different estimators of block and comb type pilot are analyzed, and different modulation and demodulation methods, QPSK and 16QAM, are compared by the simulation.

### 5.1 Simulation Scenarios

Table 5-1 shows the default OFDM system parameters. For some simulations of comparison, change of the scenario will be marked if needed.



Bandwidth	1 MHz
Frequency of carrier	1.9 GHz
Number of sub-carriers	128
Number of symbols per carrier	100
Interval of sub-carriers	7.8125 KHz
Number of multipath	5
Time delay of multipath	[0 2e-6 4e-6 8e-6 12e-6]
CP length	16
Modulation/demodulation	16QAM

Table 5-1 OFDM system simulation parameter

## 5.2 Simulation Results

Figure 5-1 shows the comparison of the required numbers of DFT and FFT operations discussed in 2.3.3. It is obviously to note the huge difference of computation complexity between DFT and FFT as bin number is increasing. Although FFT does not have big advantage compared with DFT, when the number is very small, like from 0 to about 128, but in practice, OFDM system owns a large amount number of sub-carriers, so the bin size of FFT should be very big. We could see the comparison of complexity when number is 1000 clearly. FFT plays a very important role in OFDM system for reducing the complexity and enhancing the performance.

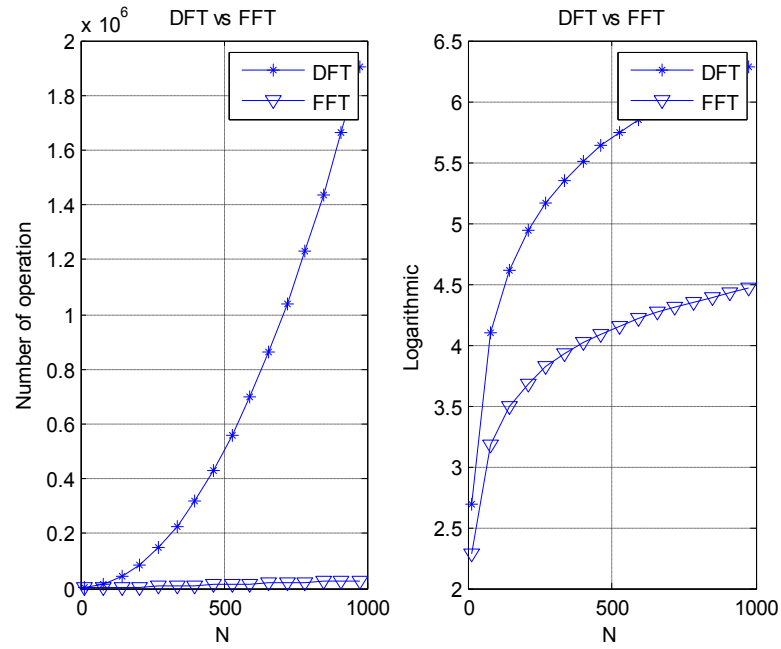


Figure 5-1 Comparison of DFT and FFT operations

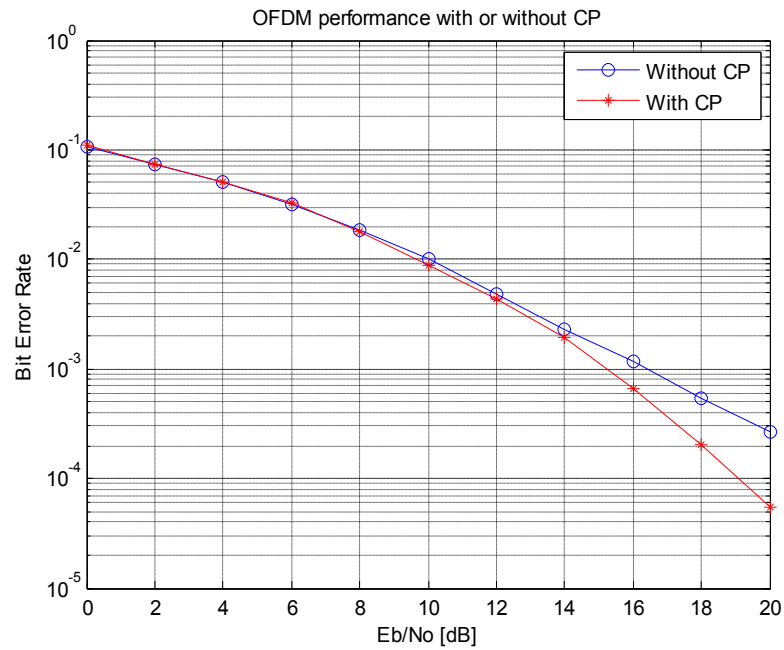


Figure 5-2 OFDM performance with or without CP

Figure 5-2 shows the OFDM performance with or without CP under multipath environment. CP length is set to 16 under condition of "With CP" and 0 under condition

of “Without CP”. When  $\frac{E_b}{N_0}$  is 10 dB, the better performance of OFDM system with CP began to be shown clearly. In multi-carrier system, CP is used to eliminate the ISI of time domain. But the length of CP can be different in different environments. Shorter CP does not yield satisfactory performance for eliminating the ISI, but longer CP takes up too much system resource to decreases the performance. But, normally, the length of CP is longer than maximum delay to alleviate the ISI.

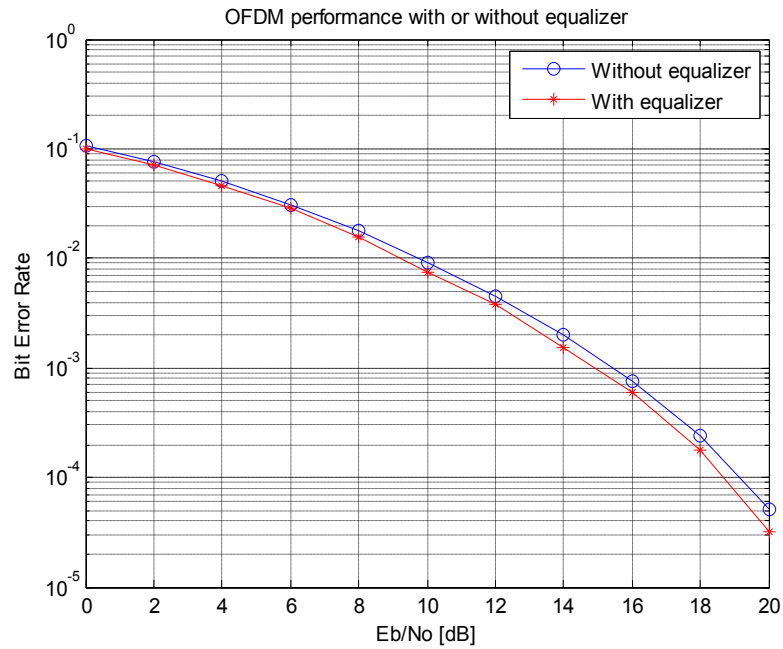


Figure 5-3 OFDM performance with or without equalizer

Figure 5-3 shows the effectiveness of a equalizer. More or less, we could note that the better performance of with equalizer. But the difference between them is very small. The reason is that CP is used to avoid the equalizer in time domain. Therefore, considering the small performance gain, increasing complexity of system by adding equalizer is not sensible.

On the other hand, equalizer in frequency domain is always used in channel estimation for eliminating the ICI. Figures 5-4 and 5-5 show the eye patterns of the received signal

before and after the equalizer. And Figure 5-6 shows the spectrum of received signal with and without the equalizer. The ZF equalizer is used in the simulation.

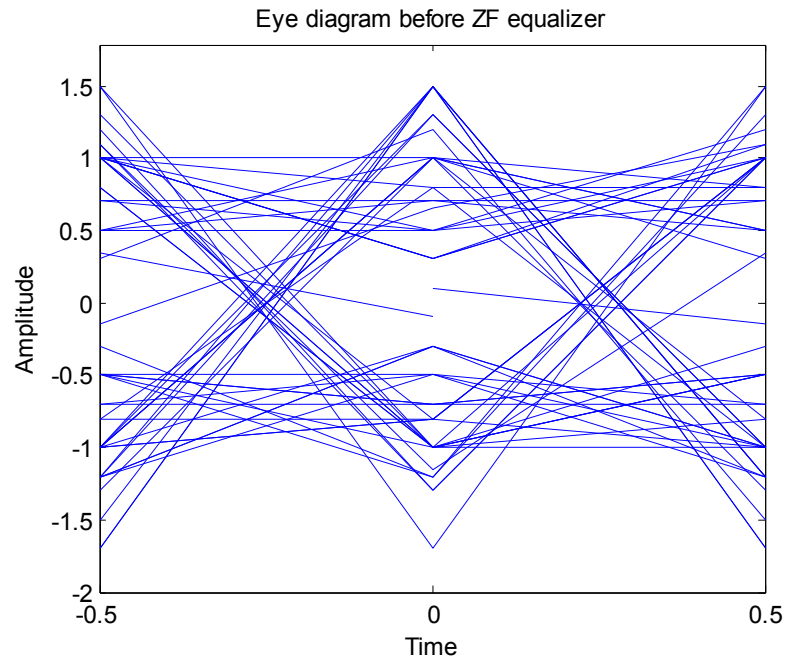


Figure 5-4 Eye pattern before ZF equalizer

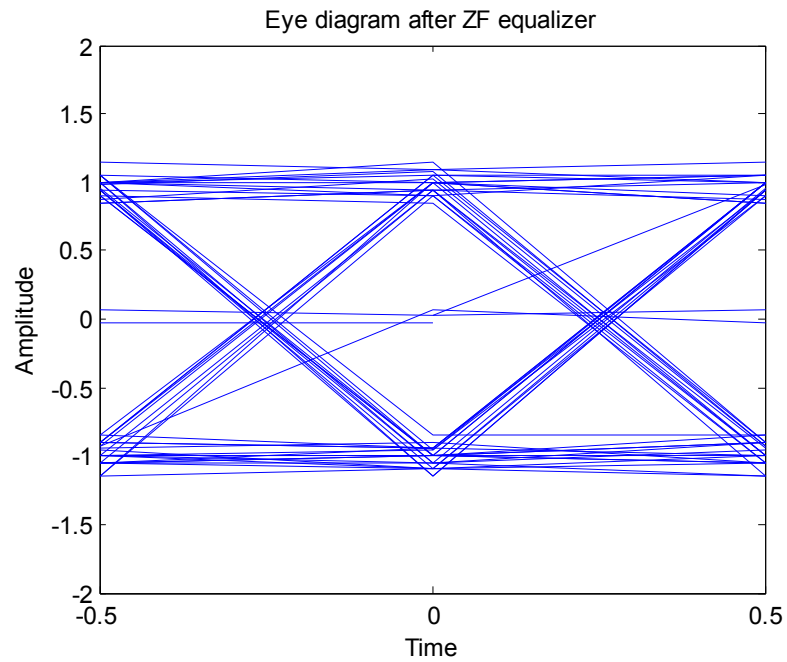


Figure 5-5 Eye pattern after ZF equalizer

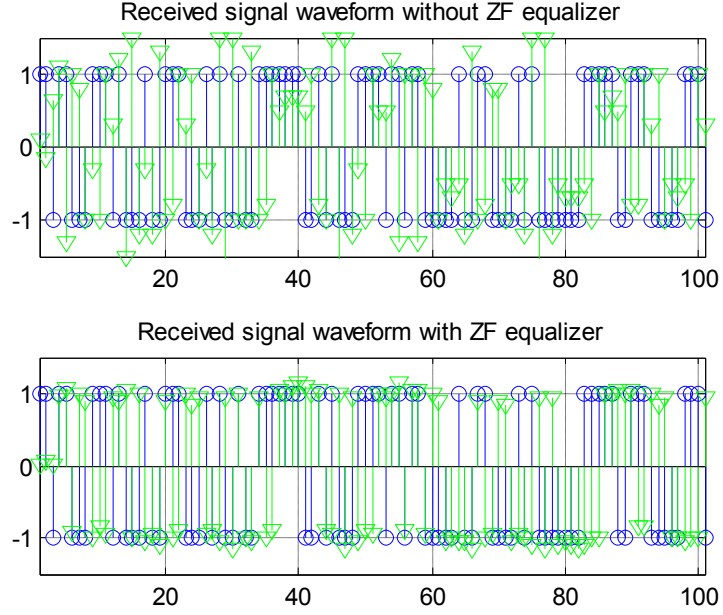


Figure 5-6 Comparison of received signal waveform with and without ZF equalizer

From the comparison of Figure 5-4 and 5-5, we could find that the eye becomes big and clearly after the ZF equalizer because of larger noise margin. Figure 5-6 also shows that the distortion of samples becomes smaller with equalizer. It means that the ZF equalizer in frequency domain can yield better performance in OFDM system. Hence, in the part of simulation for channel estimation, ZF equalizer is used in estimator.

Figure 5-7 shows the performance of OFDM system under AWGN and multipath. In this simulation, just 1 and 2 fading paths are chosen for simple comparison. The attenuation coefficients are 0.3 and 0.4 respectively. We could find that the performance under AWGN is the best. As number of path increasing, the performance gets degraded. The reason is that the fading and phase shift of received signal affected by multipath. But, because of the orthogonality of sub-carriers, OFDM system could avoid the ISI and ICI by multipath.

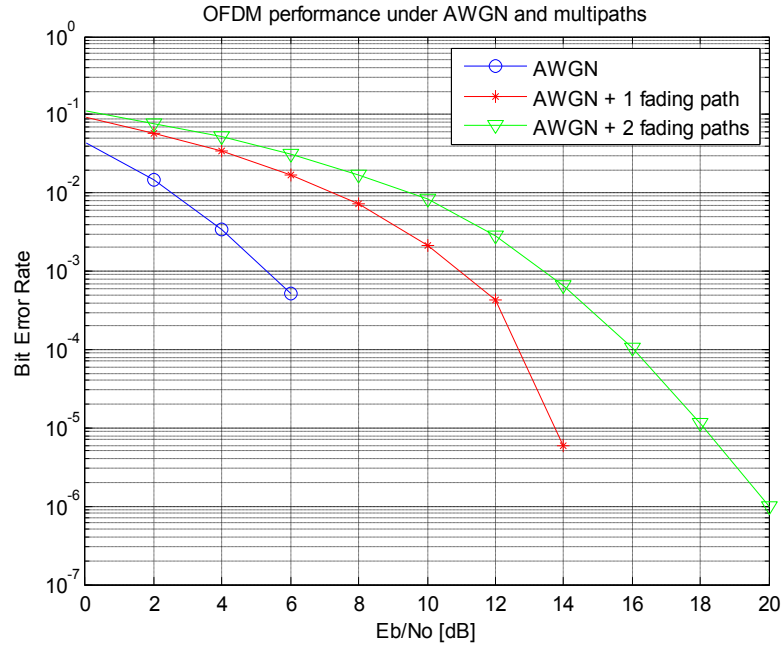


Figure 5-7 OFDM performance under AWGN and multipath

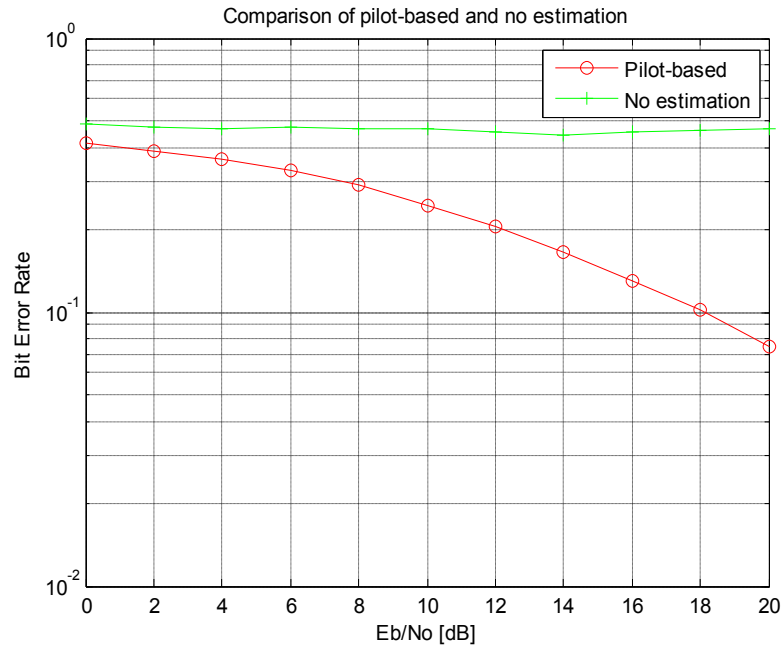


Figure 5-8 Comparison of pilot-based and no channel estimation

Figure 5-8 shows the comparison of pilot-based estimation and condition of no channel estimation. In this simulation, block type pilot and LS estimator are used. It is

obviously that the better performance of pilot-based channel estimation. As analyzed in Chapter 4, channel impulse response is estimated by algorithm of pilot-based channel estimation. So the receiver of system could get accurate received signal.

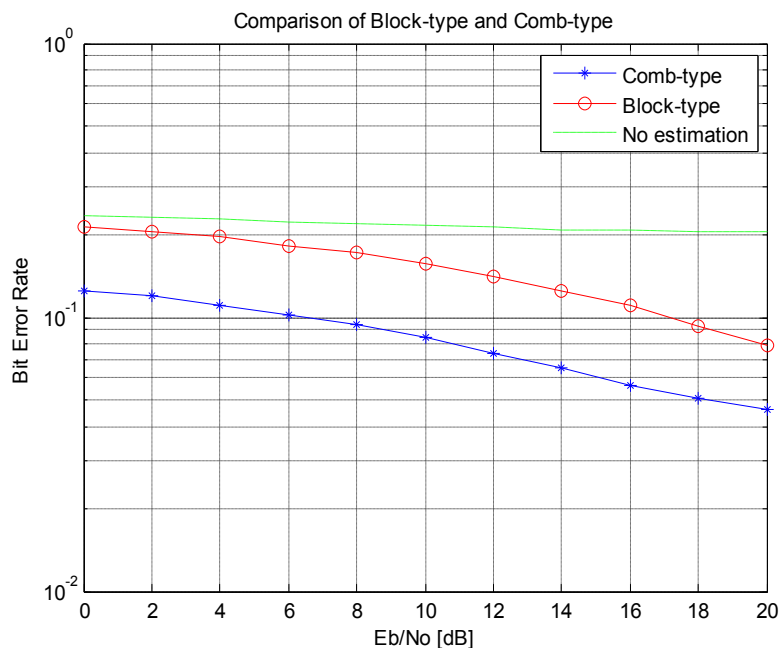


Figure 5-9 Comparison of block type and comb type pilot in fast fading channel

Figures 5-9 and 5-10 show BER performance of different pilot schemes under fast and slow fading environment. LS estimator is used for block type pilot channel estimation, and linear interpolation is used for comb type pilot channel estimation respectively. Doppler frequency shift are 5 Hz in slow fading channel, and 140 Hz in fast fading channel, respectively. In Figure 5-9, comb type pilot-based channel estimation shows better performance compared with block types. The reason is, as discussed in Chapter 4, comb type pilots are inserted in all sub-carriers of time domain in order to adapt to rapidly changing channel. But in slow fading environment, block type pilots, inserted in all sub-carriers of frequency domain, has better performance as shown in Figure 5-10. Therefore,

block type pilot and comb type pilot should be chosen for better performance in different environments, such as walking, driving, and etc..

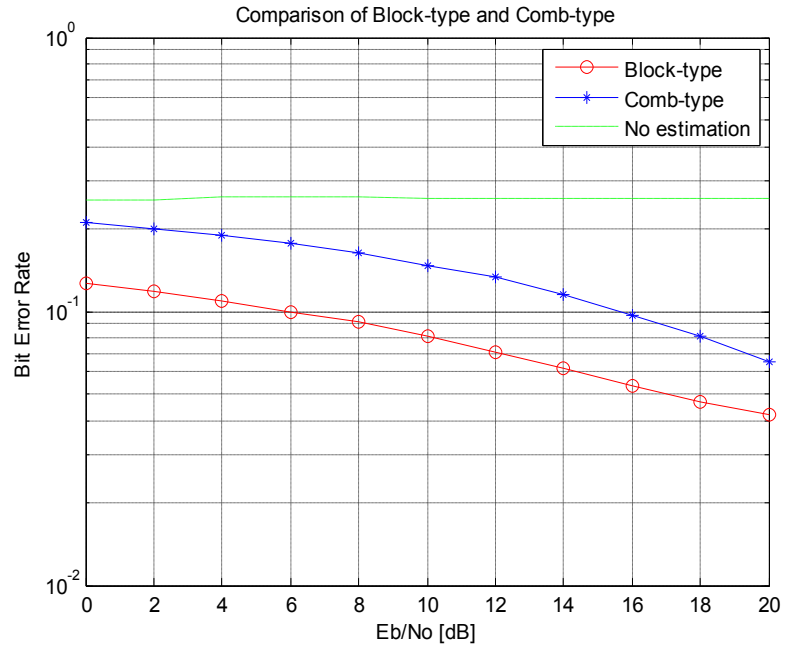


Figure 5-10 Comparison of block type and comb type pilot in slow fading channel

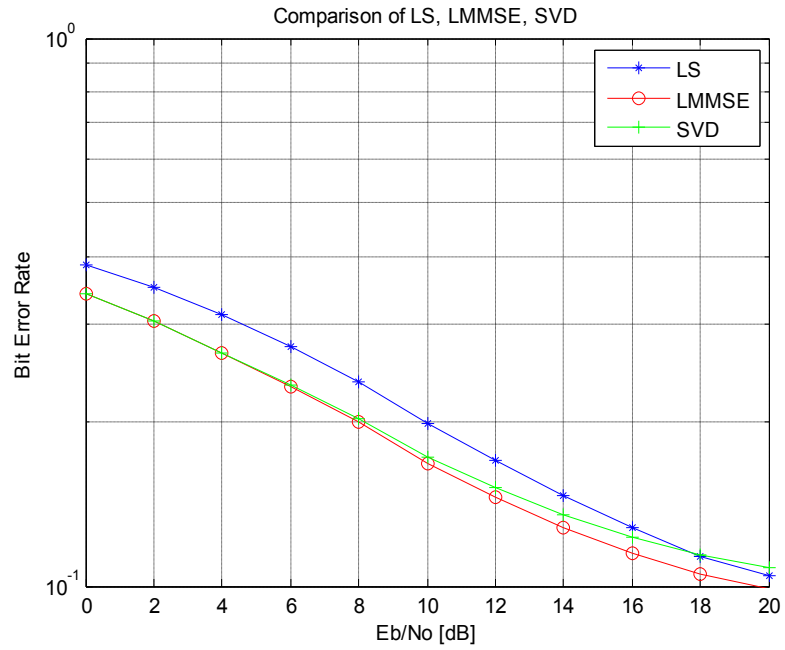


Figure 5-11 Comparison of LS, LMMSE and SVD of block type pilot



Figure 5-11 shows the comparison of BER performance of LS, LMMSE, SVD algorithms of block type pilot-based channel estimation, analyzed in Chapter 4.2. In the simulation, the  $p$  of SVD estimators is 16. Notice that the performance of LMMSE algorithm is better than LS, since influence of noise is considered in LMMSE. However, LMMSE algorithm needs much more complex computation, which reflects statistical properties of OFDM channel. In lower  $E_b/N_0$ , the performances of LMMSE and SVD algorithms are very similar. But with  $E_b/N_0$  increases, the advantage of LMMSE starts to be shown, compared with SVD algorithm. This is because the SVD algorithm, preserving only a finite number of large energy points of the channel, is reducible in its operation compared to the LMMSE algorithm. When the  $E_b/N_0$  is large, because of minimal effect from noise, the SVD algorithm filters out too much information of channel. That is why the BER performance of SVD algorithm is worse than LMMSE.

Figure 5-12 shows the comparison of SVD algorithm with different  $p$  mentioned in Equations 4-20 and 4-21. It is clear that the BER performance of  $p=5$  is worse than  $p=16$  and  $p=20$ . Because SVD algorithm just chooses the information of first  $p$  orders to do channel estimation in order to reduce complexity computation, when the  $p$  is a small number, SVD estimator filters out too much information of channel. It eventually leads to degrade the performance. We also could notice that the performances of SVD with  $p=16$  and  $p=20$  are almost identical. It means that  $p=16$  is a number large enough for  $p$  to get satisfactory performance. Further increase of  $p$ , although getting more information of channel, could not bring significant performance gain as shown in the Figure 5-12.

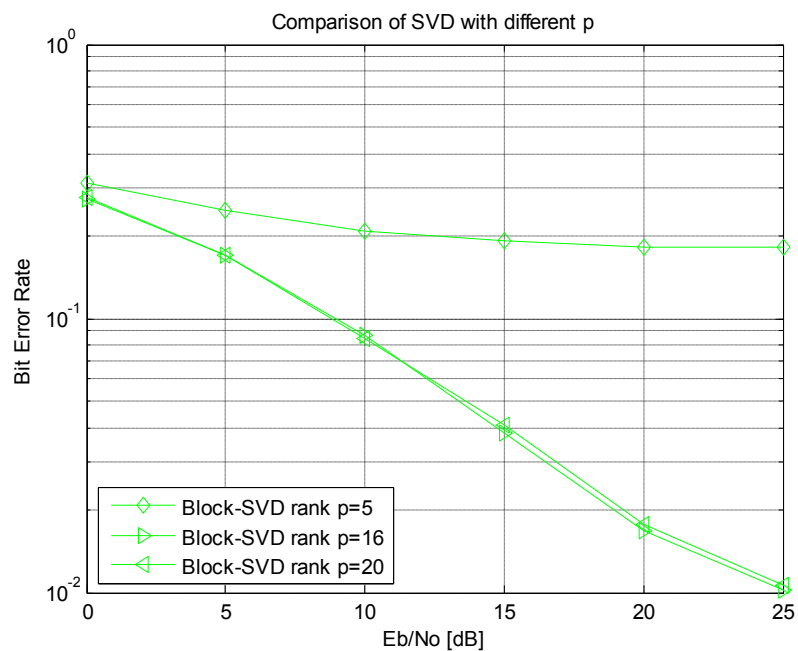


Figure 5-12 Comparison of SVD with different  $p$

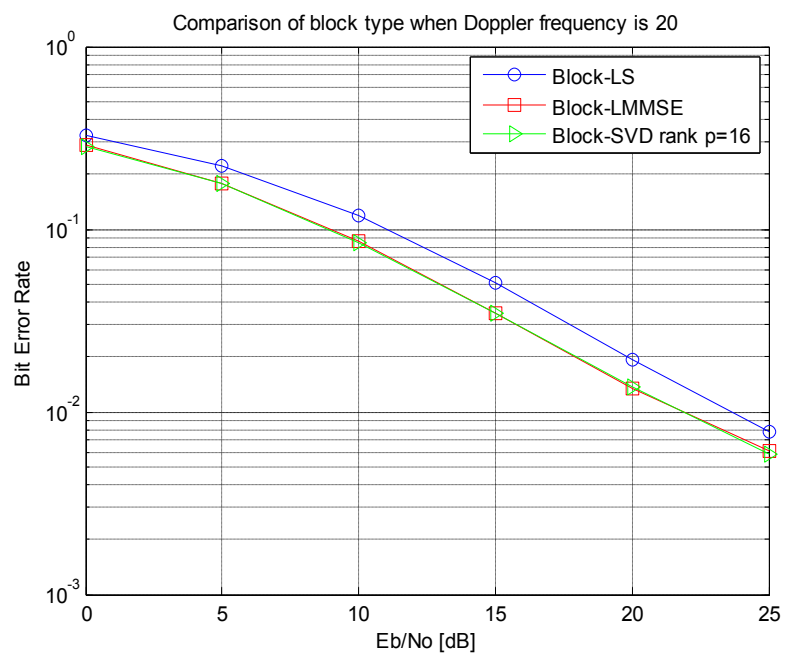


Figure 5-13 Comparison of block type when Doppler frequency is 20 Hz

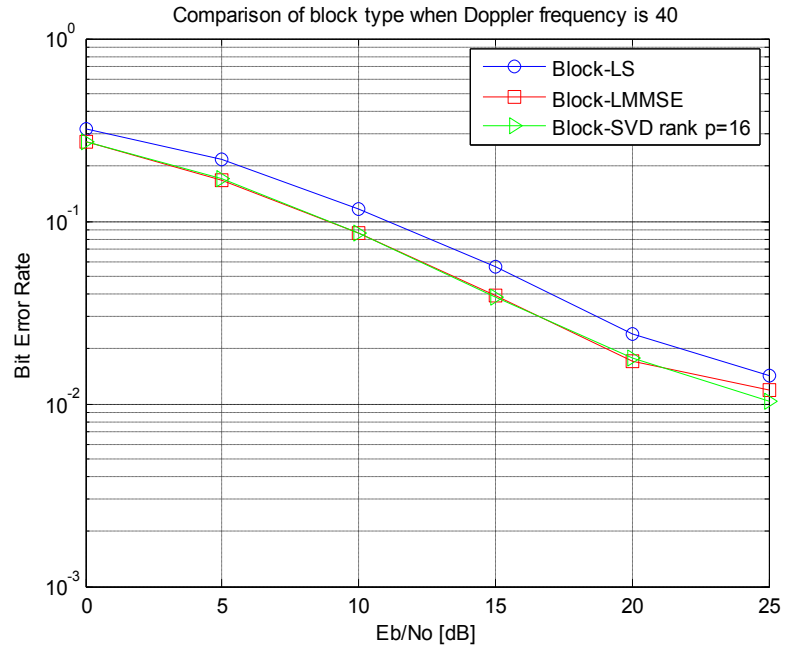


Figure 5-14 Comparison of block type when Doppler frequency is 40 Hz

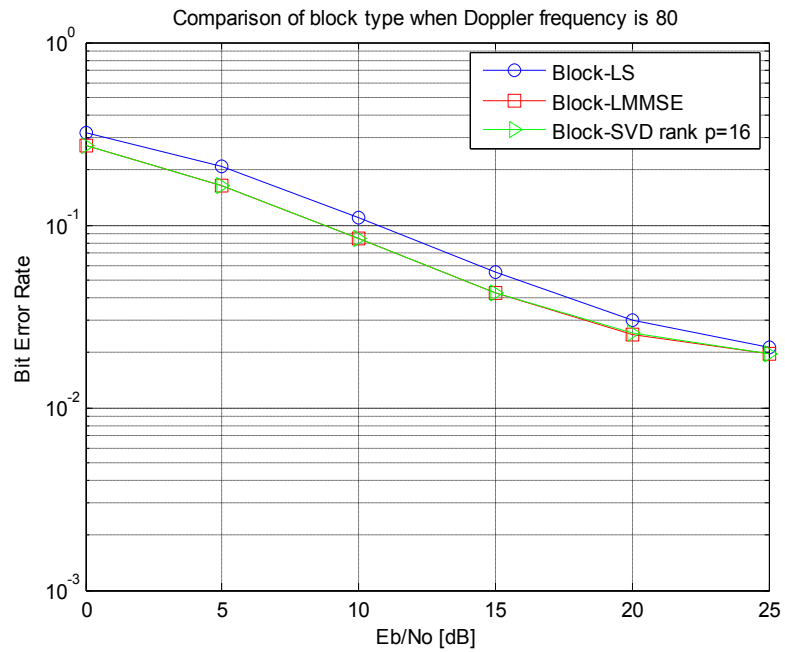


Figure 5-15 Comparison of block type when Doppler frequency is 80 Hz

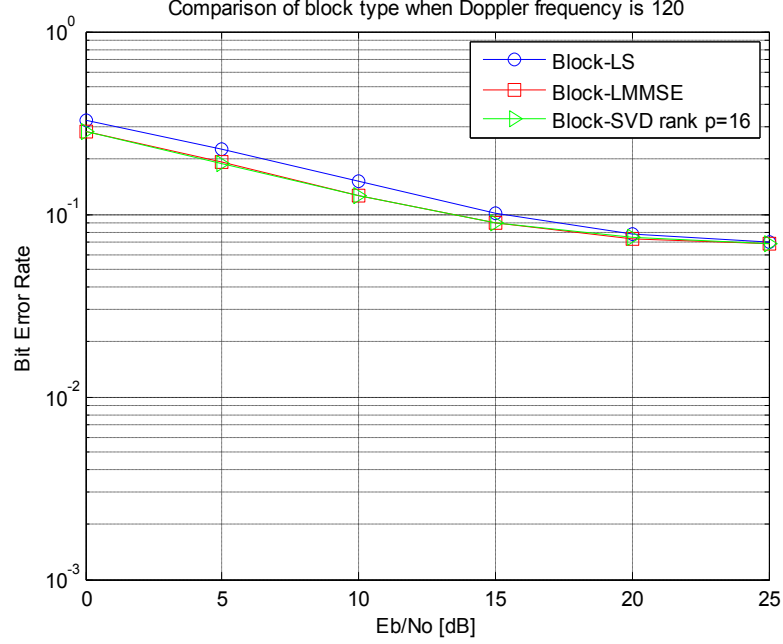


Figure 5-16 Comparison of block type when Doppler frequency is 120 Hz

Figures 5-13 to 5-16 show the performances of LS, LMMSE, SVD estimators when Doppler frequency shift are 20 Hz, 40 Hz, 80 Hz and 120 Hz, respectively. We could find that as the increasing of Doppler frequency shift, the performance curve declines significantly. That means block-type pilot based channel estimation has better performance in slow fading channel and could not adapt for the rapid changing channel.

Figure 5-17 shows the comparison of different interpolations of comb type pilot-based channel estimation. As the increase of polynomial orders, performance has significant improvement. But, at the same time, computation also will be much more complex. On the other hand, BER decreases when  $E_b/N_0$  is increasing. But when  $E_b/N_0$  increases to a certain extent, the estimation algorithm of fixed order interpolation changes little. Increasing  $E_b/N_0$  could not improve the performance significantly. Because when the  $E_b/N_0$  is large, the noise on the impact of channel estimation error is very small and even can be ignored.

And the loss of performance caused by ICI, produced by Doppler frequency shift, cannot be eliminated by some interpolation method. On the other hand, the approximation and limitation of fixed order interpolation makes estimation performance cannot be improved with the  $E_b/N_0$  increase.

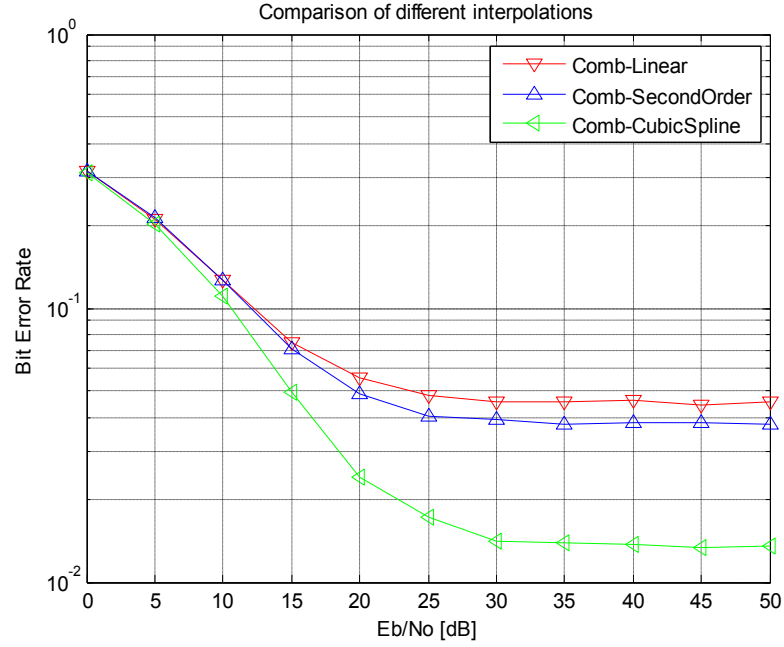


Figure 5-17 Comparison of different interpolations of comb type pilot

Figures 5-18 to 5-21 show the performance of comb type pilot-based channel estimation with different number of pilots. It is clear to see the improvement of performance with more pilots. Because increasing the number of sub-channels for the pilot is equivalent to reduce the interval between the channels, so that the correlation between two adjacent pilots increases, and estimated values of sub-channel characteristics by interpolation are more accurate. Therefore, estimated performance could be improved. However, the increasing of number of sub-channels for pilot decreases the spectrum efficiency.

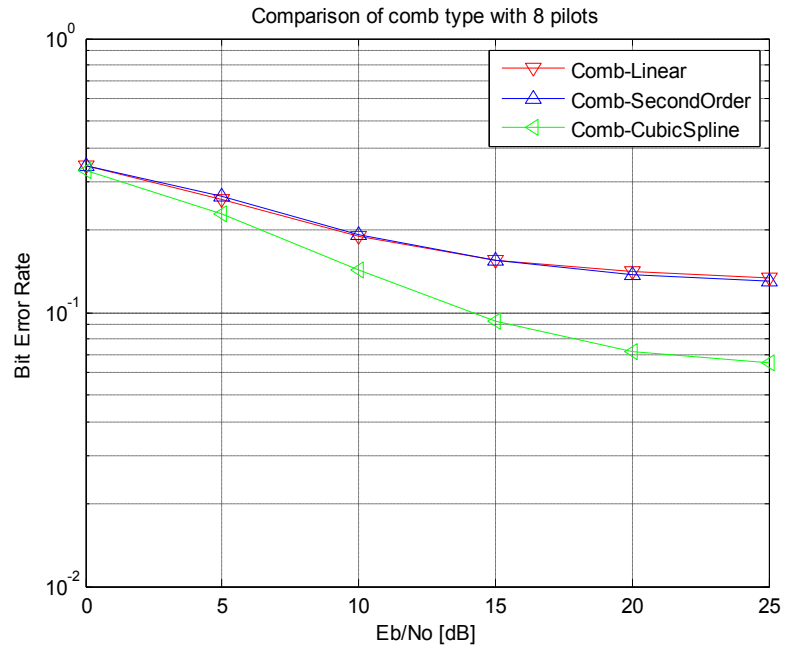


Figure 5-18 Comparison of comb type with 8 pilots

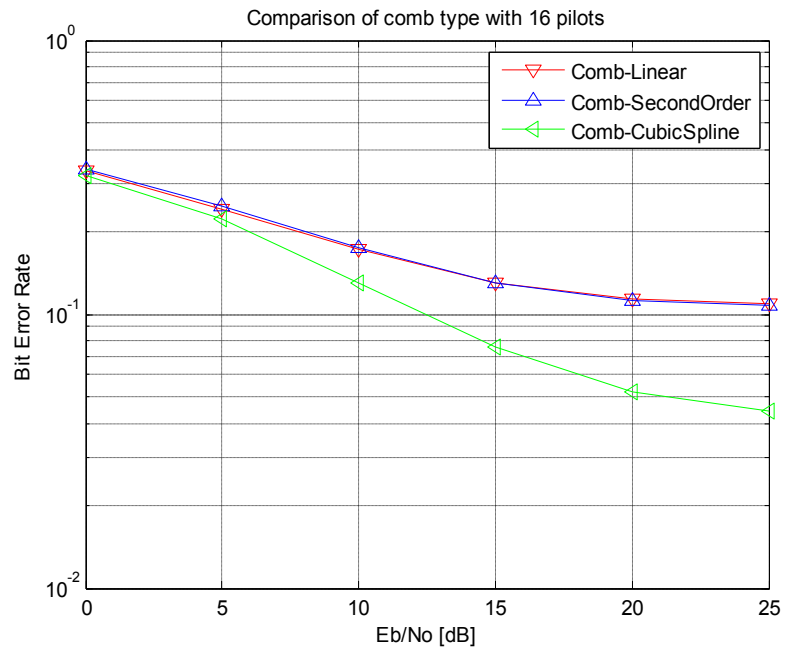


Figure 5-19 Comparison of comb type with 16 pilots

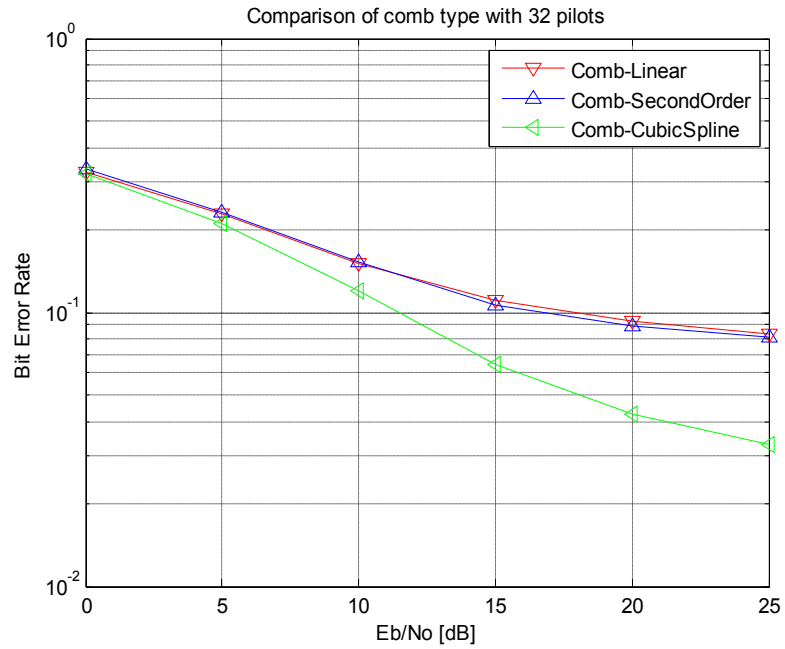


Figure 5-20 Comparison of comb type with 32 pilots

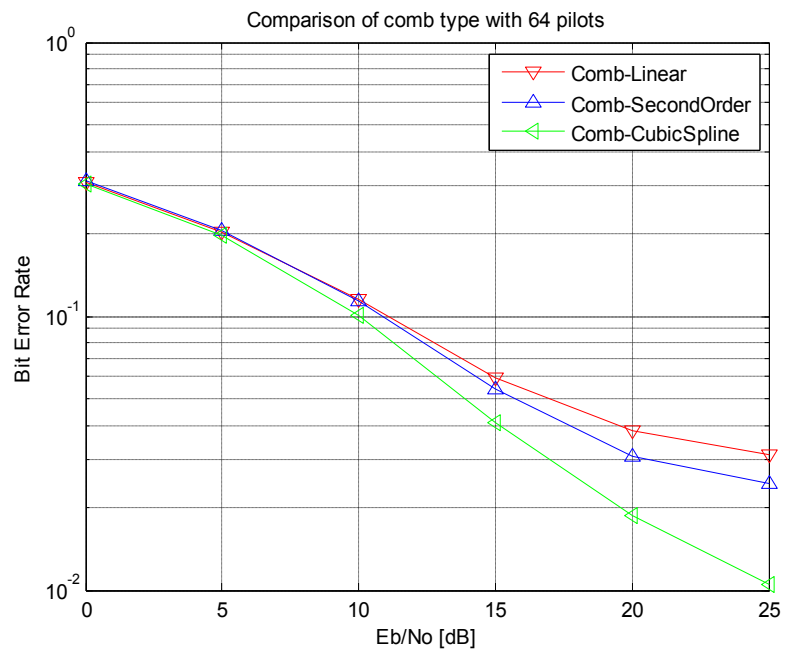


Figure 5-21 Comparison of comb type with 64 pilots

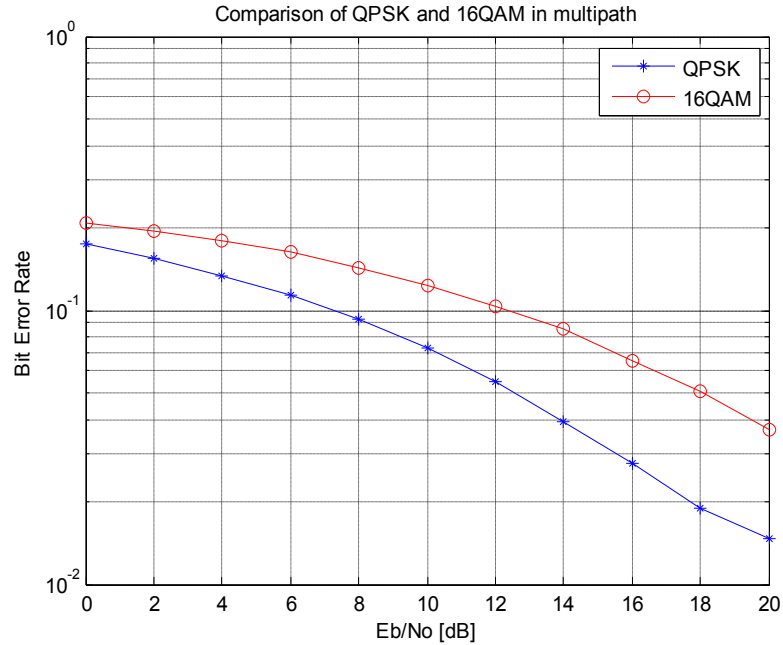


Figure 5-22 Comparison of QPSK and 16QAM under multipath

Figure 5-22 shows BER performance comparison of QPSK and 16QAM under multipath in OFDM system. Block type pilot-based channel estimation and LS algorithm are used in the simulation. The constellations of QPSK and 16QAM are shown as Figure 2-6 and 2-7 respectively. From Figure 5-22, we could find 16QAM modulation has worse BER performance. But, 16QAM has better bandwidth efficiency. Because 16QAM contains 4 bit information per symbol, and QPSK contains 2 bit information per symbol. And 16QAM is a popular modulation method in the limited bandwidth system especially.

Lastly, Figures 5-23 and 5-24 show the BER performance of block type pilot-based channel estimation in OFDM system under QPSK and 16QAM modulation methods. Figure 5-25 and 5-26 show the BER performance of comb type pilot-based channel estimation in OFDM system under QPSK and 16QAM modulation methods respectively.



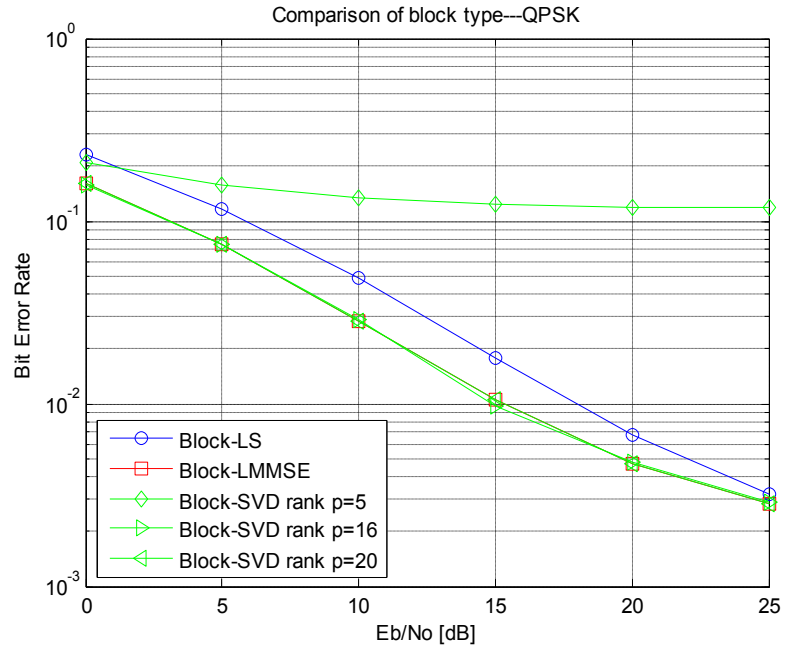


Figure 5-23 Comparison of block type under QPSK modulation

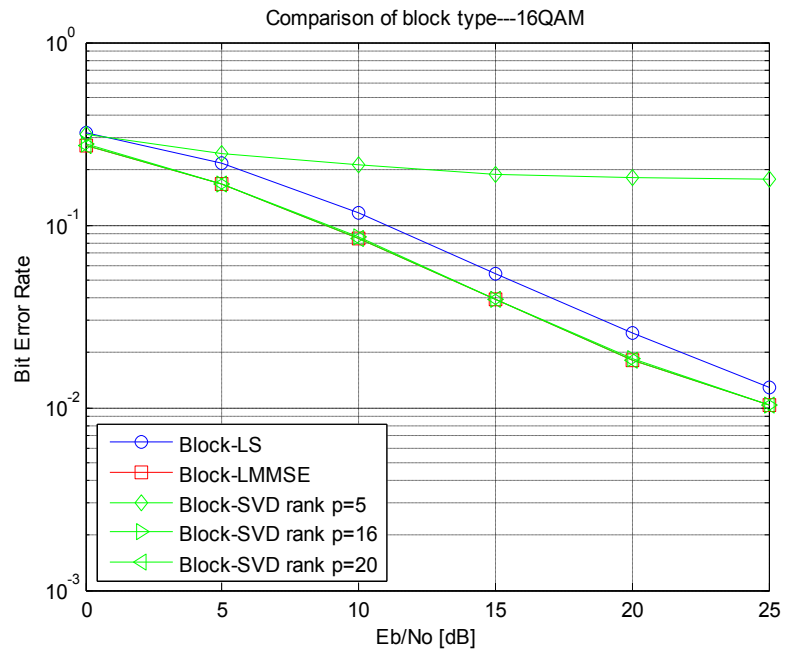


Figure 5-24 Comparison of block type under 16QAM modulation

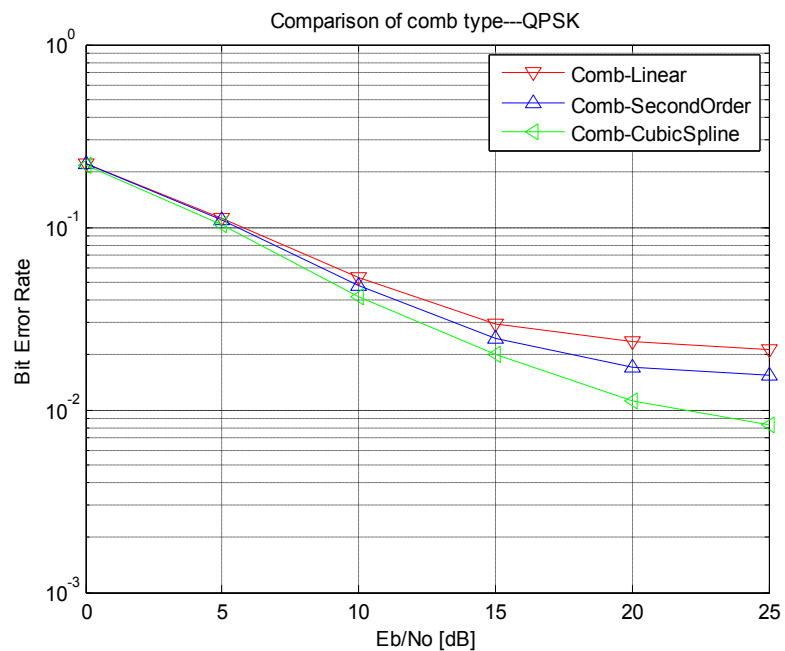


Figure 5-25 Comparison of comb type under QPSK modulation

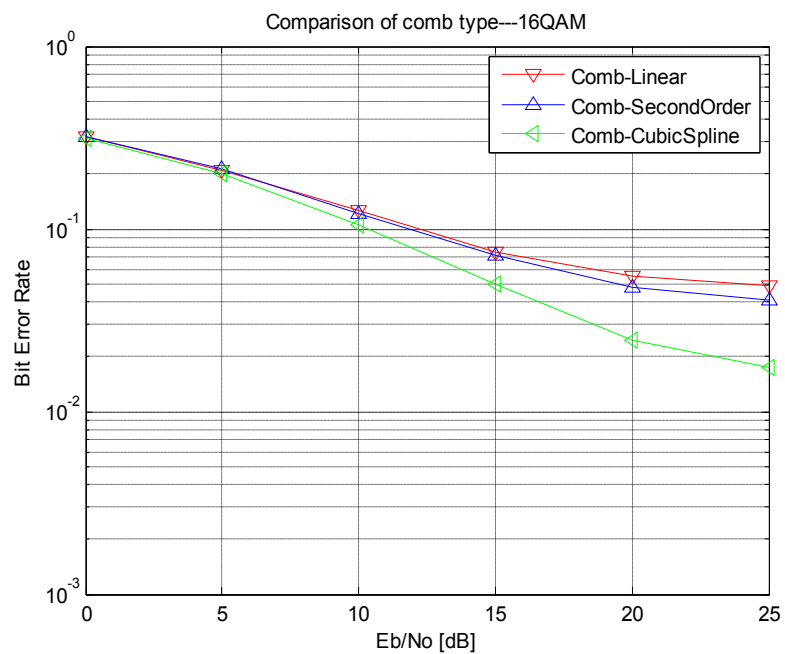


Figure 5-26 Comparison of comb type under 16QAM modulation

## Chapter 6

### Conclusion and Future Work

In this thesis, pilot-based channel estimation of OFDM system is discussed in detail. Focus has been placed on two types of pilot, block type and comb type. The thesis first introduces the OFDM wireless communication technology, history, basic principles, advantages, disadvantages and application prospects. Then, the wireless multipath channel effect on the OFDM system is analyzed theoretically, and the multipath fading channel model is given. After that, the focus on pilot-based channel estimation of OFDM is discussed.

In the OFDM system, some core techniques, such as FFT, CP, and equalizer, are analyzed and simulated. FFT plays an important role for reducing the complexity of OFDM as a multi-carrier communication system. CP is used for eliminating the ISI and avoid the equalizer in time domain. But in frequency domain, equalizer is necessary for channel estimation to reduce the ICI. ZF equalizer is simulated and used in the part of channel estimation.

In slow fading environment, block type pilot-based channel estimation in OFDM system shows better performance. LS estimator yields the worst performance but with the simplest complexity. Based on LS algorithm, LMMSE is analyzed and it shows better performance compared with LS but with more computation complexity. SVD estimator is

further simplification of LMMSE. The simulation results shows that SVD estimator has similar performance with LMMSE estimator when  $p$  is 16 or larger.

In fast fading environment, comb type pilot-based channel estimation in OFDM system has better performance. Linear interpolation, second order interpolation and cubic spline interpolation are discussed. The simulation results show that the performance becomes better as the increasing order of polynomial for interpolation. But the complexity also increases.

The effects of using different types of modulation, QPSK and 16QAM, are compared by simulation. 16QAM shows worse performance. However, for the bandwidth efficiency, 16QAM is more feasible in practice.

Based on this thesis, some of the areas having scope for further research as follows:

1. Other types of pilot. Block type and comb type pilots are two extreme types for different fading environments, slow and fast. In practice, a compromised method is needed, however.
2. This thesis is the research bases on pilot-based channel estimation. Blind channel estimation and semi-blind algorithm are worth being analyzed. Blind estimator does not need information of pilot and has high bandwidth efficiency. And semi-blind algorithm, based on blind channel estimation, does the channel estimation with small amount of pilots, in order to simplify the complexity and to adapt to changes in channel.
3. OFDM system has bottleneck of performance improvement. And other new techniques are needed to combine with OFDM. Multiple input and multiple output (MIMO) system is a popular alternative used in recent wireless communication. And MIMO-OFDM has become a research hotpot in the world.

## References

- [1] J. G. Proakis, Digital Communications, 2nd ed., McGraw-Hill, Ed. New York, 1989.
- [2] International Telecommunication Union website: [www.itu.int](http://www.itu.int).
- [3] Taewon Hwang, Chenyang Yang, Gang Wu, Shaoqian Li, Geoffrey Ye Li, “OFDM and Its Wireless Applications: A Survey”, IEEE Transactions on Vehicular Technology, May 2009, Vol. 58, No. 4, pp. 1673-1694.
- [4] Tricia J. Willink, Paul H. Wittke, “Optimization and Performance Evaluation of Multicarrier Transmission”, IEEE Transactions on Information Theory, 1997, Vol. 43, No.2, pp. 426-429.
- [5] Kenneth G. Paterson, “Generalized Reed-Muller Codes and Power Control in OFDM Modulation”, IEEE Transactions on Information Theory, 2000, Vol. 46, No. 1, pp. 104-120.
- [6] Zigang Yang, Xiaodong Wang, “A Sequential Monte Carlo Blind Receiver for OFDM Systems in Frequency-Selective Fading Channels”, IEEE Transactions on Signal Processing, 2005, Vol. 50, No. 2, pp. 271.
- [7] Hiroshi Harada, Ramjee Prasad, Simulation and Software Radio for Mobile Communications, 1st ed., Artech House, Norwood, MA, 2002.
- [8] B. Le Floch, M. Alard, C. Berrou, “Coded Orthogonal Frequency Division Multiplexing Used in Communication”, IEEE Transactions on Communications, 2003, Vol. 83, No. 6, pp. 982~984.

- [9] Wolfgang Lesch, “Impulse Response Shortening for OFDM in a Single Frequency Network”, 1998.
- [10] en.wikipedia.org.
- [11] Mehmet Kemal Ozdemir, Huseyin Arslan, “Channel Estimation for Wireless OFDM Systems”, IEEE Communications Surveys, 2nd Quarter 2007, Vol. 9, No. 2, pp. 18-48.
- [12] Meng-Han Hsieh, Che-Ho Wei, “Channel Estimation For OFDM Systems Based On Comb-Type Pilot Arrangement In Frequency Selective Fading Channels”, IEEE Transactions on Consumer Electronics, Feb 1998, Vol. 44 , No. 1, pp. 217-225.
- [13] F. Tufvesson, T. Maseng, “Pilot Assisted Channel Estimation for OFDM in Mobile Cellular Systems”, Vehicular Technology Conference, 1997 IEEE 47<sup>th</sup>, May 1997, Vol. 3, pp. 1639-1643.
- [14] R. Van Nee, R. Prasad, OFDM for Wireless Multimedia Communications, 1st ed, Artech House, Norwood, MA, 2000.
- [15] R. Negi, J. Cioffi, “Pilot Tone Selection for Channel Estimation in A Mobile OFDM System”, IEEE Transactions on Consumer Electronics, August 1998, Vol. 44, No. 3, pp. 1122-1128.
- [16] P. Hoher, S. Kaiser, P. Robertson, “Two-dimensional Pilot-Symbol-Aided Channel Estimation by Wiener Filtering”, Acoustics, Speech, and Signal Processing, 1997. ICASSP-97., 1997 IEEE International Conference, Apr 1997, Vol. 3, pp. 1845-1848.
- [17] Geoffrey Ye Li, “Pilot-symbol-aided Channel Estimation for OFDM in Wireless Systems”, IEEE Transactions on Vehicular Technology, July 2000, Vol 49 No 4, pp. 1207 – 1215.

- [18] S. Weinstein, P. Ebert, "Data Transmission by Frequency-Division Multiplexing Using the Discrete Fourier Transform", IEEE Transactions on Communication Technology, October 1971, Vol. 19 No. 5, pp. 628 – 634.
- [19] Geoffrey Ye Li, "Simplified Channel Estimation for OFDM Systems with Multiple Transmit Antennas", IEEE Transactions on Wireless Communications, January 2002, Vol. 1, No. 1, pp. 67-75.
- [20] Van de Beek, J.-J. Edfors, O. Sandell, M. Wilson, S.K. Borjesson, "On Channel Estimation in OFDM Systems", Vehicular Technology Conference, 1995 IEEE 45th, July 1995, Vol. 2, pp. 815-819.
- [21] S. Coleri, M. Ergen, A. Puri, A. Bahai, "Channel Estimation Techniques Based on Pilot Arrangement in OFDM Systems", IEEE Transactions on Broadcasting, Sep 2002, Vol. 48, No. 3, pp. 223-229.
- [22] Baoguo Yang, Zhigang Cao, K.B. Letaief, "Analysis of low-complexity windowed DFT-based MMSE channel estimator for OFDM systems", IEEE Transactions on Communications, Nov 2001, Vol. 49, No.11, pp. 1977 – 1987.
- [23] O. Edfors, M. Sandell, Van de Beek, J.-J. Wilson, S.K. Borjesson, "OFDM Channel Estimation by Singular Value Decomposition", IEEE Transactions on Communications, July 1998, Vol. 46, No.7, pp. 931-939.
- [24] S. Galih, R. Karlina, A. Irawan, T. Adiono, A. Kurniawan, Iskandar, "Low Complexity Partial Sampled MMSE Channel Estimation For Downlink OFDMA IEEE 802.16e System", Intelligent Signal Processing and Communication Systems, 2009. ISPACS 2009. International Symposium, Jan 2009, pp. 162-166.

- [25] Van de Beek, J.-J. Edfors, O. Sandell, M. Wilson, S.K. Borjesson, “On Channel Estimation in OFDM Systems”, Vehicular Technology Conference, 1995 IEEE 45th, July 1995, Vol. 2, pp. 815-819.
- [26] L., Jr. Cimini, “Analysis and Simulation of a Digital Mobile Channel Using Orthogonal Frequency Division Multiplexing”, IEEE Transactions on Communications, July 1985, Vol. 33 , No. 7, pp. 665-675.
- [27] Hussein Hijazi, Laurent Ros, “Polynomial Estimation of Time-Varying Multipath Gains with Intercarrier Interference Mitigation in OFDM Systems”, IEEE Transactions On Vehicular Technology, Jan 2009, Vol. 58, No. 1, pp. 140-151.
- [28] Knut Conradsen, “An Introduction to Statistics Vol 2”, Lecture Notes, IMM, DTU, 2005.
- [29] Y. Shen, Ed Martinez, “Channel Estimation in OFDM Systems”, Freescale Semiconductor, 2006, pp. 1-9.