Chapter 6

Digital transmission through band-limited AWGN channels

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Problems

6-16, 6-14, 6-22

Contents 5 Hrs

- ISI and zero-ISI condition

  (Ref p380-381, p393-394)

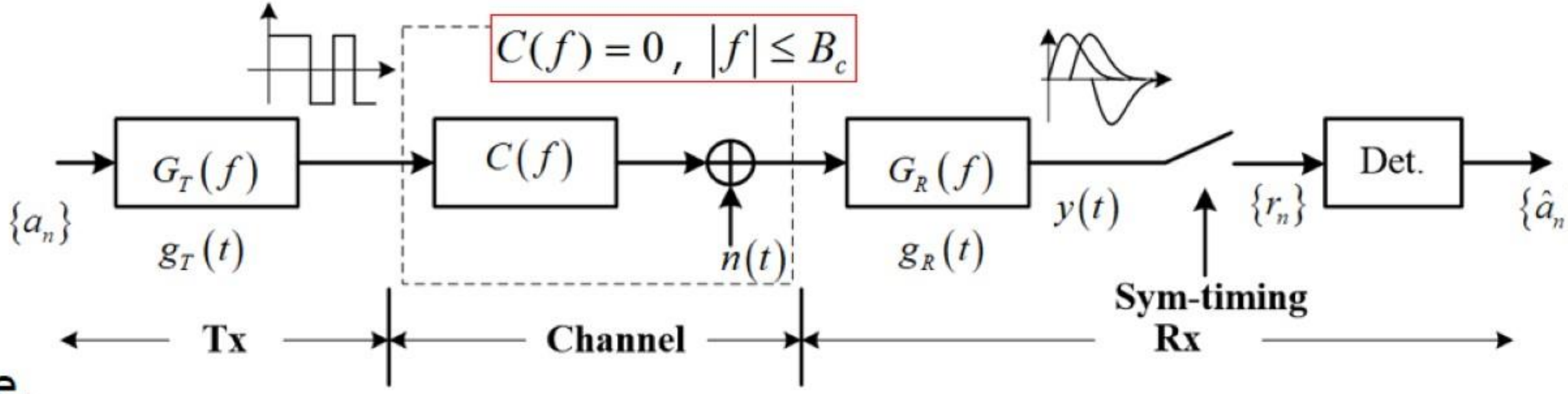
- Design of BL signals for zero-ISI

  (Ref p396-399)

- OFDM

6.1 ISI and zero-ISI condition

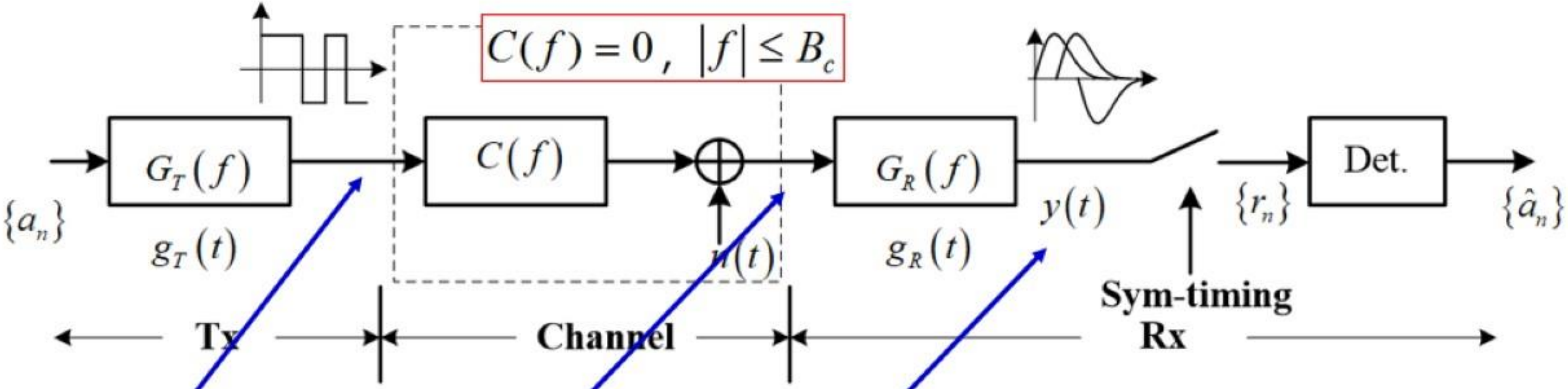
Consider the general block diagram as follows. Without loss of generality, we consider baseband (BB) case.



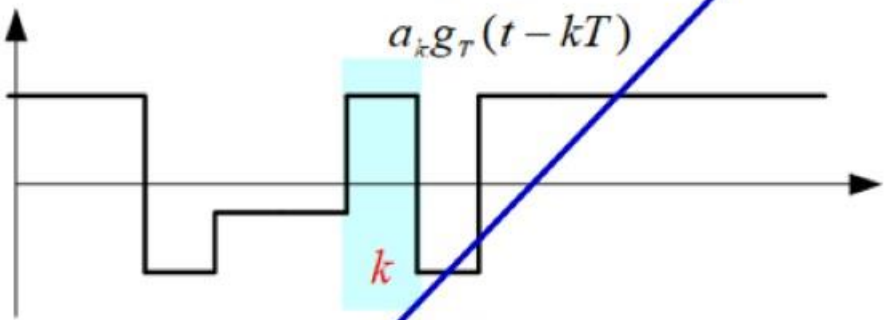
where,

- $G_T(f)$ represents the freq-characteristic of the transmission signal, which is determined by the pulse shape.
- $C(f)$ the freq-response of the channel.
- $G_R(f)$ the freq-response of the receiving filter. (eg. the MF)
- The corresponding time representations are $g_T(t)$, $c(t)$ and $g_R(t)$ respectively.

6.1 ISI and zero-ISI condition



Trans. Signal: $s(t)=\sum_{k=-\infty}^{\infty} a_k g_T(t-kT)$



Received sig: $r(t)=\sum_{k=-\infty}^{\infty} a_k g_T(t-kT)*c(t) + n(t)$

Filtered sig:

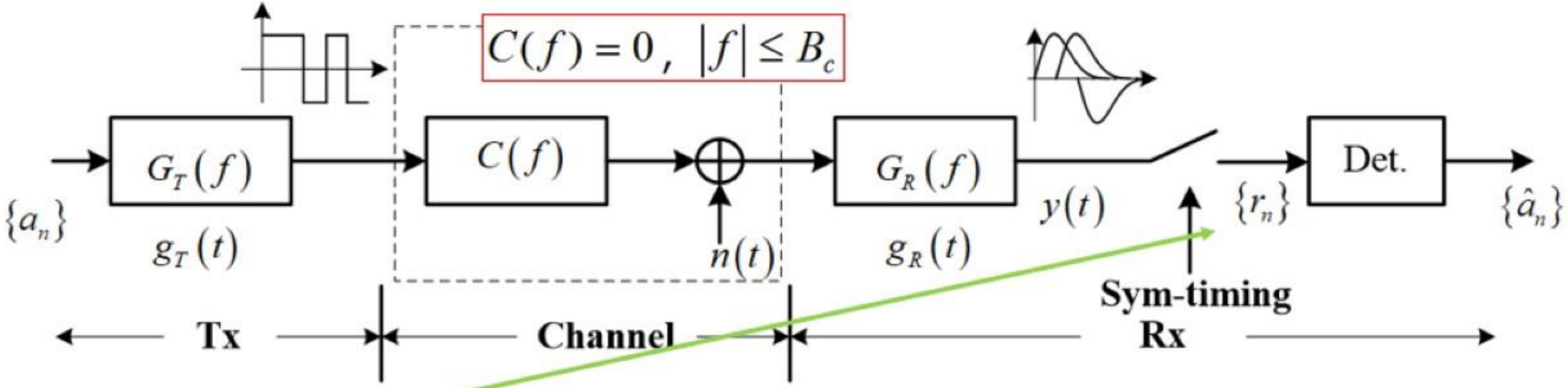
$$y(t)=r(t)*g_R(t)$$

$$=\sum_{k=-\infty}^{\infty} a_k g_T(t-kT)*c(t)*g_R(t)+n(t)*g_R(t)$$

$$=\sum_{k=-\infty}^{\infty} a_k h(t-kT)+v(t)$$

where, $h(t)=g_T(t)*c(t)*g_R(t)$ is the overall resp.

6.1 ISI and zero-ISI condition



Observation:

$$r_n = y(nT) = \sum_{k=-\infty}^{\infty} a_k h(nT-kT) + v(nT)$$

$$= \boxed{\cdots + a_{n-2}h(2T_s) + a_{n-1}h(T_s) + a_{n+1}h(-T_s) + a_{n+2}h(-2T_s)} + \boxed{a_n h(0)} + \cdots + \boxed{v(nT)}$$

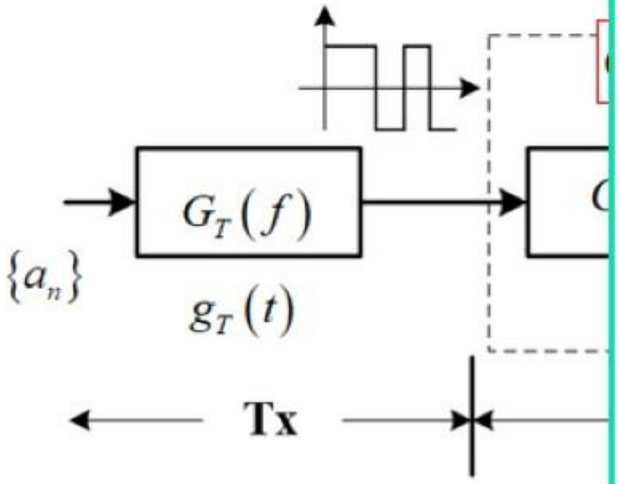
$$= a_n h(0) + \text{ISI} + v(nT)$$

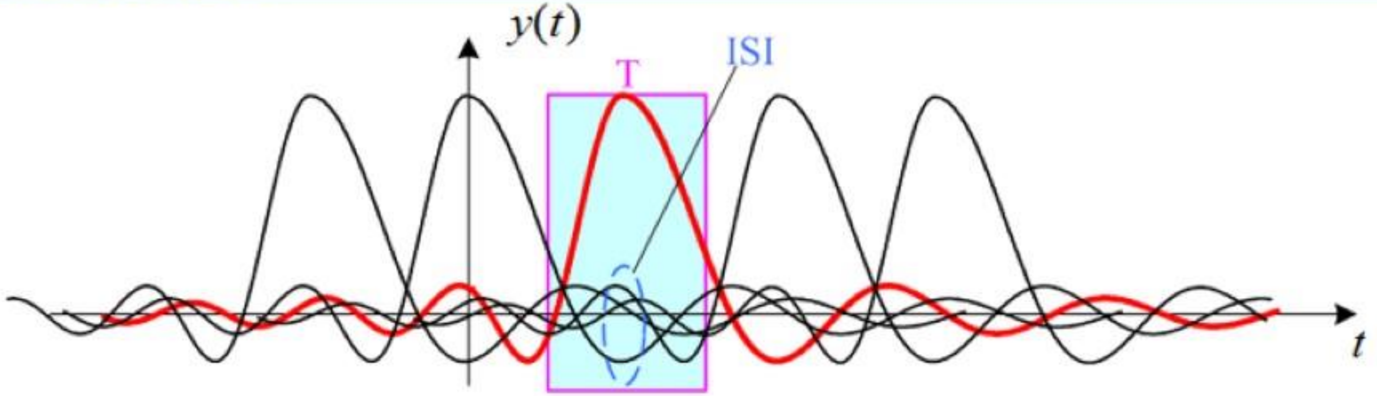
The 1st term on RHS provides the nth symbol a_n.

The 3rd term represents additive noise.

The 2nd term provides no info on a_n, and is an harmful interference, called Inter-Symbol Interference (ISI)

6.1 ISI and zero-ISI condition





The tails on both sides of $h(t)$ out of the symbol interval cause the ISI.

Observation:

$$r_n = y(nT) = \sum_{k=-\infty}^{\infty} a_k h(nT - kT) + v(nT)$$

$$= \boxed{\cdots + a_{n-2}h(2T_s) + a_{n-1}h(T_s) + a_{n+1}h(-T_s) + a_{n+2}h(-2T_s) + \cdots} + \boxed{a_n h(0)} + \boxed{v(nT)}$$

$$= a_n h(0) + \text{ISI} + v(nT)$$

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6.1 ISI and zero-ISI condition

Let $H(f)$ be the Fourier transform of h(t) and $R_s = 1/T$ be the symbol rate.

Nyquist zero-ISI theorem:

$$x_n = c\delta[n] \quad \Leftrightarrow \quad \sum_{k=-\infty}^{+\infty} H(f - kR_s) = const.$$

Observation:

$$r_n = y(nT) = \sum_{-\infty}^{\infty} a_k h(nT - kT) + v(nT)$$

$$= \boxed{\cdots + a_{n-2}\underbrace{h(2T_s)}_{0} + a_{n-1}\underbrace{h(T_s)}_{0} + a_{n+1}\underbrace{h(-T_s)}_{0} + a_{n+2}\underbrace{h(-2T_s)}_{0} + \cdots} + \boxed{a_n \underbrace{h(0)}_{c}} + \boxed{v(nT)}$$

$$= a_n h(0) + \cancel{\text{ISI}} + v(nT)$$

Zero-ISI condition: the overall communication system has to be designed such that $h(t)$ satisfies the following.

$$x_n = h(nT) = c\delta[n] = \begin{cases} c & n = 0 \\ 0 & n \neq 0 \end{cases}$$

where c is an arbitray non-zero constant.

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Let $H(f)$ be the Fourier transform of h(t) and $R_s = 1/T$ be the symbol rate.

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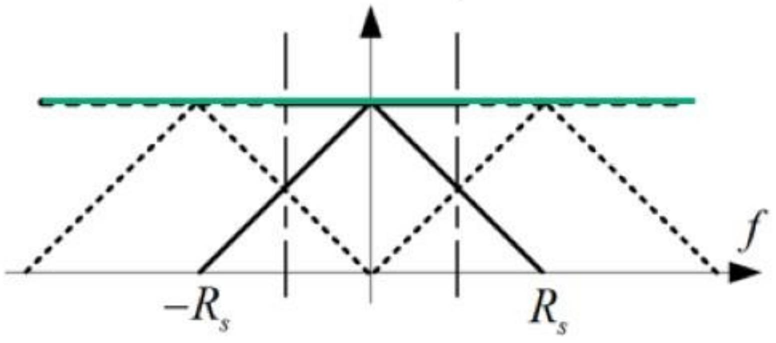
As shown in fig, the condition requires the summation yields a flat spectrum.

For $\sum_{k=-\infty}^{+\infty} H(f - kR_s)$ is periodic with R_s, we pay attention to the interval of $(-R_s/2, +R_s/2)$.



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