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Maximum Likelihood Algorithms using Iterative Maximization Methods for Joint Estimation in MIMO OFDM Systems

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Abstract: A Modern wireless broadband system of MIMO OFDM (multiple input multiple output orthogonal frequency division multiplexing) is more popular because of good data transmission rate, robustness against multipath fading and better spectral efficiency. This system provides reliable communication and wider coverage. A main challenge to MIMO OFDM system is the retrieval of channel state information (CSI) accurately and the synchronization between the transmitter section and receiver section. Proper estimation of synchronization impairments of the system increases the accuracy of Channel State Information obtained. Thus, the concept of joint estimation of system induced synchronization impairments and channel together is adopted. Hence, Maximum Likelihood Algorithms for the joint estimation of synchronization impairments and channel for MIMO-OFDM Systems has been carried out in this work. Maximum Likelihood (ML) Algorithm using grid search method for joint parameter estimation has been carried out and results are analyzed. Higher computational complexity introduced by the method of grid search is a matter of concern in this method. Thus, the method of grid search is replaced by the concept of Newton Raphson that solves the optimization problem with reduced number of computations. Performance of the estimation algorithms is studied through numerical simulations and shows that ML using Newton-Raphson method (MLNR) offers better performance than the conventional ML using grid search method. Theory states that computational complexity is highly reduced on an average implementing Newton-Raphson method.

Keywords: MIMO OFDM; Maximum Likelihood; Grid Search; MLNR.

I. INTRODUCTION

In this age of technological acceleration and sophistication, telecommunication systems mainly rely on wireless communication channels, which offer wide range of services including voice, data and multimedia packets. This creates a growing demand for enlarging the wireless link capacity and efficiency. Modern wireless communication services are supposed to offer high data-rate transmission and enlarged network capacity, at enhanced quality of service. One of the promising improvements is the migration from SISO (Single Input Single Output) systems to MIMO (Multiple Input Multiple Output) configurations [1]. MIMO configurations offers significant increase in data throughput and link range without additional bandwidth or increased transmit power. Furthermore the concept of Orthogonal Frequency Division Multiplexing (OFDM) has come on the scene as a supportive multi-channel modulation and multiplexing scheme. The integration of multiple input multiple output (MIMO) configuration and orthogonal frequency division multiplexing (OFDM) techniques has become a preferred solution for the high-rate wireless technologies because of its high spectral efficiency, robustness to frequency selective fading, enhanced diversity gain and improved system capacity. However, OFDM systems are susceptible to system induced synchronization impairments such as frequency offsets and time offset, that degrades the system performance [4]. Thus, for productive performance of MIMO OFDM systems, these impairments are to be properly estimated so as to carry out proper demodulation of the signal at the receiver section.

Over the years, research is being carried out to estimate system impairments and channel state information for MIMO OFDM communication systems [2,3]. Extensive studies have been carried out in the area of channel estimation schemes for MIMO OFDM systems, but the concept of joint estimation of channel and synchronization impairments together has not been explored considerably. Certain efforts on the joint estimation have been cited in the literature. A pilot-based method for the joint estimation of channel impulse response (CIR) and frequency offset parameters (carrier frequency offset (CFO) and sampling frequency offset (SFO)) is proposed in [9]. Here the timing error is neglected. Joint estimation of frame timing and carrier frequency offset is considered in [10], but sampling frequency offset, SFO is taken as zero. A method for jointly estimating frequency offsets (CFO and SFO) is proposed in [12], assuming known channel information and taking STE zero. Further, neglecting the effect of CFO and STE, joint ML estimations in OFDM systems for SFO and channel are also done [11]. Cramer Rao Lower bound analysis for estimating continuous parameters explains the coupling effect between these synchronization impairments and hence point out/elucidate the need for a joint estimation algorithm that considers all these synchronization impairments together with channel response. In this work, an attempt has been made to jointly consider all the three synchronization impairments along with the channel response using Maximum Likelihood Algorithms.

In the proposed method, Maximum Likelihood algorithm is implemented as an optimization concept where the estimation of the parameters, being a multidimensional optimization problem is granulated into lower dimensional search problems. The method of grid search is used for the ML estimation. Here, the ML Algorithm is granulated into two dimensional (2-D) and one dimensional (1-D) grid search. The method of grid search offers higher computational complexity. Further, as the need for estimation accuracy increases, the number of grid points specified has to be increased, that further adds to the number of computations involved and thereby increases the computational complexity, processing time involved and computation cost. Higher computational complexity introduced by the method of grid search can be resolved by making a replacement for the method of grid search. Thus, alternative methods for joint parameter estimation using ML Algorithms have been studied. Analysis states that the grid search method could be replaced with options like Newton Raphson method, scoring approach etc to reduce the number of computations involved.

II. MIMO OFDM SYSTEM MODEL

A typical MIMO OFDM system can be interpreted in three sections - Transmitter, communication channel and receiver [5]. As the name says, a MIMO system simply employs multiple antennas at terminal points, say transmitter and receiver. Here, we have N_T transmit antennas and N_R receive antennas and N subcarriers per transmit antenna, following M-ary modulation scheme. MIMO technology exploits the concept of multipath propagation as a method to multiply the radio channel capacity. The minimum of the number of transmit and receive antennas decides the factor of capacity growth. The concept of spatial multiplexing is employed for simultaneous transmission of encoded data stream, from multiple transmit antennas, reusing the space dimension. Fig. 1 show the block diagram for N_T x N_R MIMO OFDM system that employs a joint estimator for estimating the values of synchronization impairments conjointly with channel response.

The serial binary bit stream from the data source is passed to the MIMO encoder that converts it into two parallel streams (In case of 2x2 MIMO systems). The parallel signal streams enter the OFDM modulator that performs the IFFT operation. The signal then passes through the DAC and RF units, before it is put for transmission through the MIMO channel. The insertion of cyclic prefix (CP), such that length of CP, L_{cp}> =L, where L is the length of the channel impulse response, transforms the frequency-selective MIMO fading channel to multiple frequency-flat MIMO fading channels. Thus, the signal transmitted from the u_{th} radio frequency transmit antenna is faded before being received by the v_{th} receive antenna. Thus each receive antenna gets a mixture of faded signal components from all transmit antennas along with noise.

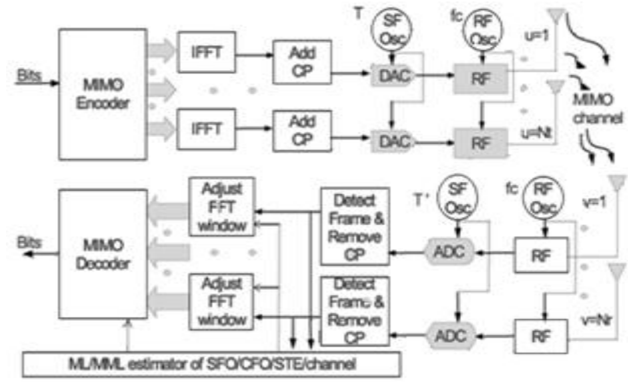

Fig 1: Block diagram of a MIMO OFDM system.

The synchronization between transmitter and receiver sections of a MIMO OFDM system has a critical role in maintaining the system performance. Synchronization in terms of time and frequency is considered. The DAC (Digital-to-Analog converter) unit at the transmitter and ADC (Analog-to-Digital converter) at the receiver should have the same clocks. If T is the sampling time at the transmitter, received signal is sampled at time T', causing a sampling time offset, SFO of ΔT=T'-T. Methods of frame time detection in the literature carries out course synchronization resulting in a symbol timing error, STE. Let the normalized STE be θ. Any mismatch in the clock timing leads to addition or removal of samples. Further, the point where the first sample of the first OFDM symbol begins, is not known to the receiver. Hence, time synchronization turns significant. Frequency offset occur due to the mismatch in local oscillator frequencies between transmitter and receiver as well as due to the effect of Doppler spread (relative motion between the transmitter and receiver) and results in a carrier frequency offset, CFO=Δfc in the signal received, where fc the radio frequency of the RF oscillator. Hence, synchronization between transmitter and receiver is important for accurate channel estimation and hence, for a superior system performance.

The CIR between the u^th transmit and v^th receive antenna is $h_{u,v}(\tau) = \sum_{l=0}^{L_{u,v}-1} h_{u,v,l}\,\delta(\tau - \tau_l)$, where $h_{u,v,l}$ denotes the channel coefficient, τ_1 denotes the l^th channel path delay ($\tau_l = lT$) and $L_{u,v}$ denotes the length of the CIR, for $u = 1,2,3,\ldots,N_T$ and $v = 1,2,3,\ldots,N_R$.Here, L_m=max($L_{u,v}$). The signal received at the v^th receive antenna is given as

$$r_v(n) = \exp\left(j2\pi\Delta f_c nT'\right)\sum_{u=1}^{N_T}\sum_{l=0}^{L_m-1} h_{u,v,l} * S_u\left(nT' - \theta T - \tau_l\right) + w_v(n) \quad (1)$$

We have

$$S_u\left(nT' - \theta T - \tau_l\right) = S_u\left(n(T + \Delta T) - \theta T - \tau_l\right)$$

$$= S_u\left((n_\eta - \theta - l)T\right) \tag{2}$$

$$\exp(j2\pi\Delta f_c n T') = \exp(j2\pi\varepsilon n(T+\Delta T)/NT) = \exp(j2\pi\varepsilon_\eta n/N) \tag{3}$$

Where $\varepsilon_\eta = \varepsilon(1+\eta)$ and $n_\eta = n(1+\eta)$.

Thus,

$$r_v(n) = \exp(j2\pi\varepsilon_\eta n/N)\sum_{u=1}^{N_T}\sum_{l=0}^{L_m-1} h_{u,v,l}\, S_u\left((n_\eta - \theta - l)T\right) + w_v(n) \tag{4}$$

The channel frequency response (CFR) is expressed as

$$\tilde{h}_{u,v}(k) = \sum_{l=0}^{L_{u,v}-1} h_{u,v,l}\exp(-j2\pi kl/N) \tag{5}$$

$$r_v(n) = \frac{\exp\left(\frac{j2\pi\varepsilon_\eta n}{N}\right)}{N}\sum_{u=1}^{N_T}\sum_{k=0}^{N-1}\exp\left(\frac{j2\pi n_\eta k}{N}\right) * \exp\left(-\frac{j2\pi\theta k}{N}\right)\tilde{h}_{u,v}(k)\tilde{x}_u(k) + w_v(n) \tag{6}$$

$$[F_1(\eta)]_{n,k} = \frac{\exp(j2\pi(n(1+\eta))/N)}{N} \tag{7}$$

$$[F_2]_{k,l} = \exp(-j2\pi lk/N) \tag{8}$$

$$r_v = D(\varepsilon,\eta)F_1(\eta)G(\theta)X\tilde{h}_v + w_v \tag{9}$$

Where,

$$D(\varepsilon,\eta) = diag\left[1, \exp\left(\frac{j2\pi\varepsilon_\eta}{N}\right), \ldots, \exp\left(\frac{j2\pi\varepsilon_\eta(N-1)}{N}\right)\right]$$

$$G(\theta) = diag\left[1, \exp\left(\frac{-j2\pi\theta}{N}\right), \ldots, \exp\left(\frac{-j2\pi(N-1)\theta}{N}\right)\right]$$

$$[\tilde{h}_v]_{NNT} \times 1 = \left[\tilde{h}_{1,v}{}^T, \tilde{h}_{2,v}{}^T, \ldots, \tilde{h}_{N_T,v}{}^T\right]^T$$

$$[X]_{N\times NNT} = [X_1, X_2, \ldots, X_{N_T}]$$

With

$$X_u = diag[\tilde{x}_u(0), \tilde{x}_u(1), \ldots, \tilde{x}_u(N-1)]$$

and

$$\tilde{h}_{u,v} = [h\tilde{\ }\_(u,v)\,(0), h\tilde{\ }\_(u,v)\,(1), \ldots, h\tilde{\ }\_(u,v)\,(N-1)]^T$$

We have CIR, denoted by $[h_{u,v}]_{L_m\times 1}$ and CFR, denoted by $[\tilde{h}_{u,v}]_{N\times 1}$ related as $\tilde{h}_{u,v} = F_2 h_{u,v}$ Thus,

$$r_v = D(\varepsilon,\eta)F_1(\eta)G(\theta)X(I_{N_T}\otimes F_2)h_v + w_v \tag{10}$$

Where, $[h_v]_{N_T L_m\times 1} = \left[h_{1,v}{}^T, h_{2,v}{}^T, \ldots, h_{N_T,v}{}^T\right]^T$

$$r = A(\varepsilon,\eta,\theta)h + w \tag{11}$$

Where,

$$A(\varepsilon,\eta,\theta) = I_{N_R}\otimes\left(D(\varepsilon,\eta)F_1(\eta)G(\theta)X(I_{N_T}\otimes F_2)\right) \tag{12}$$

III. MAXIMUM LIKELIHOOD ESTIMATION

The principle of maximum likelihood estimation (MLE), was introduced in 1920s by R.A. Fisher, which states that the probability distribution that makes the observed data ‘most likely,’ is considered as the desired probability distribution [7]. Hence, the value of the parameter vector that maximizes the likelihood functions. L(w|y) is to be selected. The resulting parameter vector, obtained by searching the multi dimensional parameter space, is called the MLE estimate. The consistency, efficiency and asymptotic normality offered by ML estimators, makes it a preferred solution for most of the multidimensional optimization problems. The likelihood function for a parameter, given outcome y is equal to the probability of the observed outcome when the parameter value is given. Here we have a set of parameters under consideration, where ϕ is the set of parameter values. Thus likelihood function L(ϕ |y)=P(Y| ϕ). Here, the outcome is the received signal r and ϕ = {ϵ,θ,ή,h}.

$$\arg\max_{\varepsilon,\eta,\theta,h} P(r|\varepsilon,\theta,\eta,h) = \arg\max_{\varepsilon,\eta,\theta,h}\frac{1}{(\pi\sigma_w^2)^{NN_R}}\exp\left\{\frac{-\|r-Ah\|_2^2}{\sigma_w^2}\right\} \tag{13}$$

Computing the log-likelihood function and simplifying, we obtain an equivalent cost function

$$\arg\min_{\varepsilon,\eta,\theta,h} J(\varepsilon,\theta,\eta,h|r) = \arg\min_{\varepsilon,\eta,\theta,h}(r-Ah)^H(r-Ah) \tag{14}$$

A. Numerical Determination of MLE

As the name says, MLE is simply the maximum of a known Likelihood function. When the values of parameter ϕ lies within a known interval, we need to simply maximize p(x; ϕ) over that interval. The safest way to do this is to perform an exhaustive search over the interval. The method of grid search, thus guarantees the most accurate estimate. As the required degree of accuracy is more, the number of grid points could be increased simply by reducing the spacing between the sample points in the interval considered. Here, a 3-D grid search is required for multiple parameter estimation [8]. In ML algorithm, the optimization problem is reduced to 2-D and 1-D grid search. However, the inherent complexity induced by the grid search method still remains. Computational complexity is proportional to the number of grid points involved.

For STE, Number of grid points, g_θ= = (θ_{max}- θ_{min})/ θ_{grid}

For CFO, Number of grid points, g_ϵ= ($\epsilon_{max}\epsilon_{min}$)/ ϵ_{grid}

For SFO, Number of grid points, $g_{ή}$= ($ή_{max}$-$ή_{max}$)/ $ή_{grid}$

Considering the ML algorithm, the computational complexity depends on the number of grid points involved in total, for both the 2-D and 1-D searches [6]. Computation complexity for 2-D search is given as $g_\epsilon g_{ή}$O($(N*NR)^3$). Computation complexity for 2-D search is given as g_θO($(N*NR)^3$). Thus, total complexity of ML algorithm is approximately ($g_\epsilon g_{ή}$ + g_θ)O($(N*NR)^3$)

B. Issue of Computational Complexity

For multi-parameter optimization problems, grid search is commonly employed as it has higher chance of giving accurate estimated values. But, since it is an exhaustive search over the specified interval of the parameter to be estimated, it creates higher complexity issues. As the interval becomes larger, the number of grid points employed increases that results in higher computational complexity and hence is graded as a potentially expensive method. As mentioned in the previous section, complexity is directly proportional to the number of grid points. Hence a reduction in the number of grid points could help to reduce the computation issues. One method to reduce the grid points is to increase the resolution of search criteria. But an increase in the resolution increases the chance of degradation in performance of the search algorithm. Another method is to select limited number of samples of the received signal for estimation. Studies reveal that this further causes performance degradation.

Estimation using all the samples of the signal received performs better than an estimation using limited number of samples. Hence, it is concluded that Complexity issues in Maximum Likelihood (ML) estimation employing the method of grid search could be compensated only at the cost of performance. Further, if the range of values is not available, a grid search simply cannot be employed. Hence, we search for alternatives that perform more effectively in a muti-dimensional exhaustive search process. Thus, we move on to iterative maximization procedures. One among the alternative methods that offer faster rate of convergence is Gradient method or simply Newton Raphson method [13]. The essential requirement of such an iterative approach is an initial guess or simply a start value for estimation. Even if the parameter is not bounded, the concept of iterative maximization holds good and stays computationally feasible. Thus Newton Raphson method turns sufficient for multivariable optimization problems.

IV. MAXIMUM LIKELIHOOD USING NEWTON RAPHSON (MLNR) METHOD

For the method of grid search, the available interval is divided into grids of required resolution and likelihood function has to be evaluated for each grid point to determine the MLE. The number of grid points gives a measure of complexity involved. Thus a method which could determine the MLE with minimum number of grid points could maintain the complexity at lower levels. The cost of computation is simply the number of computations. Thus, considering the aspects of cost, complexity and processing time, an optimization technique involving minimum number of grid points performs better. The concept of iterative maximization supports this idea. In any iterative maximization technique, we start from an initial guess, for the estimation of parameter. The MLE is first determined for the initial guess, from where multiple iterations are performed. Based on the correctness of the estimated initial guess, the number of iterations involved varies. But, on an average, the total number of computations involved is less than the case for a conventional grid search method. Moreover, iterative optimization methods maintains the complexity at convenient levels where the conventional grid search method fails, as in case of wider intervals or for estimation problems demanding higher accuracy.

Newton Raphson method is employed in the Maximum Likelihood Algorithm and the MSE is plotted. Performance comparison is done between ML algorithm employing grid search method and the MLNR method. Analysis is carried out in terms of Mean Square Error, computation complexity and processing time. Considering a function f(x) and a parameter x, MLNR algorithm can be explained as follows:

- Determine the start value, x_0.
- Determine the first derivate $f'(x)$.
- Applying Newton Raphson formula:
  $$x_{n+1} = x_n - \frac{f(x)}{f'(x)}$$
- Take the value of x_1 and repeat the above calculations using this as the initial guess.
- Repeat the procedure until the value remains consistent.
- In case of MLNR method, the process is done for maximizing the Likelihood function.

V. EXPERIMENTAL RESULTS

The performance assessment of the estimation algorithms are carried out in terms of the estimation accuracy, cost, computation complexity and processing time. Here cost and processing time generally depends on the computation complexity, which means the number of computations. Hence, the major factors to be considered for assessment are rate of estimation accuracy and computation complexity. The estimation accuracy is derived in terms of the MSE (Mean Square Error) which is calculated as,

$$MSE(p) = \frac{\sum_{i=1}^{N_{trials}} \|\hat{p}_i - p\|_2^2}{N_{trials}} \tag{15}$$

Where, p represents the actual parameter, pi represents the estimate of the parameter obtained at the i^{th} trial and N_{trials} represents the number of trials. The MSE for the carrier frequency offset is plotted against the SNR for ML and MLNR methods. The MLNR method shows significant improvement in the error performance for the entire range of SNR considered. This is further more crucial. In case of channel estimation, at lower values of SNR the improvement by MLNR algorithm is more and as the SNR increases the difference reduces. However, even for higher values of SNR the MLNR method holds good (fig 2).

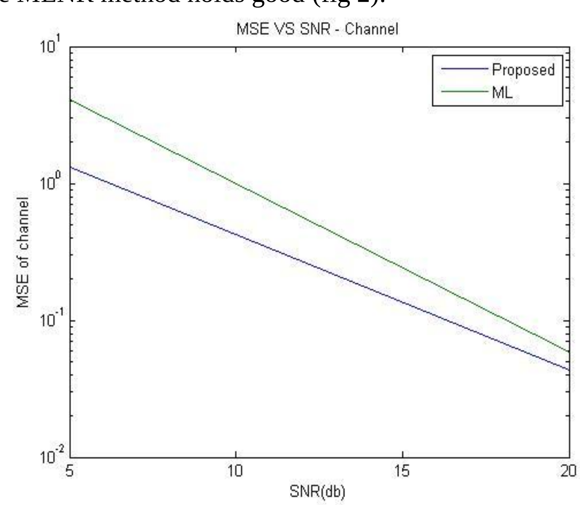

Figure2: MSE for the estimation of channel as a function of SNR (in dB) for a 2x2 MIMO OFDM system using ML and MLNR algorithm.

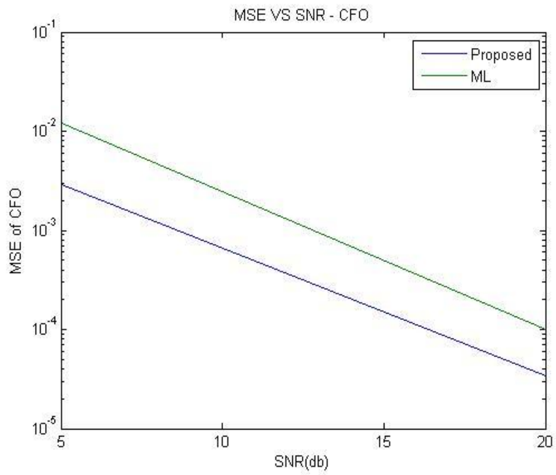

Figure 3: MSE for the estimation of CFO as a function of SNR (in dB) for a 2x2 MIMO OFDM system using ML and MLNR algorithms.

For sampling frequency offset estimation, the initial performance of MLNR method is highly superior than the ML method, and a gradual degradation occurs as the SNR increases. But on an average the MLNR method shows a significant improvement in the error performance, and is hence considered superior to the ML algorithm implementing grid search method. The processing time taken for computation is yet another factor that is used for performance assessment. Generally, the processing time involved is directly proportional to the complexity of the algorithm involved. As the proposed MLNR method is a low complexity design involving lesser number of complex multiplications, it carries the processing at smaller time duration.

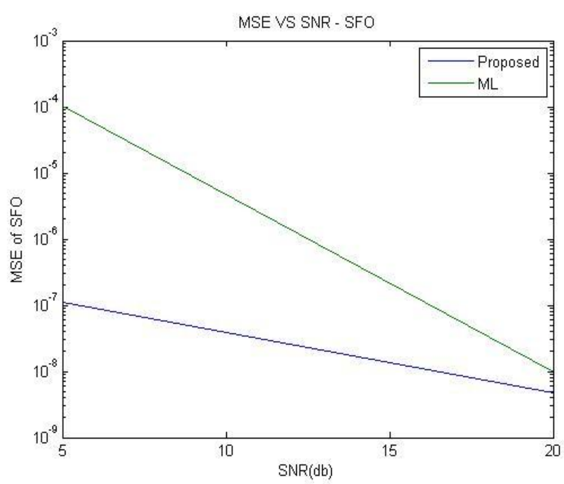

Figure 4: MSE for the estimation of SFO as a function of SNR (in dB) for a 2x2 MIMO OFDM system using ML and MLNR algorithms.

Table 1 shows the processing time required for the joint estimation using the ML algorithm and the proposed method. For smaller number of trials, the time difference is negligible (see fig 3 and 4). As the number of trials increases, the time difference becomes noticeable. Thus in real time experiments, where the estimate is obtained as an average of say 10^x trials, where x ranges from 3 to 6 the proposed method takes lesser time as compared to the ML grid search method.

TABLE 1: **Comparison of Processing Time**

No of Trials	Ml using grid search (in sec)	ML using Newton Raphson (in sec)
1	3.93	4.54
5	14.12	13.85
50	155.07	91.48
100	283.66	194.61
1000	2303.48	1767.11

VI. CONCLUSION

In this paper Maximum Likelihood (ML) algorithms and iterative maximization methods of ML algorithms have been discussed as a solution of a multivariable optimization problem. It deals with the joint estimation of synchronization impairments and channel response in a MIMO OFDM system. The system was modeled such that it shows the effect of impairments such as carrier frequency offset, sampling frequency offset and symbol timing error as well as the effect of channel. Since the impairments shows a coupling effect among themselves, the need for joint parameter estimation becomes significant. Maximum Likelihood algorithms are implemented for joint parameter estimation, which employs the concept of maximizing the likelihood function. To combat the issue of computation complexity to a greater extend, method of grid search employed in ML algorithm is replaced using iterative maximization techniques. Results and analysis states that on an average the number of computations, and therefore the cost and complexity of computation are significantly reduced using iterative maximization methods for ML algorithms at a lower MSE performance.

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