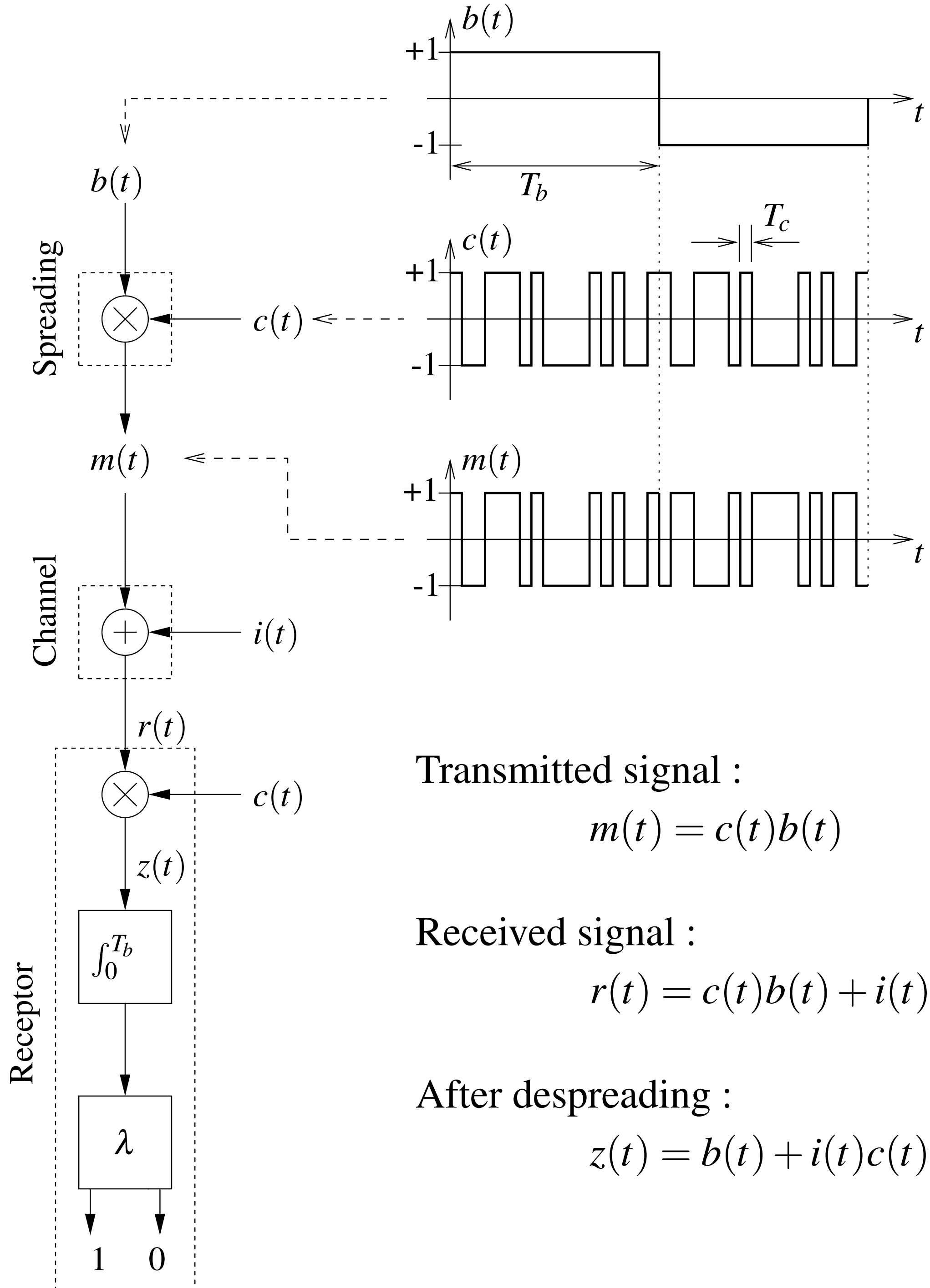
Spread spectrum

Outline :

1. Baseband
2. DS/BPSK Modulation
3. CDM(A) system
4. Multi-path
5. Exercices

1. Baseband



Transmitted signal :

$$m(t) = c(t)b(t)$$

Received signal :

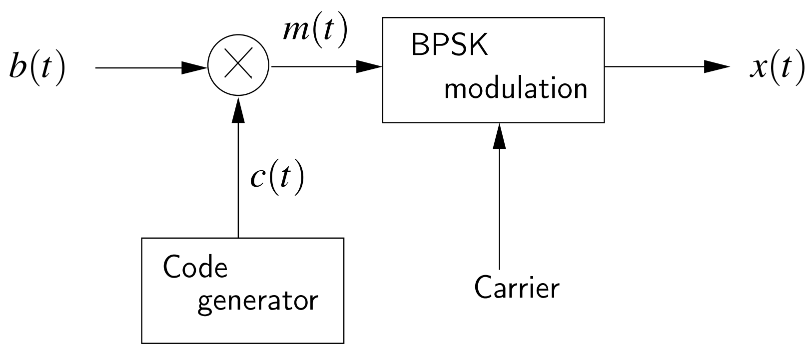
$$r(t) = c(t)b(t) + i(t)$$

After despreading :

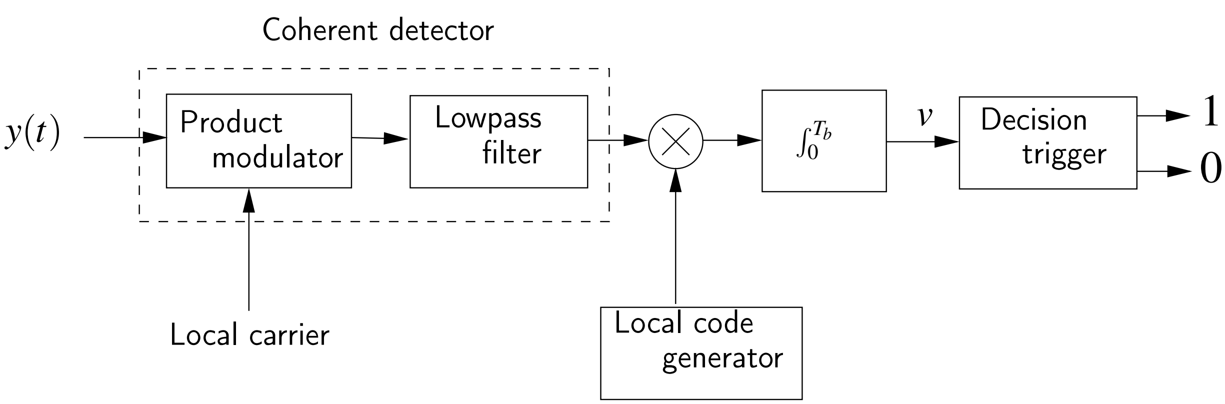
$$z(t) = b(t) + i(t)c(t)$$

2. DS/BPSK Modulation (Direct Sequence Spread Spectrum with coherent Binary Phase Shift Keying)

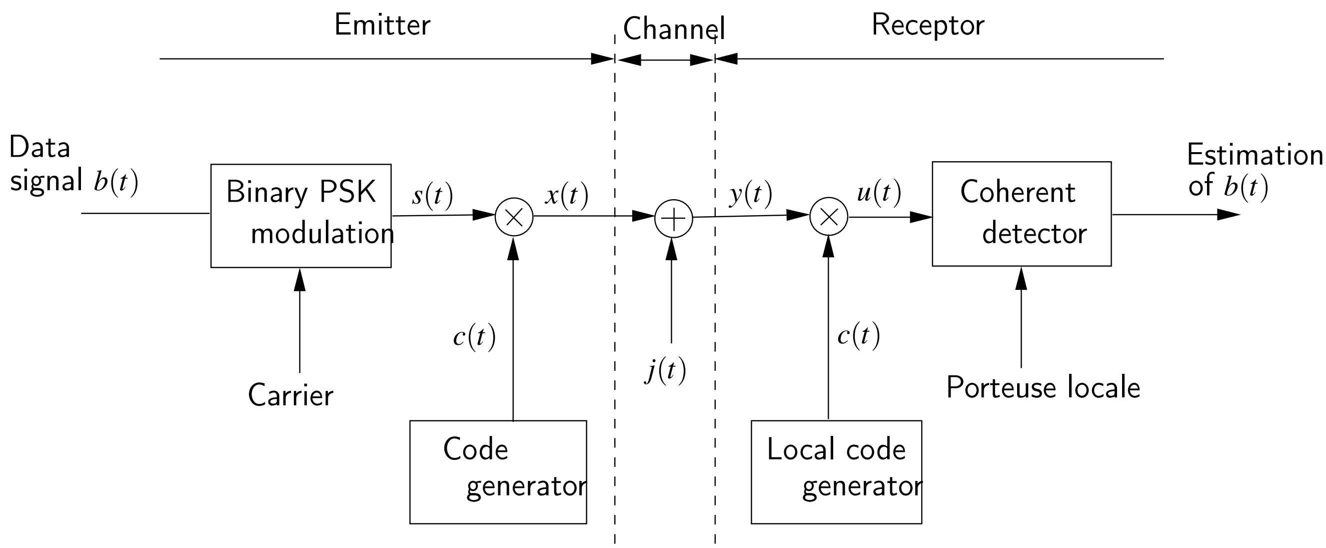
Modulator :



Demodulator :



The spreading and modulation operations are linear $\longrightarrow$ they may be permuted.



At the receptor input :

$$\begin{aligned} y(t) &= x(t) + j(t) \\ &= s(t)\,c(t) + j(t) \end{aligned}$$

where $s(t)$ = BPSK modulation of $b(t)$.

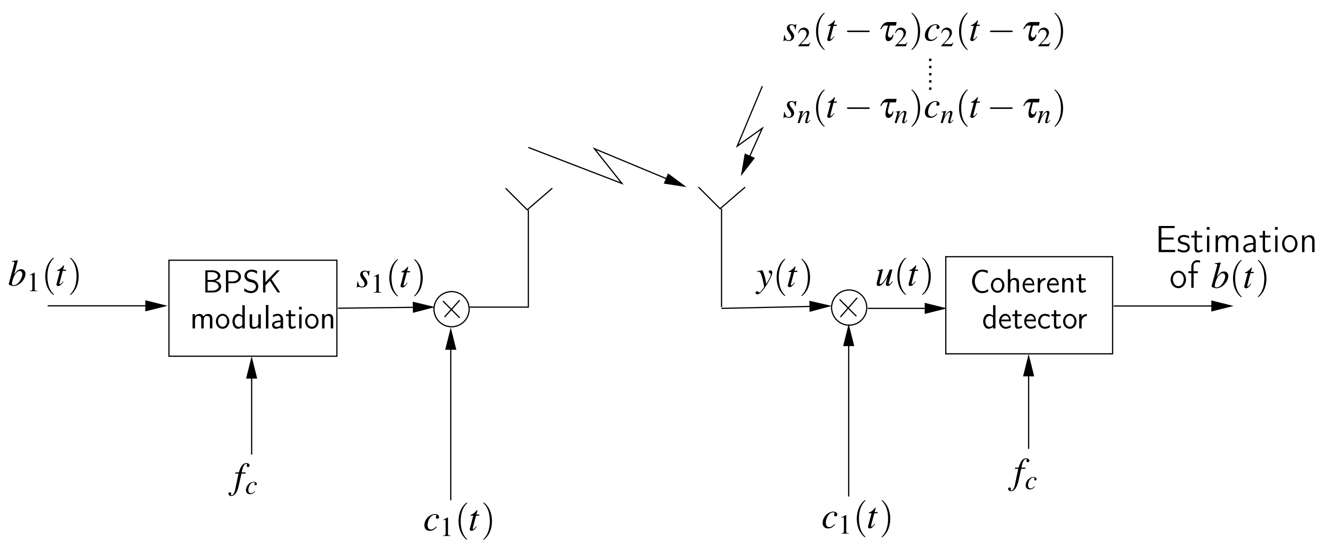
After despreading :

$$\begin{aligned} u(t) &= y(t)\,c(t) \\ &= s(t) + j(t)\,c(t) \end{aligned}$$

3. CDM(A) system

CDM = Code Division Multiplexing

CDMA = Code Division Multiple Access



Received signal :

$$\begin{aligned} y(t) &= s_1(t)\,c_1(t) + s_2(t-\tau_2)\,c_2(t-\tau_2) + \dots \\ &\quad + s_n(t-\tau_n)\,c_n(t-\tau_n) \end{aligned}$$

After despreading :

$$\begin{aligned} u(t) &= y(t)\,c_1(t) \\ &= s_1(t) + s_2(t-\tau_2)\,c_2(t-\tau_2)\,c_1(t) + \dots \\ &\quad + s_n(t-\tau_n)\,c_n(t-\tau_n)\,c_1(t) \end{aligned}$$

After BPSK demodulation :

$$
\begin{aligned}
\widetilde{b}_1(t) &= b_1(t) + b_2(t-\tau_2)\, c_2(t-\tau_2)\, c_1(t) + \ldots \\
&= + b_n(t-\tau_n)\, c_n(t-\tau_n)\, c_1(t)
\end{aligned}
$$

Through the matched filter :

$$
\begin{aligned}
v &= \int_0^{T_b} b_1(t)\, dt \\
&\quad + \int_0^{T_b} b_2(t-\tau_2)\, c_2(t-\tau_2)\, c_1(t)\, dt \rightarrow \pm T_b \Gamma_{12}(\tau_2) \\
&\quad + \ldots \\
&\quad + \int_0^{T_b} b_n(t-\tau_n)\, c_n(t-\tau_n)\, c_1(t)\, dt \rightarrow \pm T_b \Gamma_{1n}(\tau_2)
\end{aligned}
$$

$\rightarrow$ We are looking for spreading codes $c_i(t)$ which are almost uncorrelated. Idealy, we would like to have $\Gamma_{ij}(\tau) = 0$. So we use the GOLD sequences.

4. Multi-path

Received signal :

$$y(t) = s_1(t)\, c_1(t) + \alpha\, s_1(t-\tau)\, c_1(t-\tau)$$

After despreading :

$$\begin{aligned} u(t) &= y(t)\, c_1(t) \\ &= s_1(t) + \alpha\, s_1(t-\tau)\, c_1(t-\tau)\, c_1(t) \end{aligned}$$

After BPSK demodulation :

$$\widetilde{b}_1(t) = b_1(t) + \alpha\, b_1(t-\tau)\, c_1(t-\tau)\, c_1(t)$$

Through the matched filter :

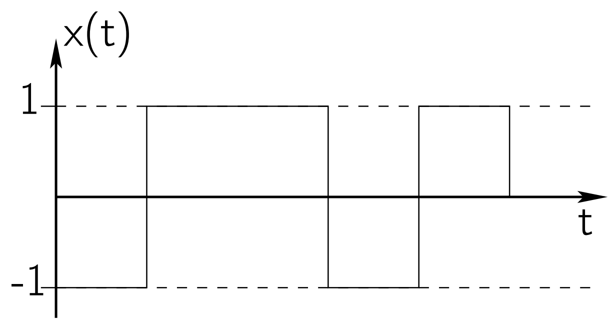
$$\begin{aligned} v &= \int_0^{T_b} b_1(t)\, dt \\ &\quad + \alpha \int_0^{T_b} b_1(t-\tau) c_1(t-\tau) c_1(t)\, dt \rightarrow \pm \alpha T_b \Gamma_{11}(\tau) \end{aligned}$$

5. Exercices

1. Let two given signals whose bandwidth are respectively equal to W and nW. Show that the product of these two signals gives a wide band signal.

2. In a spread spectrum communication system, the binary rate is $R_b = 1/T_b$ where $T_b = 4,095\,ms$. We use $T_c = 1\,\mu s$ and a BPSK modulation. In addition, the E_b/N_0 ratio leading to an error probability less than 10^{-5} is equal to 10. Determine the maximum number of simultaneous users and the bandwidth of the system.

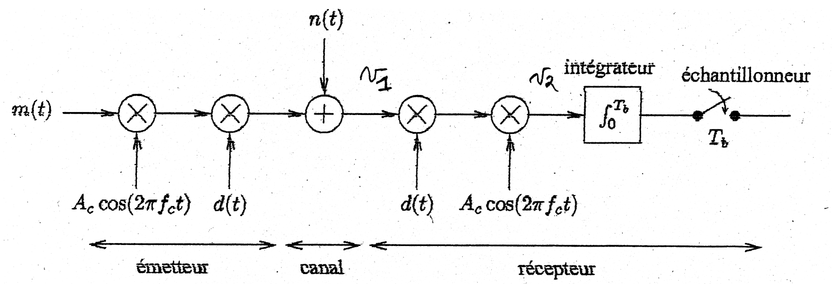
3. Given the bit sequence $0\,1\,1\,0\,1$. We will modulate it in baseband (NRZ modulation) with a rectangular modulating waveform of T_b duration and unitary amplitude. The voltage is respectively equal to $1\,[V]$ for a 1 and $-1\,[V]$ for a 0. The resulting signal $x(t)$ is then shaped like in in the drawing below and the message is transmitted at a speed of $75\,[b/s]$.



   We then decide to use a spread spectrum method where the spreading signal $g(t)$ is generated by a 4 bits shift register whose initial sequence is $1\,1\,1\,1$. The clock frequency of this circuit is $1125\,[Hz]$.

   (a) Determine the circuit diagram allowing the construction of the shift register of maximum length.(hint : use the $[4,1]$ feedback configuration)
   (b) Draw the spreaded signal for the two first bits of $x(t)$.
   (c) Determine the spread spectrum processing gain in $[dB]$.
   (d) If we then use a BPSK modulation and if the ratio between the energy per bit and the noise power is $5\,[dB]$, determine the maximum users number.
   (e) Determine the spreaded signal bandwidth.

4. We consider the spread spectrum transmission system represented by the following diagram



where
- $m(t)$ is the useful binary signal ; $m(t)$ is a NRZ signal with a $\pm V$ amplitude and a bit duration of $T_b = \frac{1}{f_b}$,
- $A_c \cos(2\pi f_c t)$ is the carrier,
- $d(t)$ is the spreading sequence ; the bit duration is $T_d = \frac{T_b}{60}$,
- $n(t)$ is an additive noise.

This question has two parts and it is possible to answer almost of all the second part without having solved the first one.

First part : Here, we will try to find the analytic expression for the noise power spectral density at the integrator output. The noise signal is $n(t) = A_n \cos(2\pi f_c t + \Theta)$ where Θ is a zero mean random phase.

(a) What is the spreading factor ?

(b) Give the analytic exppression of the $v_1(t)$ signal at the receptor input.

(c) What is the $v_2(t)$ signal at the integrator input ?

(d) If we take $f_c = \frac{600}{T_b}$, some terms in $v_2(t)$ will have a null contribution at the integrator output. What are these terms ? (Hint : (1) you should develop the cosines, (2) the terms $\cos(\Theta)$ and $\sin(\Theta)$ do not depend on the time ; they are constants on all the integration period).

(e) As all operations are linear, it is possible to neglect the terms with a null contribution starting from the integrator input. Then, what will be the simplified $v_3(t)$ signal derived from the expression of the $v_2(t)$ signal ?

(f) What is the interference term in $v_3(t)$?

(g) What is the spectral density of the interference term at the integrator input ?

(h) What is the spectral density of the interference term at the integrator output ? In the computation, you may consider that the integrator will act as an ideal lowpass filter until the f_b frequency.

Second part : We would like to compute the bit error probability. We remind you that, in the case of a classical BPSK modulation, the bit error probability P_e is

$$P_e = \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{N_0}}\right)$$

We will assume that the noise power spectral density is constant for $|f| \leq f_b$ and is equal to

$$\frac{V^2 E\left\{\cos^2(\Theta)\right\}}{\alpha f_d}$$

This spectral density is null outside the $[-f_b, f_b]$ interval. α is a constant.

(a) Compute the value of P_e. (Hint : replace E_b by its value)

(b) Which is the gain compared to the classical BPSK if we consider that Θ is a random variable uniformly distributed on the $[0, 2\pi]$ interval ?

(c) Does the gain comes from the spreading ?

Answers

1. –
2. Users number = 410. Bandwidth = 1 MHz.