Longitudinal vehicle dynamics using Simulink/Matlab

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Longitudinal vehicle dynamics using Simulink/Matlab

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Abstract This paper examines the longitudinal dynamic model of a vehicle with automatic transmission. The modeling has been done in two stages: in the first stage the dynamics of power train consisting of the engine, the torque converter, the gear box, the final drive and the wheels are considered and in the second stage the effects of external forces exerted on the vehicle are added and overall effect is investigated, the external forces included in the model are: aerodynamic drag, gravitational effects, rolling resistance and longitudinal tire effects. The objective of this paper is to present the integrated Simulink model including the dynamic of powertrain and vehicle while considering the effect of the road on the longitudinal performance of the vehicle. Simulations are conducted using a set of traffic scenarios which are likely to occur in reality, the results obtained from the simulated cases and their effects on the performance of the vehicle are examined and reported. The results reported in this paper are part of an ongoing research investigation in the design of adaptive cruise control system.

Keywords: Longitudinal vehicle dynamic, Traction force, Torque converter, Powertrain, Automatic transmission, Brake torque.

1. INTRODUCTION

A great deal of research has been carried out to investigate the dynamics of the vehicle and in particular the impact of the tire on the handling and ride performance. Research has been conducted on the tire to predict its behaviour against the various road condition, for instance Mosseau et al. (1999); Ojala (2005); Siliem (2005) used the Finite Element Method (FEM) to study the tire model and its effect on the performance of the vehicle. The Pacejka tire model (Bakker et al. (1987)) was commonly used by researchers either to develop the vehicle model or to investigate lateral and longitudinal behaviour of the tire. Lee et al. (2004) carried out the research on the tire-road friction estimation and consider this term using two different approach; cause-based which figures out the factors affecting the friction coefficient and effect-based which considers the effects caused by the friction coefficient. The latter paper developed the force estimator, a brake gain estimator, and a normal force observer to estimate the maximum friction coefficient as well as obtaining the tractive force during decelerating and accelerating for front and rear wheel. To calculate the normal force the effect of the pitch motion of the vehicle was considered, however, the normal force was assumed to be the same for the right and left wheels of each axle. Subsequently the tractive force during decelerating and accelerating was obtained for front and rear wheel. Various models for the longitudinal dynamic of a vehicle was developed and some of those were employed in designing a control system depending on the purpose of the study (Wong (2001)). For instance in studying cruise control (CC) that functions to keep the velocity constant in highway it is not necessary to consider the impact between the road and the tire due to insignificant influence of the tire on a vehicle as the longitudinal slip is small. In addition there is no mass transferred between the axles (front axle and rear axle) under this condition. However when considering other control problems such as adaptive cruise control (ACC), brake system control (see Lee and Zak (2002)) or traction control system (see Lee and Tomizuka (2003)), a more sophisticated dynamic model of a vehicle is necessary.

Occasionally in presenting the dynamic model of a vehicle the model of the tire has been excluded for simplification, despite being understood as the main realistic source to push the vehicle forward during acceleration (tractive effort) and to hold (or reduce the velocity) the vehicle when applying the brake (braking effort), instead the torque produced at the wheels through the power train is directly considered as a source of pushing the vehicle forward and subsequently the brake torque to reduce the speed of the vehicle. In reality, apart from the effect of the aerodynamic force and the gravitational force (during travelling the vehicle up/down hill) on the longitudinal performance of a vehicle, the forces developed between the tire and the road contact interface have significant impact on the performance of the vehicle.

2. VEHICLE MODEL

To design a control system a Simulink model which represents the longitudinal dynamic of the vehicle is essential . The current paper investigates the dynamic model of the vehicle considering front and rear wheels in presenting a longitudinal dynamic of the vehicle. The dynamic of the

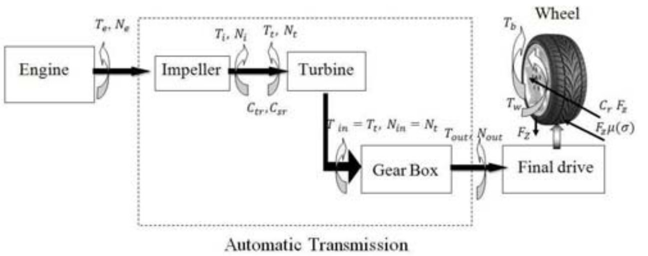


Figure 1. Transmission of the torque and velocity from the engine to the wheels

vehicle is classified into two categories: 1) the dynamic of power train comprising the engine, the torque converter, the gear box, the final drive and the wheels, 2) the dynamic of vehicle considers the external forces acting on the vehicle that includes aerodynamic drag force, gravitational force in addition rolling resistance and traction force which arises from the physical evolution between the tire and road while rotating the wheel on the road.

2.1 Powertrain model

Figure 1 shows the schematic diagram of a powertrain which demonstrates the transmission of the torque and velocity from the engine to the wheels . The torque produced by the engines is transmitted to the gearbox through the torque converter. The torque converter consists of three essential parts i.e. the impeller, the turbine and the reactor. The impeller is connected to the crankshaft that transmits the power of the engine to the turbine by the hydraulic oil inside the torque converter. In turn the turbine is connected to the output shaft of the converter which is coupled to the input shaft of the gearbox. The torque getting through the gear box varies depending on the gear ratio. Finally the output torque from the gearbox is transmitted to the wheels after passing through the final drive. The torque and the angular velocity of the wheel are effected by the brake torque applied on the wheel and other forces exerted on the wheel resulted from the interaction of the tyre and road (rolling resistance and tractive force). The modelling of the powertrain follows the method applied in MathWorks (1998). The equation determining the difference between engine torque and impeller torque is defined as (Heisler (2001)):

$$I_{ei}\dot{N}_e = T_e(u_t, N_e) - T_i \tag{1}$$

with N_e the engine speed [rpm], I_{ei} the summation of engine and impeller moment of inertia (Kgm^2), T_e the engine torque as a function of engine speed and percentage throttle position (Nm) and T_i the impeller torque (Nm). Parameters playing an important role on the performance of a torque convertor are expressed as follows: the speed ratio $C_{sr} = \frac{N_t}{N_i}$, the torque ratio $C_{tr} = \frac{T_t}{T_i}$, the efficiency $\eta_e = C_{sr} \times C_{tr}$ and the capacity factor (K-factor) K_{tc}. The capacity factor shows the ability of the converter to absorb or transmit the torque (Mosseau et al. (1999)). Figure 2 illustrates the relationship between the torque ratio, the efficiency and the capacity factor which are plotted against the speed ratio (Wang (2001)). The input torque of the converter can be defined as:

$$T_i = \left(\frac{N_i}{K_{tc}}\right)^2 \tag{2}$$

where T_i is the impeller torque (converter input torque) and N_i is the impeller speed (converter input speed). The

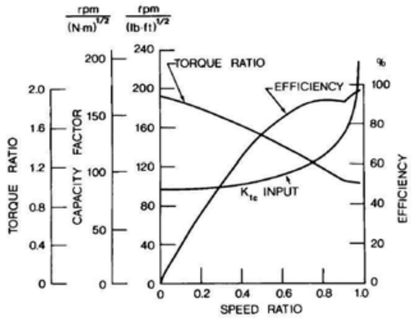


Figure 2. Performance characteristic of a torque convertor (Wang (2001))

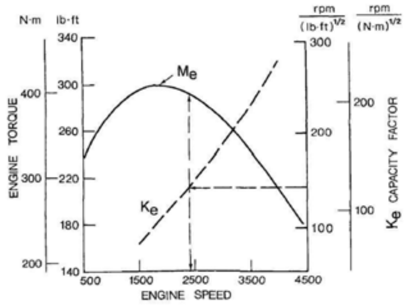


Figure 3. Capacity factor of an internal combustion engine (Wang (2001))

engine and the converter should have the same range of capacity factors ($K_e = K_{tc}$) to achieve a proper matching. The engine operating point, K_e can be obtained from Figure 3. Interpolating from Figure 2 the speed ratio C_{sr} can also be found.

The speed ratio C_{sr}, the torque ratio C_{tr} of the torque converter and K_{tc} can be determined as function of $\frac{N_t}{N_e}$. Assuming that the velocity of the impeller N_i equals to the engine speed N_e, Equation (2) is then rewritten as:

$$T_i = \left(\frac{N_e}{K_{tc}}\right)^2 \tag{3}$$

Knowing C_{tr} and C_{sr} enables us to find the output characteristic of the torque converter:

$$T_t = C_{tr} \times T_i \tag{4}$$

$$N_t = C_{sr} \times N_i \tag{5}$$

where T_t is the turbine torque and N_t is the turbine angular velocity. Knowing that the torque and speed outputs from the torque converter equal to the input characteristics of the gear box help us to find the outputs of the gearbox:

$$T_{out} = R_{tr} \times T_{in} \tag{6}$$

$$N_{out} = \frac{N_{in}}{R_{tr}} \tag{7}$$

where T_{in} and T_{out}, N_{out} and N_{in} are the transmission input and output torques, transmission output and input speeds respectively. R_{tr} is the transmission ratio which varies with the gear setting. Multiplying the transmission output torque by the final drive ratio determines the wheel torque, T_w. To present the model of the engine in the simulation, a look up table is applied which defines the amount of engine torque vs. engine rotation speed (rpm) and the percentage of throttle opening. The engine map

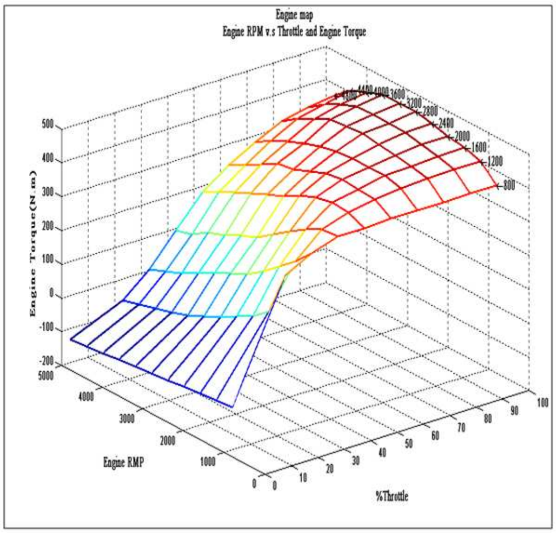


Figure 4. Engine map (engine torque vs. %throttle and engine RPM)

was taken from the Matlab Simulink (MathWorks (1998)) with converting the unit from $Ft.lb$ to Nm (Figure 4). In the model of power train, implementation the gear shift is done through the shift logic based on the threshold calculated by the respective block for up-shift and down-shift.

2.2 Vehicle dynamic terminology

Rolling resistance The rolling resistance is produced due to the deflection of the tire while rotating on the road causing the distribution of the normal pressure in the leading half of the contact interface of the tire being higher than pressure in the trailing half, which causes the moment to be produced at the rotational axis of the tire. Thus the horizontal equivalent force is created at the contact point of the tire and the road interface which is known as rolling resistance. The rolling resistance depends on the different parameters which most importantly include the tire shoulder temperature, ambient temperature, tire diameter, road condition, inflation pressure of the tire and type of the tire (Wang (2001)). Many empirical relationships are introduced for rolling resistance coefficient as function of the velocity for various types of the tire and road conditions. The value of rolling resistance for passenger cars and truck vehicles for various tires are given as a function of velocity v in Table 1. In (Zoroofi (2008)) the rolling resistance was given as:

$$C_r = 0.01\left(1 + \frac{v}{100}\right) \tag{8}$$

Table 1. Coefficient of rolling resistance for different type of tire (Wang (2001))

Type of tire	C_r	Effective working range (Up to)
Radial-ply passenger car	0.0136+0.40e-7	150 Km/h
Bias-ply passenger car	0.0169+0.19e-9	150 Km/h
Radial-ply truck	0.006+0.23e-6	100 Km/h
Bias-ply truck	0.0007+0.45e-6	100 Km/h

Gravitational force Gravitational force acts on a vehicle going uphill and downhill and sign of this force

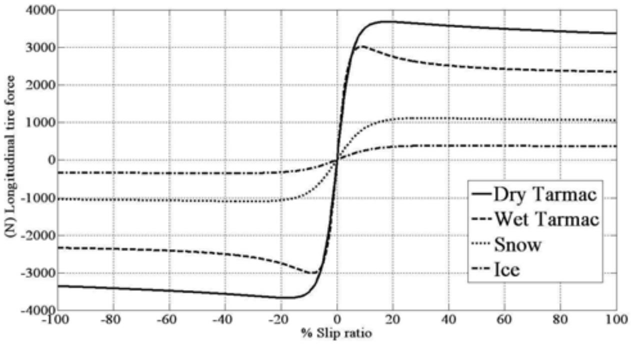


Figure 5. Longitudinal tire force vs. percentage slip ratio corresponding to different road condition

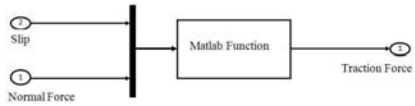


Figure 6. Employing Matlab Function for calculation of traction force

depends on whether the vehicle is going up or downhill. So it is negative when the vehicle goes uphill and positive otherwise.

$$F_s = \pm mg \sin\theta \tag{9}$$

Where m is the total mass of the vehicle, g is known gravitational acceleration, and θ varyies depending on inclination of the road and it is so-called road slope.

Aerodynamic force As the aerodynamic drag force is a function of the square of the speed, thus it is the most significant parameter affecting the performance of a car at speed of more than 48 km/h (Wang (2001)). This force is determined by the following function:

$$F_{aero} = \frac{1}{2}\rho A C_d (v + v_w)^2 \tag{10}$$

where $\rho = 1.225kg/m^2$ is the air density, C_d is the drag coefficient depending on the body shape, v is velocity of vehicle, v_w is the air velocity which could be neglected in the calculations and A is the maximum vehicle cross area (m^2). In (Wang (2001)) an approximated relationship for this parameter has been determined as a function of the car mass m_v (kg) in the range of 800-2000 kg:

$$A = 1.6 + 0.00056(m_v - 765) \tag{11}$$

Table 2. Coeffients of the Pacejka model for different road condition

Surface	B	C	D	E
Dry Tarmac	10	1.9	1	0.97
Wet Tarmac	12	2.3	0.82	1
Snow	5	2	0.3	1
Ice	4	2	0.1	1

Traction force (Tire model) Tire is the main source the road forces acting on the vehicle. During acceleration between the tire and the road interface the primary force is generated which plays the main role to push the vehicle forward (traction force) and subsequently reducing the speed of the vehicle during braking (braking effort) which is known as the longitudinal force. In this paper the Pacejka tire model is utilized to calculate longitudinal force based on the percent longitudinal slip. The Pacejka model which was applied in Short et al. (2004) calculates the friction coefficient as a function of slip ratio σ:

$$\mu(\sigma)=D \sin \left(C \tan^{-1} \left(B\sigma - E \left(B\sigma - \tan^{-1} (B\sigma)\right)\right)\right) \quad (12)$$

The value of B, C, D and E for different road types are shown in Table 2. Having the friction coefficient $\mu(\sigma)$ and the normal force F_z exerting on the wheel, the longitudinal traction force F_x is then calculated as follows:

$$F_x = \mu(\sigma) F_z \quad (13)$$

The slip ratio is different for the case when the vehicle is braking or accelerating, as given by following relationship:

$$\sigma = \begin{cases} \dfrac{r\omega - v}{v} & \text{during braking} \\ \dfrac{r\omega - v}{r\omega} & \text{during acceleration} \end{cases} \quad (14)$$

where r is the wheel radius, v is the transitional velocity of wheel and equivalent to the velocity of the vehicle (m/s) and ω is rotational velocity of the wheel (rad/s).

Presented model of the tire was plotted assuming a force of 3675 N acting on each wheel for a vehicle with total mass of 1500 kg (Figure 5). The Pacejka model is implemented using the Matlab function block in Simulink. The Normal force and slip ratio obtained from the respective Simulink blocks are employed as the inputs of the Matlab function (Figure 6) to calculate the longitudinal force which subsequently is used by other block involving the vehicle dynamic and wheel model.

Brake model A simple model to present a brake system that feed the braking torque as function of the brake pedal into the vehicle model is used. The amount of the braking torque is varied in order to control the perfomance of the vehicle in various situations. According to the model of brake system introduced by Gerdes et al. (1993) the braking torque is obtained by finding out the amount of pressure produced behind the brake disk P_{bi} while applying the brake pedal. In this model the value of u_b is varied between 0-100 which indicates the position of the brake pedal (Short et al. (2004)).

$$P_{bi} = 1.5 K_{ci} u_{bi} - \tau_{bs} \dot{P}_{bi} \qquad i = r, f \quad (15)$$

τ_{bs} is the lumped lag obtained by combining two lags relating to the dynamic of the servo valve and the hydraulic system, K_c is pressure gain and the notaitions of r and f denote the rear and front wheel, respectively.The pressure function can be used in the form of a transfer function in Simulink model. For doing so the Laplace transform is taken of the pressure equation as below:

$$P_{bi}(s) = \frac{1.5 K_{ci} u_{bi}}{1 + \tau_{bs} s} \qquad i = r, f \quad (16)$$

The brake torque being applied for each of the front and the rear wheel are different because of the load transfers during braking. This behaviour is introduced by determining a different value of K_b for each axle. Also the brake torque T_{bi} (Nm) is a function of the wheel velocity:

$$T_{bi} = P_{bi} K_{bi} min(1, \frac{\omega_i}{0.001}) \qquad i = r, f \quad (17)$$

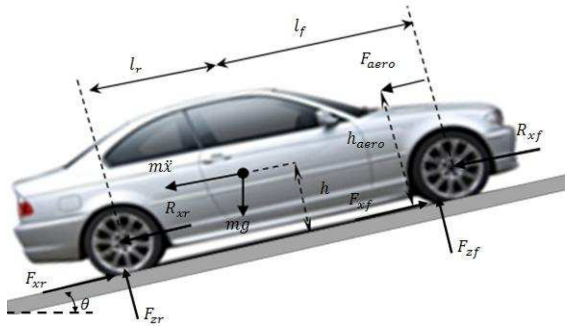


Figure 7. Longitudinal force exerting on vehicle when traveling on the inclined road

2.3 Longitudinal model of the vehicle

In order to develop an integrated model of the vehicle dynamic, the Simulink blocks representing the dynamic of each parts (vehicle dynamic, wheel dynamic, longitudinal traction force, normal force and tire slip) need to be incorporated as illustrated in Figure 8.

Vehicle dynamic Suppose a vehicle travels up/downhill as illustrated in Figure 7, thus by applying the second Newton's law which is based on balancing the forces acting on the vehicle, the acceleration of the vehicle $\ddot{x}$ can be obtained (Rajamani (2006) and et. al. (2001)):

$$m\ddot{x} = F_{xr} + F_{xf} - F_{aero} - R_{xf} - R_{xr} \pm mg \sin(\theta) \quad (18)$$

where F_{xr} and F_{xf} express the longitudinal traction force for the front and rear wheel respectively, F_{aero} is the aerodynamic force, R_{xf} and R_{xr} are the rolling resistance produced on each wheel (Rajamani (2006)):

$$R_{xf} + R_{xr} = C_r (F_{zf} + F_{zr}) \quad (19)$$

where C_r is the rolling resistance coefficient and F_z is the normal force, f and r denote front and rear wheel, respectively. The normal forces are given as

$$F_{zf} = \frac{-F_{aero} h_{aero} - m\ddot{x} - mgh \sin(\theta) + mgl_r \cos(\theta)}{l_r + l_f}$$

$$F_{zr} = \frac{F_{aero} h_{aero} + m\ddot{x} + mgh \sin(\theta) + mgl_f \cos(\theta)}{l_r + l_f}$$

where l_f and l_r are respectively the distance from the vehicle center of gravity (c.g.) to the front and the rear wheel axle of the vehicle. h is the height of (c.g.) relative to the road surface and θ is defined as the road slope.

Consequently By integrating of the acceleration, the velocity of the vehicle will be obtained.

Wheel dynamic Assuming front wheels drive, the dynamic of the wheels involving front and rear axle can be described as below (Rajamani (2006)):

$$J_{wf} \dot{\omega}_f = T_w - T_b - r_w F_{xf} \quad (20)$$

$$J_{wr} \dot{\omega}_r = -T_b - r_w F_{xr} \quad (21)$$

where J_w is moment of inertia of the wheels, $\dot{\omega}$ is the angular acceleration of the wheel which by integration, the rotational velocity of each set of wheels can be obtained, T_w is the wheel torque and T_b is brake torque. The subscript r and f denote the rear and front wheel respectively.

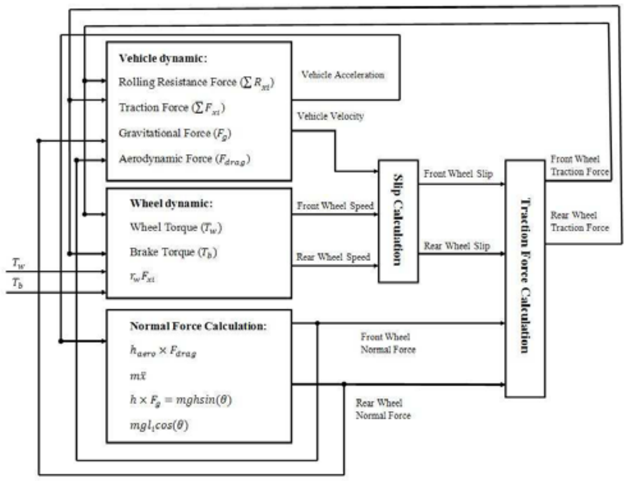


Figure 8. The schematic diagram of the Simulink blocks including vehicle dynamic, wheel dynamic, longitudinal traction force, slip calculation and normal force

3. TESTING SCENARIO

The model generated is tested using the following scenarios and road conditions which realistically may occur:

Position variation of the throttle opening: This is applied depending on the different situation taking place on the road, such as slowing down of other vehicle ahead, the driver needs to take foot of the accelerator pedal without applying the brake and increases the speed of the car after passing that situation. ACC is an example of the systems working with this characteristic (Figure 11).

Different road condition: Since the performance of the vehicle on different road conditions is distinct, because of the interaction between the road and tire, the model is examined on two different road conditions namely the dry road and one covered with the snow(Figure 10).

Sudden braking situation: This condition is to examine the reaction of the vehicle against sudden braking. This scenario can be introduced to design and test the brake control system such as ABS (anti brake system).

Stop and Go: This situation may arise in a traffic jam where the vehicle comes to standstill from a certain velocity and after stopping several times, start moving again. This scenario is introduced to test the performance of the model over stop and go situation which is more likely to occur in urban area and where there are traffic jams (Figure 9).

4. SIMULATION

The model of the vehicle dynamic was formulated and Simulink model was constructed in previous section. Now the result of the simulation is obtained by defining different scenarios. The schematic diagram of the Simulink blocks consisting of the vehicle model, front wheels and rear wheels and the traction force is shown in Figure 8. The velocity for each wheel and velocity of the vehicle are calculated during running the simulation at each time step. The velocity calculated by each block enters the slip calculation block to obtain the slip associated with each wheel. These blocks calculate the amount of the slip for each wheel by applying the equations (14), and subsequently these values are used by traction force block to obtain the longitudinal effective force which is the force produced between the tire and the road interface at the contact point causeing the vehicle to move forward. The direction of this force changes when applying the brake which opposes the direction of the motion of the vehicle (Figure 5). The normal force acting on the front wheel is reduced when accelerating due to the load transferring into the front axle, while this force increases on the rear wheel. Unlike during acceleration, the front axle takes more load than the rear one because of reversing the direction of inertia force acting on the vehicle when applying the brake.The result in Figure 11-a illustrates the velocity of the vehicle for the specified throttle opening position. Due to the dynamic of the brake model presented here the velocity of the vehicle and each wheels reach zero at the same time. To verify the performance of the model for the situation which the vehicle goes into standstill and after a while start moving, a test was implemented through introducing the condition specified by the throttle opening position and the brake pedal. In this condition, it was assumed that the driver initially pushed the accelerator pedal for 4 sec. After this time, driver applied the brake to slow down the vehicle until it reaches to standstill. The car would be stopped for a few second and again start moving. As Idling causes small torque to be transmitted to the wheels of the vehicle including the automatic transmission, the small braking torque is necessary to be applied in

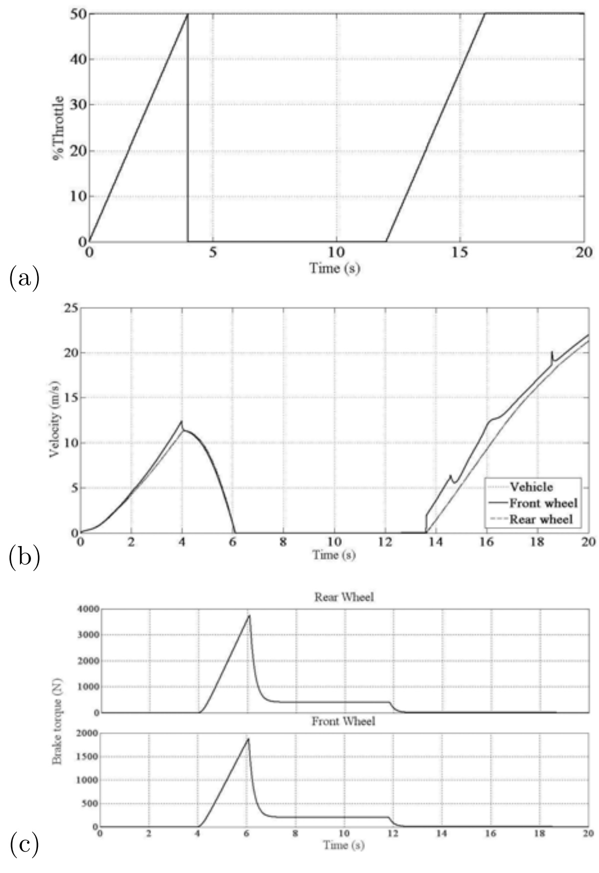
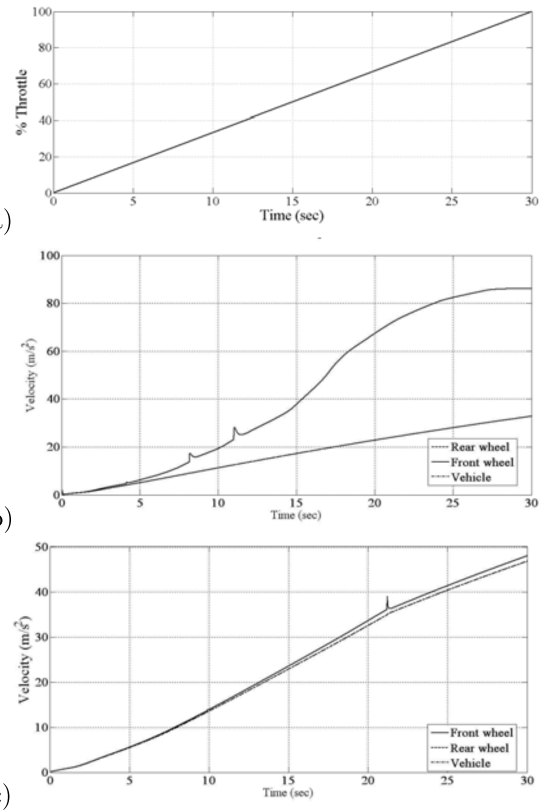


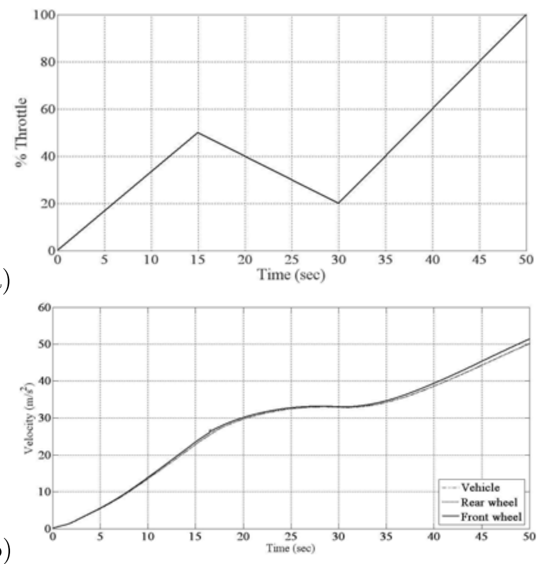
Figure 9. (a) Opening position of the throttle, (b) Velocity of the vehicle, rear and front wheel during travelling, (c) The applied braking torque

order to keep it fully stopped (Figure 9-c). Also to verify whether the performance of the simulation on the different road condition is realistic, the test was implemented. the result in Figure 10-b shows considerable difference between



(a) (b) (c)

Figure 10. (a) Opening position of the throttle, (b) Velocity of the vehicle, rear and front wheel during traveling on the road covered with snow, (c) Velocity of the vehicle, rear and front wheel during travelling on the dry road



(a) (b)

Figure 11. (a) Opening position of the throttle, (b) Velocity of the vehicle, rear and front wheel during traveling on the road against the opening position of the throttle

the velocity of the vehicle and the wheel, This is due to small amount of the slip between tire and road interface covered with snow, Thus the driven wheel(front wheel) starts spinning which in turn the force causing the vehicle to move forward (traction force) decreases. But it can be seen the identical behavior between the vehicle and each wheels while travelling on the dry road Figure 10-c.

5. CONCLUSION

The dynamic longitudinal model of a vehicle is constructed using SIMULINK, for the purpose of developing an ACC system in different traffic scenarios under various road conditions. The model includes the power train, rolling resistance, gravitational effects, aerodynamics and tractive forces as well as the vehicle, brake and wheel dynamic effects. The input action is characterized by the throttle and brake pedal setting and the output by the velocity of the vehicle. The simulation results show a good correlation between the variations the throttle setting against time with the velocities of the vehicle in comparison with other vehicle dynamic model presented in the literatures by applying the same data.

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