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# D-H Parameters Augmented with Dummy Frames for Serial Manipulators containing Spatial Links

Amanpreet Singh, Ashish Singla\* and Sanjeev Soni

**Abstract**— Conventional kinematic studies of serial manipulators involve the proper selection of coordinate frames of reference at appropriate positions. The standard practice, being used in the past, is the use of Denavit-Hartenberg (D-H) algorithm for assigning coordinate frames. However, it has been observed that when an open kinematic chain contains a *spatial link* with two consecutive joint axes at right angle to each other, the forward kinematics derived with D-H algorithm comes geometrically inconsistent. A typical spatial link involves more than one non-zero link/joint parameters, which are not being accounted for, in the corresponding D-H parameter table. Forward kinematic study of manipulators involving spatial links with two consecutive joint axes at right angles to each other leads to recognizable deficiency of the D-H algorithm, as one of its practical limitation. In the present work, the concept of *dummy frames* is proposed to eliminate this deficiency. The proposed concept is demonstrated successfully for the case study of a *Manipulator for Medical Application (MMA)*, which is a seven degrees-of-freedom (DOF) manipulator containing spatial links. Both geometrical and physical validation is performed to ensure the efficacy of the proposed concept.

**Keywords:** *Dummy Frames, spatial link, serial manipulator, medical surgery, D-H algorithm.*

## I. INTRODUCTION

In 1955, Jacques Denavit and Richard S Hartenberg [1] introduced a consistent and concise notation to assign reference coordinate frames to serial mechanisms. In this notation, four parameters, popularly known as *D-H parameters*, are defined to provide the geometric description to serial mechanisms. Out of the four, two are known as *link parameters*— link length ( $a_i$ ) and link twist ( $\alpha_i$ ), describes the relative location of two attached axes in space; while remaining two are described as *joint parameters*— joint offset ( $d_i$ ) and joint angle ( $\theta_i$ ), describes connection of any link to its neighboring link [2]. Two modifications of this notation have been reported in literature — first by Kahn and Roth [3] and second by Featherstone [4]. Objective of these modifications was to make the parameter identification simpler [5]. First modified notation is known as *distal* variant of D-H notation, while second is known as the *proximal* variant. The distal variant is popularized by Paul [6], whereas proximal variant is popularized by Craig [2]. It has been found that the distal variant is used more widely as compared to its original and proximal counterparts [7].

Original D-H notation and its variants differ about the placement of joint axes, along which all the D-H parameters are assigned. In first modified notation (distal variant), joint offset  $d_i$  and joint angle  $\theta_i$  were measured about  $z_{i-1}$ ,

whereas link length  $a_i$  and link twist  $\alpha_i$  were measured about  $x_i$ . Whereas in second modified notation (proximal variant), joint offset  $d_i$  and joint angle  $\theta_i$  were measured about  $z_i$ , whereas link length  $a_i$  and link twist  $\alpha_i$  were measured about  $x_i$ . However, in original D-H notation, joint offset  $d_i$  and joint angle  $\theta_i$  are measured about  $z_i$  whereas link length  $a_i$  and link twist  $\alpha_i$  were measured about  $x_{i+1}$ . A comparison of original and its two variants has been made by Lipkin [7].

An alternative approach to D-H notation has been proposed by Corke [8]. According to this approach, manipulator kinematics can be described as a series of fundamental translations and rotations (walk through) as the first step. Thereafter, a set of algebraic rules are applied as second step to convert these fundamental transformations into D-H form. Proposed approach is different from D-H notation in the sense that there is no constraint on the axes about which these fundamental transformations take place. Another alternative approach, known as *successive screw displacement* method, presented and compared with D-H notation by Rocha et al. [5]. It was observed by the authors that screw based method is useful as in this approach parameters identification is simple. Only two reference frames are required for the entire chain, which can be chosen randomly. However, the proposed approach uses non-minimal parameter representation, less popular as compared to D-H notation for geometric description of kinematic chains.

It has been observed by Seth and Oicker [9] that it is difficult to apply D-H notation to robots with links having more than two joints or tree and closed-loop robot structures, which leads to ambiguities in D-H notation. The authors have proposed a new notation, known as S-U notation that is applicable to any mechanism and helps to remove the ambiguities in D-H notation. However, proposed approach uses non-minimal (seven) parameters to describe a link, due to which it is limited to close-loop structures only [10]. A similar approach has been developed by Khalil and Kleinfinger [11], which can be applied to open as well as closed-loop structures. This approach uses four parameters to define a link having two joints. However, if a robot has links with more than two joints then two additional parameters may be required. A new method for kinematic analysis of manipulators using *zero reference* position has been proposed by Gupta [12]. Thomas et al. [13], proposed a new notation based on graph representation and is an extension of most

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widely used D-H notation. Proposed notation is unified in the sense that it can be applied to parallel, serial, hybrid robots or articulated machine tools as well. A method of kinematic modeling of forward kinematics formulation, based on Seth-Uicker notation has been proposed by B. Bonagrdt [14] and was implemented to a new software named CAD-2-SIM.

In the present work, a *recognizable deficiency* in D-H notation has been observed during kinematic study of *Manipulator for Medical Application* (MMA). In order to eliminate this deficiency, concept of *dummy frames* has been proposed and validated both geometrically as well as physically as a case study of MMA. It is a device operated by the surgeon in robotic assisted surgery. It provides the surgeon with all the independent motions of a human arm. The function of this manipulator is to record the actions made by surgeon, while performing surgery. Thereafter, recorded motion is transferred to patient side manipulator.

## II. KINEMATIC STUDY

Figure 1-(a), shows line diagram of Manipulator for Medical Application, which is an open kinematic chain, consists of 1 fixed and 9 moving rigid links, out of which links numbered as  $L_1, L_2, L_3, L_{3a}$ , and  $L_{3b}$  are planar while  $L_4, L_5, L_6$ , and  $L_7$  are spatial links. Kinematic chain contains two types of joints: i) a ternary joint between links  $L_2, L_3$  and  $L_{3a}$ . ii) binary joints between rests of links. All joints are of revolute type.

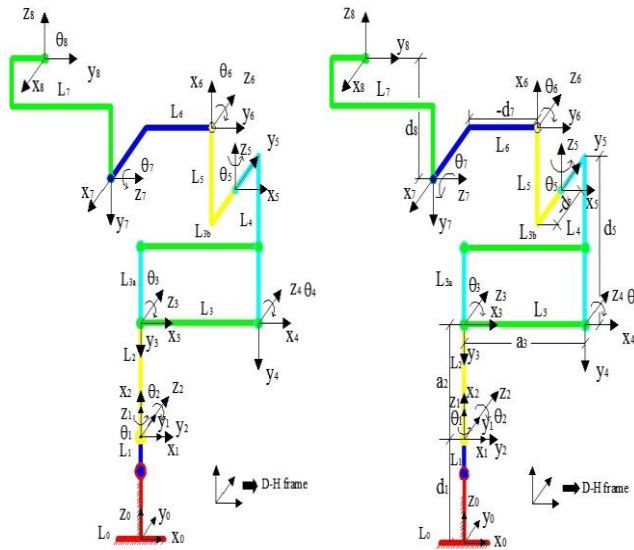


Fig. 1-(a) shows the line diagram and Fig. 1-(b) shows the line diagram with D-H parameters marked, with attached reference frames according to proximal variant of D-H convention in assumed home position.

To provide the geometric description of a kinematic chain, methods like distal and proximal variants of D-H notation [7], screw based displacement method [5], simple and systematic method to generate D-H parameters [8], S-U notation [9, 15], modified D-H notation [4] etc. are available in the literature.

Due to minimal parameters representation, proximal variant of D-H notation has been preferred in the present work to assign reference coordinates frames to the links of MMA as shown in Fig. 1-(a). Thereafter, corresponding D-H parameters has been calculated, as shown in Fig. 1-(b) and are given in Table 1.

TABLE 1: D-H PARAMETERS OF MMA IN HOME POSITION.

S. No	Link Length ( $a_{i-1}$ ) (mm)	Joint Distance ( $d_i$ ) (mm)	Link Twist ( $\alpha_{i-1}$ ) (deg.)	Joint Variable ( $\theta_i$ ) (deg.)	Home Position
1.	$a_0 = 0$	$d_1 = 137.77$	$\alpha_0 = 0$	$\theta_1$	0
2.	$a_1 = 0$	$d_2 = 0$	$\alpha_1 = -90$	$\theta_2$	-90
3.	$a_2 = 187.46$	$d_3 = 0$	$\alpha_2 = 0$	$\theta_3$	90
4.	$a_3 = 160.76$	$d_4 = 0$	$\alpha_3 = 0$	$\theta_4$	0
5.	$a_4 = 0$	$d_5 = 67.82$	$\alpha_4 = 90$	$\theta_5$	0
6.	$a_5 = 0$	$d_6 = -35.07$	$\alpha_5 = -90$	$\theta_6$	-90
7.	$a_6 = 0$	$d_7 = -38.40$	$\alpha_6 = -90$	$\theta_7$	90
8.	$a_7 = 0$	$d_8 = 63.79$	$\alpha_7 = 90$	-	-

## III. AMBIGUITY IN D-H NOTATION

A forward kinematic model is developed by multiplying  $4 \times 4$  homogeneous link coordinate transformation matrix (refer Appendix B). Developed model gives the position and orientation of tool-tip, given the joint angles. Thereafter, D-H parameters calculated in section 2 are substituted in derived model. However, discrepancies has been observed in tool-tip position obtained from kinematic model as compared to one obtained from physical prototype of MMA. This geometrical inconsistency arises because all the segments of spatial links  $L_4, L_5, L_6$ , and  $L_7$  does not get accounted for into corresponding D-H parameters Table 1, as can be seen in Fig. 1-(b). This leads to *recognizable deficiency* of D-H notation in case of spatial links, where two consecutive joint axes at right angles to each other. A detailed analysis has been made by considering a spatial link  $L_4$  taken from Fig. 1-(b), separately with two possible joint axes orientations, as can be seen in Figs. 3-(b) and 3-(c). Figure 3-(a) shows a solid spatial link to which joint axes are assigned with two possibilities— a) parallel to each other, b) at right angles to each other.

Now, if a mechanism contains a spatial link with parallel joint axes, as in case of four-axis SCARA robot (refer Figure 2) [16], D-H notation comes geometrically consistent as both segments of spatial link get accounted for into corresponding D-H parameters. However, in case of a spatial link with two consecutive joint axes at right angle to each other, all the segments of spatial link does not get accounted for into corresponding D-H parameter table (refer Table 2).

TABLE 2: D-H PARAMETERS OF LINK  $L_4$ , SHOWN IN FIG. 3-(C)

S.No	Link Length ( $a_{i-1}$ )	Joint Distance ( $d_i$ )	Link Twist ( $\alpha_{i-1}$ ) (deg.)	Joint Variable ( $\theta_i$ )	Home Position (deg.)
1.	$a_4 = 0$	$d_5$	-90	$\theta_5$	0

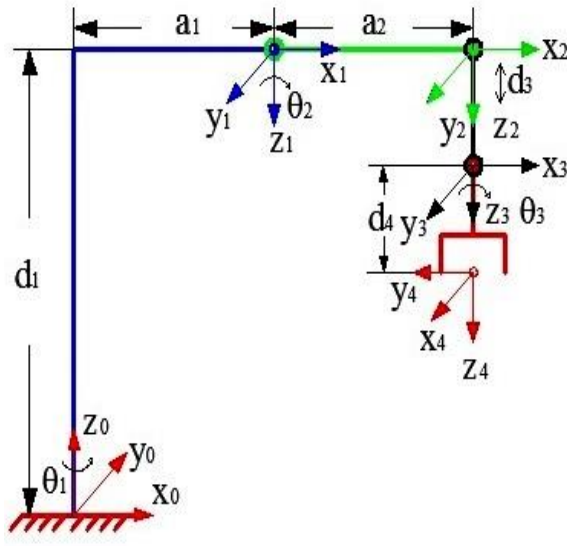


Fig. 2: Line diagram of 4-axis SCARA robot.

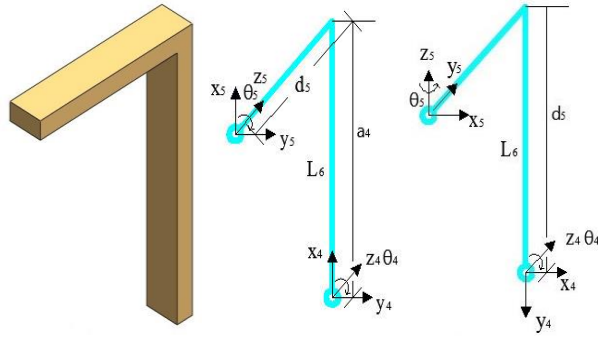


Fig. 3-(a) shows a solid spatial link, Fig. 3-(b) shows a spatial link with parallel joint axes and Fig. 3-(c) shows recognizable deficiency of proximal variant of original D-H convention.

On the similar basis, if the remaining spatial links of the MMA are observed, geometrical inconsistency is found in all of them. This inconsistency can be observed from Table 1, where the entries  $a_5$ ,  $a_6$  and  $a_7$  are coming as zero, describing that all the parameters of spatial links  $L_4$ ,  $L_5$ ,  $L_6$  and  $L_7$  are not accounted for. The geometric validation of this inconsistency is given in later section.

#### IV. IMPROVEMENT IN D-H NOTATION

##### A. Concept of Virtual Link

Now, in order to eliminate the deficiency of D-H notation as reported in Section 3, the *concept of virtual link* has been applied. According to the fundamentals of kinematics that motion study does not depend upon shape of link. So, a virtual link is generally assumed by joining end points of a spatial link.

An attempt is made to use virtual link because both of the segments of spatial link get accounted for into corresponding length calculation as  $s = \sqrt{a^2 + b^2}$ , according to Pythagoras theorem. Taking virtual link in account, the plane of motion of original spatial link got changed, which is not desirable in surgical applications. That is why, the concept of virtual link has not been applied.

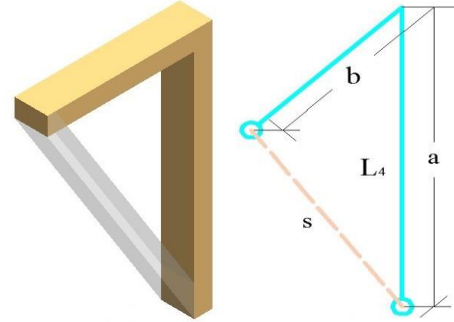


Fig. 4-(a) shows the solid spatial and corresponding virtual link, and Fig. 4-(b) shows geometric description of spatial link.

##### B. Concept of Dummy Frames

However, the reported deficiency can be successfully eliminated by introducing a new concept, which is to the best of authors knowledge, has not been used before in literature. The introduced concept is coined as the concept of **Dummy Frames**. When this concept is applied to a spatial link, as shown in Fig. 5-(b), it accounts for all the segments of spatial link into corresponding D-H parameters (refer Table 3) and eliminates the deficiency of D-H notation. Dummy frame is different from D-H frame in the sense that it is placed at a point where there is no motion, whereas a D-H frame is placed at a point where motion is taking place. Absence of motion at dummy frame does not imply that joint angle corresponding to dummy frame vanishes rather it becomes a constant, whose value can be determined (as angle from  $x_{i-1}$  to  $x_i$  about  $z_i$ ) according to D-H algorithm.

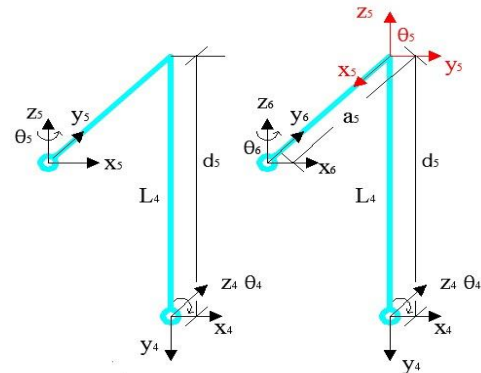


Fig. 5-(a) shows the spatial link  $L_4$  taken from Fig. 1-(b), and Fig. 5-(b) shows the spatial link  $L_4$  with dummy frame (in red) assignment.

### Rules to Assign Dummy Frames:

1. First of all, z-axis is assigned in the same direction as that of next joint axis.
2. Next, assign x-axis according to D-H notation.
3. Use right hand thumb rule to assign y-axis.

Figure 5-(b) shows the demonstration of concept of dummy frame (in red). In the present work, four dummy frames are attached to Manipulator for Medical Application. Hence, four constants are present in Table 4. As there is no motion taking place about any of the axis of assigned dummy frame, that is why, they are known as dummy frame.

TABLE 3: D-H PARAMETERS OF SPATIAL LINK  $L_4$  AUGMENTED WITH DUMMY FRAME, SHOWN IN FIG. 5-(B)

S.No	Link Length ( $a_{i-1}$ )	Joint Distance ( $d_i$ )	Link Twist ( $\alpha_{i-1}$ ) (deg.)	Joint Variable ( $\theta_i$ ) (deg.)	Home Position (deg.)
1.	$a_5$	$d_5$	0	$\theta_5$	-90

### C. D-H parameters of Manipulator for Medical Application augmented with Dummy Frames

Figure 6-(a) shows the line diagram of Manipulator for Medical Application with assigned dummy frames (shown in red) in addition to D-H frames and Fig. 6-(b) shows the line diagram with marked new D-H parameters, given in Table 4.

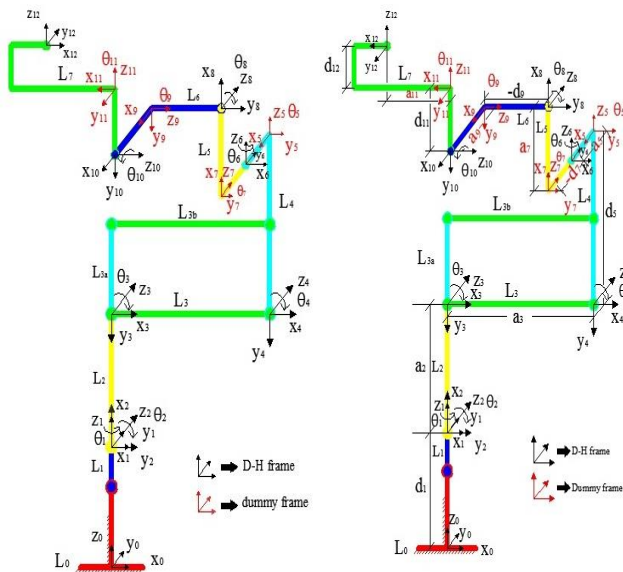


Fig. 6-(a) shows the line diagram with frames attached to links of MMA and Fig. 6-(b) shows the D-H parameters augmented with dummy frames.

At this point, a comparison of Figs. 1-(b) and 6-(b) can be made and it can be observed that segments of spatial links  $L_4$ ,  $L_5$ ,  $L_6$ , and  $L_7$  which are not get accounted for into

corresponding D-H parameters with conventional D-H notation, are now get accounted into D-H parameters.

TABLE 4: D-H PARAMETERS OF MMA WITH DUMMY FRAMES.

S. No	Link Length ( $a_{i-1}$ ) (mm)	Joint Distance ( $d_i$ ) (mm)	Link Twist ( $\alpha_{i-1}$ ) (deg.)	Joint Var. ( $\theta_i$ ) (deg.)	Home Postn. (deg.)
1.	$a_0 = 0$	$d_1 = 137.77$	$\alpha_0 = 0$	$\theta_1$	0
2.	$a_1 = 0$	$d_2 = 0$	$\alpha_1 = -90$	$\theta_2$	-90
3.	$a_2 = 187.46$	$d_3 = 0$	$\alpha_2 = 0$	$\theta_3$	90
4.	$a_3 = 160.76$	$d_4 = 0$	$\alpha_3 = 0$	$\theta_4$	0
5.	$a_4 = 0$	$d_5 = 67.82$	$\alpha_4 = 90$	$\theta_5 = C$	-90
6.	$a_5 = 20.11$	$d_6 = 0$	$\alpha_5 = 0$	$\theta_6$	90
7.	$a_6 = 0$	$d_7 = -35.07$	$\alpha_6 = -90$	$\theta_7 = C$	-90
8.	$a_7 = 25.24$	$d_8 = 0$	$\alpha_7 = 0$	$\theta_8$	0
9.	$a_8 = 0$	$d_9 = -38.40$	$\alpha_8 = -90$	$\theta_9 = C$	90
10.	$a_9 = 16.09$	$d_{10} = 0$	$\alpha_9 = 0$	$\theta_{10}$	0
11.	$a_{10} = 0$	$d_{11} = 27.29$	$\alpha_9 = 90$	$\theta_{11} = C$	-90
12.	$a_{11} = 118.73$	$d_{12} = 36.50$	$\alpha_{10} = 90$	-	-

### V. GEOMETRIC VALIDATION

To ensure the efficacy of proposed concept (Dummy Frame) a kinematic model has been developed in Matlab environment. Also, a physical prototype of Manipulator for Medical Application has been made. Calculated D-H parameters augmented with dummy frames are substituted to kinematic model, gives position and orientation of tool-tip. Also, position and orientation are measured with physical prototype. It has been observed that results given by kinematic model and physical prototype, found with close agreement with each other. The comparison of both the results is given in Table 7. The slight variation found in the coordinates is due to the human error involved while measuring the MMA coordinates manually. It is expected that the above mentioned error will reduce significantly if more sensitive measuring techniques were adopted like Coordinate Measuring Machine (CMM) or vision-based techniques.

#### A. Geometrical Validation with D-H Notation

To validate the D-H parameter table, the tip coordinates of MMA are geometrically calculated, as shown in Fig. 6-(b) and are given as

TABLE 5: GEOMETRICAL VALIDATION WITH D-H NOTATION

Coordinate	Desirable tip coordinate obtained geometrically from Fig. 6-(b).	Position of tip given by forward kinematic model
X	$a_3 - d_9 - a_{11}$	$a_3 - d_7$
Y	$-(a_5 + d_7 + a_9)$	$-d_6$
Z	$d_1 + a_2 + d_5 + a_7 + d_{11} + d_{12}$	$d_1 + a_2 + d_5 + d_8$



It can be seen from Figs. 1-(b) and 6-(b) that  $d_8 = d_{11} + d_{12}$ .

### B. Geometrical Validation with concept of Dummy Frame

TABLE 6: GEOMETRICAL VALIDATION WITH IMPROVED (DUMMY FRAME) D-H NOTATION

Coordinate	Desirable tip coordinate obtained geometrically from Fig. 6-(b).	Position of tip given by forward kinematic model
X	$a_3 - d_9 - a_{11}$	$a_3 - d_9 - a_{11}$
Y	$-(a_5 + d_7 + a_9)$	$-(a_5 + d_7 + a_9)$
Z	$d_1 + a_2 + d_5 + a_7 + d_{11} + d_{12}$	$d_1 + a_2 + d_5 + a_7 + d_{11} + d_{12}$

Tables 5 and 6, shows the geometrical validation results with, conventional D-H notation and D-H notation incorporated with dummy frames. From Table 5 it can be observed that position given by forward kinematic model is inconsistent with one obtained from geometric line diagram (Fig. 6-(b)). As the all segments are not marked in Fig. 1-(b), so desirable tip position is obtained from Fig. 6-(b). However, when concept of dummy frames is implemented, as seen in Fig. 6-(a), position given by the kinematic model is completely consistent with one obtained from geometric line diagram (Fig. 6-(b)).

TABLE 7: COMPARISON OF VALIDATION RESULTS

Mapping		With Kinematic Model			With Physical Prototype		
From	To	X (mm)	Y (mm)	Z (mm)	X (mm)	Y (mm)	Z (mm)
4 <sup>th</sup>	0 <sup>th</sup>	160.7	0	325.2	158.5	0	323.5
5 <sup>th</sup>	0 <sup>th</sup>	160.7	0	393.1	158.5	0	391.0
6 <sup>th</sup>	0 <sup>th</sup>	160.7	-20.1	393.1	158.5	-22.9	391.0
8 <sup>th</sup>	0 <sup>th</sup>	160.7	-55.2	418.3	158.5	-56.6	417.2
9 <sup>th</sup>	0 <sup>th</sup>	122.4	-55.2	418.3	124	-56.6	417.5
12 <sup>th</sup>	0 <sup>th</sup>	3.6	-71.3	482.1	3.6	-72.9	481.5

## VI. CONCLUSION

D-H notation is a powerful tool for giving the geometric description to an open kinematic chain. However, an ambiguity in it is observed by Seth and Uicker [9] in case of closed robots. Also, in the present work, an ambiguity has been observed in D-H notation, in case of spatial links with two consecutive joint axes at right angle to each other. Due to this, all the segments of spatial link does not get accounted for into corresponding D-H parameters and this ambiguity is demonstrated in kinematic study of Manipulator for Medical Application. The amount of error in any given configuration is deterministic in nature, that is why it can be stated as a recognizable deficiency in D-H notation. In order to eliminate this ambiguity, concept of dummy frames has been proposed which is to the best of authors knowledge, has not been used before in literature. The efficacy of the proposed concept has been demonstrated successfully in the field of surgical robots, using a MMA prototype, to obtain geometrical as well as physical consistency of the kinematic model. To file the

patent, the details of MMA prototype has been kept confidential.

## Appendix A

Details of *proximal* variant of D-H notation is given in this appendix which is as follows:

1. Draw line diagram of an open kinematic chain, identify and draw all the motion axis with lines of infinite length.
2. Mark these motion axis as z-axis.
3. Next step is to assign x-axis, which is normal at the point of intersection of  $z_i$  and  $z_{i+1}$ . In case, if they don't intersect then x-axis will be drawn such that it is common normal to  $z_i$  and  $z_{i+1}$ .
4. Finally, assign y-axis as given by *right hand thumb* rule.
5. Thereafter, calculate D-H parameters which are as follows:
  - Link twist angle ( $\alpha_i$ ): Angle from  $z_i$  to  $z_{i+1}$  about  $x_i$ .
  - Link length ( $a_i$ ): Distance from  $z_i$  to  $z_{i+1}$  along  $x_i$ .
  - Joint distance ( $d_i$ ): Distance from  $x_{i-1}$  to  $x_i$  along  $z_i$ .
  - Joint variable ( $\theta_i$ ): Angle from  $x_{i-1}$  to  $x_i$  about  $z_i$ .

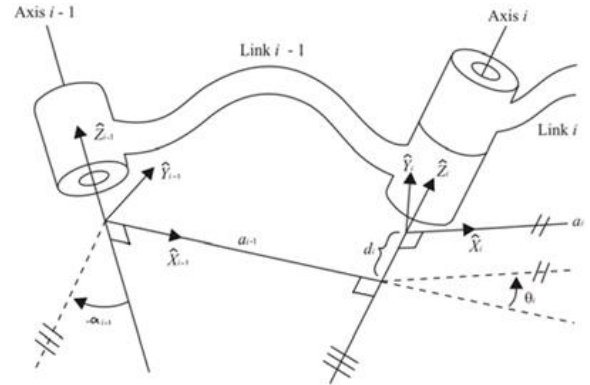


Fig. 7: Geometric Description of D-H parameters [2].

## Appendix B

The general homogeneous link coordinate transformation matrix [2] comes directly from D-H parameters:

$${}_{i-1}^iT = \begin{bmatrix} \cos \theta_i & \sin \theta_i & 0 & a_{i-1} \\ \sin \theta_i \cos \alpha_{i-1} & \cos \theta_i \cos \alpha_{i-1} & -\sin \alpha_{i-1} & -\sin \alpha_{i-1} d_i \\ \sin \theta_i \sin \alpha_{i-1} & \cos \theta_i \sin \alpha_{i-1} & \cos \alpha_{i-1} & \cos \alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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