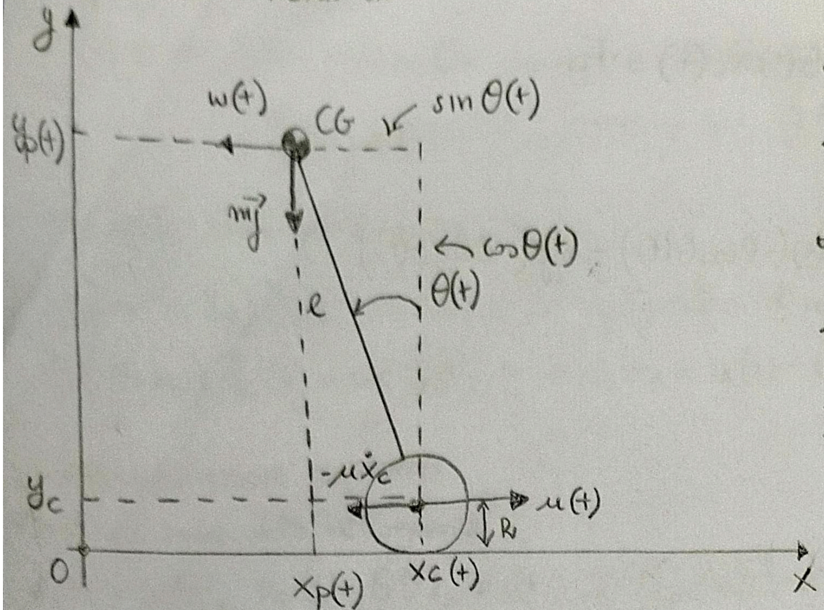
MODELLAZIONE MATEMATICA



• Cinematica Pendolo

$$\begin{cases} x_p(t) = x_c(t) - l\sin\theta(t) \\ y_p(t) = R_r + l\cos\theta(t). \end{cases}$$

deriviamo rispetto al tempo,

$$\begin{cases} \dot{x}_p(t) = \dot{x}_c(t) - l\cos\theta(t)\dot{\theta}(t) \\ y_p(t) = -l\sin\theta(t)\dot{\theta}(t) \end{cases}$$

Identifichiamo le forze sui singoli componenti.

$$\text{Cart}\begin{cases} \Sigma F_x = u(t) - \mu\dot{x}_c(t) \\ \Sigma F_y \text{ trascurate.} \end{cases}$$

$$\text{Pendolo}\begin{cases} \Sigma F_x = -w(t) \text{ disturbo} \\ \Sigma F_y = -mg \text{ gravità.} \end{cases}$$

$$\text{Cart.}\begin{cases} x_c(t) = f_1(x_c(t);\theta(t)) = x_c(t) \\ y_c(t) = f_2\underbrace{(x_c(t),\theta(t))}_{\text{coordinate generalizzate.}} = R_r. \end{cases}$$

→ RIFORMULAZIONE LAGRANGIANA NSL

$$\frac{\partial}{\partial t}\frac{\partial}{\partial \dot{q}_r}K - \frac{\partial}{\partial q_r}K = \sum_{i=1}^{m}\left(F_{ix}\frac{\partial f_i}{\partial q_r} + F_{iy}\frac{\partial f_i}{\partial q_r}\right),$$ con q_r, $r=1,2$ coordinate generalizzate.

$$\text{Pendolo}\begin{cases} x_p(t) = f_2(x_c(t);\theta(t)) = x_c(t) - l\sin\theta(t) \\ y_p(t) = f_2(x_c(t),\theta(t)) = R_r + l\cos\theta(t). \end{cases}$$

• Definiamo l'energia cinetica del modello complessivo.

$$K(x_c(t);\theta(t)) = \frac{1}{2}M\dot{x}_c^2(t) + \frac{1}{2}m\left(\dot{x}_c(t) - l\cos\theta(t)\dot{\theta}(t)\right)^2 + \frac{1}{2}m\left(-l\sin\theta(t)\dot{\theta}(t)\right)^2 + \frac{1}{2}I_p\dot{\theta}^2(t)$$

determiniamone le variazioni,

$$\frac{\partial}{\partial\dot{\theta}}K = m\left(\dot{x}_c(t) - l\cos\theta(t)\dot{\theta}(t)\right)\left(-l\cos\theta(t)\right) + m\left(-l\sin\theta(t)\dot{\theta}(t)\right)\left(-l\sin\theta(t)\right) + I_p\dot{\theta}(t) =$$
$$= -ml\cos\theta(t)\dot{x}_c(t) + ml^2\cos^2\theta(t)\dot{\theta}(t) + ml^2\sin^2\theta(t)\dot{\theta}(t) + I_p\dot{\theta}(t) =$$
$$= (ml^2 + I_p)\dot{\theta}(t) - ml\cos\theta(t)\dot{x}_c(t).$$

$$\frac{\partial}{\partial t}\frac{\partial}{\partial\dot{\theta}}K = (ml^2 + I_p)\ddot{\theta}(t) - ml\cos\theta(t)\ddot{x}_c(t) + ml\sin\theta(t)\dot{\theta}(t)\dot{x}_c(t)$$

$$\frac{\partial}{\partial\theta}K = m\left(\dot{x}_c(t) - l\cos\theta(t)\dot{\theta}(t)\right)\left(l\sin\theta(t)\dot{\theta}\right) + m\left(-l\sin\theta(t)\dot{\theta}(t)\right)\left(-l\cos\theta(t)\dot{\theta}(t)\right)$$
$$= ml\sin\theta(t)\dot{\theta}(t)\dot{x}_c(t) - ml^2\sin\theta\cos\theta\dot{\theta}^2(t) + ml^2\sin\theta\cos\theta\dot{\theta}^2(t) =$$
$$= ml\sin\theta(t)\dot{\theta}(t)\dot{x}_c(t).$$

infine, otteniamo che

$$\frac{\partial}{\partial t}\frac{\partial}{\partial \dot\theta}K - \frac{\partial}{\partial \theta}K = (m\ell^2 + I_p)\ddot\theta(t) - m\ell\cos\theta(t)\ddot x_c(t) = F_\theta$$

con F_θ forze generalizzate.

$$F_\theta = \sum_{i=1}^{2}\left(F_{ix}\frac{\partial \delta_i}{\partial \theta} + F_{iy}\frac{\partial \delta_i}{\partial \theta}\right) = -w(t)(-\ell\cos\theta(t)) - mg(-\ell\sin\theta(t))$$

$$\frac{\partial \delta_1}{\partial \theta} = \frac{\partial \delta_1}{\partial \theta} = 0.$$

$$\frac{\partial \delta_2}{\partial \theta} = -\ell\cos\theta(t), \quad \frac{\partial \delta_2}{\partial \theta} = -\ell\sin\theta(t)$$

1a Equazione Dinamica Sistema

- $(m\ell^2 + I_p)\ddot\theta(t) - m\ell\cos\theta(t)\ddot x_c(t) - mg\ell\sin\theta(t) = w(t)\ell\cos\theta(t).$

Determiniamo le variazioni di K rispetto a x_c

$$\frac{\partial}{\partial \dot x_c}K = M\dot x_c(t) + m(\dot x_c(t) - \ell\cos\theta(t)\dot\theta(t)) =$$
$$= (M+m)\dot x_c(t) - m\ell\cos\theta(t)\dot\theta(t).$$

$$\frac{\partial}{\partial x_c}K = 0.$$

$$\frac{\partial}{\partial t}\frac{\partial}{\partial \dot x_c}K = (M+m)\ddot x_c(t) - m\ell\cos\theta(t)\ddot\theta(t) + m\ell\sin\theta(t)\dot\theta^2(t) = F_{x_c}$$

$$F_{x_c} \triangleq \sum_{i=1}^{2}\left(F_{ix}\frac{\partial \delta_i}{\partial x_c} + F_{iy}\frac{\partial \delta_i}{\partial x_c}\right) =$$

$$= u(t) - \mu\dot x_c(t) - w(t).$$

$$\frac{\partial \delta_1}{\partial x_c} = 1, \quad \frac{\partial \delta_1}{\partial x_c} = 0.$$

$$\frac{\partial \delta_2}{\partial x_c} = 1, \quad \frac{\partial \delta_2}{\partial x_c} = 0$$

Modello Non Lineare Sistema in ANALISI

- $(m\ell^2 + I_p)\ddot\theta(t) - m\ell\cos\theta(t)\ddot x_c(t) - mg\ell\sin\theta(t) = w(t)\ell\cos\theta(t)$
- $(M+m)\ddot x_c(t) - m\ell\cos\theta(t)\ddot\theta(t) + m\ell\sin\theta(t)\dot\theta^2(t) + \mu\dot x_c(t) = u(t) - w(t).$

• Linearizzazione in O_x.

Ricordiamo che linearizzare in O_x equivale ad effettuare le seguenti approssimazioni

$$\sin\theta(t) \approx \theta(t)\ ;\ \cos\theta(t) \approx 1\ ;\ \dot{\theta}^2 \approx 0.$$

Il modello linearizzato in O_x, sarà:

$$1)\ (m\ell^2 + I_p)\ddot{\theta}(t) - m\ell\ddot{x}_c(t) - mg\ell\theta(t) = \ell\, w(t)$$

$$2)\ (M+m)\ddot{x}_c(t) - m\ell\ddot{\theta}(t) + \mu\dot{x}_c(t) = u(t) - w(t).$$

• RAPPRESENTAZIONE IN STATO

Prendiamo come stato le seguenti:

$x_1(t) = \theta(t),\ x_2(t) = \dot{\theta}(t),\ x_3(t) = x_c(t),\ x_4(t) = \dot{x}_c(t)$.

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = \dfrac{1}{(m\ell^2 + I_p)}\left\{ mg\ell\, x_1(t) + \ell w(t) + m\ell\,\dot{x}_4(t) \right\} \\ \dot{x}_3(t) = x_4(t) \\ \dot{x}_4(t) = \dfrac{1}{(M+m)}\left\{ -\mu x_4(t) + u(t) - w(t) + m\ell\,\dot{x}_2(t) \right\}. \end{cases}$$

sostituendo le relative espressioni,

$$\dot{x}_1(t) = x_2(t)$$

$$\dot{x}_2(t) = \frac{1}{(m\ell^2+I_p)}\left\{ mg\ell\, x_1(t) + \ell w(t) + \frac{m\ell}{(M+m)}\left[-\mu x_4(t) + u(t) - w(t) + m\ell\,\dot{x}_2(t) \right] \right\}$$

$$= \frac{mg\ell}{(m\ell^2+I_p)}x_1(t) + \frac{\ell}{(m\ell^2+I_p)}w(t) - \frac{m\ell\mu}{(m\ell^2+I_p)(M+m)}x_4(t) + \frac{m\ell}{(m\ell^2+I_p)(M+m)}u(t) +$$

$$- \frac{m\ell}{(m\ell^2+I_p)(M+m)}w(t) + \frac{(m\ell)^2}{(m\ell^2+I_p)(M+m)}\dot{x}_2(t).$$

ovvero,

$$\left[1 - \frac{(m\ell)^2}{(m\ell^2+I_p)(M+m)}\right]\dot{x}_2(t) = \frac{mg\ell}{(m\ell^2+I_p)}x_1(t) - \frac{m\ell\mu}{(m\ell^2+I_p)(M+m)}x_4(t) +$$

$$+ \frac{m\ell}{(m\ell^2+I_p)(M+m)}u(t) + \left[\frac{\ell}{(m\ell^2+I_p)} - \frac{m\ell}{(m\ell^2+I_p)(M+m)}\right]w(t)$$

poi,

$$\dot{x}_3(t) = x_4(t).$$

ed infine

$$\dot{x}_4(t) = \frac{1}{(M+m)}\left\{-\mu x_4(t) + u(t) - w(t) + \frac{m\ell}{(m\ell^2+I_p)}\left[mg\ell x_1(t) + \ell w(t) + m\ell \dot{x}_4(t)\right]\right\}$$

$$= -\frac{\mu}{(M+m)}x_4(t) + \frac{1}{(M+m)}u(t) - \frac{1}{(M+m)}w(t) + \frac{(m\ell)^2 g}{(m\ell^2+I_p)(M+m)}x_1(t) +$$

$$+ \frac{m\ell^2}{(m\ell^2+I_p)(M+m)}w(t) + \frac{(m\ell)^2}{(m\ell^2+I_p)(M+m)}\dot{x}_4(t)$$

raggruppando a primo membro,

$$\left[1 - \frac{(m\ell)^2}{(m\ell^2+I_p)(M+m)}\right]\dot{x}_4(t) = -\frac{\mu}{(M+m)}x_4(t) + \frac{(m\ell)^2 g}{(m\ell^2+I_p)(M+m)}x_1(t) +$$

$$+ \frac{1}{(M+m)}u(t) + \left[\frac{m\ell^2}{(m\ell^2+I_p)(M+m)} - \frac{1}{(M+m)}\right]w(t).$$

in forma compatta,

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ A_{21} & 0 & 0 & A_{24} \\ 0 & 0 & 0 & 1 \\ A_{41} & 0 & 0 & A_{44} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ B_2 \\ 0 \\ B_4 \end{bmatrix}u(t) + \underbrace{\begin{bmatrix} 0 \\ D_2 \\ 0 \\ D_4 \end{bmatrix}}_{\text{disturbo}} w(t)$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}.$$