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# **E344 Assignment 2**

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Report submitted in partial fulfilment of the requirements of the module  
Design (E) 344 for the degree Baccalaureus in Engineering in the Department of Electrical  
and Electronic Engineering at Stellenbosch University.

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# Nomenclature

## Variables and functions

$p(x)$	Probability density function with respect to variable $x$ .
$P(A)$	Probability of event $A$ occurring.
$\varepsilon$	The Bayes error.
$\varepsilon_u$	The Bhattacharyya bound.
$B$	The Bhattacharyya distance.
$s$	An HMM state. A subscript is used to refer to a particular state, e.g. $s_i$ refers to the $i^{\text{th}}$ state of an HMM.
$\mathbf{S}$	A set of HMM states.
$\mathbf{F}$	A set of frames.
$\mathbf{o}_f$	Observation (feature) vector associated with frame $f$ .
$\gamma_s(\mathbf{o}_f)$	A posteriori probability of the observation vector $\mathbf{o}_f$ being generated by HMM state $s$ .
$\mu$	Statistical mean vector.
$\Sigma$	Statistical covariance matrix.
$L(\mathbf{S})$	Log likelihood of the set of HMM states $\mathbf{S}$ generating the training set observation vectors assigned to the states in that set.
$\mathcal{N}(\mathbf{x} \mu, \Sigma)$	Multivariate Gaussian PDF with mean $\mu$ and covariance matrix $\Sigma$ .
$a_{ij}$	The probability of a transition from HMM state $s_i$ to state $s_j$ .
$N$	Total number of frames or number of tokens, depending on the context.
$D$	Number of deletion errors.
$I$	Number of insertion errors.
$S$	Number of substitution errors.

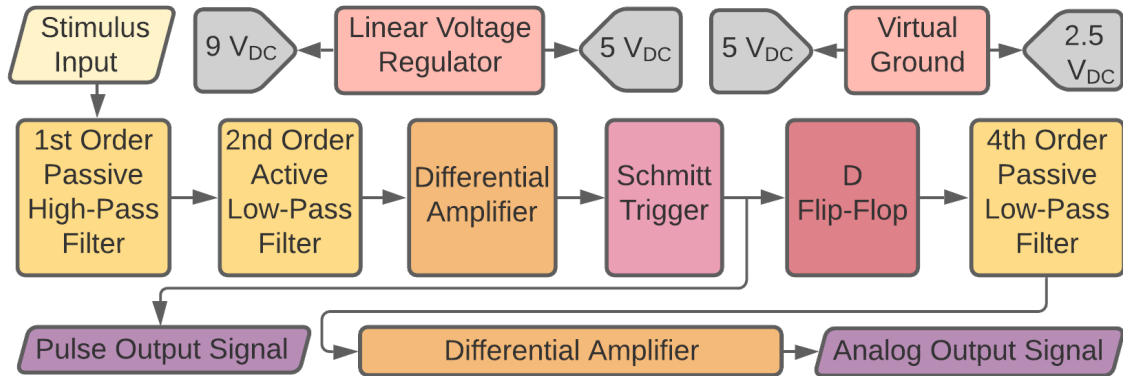
## Acronyms and abbreviations

AE	Afrikaans English
AID	accent identification
ASR	automatic speech recognition
AST	African Speech Technology
CE	Cape Flats English
DCD	dialect-context-dependent
DNN	deep neural network
G2P	grapheme-to-phoneme
GMM	Gaussian mixture model
HMM	hidden Markov model
HTK	Hidden Markov Model Toolkit
IE	Indian South African English
IPA	International Phonetic Alphabet
LM	language model
LMS	language model scaling factor
MFCC	Mel-frequency cepstral coefficient
MLLR	maximum likelihood linear regression
OOV	out-of-vocabulary
PD	pronunciation dictionary
PDF	probability density function
SAE	South African English
SAMPA	Speech Assessment Methods Phonetic Alphabet

# Chapter 1

## System design

### 1.1. System overview



**Figure 1.1:** System Diagram

A heart-rate sensor is designed to receive an input signal from which pulses and analogue values are generated, corresponding to heart-beats and the heart-rate respectively. The aforementioned is achieved by voltage regulation, signal conditioning, pulse generation and conversion to analogue, as shown in figure 1.1. The circuit is powered by a voltage regulator [1] which can supply 100 mA, of which 12.8 mA is used for the temperature sensor - see E344 Assignment 1 [1] - leaving 87.2 mA at 5 V, mandating conservative current use. The input signal has an amplitude of insufficient magnitude for conversion, and is subject to noise, necessitating signal conditioning: a first order passive high-pass filter and a second order active low-pass filter attenuate both high- and low-frequencies. Chosen with maximal simplicity in mind to reduce cost and complexity, the filters still performing adequately. Thereafter, a differential amplifier produces a signal with a large amplitude and little noise. A Schmitt Trigger then outputs a pulse signal with a frequency corresponding to the heart-rate. The Schmitt Trigger was chosen as it provides a noise margin via hysteresis. An analogue voltage output is required for the microcontroller - filtering and peak detection using diodes was considered but discarded, as non-linear diodes result in extremely slow simulation. Rather, the pulse output signal was converted to a pulse-width modulated signal, where the frequency of the former determines the duty cycle of the latter. This was done as PWM signals lend themselves to conversion-to-analogue by simple filtering. The PWM signal was obtained by using a D Flip-Flop and a RC-circuit (see section 2.2). A third order passive RC filter is then used - passive components reduce current usage and simulation time. The filter is of high order as to minimize noise, while meeting the settling time requirement. Finally, the signal was amplified to achieve the required range.



# Chapter 2

## Heart rate sensor

### 2.1. Introduction

Circuits pertaining to signal conditioning, pulse signal and analogue output generation will be discussed. Conditioning is done via filtering and amplification; active filters provide high input and low output impedance, and a large Q-factor [2]. A differential amplifier is suitable for amplification, as the gain is referenced against a customizable voltage [3]. A Schmitt Trigger is well-suited for pulse generation, as it provides a noise margin [4]. PWM signals can be converted to analogue using filtering [5].

### 2.2. Design

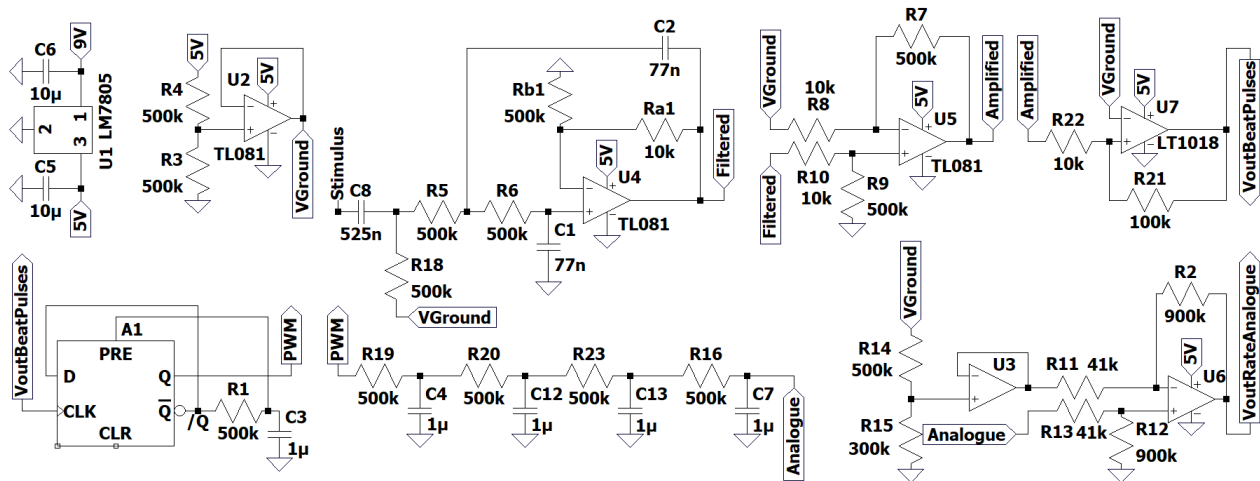


Figure 2.1: Complete Circuit enlarge this

The complete circuit is shown upfront in figure 2.1 to aid with explanation. The largest resistor in sub-circuits is always chosen to be 500 k $\Omega$ , as to reduce current usage. The design process now follows. The FFT of the stimulus input in figure 2.2 has information residing between 0.8 to 2.5 Hz, corresponding to 50 and 150 BPM respectively. The other peaks represent noise at 0.25 Hz, as well as at twice and three times the message signal, and higher. Noise distorts square wave output,

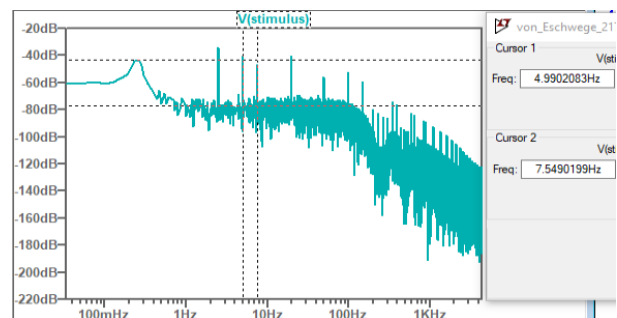


Figure 2.2: Fast fourier transform of stimulus

necessitating filtering. A first order passive high-pass filter, cutoff frequency 0.606 Hz, attenuates the low frequency noise.  $R_{18} = 500 \text{ k}\Omega$ , thus  $C_8 = 525 \text{ nF}$  according to  $f_c = \frac{1}{2\pi RC}$ . The capacitor is connected to a virtual ground of 2.5 V, centering the signal around 2.5 V. A second order active low-pass filter, cutoff frequency 4.1 Hz, filters out high frequency noise.  $R_5 = R_6 = 500 \text{ k}\Omega$ ,  $C_1 = C_2 = 77 \text{ nF}$  - see aforementioned formula. **mention amplitudes** Cutoff frequencies were selected to remove noise maximally while minimally affecting heart-rate data. The signal should reside slightly above 2.5 V to facilitate amplification (to be discussed). Thus,  $R_{b1} = 500 \text{ k}\Omega$  and  $R_{a1} = 10 \text{ k}\Omega$  since  $A_v = 1 + \frac{R_A}{R_B}$  [6]. The TL081 op-amp is used, as it is less expensive than the TLC2272 [7]. A filter output with DC offset slightly above 2.5 V allows for the use of a differential amplifier with the existing virtual ground connected to the negative input, thus removing the need for additional circuitry otherwise required. The signal is amplified according to  $V_{OUT} = \frac{R_a}{R_b} (V_2 - V_1)$  [3], where  $R_a$  corresponds to  $R_7$  and  $R_9$ , and  $R_b$  to  $R_8$  and  $R_{10}$ . The gain of 50 was selected to again provide a DC offset of 2.5 V, as it facilitates implementation of the comparator (to be discussed). Since the amplified signal has an amplitude of only 1.66 V, the inexpensive TL081 was chosen despite having a smaller output range. Next, the signal is fed into a Schmitt Trigger comparator - the LT1018 was chosen as it allows for output very close to 0 and 5V, and has a high gain. 5 V is produced if the input exceeds the upper trip point (UTP) and 0 V if the input falls below the lower trip point (LTP) [4]. The range between the UTP and LTP is referred to as the hysteresis width ( $w_h$ ) and serves as a noise margin [4] around the reference voltage,  $V_{REF_{comparator}}$ . The DC offset of 2.5 V of the amplified signal requires  $V_{REF_{comparator}} = 2.5 \text{ V}$ , once again allowing for the use of the existing virtual ground (instead of additional circuitry) at the negative input of the comparator. Pulse duration is designed for the highest frequency, as it will then also meet the requirement for lower frequencies. Since 150 BPM corresponds to a period of 0.4 s,  $w_h$  should be selected to ensure an output pulse width of at least 150 ms. The amplified signal approximates a sinusoid - substitute values of  $t$  that differ by 200 ms for  $t_1$  and  $t_2$  to account for noise:

$$y_1 = 1.66 \sin(2\pi(2.5)t_1) + 2.5 \quad (2.1)$$

$$y_2 = 1.66 \sin(2\pi(2.5)t_2) + 2.5 \quad (2.2)$$

Subtracting 2.2 from 2.1 yields  $y_1 - y_2 = w_h = 0.52 \text{ V}$  for  $t_1 = 0.01 \text{ s}$  and  $t_2 = 0.21 \text{ s}$ .  $w_h$  also is an order of magnitude larger than the highest input signal noise levels, increasing design fidelity. Now,  $UTP = 2.75 \text{ V}$ ,  $LTP = 2.25 \text{ V}$ ,  $UTP = V_{REF} + \beta V_{CC}$  and  $LTP = V_{REF} - \beta V_{CC}$ , thus  $\beta = 0.05$  [4]. Further,  $\beta = \frac{R_{22}}{R_{22} + R_{21}}$ . Thus  $R_{21} = 190 \text{ k}\Omega$  and  $R_{22} = 10 \text{ k}\Omega$ . ( $R_{21}$  was later adjusted to  $100 \text{ k}\Omega$  to account for loading effects). A one-shot was considered to extend the pulses, but was omitted as the requirement could be met without it. A square wave is thus produced, where the frequency of the pulses represent the heart-rate.

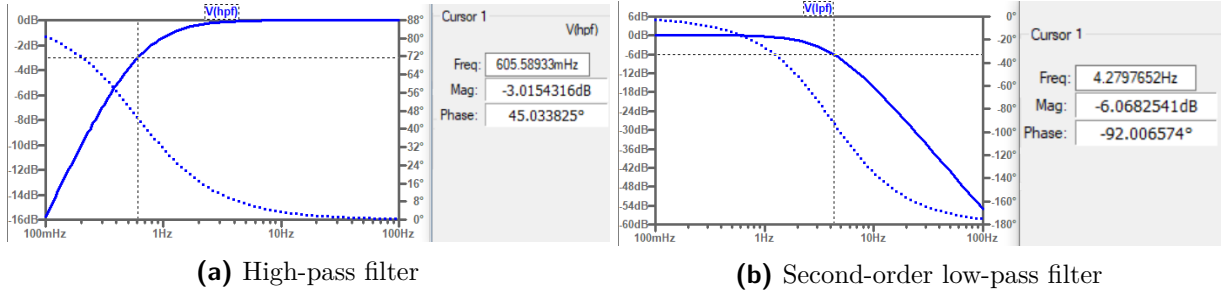
Obtaining an output suitable for a microcontroller presents itself as a problem of converting

frequency to analogue. The decision was made to obtain a PWM signal from the FM pulse signal, as PWM signals lend themselves more readily to conversion-to-analogue using a simple low-pass RC filter. Pulse-width modulation was achieved by means of a D flip-flop. The essence of the design is to firstly keep the output high continually, but to then drive the output low for a fixed time, once per period. Since the period of the clock signal varies against BPM, driving low for a fixed amount of time results in a different ratio of the pulse being low at different frequencies, producing a varying duty cycle. Specifically, the previously generated pulse signal is used as a clock signal for the D flip-flop, and the not-Q output is fed back into the data line. This would normally drive the output high for all time  $t$ ; thus a RC circuit is added of which the capacitor is charged by the not-Q output, and connected to the SET-line of the flip-flop. Once the capacitor charges to 2.5 V, Q is pulled high and not-Q low until the next rising edge of the clock input inverts Q and not-Q. The result is a slightly delayed PWM signal, with a large duty cycle at high frequencies, and vice-versa. This configuration was preferred above the standard one-shot setup, as it results in a consistent PWM output as long as the capacitor charge time is selected to be shorter than the narrowest pulse of the clock signal. See section 2.3, figure ?? for output graphs. calculations follow. Selecting  $R1 = 500\text{ k}\Omega$  gives  $C = 1\mu$ , according to the capacitor charge/discharge exponential equations, of which the extended solution (using a Matlab script) is given in Appendix C for the sake of brevity. Having obtained a PWM signal, a fourth order passive low-pass filter produces analogue values in the range of 0.95 to 1.2 V. A cutoff frequency of 0.5 Hz ensures maximal attenuation of noise while maintaining a 5% settling time of 10 seconds. Therefore resistor values are  $500\text{ k}\Omega$  and capacitor values are  $1\mu$ . Finally, the analogue output is amplified by means of a differential amplifier, using a TLC2272, with a negative input at 0.938 V. A gain of 20 produces a sufficient output range. Resistor values were chosen to be larger as to minimize loading effects. Thus,  $R2 = R12 = 900\text{ k}\Omega$  and  $R11 = R13 = 45\text{ k}\Omega$  according to  $V_{OUT} = \frac{R_a}{R_b} (V_2 - V_1)$  [3]. Loading effects still had a slight effect, and required adapting R11 and R13 to  $41\text{ k}\Omega$ . An analogue transducer is thus designed, and can be connected to a microcontroller.

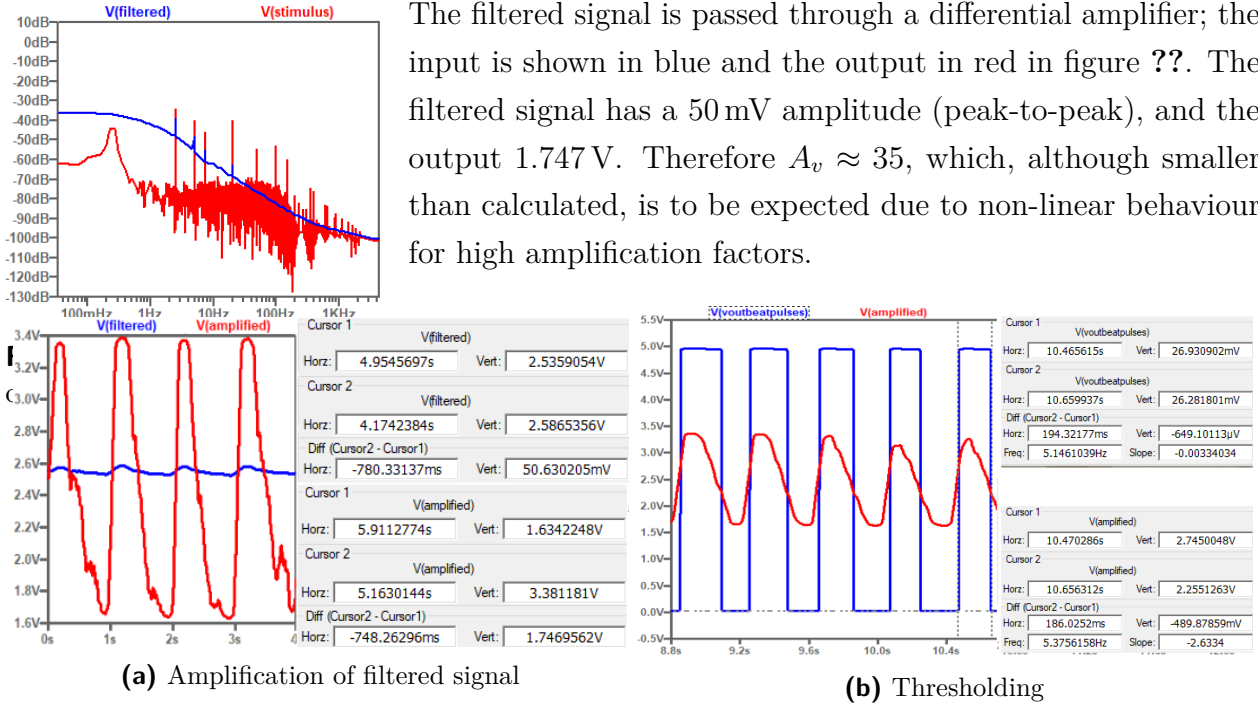
Total current is calculated using  $1.4\text{ mA}$  per TL081 [8],  $2.2\text{ mA}$  per TLC2272 [9] and  $110\mu\text{A}$  per LT1018 [10]. Non-trivial resistors are all approximated using  $5\text{ V}$  over, as to err on the side of caution:  $4(1.4m) + (2.2m) + (110n) + 16\left(\frac{5}{500k}\right) + 3\left(\frac{5}{10k}\right) + 2\left(\frac{5}{41k}\right) + \left(\frac{5}{100k}\right) = 9.75\text{ mA}$ . Voltage regulator current is considered in E344 Assignment 1 [1].  $100m - 9.75m - 11.39m$  leaves  $78.86\text{ mA}$  for Assignment 3.

## 2.3. Results

The frequency response of both the high- and low-pass filters are  $-3\text{ dB}$  at  $0.606\text{ Hz}$  and  $-6\text{ dB}$  at  $4.3\text{ Hz}$  respectively, as shown in figures 2.3a and 2.3b, thus matching the design calculations almost perfectly. Comparison of filter input and output in figure 2.4 shows a drastic reduction in noise.



**Figure 2.3:** Frequency response of filters



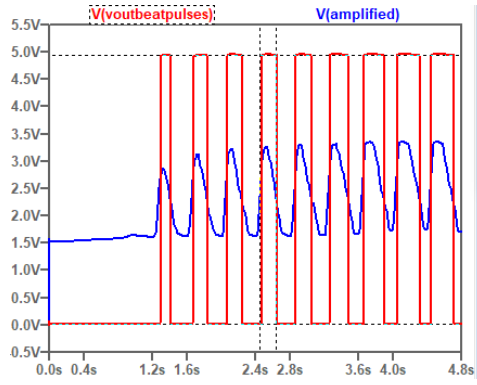
**Figure 2.5:** Signal Conditioning

Input and output of a LT1018 comparator, used for thresholding, is shown in figure ?? . See  $UTP = 2.75\text{ V}$  and  $LTP = 2.25\text{ V}$  in figure ?? , giving a hysteresis width of  $0.5\text{ V}$ , exactly as calculated.

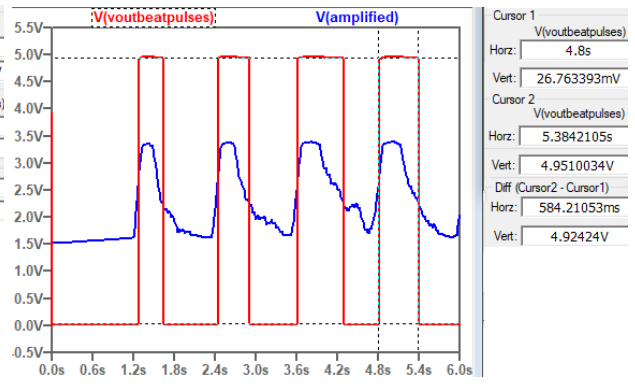
The output ranges from  $0.026$  to  $4.95\text{ V}$ , with a pulse duration in range  $189$  to  $584\text{ ms}$  in figures 2.6a and 2.6b, meeting the  $150\text{ ms}$  requirement. The design remains consistent for specification deviations of  $\pm 10\%$  amplitude and a DC offset variation of  $\pm 0.2\text{ V}$ , as the high-pass filter removes the DC component and the filter has a cutoff frequency low enough to greatly attenuate noise.

For the analogue transducer, a PWM signal is obtained. Section 2.2 explains the interrelation of the signals, where `voutbeatpulses`, `pwm` and `rc` in figure ?? correspond to the clock signal,  $Q$ , and the charging capacitor voltage respectively. The  $5\%$  settling time is  $9.45\text{ s}$ . The duty cycle is  $73\%$  at  $60\text{ BPM}$  and decreases to  $56\%$  at  $150\text{ BPM}$ . Finally, amplification and filtering of the PWM signal produces analogue output, range  $4.2\text{ V}$  (2.7b).

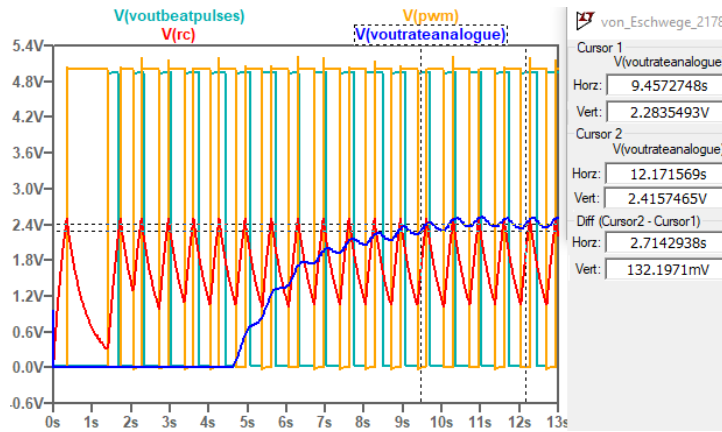
In figure 2.8, total current draw is measured through  $R_{\text{sense}}$ , averaging  $12.9\text{ mA}$ , meeting the



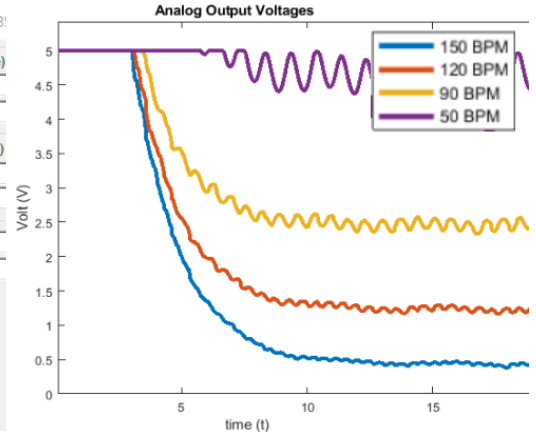
(a) 150 BPM



(b) 50 BPM



(a) PWM and settling time



(b) Analogue voltage output

bonus requirement of 15 mA. When combined with Assignment 1, the combined current of 25.8 mA still is far less than the maximum current rating of 100 mA for the voltage regulator [1].

## 2.4. Summary

Concluding, it has been shown that the design performs as expected, meeting the all requirements. Output pulse width is broader than required, analogue output is generated with discrete, non-overlapping values exceeding the required range, while settling to a final value according to specification. Current draw remains sub-15 mA. Upon implementation of the microcontroller, be cognizant of the fact that analogue output values scale in a slightly non-linear fashion. This is not a problem, as it can be easily accounted for in software.

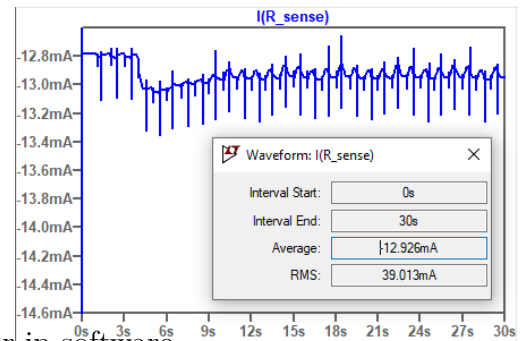


Figure 2.8: Total current usage

# Chapter 3

## System and conclusion

### 3.1. System

The design of the heart-rate sensor goes to show that a noisy input signal could effectively be converted to a square wave output as well as an analogue output, allowing for reliable interfacing of real-world measurements with digital systems. The heart-rate sensing circuit can now be integrated with the remainder of the health monitoring system by connecting the analogue output to the microcontroller input, to which the temperature sensor will also be connected - see E344 Assignment 1 [1]. The modular design of different parts of the health-monitoring system allows for simplification of both design and debugging. Due to the second order low-pass filter, the heart-rate sensor is remarkably robust with respect to variations in noise, attenuating noise levels to below 6% of the signal amplitude before creating a pulse output. The design also performs well on a variety of the normalised input data sets. For microcontroller integration, a log-linear scale can be used to obtain a linear analogue reading, which can be calibrated using

$$\text{ADC}_{\text{val}} = (2^{10} - 1) \frac{V_{in}}{V_{ref}} \rightarrow V_{in} = \frac{(\text{ADC}_{\text{val}})(V_{ref})}{2^{10} - 1}$$

The slope of the frequency increase is  $\frac{0.8-2.5}{4.7-0.5} = -0.404$ . Thus, for 150 BPM:

$$f = 2.5 = -0.404V + C = -0.404(0.5) + C \rightarrow C = 2.702$$

Therefore, the frequency can be calculated by

$$f = (-0.404) \frac{\text{ADC}_{\text{val}}(V_{ref})}{2^{10} - 1} + 2.702$$

The quantisation error is  $f_{err} = \frac{2.5-0.8}{2^{10}} / (2.5 - 0.8) * 100 = 0.1\%$

Concluding, it has been shown that the combination of the voltage regulator and the temperature sensing circuitry works very well to achieve the desired result. Since the cascaded filters ensure extremely low noise levels, the ADC should be capable of distinguishing the measured temperature to a very high degree of accuracy. The system uses more of the less costly components, and is very power efficient, all the while meeting all of the bonus requirements. It is thus safe to say that the objective of the design has been met successfully.

## 3.2. Lessons learnt

1. LTSpice is a blessing, but can be an absolute nightmare with simulations. I have created a rough metric, and it goes to show that 30% of my time was spent on circuit design, 10% on report writing, and 60% on debugging LTSpice simulation problems. I entered E-Design with the belief that simulation would greatly accelerate the pace of the subject, as soldering was eliminated, but I stand firm in my belief that I could have built a practical circuit much faster, as 'timestep too small' does not apply in the beauty of real-world continuity. However, maybe I just stand firm in this belief because I have not dealt with the problems arising from burnt-out components, messy soldering, melted PCB tracks and exploding diodes.
2. I believe to have greatly improved with regards to modularising and debugging not only circuits, but systems in general. Treating everything as a small problem to be solved went a long way towards improving conceptual understanding of component interaction.
3. Don't go surfing for the entirety of the first week of the term. I said this in Assignment 1 as well, and lost a bunch of marks for it, but it is still applicable and true, so I'll say it again.
4. If I could have it all over again, I wouldn't have texted my circuit; she did not reply.

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- [9] *TLC227x: Advanced LinCMOS Rail-to-Rail Operational Amplifiers*, TL2272 Datasheet, Texas Instruments, 2016.
- [10] *LT1018 Micropower Dual Comparator*, LT1018 Datasheet, Linear Technology.



# Appendix A

## Social contract




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### E-design 344 Social Contract

2020


The purpose of this document is to establish commitment between the student and the organisers of E344. Beyond the commitment made here, it is not binding.

In the months preceeding the term, the lecturer (Thinus Booysen) and the Teaching Assistant (Michael Ritchie) spent countless hours to prepare for E344 to ensure that you get your money's worth and that you are enabled to learn from the module and demonstrate and be assessed on your skills. We commit to prepare for the module, to set the tests and assessments fairly, to be reasonably available, and to provide feedback and support as best and fast we can. We will work hard to give you the best opportunity to learn from and pass analogue electronic design E344.

Signature: .....  ..... Date: ..... 13 July 2020 .....

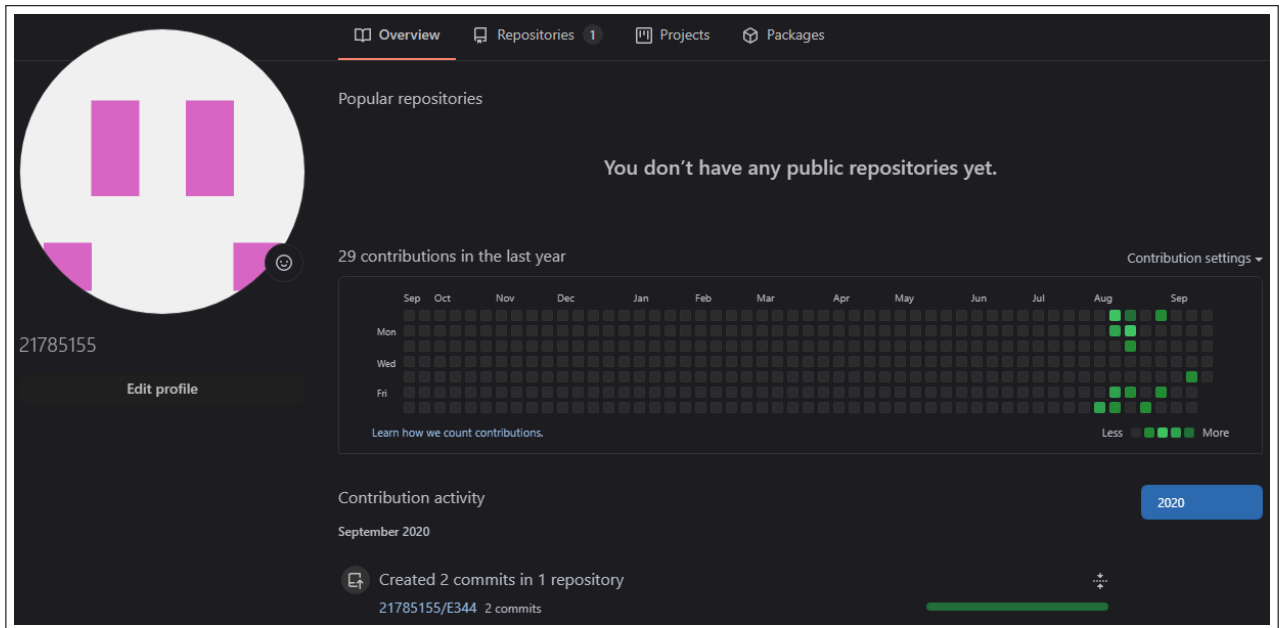
I, ..... **Daniel Heinrich von Eschwege** ..... have registered for E344 of my own volition with the intention to learn of and be assessed on the principals of analogue electronic design. Despite the potential publication of supplementary videos on specific topics, I acknowledge that I am expected to attend the lectures and lab sessions to make the most of these appointments and learning opportunities. Moreover, I realise I am expected to spend the additional requisite number of hours on E344 as specified in the yearbook.

I acknowledge that E344 is an important part of my journey to becoming a professional engineer, and that my conduct should be reflective thereof. This includes doing and submitting my own work, working hard, starting on time, and assimilating as much information as possible. It also includes showing respect towards the University's equipment, staff, and their time.

Signature: .....  ..... Date: ..... 24/09/2020 .....

# Appendix B

## GitHub Activity Heatmap



# Appendix C

## Stuff you want to include

$R1 = 500\text{ k}\Omega$ .  $\tau = RC$ . The capacitor charge oscillates between  $V_L$  and  $V_H$ .  $V_H = 2.5\text{ V}$ .  $V_L$  is reached for the first time at  $t_{L1}$  and  $V_H$  at  $t_{H1}$ .  $V_L$  is then reached at  $t_{L2}$ . For 150 BPM (or 2.5 Hz), the pulse drives high for 0.2 ms. Since the capacitor has to charge faster than 0.2 ms, a charge time of 0.16 ms was selected to add a 20% margin, accounting for noise. Thus  $t_{H1} - t_{L1} = 0.16$  and  $t_{L2} - t_{H1} = 0.4 - 0.16 = 0.24$  for the 150 BPM signal. Finally,

$$V_L = 5 \left( 1 - e^{-\frac{t_{L1}}{\tau}} \right)$$

$$V_H = 5 \left( 1 - e^{-\frac{t_{H1}}{\tau}} \right)$$

$$V_L = V_H \left( e^{-\frac{t_{L1}}{\tau}} \right)$$

giving  $C = 1\mu$ , when solving using the following MatLab script:

```
syms t
syms t11
syms th1
syms t12
syms v1
Vh = 2.5
R = 500000
A = v1==5*(1-exp(-t11/t))
B = Vh==5*(1-exp(-th1/t))
C = v1==Vh*(exp(-(t12-t11)/t))
D = th1 - t11 == 0.16
E = t12 - th1 == 0.24
S = solve([A, B, C, D, E],[t, t11, th1, t12, v1])
S.t
S.t11
S.th1
S.t12
v1
C = R/t
```

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