Falling of a Pencil in Axion Enviroment

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Abstract

This note investigate the motion of a rigid pencil on a horizontal table, assuming that all mass oscillates coherently from the coupling to a homogeneous axion background. The pencil is initially erect, then it undergoes a spontaneous symmetry breaking: it falls, the question is that whether this pencil will slide during this falling process. It is shown that the condition of sliding can be modified greatly, depends on the magnitude of coupling, its oscillation frequency and the phase of oscillation.

CONTENTS

I. THE PROBLEM

The situation is depicted in the figure below. The pencil is modeled as a straight rigid rod with uniform mass ditribution, mass $m(t) = 1 + \beta\cos(\mu t + \phi)$, where β, μ, ϕ are constants, and length $l = 1$ (also we work in the unit $g = 1$). As the pencil falls, θ increases from 0 to $\pi/2$.

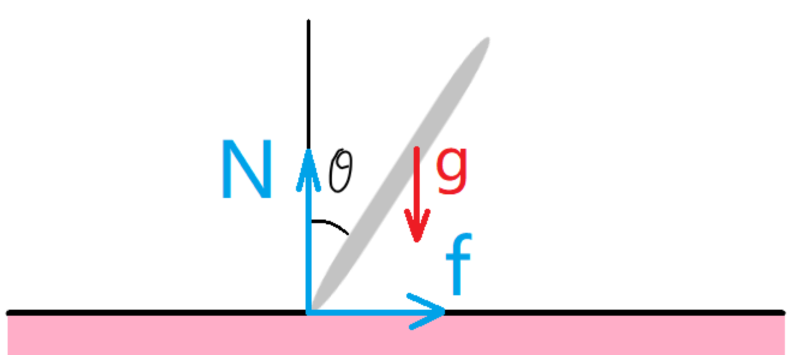

FIG. 1. The problem set-up.

To demonstrate the effect of $m(t)$, we do a lagrangian analysis for the frictionless case,

$$L = \frac{1}{2}\left(\frac{1}{12}ml^2\right)\dot{\theta}^2 + \frac{1}{2}mv^2 - \frac{1}{2}mgl\cos\theta \tag{1}$$

where $\mathbf{v}$ is the velocity of the mass center. Now the rotation dynamics reads

$$\frac{d}{dt}\left(\frac{1}{12}ml^2\dot{\theta}\right) = \frac{1}{2}mgl\sin\theta \tag{2}$$

or

$$\frac{\dot{m}}{m}\dot{\theta} + \ddot{\theta} = 6\sin\theta \tag{3}$$

The vertical motion of the mass center is

$$\frac{d}{dt}\left(m\frac{d}{dt}(\frac{1}{2}l\cos\theta)\right) = N - mg \tag{4}$$

or

$$N/m = 1 - \frac{1}{2}\sin\theta\left(\ddot{\theta} + \frac{\dot{m}}{m}\dot{\theta} + \cot\theta\dot{\theta}^2\right) = 1 - \frac{1}{2}\left(6\sin^2\theta + \cos\theta\dot{\theta}^2\right) \tag{5}$$

where N is the normal force exerted by the table. Now incorporating friction from the table, these equations do not change, but horizontal motion of the mass center does change. Assuming the contacting point doesn't slide, we have

$$\frac{d}{dt}\left(m\frac{d}{dt}(\frac{1}{2}l\sin\theta)\right) = f \tag{6}$$

or

$$f/m = \frac{1}{2}\cos\theta\left(\ddot{\theta} + \frac{\dot{m}}{m}\dot{\theta} - \tan\theta\dot{\theta}^2\right) = \frac{1}{2}\left(6\sin\theta\cos\theta - \sin\theta\dot{\theta}^2\right) \tag{7}$$

where f is the friction force. In the case of static mass, if the pencil does not slide, the total energy of the system

$$H = \frac{1}{2}\left(\frac{1}{12}ml^2\right)\dot{\theta}^2 + \frac{1}{2}mgl\cos\theta = \frac{1}{2}mgl \tag{8}$$

is conserved, then f, N are fixed as functions of the angle and time, the result is

$$\begin{aligned}\dot{\theta} &= \sqrt{12\,(1-\cos\theta)}\\ N/mg &= 1 - 3\sin^2\theta + 6\cos^2\theta - 6\cos\theta\\ f/mg &= 9\sin\theta\cos\theta - 6\sin\theta\end{aligned} \tag{9}$$

The slide cannot happen if $r \equiv f/N$ is smaller then the critical friction factor of the table. Note $N(\theta)$ diverge near 28°, hence the sliding is unavoidable.

If mass varies with time, however, the Hamiltonian is no longer conserved, we have to solve the equation of motion (3) directly, which for the mass oscillation model above, is explicitly

$$\ddot{\theta} + \frac{-\beta\mu \sin(\mu t + \phi)}{1 + \beta \cos(\mu t + \phi)}\dot{\theta} - 12 \sin\theta = 0 \tag{10}$$

and calculate f, N using (5) and (7).

II. RESULTS AND DISCUSSION

We examine at first the case of slow oscillation, when the time scale of falling is much shorter then $1/\mu$, hence

$$\ddot{\theta} + \frac{-\beta\mu(\sin\phi + \mu\cos\phi\, t)}{1 + \beta(\cos\phi - \mu\sin\phi\, t)}\dot{\theta} - 12 \sin\theta = 0 \tag{11}$$

In this case we find numerically that $r(\theta)$ qualitatively does not change.

In the high-frequency regime, the friction term actually acts as a driving force, and the effects of β and ϕ are way more significant. Some results are shown below. For very high frequency, the overal trend is that r is enhanced early on, so that the pencil is more likely to slide.

For mapping to a real experiment, consider the time scale of falling without slide, in the static mass case it's shorter than (since the slide is unavoidable)

$$\int_0^{\pi/2} \frac{d\theta}{\dot{\theta}} = \sqrt{\frac{l}{g}} \int_0^{\pi/2} \frac{d\theta}{\sqrt{12(1-\cos\theta)}} \approx 7.8\sqrt{\frac{l}{g}} \approx 2.5\sqrt{l/\mathrm{m}} \tag{12}$$

which translates into a frequency scale $f = 0.4/\sqrt{l/\mathrm{m}}$.

The oscillation frequency is given by the particle mass

$$\mu = \text{mass} \times \frac{c^2}{\hbar} = \frac{\text{mass} \times c^2}{\text{eV}} 1.5 \times 10^{15}\,\text{Hz} \tag{13}$$

Thus, for an ultralight particle with mass $\sim 10^{-20}$ eV, the freuency is matched for a pencil of length 10^8 meters (at least). But for larger mass, e.g., 10^{-10} eV, the oscaillation is much faster then the falling even for a normal-sized pencil, in this case the only problem is the coupling being too weak.

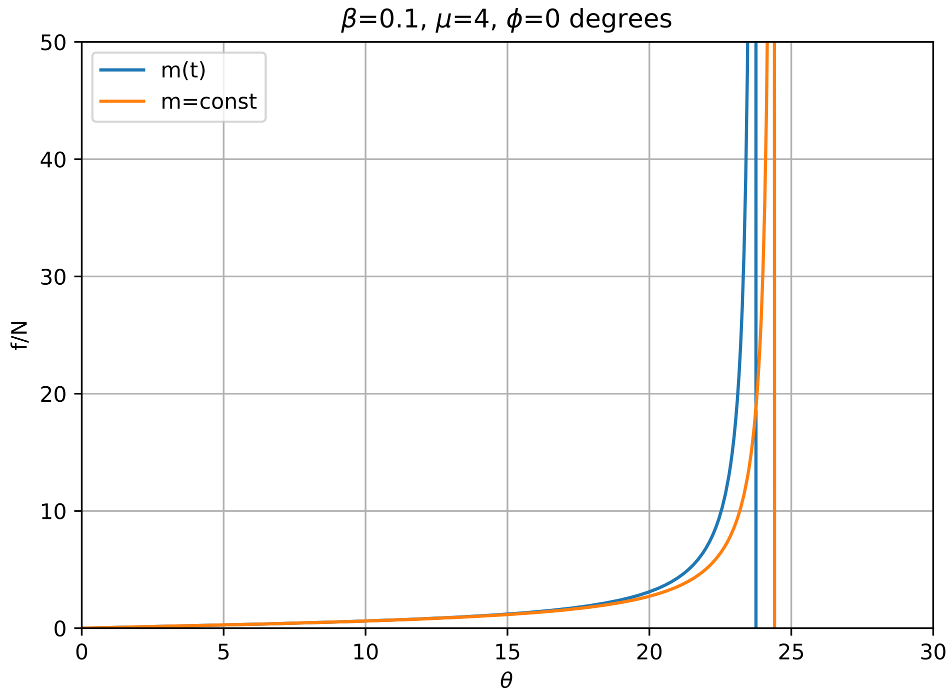

FIG. 2. Small coupling

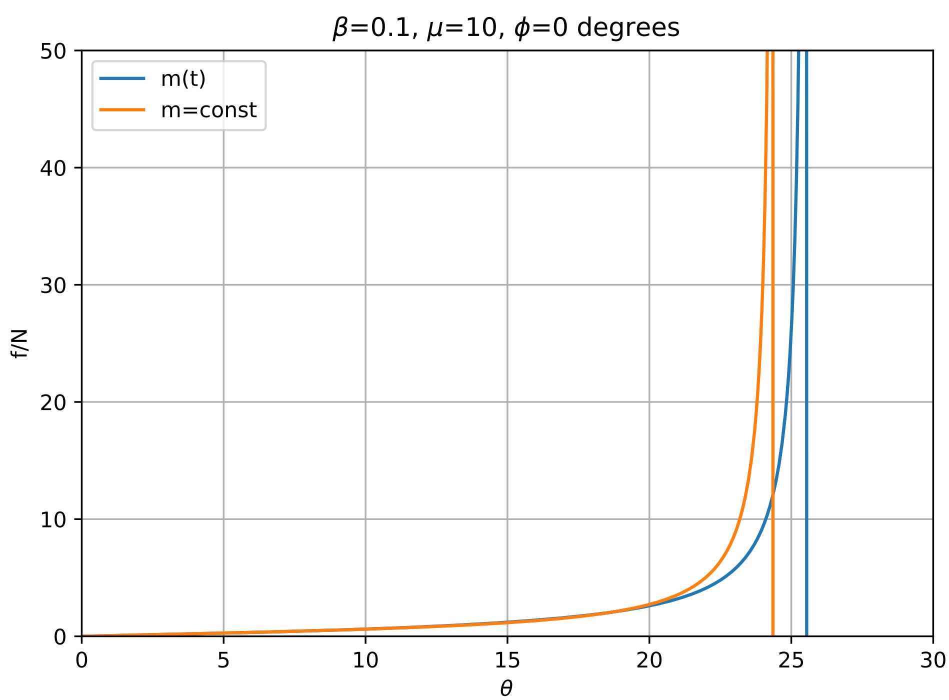

FIG. 3. Small coupling

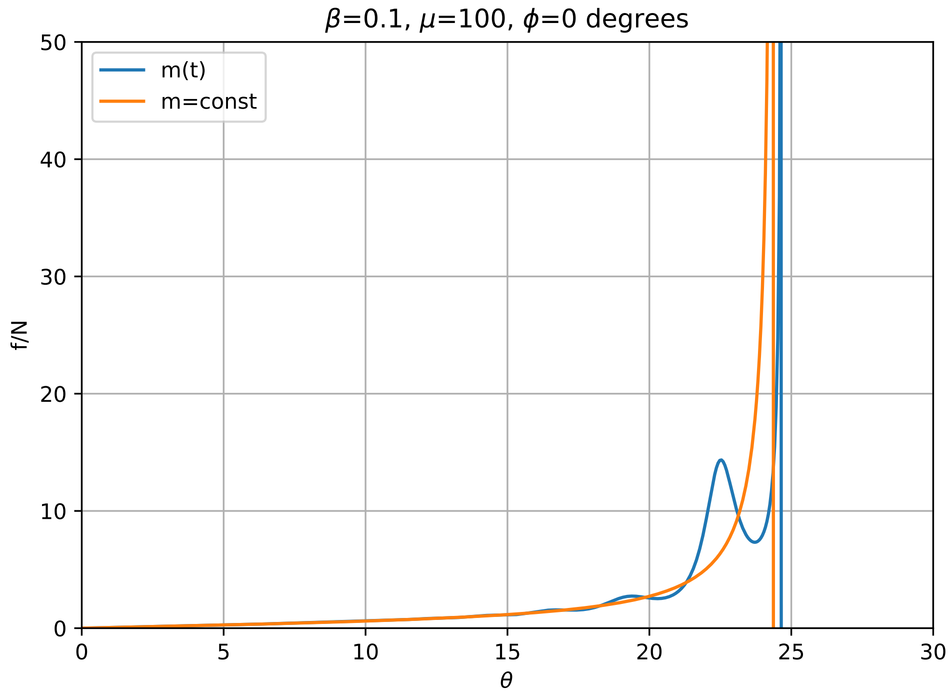

FIG. 4. Small coupling, high frequency.

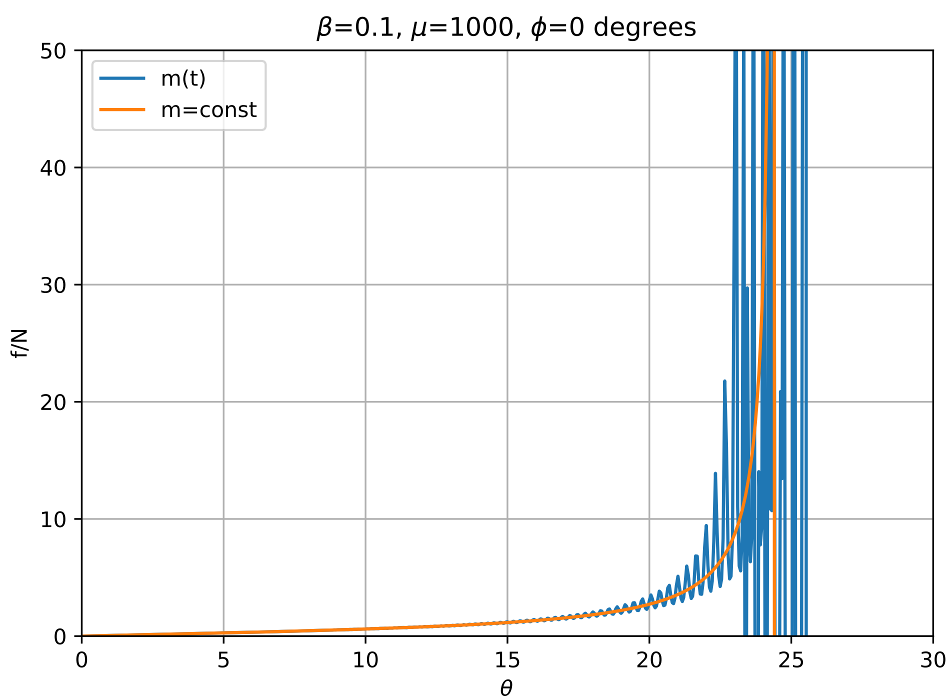

FIG. 5. Small coupling, even higher frequency.

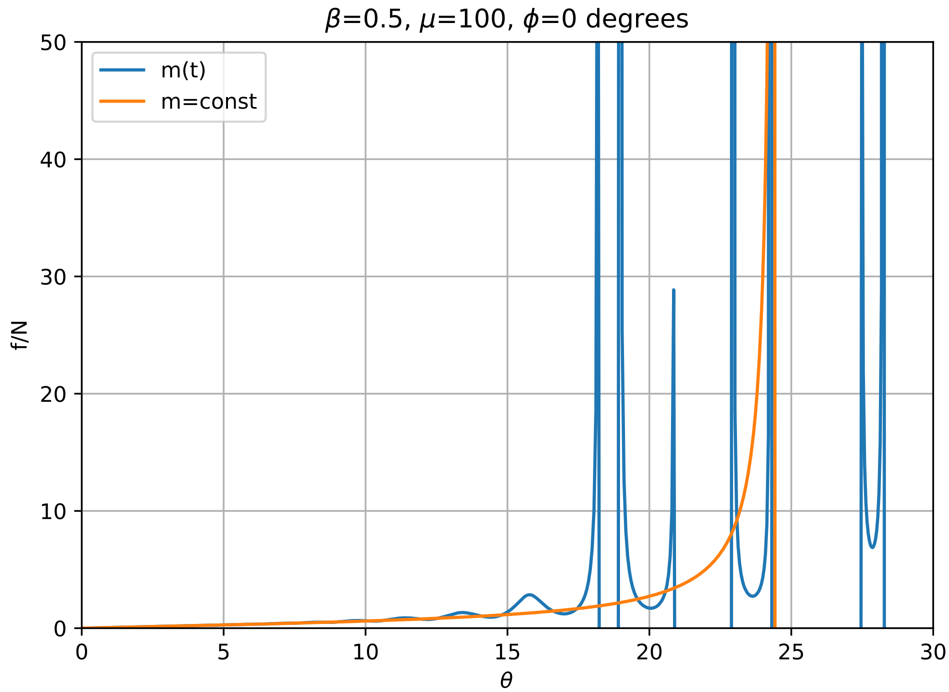

FIG. 6. Large coupling, high frequency.