Falling of a Pencil in Axion Enviroment

Tesler Grimm

Oort Cloud

(Dated: December 4, 2022)

Abstract

This note investigate the motion of a rigid pencil on a horizontal table, assuming that all mass oscillates coherently from the coupling to a homogeneous axion background. The pencil is initially erect, then it undergoes a spontaneous symmetry breaking: it falls, the question is that whether this pencil will slide during this falling process. It is shown that the condition of sliding remains same for arbitrary $m(t)$.

CONTENTS

I. THE PROBLEM

The situation is depicted in the figure below. The pencil is modeled as a straight rigid rod with uniform mass ditribution, mass $m(t) = 1 + \beta\cos(\mu t)$ and length $l = 1$. As the pencil falls, θ increases from 0 to $\pi/2$.

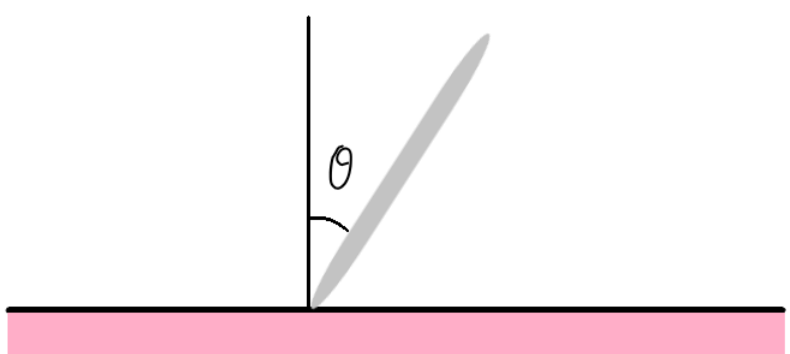

FIG. 1. The problem set-up.

To demonstrate the effect of $m(t)$, we do a lagrangian analysis for the frictionless case,

$$L = \frac{1}{2}\left(\frac{1}{12}ml^2\right)\dot{\theta}^2 + \frac{1}{2}mv^2 - mgl\cos\theta \tag{1}$$

where $\mathbf{v}$ is the velocity of the mass center. Now the rotation dynamics reads

$$\frac{d}{dt}\left(\frac{1}{12}ml^2\dot{\theta}\right) = mgl\sin\theta \tag{2}$$

or

$$\frac{\dot{m}}{m}\dot{\theta} + \ddot{\theta} = 12g\sin\theta \tag{3}$$

The vertical motion of the mass center is

$$\frac{d}{dt}\left(m\frac{d}{dt}(\frac{1}{2}l\cos\theta)\right) = N - mg \tag{4}$$

or

$$\ddot{\theta} + \cot\theta\dot{\theta}^2 + \frac{\dot{m}}{m}\dot{\theta} = \frac{2(mg-N)}{m\sin\theta} \tag{5}$$

where N is the normal force exerted by the table. Now incorporating friction from the table, these equations do not change, but horizontal motion of the mass center does change. Assuming the contacting point doesn't slide, we have

$$\frac{d}{dt}\left(m\frac{d}{dt}(\frac{1}{2}l\sin\theta)\right) = f \tag{6}$$

or

$$\ddot{\theta} - \tan\theta\dot{\theta}^2 + \frac{\dot{m}}{m}\dot{\theta} = \frac{2f}{m\cos\theta} \tag{7}$$

where f is the friction force. If the pencil does not slide, the total energy of the system

$$H = \frac{1}{2}\left(\frac{1}{3}ml^2\right)\dot{\theta}^2 + mgl\cos\theta = mg \tag{8}$$

is conserved. Then we can express f, N as functions of the angle and time, the result is

$$\begin{aligned}
\dot{\theta} &= \sqrt{6\left(1-\cos\theta\right)g} \\
\ddot{\theta} &= 12g\sin\theta - \frac{\dot{m}}{m}\sqrt{6\left(1-\cos\theta\right)g} \\
f/mg &= \frac{\cos\theta}{2}\left[12\sin\theta - 6\tan\theta\left(1-\cos\theta\right)\right] \\
N/mg &= 1 - \frac{\sin\theta}{2}\left[12\sin\theta + 6\cot\theta(1-\cot\theta)\right]
\end{aligned} \tag{9}$$

The slide cannot happen if f/N is smaller then the critical friction factor of the table.

II. DISCUSSION

It turns out that f, N are simply proportional to mg, hence for f/N the situation is completely same as compared to the constant mass case, which has been analysed before. However, once the pencil starts to slide, we expect the situation to change, this is left for futural study.