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title.jpg

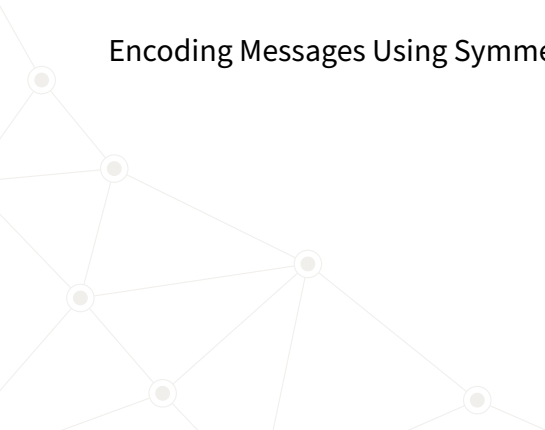
POLYGONS

Adam Klepáč

October 2, 2023

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Encoding Messages Using Symmetries



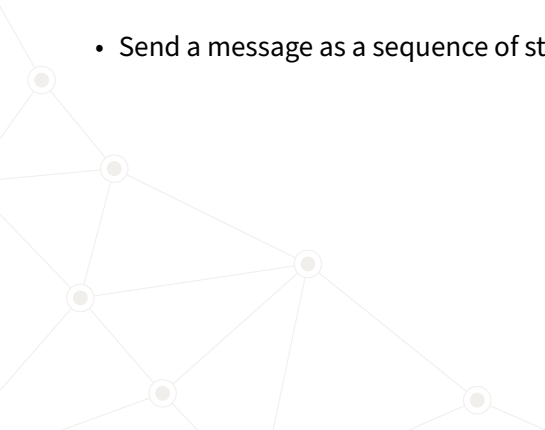
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ENCODING MESSAGES USING SYMMETRIES



ENCODING MESSAGES – GENERAL IDEA

- Send a message as a sequence of strings which **can be decoded on the other side**.



ENCODING MESSAGES – GENERAL IDEA

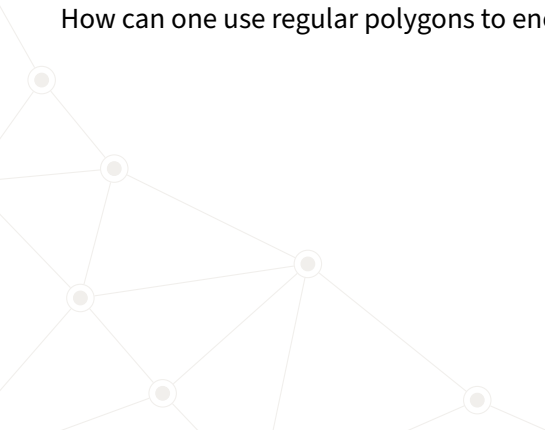
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- The decoding should require a 'key' which is **statistically impossible to determine quickly**.

ENCODING MESSAGES – GENERAL IDEA

- Send a message as a sequence of strings which **can be decoded on the other side**.
- The decoding should require a 'key' which is **statistically impossible to determine quickly**.
- The encoding and decoding must be done procedurally – requires a system with concrete rules and a **limited** (but huge) **number of combinations**.

ENCODING MESSAGES – REGULAR POLYGONS

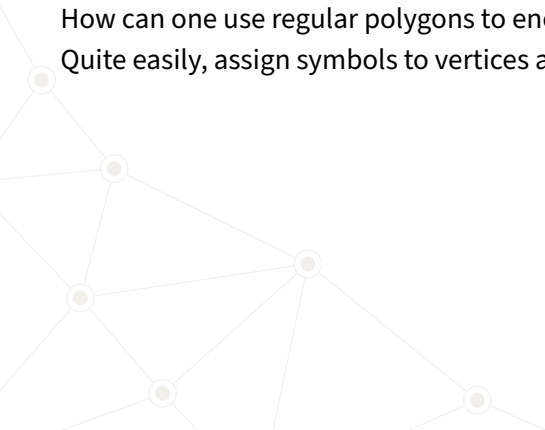
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ENCODING MESSAGES – REGULAR POLYGONS

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Quite easily, assign symbols to vertices and choose one **main** vertex.

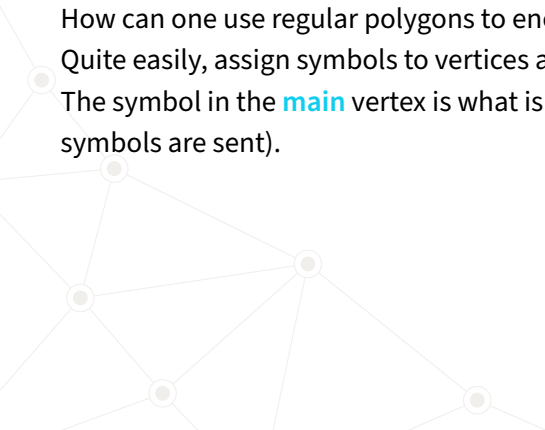


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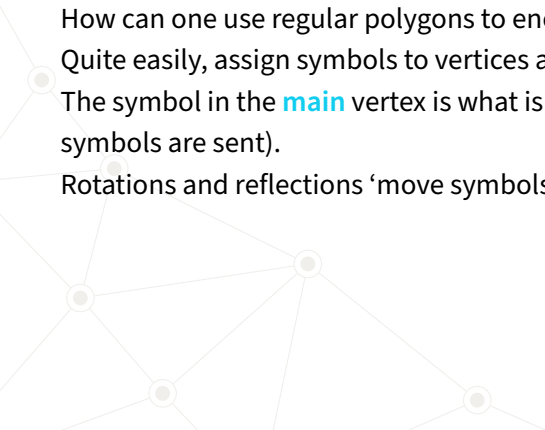
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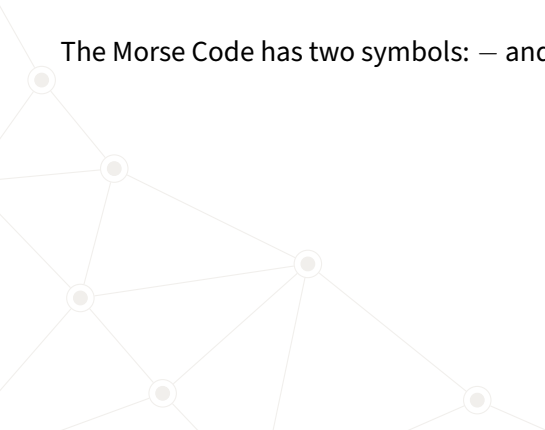
The symbol in the **main** vertex is what is sent during each tick (time interval during which symbols are sent).

Rotations and reflections ‘move symbols between vertices’.

This means that after applying a rotation or reflection, a (in most cases) different symbol will appear in the main vertex.

ENCODING MESSAGES – MORSE CODE

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We also need to be able to determine spaces between words – we can assign one vertex a ‘blank’, meaning space.

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This means that to send messages written Morse Code using regular polygons, we need three vertices – a triangle.

ENCODING MESSAGES – MORSE CODE

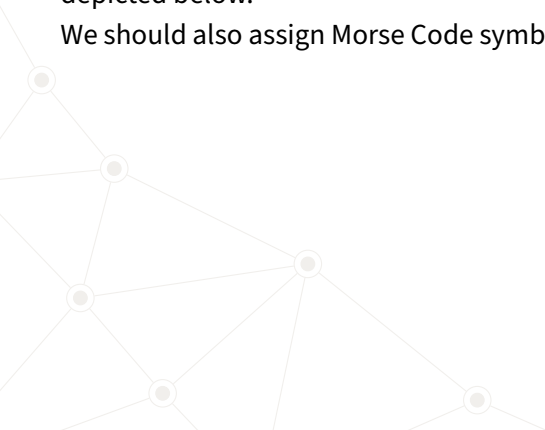
Let the top vertex be main and let's choose a rotation $r = \curvearrowright 120^\circ$ and a reflection s depicted below.



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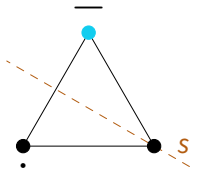
We should also assign Morse Code symbols to two of the vertices. We can do so randomly.



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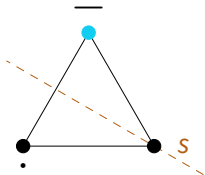
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We should also assign Morse Code symbols to two of the vertices. We can do so randomly.



The **top vertex** is the main one, which means that the symbol above it will get sent.

ENCODING MESSAGES – MORSE CODE

The important property of symmetries is that two different sequences can give the same symmetry.



ENCODING MESSAGES – MORSE CODE

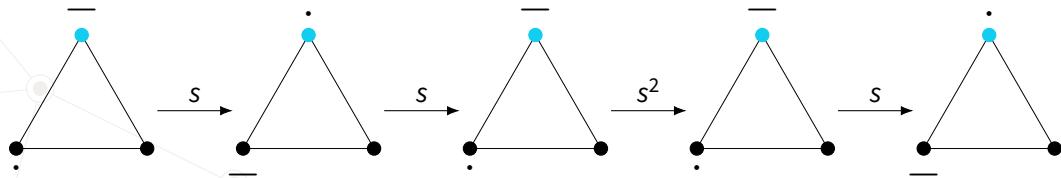
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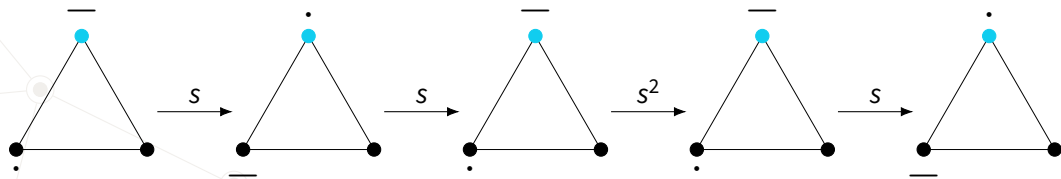
For example, we can send the letter P ($\cdot - - \cdot$) like this:



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For example, we can send the letter P ($\cdot - - \cdot$) like this:

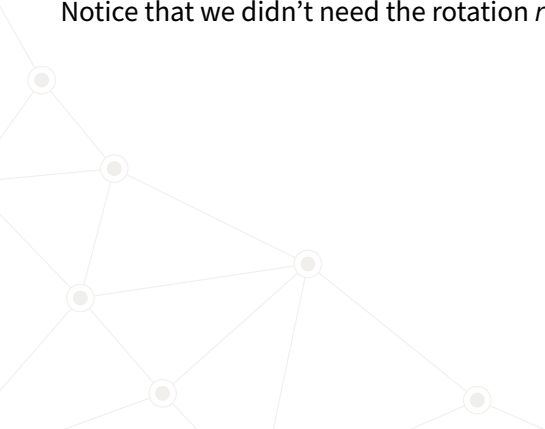


Or, as a sequence

$S, S, S^2, S.$

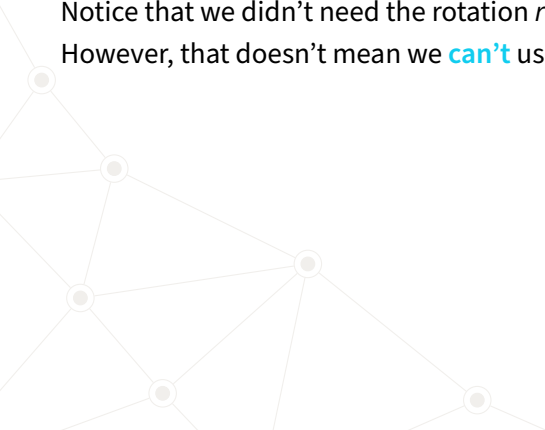
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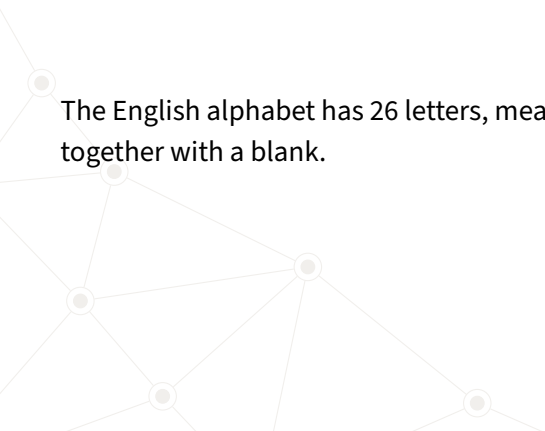
Actually, it's often beneficial to encode messages using different combinations of symmetries for security.

The letter $\cdot - - \cdot$ can also be sent for example like this

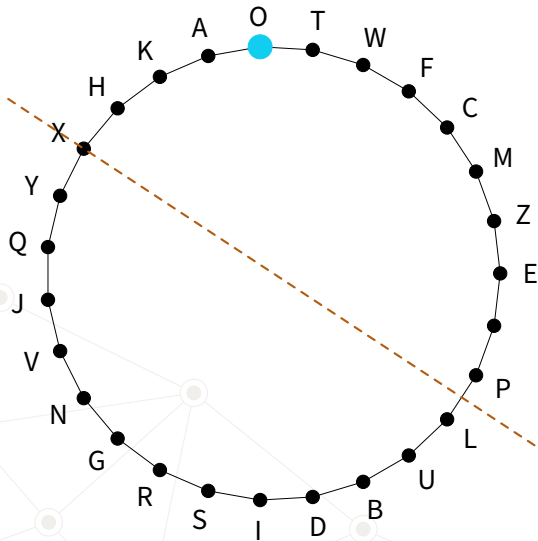
$$r^2, sr, sr^2, rsr.$$

ENCODING MESSAGES – ALPHABET

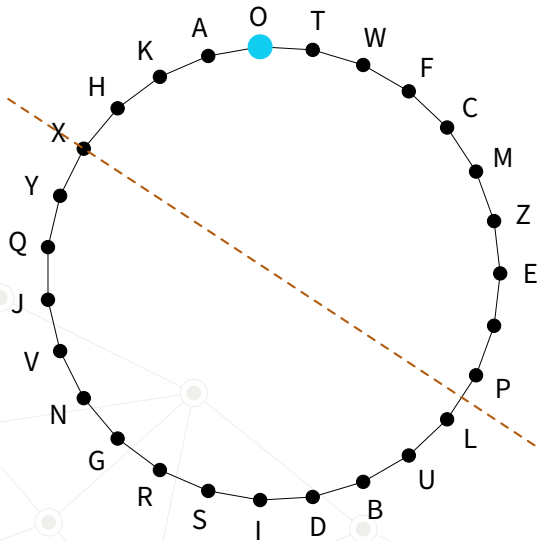
The English alphabet has 26 letters, meaning we need a 27-gon to represent all of them together with a blank.

A decorative geometric pattern in the bottom-left corner of the slide. It consists of several light gray lines connecting circular nodes. The nodes are small circles with a light gray outline and a darker gray center. The lines form a network of triangles and quadrilaterals, extending from the left edge towards the bottom right.

ENCODING MESSAGES – ALPHABET

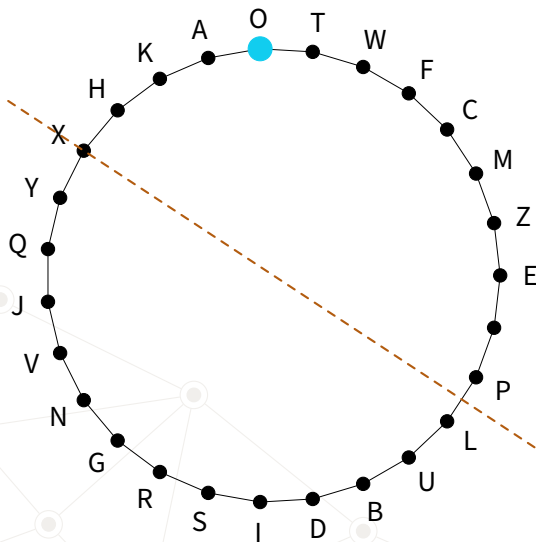


ENCODING MESSAGES – ALPHABET



Let's start with $r = \bigcirc \frac{2}{27} \cdot 360^\circ$ and s as on the picture.

ENCODING MESSAGES – ALPHABET



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One way to encode the word POLY-GONS would be

$$sr^5, r^{10}sr^{15}, r^5, sr, r^7s, r^5sr^{13}, sr, r^3s$$