Congruences & CRT

3.AB PreIB Maths – Exam A

Unless specified otherwise, you are to **always** (at least briefly) explain your reasoning. Even in closed questions.

Congruences

a) For which values of $k \in \mathbb{Z}$ with $1 \leq k \leq 7$ does there exist a solution to the congruence [35 %]

$$k \cdot x \equiv 4 \pmod 8?$$

Write those down and **state how you found them**.

b) Three mathematicians are walking up a tower and wish to count the number of stairs leading to the top. They sit down and try to think of a plan. [20 %]

As is universally the case, the mathematicians can't count up to more than ten. Knowing that there are likely at most five hundred stairs, devise a way for the mathematicians to determine the number of stairs.

Explain thoroughly how you arrived at your method and make sure to explain why it works.

Chinese Remainder Theorem

Solve the following system of congruences. [45 %]

$$\begin{aligned} x &\equiv 0 \pmod 7 \\ x &\equiv 2 \pmod{11} \\ x &\equiv 4 \pmod{13} \end{aligned}$$

Explain briefly why there is only one solution smaller than $7 \cdot 11 \cdot 13$.