Logic & Set Theory

2.AB PreIB Maths – Exam B

Unless specified otherwise, you are to **always** (at least briefly) explain your reasoning. Even in closed questions.

Logic – propositions and operators

For which truth values of the propositions p, q, r, s is the proposition [25 %]

$$(p \wedge q) \wedge (r \wedge s)$$

true? **Elaborate** your process and be sure that you got **all** the possible quadruplets.

Bonus Problem [10 %]

Determine the negation of the proposition from the previous exercise. In other words, simplify

$$\neg((p \wedge q) \wedge (r \wedge s)).$$

Hint: Remember that $\neg(p \wedge q) = \neg p \vee \neg q$. First negate the $\wedge$ between the two propositions in parentheses and then negate them as well.

Basic set operations

Given sets $A = \{1, 2, 3, 4, 5\}$ and $B = \{4, 5\}$, find **all** the sets C that satisfy **both** of the conditions [35 %]

$$C \subseteq A \quad \text{and} \quad C \cap B = \{\}.$$

Don't forget that empty set ($\{\}$) is a subset of any set.

Bonus Problem

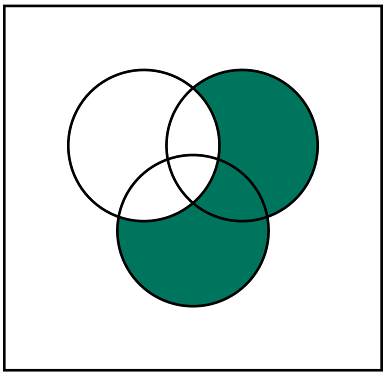
[10 %]

Take the set C from the previous question and define it using logical operators only. This means converting the two conditions forced upon C to their logical operator form.

The solution should be in the form $C = \{x \mid p(x)\}$ where $p(x)$ is some proposition using $x \in A$ and $x \in B$.

Venn diagrams

a) Given the Venn diagram below, determine the set which it represents. You **don't** have to provide an **explanation**. [20 %]



b) Draw a Venn diagram for the following expression: [20 %]

$$(A \setminus B) \cap (B \cup C).$$

You **don't** have to explain anything.

Bonus Problem

[10 %]

We denote the **size** (the number of elements) of a set A as $|A|$. So, for a set $S = \{1, 2, 3\}$ it is true that $|S| = 3$. The expression $|A| + |B| + |C|$ can be interpreted as *"counting all the elements from all the sets"*. In each of the sectors of the following Venn diagram write **how many times** are the elements from that section counted in the expression $|A| + |B| + |C|$. For example, the elements from $A \cap B$ are counted **twice** because they are both part of A and of B.