Logic & Set Theory

3.AB PreIB Maths – Resit exam

Unless specified otherwise, you are to **always** (at least briefly) explain your reasoning. Even in closed questions.

Logic – propositions and conjunctions.

a) Complete the truth table below.   [15 %]

p	q	$p \wedge \neg q$
1	1	
1	0	
0	1	0
0	0	0

In other words: evaluate the proposition $p \wedge \neg q$, for the truth values of p and q corresponding for the omitted lines in the truth table.

b) Complete the blank square in proposition   [10 %]

$$p \;\square\; \neg q$$

with some logical conjunction ($\wedge, \vee, \Rightarrow, \Leftrightarrow$) so it equivalent $\neg(p \Rightarrow q)$. Two statements are equivalent if their truth tables are the same.

For convenience the truth table of implication is stated below.

p	q	$p \Rightarrow q$
1	1	1
1	0	0
0	1	1
0	0	1

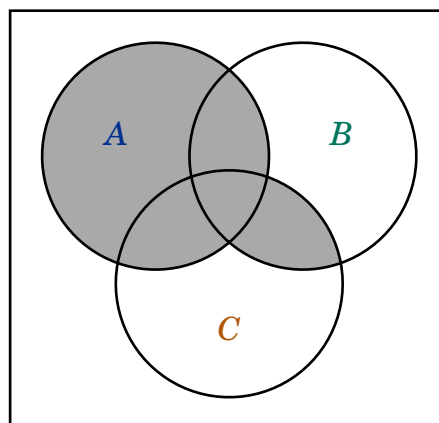**Explain** your choice.

Basic set operations.

a) Given sets $A = \{c, c, c, b, b, a\}$ and $B = \{a, b, c\}$, determine the statements   [15 %]

$$A \subseteq B \text{ and } B \subseteq A.$$

Explain your method.

Bonus (+10%): if both the statements are true there is something to be concluded about A and B. **Explain** what it is.

b) Write an expression (using set operations) for the shaded are on the diagram   [10 %]
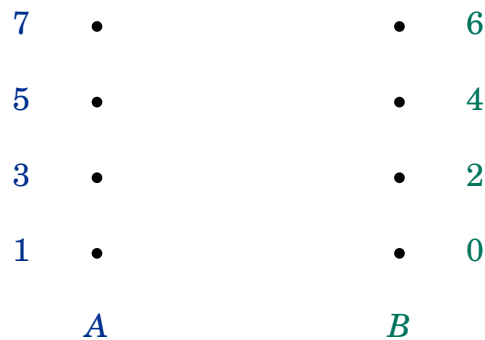below.

Cartesian product and relations.

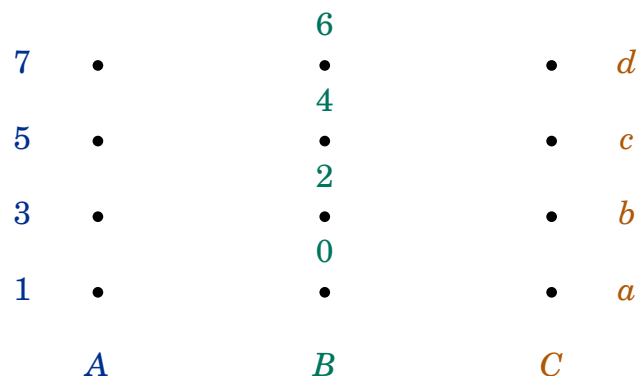a) On the diagram below draw the relation R from A to B for   [15 %]

$A = \{1, 3, 5, 7\}$, $B = \{0, 2, 4, 6\}$ and $R = \{(1, 2), (3, 6), (5, 0)\}$.

```
7   •              •   6

5   •              •   4

3   •              •   2

1   •              •   0

    A              B
```

b) Draw again the relation R from the previous exercise together with the relation   [10 %]
$S = \{(0, a), (2, c), (4, d)\}$ between sets B and $C = \{a, b, c, d, \}$.

```
            6
7   •       •       •   d
            4
5   •       •       •   c
            2
3   •       •       •   b
            0
1   •       •       •   a

    A       B       C
```

Now is your task to compose the relations R and S into one relation T that goes from A to C. This means that T firstly applies R to get from A to B. Then on all of the results of R (end of an every arrow from a)) applies S which gets it from B to C. At the end T forgets the element from B and ends up only with the beginning and the ending of the journey. **Write down** T.

Equivalence.

a) One of the examples of a equivalence is **'what flavor of ice cream'** each person   [15 %]
   likes the most. Verify that it is truly equivalence. In other words: it has to satisfy
   - **reflexivity**: every element is equivalent to itself;
   - **symmetry**: if a is equivalent to b, then b is equivalent to a;
   - **transitivity**: if a is eq. to b and b is eq. to c, then a is eq. to c.

b) Come up with at **least three** other equivalences on the set of all people. Try   [10 %]
   to estimate the number of equivalence classes they create. For the maximum
   credit there should be one that creates **over 100** of partitions and also one that
   creates fewer than two.

   You **can not** use the equivalence from part a).