

Bonus Problems – PrelB 3.AB 3 & 4

Triangulations and Symmetries of Regular Polygons

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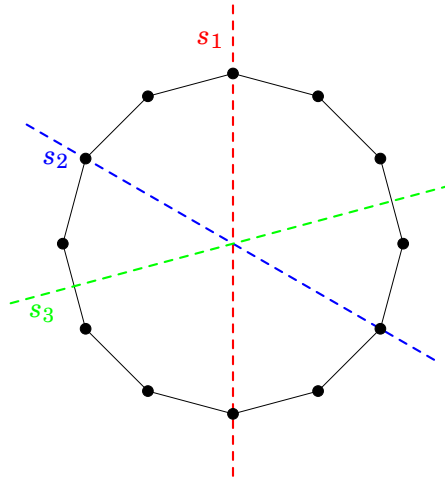
YOU WILL BE ASKED TO PRESENT YOUR SOLUTIONS ORALLY. YOU DON'T NEED TO WRITE ANYTHING DOWN. ONLY YOUR UNDERSTANDING OF THE PROBLEMS AND THE PROVIDED SOLUTION IS EVALUATED. CORRECTNESS OF THE RESULTS IS IMMATERIAL.

Triangulations

The graph of triangulations of a regular heptagon has 42 triangulations. By a **cycle** in the graph, I mean a path (of flips) which ends where it started. That is, it's a sequence of flips which returns to the original triangulation. Find one of the **shortest** (having the least number of flips) cycles in the graph of triangulations of the heptagon. **Explain very precisely WHY the cycle you've found is the shortest.** Please, do **not** draw the whole graph of triangulations...

Symmetries

In the regular dodecagon (12 vertices), you're given 3 reflections – s_1 , s_2 and s_3 .



Can I get s_3 by combining s_1 and s_2 ? By a ‘combination’ I mean any sequence (however long) of s_1 ’s and s_2 ’s, for example $s_1s_1s_2s_1s_2s_2$. If yes, show how. If not, explain why. **In any case, be precise and thorough in describing your solution.**