



PYTHAGOREAN THEOREM

Adam Klepáč

November 6, 2023



CONTENTS



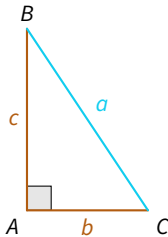
Pythagorean Theorem

Applications

RIGHT TRIANGLE

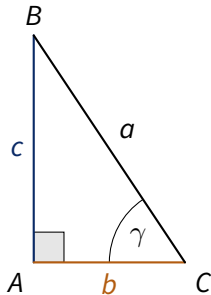
A triangle is called **right**, if one of its angles is a right angle (90°).

Right triangles have been of special import in many fields and so their sides have unique names:



The **short sides** are called *catheti* and the **long side** is called *hypotenuse*.

RIGHT TRIANGLE

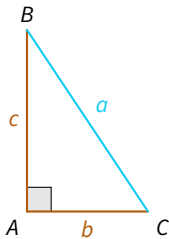


With respect to a chosen angle γ , the side c is called *opposite* and b is called *adjacent*.

PYTHAGOREAN THEOREM

The background of the slide is composed of three large, solid-colored triangles that meet at a central point. A yellow triangle is on the left, a cyan triangle is on the right, and a green triangle is at the bottom. The top portion of the slide is white.

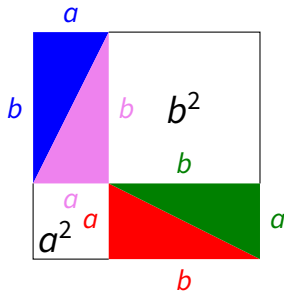
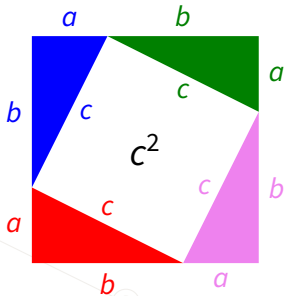
PYTHAGOREAN THEOREM



Given a right triangle, the **Pythagorean Theorem** says that

$$a^2 = b^2 + c^2.$$

PYTHAGOREAN THEOREM – PROOF



PYTHAGOREAN THEOREM – PROBLEM 1

What is the length of the hypotenuse in a right triangle if the catheti are 5 and 12?

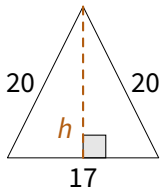
We simply calculate (h means hypotenuse)

$$h^2 = 5^2 + 12^2 = 169$$

$$h = \sqrt{169} = 13.$$

PYTHAGOREAN THEOREM – PROBLEM 2

Find the height of the following isosceles triangle:



We have two right triangles next to each other – each one with hypotenuse of length 20 and one cathetus of length $17/2 = 8.5$. So, we know that

$$20^2 = 8.5^2 + h^2,$$

and thus $h^2 = 20^2 - 8.5^2 = 327.75$ and $h = \sqrt{327.75} = 18.1$.

APPLICATIONS

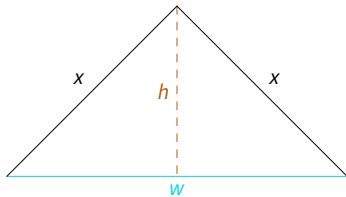
The background features three large, overlapping triangles. A yellow triangle is on the left, a cyan triangle is on the right, and a green triangle is at the bottom center where the other two overlap.

ARCHITECTURE & CONSTRUCTION

When constructing a roof, you typically only know the **height** and **width**.

Using Pythagorean Theorem, you can also calculate the length of the diagonal slope and its area to cut out supporting beams.

For example, if the **height** of the roof is 3 meters and its **width** is 6 meters,



the length of the slope can be calculated as $x^2 = h^2 + (w/2)^2 = 9 + 9 = 18$ and taking the square root to get $x = 4.24$ meters.

ARCHITECTURE & CONSTRUCTION

A triangle with side lengths a, b, c is a right triangle if and only if the Pythagorean Theorem $a^2 + b^2 = c^2$ holds.

This can be used to efficiently measure right angles. If we know, for instance, that a triangle with side lengths 3, 4 and 5 is right, we can set out a triangle made of strings of these lengths to lay out a foundation or construct a corner between walls.

For example, one can make sure that the triangles of side lengths 5, 12, 13 and 20, 21, 29 are right because

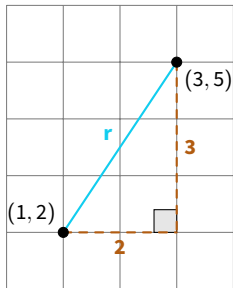
$$5^2 + 12^2 = 25 + 144 = 169 = 13^2,$$

$$20^2 + 21^2 = 400 + 441 = 841 = 29^2.$$

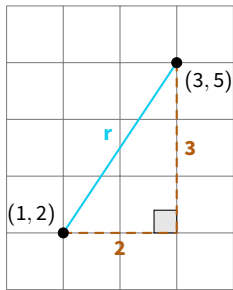
NAVIGATION

The Pythagorean Theorem is useful when calculating distances between points in the plane or in space.

For example, to calculate the distance between $(1, 2)$ and $(3, 5)$, one sets up a right triangle like this:



NAVIGATION



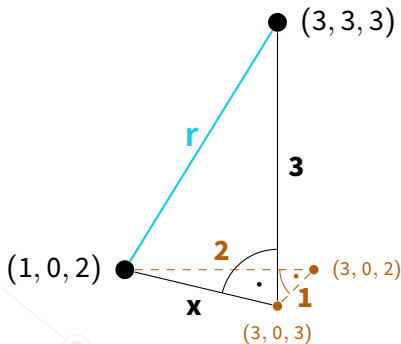
So, the distance from (1, 2) to (3, 5) satisfies

$$r^2 = 2^2 + 3^2,$$

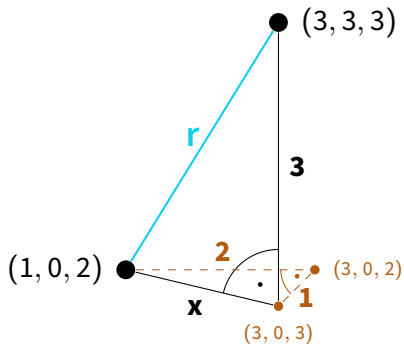
$$r = \sqrt{13}.$$

NAVIGATION

Distances in 3D are measured the same way, one just has to set up two right triangles. For example, let's measure the distance from $(1, 0, 2)$ to $(3, 3, 3)$. We set up our right triangles:



NAVIGATION



From this, we can calculate

$$x = \sqrt{1^2 + 2^2} = \sqrt{5}, \quad r = \sqrt{3^2 + (\sqrt{5})^2} = \sqrt{9 + 5} = \sqrt{14}.$$