

Logic & Set Theory

3.AB PrelB Maths – Resit exam

Unless specified otherwise, you are to **always** (at least briefly) explain your reasoning. Even in closed questions.

Logic – propositions and conjunctions.

a) Complete the truth table below.

[15 %]

p	q	$p \wedge \neg q$
1	1	<input type="checkbox"/>
1	0	<input type="checkbox"/>
0	1	0
0	0	0

In other words: evaluate the proposition $p \wedge \neg q$, for the truth values of p and q corresponding for the omitted lines in the truth table.

b) Complete the blank square in proposition

[10 %]

$$p \quad \square \quad \neg q$$

with some logical conjunction so it the same as $\neg(p \Rightarrow q)$. Two statements are the same if their truth tables are the same. You may choose from:

- **and:** \wedge ;
- **or:** \vee ;
- **implies:** \Rightarrow ;
- **if and only if:** \Leftrightarrow .

Explain your choice.

Basic set operations.

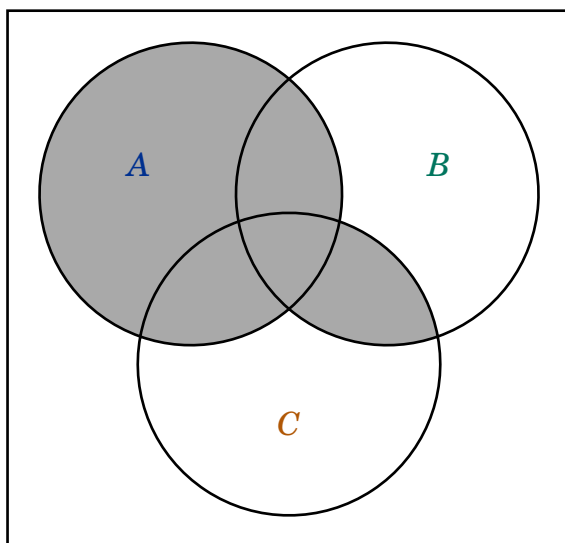
- a) Given sets $A = \{c, c, c, b, b, a\}$ and $B = \{a, b, c\}$, determine the statements [15 %]

$$A \subset B \text{ and } B \subset A.$$

Explain your method.

Bonus (+10%): if both the statements are true there is something to be concluded about A and B . **Explain** what it is.

- b) Write an expression (using set operations) for the shaded are on the diagram [10 %]
below.

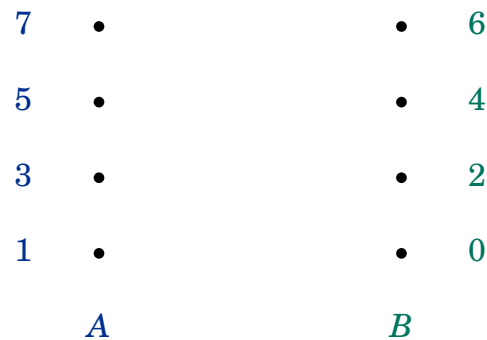


Cartesian product and relations.

a) On the diagram below draw the relation R from A to B for

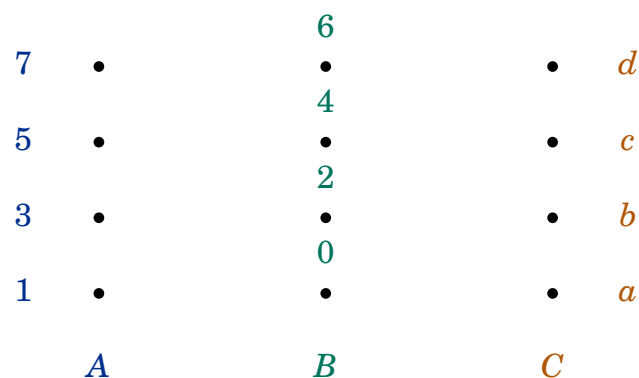
[15 %]

$A = \{1, 3, 5, 7\}$, $B = \{0, 2, 4, 6\}$ and $R = \{(1, 2), (3, 6), (5, 0)\}$.



b) Draw again the relation R from the previous exercise together with the relation $\hat{R} = \{(0, a), (2, c), (4, d)\}$ between sets B and $C = \{a, b, c, d, \}$.

[10 %]



Now write down the relation that firstly “follows” R from A to B and then “follows” \hat{R} from B to C .

Equivalence.

- a) One of the examples of a equivalence is people '**being the same age**'. Verify [15 %]
that it is truly equivalence. In other words: it has to satisfy
- **reflexivity**: every element is equivalent to itself;
 - **symmetry**: if a is equivalent to b , then b is equivalent to a ;
 - **transitivity**: if a is eq. to b and b is eq. to c , then a is eq. to c .

- b) Come up with at **least three** other equivalences on the set of all people. Try [10 %]
to estimate the number of equivalence classes they create. For the maximum
credit there should be one that creates **over 100** of partitions and also one that
creates less then two.

You **can not** use the equivalence from part a).