Systems of Linear Equations

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Functions

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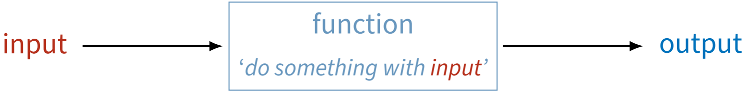

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input ⟶ function *'do something with input'* ⟶ output

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input → **function** *'do something with input'* → output

We'll call the data that **a function receives**, inputs and the **data it gives back**, outputs. Inputs and outputs need not necessarily be just 'one object', they can be for example lists of numbers.

Functions – Example

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$$\begin{matrix} x_1, x_2, \ldots, x_n \\ n \end{matrix} \longrightarrow \boxed{\begin{matrix} \text{average} \\ \textit{`sum all the numbers} \\ \textit{and divide them by n'} \end{matrix}} \longrightarrow \frac{x_1 + x_2 + \ldots + x_n}{n}$$

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$$\text{omelette} \longrightarrow \boxed{\text{ingredients}} \longrightarrow \begin{array}{c}\text{eggs}\\ \text{salt}\\ \text{butter}\end{array}$$

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Function Composition

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Of course, composition is only possible if the output of the first function is a valid input for the second.

For instance, you could hardly compose the ingredients function with the average function.

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Considering two functions

$$a, b \longrightarrow \boxed{\text{sum}} \longrightarrow a + b$$

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What would the output of this composition look like

$$a \longrightarrow \boxed{\text{times two}} \longrightarrow \boxed{\text{sum}} \longrightarrow ?$$

Function Composition

So, the **order of the composition matters**! Here are a few examples of compositions which make sense:

omelette $\longrightarrow$ $\boxed{\text{ingredients}}$ $\longrightarrow$ $\boxed{\text{total price}}$ $\longrightarrow$ \$4.37

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We can of course compose **as many functions** as we like. An example of this:

omelette → $\boxed{\text{ingredients}}$ → $\boxed{\text{total price}}$ → $\boxed{\text{times two}}$ → \$8.74

Functions – Notation

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we simply write **function**(input) = output.

You're probably used to seeing function written like $f(x) = y$. The picture corresponding to this is

$$x \longrightarrow \boxed{f} \longrightarrow y$$

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For example, if f and g are two functions, their composition $f \circ g$ corresponds to this picture

$$x \longrightarrow \boxed{g} \longrightarrow \boxed{f} \longrightarrow y$$

that is, **first g, then f**.

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Real Functions

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- $g(x) = \tan^6(\log^{\sin(x^2+4)}(\frac{5x^3-2}{9x^7}))$,

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where $x \in \mathbb{R}$. Or, using pictures,

$$x \longrightarrow \boxed{f} \longrightarrow 0$$

$$x \longrightarrow \boxed{g} \longrightarrow \tan^6(\log^{\sin(x^2+4)}(\frac{5x^3-2}{9x^7}))$$

Operations On Real Functions

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- $(f \circ g)(x) = 2\left(\frac{1}{1+x}\right)^2 + 7$ and
- $(g \circ f)(x) = \frac{1}{1+(2x^2+7)}$.