Number Sets & GCD

3.AB PreIB Maths – Exam A

Unless specified otherwise, you are to **always** (at least briefly) explain your reasoning. Even in closed questions.

Natural Numbers

a) Thus far the addition and multiplication of natural numbers have been defined. Now the **exponentiation** is presented in two rules: [20 %]

$$\begin{aligned} &1) \quad a^0 = 1 \\ &2) \quad a^{\mathrm{succ}(b)} = a^b \cdot a \end{aligned}$$

Using **only these two rules** (and all your other knowledge about multiplication and addition), evaluate the following expressions.

- 2^4
- 3^3

b) **Generalise** your method from part a) to calculate a^b for **any** $a, b \in \mathbb{N}$. [10 %]

Integers & Rationals

a) Connect all pairs belonging to the **same equivalence class** and write down the value of the **represented integer** for each class. [20 %]

$(1,4)$ $(2,3)$ $(1,2)$

$(6,9)$ $(12,10)$ $(120,123)$

$(0,1)$ $(7,5)$ $(7,8)$

b) You are given two pairs of natural numbers: (a',b') and (a,b) from the **same equivalence class** (they represent the same integer value). Show that their respective **sums** with some pair (c,d) also belong to the **same equivalence class**. [10 %]

In other words, show that if $[(a,b)]_E = [(a',b')]_E$, then

$$[(a,b)]_E + [(c,d)]_E = [(a',b')]_E + [(c,d)]_E$$

Hint: The pairs (a,b) and (a',b') represent the same integer if (informally) '$a-b=a'-b'$'.

Divisibility & GCD

a) Find all numbers smaller than 100 that have **exactly 3 divisors**. Do **not** proceed by trial and error (this method would result in 0 %). [20 %]

b) Compute $\gcd(358758, 318402)$. Write down performed calculations **in full detail**. [20 %]