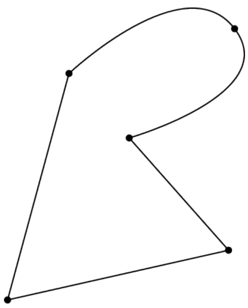
Convex Polygons and Their Symmetries

3.AB PreIB Maths – Mock Exam

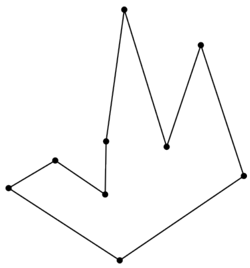
Unless specified otherwise, you are to **always** (at least briefly) explain your reasoning. Even in closed questions.

1. Definition of a polygon.

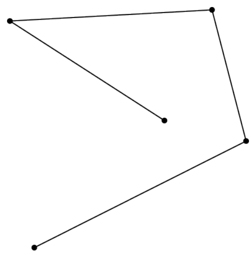
   (a) Which of these shapes *are not* polygons? **Explain**. [10 %]



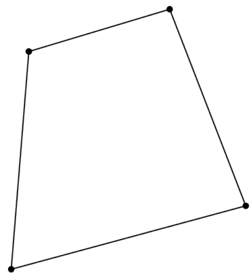
Option A.



Option B.



Option C.

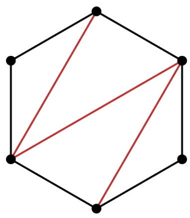


Option D.

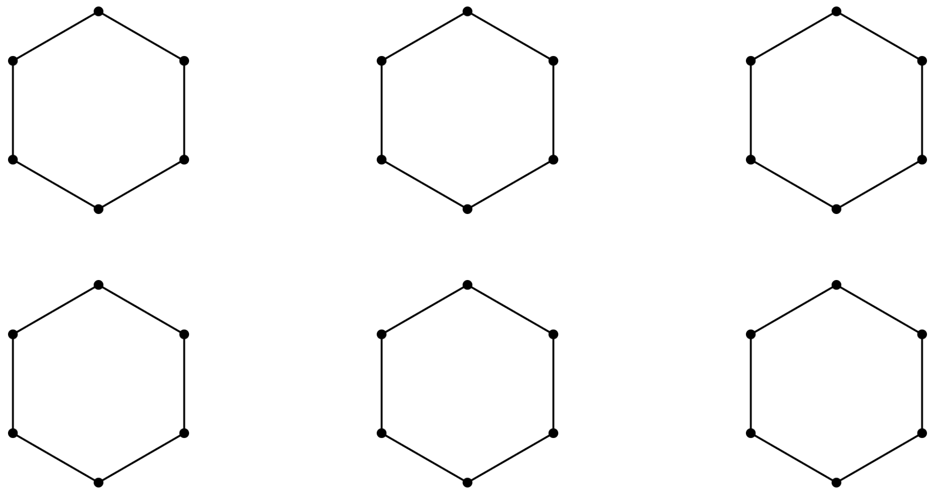
   (b) The sum of the sizes of all internal angles in a *convex* polygon on n vertices is $(n-2)\cdot 180°$. How about *non-convex* polygons? Is there a number the sum of the sizes of internal angles in a non-convex polygon on n vertices **cannot exceed**, or can it be infinite? Attempt to find such a number or construct a counterexample. [10 %]

2. Triangulations of convex polygons.

   (a) Draw all triangulations of the hexagon *that can be reached in one flip* from the one shown below. Use the provided shapes (not all of them necessarily). **No explanation required**. [10 %]

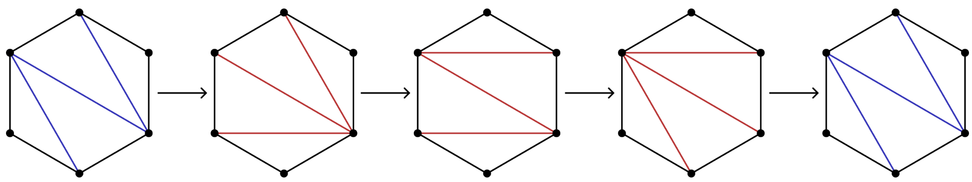


The initial triangulation.



Shapes to draw diagonals into.

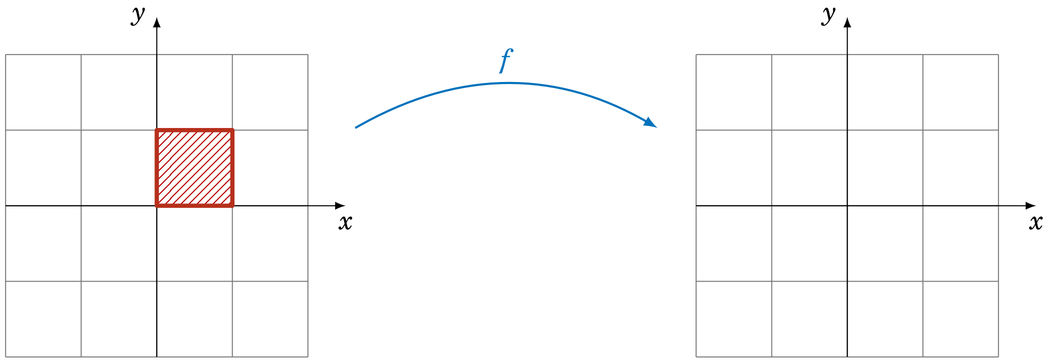
   (b) The minimum number of flips to get from one triangulation of the hexagon to the *same triangulation* **without flipping the same diagonal twice in a row** is four. One such path is depicted below. [10 %]



   Try to argue that *there are only two paths* of four flips, this one and its reverse. *Notice that the 'middle' diagonal remained stable.*

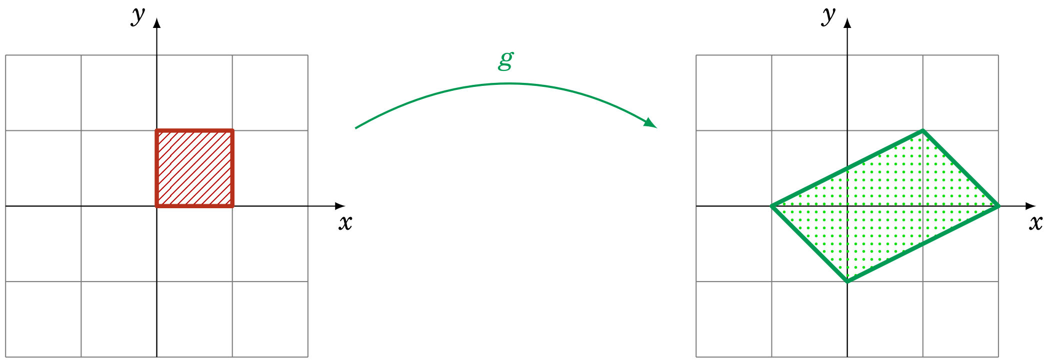
3. Plane transformations.

   (a) Find out the *image* (the resulting shape when transformed) of a square (depicted below) under the plane transformation $f(x, y) = (y, -x)$. **Provide a short explanation.** [10 %]



The initial square. Draw the resulting shape here.

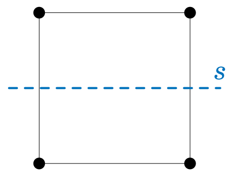
   (b) Below, you see a unit square transformed by a plane transformation g. Figure out *one possible prescription* of this transformation, that is, write the coordinates of the point $g(x, y)$ using x and y. [10 %]



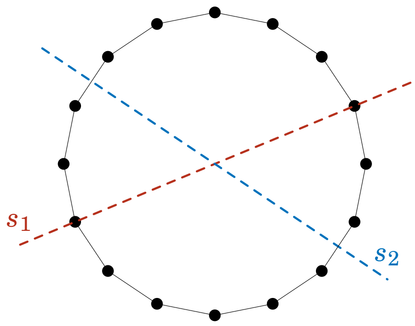
The initial square. The transformed square.

4. Symmetries of regular polygons.

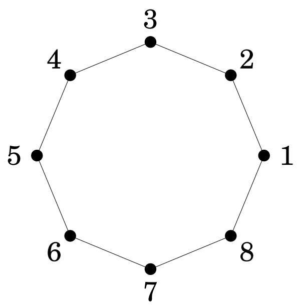
   (a) Given two symmetries of the *square* – the rotation $r = ⟲ 180°$ by 180° counter-clockwise and the reflection s drawn below – determine (using any method you wish) the composition rs. **Explain**. [10 %]



   (b) Given two symmetries of the hexakaidecagon (16 vertices) – the reflections s_1 and s_2 depicted below – compute (using any method you wish) the composition $s_1 s_2$. **Explain**. [10 %]



(c) Select those of the following four pairs of symmetries of the regular octagon (8 vertices) that *generate all* of its symmetries. **No explanation necessary**. [10 %]



Picture of the octagon for reference.

- ◯ the rotation $r = \circlearrowleft 2 \cdot 360°/8$ and the reflection s over the line passing through vertices 4 and 8,
- ◯ the rotation $r_1 = \circlearrowleft 3 \cdot 360°/8$ and the rotation $r_2 = \circlearrowleft 5 \cdot 360°$,
- ◯ the reflection s_1 over the line passing through the midpoints of 23 and 67 and the reflection s_2 over the line passing through the midpoints of 45 and 18,
- ◯ the rotation $r = \circlearrowleft 7 \cdot 360°/8$ and the reflection s over the line passing through vertices 3 and 7.

(d) Given reflections s_1 and s_2 of the heptagon (7 vertices), compose them (and *only* them) to create the reflection s_3 illustrated below. **Explain**. [10 %]