Number Sets & GCD

3.AB PreIB Maths – Exam B

Unless specified otherwise, you are to **always** (at least briefly) explain your reasoning. Even in closed questions.

Natural Numbers

a) Remember that we defined **addition** and **multiplication** by [20 %]

$$\mathrm{succ}(n) = n + 1 \qquad n \cdot 1 = n$$
$$\mathrm{succ}(n + m) = n + \mathrm{succ}(m) \qquad n \cdot \mathrm{succ}(m) = n \cdot m + n$$

Using **only** those axioms calculate:

- $2 \cdot 3$
- $1 + (2 \cdot 2)$

b) Assuming $x + y = y + x$, show that $x + \mathrm{succ}(y) = \mathrm{succ}(y) + x$. In your proof use only the **axioms** that **define addition**. [10 %]

Integers & Rationals

a) Connect the pairs of **integers** that correspond to the **same equivalence class** and write down the value of the represented **rational number**. [20 %]

$(2,20)$	$(5,50)$	$(35,7)$
$(-15,-3)$	$(10,2)$	$(-50,-2)$
$(-2,2)$	$(-4,4)$	$(100,4)$

b) Integers and rationals share some similarities in their definition. They are defined as **equivalence classes** on $\mathbb{N} \times \mathbb{N}$ and $\mathbb{Z} \times \mathbb{Z}$, respectively. Define **at least one** additional equivalence on $\mathbb{N} \times \mathbb{N}$ and one on $\mathbb{Z} \times \mathbb{Z}$. Comment on the equivalence classes, **how many are there**? Do they have a specific shape? [10 %]

The two trivial equivalences are equality (a is equivalent to b if $a = b$) and the equivalence where all pairs of natural numbers (or integers) belong to the same equivalence class. **These won't count** as valid solutions.

Divisibility & GCD

a) Some **natural number** n can be decomposed into primes as $n = p_1 \cdot p_2 \cdot \ldots \cdot p_k$. [20 %]
Describe a method how to use the primes $p_1, p_2, \ldots, p_k$ to find **all the divisors** of n.

b) Compute $\gcd(1029, 1617)$. Write down performed calculations **in full detail**. [20 %]