Polygons & Transformations Cheatsheet

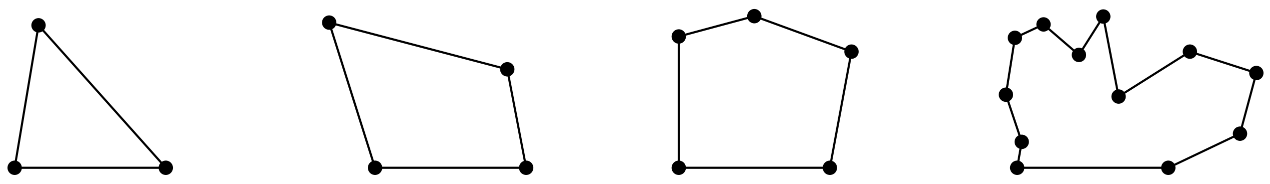
3.AB PreIB Math

Adam Klepáč

Polygons

Polygon is a closed 2D shape made only of segments. We call the endpoints of those segments, vertices, and the segments themselves, edges.

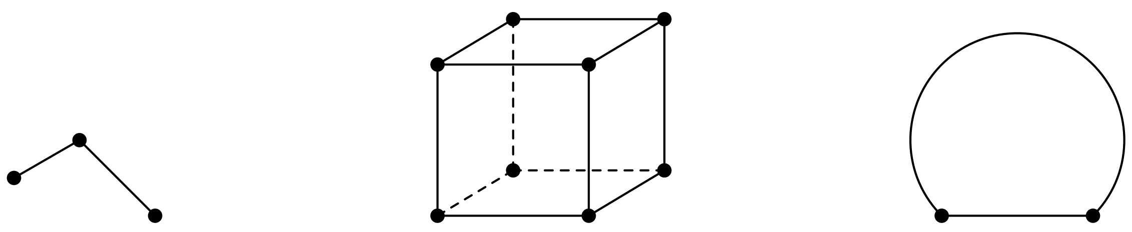
Examples



Triangle Quadrilateral Pentagon Dodecagon

Polygons with n sides are called n-gons.

Counterexamples



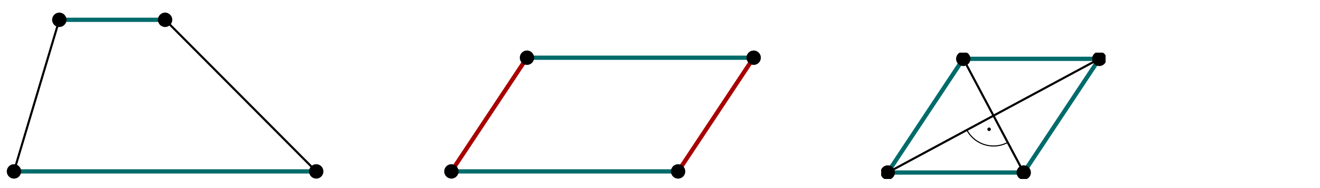
Not closed 3D Not straight

Convex Polygons

A polygon is called convex if it has no internal angle greater than 180°.

Convex NOT convex

Special types of convex polygons



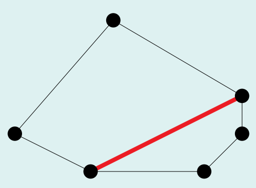
Trapezoid/Trapezium
A convex quadrilateral with at least two parallel sides.

Parallelogram
A convex quadrilateral with two pairs of parallel sides.

Rhombus
An **equilateral** (all sides of the same length) parallelogram.

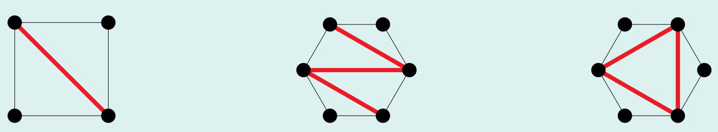
Diagonals & Triangulations

A diagonal in a **convex** polygon is a segment connecting two of its **non-adjacent** vertices.

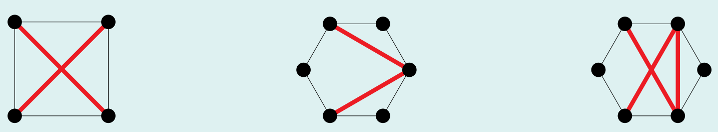


Diagonal in a convex hexagon.

A triangulation of a **convex** polygon is its division into triangles by **non-intersecting** diagonals.



Examples of triangulations.



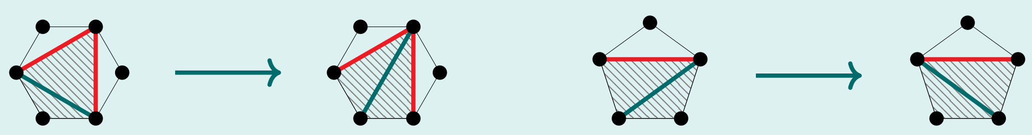
Counterexamples of triangulations.

The total number of different triangulations of a convex n-gon is

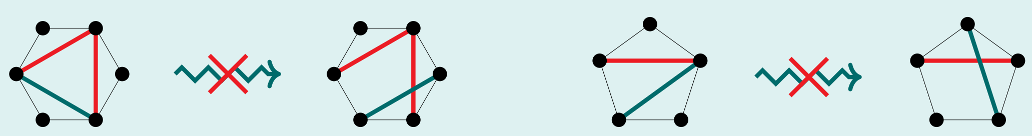
$$\frac{n \cdot (n+1) \cdot \ldots \cdot (2n-4)}{(n-2)!},$$

which you **of course don't have to remember**. Interestingly enough, every triangulation can be transformed into any other by a series of flips.

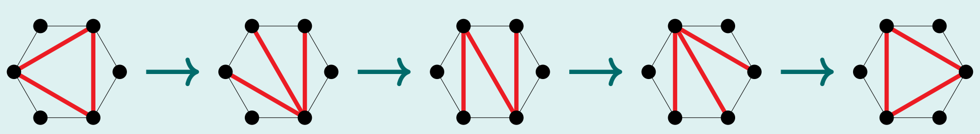
A flip is a swap of one diagonal for the other in a chosen quadrilateral so that the **result is again a triangulation**.



Examples of flips.



Counterexamples of flips.



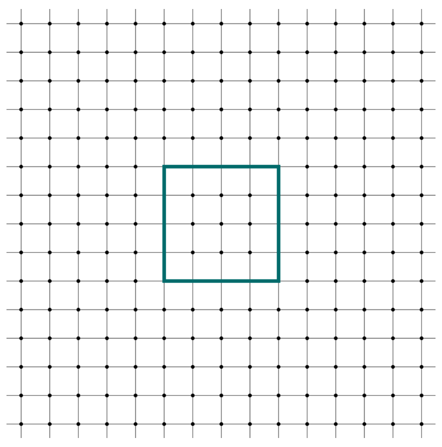
Passage from one triangulation to another through a series of flips.

I encourage you to think about how to determine the number of flips necessary to pass from one triangulation to another. Can I have made the passage above in fewer flips?

Plane Transformations

The plane is basically just the set $\mathbb{R}^2$ of all **pairs of real numbers**. A pair $(x, y) \in \mathbb{R}^2$ is typically called a point. Then, a plane transformation is a **function** which maps points to points. In symbols, it's a function $\mathbb{R}^2 \to \mathbb{R}^2$.

We can visualise what a transformation does for example by look at the image of a square (or an entire grid).



The transformation $(x, y) \mapsto (100(\sin x + \cos y), 100(\cos x + \sin y))$.

References (opcional)

[1] Claude E. Shannon.
A mathematical theory of communication.
Bell System Technical Journal, 27(3):379–423, 1948.