Statistics

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The collective information about a system's past state is called **data**.
It assigns **probabilities** to each possible future state of system based on data.
It also assigns probabilities to the **possibility of wrong prediction**.

Example – Biased Coin?

We throw a coin 10 times with the following outcome:

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  - We got 7 heads out of 10 tosses, so the probability for the next toss being heads is $7/10$.
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  - We got 7 heads out of 10 tosses, so the probability for the next toss being heads is $7/10$.
- Is this coin is **biased towards** ‘heads’/‘tails’ with *allowed probability of error* α?
  - **No**, for $\alpha = 0.05$.
  - **Yes**, for $\alpha = 0.2$.

Contents

Visualizing Discrete Data

Data

What Do We Mean By Data?

Data

Two sets (called *inputs* and *outputs*) describing the studied system.

Example – Junctions

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An **output** is the number of traffic accidents in a given day.

Example – First Baby

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We study the age that women bear children for the first time across Europe.
An **input** would be a name of a European country.
An **output** is the average age of a first-time mother in that country.

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Types of Data

Discrete Data vs. Continuous Data

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- There are only *finitely many* countries on a continent.

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More often than not, the inputs in a continuous data are **moments in time** or **coordinates in space**.

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- Another example is the density of air per cubic meter.
  - Input: Coordinates of a unit cube in space.
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  - The data is a function $f : \mathbb{R}^3 \to \mathbb{R}$.

Visualizing Discrete Data

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Input	1	2	3	4	5	6	7	8	9	10
Output	180	169	191	177	175	181	171	153	180	183

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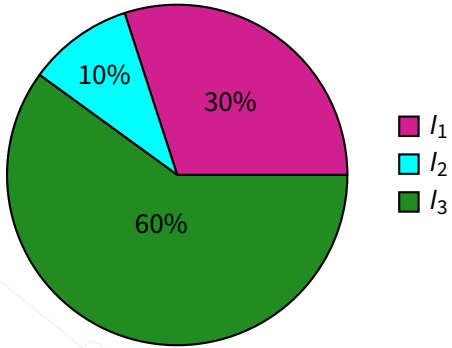
Suppose we have three inputs – I_1, I_2 and I_3 – with three outputs – 30%, 10% and 60%.
Pie chart of this data looks like this



Pie Chart – Examples

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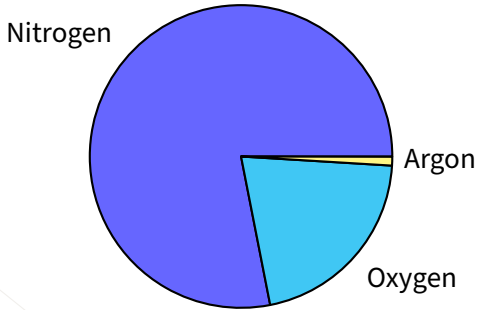
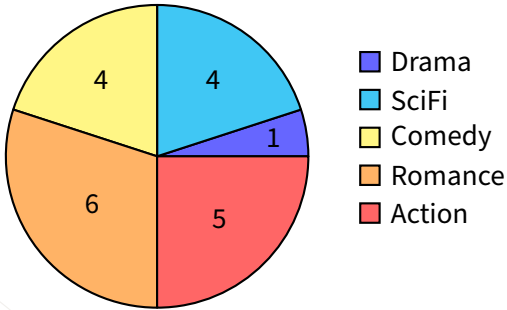
For instance, here is a pie chart of the composition of *air*.



Pie Chart – Examples

Favourite type of movie as determined by a survey.



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Especially useful when your inputs are ordered and when you expect the data to follow a certain trend – it can be easily approximated by a polygonal curve.
Also very good for comparing more outputs for the same inputs.

Bar Chart – Example

Suppose we count the number of customers in our shop over each hour. If we're open from 8 AM to 5 PM, a bar chart of such an experiment can look like this:

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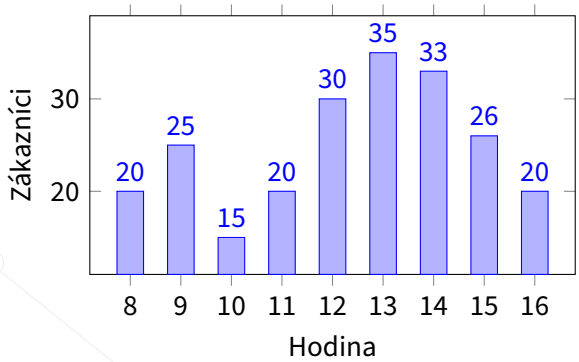

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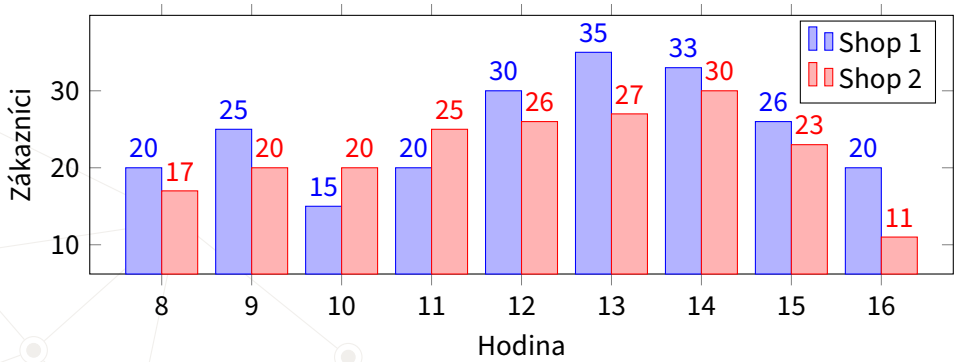

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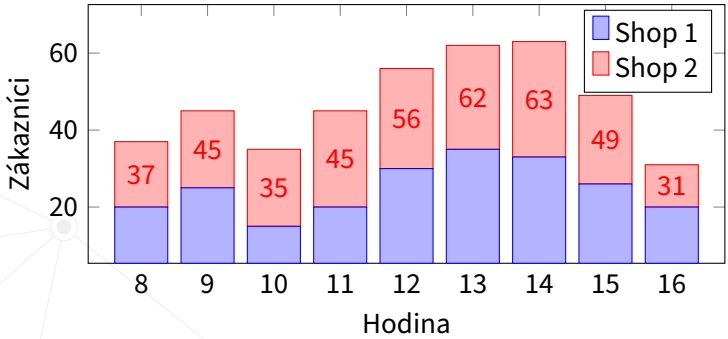

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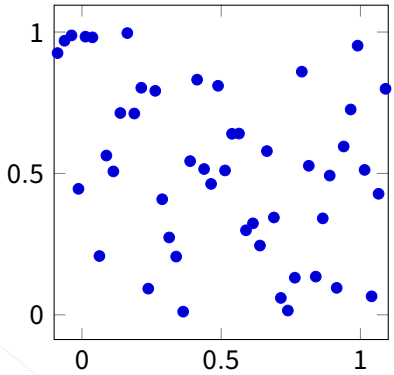

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