Number Sets & GCD

3.AB PreIB Maths – Real Exam

Unless specified otherwise, you are to **always** (at least briefly) explain your reasoning. Even in closed questions.

Natural Numbers

a) So far the addition and multiplication of natural numbers were defined. Now the **exponentiation** is presented in two axioms : [15 %]

$$\begin{aligned} &1) && a\text{^}0 = 1 \\ &2) && a\text{^}\,\mathrm{succ}(b) = a\text{^}b \cdot b \end{aligned}$$

Using **only those two axioms** (and all your other knowledge about multiplication) evaluate the following expressions. You can denote exponentiation in the traditional form as a^b.

- 2^5
- 1^98
- 5^3

b) **Generalise** your method from part a) to calculate $a\text{^}b$ for **any** $a, b \in \mathbb{N}$. [15 %]

Integers & Rationals

a) Connect the pairs that correspond to the **same equivalence classes** and write down their **values**. [20 %]

$(2,3)$	$(3,2)$	$(5,3)$
$(8,6)$	$(9,10)$	$(122,123)$
$(2,0)$	$(5,4)$	$(7,8)$

b) Define **exponentiation** for **rationals numbers**. Remember that multiplication is repeated addition (the same way exponentiation is repeated multiplication) and is defined as: [10 %]

$$[(a,b)]_E \cdot [(c,d)]_E = [(a \cdot c + b \cdot d, b \cdot c + a \cdot d)]_E$$

Hint: Try to continue the pattern from multiplication and use the fact that you can exponentiate two natural numbers.

Divisibility & GCD

a) Find **all the natural numbers** from **2** to **30** that share the **minimum number** of divisors. [20 %]

b) Compute $\gcd(410, 240)$. Write down performed calculations **in full detail**. [20 %]