

Congruences & CRT

3.AB PrelB Maths – Real Exam

Unless specified otherwise, you are to **always** (at least briefly) explain your reasoning. Even in closed questions.

Congruences

- a) For which values of $k < 10$ there exists a solution to the congruence

[35 %]

$$k \cdot x \equiv 1 \pmod{10}?$$

Write those down or **state how to find them**.

- b) Three mathematicians are walking up a tower, wanting to count all the stairs they sit down at the bottom and try to think of a plan. “I’ve got it” one exclaims. “Each of us will only count stairs up to some constant and then go back to one. At the top we will surely somehow calculate the result.” Unfortunately yet unsurprisingly none of them can count higher than 10.

[20 %]

Choose the three constants such that they can succeed at any tower up to 500 stairs, also **briefly describe** the method for calculating the result at the end.

Chinese Remainder Theorem

Solve the following system of congruences.

[45 %]

$$x \equiv 7 \pmod{10}$$

$$x \equiv 2 \pmod{11}$$

$$x \equiv 5 \pmod{13}$$

Explain why there is only one solution smaller than $10 \cdot 11 \cdot 13$.