Number Sets & GCD

3.AB PreIB Maths – Exam B

Unless specified otherwise, you are to **always** (at least briefly) explain your reasoning. Even in closed questions.

Natural Numbers

a) Remember that we defined **addition** and **multiplication** as: [20 %]

$$\mathrm{succ}(n) = n + 1 \qquad n \cdot 0 = 0$$
$$\mathrm{succ}(n + m) = n + \mathrm{succ}(m) \qquad \mathrm{succ}(n \cdot m) = n \cdot m + m$$

Using **only** those axioms calculate:

- $2 \cdot 3$
- $1 + (2 \cdot 2)$

b) Assuming $x + y = y + x$, show that $x + \mathrm{succ}(y) = \mathrm{succ}(y) + x$. In your proof use only the **axioms** that **define addition**. [10 %]

Integers & Rationals

a) Connect the pairs of **integers** that correspond to the **same equivalence class** and write down the value of the represented **rational number**. [20 %]

$(2, 20)$	$(5, 50)$	$(35, 7)$
$(-15, -3)$	$(10, 2)$	$(-50, -2)$
$(-2, 2)$	$(-4, 4)$	$(100, 4)$

b) Integers and rationals share some similarities in their definition. They are defined as **equivalence classes** on $\mathbb{N} \times \mathbb{N}$ and $\mathbb{Z} \times \mathbb{Z}$, respectively. Define **at least one** additional equivalence on $\mathbb{N} \times \mathbb{N}$ and one on $\mathbb{Z} \times \mathbb{Z}$. Comment on the equivalence classes, **how many are there**? Do they have a specific shape? [10 %]

The two trivial equivalences are equality (a is equivalent to b if $a = b$) and the equivalence where all pairs of natural numbers (or integers) belong to the same equivalence class. **These won't count** as valid solutions.

Divisibility & GCD

a) Some **natural number** n can be decomposed into primes as $n = p_1 \cdot p_2 \cdot \ldots \cdot p_k$. [20 %]
Describe a method how to use the primes $p_1, p_2, \ldots, p_k$ to find **all the divisors** of n.

b) Compute gcd$(1029, 1617)$. Write down performed calculations **in full detail**. [20 %]