Congruences & CRT

3.AB PreIB Maths – Mock Exam

Unless specified otherwise, you are to **always** (at least briefly) explain your reasoning. Even in closed questions.

Congruences

a) Does the congruence [25 %]

$$3 \cdot x \equiv 7 \pmod 9$$

have a solution? **Explain properly**.

b) A group of 13 pirates managed to steal a chest with golden coins. They tried to split the coins evenly into 13 piles but 10 coins remained. This provoked a fight and one of the pirates was stabbed dead. The 12 remaining pirates tried to split the coins evenly again but this time 3 coins remained. Yet another pirate died in the ensuing fight and the remaining 11 pirates finally managed to split the treasure evenly among themselves. At least how many coins did the chest contain? [25 %]

You only have to describe the **method** of solving this problem. You **don't need to** calculate the exact number.

Chinese Remainder Theorem

Solve the following system of congruences. [50 %]

$$\begin{aligned} x &\equiv 3 \pmod{9} \\ x &\equiv 5 \pmod{10} \\ x &\equiv 2 \pmod{11} \end{aligned}$$

Explain why there is only one solution smaller than $9 \cdot 10 \cdot 11$.