# Systems of Linear Equations

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# Functions

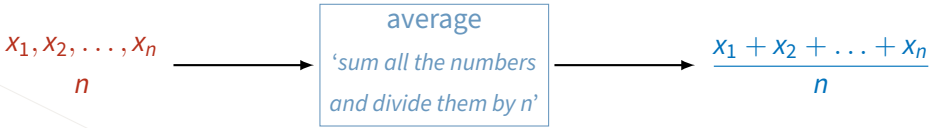
# What Is A Function?

Intuitively, a function is a **box** which receives data and gives some data back.


We'll call the data that **a function receives**, inputs and the **data it gives back**, outputs. Inputs and outputs need not necessarily be just 'one object', they can be for example lists of numbers.

# Functions – Example

A function which returns the **average** of a given set of numbers receives the numbers and also their count as input and returns the **average** as output.



# Functions – Example

We can also consider ‘non-mathematical’ functions. Like a function which receives a type of meal and returns the ingredients.

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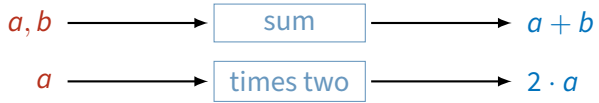
# Function Composition

# Function Composition

If we have **two functions**, we can **in certain cases** ‘compose’ them.
**Composition** simply means that **one function follows the other** – in other words, the output of the first function is the input of the second.
Of course, **composition** is only possible if the output of the first function is a valid input for the second.
For instance, you could hardly compose the **ingredients** function with the **average** function.

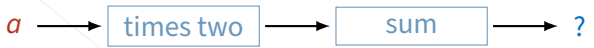
# Function Composition

Considering two functions



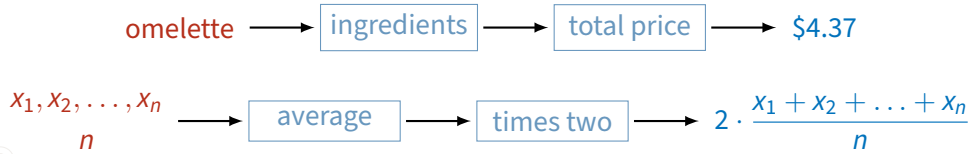

their composition can look like this


What would the output of this composition look like



# Function Composition

So, the **order of the composition matters**! Here are a few examples of compositions which make sense:




We can of course compose **as many functions** as we like. An example of this:

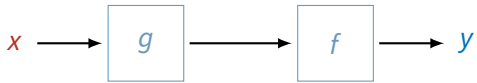
# Functions – Notation

Drawing pictures like this would be cumbersome. Instead of


we simply write **function**(input) = output.
You're probably used to seeing function written like $f(x) = y$. The picture corresponding to this is

# Functions – Notation

The symbol used for function composition is ∘. It is however a little confusing because it is written in order 'from right to left'.
For example, if $f$ and $g$ are two functions, their composition $f \circ g$ corresponds to this picture




that is, **first $g$, then $f$**.

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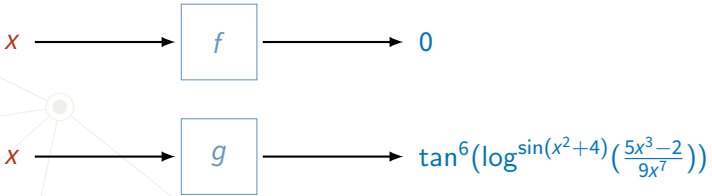
# Real Functions

# Real Function

A **real function** is simply a function whose **input** and **output** are both real numbers.
Examples of such functions are

- $f(x) = 0$,
- $g(x) = \tan^6(\log^{\sin(x^2+4)}(\frac{5x^3-2}{9x^7}))$,

where $x \in \mathbb{R}$. Or, using pictures,



# Operations On Real Functions

As both the input and the output of a real function is a real number, **we can always compose real functions**.

However, that **doesn't mean that the order doesn't matter!** Different order of composition gives different functions.

For example, take $f(x) = 2x^2 + 7$ and $g(x) = \frac{1}{1+x}$. Then,

- $(f \circ g)(x) = 2\left(\frac{1}{1+x}\right)^2 + 7$ and
- $(g \circ f)(x) = \frac{1}{1+(2x^2+7)}$.

# Operations On Real Functions

Real functions can also be **added** and **multiplied**, just like real numbers.
This involves simply adding or multiplying their respective outputs. For the functions $f(x) = 2x^2 + 7$ and $g(x) = \frac{1}{1+x}$, their

- **sum** is the function with output

$$(f + g)(x) = f(x) + g(x) = 2x^2 + 7 + \frac{1}{1+x}.$$

- **product** is the function with output

$$(f \cdot g)(x) = f(x) \cdot g(x) = (2x^2 + 7) \cdot \left(\frac{1}{1+x}\right) = \frac{2x^2 + 7}{1+x}.$$

# Graphs

As real functions have real numbers as inputs and outputs, they can be easily **graphed**. **Graphing** a real function $f$ simply means drawing the points $(x, f(x))$ or $(input, output)$ in some chosen coordinate system.
We typically use the **Cartesian** coordinate system with two axes (one for input and one for output) that are mutually perpendicular. These are often called the $x$-axis and the $y$-axis. However, later, we'll also use the **polar** coordinate system where every point is instead determined by its angle and distance from the origin of the system.

# Graphs

The functions $f(x) = 2x^2 + 7$ and $g(x) = \frac{1}{1+x}$ have the following (parts of) graphs: