Congruences & CRT

3.AB PreIB Maths – Exam B

Unless specified otherwise, you are to **always** (at least briefly) explain your reasoning. Even in closed questions.

Congruences

a) Connect together all the numbers which are **always** equal modulo n for some $n \in \mathbb{Z}$. [25 %]

3	$n-1$	-3
$n-3$	-1	$3n^2-n-1$
$3n+3$	$10n$	0

b) After finishing a meal in a restaurant, the waitress brings your check. You realize that you can either pay with **thirty-one four-dollar bills** or with **twenty-five five-dollar bills**, with the change in both scenarios being **smaller** than the bill used for payment. [35 %]

Calculate the possible prices of the meal.

Chinese Remainder Theorem

Solve the following system of congruences. [40 %]

$$x \equiv 6 \pmod 7$$
$$x \equiv 3 \pmod{11}$$
$$x \equiv 9 \pmod{10}$$

Explain briefly why there is only one solution smaller than $7 \cdot 11 \cdot 10$.