Pythagorean Theorem

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Right Triangle

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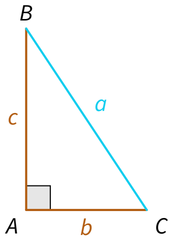
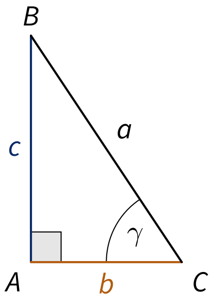
RIGHT TRIANGLE

A triangle is called **right**, if one of its angles is a right angle (90°).
Right triangles have been of special import in many fields and so their sides have unique names:



The short sides are called *catheti* and the long side is called *hypothenuse*.

Right Triangle



With respect to a chosen angle γ, the side c is called *opposite* and b is called *adjacent*.

Pythagorean Theorem

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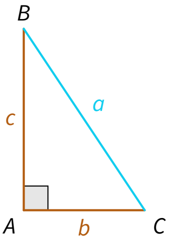
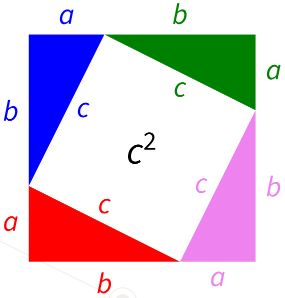


Given a right triangle, the **Pythagorean Theorem** says that

$$a^2 = b^2 + c^2.$$

Pythagorean Theorem – Proof



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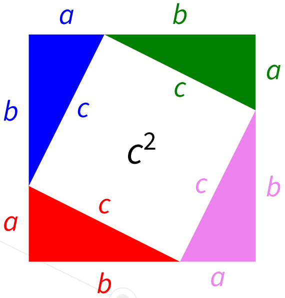

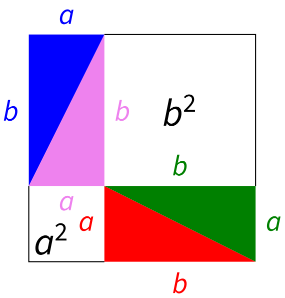

Pythagorean Theorem – Problem 1

What is the length of the hypotenuse in a right triangle if the catheti are 5 and 12?

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What is the length of the hypotenuse in a right triangle if the catheti are 5 and 12?

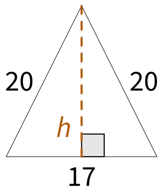
We simply calculate (h means hypotenuse)

$$h^2 = 5^2 + 12^2 = 169$$

$$h = \sqrt{169} = 13.$$

Pythagorean Theorem – Problem 2

Find the height of the following isosceles triangle:



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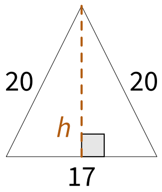

We have two right triangles next to each other – each one with hypotenuse of length 20 and one cathetus of length $17/2 = 8.5$.

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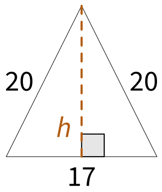

We have two right triangles next to each other – each one with hypotenuse of length 20 and one cathetus of length $17/2 = 8.5$. So, we know that

$$20^2 = 8.5^2 + h^2,$$

and thus $h^2 = 20^2 - 8.5^2 = 327.75$ and $h = \sqrt{327.75} = 18.1$.

Applications

Architecture & Construction

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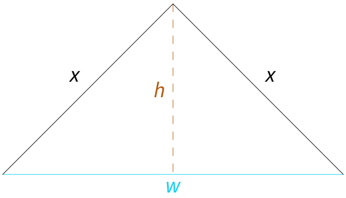
When constructing a roof, you typically only know the height and width.
Using Pythagorean Theorem, you can also calculate the length of the diagonal slope and its area to cut out supporting beams.
For example, if the height of the roof is 3 meters and its width is 6 meters,



the length of the slope can be calculated as $x^2 = h^2 + (w/2)^2 = 9 + 9 = 18$ and taking the square root to get $x = 4.24$ meters.

Architecture & Construction

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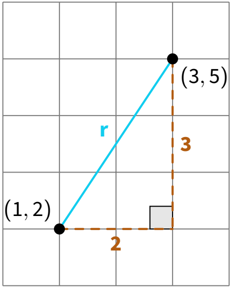
For example, one can make sure that the triangles of side lengths 5, 12, 13 and 20, 21, 29 are right because

$$5^2 + 12^2 = 25 + 144 = 169 = 13^2,$$
$$20^2 + 21^2 = 400 + 441 = 841 = 29^2.$$

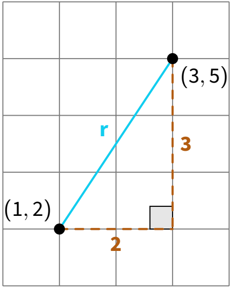
Navigation

The Pythagorean Theorem is useful when calculating distances between points in the plane or in space.

For example, to calculate the distance between $(1, 2)$ and $(3, 5)$, one sets up a right triangle like this:



NAVIGATION



So, the distance from (1, 2) to (3, 5) satisfies

$$r^2 = 2^2 + 3^2,$$
$$r = \sqrt{13}.$$

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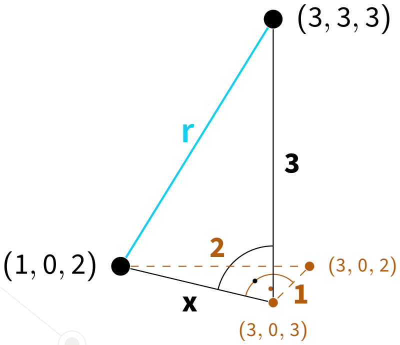
Distances in 3D are measured the same way, one just has to set up two right triangles.

NAVIGATION

Distances in 3D are measured the same way, one just has to set up two right triangles. For example, let's measure the distance from $(1, 0, 2)$ to $(3, 3, 3)$. We set up our right triangles:



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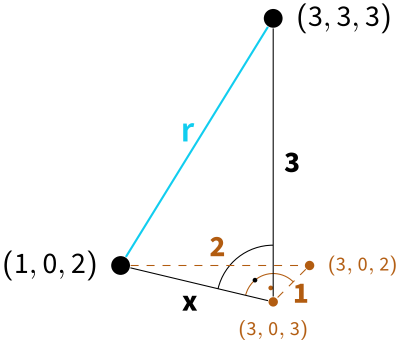

From this, we can calculate

$$x = \sqrt{1^2 + 2^2} = \sqrt{5}, \quad r = \sqrt{3^2 + (\sqrt{5})^2} = \sqrt{9 + 5} = \sqrt{14}.$$