

Number Sets & GCD

3.AB PreIB Maths – Real Exam

Unless specified otherwise, you are to **always** (at least briefly) explain your reasoning. Even in closed questions.

Natural Numbers

- a) So far the addition and multiplication of natural numbers were defined. Now the **exponentiation** is presented in two axioms : [15 %]

$$1) \quad a^0 = 1$$

$$2) \quad a^{\text{succ}(b)} = a^b \cdot b$$

Using **only those two axioms** (and all your other knowledge about multiplication) evaluate the following expressions. You can denote exponentiation in the traditional form as a^b .

- 2^5
- 5^3

- b) **Generalise** your method from part a) to calculate a^b for **any** $a, b \in \mathbb{N}$. [15 %]

Integers & Rationals

- a) Connect the pairs that correspond to the **same equivalence classes** and write down the value of **represented integer**. [20 %]

 $(2, 3)$ $(3, 2)$ $(5, 3)$ $(8, 6)$ $(9, 10)$ $(122, 123)$ $(2, 0)$ $(5, 4)$ $(7, 8)$

- b) You are given two elements: $[(a', b')]_E$ and $[(a, b)]_E$ from the **same equivalence class** (they represent the same integer value). Show that their respective **sum** with some element $[(c, d)]_E$ is always the **same**. In other words show that [10 %]

$$[(a, b)]_E + [(c, d)]_E = [(a', b')]_E + [(c, d)]_E$$

Hint: Two elements are equivalent under E if they have the **same difference**.

Divisibility & GCD

a) Find a number that has **exactly 3 prime divisors** or show that such a number can not exist. [20 %]

b) Compute $\text{gcd}(410, 240)$. Write down performed calculations **in full detail**. [20 %]