Logic & Set Theory

3.AB PreIB Maths – Exam A

Unless specified otherwise, you are to **always** (at least briefly) explain your reasoning. Even in closed questions.

Logic – propositions and conjunctions.

a) For each value of p write down the value of: [15 %]

$$p \vee \neg p.$$

You **don't** have to show your method.

b) Decide whether the proposition: [10 %]

$$(p \Rightarrow q) \vee \neg(p \Rightarrow q)$$

is true regardless of the values of p and q.

Basic set operations.

a) Given sets $A = \{😎, 🍩, 😈, 🪦\}$, $B = \{🍩, 🪦, 🍣\}$ and $C = \emptyset$, determine the set [15 %]

$$(A \cup B) \cap C.$$

Explain your method.

b) Decide whether [10 %]

$$(A \cup B) \cap C = A \cup (B \cap C)$$

for any sets A, B, C. **Explain**.
Hint: Use Venn diagrams.

Cartesian product and relations.

a) You are given [15 %]

$$A = \{1, 2\},\ B = \{a, b, c\} \text{ and } R = \{(2, a), (2, b)\},$$

where R is a relation from A to B. Provide at least two other relations from A to B that are different from the relation R.

b) How many relations are there from A to B if [10 %]

$$A = \{5\} \text{ and } B = \{\check{e}, \check{s}, \check{c}, \check{r}, \check{z}\}.$$

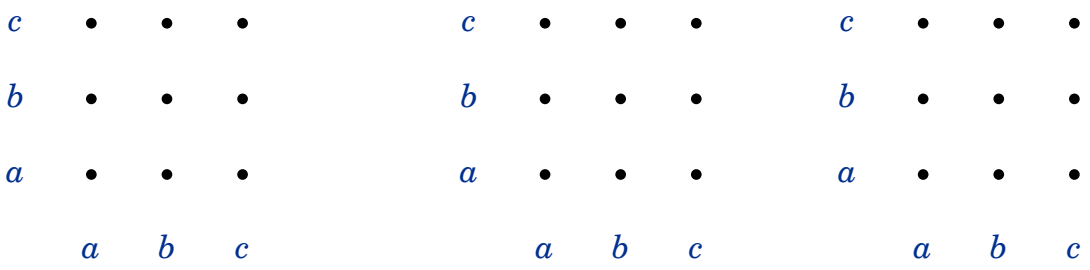
Hint: It is **not** necessary to write all of them. A simple argument suffices.

Equivalence.

a) For each of the following relations decide if they are equivalence on $A = \{a,b,c\}$ or not. You **don't** need to **explain anything**. [15 %]

- ☐ $R = \{(a,a),(b,b),(c,c)\}$
- ☐ $R = \{(a,b),(b,a),(a,a),(b,b),(c,c)\}$
- ☐ $R = \{(1,2),(2,3),(1,3)\}$
- ☐ $R = \{(a,a),(b,b),(c,c),(a,b),(b,c),(b,c),(b,a)\}$

You may use the empty diagrams below to draw the relations from above.



b) To every point in the visualization of the equivalences from part a) assign one defining feature of equivalence that demands its existence in the equivalence. [10 %]

For example: This specific pair is present because otherwise the symmetry property would not be satisfied.

Hint: Try assigning only the reflexivity and symmetry. The geometrical representation of transitivity is harder to see.