Number Sets & GCD

3.AB PrelB Maths – Exam C

Unless specified otherwise, you are to **always** (at least briefly) explain your reasoning. Even in closed questions.

Natural Numbers

a) The **multiplication** of natural numbers is defined by these two rules: [20 %]

1) $n \cdot 1 = 1$
2) $n \cdot \text{succ}(m) = n \cdot m + n$

where n and m are any natural numbers.

Using **only these two rules** and your knowledge about **addition** evaluate the following expressions.

- $3 \cdot 3$
- $2 \cdot 5$

b) Assuming $x \cdot y = y \cdot x$, show that $x \cdot \text{succ}(y) = \text{succ}(y) \cdot x$. Use **only** the rules that define multiplication from part a). [10 %]

Integers & Rationals

a) Connect all pairs belonging to the **same equivalence class** and write down the value of the **represented integer** for each class. [20 %]

$(1,1)$ $(2,4)$ $(2,2)$

$(6,8)$ $(10,10)$ $(121,120)$

$(0,2)$ $(7,6)$ $(8,7)$

b) You are given two pairs of natural numbers: (a',b') and (a,b) from the **same equivalence class** (they represent the same integer value). Show that their respective **products** with some pair (c,d) also belong to the **same equivalence class**. [10 %]

In other words, show that if $[(a,b)]_E = [(a',b')]_E$, then

$$[(a,b)]_E \cdot [(c,d)]_E = [(a',b')]_E \cdot [(c,d)]_E$$

Hint: The pairs (a,b) and (a',b') represent the same integer if (informally) '$a-b=a'-b'$'.

Divisibility & GCD

a) Find all numbers smaller than 50 that have **exactly 4 divisors**. Do **not** proceed by trial and error (this method would result in 0 %). [20 %]

b) Compute $\gcd(7205, 7535)$. Write down performed calculations **in full detail**. [20 %]