Logic & Set Theory

2.AB PreIB Maths – Exam C

Unless specified otherwise, you are to **always** (at least briefly) explain your reasoning. Even in closed questions.

Logic – propositions and operators

Is the proposition [25 %]

$$(p \Rightarrow q) \vee \neg q$$

a **tautology**? Meaning, is it **always true** regardless of p and q being true or false? **Explain**.

Bonus Problem [10 %]

Consider a new logical operator $\oplus$ given by the following truth table:

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Write the proposition $p \oplus q$ using only the standard logical operators $\neg$, $\wedge$ and $\vee$.

Basic set operations

Given sets $A = \{a, b, c, d, e\}$, $B = \{b, e\}$ and $C = \{a, d, f\}$, use set operations (whichever you wish) on A, B and C to create the sets [35 %]

$$\{b, e, f\} \quad \text{and} \quad \{a, d\}.$$

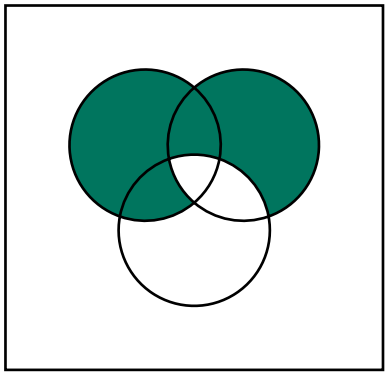
You **don't** have to **explain** your method.

Bonus Problem [10 %]

Consider the logical operator $\oplus$ from the previous bonus problem. Determine the set $A \triangle C = \{x \mid x \in A \oplus x \in C\}$ where A and C are defined above. Give some **comments** on the method you used to obtain the set.

Venn diagrams

a) Given the Venn diagram below, determine the set which it represents. You **don't** have to provide an **explanation**. [20 %]



b) Draw a Venn diagram for the following expression: [20 %]

$$(A \cap B \cap C) \cup (B \setminus C)$$

You **don't** have to **explain** anything.

Bonus Problem [10 %]

The **complement** of a set X inside a set Y is defined as $Y \setminus X$. Draw a Venn diagram of the **complement** of the set $(A \setminus B) \cap C$ inside $A \cup B \cup C$.