

# Logic & Set Theory

## 2.AB PreIB Maths – Exam D

Unless specified otherwise, you are to **always** (at least briefly) explain your reasoning. Even in closed questions.

### Logic – propositions and operators

Is the proposition

[25 %]

$$(p \wedge q) \vee (\neg p \wedge \neg q)$$

a **tautology**? Meaning, is it **always true** regardless of  $p$  and  $q$  being true or false? **Explain.**

### Bonus Problem

[10 %]

Consider a new logical operator  $\odot$  given by the following truth table:

$p$	$q$	$p \odot q$
$T$	$T$	$T$
$T$	$F$	$T$
$F$	$T$	$F$
$F$	$F$	$T$

Write the proposition  $p \odot q$  using only the standard logical operators  $\neg$ ,  $\wedge$ ,  $\vee$ ,  $\Rightarrow$  and  $\Leftrightarrow$ .

**Basic set operations**

Given sets  $A = \{1, 2, 3, 4, 5\}$ ,  $B = \{2, 4, 5\}$  and  $C = \{1, 4, 5\}$ , determine the sets

[35 %]

$$A \setminus (B \cup C) \text{ and } (A \cup B) \cap C$$

You **don't** have to **explain** your method.

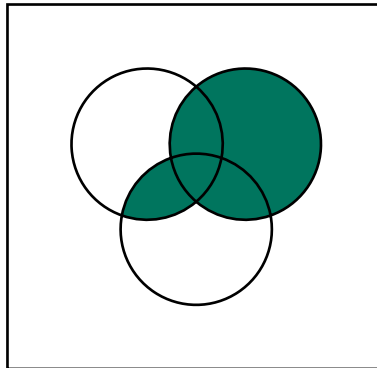
**Bonus Problem**

[10 %]

Consider the logical operator  $\odot$  from the previous bonus problem. Determine the set  $\{x \mid x \in A \odot x \in B\}$  where  $A$  and  $B$  are defined above. Give some **comments** on the method you used to obtain the set.

**Venn diagrams**

- a) Given the Venn diagram below, determine the set which it represents. You **don't** have to provide an **explanation**. [20 %]



- b) Draw a Venn diagram for the following expression: [20 %]

$$(B \cup C) \setminus (A \cap C)$$

You **don't** have to **explain** anything.

**Bonus Problem**

[10 %]

The **complement** of a set  $X$  inside a set  $Y$  is defined as  $Y \setminus X$ . Draw a Venn diagram of the **complement** of the set  $(A \cap B) \cup C$  inside  $A \cup B \cup C$ .