# Logic & Set Theory

## 3.AB PreIB Maths – Exam A

Unless specified otherwise, you are to **always** (at least briefly) explain your reasoning. Even in closed questions.

### Logic – propositions and conjunctions.

a) For each **truth value** of $p$ write down the **truth value** of the proposition [15 %]

$$p \vee \neg p.$$

You **don't** have to show your method.

b) Decide whether the proposition [10 %]

$$(p \Rightarrow q) \vee \neg(p \Rightarrow q)$$

is **always true** regardless of the truth values of $p$ and $q$.

## Basic set operations.

a) Given sets $A = \{$😎, 🍩, 😈, 🪦$\}$, $B = \{$🍩, 🪦, 🥌$\}$ and $C = \emptyset$, determine the set [15 %]

$$(A \cup B) \cap C.$$

**Explain** your method.

b) Decide whether [10 %]

$$(A \cup B) \cap C = A \cup (B \cap C)$$

for any sets $A$, $B$, $C$. **Explain**.
**Hint**: Use Venn diagrams.

## Cartesian product and relations.

a) You are given [15 %]

$$A = \{1, 2\},\ B = \{a, b, c\}\ \text{ and }\ R = \{(2, a), (2, b)\},$$

where $R$ is a relation from $A$ to $B$. Provide at least two other relations from $A$ to $B$ that are different from the relation $R$.

b) How many relations are there from $A$ to $B$ if [10 %]

$$A = \{5\} \text{ and } B = \{\text{ě}, \text{š}, \text{č}, \text{ř}, \text{ž}\}.$$

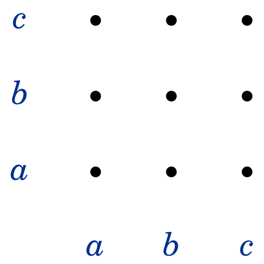
**Hint:** It is **not** necessary to write all of them. A simple argument suffices.

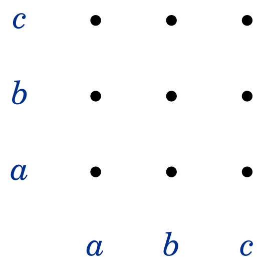
## Equivalence.

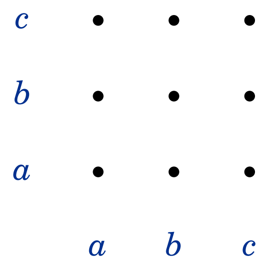
a) For each of the following relations decide if they are an equivalence on the set $A = \{a, b, c\}$ or not. You **don't** need to **explain anything**. [15 %]

- ☐ $R = \{(a,a), (b,b), (c,c)\}$
- ☐ $R = \{(a,b), (b,a), (a,a), (b,b), (c,c)\}$
- ☐ $R = \{(1,2), (2,3), (1,3)\}$
- ☐ $R = A \times A$
- ☐ $R = \{(a,a), (b,b), (c,c), (a,b), (b,c), (b,c), (b,a)\}$

You may use the empty diagrams below to draw the relations from above.




b) Recall that the relation of *equivalence* is given by three conditions: [10 %]

- **reflexivity**: every element is equivalent to itself;
- **symmetry**: if $a$ is equivalent to $b$, then $b$ is equivalent to $a$;
- **transitivity**: if $a$ is eq. to $b$ and $b$ is eq. to $c$, then $a$ is eq. to $c$.

To every point in the visualization of the equivalences from part a) assign one defining condition of equivalence that forces its presence in the equivalence.

For example: *'This specific pair is present because otherwise the symmetry property would not be satisfied'*.

**Hint:** Try assigning only the reflexivity and symmetry conditions. The geometrical representation of transitivity is harder to see.