Probability

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January 26, 2024

Tree Diagrams

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In some sense, they allow **dependent events** to **become independent** and compute the probability of the successive occurrence of such events by simple multiplication.

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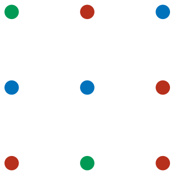
- connecting successive events by lines;
- drawing the events from top to bottom in chronological order.

Tree Diagram – Example

Imagine again a set of 9 balls of three different colors.



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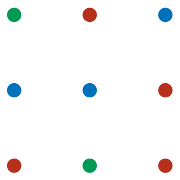

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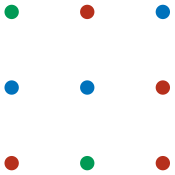

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We'll perform the same computation using a **tree diagram**.

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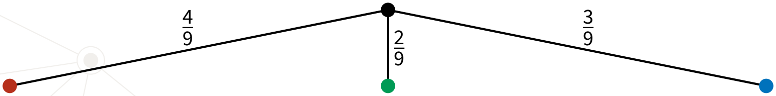
We represent this in the tree diagram like this:



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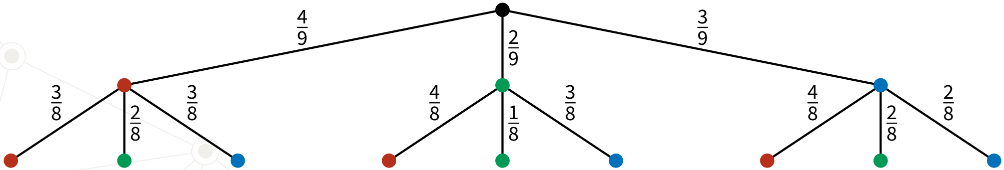
The full tree diagram of this probabilistic experiment would look like this:



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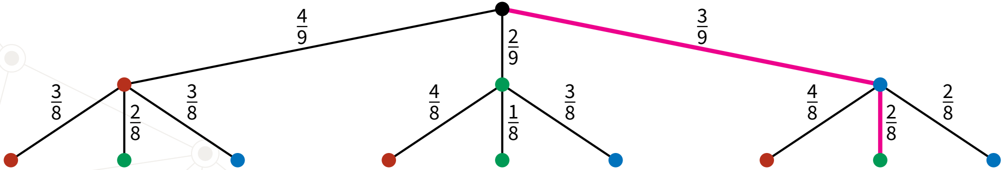

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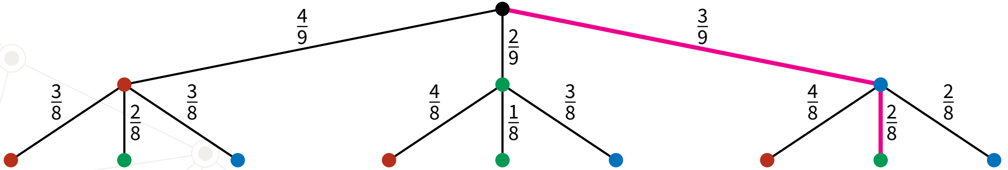

Hence, we know its probability to be $\frac{3}{9} \cdot \frac{2}{8} = \frac{1}{12}$.

Tree Diagram – Problem #1

A football team wins its matches with a probability of 0.7.

Using a tree diagram, find the probability that they win at least 1 of their next three matches.

Tree Diagram – Problem #2

Anna and Rob take their driving tests on the same day. The probability of Anna passing her driving test is 0.7. The probability of both Anna and Rob passing is 0.35.

1. Work out the probability of Rob passing his driving test.
2. Work out the probability of both Anna and Rob failing their driving tests.

Tree Diagram – Problem #3

You roll a dice three times. What's the probability that the sum of all the rolled numbers is 12 assuming

1. the first rolled number is 3.
2. the second rolled number is 5.

Use tree diagrams to solve the problem.

Tree Diagram – Problem #4

In a factory, three machines – A, B and C – are used to make biscuits.

Machine A makes 25 % of the biscuits, B makes 45 % and C the rest. In addition, about 2 % of all the biscuits made by A are broken, 3 % of those made by B are broken and 5 % of those made by C are broken.

1. Draw a tree diagram representing the problem.
2. Calculate the probability that a randomly picked biscuit made by machine A is not broken.
3. Calculate the probability that a randomly picked biscuit is broken.
4. Assuming that a biscuit is broken, what's the probability it was **not** made by machine B?

Monty Hall Problem

Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2?" Is it to your advantage to switch your choice?