

Homework – PreIB 3.AB 3 & 4

Structures and Operations

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DON'T FORGET TO EXPLAIN EVERYTHING EVEN IF YOU THINK IT'S OBVIOUS!

Natural Numbers

Exponentiation of two natural numbers $n, m \in \mathbb{N}$ is defined by the following two formulae:

- $n^0 = 1$,
- $n^{s(m)} = n^m \cdot n$.

Explain **very clearly** how to calculate n^m using **only** the two rules above.

Hint: This process is very similar to the definition of *addition* and *multiplication* on natural numbers.

Answer the following questions:

1. Is exponentiation *commutative*, that is, is it true that $n^m = m^n$ for all pairs of natural numbers $n, m \in \mathbb{N}$? If yes, explain why. If not, provide a counterexample.
2. Is exponentiation *associative*, that is, is it true that $n^{(m^k)} = (n^m)^k$ for all triples of natural numbers $n, m, k \in \mathbb{N}$? If yes, explain why. If not, provide a counterexample.
3. Is exponentiation an *operation* (by definition) on natural numbers? Explain.

Operations

On the set $X = \{a, b, c, d\}$ there are two operations given by the following picture.

