Statistics

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September 26, 2023

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The collective information about a system's past state is called **data**.
It assigns **probabilities** to each possible future state of system based on data.
It also assigns probabilities to the possibility of **possibility of wrong prediction**.

Example – Biased Coin?

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- Is this coin is **biased towards** 'heads'/'tails' with *allowed probability of error* α?
  - **No**, for $\alpha = 0.05$.
  - **Yes**, for $\alpha = 0.2$.

Contents

Data

What Do We Mean By Data?

Data

Two sets (called *inputs* and *outputs*) describing the studied system.

Example – Junctions

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An **output** is the number of traffic accidents in a given day.

Example – First Baby

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An **input** would be a name of a European country.

An **output** is the average age of a first-time mother in that country.

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Types of Data

Discrete Data vs. Continuous Data

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- There are only *finitely many* countries on a continent.

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More often than not, the inputs in a continuous data are **moments in time** or **coordinates in space**.

Continuous Data – Examples

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- Another example is the density of air per cubic meter.
  - Input: Coordinates of a unit cube in space.
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  - The data is a function $f : \mathbb{R}^3 \to \mathbb{R}$.

Visualizing Discrete Data

Tables

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Input	1	2	3	4	5	6	7	8	9	10
Output	180	169	191	177	175	181	171	153	180	183

Pie Chart

Only usable if your outputs **total a predetermined number**, typically *percentages*.

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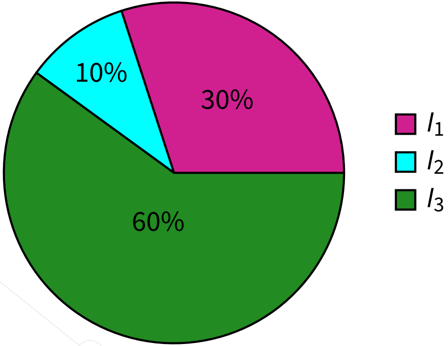
Pie chart of this data looks like this



Pie Chart – Examples

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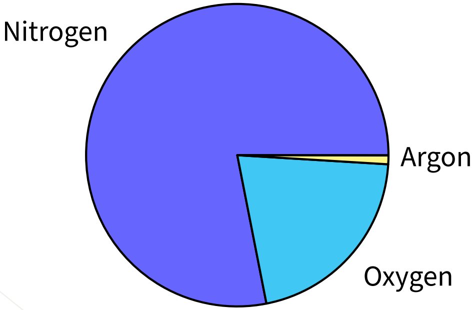
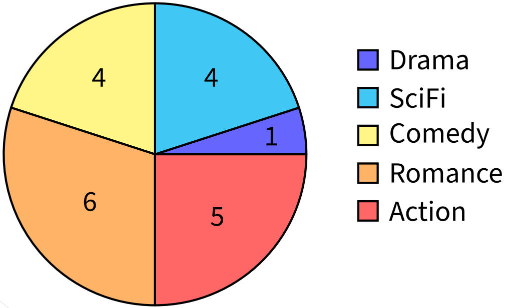
For instance, here is a pie chart of the composition of *air*.



PIE CHART – EXAMPLES

Favourite type of movie as determined by a survey.



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Also very good for comparing more outputs for the same inputs.

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Suppose we count the number of customers in our shop over each hour. If we're open from 8 AM to 5 PM, a bar chart of such an experiment can look like this:

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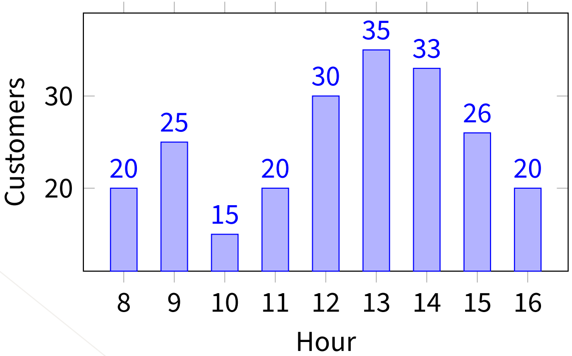

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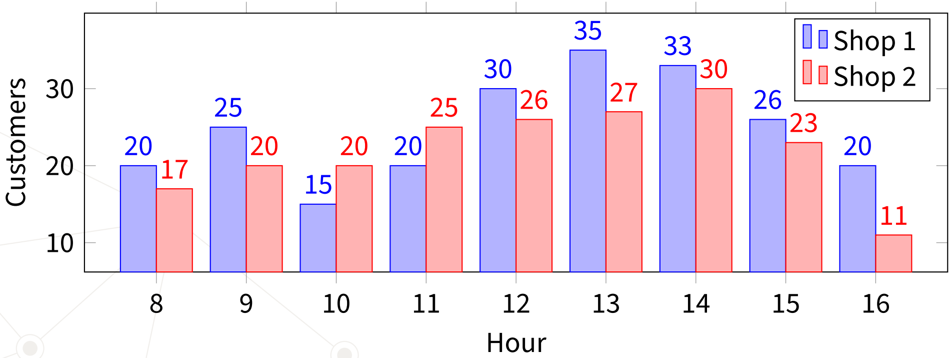

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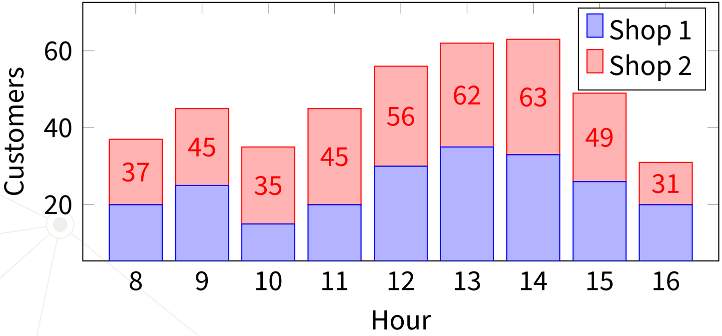

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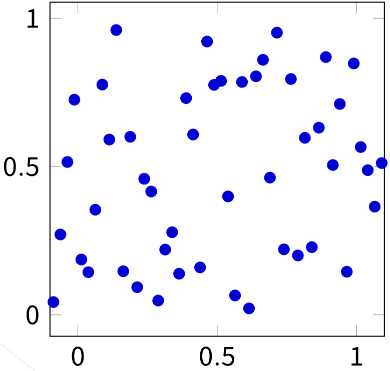

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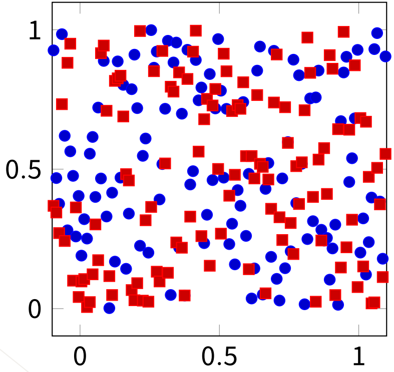

Mean – Median – Deviation – Correlation

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- the expected value of the next experiment (the **mean**),
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- dependence on any other data (the **correlation**).

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The Mean

Types of Mean

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Types of Mean – Arithmetic Mean

Arithmetic Mean

The **arithmetic mean** is the sum of outputs divided by their number. If $x_1, \ldots, x_n$ are the outputs, their arithmetic mean (often denoted $\bar{x}$) is

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While comparing the heights of two people, we care about the **absolute** difference in centimetres.

For example, if this is our data

Input	1	2	3	4	5
Output	165	161	164	172	168,

we conclude that the expected height of a randomly chosen 15-year-old British male is

$$\frac{165 + 161 + 164 + 172 + 168}{5} = 166.$$

Types of Mean – Geometric Mean

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The **geometric mean** is the n-th root of the product of n outputs. That is, if $x_1, \ldots, x_n$ are the outputs, their geometric mean is

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Input	India	China	Japan	South Korea	Mongolia	Taiwan
Output	1.328	1.118	0.991	1.100	1.366	1.078

This means that the expected increase in population in a randomly chosen Asian country is

$$\sqrt[6]{(1.328 \cdot 1.118 \cdot 0.991 \cdot 1.100 \cdot 1.366 \cdot 1.078)} = 1.156.$$

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Output	65 km/h	52 km/h	71 km/h	60 km/h	62 km/h	53 km/h,

then the average speed of the train across the whole track is

$$\frac{6}{\frac{1}{65}+\frac{1}{52}+\frac{1}{71}+\frac{1}{60}+\frac{1}{62}+\frac{1}{53}} = 59.78 \text{ km/h}.$$

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If you summed up all the distances between stations and divided them by the total time, you would get the **harmonic mean!**

The Median

Median

The **median** is the value which lies exactly in the middle of a dataset. It is essentially the value separating the lower and upper half of outputs. If $x_1, \ldots, x_n$ are the outputs, the median is

$$\mathrm{median}(x) := \begin{cases} x_{(n+1)/2} & \text{if n is odd,} \\ \frac{x_{n/2}+x_{n/2+1}}{2} & \text{if n is even.} \end{cases}$$

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For example, imagine we're trying to determine the distance from our measuring station to the hypocentre of an earthquake.

We can detect where the quake is strongest, giving us this data:

Input	1	2	3	4	5	6
Output	1 km	2 km	2 km	2 km	3 km	14 km

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The median of this dataset is 2 km which is a much better estimate of a 'centre' than for example the arithmetic mean, being equal to 4, is.

Also, the mean and the median cannot be 'too far' apart and the median requires at most two values to calculate, making it a very resource efficient approximation of the mean.

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- the **average absolute deviation** (a measure of actual 'difference').

A **very important distinction** is that the *standard deviation* concerns **future** measurements while the *average absolute deviation* concerns **past** measurements.

Types of Deviation – Standard Deviation

Standard Deviation

The **standard deviation** measures the dispersion of a set of values. Basically, it measures how likely the data is to concentrate around the mean. If $x_1, \ldots, x_n$ are the outputs and $\bar{x}$ is their **arithmetic** mean, then their standard deviation is

$$\sigma := \sqrt{\frac{1}{n}((x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \ldots + (x_n - \bar{x})^2)}.$$

Standard Deviation – Example

Let us repeat the height experiment. We measured the heights of 5 15-year-old British males to try to determine the national average. This is the data:

Input	1	2	3	4	5
Output	165	161	164	172	168

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Input	1	2	3	4	5
Output	165	161	164	172	168

We computed the arithmetic mean to be 166. This means that the standard deviation of this data is

$$\sigma = \sqrt{\frac{1}{5}((165-166)^2 + (161-166)^2 + (164-166)^2 + (172-166)^2 + (168-166)^2)}$$
$$= 3.742,$$

meaning we can expect most new values to concentrate 3.742 cm around 166 cm.

Types of Deviation – Average Absolute Deviation

Average Absolute Deviation

The **average absolute deviation** is the average of the absolute deviations from a chosen central point (typically the mean). If $x_1, \ldots, x_n$ are the outputs and $\bar{x}$ is the chosen central point, then the average absolute deviation of this dataset is

$$\frac{|x_1 - \bar{x}| + |x_2 - \bar{x}| + \ldots + |x_n - \bar{x}|}{n}.$$

Average Absolute Deviation – Example

If we return to the height experiment yet again, we can calculate that the average absolute deviation of the data (with the central point being the arithmetic mean)

Input	1	2	3	4	5
Output	165	161	164	172	168

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$$\frac{|165-166|+|161-166|+|164-166|+|172-166|+|168-166|}{5}=3.2,$$

meaning that the measured heights differ on average by 3.2 cm from the calculated arithmetic mean.

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- negative correlation means that the two series of outputs **contradict** each other;
- zero correlation means that the two series of outputs are **unrelated**;
- positive correlation means that the two series of outputs **influence** each other.

Computing Correlation

Correlation Formula

If $x_1, \ldots, x_n$ and $y_1, \ldots, y_n$ are two series of outputs for the same inputs with means $\bar{x}$ and $\bar{y}$, their correlation is

$$\operatorname{cor}(x, y) := \frac{(x_1 - \bar{x})(x_2 - \bar{x}) \cdots (x_n - \bar{x})(y_1 - \bar{y})(y_2 - \bar{y}) \cdots (y_n - \bar{y})}{\sqrt{(x_1 - \bar{x})^2 (x_2 - \bar{x})^2 \cdots (x_n - \bar{x})^2 (y_1 - \bar{y})^2 (y_2 - \bar{y})^2 \cdots (y_n - \bar{y})^2}}.$$

Interpreting Correlation – Table

A crude interpretation of correlation is given in the following table:

Coefficient	Strength	Type
-0.7 to -1	Very strong	Negative
-0.5 to -0.7	Strong	Negative
-0.3 to -0.5	Moderate	Negative
0 to -0.3	Weak	Negative
0 to 0.3	Weak	Positive
0.3 to 0.5	Moderate	Positive
0.5 to 0.7	Strong	Positive
0.7 to 1	Very strong	Positive

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