Number Sets & GCD

3.AB PreIB Maths – Resit Exam

Unless specified otherwise, you are to **always** (at least briefly) explain your reasoning. Even in closed questions.

Natural Numbers

a) Remember that we defined **addition** and **multiplication** as: [20 %]

$$\mathrm{succ}(n) = n + 1 \qquad\qquad n * 0 = 0$$

$$\mathrm{succ}(n + m) = n + \mathrm{succ}(m) \qquad \mathrm{succ}(n * m) = n * m + m$$

Using **only** those axioms calculate:

- $2 \cdot 3$
- $1 + (2 \cdot 2)$

b) Assuming $x + y = y + x$, show that $x + \mathrm{succ}(y) = \mathrm{succ}(y) + x$. In your prove use only those **axioms** that **define addition**. [10 %]

Integers & Rationals

a) Connect the pairs that correspond to the **same equivalence classes** and write down the value of **represented rational**. [20 %]

$(2, 20)$	$(5, 50)$	$(35, 28)$
$(10, 8)$	$(25, 2)$	$(-50, -2)$
$(-2, 2)$	$(4, 4)$	$(7, 8)$

b) Integers and rationals share some similarities on their definition. They are defined as **equivalence classes** on $\mathbb{N} \times \mathbb{N}$ and $\mathbb{Z} \times \mathbb{Z}$ respectively. Create **at least one equivalence** on $\mathbb{N} \times \mathbb{N}$ and one on $\mathbb{Z} \times \mathbb{Z}$. Comment on the equivalence classes, **how many are there**? do they have a specific shape? [10 %]

For example one equivalence A may be: $a\mathrm{A}b$ if $a = b$. Other example B is: $a\mathrm{B}b$ for all a, b either in $\mathbb{N}$ or $\mathbb{Z}$. These are the trivial equivalences so they will **not** earn you any points.

Divisibility & GCD

a) Some **natural number** n can be decomposed into primes as $n = p_1 \cdot p_2 ... p_k$. Use the primes $p_1, p_2, ..., p_k$ to find **all the divisors** of n. [20 %]

b) Compute $\text{gcd}(1029, 1617)$. Write down performed calculations **in full detail**. [20 %]