Logic & Set Theory

3.AB PreIB Maths – Mock Exam

Unless specified otherwise, you are to **always** (at least briefly) explain your reasoning. Even in closed questions.

Logic – propositions and conjunctions.

a) Supposing a proposition p is false and another proposition q is also false, is the proposition [15 %]

$$(p \Rightarrow q) \vee q$$

true or false? **Explain**.

b) Fill the propositions p and q (you may not need both) in the blanks so that the proposition [10 %]

$$(\neg p \Rightarrow \square) \Leftrightarrow (\square \vee q)$$

is **always** true independently of whether p and q are themselves true or false. **Check that your answer is correct.**

Basic set operations.

a) Given sets $A = \{2, 3, 5\}$, $B = \{3, 4, 5\}$ and $C = \{1, 2, 3, 4\}$, determine the set [15 %]

$$(A \cup B) \cap C.$$

You **don't** have to provide any **explanation**.

b) Show that [10 %]

$$(A \cup B) \cup C = A \cup (B \cup C)$$

for any sets A, B, C. **Explain**.
Hint: Use Venn diagrams.

Cartesian product and relations.

a) Mark each of the following sets **that is a relation** from A to B, where [15 %]

$$A = \{1, 2\} \text{ and } B = \{a, b, c\}.$$

You **don't** need to **explain anything**.

☐ $R = \{(1, a), (1, b), (2, c)\}$

☐ $R = \{(a, 2), (b, 1)\}$

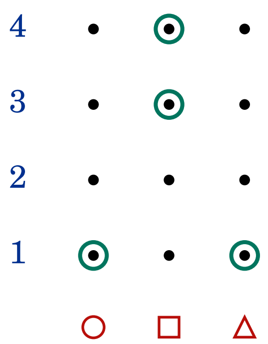
☐ $R = \{(1, 2), (1, b), (2, 2)\}$

☐ $R = \{(2, a), (2, b)\}$

☐ $R = \{(a, b), (a, c)\}$

b) A relation R is called a *function* if every element $b \in B$ is related to **at most one** element $a \in A$. That is, if aRb, then a is **not** related to any other element $\hat{b} \in B$. [10 %]

Below, you see a picture of a relation R from the set $A = \{\bigcirc, \square, \triangle\}$ to the set $B = \{1, 2, 3, 4\}$. Is R a *function*? **Why**?



Equivalence.

a) Is the relation [15 %]

$$E = \{(a,a),(b,b),(c,c),(c,d),(d,c),(b,d)\}$$

an **equivalence** on the set $A = \{a, b, c, d\}$? If not, add as few pairs to it as necessary to make it into an equivalence. **Explain**.

b) **How many different** equivalences are there on the set $B = \{1, 2, 3, 4, 5\}$ that partition it into **exactly** 3 distinct classes of equivalence? **Explain**. [10 %]