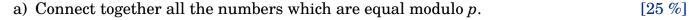# Congruences & CRT
3.AB PreIB Maths – Exam B

> Unless specified otherwise, you are to **always** (at least briefly) explain your reasoning. Even in closed questions.

Congruences

a) Connect together all the numbers which are equal modulo p. [25 %]

$$0 \qquad 1 \qquad 2p$$

$$p + 2 \qquad -1 \qquad 2$$

$$8p + 1 \qquad 10p \qquad 3p - 1$$

b) After finishing a meal in a restaurant, the waitress brings your check. You [35 %] realize that you can either pay with **eleven five-dollar bills** or with **twenty-eight four-dollar bills**, with the change in both scenarios being **smaller** then the **bill used for payment**.

   Calculate the possible prices of the meal.

Chinese Remainder Theorem

Solve the following system of congruences.   [40 %]

$$x \equiv 1 \quad (\text{mod } 5)$$
$$x \equiv 4 \quad (\text{mod } 12)$$
$$x \equiv 2 \quad (\text{mod } 7)$$

Explain briefly why there is only one solution smaller than $5 \cdot 12 \cdot 7$.