Polygons

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October 2, 2023

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The endpoints of those segments are called **vertices**.

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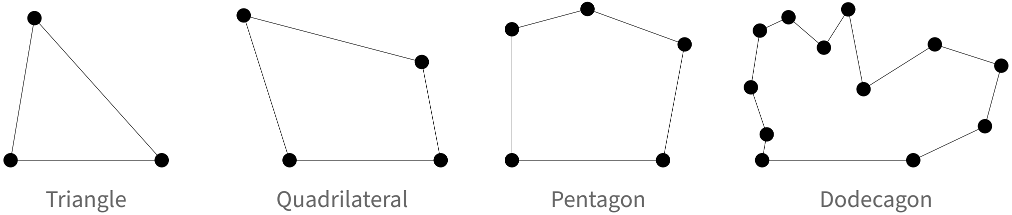
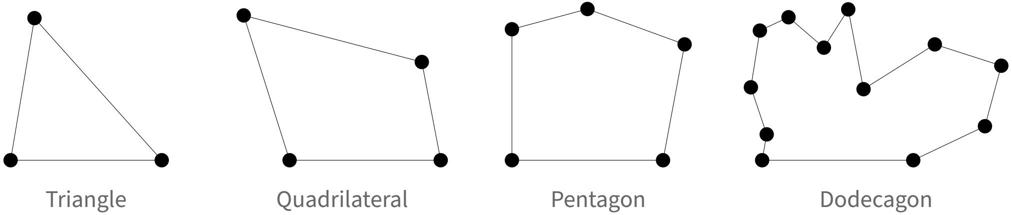
The endpoints of those segments are called **vertices**.

The segments themselves are called **edges**.

General Polygons – Examples

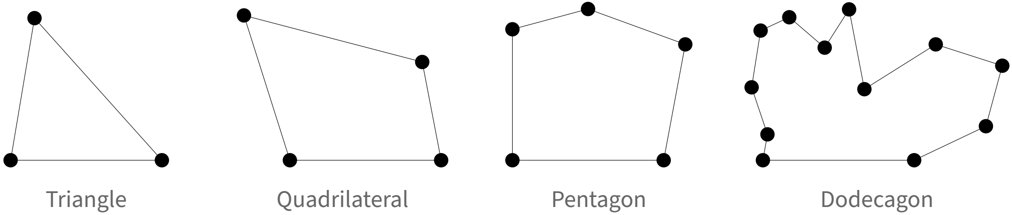


General Polygons – Examples



A polygon with $n \in \mathbb{N}$ sides is called an n-gon.

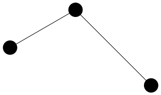
General Polygons – Examples



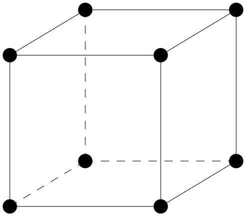
A polygon with $n \in \mathbb{N}$ sides is called an n-gon.

For example a polygon with 123456 sides is called a 123456-gon or decadismyriatrischilliatetrahectapentacontakaihexagon.

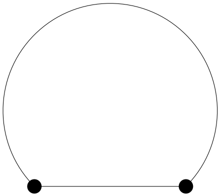
General Polygons – Counterexamples



Not closed



3D



Not straight

General Polygons – Convexity

Convex Polygon

A polygon is called **convex** if it has no internal angle greater than 180°.

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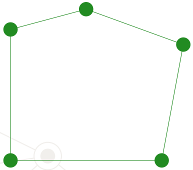
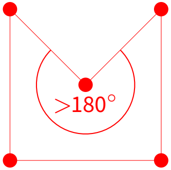


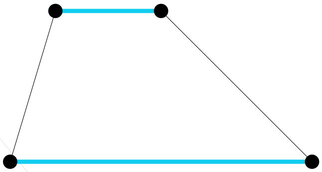
Convex



NOT convex

Convex Polygons

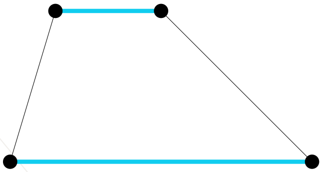
Convex Polygons – Special Types



Trapezoid/Trapezium

A convex quadrilateral with at least two parallel sides.

CONVEX POLYGONS – SPECIAL TYPES



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A convex quadrilateral with two pairs of parallel sides.

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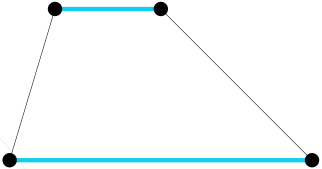

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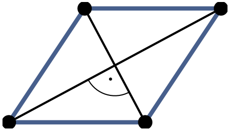


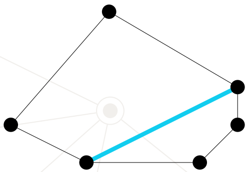
Rhombus

An **equilateral** (all sides of the same length) parallelogram.

Convex Polygons – Diagonals

Diagonal in a Convex Polygon

A diagonal of a **convex** polygon is a segment connecting two of its non-adjacent vertices.



Diagonal in a convex hexagon.

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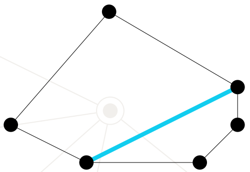

Diagonal in a convex hexagon.

Voluntary HW: How many different diagonals does a convex n-gon have?

Convex Polygons – Triangulations

Triangulation of a Convex Polygon

A triangulation of a **convex** polygon is its division into triangles by non-intersecting diagonals.

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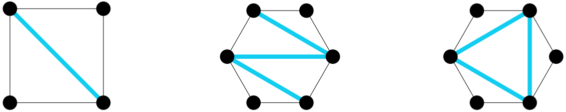

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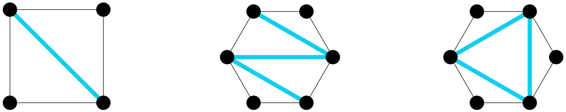

Examples of triangulations.

Voluntary HW: How many different triangulations of an n-gon are there?

Convex Polygons – Triangulations

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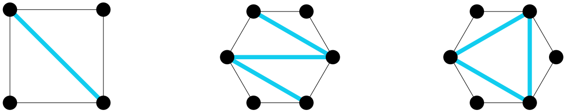
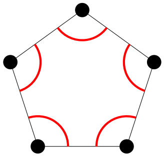


Examples of triangulations.

Voluntary HW: Find a **non-convex** polygon which **cannot** be triangulated.

Convex Polygons – Internal Angles



Internal angles of a pentagon.

Question: What is the sum of internal angles of a convex n-gon?

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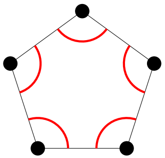

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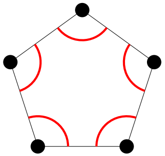

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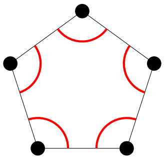

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- For a pentagon, it's 540°.

Convex Polygons – Internal Angles

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We choose the first triangle – it covers 3 vertices.

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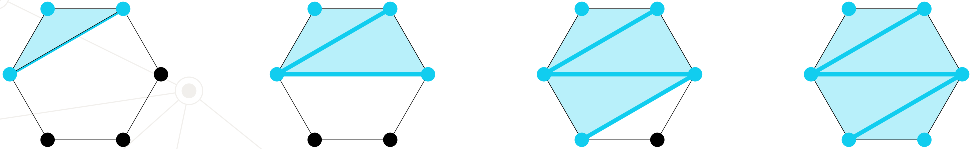

Construction of a triangulation of a hexagon.

Convex Polygons – Internal Angles

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The sum of all internal angles in a triangle is 180°.

Convex Polygons – Internal Angles

A convex n-gon is divided into $n-2$ triangles.

The sum of all internal angles in a triangle is 180°.

Sum of Internal Angles in a Convex Polygon

The sum of all internal angles of a convex n-gon is $(n-2)\cdot 180^\circ$.

Regular Polygons

Definition

Regular Polygon

A **regular polygon** is a convex polygon whose sides all have the same length and whose internal angles all have the same size.

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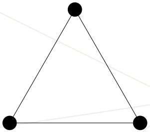
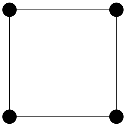
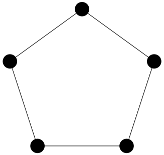


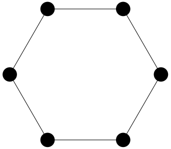
Equilateral triangle
(regular trigon)



Square (regular tetragon)



Regular pentagon



Regular hexagon

Review – Plane Transformations

Rotation

Rotation of a polygon consists of well ... rotating each of its points by a fixed angle around a fixed point (called *anchor*).

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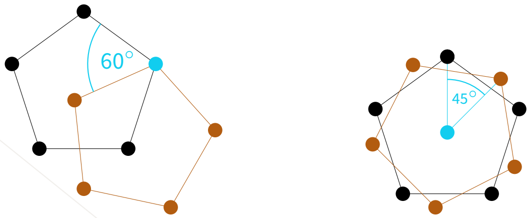

Examples of rotations.

Review – Plane Transformations

Reflection

Reflection of a polygon consists of ‘mirroring’ each of its points through a given line (called *axis of reflection*).

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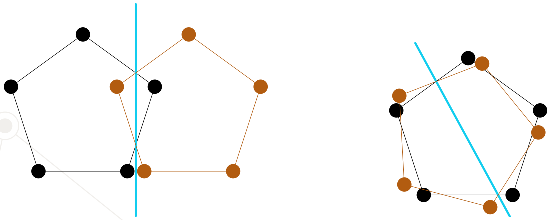

Examples of reflections.

Review – Plane Transformations

Point Symmetry

Point symmetry of a polygon consists of ‘mirroring’ each of its points through a given point (called *center of symmetry*).

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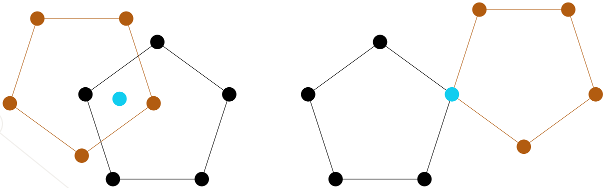

Examples of point symmetries.

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  - for n even over lines passing through opposite vertices
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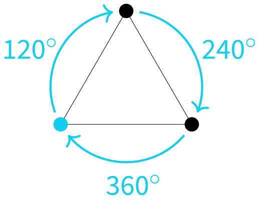
- point symmetries
  - only through the 'centre' – the point where its axes of symmetry intersect – in case n is even

Symmetries of Regular Polygons



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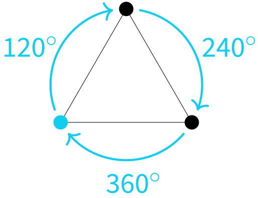

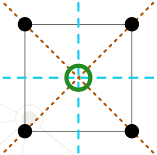

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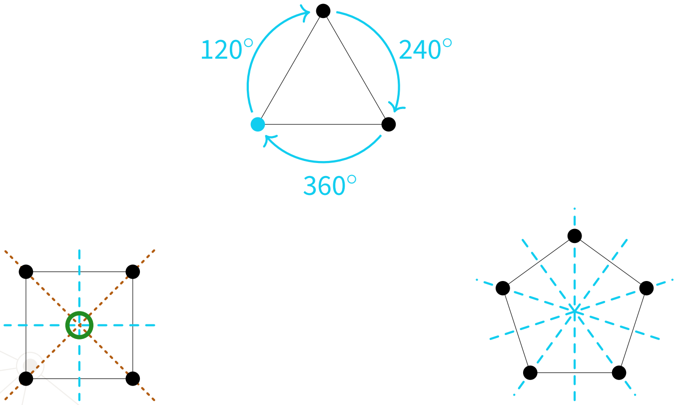

Examples of regular polygon symmetries

Cryptography on Regular Polygons

Chaining Symmetries

Given two symmetries, s_1 and s_2 of a regular polygon, one can apply them one after the other (‘compose’ them, like functions).

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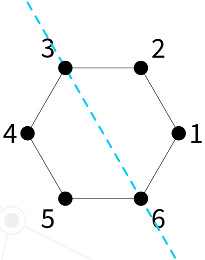
We'll denote this composition simply by $s_1 s_2$.

Chaining Symmetries – Example



Example of a chain of symmetries.

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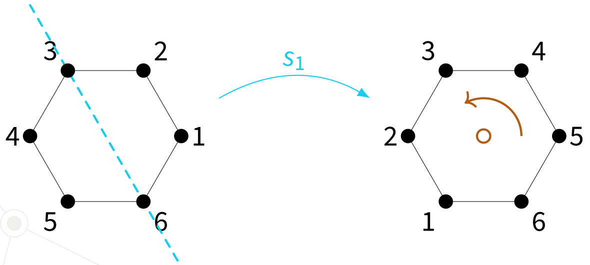

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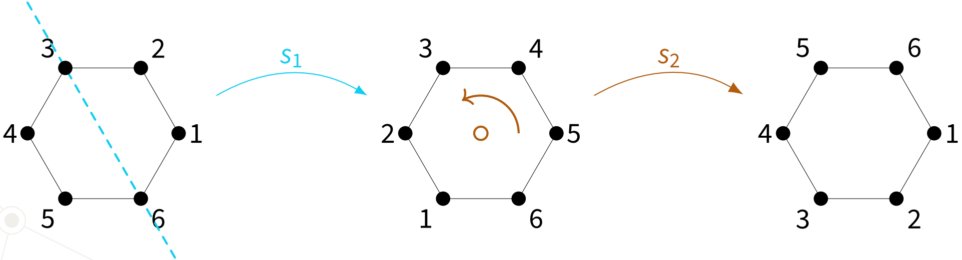

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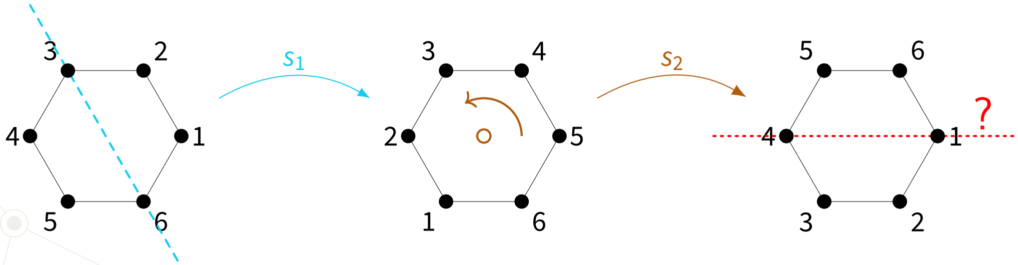

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For example,

- if s_1 is any reflectional symmetry and s_2 is a rotation by 60° counter-clockwise, then $s_2^3 s_1$ (s_2^3 means $s_2 s_2 s_2$) reflects a hexagon through a line perpendicular to the line of s_1.

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- if s_1 is a rotation by 120° clockwise and s_2 is a reflection through a vertical line passing through the top vertex, then $s_1 s_2$ is a reflection through the line given by the rotation of the line of s_2 60° clockwise.

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- any reflection (we'll denote it s).

Chaining Symmetries – Triangle

Let r be the rotation by 120° and s any reflectional symmetry.

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- The other two rotational symmetries are r^2 and r^3.
- The other two reflectional symmetries are rs and r^2s.
- Therefore, all the symmetries of an equilateral triangle are

$$\{r, r^2, r^3, s, rs, r^2s\}.$$

Chaining Symmetries – General Algorithm

In general, to create all symmetries, one needs a rotation by an angle $k \cdot 360^\circ / n$ where k **doesn't share a prime factor** with n (in other words, the fraction $\frac{k}{n}$ cannot be simplified) and any one reflectional symmetry.

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Why?

- If k shares factors with n, then you can never get rotation by $360^\circ / n$.

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- Two symmetries cannot in general produce every rotation.
- Two rotations can never produce a symmetry.

Chaining Symmetries – General Algorithm

You're given a rotation r by $k \cdot 360^\circ / n$ such that k doesn't share factors with n and a reflectional symmetry s.

Chaining Symmetries – General Algorithm

You're given a rotation r by $k \cdot 360^\circ / n$ such that k doesn't share factors with n and a reflectional symmetry s.

If you need to calculate a rotation, then

1. First measure the angle **counter-clockwise**.

Chaining Symmetries – General Algorithm

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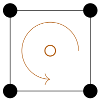
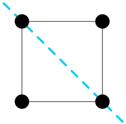
Voluntary HW: *Why* does this algorithm work?

Chaining Symmetries – Algorithm Example

We're given two symmetries of the square:



rotation r by 270°



reflection s over the first diagonal

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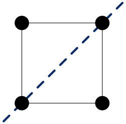

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2. The angle between the two diagonals is 90° in any direction.
3. Repeating the rotation from step 1 two times (that is, $b = 2$) and then using s gives the desired symmetry – in this case it's $(r^3)^2 s = r^6 s$. Of course, r^4 is rotation by 360° which does nothing, so the final symmetry is $r^2 s$.

1

Encoding Messages Using Symmetries

Encoding Messages – General Idea

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- The decoding should require a ‘key’ which is **statistically impossible to determine quickly**.
- The encoding and decoding must be done procedurally – requires a system with concrete rules and a **limited** (but huge) **number of combinations**.

Encoding Messages – Regular Polygons

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This means that after applying a rotation or reflection, a (in most cases) different symbol will appear in the main vertex.

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This means that to send messages written Morse Code using regular polygons, we need three vertices – a triangle.

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Let the top vertex be main and let's choose a rotation $r = \circlearrowleft 120^\circ$ and a reflection s depicted below.

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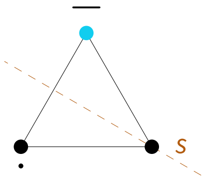

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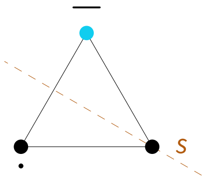

The top vertex is the main one, which means that the symbol above it will get sent.

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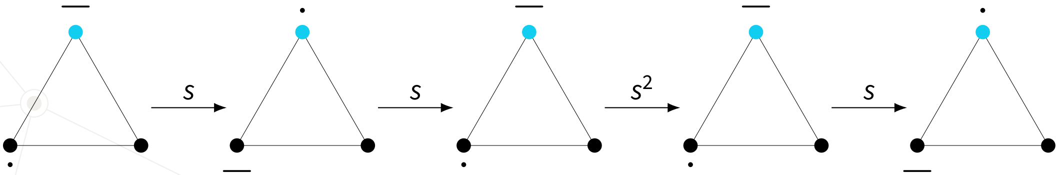
For example, we can send the letter P (· — — ·) like this:



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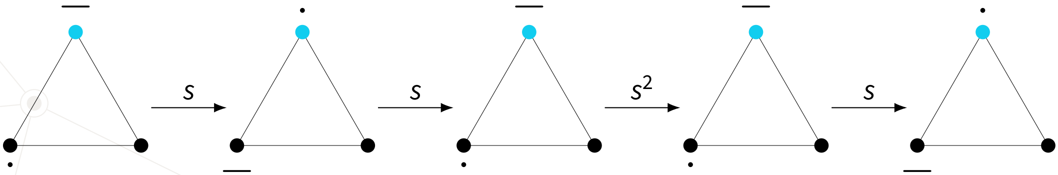

Or, as a sequence

$$s, s, s^2, s.$$

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The letter $\cdot - - \cdot$ can also be sent for example like this

$$r^2, sr, sr^2, rsr.$$