Logic & Set Theory

2.AB PreIB Maths – Mock Exam

> Unless specified otherwise, you are to **always** (at least briefly) explain your reasoning. Even in closed questions.

Logic – propositions and operators

Supposing a proposition p is false and another proposition q is also false, is the proposition

[25 %]

$$(p \Rightarrow q) \vee q$$

true or false? **Elaborate**.

Bonus Problem

[10 %]

Fill the propositions p and q (you may not need both) in the blanks so that the proposition

$$(\neg p \Rightarrow \boxed{}) \Leftrightarrow (\boxed{} \vee q)$$

is **always** true independently of whether p and q are themselves true or false. **Check that your answer is correct.**

Basic set operations

Given sets $A = \{2, 3, 5\}$, $B = \{3, 4, 5\}$ and $C = \{1, 2, 3, 4\}$, determine the sets [35 %]

$$(A \cup B) \cap C \quad \text{and} \quad C \setminus (A \cap B).$$

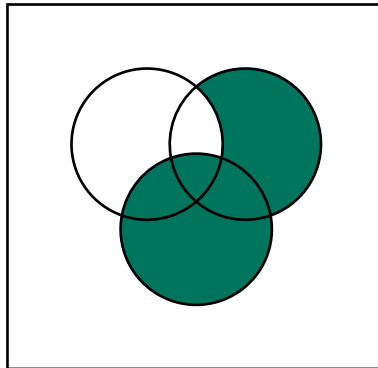You **don't** have to provide any **explanation**.

Bonus Problem [10 %]

There is a set operation called **symmetric difference** of A and B and written as $A \triangle B$. It can be defined like this:

$$A \triangle B = (A \setminus B) \cup (B \setminus A).$$

Define this set using **logical operators only**. That is, find a proposition $p(x)$ (consisting of the propositions $x \in A$ and $x \in B$) such that $A \triangle B = \{x \mid p(x)\}$.

Venn diagrams

a) Given the Venn diagram below, determine the set which it represents. You **don't**   [20 %]
   have to provide an **explanation**.

b) Draw a Venn diagram for the following expression: [20 %]

$$(A \cap B) \cup (A \setminus C).$$

You **don't** have to explain anything.

Bonus Problem   [10 %]

Prove (using Venn diagrams, logic or anything else) that

$$(A \cap C) \setminus (A \cap B) \subseteq C \setminus B$$

for any three sets A, B and C.