Logic & Set Theory

3.AB PreIB Maths – Exam A

Unless specified otherwise, you are to **always** (at least briefly) explain your reasoning. Even in closed questions.

Logic – propositions and conjunctions.

a) For each **truth value** (i.e. *true* or *false*) of p write down the **truth value** of the proposition [15 %]

$$p \vee \neg p.$$

You **don't** have to show your method.

b) Decide whether the proposition [10 %]

$$(p \Rightarrow q) \vee \neg(p \Rightarrow q)$$

is **always true** regardless of the truth values of p and q. **Explain**.

Basic set operations.

a) Given sets $A = \{$😎, 🍩, 😈, 🪦$\}$, $B = \{$🍩, 🪦, 🥌$\}$ and $C = \emptyset$, determine the set [15 %]

$$(A \cup B) \cap C.$$

Explain your method.

b) Decide whether [10 %]

$$(A \cup B) \cap C = A \cup (B \cap C)$$

for any sets A, B, C. **Explain**.
Hint: Use Venn diagrams.

Cartesian product and relations.

a) You are given [15 %]

$$A = \{1, 2\},\ B = \{a, b, c\} \text{ and } R = \{(2, a), (2, b)\},$$

where R is a relation from A to B. Provide at least two other relations from A to B that are different from the relation R. You **don't** have to **explain anything**.

b) How many relations are there from A to B if [10 %]

$$A = \{5\} \text{ and } B = \{\text{ě}, \text{š}, \text{č}, \text{ř}, \text{ž}\}?$$

Hint: It is **not** necessary to write all of them. A simple argument suffices.

Equivalence.

a) For each of the following relations decide if they are an equivalence on the set $A = \{a, b, c\}$ or not. You **don't** need to **explain anything**. [15 %]

☐ $R = \{(a,a), (b,b), (c,c)\}$

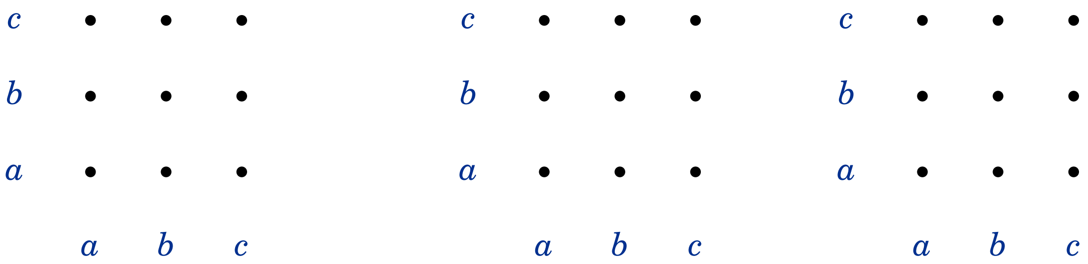
☐ $R = \{(a,b), (b,a), (a,a), (b,b), (c,c)\}$

☐ $R = \{(1,2), (2,3), (1,3)\}$

☐ $R = A \times A$

☐ $R = \{(a,a), (b,b), (c,c), (a,b), (b,c), (c,b), (b,a)\}$

You may use the empty diagrams below to draw the relations from above.



b) Recall that the relation of *equivalence* is given by three conditions: [10 %]

- **reflexivity**: every element is equivalent to itself;
- **symmetry**: if a is equivalent to b, then b is equivalent to a;
- **transitivity**: if a is eq. to b and b is eq. to c, then a is eq. to c.

To every point in the visualization of the equivalences from part a) assign one defining condition of equivalence that forces its presence in the equivalence.

For example: *'This specific pair is present because otherwise the symmetry property would not be satisfied'*.

Hint: Try assigning only the reflexivity and symmetry conditions. The geometrical representation of transitivity is harder to see.