Math Exam – PreIB 3.AB 2 & 3
Quadratic Functions & Equations

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DON'T FORGET TO EXPLAIN EVERYTHING EVEN IF YOU THINK IT'S OBVIOUS!

Some of the numbers arising in the solutions are quite large. Use a calculator (on your phone/tablet) if you wish.

Meet Don Corleone, a crime boss. Don Corleone runs a fake car rental service to cover a tiny part of his profits from rigged slot machines and usury. Don Corleone's personal accountant has stolen a small set of data assembled over a three weeks period from an actual car rental company. He knows the average price per car per day of rental and the profit made each week.

You, being the notorious underground mathematician, have been hired to perform various calculations on this data. You'd better do your job well!

1. Don Corleone's accountant has presented you with the following table.

	Average price per car per day	Profit
Week 1	$75	$13200
Week 2	$65	$11220
Week 3	$95	$15360

   Since you have only three pieces of information, model Don Corleone's fake profits as a quadratic function P. That is, find a quadratic function labelled P whose inputs are the average prices and outputs are the profits from the table above.

2. Don Corleone wishes to cover as large an amount of his profits as possible by the fake car rental company. Yet, he needs to make the forged accounts believable. **Find the average price** per rental of one car per day **which would generate the most profit** based on the provided data.

3. A court decision has forced Don Corleone to supply additional data about his fake company's operations; in particular, the accounts of the average maintenance cost per one car per week.

   Denote the average rental price variable by p and the number of cars rented on average per week by c. The expected situation is that **the higher the rental price, the fewer cars you rent**. That is, you can approximate c **as a linear function in** p. You know that the total **profit is then a quadratic function in** p. Concretely,

   $$\text{profit} = \text{cars rented} \cdot \text{price per car} - \text{cars rented} \cdot \text{maintenance cost}$$
   $$= c(p) \cdot p - c(p) \cdot m,$$

   where m is the unknown average maintenance cost per car per week. Can you determine the maintenance cost without knowing the number of cars rented? If yes, how? If not, why?

4. In the end, Don Corleone's accountant has managed to also steal the data containing the number of cars rented each week. It is given in the following table.

	Cars rented
Week 1	300
Week 2	330
Week 3	240

   Using this table, **interpret the** number of cars rented each week **as a linear function in price** p, that is, determine $c(p)$.

5. Finally, now that you know $c(p)$, **determine the maintenance cost** per car per week, m.
   Hint: Can you perhaps factor out $c(p)$ from the formula

   $$P(p) = c(p) \cdot p -$$