

# Number Sets & GCD

## 3.AB PreIB Maths – Exam C

Unless specified otherwise, you are to **always** (at least briefly) explain your reasoning. Even in closed questions.

### Natural Numbers

a) The **multiplication** of natural numbers is defined by these two rules:

[20 %]

$$1) \quad n \cdot 1 = 1$$

$$2) \quad n \cdot \text{succ}(m) = n \cdot m + n$$

where  $n$  and  $m$  are any natural numbers.

Using **only these two rules** and your knowledge about **addition** evaluate the following expressions.

- $3 \cdot 3$
- $2 \cdot 5$

b) Assuming  $x \cdot y = y \cdot x$ , show that  $x \cdot \text{succ}(y) = \text{succ}(y) \cdot x$ . Use **only** the rules that define multiplication from part a).

[10 %]

## Integers & Rationals

- a) Connect all pairs belonging to the **same equivalence class** and write down the value of the **represented integer** for each class. [20 %]

(1, 1)

(2, 4)

(2, 2)

(6, 8)

(10, 10)

(121, 120)

(0, 2)

(7, 6)

(8, 7)

- b) You are given two pairs of natural numbers:  $(a', b')$  and  $(a, b)$  from the **same equivalence class** (they represent the same integer value). Show that their respective **products** with some pair  $(c, d)$  also belong to the **same equivalence class**. [10 %]

In other words, show that if  $[(a, b)]_E = [(a', b')]_E$ , then

$$[(a, b)]_E \cdot [(c, d)]_E = [(a', b')]_E \cdot [(c, d)]_E$$

**Hint:** The pairs  $(a, b)$  and  $(a', b')$  represent the same integer if (informally) ' $a - b = a' - b'$ '.

**Divisibility & GCD**

- a) Find all numbers smaller than 50 that have **exactly 4 divisors**. Do **not** proceed by trial and error (this method would result in **0 %**). [20 %]

- b) Compute **gcd**(7205, 7535). Write down performed calculations **in full detail**. [20 %]