Statistics

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The collective information about a system's past state is called **data**.
It assigns **probabilities** to each possible future state of system based on data.
It also assigns probabilities to the **possibility of wrong prediction**.

Example – Biased Coin?

We throw a coin 10 times with the following outcome:

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- Is this coin is **biased towards** 'heads'/'tails' with *allowed probability of error* α?
  - **No**, for $\alpha = 0.05$.
  - **Yes**, for $\alpha = 0.2$.

Contents

Data

What Do We Mean By Data?

Data

Sets (called *inputs* and *outputs*) describing the studied system. There is typically only one set of inputs and possibly multiple sets of outputs.

Example – Junctions

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An **output** is the number of traffic accidents in the given day on the given junction.

Example – First Baby

We study the age that women bear children for the first time across Europe.

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An **input** would be a name of a European country.

An **output** is the average age of a first-time mother in that country.

1

Types of Data

Discrete Data vs. Continuous Data

Discrete Data

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- There are only *finitely many* countries on a continent.

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More often than not, the inputs in a continuous data are **moments in time** or **coordinates in space**.

Continuous Data – Examples

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- Another example is the density of air per cubic meter.
  - Input: Coordinates of a unit cube in space.
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  - The data is a function $f : \mathbb{R}^3 \to [0, \infty)$.

Visualizing Discrete Data

Tables

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Input	1	2	3	4	5	6	7	8	9	10
Output	180	169	191	177	175	181	171	153	180	183

Pie Chart

Only usable if your outputs **total a predetermined number**, typically *percentages*.

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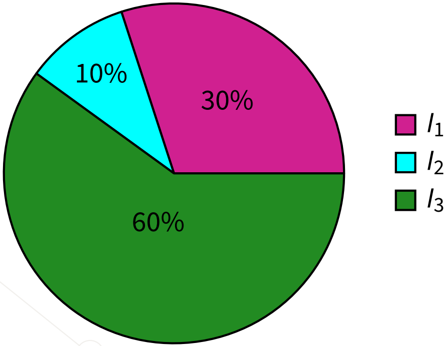
Pie chart of this data looks like this



Pie Chart – Examples

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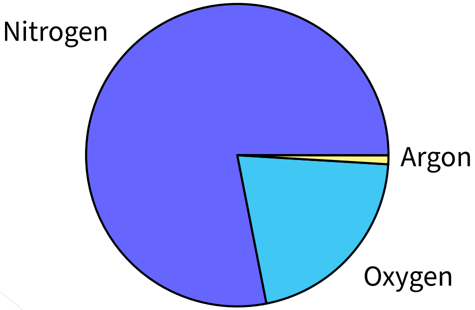
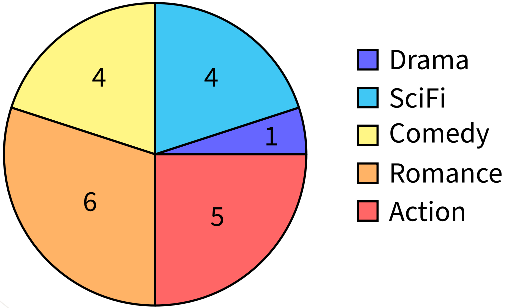
For instance, here is a pie chart of the composition of *air*.



Pie Chart – Examples

Favourite type of movie as determined by a survey.



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Also very good for comparing more outputs for the same inputs.

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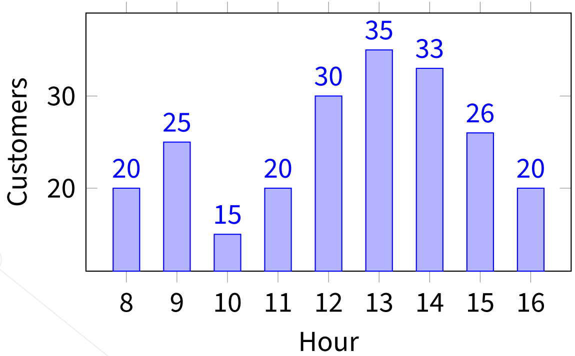

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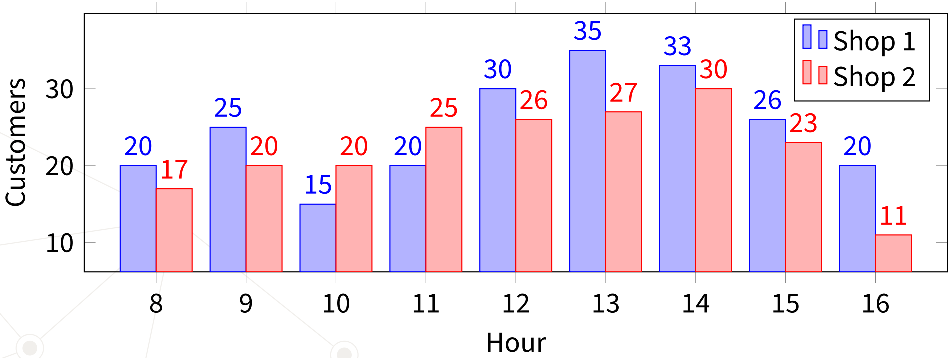

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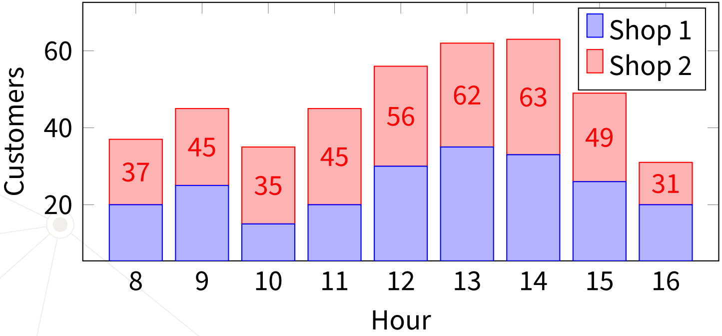

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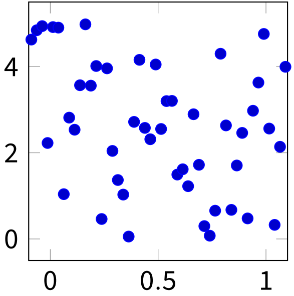

Here, the *x*-axis represents time (0 to 1s) and the *y*-axis represents one coordinate of the molecule (say the box is a 5x5x5 cube).

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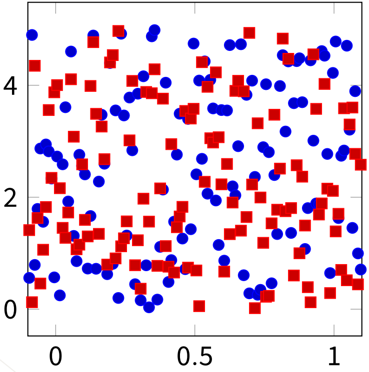

Mean – Median – Deviation – Correlation

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- the ‘middle’ value of the outputs regardless of proportion (the **median**),
- the expected/observed measure of difference of observed values from the mean (the **deviation**),
- dependence on any other data (the **correlation**).

1

The Mean

Types of Mean

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Types of Mean – Arithmetic Mean

Arithmetic Mean

The **arithmetic mean** is the sum of outputs divided by their number. If $x_1, \ldots, x_n$ are the outputs, their arithmetic mean (often denoted $\bar{x}$) is

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While comparing the heights of two people, we care about the **absolute** difference in centimetres.

For example, if this is our data

Input	1	2	3	4	5
Output	165	161	164	172	168,

we conclude that the expected height of a randomly chosen 15-year-old British male is

$$\frac{165 + 161 + 164 + 172 + 168}{5} = 166.$$

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The **geometric mean** is the n-th root of the product of n outputs. That is, if $x_1, \dots, x_n$ are the outputs, their geometric mean is

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Input	India	China	Japan	South Korea	Mongolia	Taiwan
Output	1.328	1.118	0.991	1.100	1.366	1.078

then the expected increase in population in a randomly chosen Asian country is

$$\sqrt[6]{(1.328 \cdot 1.118 \cdot 0.991 \cdot 1.100 \cdot 1.366 \cdot 1.078)} = 1.156.$$

Types of Mean – Harmonic Mean

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Harmonic Mean – Example

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Output	65 km/h	52 km/h	71 km/h	60 km/h	62 km/h	53 km/h,

then the average speed of the train over the whole track is

$$\frac{6}{\frac{1}{65}+\frac{1}{52}+\frac{1}{71}+\frac{1}{60}+\frac{1}{62}+\frac{1}{53}} = 59.78 \text{ km/h}.$$

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Actually, here the arithmetic mean is 60.5 km/h which is not just an *inadequate* estimate, it's simply **the wrong answer!**

If you summed up all the distances between stations and divided them by the total time, you would get the **harmonic mean!**

2

The Median

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Median

The **median** is the value which lies exactly in the middle of a dataset. It is essentially the value separating the lower and upper half of outputs. If $x_1, \ldots, x_n$ are the outputs **ordered from least to greatest**, the median is

$$\mathrm{median}(x) := \begin{cases} x_{(n+1)/2} & \text{if n is odd,} \\ \frac{x_{n/2}+x_{n/2+1}}{2} & \text{if n is even.} \end{cases}$$

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For example, imagine we're trying to determine the distance from our measuring station to the hypocentre of an earthquake.

We can detect where the quake is strongest, giving us this data:

Input	1	2	3	4	5	6
Output	1 km	2 km	2 km	2 km	3 km	14 km

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The median of this dataset is 2 km which is a much better estimate of a 'centre' than for example the arithmetic mean, being equal to 4, is.

Also, the mean and the median cannot be 'too far' apart and the median requires at most two values to calculate, making it a very resource efficient approximation of the mean.

3

The Deviation

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- the **standard deviation** (a measure of 'dispersion'),
- the **average absolute deviation** (a measure of actual 'difference').

A **very important distinction** is that the *standard deviation* concerns **future** measurements while the *average absolute deviation* concerns **past** measurements.

Types of Deviation – Standard Deviation

Standard Deviation

The **standard deviation** measures the dispersion of a set of values. Basically, it measures how likely the data is to concentrate around the mean. If $x_1, \ldots, x_n$ are the outputs and $\bar{x}$ is their **arithmetic** mean, then their standard deviation is

$$\sigma := \sqrt{\frac{1}{n}((x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \ldots + (x_n - \bar{x})^2)}.$$

Standard Deviation – Example

Let us repeat the height experiment. We measured the heights of 5 15-year-old British males to try to determine the national average. This is the data:

Input	1	2	3	4	5
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We computed the arithmetic mean to be 166. This means that the standard deviation of this data is

$$\sigma = \sqrt{\frac{1}{5}((165-166)^2 + (161-166)^2 + (164-166)^2 + (172-166)^2 + (168-166)^2)}$$
$$= 3.742,$$

meaning we can expect most new values to concentrate 3.742 cm around 166 cm.

Average Absolute Deviation

The **average absolute deviation** is the average of the absolute deviations from a chosen central point (typically the mean). If $x_1, \ldots, x_n$ are the outputs and $\bar{x}$ is the chosen central point, then the average absolute deviation of this dataset is

$$\frac{|x_1 - \bar{x}| + |x_2 - \bar{x}| + \ldots + |x_n - \bar{x}|}{n}.$$

AVERAGE ABSOLUTE DEVIATION – EXAMPLE

If we return to the height experiment yet again, we can calculate that the average absolute deviation of the data (with the central point being the arithmetic mean)

Input	1	2	3	4	5
Output	165	161	164	172	168

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meaning that the measured heights differ on average by 3.2 cm from the calculated arithmetic mean.

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- zero correlation means that the two series of outputs are **unrelated**;
- positive correlation means that the two series of outputs **influence** each other.

Computing Correlation

Correlation Formula

If $x_1, \ldots, x_n$ and $y_1, \ldots, y_n$ are two series of outputs for the same inputs with means $\bar{x}$ and $\bar{y}$, their correlation is

$$\mathrm{cor}(x, y) := \frac{(x_1 - \bar{x})(x_2 - \bar{x}) \cdots (x_n - \bar{x})(y_1 - \bar{y})(y_2 - \bar{y}) \cdots (y_n - \bar{y})}{\sqrt{(x_1 - \bar{x})^2 (x_2 - \bar{x})^2 \cdots (x_n - \bar{x})^2 (y_1 - \bar{y})^2 (y_2 - \bar{y})^2 \cdots (y_n - \bar{y})^2}}.$$

Interpreting Correlation – Table

A crude interpretation of correlation is given in the following table:

Coefficient	Strength	Type
-0.7 to -1	Very strong	Negative
-0.5 to -0.7	Strong	Negative
-0.3 to -0.5	Moderate	Negative
0 to -0.3	Weak	Negative
0 to 0.3	Weak	Positive
0.3 to 0.5	Moderate	Positive
0.5 to 0.7	Strong	Positive
0.7 to 1	Very strong	Positive

Interpreting Correlation – Chart

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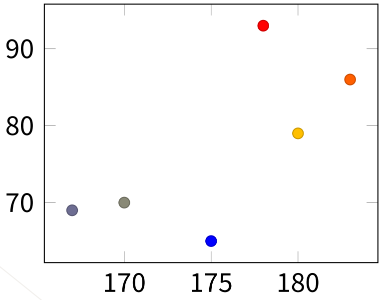

Frequency Distribution

What Is Frequency Distribution?

Frequency Distribution

A **frequency** of a value is the number of times it occurs in a dataset. A **frequency distribution** is the number of times each variable occurs in a dataset.

TYPES OF FREQUENCY DISTRIBUTIONS

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- **Relative frequency distribution**: the proportion of each value or class interval of a variable. Useful for any type of data **if we care about comparing frequencies** rather than amounts.
- **Cumulative frequency distribution**: the sum of frequencies less than or equal to each value or class interval of a variable. Useful when we want to understand how often observations fall below certain values.

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  - **Continuous data** represents measurements of uncountable values like density, volume or time.

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  - **Ordinal data** represents groups that are ranked – like finishing place in a race.

Ungrouped Frequency Data – Table

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    - For **ordinal** variables, the values should be ordered from smallest to largest.
    - For **nominal** variables, the rows can be ordered arbitrarily.
2. Count the **frequencies**.

Ungrouped Frequency Data – Example 1

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Ungrouped Frequency Data – Example 1

A gardener sets up a bird feeder in his backyard. He wishes to know which type of bird species visit the feeder the most. His observations are in the following table:

Species	Frequency
Chickadee	3
Dove	1
Finch	4
Grackle	2
Sparrow	4
Starling	2

Ungrouped Frequency Data – Example 2

We observe how many times a specific type of tram (based on age) stops at a chosen station each day.

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This experiment may yield a table like this:

Type	Frequency
1990	6
1996	11
2005	3
2017	5

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     It is typically beneficial to round this value to an integer.
   - Calculate the **class intervals**. Each interval is of the form [lower limit, lower limit $+$ width). Simply divide the outputs into these intervals.

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$$52, 34, 32, 29, 63, 40, 46, 54, 36, 36, 24, 19, 45, 20, 28, 29, 38, 33, 49, 37.$$

We calculate the range as highest $-$ lowest $= 63 - 19 = 44$.
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and round it up to 10.

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and round it up to 10.
Therefore, we have the following intervals

$$[19, 29), [29, 39), [39, 49), [49, 59), [59, 69).$$

Grouped Frequency Data – Example

Counting the numbers of outputs falling into each of those intervals gives the table:

Age	Frequency
19 – 28	4
29 – 38	9
39 – 48	3
49 – 58	3
59 – 68	1

Relative Frequency Data – Table

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2. To each output add another column to represent **relative frequencies**.

Relative Frequency Data – Example

In our gardener example, the relative frequency table would look like this:

Species	Frequency	Relative Frequency
Chickadee	3	$\frac{3}{3+1+4+2+4+2} = 0.19$
Dove	1	0.06
Finch	4	0.25
Grackle	2	0.13
Sparrow	4	0.25
Starling	2	0.13

Cumulative Frequency Data – Table

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1. Create an ungrouped or grouped frequency table for **an ordinal or quantitative variable**. Cumulative frequencies make no sense for nominal variables because they're not ordered.
2. Add another column for each output with **cumulative frequency**. The cumulative frequency is the number of observations less than or equal to a certain value or class interval.

Cumulative Frequency Data – Example

Going back to our example of a sociological survey. The cumulative frequency table of the age of survey participants would look like this:

Age	Frequency	Cumulative Frequency
19 – 28	4	4
29 – 38	9	9 + 4 = 13
39 – 48	3	9 + 4 + 3 = 16
49 – 58	3	19
59 – 68	1	20