Logic & Set Theory

3.AB PreIB Maths – Exam B

Unless specified otherwise, you are to **always** (at least briefly) explain your reasoning. Even in closed questions.

Logic – propositions and conjunctions.

a) Complete the truth table below. [15 %]

p	q	$p \wedge \neg q$
1	1	
1	0	
0	1	0
0	0	0

In other words: evaluate the proposition $p \wedge \neg q$ for the truth values of p and q corresponding to the first two lines of the truth table. You **don't** have to **explain anything**.

b) Complete the blank square in proposition [10 %]

$$p \; \square \; \neg q$$

with some logical conjunction ($\wedge, \vee, \Rightarrow, \Leftrightarrow$) to make it *equivalent* to $\neg(p \Rightarrow q)$. Two statements are *equivalent* if their truth tables are the same.

For convenience the truth table of implication is shown below.

p	q	$p \Rightarrow q$
1	1	1
1	0	0
0	1	1
0	0	1

Explain your choice.

Basic set operations.

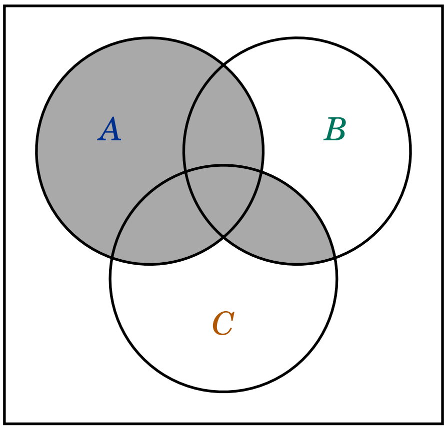
a) Given sets $A = \{c, c, c, b, b, a\}$ and $B = \{a, b, c\}$, determine the statements [15 %]

$$A \subseteq B \text{ and } B \subseteq A.$$

Explain your method.

Bonus (+10%): if both the statements are true there is something to be concluded about A and B. **Explain** what it is.

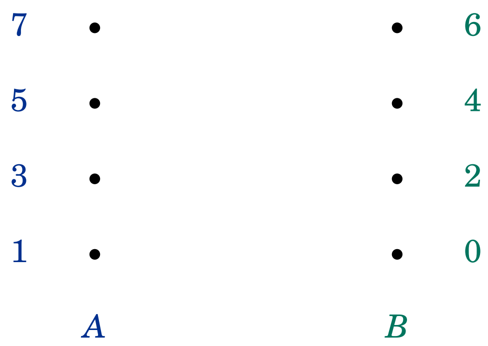
b) Write an expression (using set operations) for the shaded are on the diagram below. [10 %]



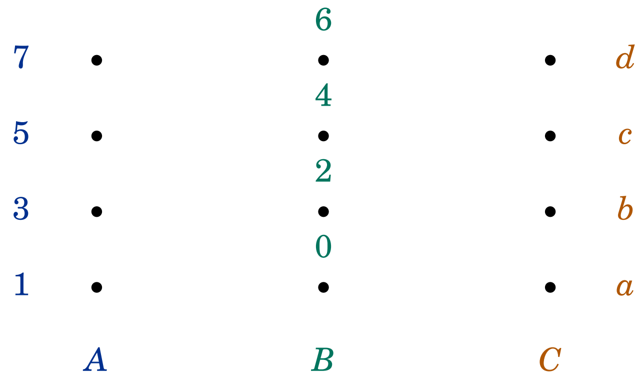
Cartesian product and relations.

a) On the diagram below draw the relation R from A to B for [15 %]

$$A = \{1, 3, 5, 7\}, B = \{0, 2, 4, 6\} \text{ and } R = \{(1, 2), (3, 6), (5, 0)\}.$$



b) Draw again the relation R from the previous exercise together with the relation $S = \{(0, a), (2, c), (4, d)\}$ between sets B and $C = \{a, b, c, d, \}$. [10 %]



Now is your task to compose the relations R and S into one relation T that goes from A to C. This means that T firstly applies R to get from A to B. Then on all of the results of R (end of an every arrow from a)) applies S which gets it from B to C. At the end T forgets the element from B and ends up only with the beginning and the ending of the journey. **Write down** T.

Equivalence.

a) One of the examples of a equivalence is **'what flavor of ice cream'** each person likes the most. Verify that it is truly equivalence. In other words: it has to satisfy [15 %]

- **reflexivity**: every element is equivalent to itself;
- **symmetry**: if a is equivalent to b, then b is equivalent to a;
- **transitivity**: if a is eq. to b and b is eq. to c, then a is eq. to c.

b) Come up with at **least three** other equivalences on the set of all people. Try to estimate the number of equivalence classes they create. For the maximum credit there should be one that creates **over 100** of partitions and also one that creates fewer than two. [10 %]

You **can not** use the equivalence from part a).