

# Logic & Set Theory

## 2.AB PreIB Maths – Exam A

Unless specified otherwise, you are to **always** (at least briefly) explain your reasoning. Even in closed questions.

### Logic – propositions and operators

Given the proposition

[25 %]

$$(q \wedge \neg p) \Leftrightarrow p,$$

determine when it is true (for which truth values of  $p$  and  $q$ ) and when it is false.

**Explain your answer.**

**Hint:** You may use a truth table if you wish.

### Bonus Problem

[10 %]

Every logical proposition can be expressed using only the operators  $\neg$ ,  $\wedge$  and  $\vee$ . Do this for the implication, that is, find a proposition which uses only the operators  $\neg$ ,  $\wedge$  and  $\vee$  and is equivalent to  $p \Rightarrow q$ . **Explain your answer.**

**Basic set operations**

Given sets  $A = \{b, c, e\}$ ,  $B = \{a, c, d\}$  and  $C = \{a, b, c, d\}$ , determine the sets

[35 %]

$$(A \setminus B) \cup C \quad \text{and} \quad A \cap (B \cap C).$$

You **don't** have to provide any **explanation**.

**Bonus Problem**

[10 %]

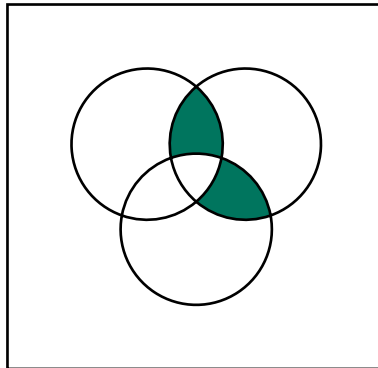
For a set  $A$ , there exists a set that contains all the subsets of  $A$  and it's denoted  $2^A$ . For example, if  $A = \{a, b\}$ , then

$$2^A = \{\{\}, \{a\}, \{b\}, \{a, b\}\}.$$

Try to explain **why** the set of all subsets is denoted  $2^A$ . How many elements does  $2^A$  have if  $A$  has  $n$  elements? **Explain your reasoning.**

**Venn diagrams**

- a) Given the Venn diagram below, determine the set which it represents. You **don't** have to provide an **explanation**. [20 %]



- b) Draw a Venn diagram for the following expression: [20 %]

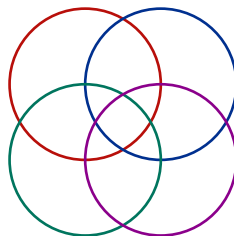
$$(A \cap C) \setminus B.$$

You **don't** have to explain anything.

**Bonus Problem**

[10 %]

Drawing a Venn diagram with four sets is quite hard and cannot be done using circles only. Indeed, consider the following picture for sets  $A$ ,  $B$ ,  $C$  and  $D$ :



Would you be able to find a combination of set operations on  $A$ ,  $B$ ,  $C$  and  $D$  which doesn't have a corresponding region in the diagram above? You **don't** have to provide an **explanation**.