Probability

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Tree Diagrams

Tree Diagrams

Tree diagrams are essentially a schematic representation of all the possible outcomes of a probabilistic experiment.
They provide a good way to visualize a problem and compute probabilities.
In some sense, they allow **dependent events** to **become independent** and compute the probability of the successive occurrence of such events by simple multiplication.

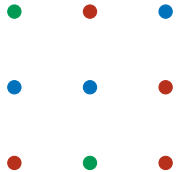
Drawing Tree Diagrams

We draw tree diagrams by

- representing events/outcomes by ‘points’ or ‘dots’;
- connecting successive events by lines;
- drawing the events from top to bottom in chronological order.

Tree Diagram – Example

Imagine again a set of 9 balls of three different colors.

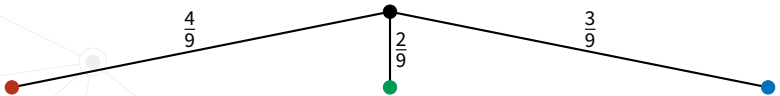


We computed the probability that the **second** ball we pick at random is red assuming the first ball was also red.
We'll perform the same computation using a **tree diagram**.

Tree Diagram – Example

We start by drawing the 'root' of the three. Just a simple dot which will then branch into different events.

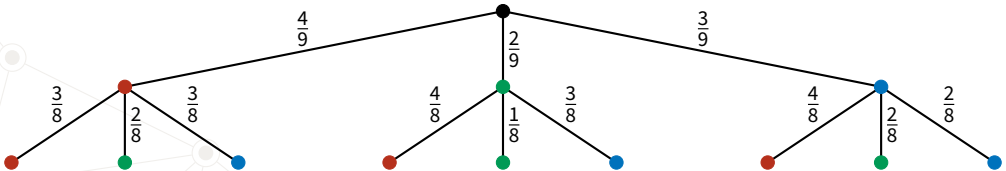
The first event is 'a randomly chosen ball is red'. There are three possible outcomes of a random choice – red, green or blue, each of different probability.
We represent this in the tree diagram like this:



Tree Diagram – Example

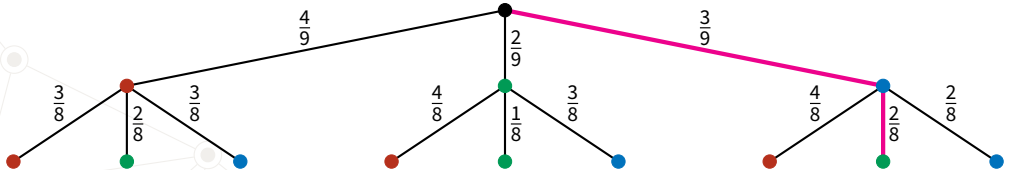
The next ball can be again red, green or blue. But, this event follows after a previous random choice of a ball, so the number of balls remaining is 8 and their colours depend on the previous event.

The full tree diagram of this probabilistic experiment would look like this:



Tree Diagram – Example

The great advantage of the tree diagram is that the probability of any events is just the product of the probabilities of the events that come before and this one.
For example, suppose we want to calculate the probability that a green ball is picked after a blue one has been.
This event corresponds to **this path** in the tree:



Hence, we know its probability to be $\frac{3}{9} \cdot \frac{2}{8} = \frac{1}{12}$.

Tree Diagram – Problem #1

A football team wins its matches with a probability of 0.7.

Using a tree diagram, find the probability that they win at least 1 of their next three matches.

Tree Diagram – Problem #2

Anna and Rob take their driving tests on the same day. The probability of Anna passing her driving test is 0.7. The probability of both Anna and Rob passing is 0.35.

1. Work out the probability of Rob passing his driving test.
2. Work out the probability of both Anna and Rob failing their driving tests.

Tree Diagram – Problem #3

You roll a dice three times. What's the probability that the sum of all the rolled numbers is 12 assuming

1. the first rolled number is 3.
2. the second rolled number is 5.

Use tree diagrams to solve the problem.