Number Sets & GCD

3.AB PreIB Maths – Mock Exam

Unless specified otherwise, you are to **always** (at least briefly) explain your reasoning. Even in closed questions.

Natural Numbers

We define the *ordering* $\leq$ on $\mathbb{N}$ by $n \leq m$ if $n \subseteq m$. Remember that natural numbers are just sets so we say that **a natural number n is smaller than m if it is a subset**. [30 %]

a) Show that $\leq$ is truly an *ordering* on $\mathbb{N}$. This means that
   - it is **reflexive**: $n \leq n$ for every natural number $n \in \mathbb{N}$;
   - it is **anti-symmetric**: if $n \leq m$ and also $m \leq n$, then necessarily $n = m$ for any two numbers $n, m \in \mathbb{N}$.

b) Show that if $n \leq m$, then also $\mathrm{succ}(n) \leq \mathrm{succ}(m)$.
   Hint: use the formula for successor.

c) Remember that **addition** on $\mathbb{N}$ is defined using two rules:
   (1) $n + 1 = \mathrm{succ}(n)$
   (2) $n + \mathrm{succ}(m) = \mathrm{succ}(n + m)$

   Part b) can be rewritten using rule (1) in the **definition of addition** as $n + 1 \leq m + 1$. Use this fact together with rule (2) of addition to prove that

   $$\text{if } n \leq m, \text{ then } n + 2 \leq m + 2.$$

   Hint: Remember that $2 = \mathrm{succ}(1)$ and make use of the result in part b).

Integers & Rationals

a) Check all pairs of natural numbers **that represent the integer** -3. [15 %]

- ☐ $(1, 3)$
- ☐ $(-3, 0)$
- ☐ $(2, 5)$
- ☐ $(10, 7)$
- ☐ $(5, 8)$

You **don't have to explain** anything.

b) Assume that the pair (a, b) represents a **positive** integer y, which is equal to $2 \cdot x$. Find a pair that represents x. Remember: the elements of the pair are **natural numbers**, you can't subtract or divide them. **Explain**. [15 %]
Hint: The fact that (a, b) represents $2 \cdot x$ can be informally expressed by the equation '$a - b = 2 \cdot x$'. Use this equation to express x as a difference of two natural numbers.

Divisibility & GCD

a) Find a natural number **smaller than 40** with the **highest number of divisors**. Explain. Do **not** proceed by trial and error (doing so will not yield any points). [20 %]
Hint: What does the number of primes in a prime decomposition tell you about the number of divisors?

b) Compute $\gcd(1968, 928)$. Write down performed calculations **in full detail**. [20 %]