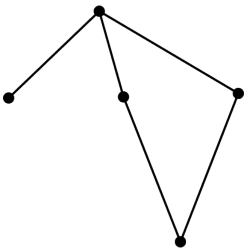
Convex Polygons and Their Symmetries

3.AB PreIB Maths – Exam B

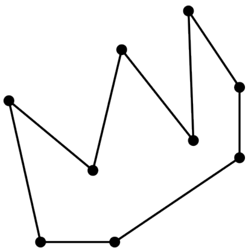
Unless specified otherwise, you are to **always** (at least briefly) explain your reasoning. Even in closed questions.

1. Definition of a polygon.

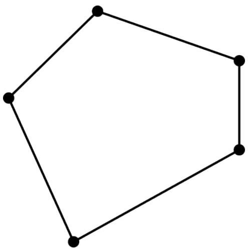
   (a) Which of these shapes *are not* polygons? **Explain**. [10 %]



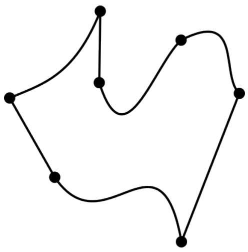
Option A.



Option B.

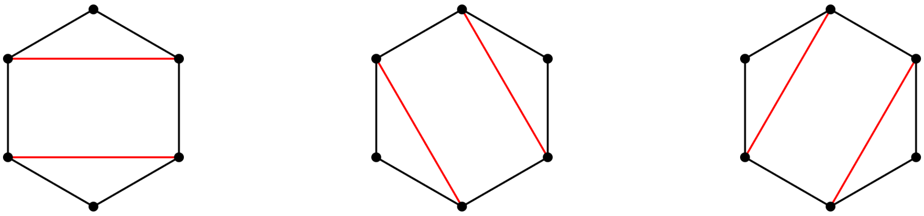


Option C.



Option D.

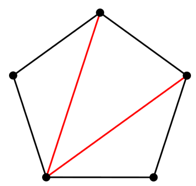
   (b) Try to count the number of pairs of *parallel* diagonals in a **regular** polygon on n vertices. For example, the hexagon has three such pairs: [10 %]



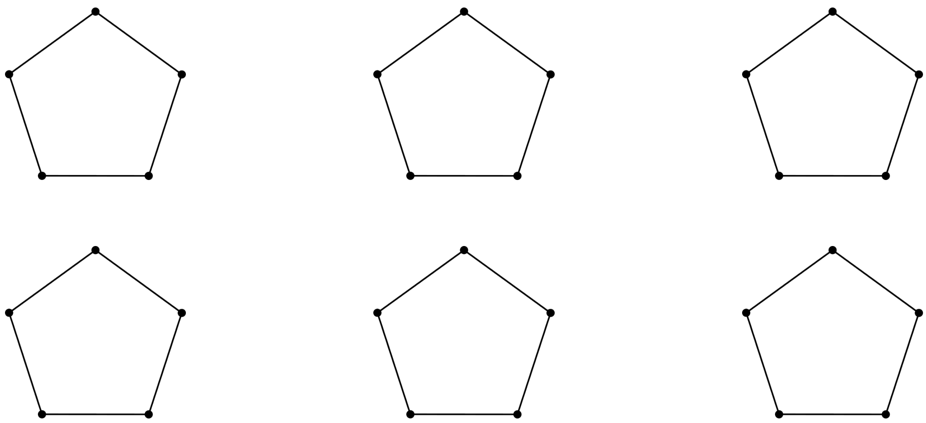
Hint: Distinguish polygons with even number of vertices from those with odd.

2. Triangulations of convex polygons.

   (a) Draw all triangulations of the pentagon *that can be reached in one flip* from the one shown below. Use the provided shapes (not all of them necessarily). **No explanation required**. [10 %]

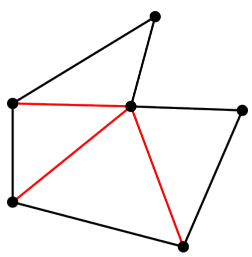


The initial triangulation.



Shapes to draw diagonals into.

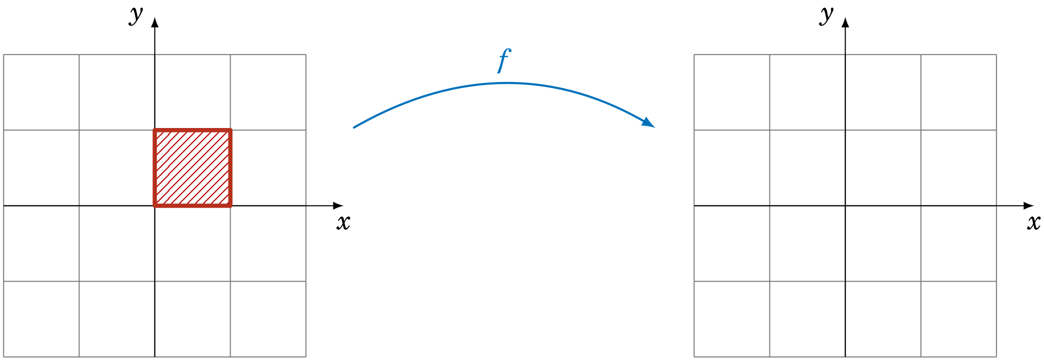
   (b) Can every *non-convex* polygon also be triangulated, that is, divided into triangles by non-intersecting diagonals? Try to think of an argument or provide a counterexample. [10 %]



A triangulation of a non-convex hexagon.

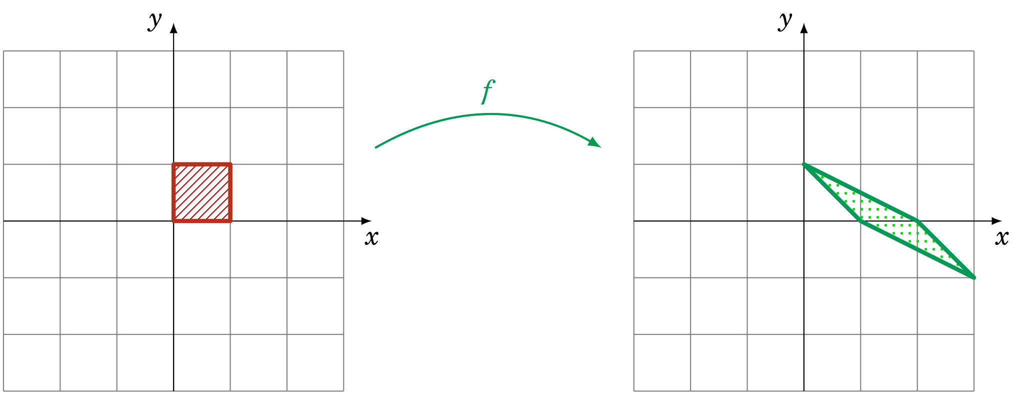
3. Plane transformations.

   (a) Find out the *image* (the resulting shape when transformed) of a square (depicted below) under the plane transformation $f(x, y) = (x, y - x)$. **Provide a short explanation**. [10 %]



The initial square. Draw the resulting shape here.

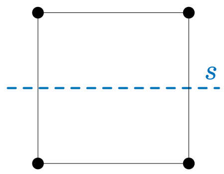
   (b) Below, you see a unit square transformed by the plane transformation f. The function f sends the x-coordinate of every point to $1 - y + 2x$. What does it send the y-coordinate to? **Explain**. [10 %]



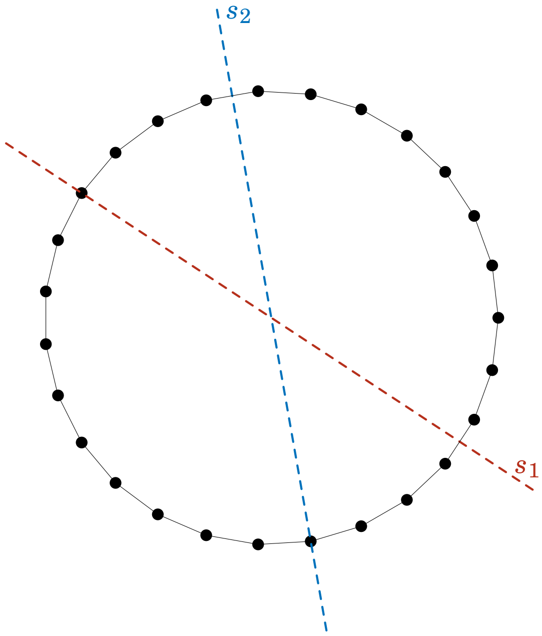
The initial square. The square transformed by f.

4. Symmetries of regular polygons.

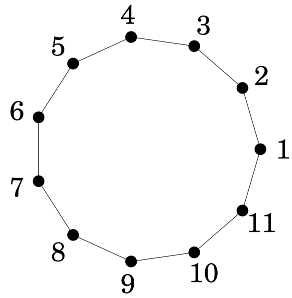
   (a) Given two symmetries of the *square* – the rotation $r = \circlearrowleft 90°$ by 90° counter-clockwise and the reflection s drawn below – determine (using any method you wish) the composition rs. **Explain**. [10 %]



   (b) Given two symmetries of the icosiheptagon (27 vertices) – the reflections s_1 and s_2 depicted below – compute (using any method you wish) the composition $s_2 s_1$. **Explain**. [10 %]



(c) Select those of the following four pairs of symmetries of the regular hendecagon (11 vertices) that *generate all* of its symmetries. **No explanation necessary**. [10 %]



Picture of the hendecagon for reference.

- ◯ the reflection s over the line passing through vertex 1 and the midpoint of 67 and the rotation $r = \circlearrowleft 3 \cdot 360°/11$,
- ◯ the rotation $r_1 = \circlearrowleft 5 \cdot 360°/11$ and the rotation $r_2 = \circlearrowleft 7 \cdot 360°/11$,
- ◯ the rotation $r = \circlearrowleft 7 \cdot 360°/11$ and the reflection s over the line passing through vertex 4 and the midpoint of $9, 10$,
- ◯ the reflection s_1 over the line passing through vertex 2 and the midpoint of 78 and the reflection s_2 over the line passing through vertex 3 and the midpoint of 89.

(d) Given reflections s_1 and s_2 of the octagon (8 vertices), compose them (and *only* them) to create the reflection s_3 illustrated below. **Explain**. [10 %]