Number Sets & GCD

3.AB PreIB Maths – Real Exam

Unless specified otherwise, you are to **always** (at least briefly) explain your reasoning. Even in closed questions.

Natural Numbers

a) So far the addition and multiplication of natural numbers were defined. Now the **exponentiation** is presented in two axioms : [15 %]

$$\begin{aligned} &1) \quad a\text{^}0 = 1 \\ &2) \quad a\text{^}\,\mathrm{succ}(b) = a\text{^}b \cdot b \end{aligned}$$

Using **only those two axioms** (and all your other knowledge about multiplication) evaluate the following expressions. You can denote exponentiation in the traditional form as a^b.

- 2^5
- 5^3

b) **Generalise** your method from part a) to calculate $a\text{^}b$ for **any** $a, b \in \mathbb{N}$. [15 %]

Integers & Rationals

a) Connect the pairs that correspond to the **same equivalence classes** and write down the value of **represented integer**. [20 %]

$(2,3)$	$(3,2)$	$(5,3)$
$(8,6)$	$(9,10)$	$(122,123)$
$(2,0)$	$(5,4)$	$(7,8)$

b) You are given two elements: $[(a',b')]_E$ and $[(a,b)]_E$ from the **same equivalence class** (they represent the same integer value). Show that their respective **sum** with some element $[(c,d)]_E$ is always the **same**. In other words show that [10 %]

$$[(a,b)]_E + [(c,d)]_E = [(a',b')]_E + [(c,d)]_E$$

Hint: Two elements are equivalent under E if they have the **same difference**.

Divisibility & GCD

a) Find a number that has **exactly 3 prime divisors** or show that such a number can not exist. [20 %]

b) Compute $\gcd(410, 240)$. Write down performed calculations **in full detail**. [20 %]