Polygons & Transformations Cheatsheet
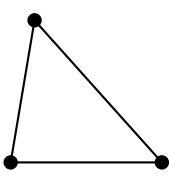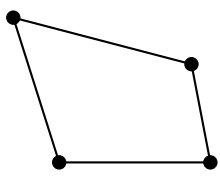
3.AB PreIB Math

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Polygons

Polygon is a **closed** 2D shape **made only of segments**. We call the endpoints of those segments, **vertices**, and the segments themselves, **edges**.
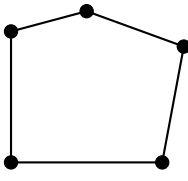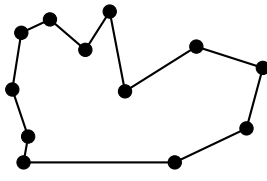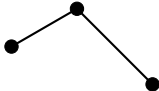
Examples

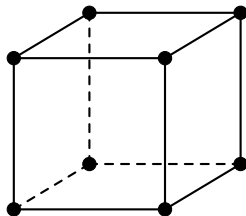

Triangle   Quadrilateral   Pentagon   Dodecagon

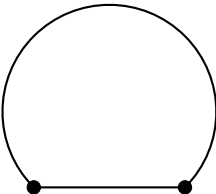Polygons with n sides are called **n-gons**.

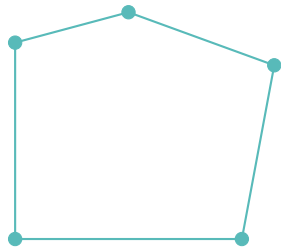Counterexamples

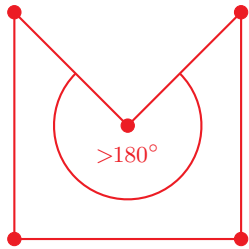Not closed   3D   Not straight

Convex Polygons

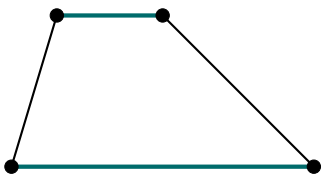A polygon is called **convex** if it has no internal angle greater than 180°.
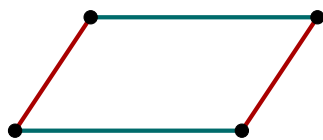

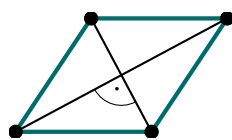Convex   NOT convex

Special types of convex polygons

Trapezoid/Trapezium
A convex quadrilateral with at least two parallel sides.

Parallelogram
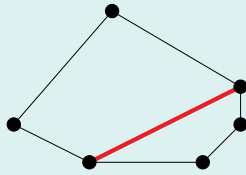A convex quadrilateral with two pairs of parallel sides.

Rhombus
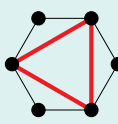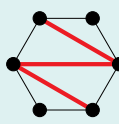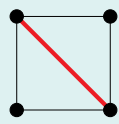An **equilateral** (all sides of the same length) parallelogram.

Diagonals & Triangulations

A **diagonal** in a **convex** polygon is a segment connecting two of its **non-adjacent** vertices.
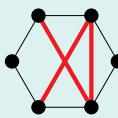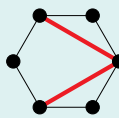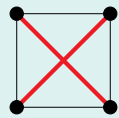

Diagonal in a convex hexagon.

A **triangulation** of a **convex** polygon is its division into triangles by **non-intersecting** diagonals.
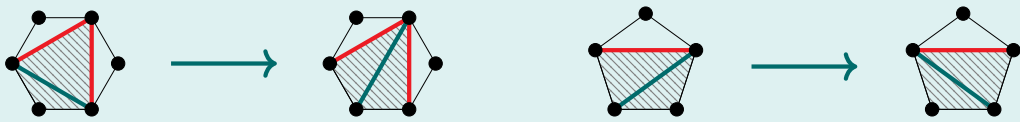

Examples of **triangulations**.

Counterexamples of **triangulations**.

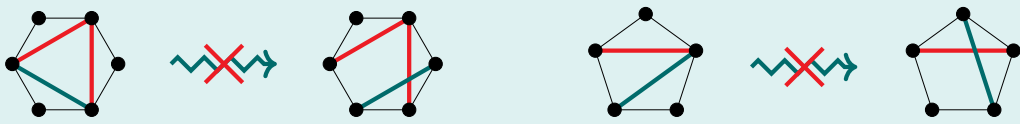The total number of different triangulations of a convex n-gon is

$$\frac{n \cdot (n+1) \cdot \ldots \cdot (2n-4)}{(n-2)!},$$

which you **of course don't have to remember**. Interestingly enough, every triangulation can be transformed into any other by a series of **flips**.
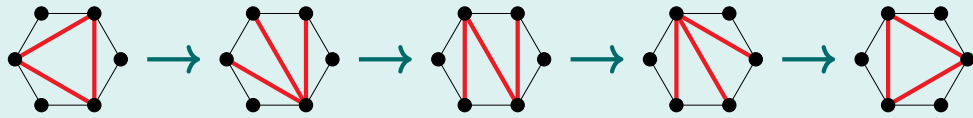
A **flip** is a swap of one diagonal for the other in a chosen quadrilateral so that the **result is again a triangulation**.

Examples of **flips**.

Counterexamples of **flips**.

Passage from one triangulation to another through a series of **flips**.

I encourage you to think about how to determine the number of flips necessary to pass from one triangulation to another. Can I have made the passage above in fewer flips?

Plane Transformations

The **plane** is basically just the set $\mathbb{R}^2$ of all **pairs of real numbers**. A pair $(x, y) \in \mathbb{R}^2$ is typically called a **point**. Then, a plane **transformation** is a **function** which maps points to points. In symbols, it's a function $\mathbb{R}^2 \to \mathbb{R}^2$.

References (opcional)

[1] Claude E. Shannon.
    A mathematical theory of communication.
    Bell System Technical Journal, 27(3):379–423, 1948.