Number Sets & GCD

3.AB PrelB Maths – Exam A

Unless specified otherwise, you are to **always** (at least briefly) explain your reasoning. Even in closed questions.

Natural Numbers

a) Thus far the addition and multiplication of natural numbers have been defined. [20 %] Now the **exponentiation** is presented in two rules:

1)
$$a^0 = 1$$

2)
$$a^{\operatorname{succ}(b)} = a^b \cdot a$$

Using **only these two rules** (and all your other knowledge about multiplication and addition), evaluate the following expressions.

- 2⁴
- 33

b) Generalise your method from part a) to calculate a^b for any $a, b \in \mathbb{N}$.

[10 %]

Integers & Rationals

a) Connect all pairs belonging to the **same equivalence class** and write down the value of the **represented integer** for each class.

$$(1,4)$$
 $(2,3)$ $(1,2)$

$$(6,9) (12,10) (120,123)$$

$$(0,1) (7,5) (7,8)$$

b) You are given two pairs of natural numbers: (a',b') and (a,b) from the **same** equivalence class (they represent the same integer value). Show that their respective sums with some pair (c,d) also belong to the same equivalence class.

In other words, show that if $[(a,b)]_E = [(a',b')]_E$, then

$$[(a,b)]_E + [(c,d)]_E = [(a',b')]_E + [(c,d)]_E$$

Hint: The pairs (a,b) and (a',b') represent the same integer if (informally) (a-b)=a'-b'.

Divisibility & GCD

a) Find all numbers smaller than 100 that have **exactly 3 divisors**. Do **not** [20 %] proceed by trial and error (this method would result in 0 %).

b) Compute gcd(358758, 318402). Write down performed calculations in full [20 %] detail.