

Logic & Set Theory

2.AB PreIB Maths – Exam A

Unless specified otherwise, you are to **always** (at least briefly) explain your reasoning. Even in closed questions.

Logic – propositions and operators

Given the proposition

[25 %]

$$(q \wedge \neg p) \Leftrightarrow p,$$

determine when it is true (for which truth values of p and q) and when it is false.

Explain your answer.

Hint: You may use a truth table if you wish.

Bonus Problem

[10 %]

Every logical proposition can be expressed using only the operators \neg , \wedge and \vee . Do this for the implication, that is, find a proposition which uses only the operators \neg , \wedge and \vee and is equivalent to $p \Rightarrow q$. **Explain your answer.**

Basic set operations

Given sets $A = \{b, c, e\}$, $B = \{a, c, d\}$ and $C = \{a, b, c, d\}$, determine the sets

[35 %]

$$(A \setminus B) \cup C \quad \text{and} \quad A \cap (B \cap C).$$

You **don't** have to provide any **explanation**.

Bonus Problem

[10 %]

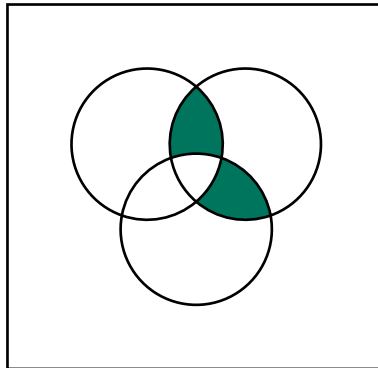
For a set A , there exists a set that contains all the subsets of A and it's denoted 2^A . For example, if $A = \{a, b\}$, then

$$2^A = \{\{\}, \{a\}, \{b\}, \{a, b\}\}.$$

Try to explain **why** the set of all subsets is denoted 2^A . How many elements does 2^A have if A has n elements? **Explain your reasoning.**

Venn diagrams

- a) Given the Venn diagram below, determine the set which it represents. You **don't** have to provide an **explanation**. [20 %]



- b) Draw a Venn diagram for the following expression: [20 %]

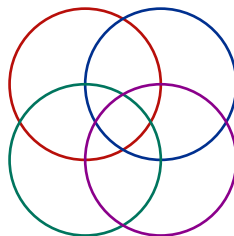
$$(A \cap C) \setminus B.$$

You **don't** have to explain anything.

Bonus Problem

[10 %]

Drawing a Venn diagram with four sets is quite hard and cannot be done using circles only. Indeed, consider the following picture for sets A , B , C and D :



Would you be able to find a combination of set operations on A , B , C and D which doesn't have a corresponding region in the diagram above? You **don't** have to provide an **explanation**.