RESEARCH STATEMENT

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ABSTRACT. Following the construction of d-representation-finite algebras in [2] and the description of the correspondence between certain types of cluster algebras and triangulations of bordered surfaces with marked points in [1], links have appeared connecting d-representation finite algebras to higher dimensional variants of said surface. One such link was discovered in [3] between higher Auslander algebras of the path algebra of linearly oriented Dynkin quiver A_n and cyclic polytopes. I wish to further study such kinds of connections, starting with the establishment of a similar type of link for path algebras of quivers of type D_n which, in the low-dimensional case, correspond to once punctured polygons; then, with a touch of expectation and naïvety, broadening it to include (special types of) cluster algebras not necessarily representation-finite.

1. Introduction

This text serves primarily as an overview of relevant concepts regarding cluster algebras, bordered surfaces with marked points, higher dimensional cluster categories and d-representation-finite algebras interwoven with ideas of possible generalizations and caveats tied to such endeavour. So far, I have only scratched the surface of this topic, hence very few original results are present.

In Section 2, I give a summary of the theory of bordered surfaces with marked points. Section 3 is dedicated to (normalized skew-symmetrizable) cluster algebras and their connection to bordered surfaces with marked points is drawn. Sections 4 and 5 define d-representation-finite algebras and higher cluster categories, respectively. Section 6 summarizes relevant results from [3], regarding a higher-dimensional kind of connection described in Section 3. Finally, Section 7 is riddled with (splinters of) steps towards generalizations of the content of Section 6.

References

- [1] Sergey Fomin, Michael Shapiro, and Dylan Thurston, Cluster algebras and triangulated surfaces. Part I: Cluster complexes, Acta Mathematica, 201:83-146, 2008.
- [2] Osamu Iyama, Cluster tilting for higher Auslander algebras, Adv. Math. 226 (2011), no. 1, 1–61.
- [3] Steffen Oppermann and Hugh Thomas, Higher dimensional cluster combinatorics and representation theory, Acta Mathematica, 201:83-146, 2008.

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