COMBINATORIAL PROPERTIES OF HIGHER CLUSTER CATEGORIES

ADAM KLEPÁČ

Summary

The main objective of the project is to understand and describe the combinatorics of higher-dimensional analogues of the path algebras (in the sense of higher representation theory) of affine Dynkin quivers and their associated higher cluster categories, possibly and hopefully aiding in an eventual broader comprehension of the combinatorial structure of n-representation-infinite algebras.

The first year will be dedicated to the study of the combinatorics of tilting modules in higher Auslander algebras of the path algebras of type D and E Dynkin quivers and the cluster tilting objects of their associated higher cluster categories. A full combinatorial description is already in place for these algebras and categories of Dynkin type A by means of triangulations of cyclic polytopes. It remains to extend this theory to also include types D and E.

In the next two years, I aim to generalize this theory further by studying path algebras of affine Dynkin quivers (of types $\tilde{A}$, $\tilde{D}$ and $\tilde{E}$) which are no longer representation-finite and thus the concept of higher Auslander algebras is not defined in this case. Due to the apparent proximity of these path algebras to those of pure Dynkin type, it seems reasonable to expect that a generalized version of a higher Auslander algebra can be constructed. For instance, there already exists a construction of n-representation-infinite algebras of type $\tilde{A}$. The search for a combinatorial description of the tilting modules of these algebras will mainly involve computing their Auslander-Reiten quivers and finding a regular criterion which can be used to check whether a given module is tilting. I intend to first find such a combinatorial model in the low-dimensional setting and see if it propagates well to higher dimensions. If it does not, it likely means the model is wrong, but it can also point to errors in my conception of the generalized higher Auslander algebras. Finally, for these algebras, I hope to find a suitable construction of a cluster category whose cluster tilting objects will share the same combinatorial model as the tilting modules of its associated algebra and, last but not least, its cluster tilting objects will in the low-dimensional case also correspond to the clusters of the connected cluster algebra. Higher cluster categories have already been defined and studied. I suspect the right construction will turn out to be a certain distinguished d-cluster tilting subcategory of the higher cluster category associated to the algebra in question just as it is the case for higher Auslander algebras of Dynkin type A.