COMBINATORIAL PROPERTIES OF HIGHER CLUSTER CATEGORIES

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Methods of Research

The majority of the first year will be spent on research objective 1 – the combinatorial description of higher Auslander algebras of path algebras of Dynkin type D and E and their associated cluster categories – followed by a thorough study of the theory of generalized cluster categories (mainly [Ami09, Sections 3 and 4], [Ami08, Chapters 5, 6 and 7] and [Guo12, Chapters 2 and 3]), of n-representation-infinite algebras ([HIO14, Sections 1-5]) and n-angulated categories (the entirety of [GKO13]).

The correspondence between quivers of Dynkin type D and triangulations of discs with marked points on their boundaries and one in their interior is bound to prove crucial for this endeavour. Unlike in the case of triangulations of discs with marked points lying only on their boundaries (giving rise to quivers of type A), the existence of an inner marked point naturally leads to the concept of a self-folded triangle (a triangle with only two distinct sides). The first relatively non-trivial task will entail generalizing this concept to higher dimensions and understanding the triangulations of a cyclic polytope with a distinguished point in its interior.

In [OT11, Section 3], an orientation of A_n is chosen that guarantees the equality $\mathrm{Hom}_{A_n^1}(P_i, P_j) = 0$ whenever $i > j$ (here P_i denotes the projective cover of the simple module S_i concentrated in vertex i). With the right approach, this choice makes the study of partial tilting modules of A_n^d contained in add ${}_{A_n^d}M$ (here ${}_{A_n^d}M$ denotes the cluster tilting module of A_n^d) smooth and elegant. Such an orientation cannot be chosen for the quivers of type D or E and thus irregularities will inevitably occur and need to be dealt with. More irregularities lurk within the connection between bistellar flips of the triangulations of cyclic polytopes and mutations of basic tilting modules of A_n^d detailed in [OT11, Section 4]. The issue lies in the fact that the folded edges in self-folded triangles cannot be flipped. This situation does not arise in the study of triangulations of discs without internal marked points and is likely to propagate to higher dimensions.