RESEARCH STATEMENT

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Abstract. Following the construction of d-representation-finite algebras in [2] and the description of the correspondence between certain types of cluster algebras and triangulations of bordered surfaces with marked points in [1], links have appeared connecting d-representation finite algebras to higher dimensional variants of said surface. One such link was discovered in [3] between higher Auslander algebras of the path algebra of linearly oriented Dynkin quiver A_n and cyclic polytopes. I wish to further study such kinds of connections, starting with the establishment of a similar type of link for path algebras of quivers of type D_n which, in the low-dimensional case, correspond to once punctured polygons; then, with a touch of expectation and naïvety, broadening it to include (special types of) cluster algebras not necessarily representation-finite.

1. Introduction

This text serves primarily as an overview of relevant concepts regarding cluster algebras, bordered surfaces with marked points, higher dimensional cluster categories and d-representation-finite algebras interwoven with ideas of possible generalizations and caveats tied to such endeavour. So far, I have only scratched the surface of this topic, hence very few original results are present.

In Section 2, I give a summary of the theory of bordered surfaces with marked points. Section 3 is dedicated to (normalized skew-symmetrizable) cluster algebras and their connection to bordered surfaces with marked points is drawn. Sections 4 and 5 define d-representation-finite algebras and higher cluster categories, respectively. Section 6 summarizes relevant results from [3], regarding a higher-dimensional kind of connection described in Section 3. Finally, Section 7 is riddled with (splinters of) steps towards generalizations of the content of Section 6.

2. Bordered Surfaces with Marked Points

This section is a brief summary of [1], Section 2.

Definition 2.1 (Bordered surface with marked points)**.** Let $\mathbf{S}$ be a connected oriented 2-dimensional Riemann surface with boundary. We fix a finite set $\mathbf{M}$ of *marked points* in the closure of $\mathbf{S}$. Marked points lying in the interior of $\mathbf{S}$ are called *punctures*. The pair $(\mathbf{S}, \mathbf{M})$ is called a *bordered surface with marked points* if the following additional technical conditions are satisfied.

- The set $\mathbf{M}$ is non-empty.
- The pair $(\mathbf{S}, \mathbf{M})$ is not
  - a sphere with one or two punctures;
  - a monogon with zero or one puncture;
  - a digon without punctures;
  - a triangle without punctures.

Here, the term n-gon denotes a disk with n marked points on its boundary. Moreover, sphere with three punctures is also often excluded.

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Definition 2.2 (Arc)**.** An arc γ in a bordered surface with marked points $(\mathbf{S}, \mathbf{M})$ is a curve in $\mathbf{S}$ such that

- its endpoints are marked points;
- γ does not intersect itself, except that its endpoints may coincide;
- except for its endpoints, γ is disjoint from $\mathbf{M}$ and from the boundary of $\mathbf{S}$;
- γ is not contractible into $\mathbf{M}$ or into the boundary of $\mathbf{S}$.

We are interested in triangulations of $(\mathbf{S}, \mathbf{M})$. Vaguely speaking, triangulation is a division of $\mathbf{S}$ into 'triangles' by a series of 'cuts'. Here, 'triangles' are either disks with three marked points on their boundaries or, so-called *self-folded* triangles, once-punctured monogons with an arc connecting the unique marked point to the unique puncture. See figure 1.

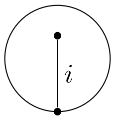

FIGURE 1. A self-folded triangle.

Definition 2.3 (Isotopy)**.** Let γ_1, γ_2 be two arcs in $(\mathbf{S}, \mathbf{M})$. An *isotopy* between γ_1 and γ_2 is a homotopy H between γ_1 and γ_2 such that $H(x,t)$ is an embedding for each fixed $t \in [0,1]$. Isotopy is an equivalence relation on the set of all arcs in $(\mathbf{S}, \mathbf{M})$.

In the following text, each arc in $(\mathbf{S}, \mathbf{M})$ is considered up to isotopy.

Definition 2.4 (Compatibility of arcs)**.** Two arcs in $(\mathbf{S}, \mathbf{M})$ are called *compatible* if they (up to isotopy) do not intersect each other in the interior of $\mathbf{S}$.

Proposition 2.5. *Any collection of pairwise compatible arcs can be realized by curves in their respective isotopy classes which do not intersect in the interior of* $\mathbf{S}$.

Definition 2.6 (Ideal triangulation)**.** A maximal collection of pairwise compatible arcs is called an *ideal triangulation.* In fact, definition 2.1 excludes all cases where $(\mathbf{S}, \mathbf{M})$ cannot be triangulated. The arcs of an ideal triangulation cut $\mathbf{S}$ into *ideal triangles.* The three sides of an ideal triangle need not be distinct, leading to self-folded triangle, and two triangles can share more than one side.

REFERENCES

[1] Sergey Fomin, Michael Shapiro, and Dylan Thurston, *Cluster algebras and triangulated surfaces. Part I: Cluster complexes*, Acta Mathematica, 201:83-146, 2008.

[2] Osamu Iyama, *Cluster tilting for higher Auslander algebras*, Adv. Math. 226 (2011), no. 1, 1–61.

[3] Steffen Oppermann and Hugh Thomas, *Higher dimensional cluster combinatorics and representation theory*, Acta Mathematica, 201:83-146, 2008.