



INSTITUTE FOR DEFENSE ANALYSES

## **Space-Filling Experimental Design and Surrogate Models for U.S. Department of Defense Modeling and Simulation Evaluation**

John T. Haman, Project Leader

Curtis G. Miller

OED Draft

August 2023

Approved for public release; distribution  
unlimited.

IDA Document 1038122



The Institute for Defense Analyses is a nonprofit corporation that operates three Federally Funded Research and Development Centers. Its mission is to answer the most challenging U.S. security and science policy questions with objective analysis, leveraging extraordinary scientific, technical, and analytic expertise.

#### About This Publication

This work was conducted by the Institute for Defense Analyses (IDA) under contract HQ0034-19-D-0001, Task BD-9-2299(90), "Test Science: Methods Advancements" for the Office of the Director, Operational Test and Evaluation. The views, opinions, and findings should not be construed as representing the official position of either the Department of Defense or the sponsoring organization.

#### Acknowledgments

The IDA Technical Review Committee was chaired by Dr. V. Bram Lillard and consisted of Dr. Emily Heuring, Dr. John T. Haman, Dr. Rebecca M. Medlin, Mr. Chris Dimpasok, Dr. Keyla Pagan-Rivera, and Dr. Addison Adams from the Operational Evaluation Division.

#### For more information:

Dr. John Haman, Project Leader  
[jhaman@ida.org](mailto:jhaman@ida.org) • (703) 845-2132

Dr. V. Bram Lillard, Director, Operational Evaluation Division  
[vlllard@ida.org](mailto:vlllard@ida.org) • (703) 845-2230

#### Copyright Notice

© 2023 Institute for Defense Analyses  
730 East Glebe Road, Alexandria, Virginia 22305 • (703) 845-2000

This material may be reproduced by or for the U.S. Government pursuant to the copyright license under the clause at DFARS 252.227-7013 [Feb. 2014].

INSTITUTE FOR DEFENSE ANALYSES

IDA Document 1038122

**Space-Filling Experimental Design and Surrogate Models for U.S. Department of Defense Modeling and Simulation Evaluation**

John T. Haman, Project Leader

Curtis G. Miller

## Executive Summary

---

Operational testing (OT) provides important information on the capability of equipment acquired by the United States Department of Defense (DoD). Operational testing has been a vital component of the acquisition process since Congress reformed OT conduct with the Department of Defense Authorization Act of 1984. Computer modeling and simulation (M&S) can help address the small sample size problem in live, operational testing, but M&S must be validated in order for stakeholders to believe M&S makes meaningful predictions about real-world outcomes.

The Institute for Defense Analyses (IDA) has published documents providing guidance to the OT community on statistical validation of M&S, including a handbook and papers on the use of space-filling designs and statistical surrogates (also known as metamodels). However, important questions remain that need to be answered in order to best validate M&S.

In particular, operational testers need better recommendations on sample size selection, determining the number of replicates in a design (if any), validating with small real-world sample sizes, and incorporating statistical

surrogates into hypothesis tests that help determine whether M&S outcomes match real-world outcomes or not.

We hope the larger statistical community can help contribute answers to these questions. This presentation was given at the Joint Statistical Meeting 2023 in Toronto, Canada.



# **Space-Filling Experimental Design and Surrogate Models for U.S. Department of Defense Modeling and Simulation Evaluation**

Dr. Curtis Miller

August 9, 2023

**Institute for Defense Analyses**  
730 East Glebe Road • Alexandria, Virginia 22305

# We want statisticians to appreciate...



*... why operational testing matters*



*... why computer modeling and simulation matters to operational testing*



*... the challenges in planning computer modeling and simulation studies*



*... the challenges in statistically validating a computer model*

Why does operational testing matter?



# Operational testing studies DOD system effectiveness in warfighting conditions

97 STAT. 614

PUBLIC LAW 98-94—SEPT. 24, 1983

Public Law 98-94  
98th Congress

An Act

Sept. 24, 1983

[S. 675]

Department of  
Defense  
Authorization  
Act, 1984.

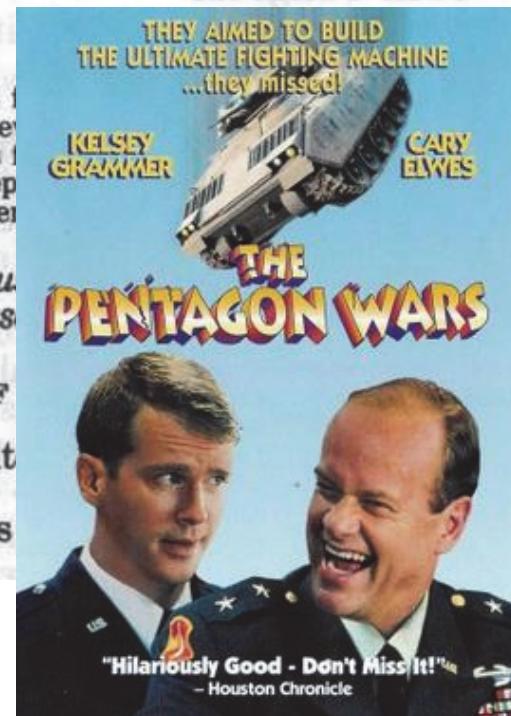
To authorize appropriations for fiscal year 1984 for the Department of Defense, for research, development, test, and evaluation, for maintenance, to prescribe personnel strengths for the Armed Forces and for civilian employees of the Department, and for other purposes; and for appropriations for such fiscal year for civil defense.

*Be it enacted by the Senate and House of Representatives of the United States of America in Congress assembled,*

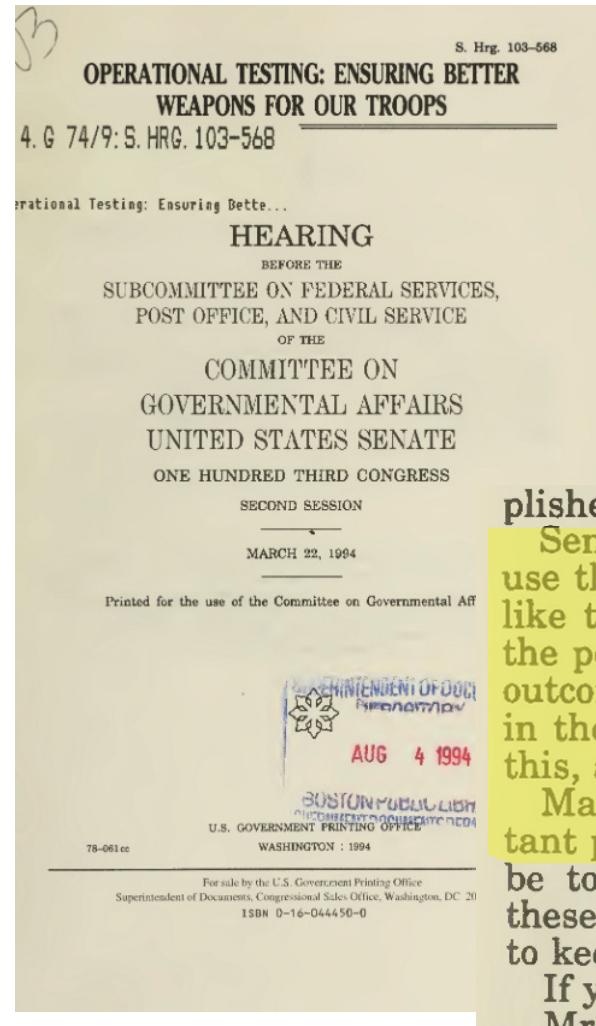
SHORT TITLE; TABLE OF

SECTION 1. (a) This Act may be cited as the "Department of Defense Authorization Act, 1984".

(b) The table of contents for this Act is



# OT results matter to both Congress and the warfighter



**Sen. D. Pryor (D-AK)**



**Sen. W. Roth (R-DE)**

plished.

Senator PRYOR. Now, you know, testing is not a very—I hate to use the word “sexy,” but it is not a very high priority item it seems like today. There are not a lot of people interested in this, except the people in the battlefield. They are going to be interested in the outcome of all of this, and they are certainly going to be interested in the outcome of S. 1587. They will also be interested in whether this, as we call it, Mack truck amendment, remains in the bill.

Making certain that these weapons work is a very, very important part of our military preparedness. Whoever our enemies might be today, we had better believe that their intelligence knows if these weapons work or don’t work, and I think that we have got to keep that in mind throughout this whole process.

If you would please continue.

Mrs. PRESTON. Let me quickly go through some of the other

# OT&E should reduce risk and uncertainty regarding the performance of systems in wartime

GAO

United States General Accounting Office

Report to the Honorable  
William V. Roth and the Honorable  
Charles E. Grassley, U.S. Senate

October 1997

## TEST AND EVALUATION

### Impact of DOD's Office of the Director of Operational Test and Evaluation



number of unknowns prior to the decision to begin full production, while program and service officials typically sought less testing and were willing to accept greater risk when making production decisions. The additional testing DOT&E advocated, often over the objections of service testers, served to meet the underlying objectives of operational testing—to reduce the uncertainty and risk that systems entering full-rate production would not fulfill their requirements.

GAO/NSIAD-98-22

Why does computer modeling and simulation matter to OT?



# Small data problems are alive and well in operational testing

*SM-3 Blk IIA*



*Next Generation Interceptor*



*PAC-3*



***Missile tests can cost \$10Ms to \$100Ms  
per shot***

<https://missiledefenseadvocacy.org/missile-defense-systems-2/missile-defense-systems/missile-interceptors-by-cost/>

# Small data problems are alive and well in operational testing

*SM-3 Blk IIA*



*Next Generation Interceptor*



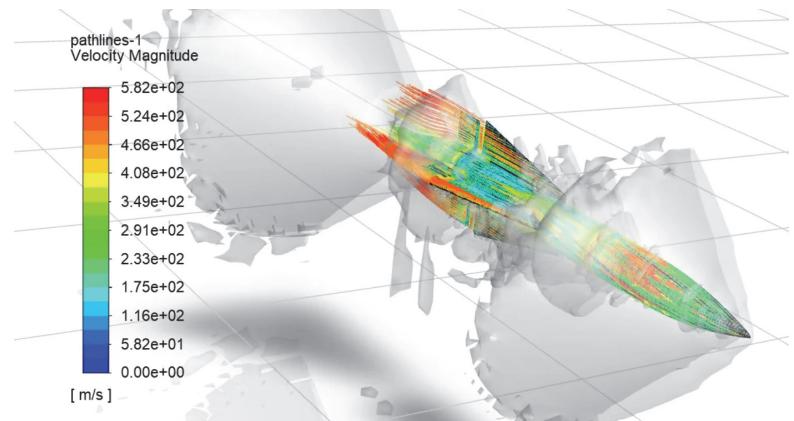
*PAC-3*



COST AND ASSET  
AVAILABILITY GENERATE  
DEMAND FOR M&S

**Missile tests can cost \$10Ms to \$100Ms  
per shot**

# Modeling and simulation comes in a variety of flavors with unique statistical considerations



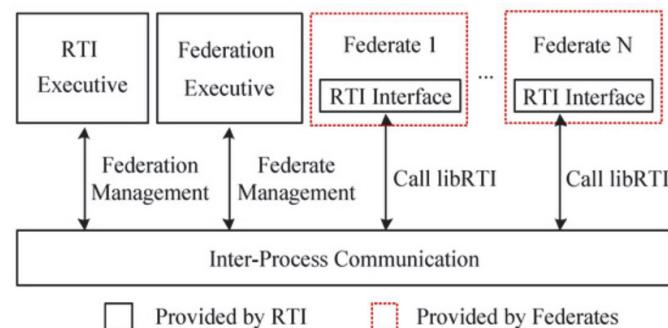
## Digital Simulation

[https://www.reddit.com/r/dcsworld/comments/mqmvy3/lowquality\\_steadystate\\_cfd\\_simulation\\_of\\_an\\_aim54](https://www.reddit.com/r/dcsworld/comments/mqmvy3/lowquality_steadystate_cfd_simulation_of_an_aim54)



## Hardware in the Loop

<https://testscience.org>

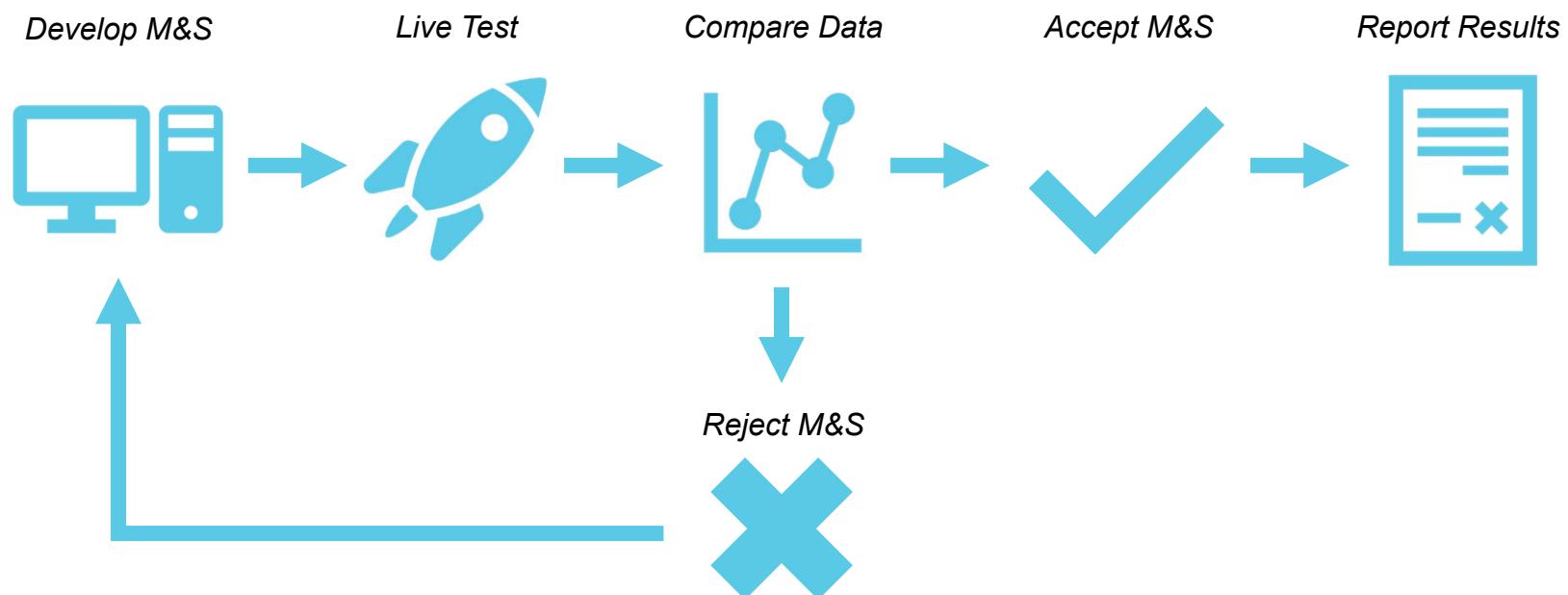


M&S – Modeling and Simulation

## Federation

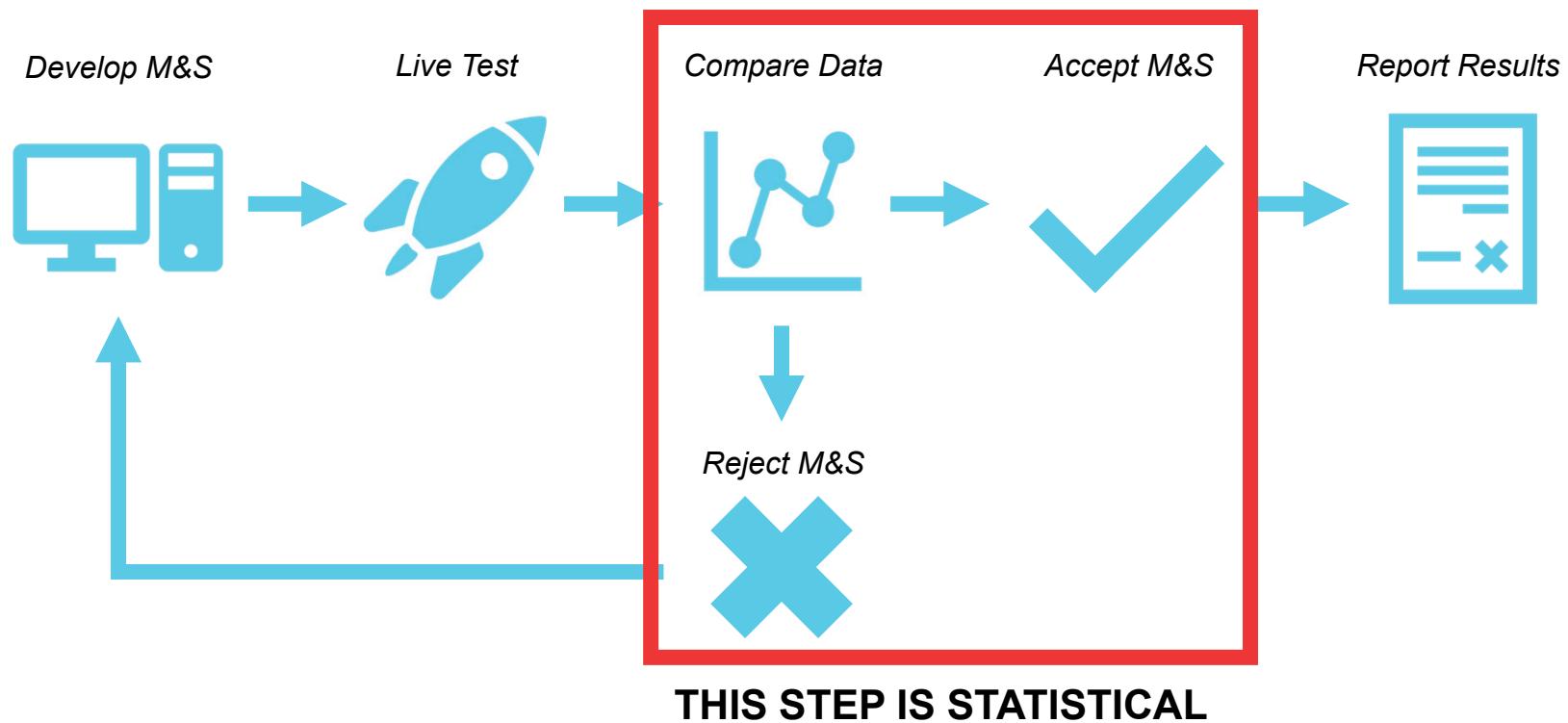
<https://doi.org/10.3390/electronics9030540>

If M&S outputs will be used for predicting OT results,  
we must compare M&S outputs to live test data



M&S – Modeling and Simulation; OT – Operational Testing

If M&S outputs will be used for predicting OT results,  
we must compare M&S outputs to live test data



M&S – Modeling and Simulation; OT – Operational Testing

If M&S outputs will be used for predicting OT results,  
we must compare M&S outputs to live test data



M&S – Modeling and Simulation; OT – Operational Testing

What are the challenges in planning computer M&S studies?



# IDA publications introduce and recommend M&S DOE best practices

Approved for public release; distribution is unlimited.

INSTITUTE FOR DEFENSE ANALYSES

**IDA**

**Space Filling Designs for  
Modeling & Simulation Validation**

Heather Wojton, Project Leader

Kelly Avery  
Han Yi  
Curtis Miller

June 2021  
Approved for Public Release.  
Distribution Unlimited.  
IDA Document NS D-21562  
Log: H 2021-000048

INSTITUTE FOR DEFENSE ANALYSES  
4830 Mark Center Drive  
Alexandria, Virginia 22311-1582

Approved for public release; distribution is unlimited.

INSTITUTE FOR DEFENSE ANALYSES

**IDA**

**Handbook on Statistical Design &  
Analysis Techniques for  
Modeling & Simulation Validation**

Heather Wojton, Project Leader

Kelly M. Avery  
Laura J. Freeman  
Samuel H. Parry  
Gregory S. Whittier  
Thomas H. Johnson  
Andrew C. Flack

February 2019  
Approved for public release.  
Distribution is unlimited.  
IDA Document NS D-10455  
Log: H 2019-000044

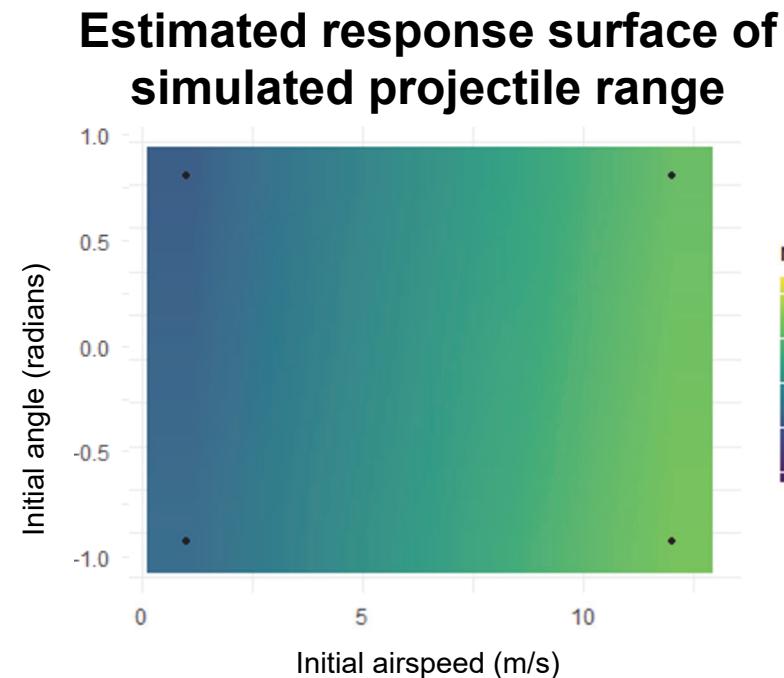
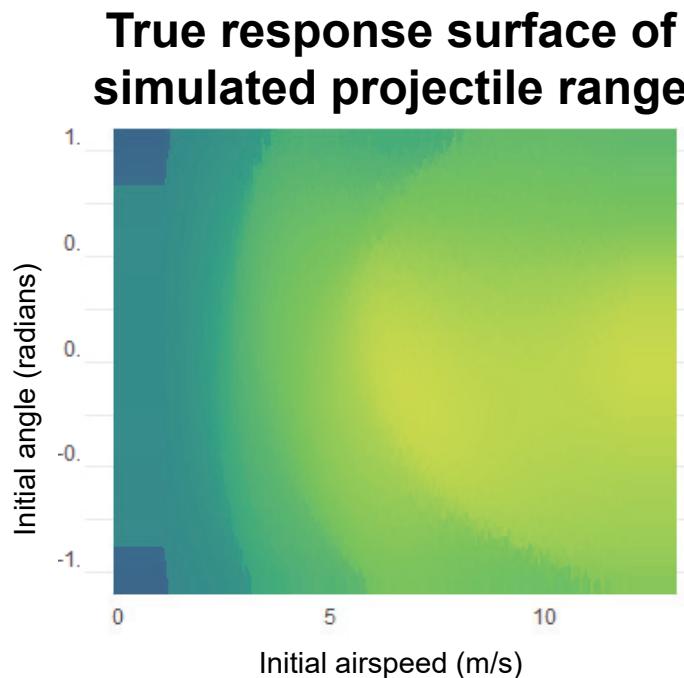
INSTITUTE FOR DEFENSE ANALYSES  
4830 Mark Center Drive  
Alexandria, Virginia 22311-1582

DOE – Design of experiments; IDA – Institute for Defense Analyses; SFD – Space-Filling Design

[https://testscience.org/wp-content/uploads/formidable/20/SFD\\_Literature\\_Review\\_Final.html](https://testscience.org/wp-content/uploads/formidable/20/SFD_Literature_Review_Final.html)

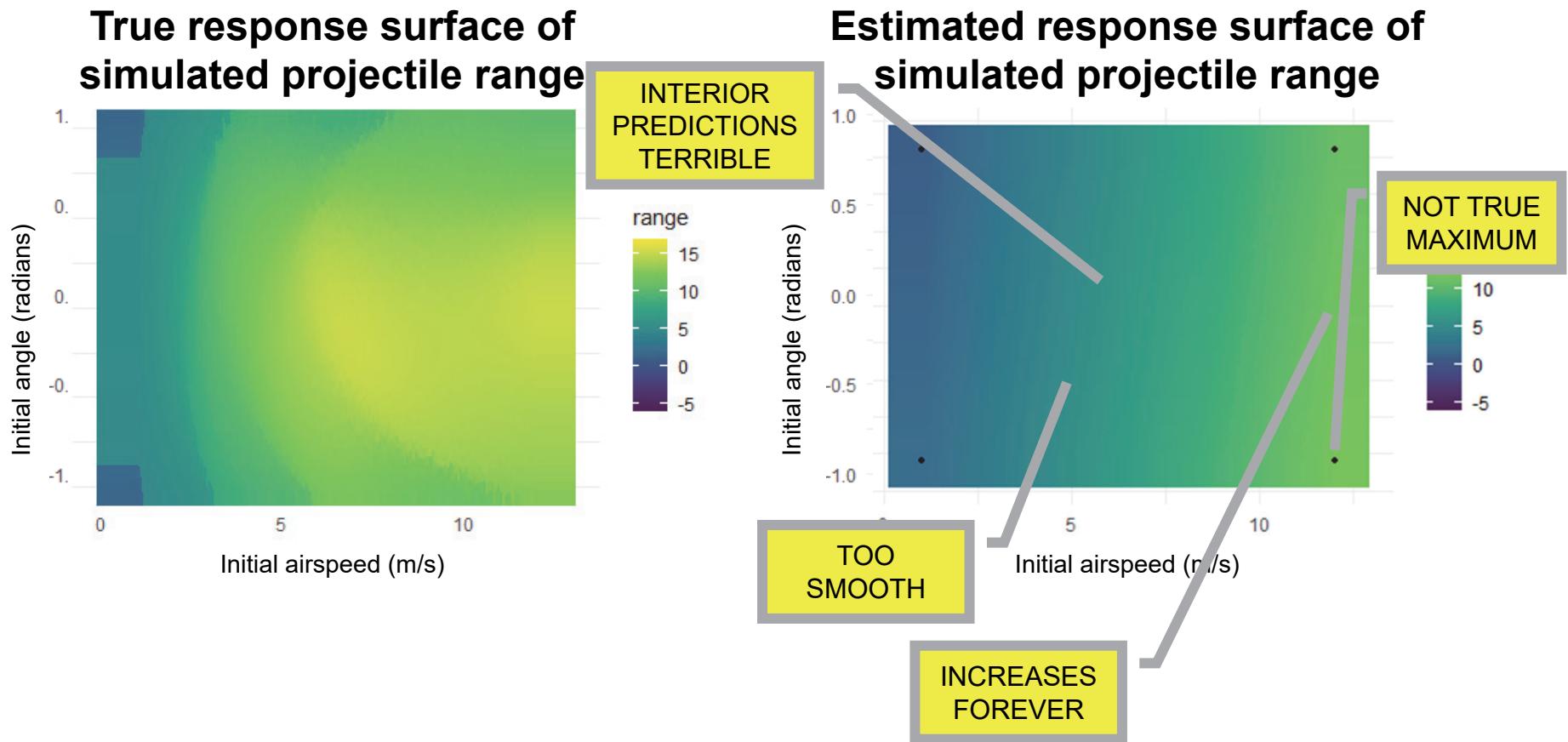
# Space-filling design of experiments helps recover more trends in simulation outputs

A **factorial** design with a simple **linear model** fitted will not accurately describe the M&S system's behavior.



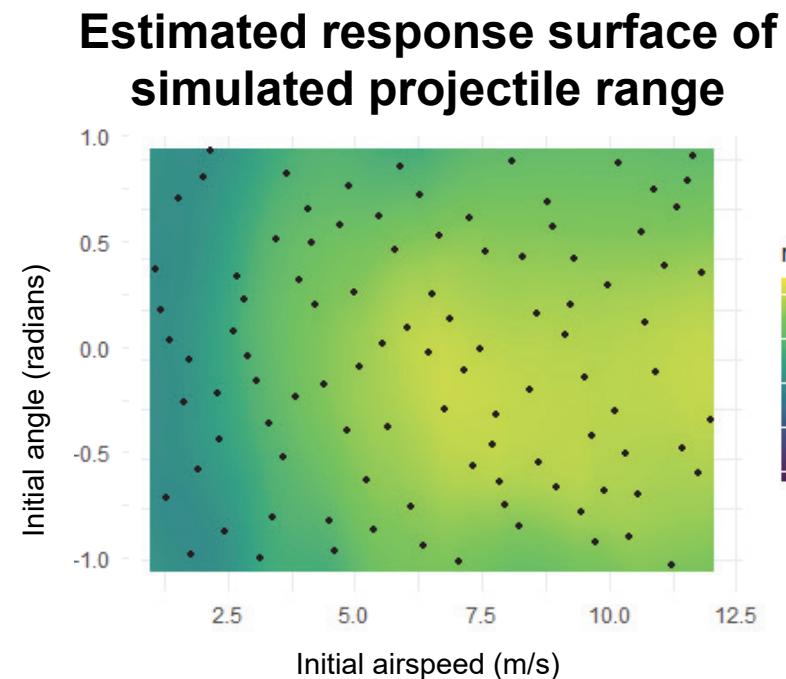
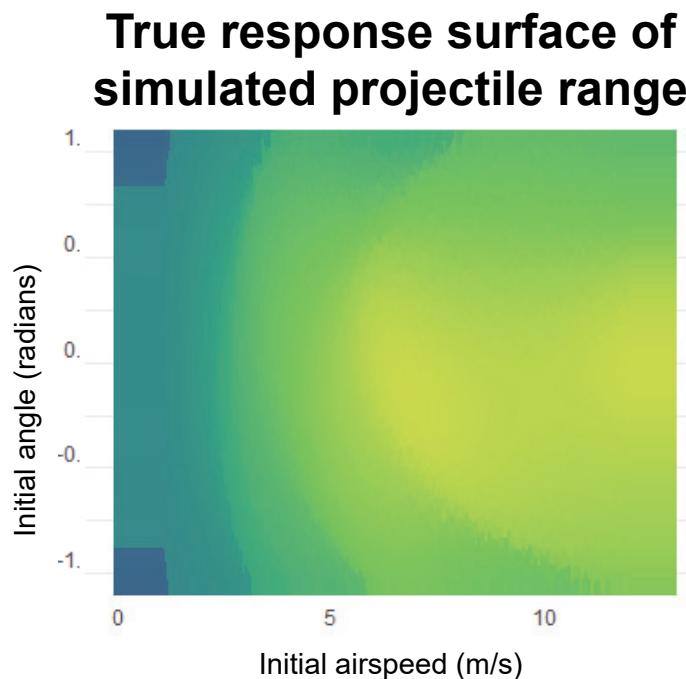
# Space-filling design of experiments helps recover more trends in simulation outputs

A **factorial** design with a simple **linear model** fitted will not accurately describe the M&S system's behavior.



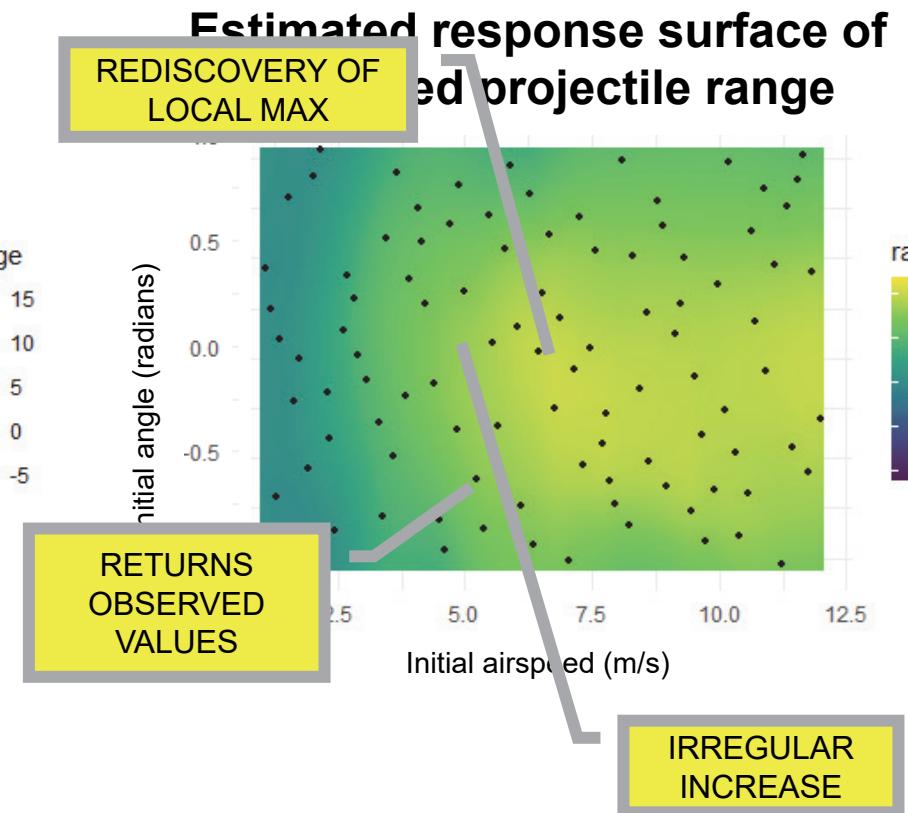
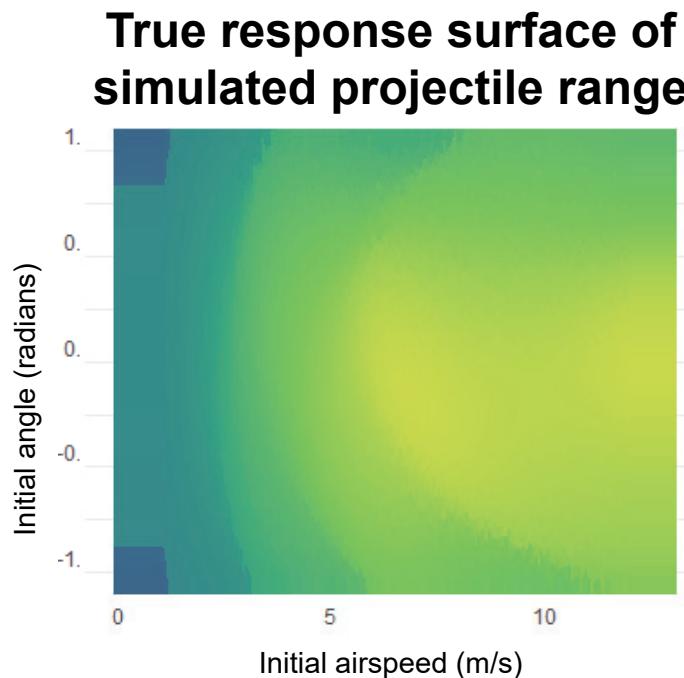
# Space-filling design of experiments helps recover more trends in simulation outputs

Analyzing the flights with a **Gaussian Process model** via a **Space-Filling Design** yields a good approximation to simulation output.



# Space-filling design of experiments helps recover more trends in simulation outputs

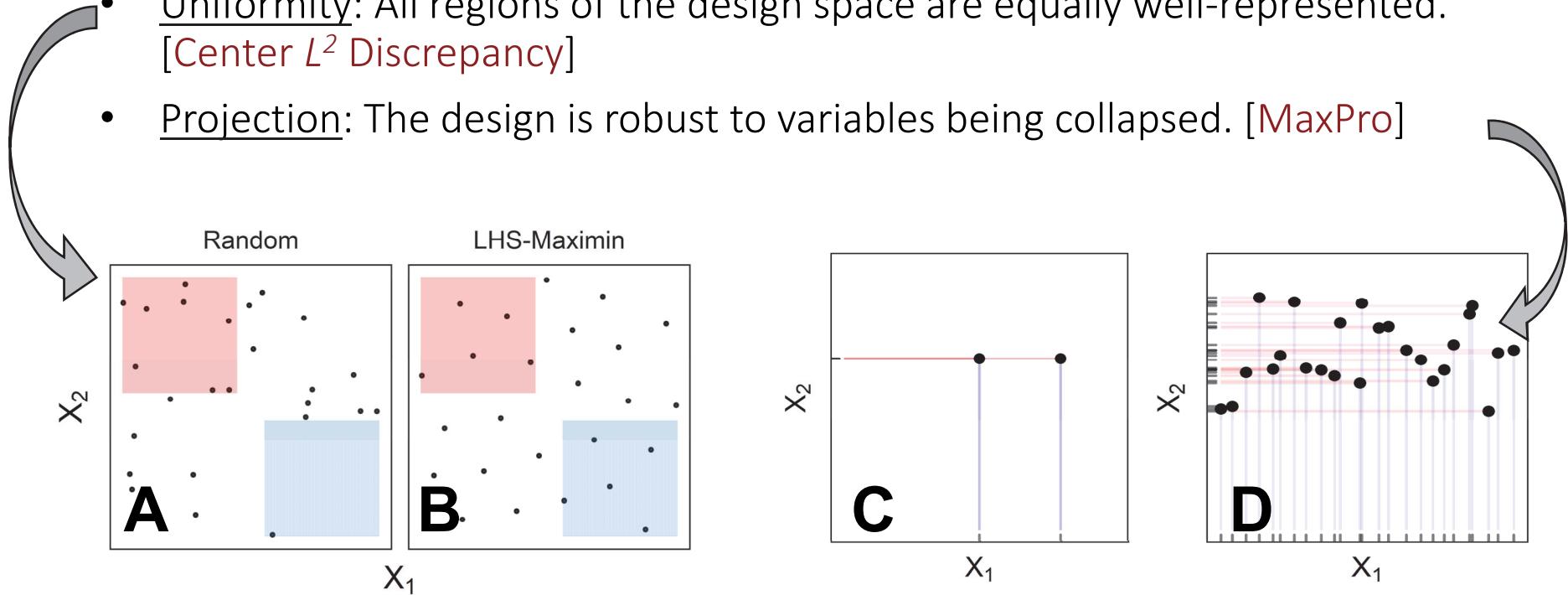
Analyzing the flights with a **Gaussian Process model** via a **Space-Filling Design** yields a good approximation to simulation output.



# Just like with classical DOE, there are quantitative ways to evaluate a specific design

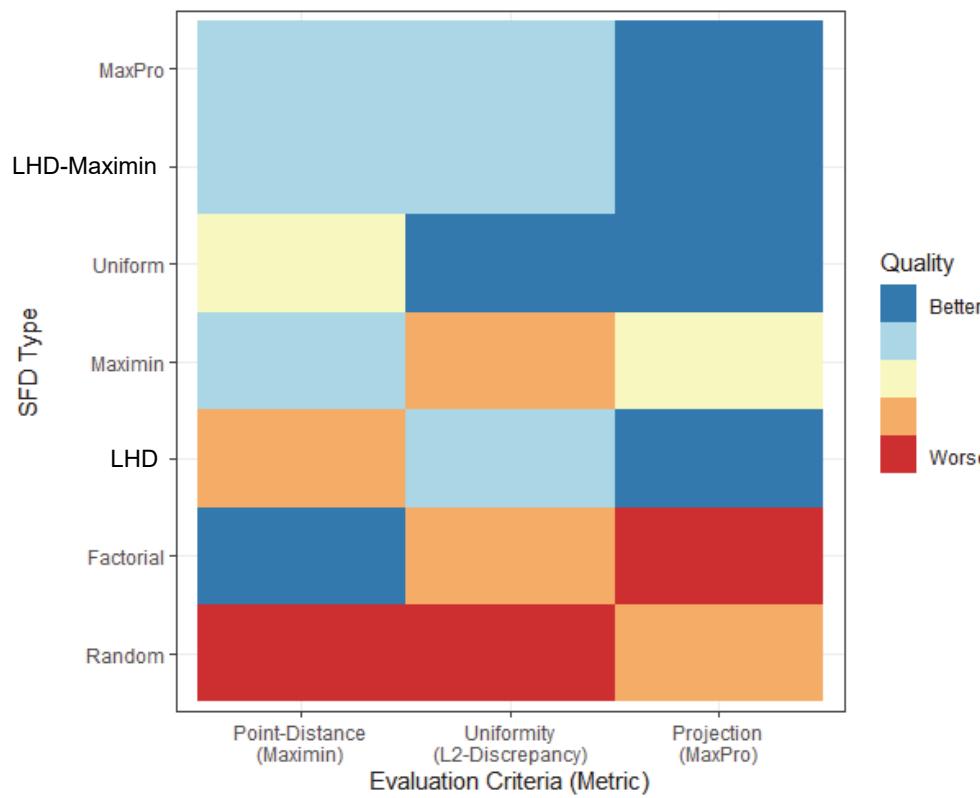
Many criteria exist, but it is particularly important that an SFD satisfy the following three criteria in order to be useful:

- Point-distance: Samples are placed as far apart from each other as possible. [Maximin]
- Uniformity: All regions of the design space are equally well-represented. [Center  $L^2$  Discrepancy]
- Projection: The design is robust to variables being collapsed. [MaxPro]

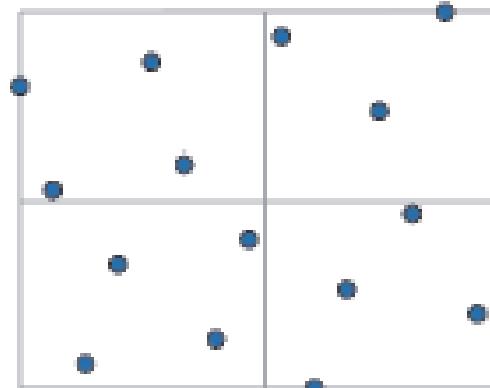


# I prefer maximin sliced Latin hypersquare designs (Maximin SLHD) and MaxPro SFDs

General recommendations:  
Maximin (Sliced) LHD or MaxPro

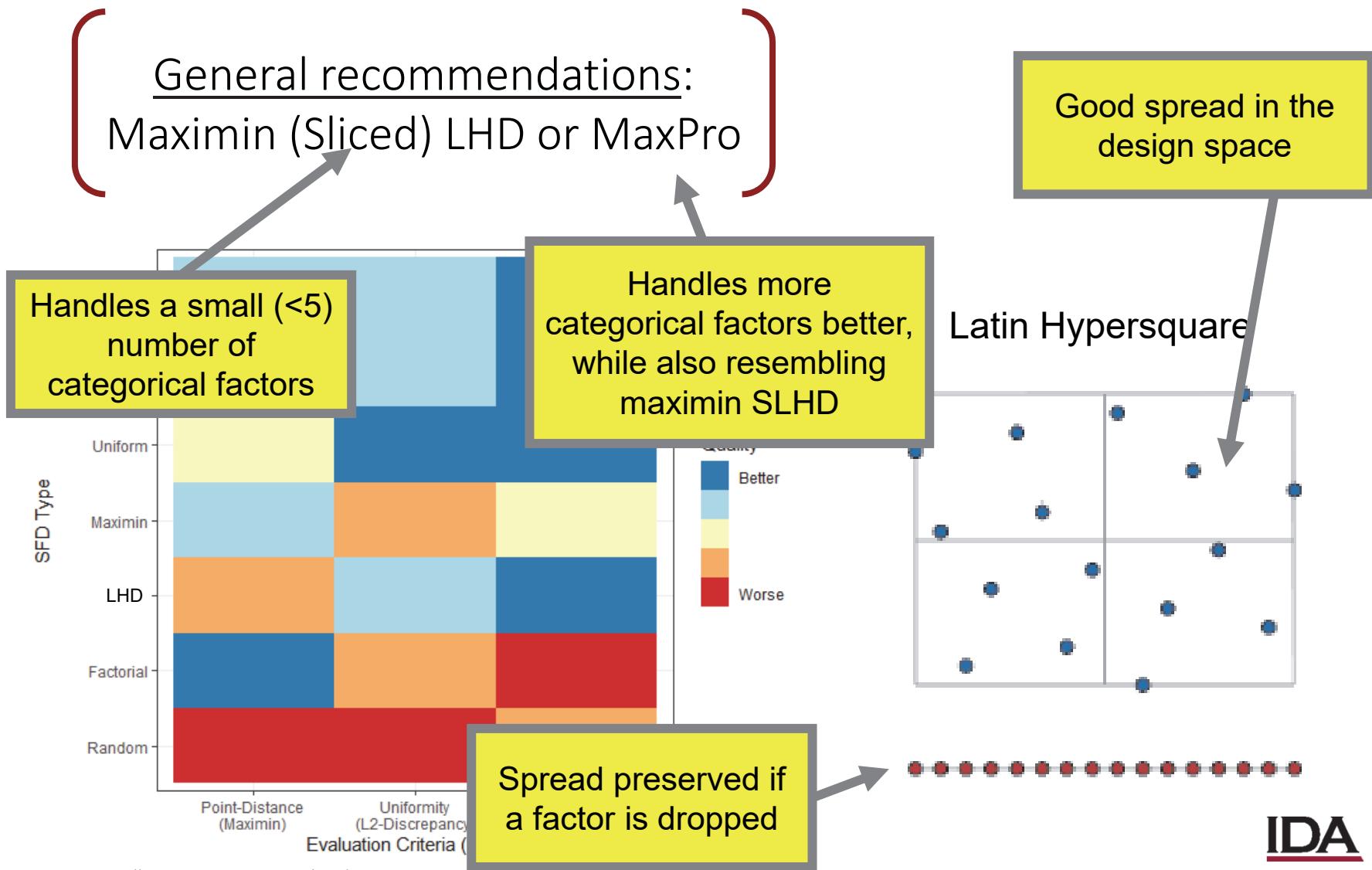


Latin Hypersquare



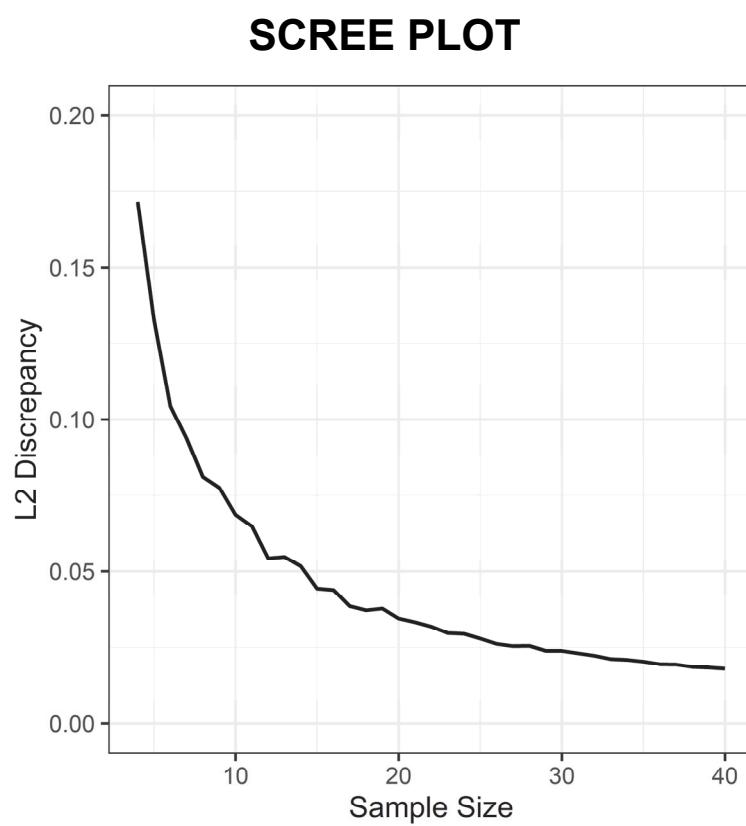
SFD – Space-Filling Design; SLHD – Sliced Latin Hypersquare Design

# I prefer maximin sliced Latin hypersquare designs (Maximin SLHD) and MaxPro SFDs



SFD – Space-Filling Design; SLHD – Sliced Latin Hypersquare Design

# How should we choose the sample size when generating an SFD?



$$n = 10d$$

## Choosing the Sample Size of a Computer Experiment: A Practical Guide

Jason L. LOEPPKY

Mathematics, Statistics, and Physics  
University of British Columbia, Okanagan  
Kelowna, BC V1V 1V7  
Canada  
(jason@stat.ubc.ca)

Jerome SACKS

National Institute of Statistical Sciences  
Research Triangle Park, NC 27709  
(sacks@niss.org)

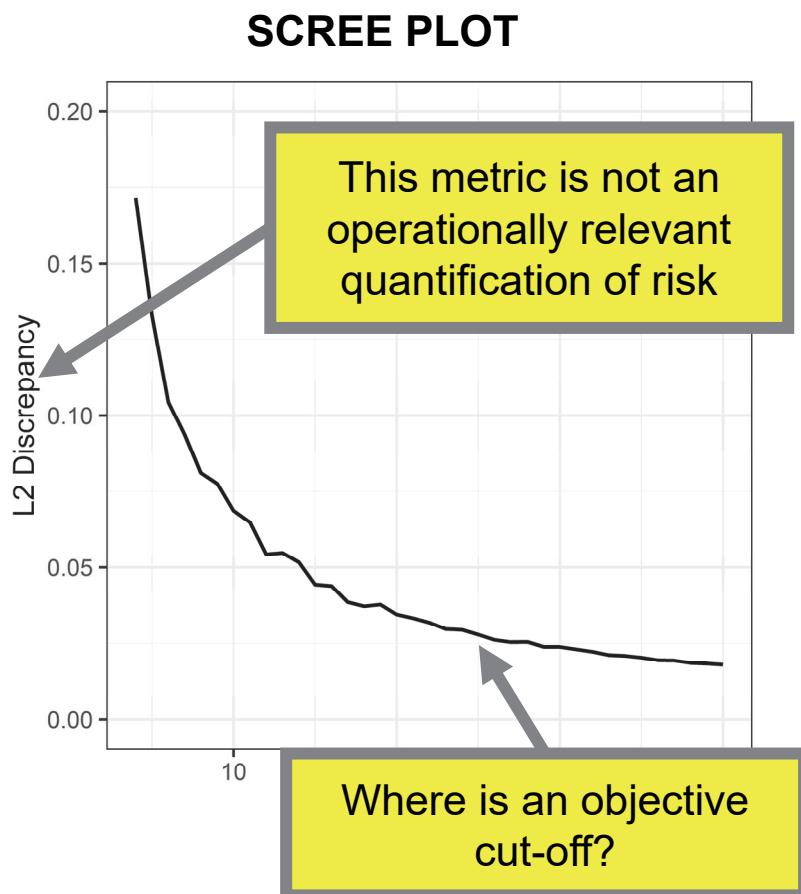
William J. WELCH

Department of Statistics  
University of British Columbia  
Vancouver, BC V6T 1Z2  
Canada  
(will@stat.ubc.ca)

We provide reasons and evidence supporting the informal rule that the number of runs for an effective initial computer experiment should be about 10 times the input dimension. Our arguments quantify two key characteristics of computer codes that affect the sample size required for a desired level of accuracy when approximating the code via a Gaussian process (GP). The first characteristic is the total sensitivity of a code output variable to all input variables; the second corresponds to the way this total sensitivity is distributed across the input variables, specifically the possible presence of a few prominent input factors and many impotent ones (i.e., effect sparsity). Both measures relate directly to the correlation structure in the GP approximation of the code. In this way, the article moves toward a more formal treatment of sample size for a computer experiment. The evidence supporting these arguments stems primarily from a simulation study and via specific codes modeling climate and ligand activation of G-protein.

KEY WORDS: Curse of dimensionality; Effect sparsity; Gaussian process; Latin hypercube design; Prediction accuracy; Random function.

# How should we choose the sample size when generating an SFD?



Seems too small in stochastic cases

$$n = 10d$$

## Choosing the Sample Size of a Computer Experiment: A Practical Guide

Jason L. LOEPPKY

Mathematics, Statistics, and Physics  
University of British Columbia, Okanagan  
Kelowna, BC V1V 1V7  
Canada  
(jason@stat.ubc.ca)

Jerome SACKS

National Institute of Statistical Sciences  
Research Triangle Park, NC 27709  
(sacks@niss.org)

William J. WELCH

Department of Statistics  
University of British Columbia  
Vancouver, BC V6T 1Z2  
Canada  
(will@stat.ubc.ca)

We provide reasons and evidence supporting the claim that the initial computer experiment should be able to identify key characteristics of computer codes that are important when approximating the code via a Gaussian process (GP). We consider a code output variable to all input variables distributed across the input variables, specifically the active and many impotent ones (i.e., effect sparsity), and their effect in the GP approximation of the code. In particular, we study the sample size for a computer experiment. This is done via a simulation study and via specific codes.

KEY WORDS: Curse of dimensionality; Computer experiments; Prediction accuracy; Radial basis functions; Space-filling designs

**ADJACENT QUESTION:  
WHEN SHOULD  
REPLICATES BE USED  
(AND HOW MANY)?**

The minimum may be all you get

What are the challenges in statistically validating a computer model?



# IDA publications and presentations introduce and recommend surrogate modeling best practices



INSTITUTE FOR DEFENSE ANALYSES

## Metamodeling Techniques for Verification and Validation of Modeling and Simulation Data

John T. Haman, Project Leader

Curtis G. Miller

September 2022  
This publication has not been approved for public release or distribution and release.  
Reproduction or use of this material is not authorized without prior permission from the responsible IDA Division Director.

IDA Paper P-33230  
Log: H 2022-000374

INSTITUTE FOR DEFENSE ANALYSES  
730 East Glebe Road  
Alexandria, Virginia 22305

<https://testscience.org>



## Space Filling Designs and Metamodeling for Understanding Modeling & Simulation Behavior

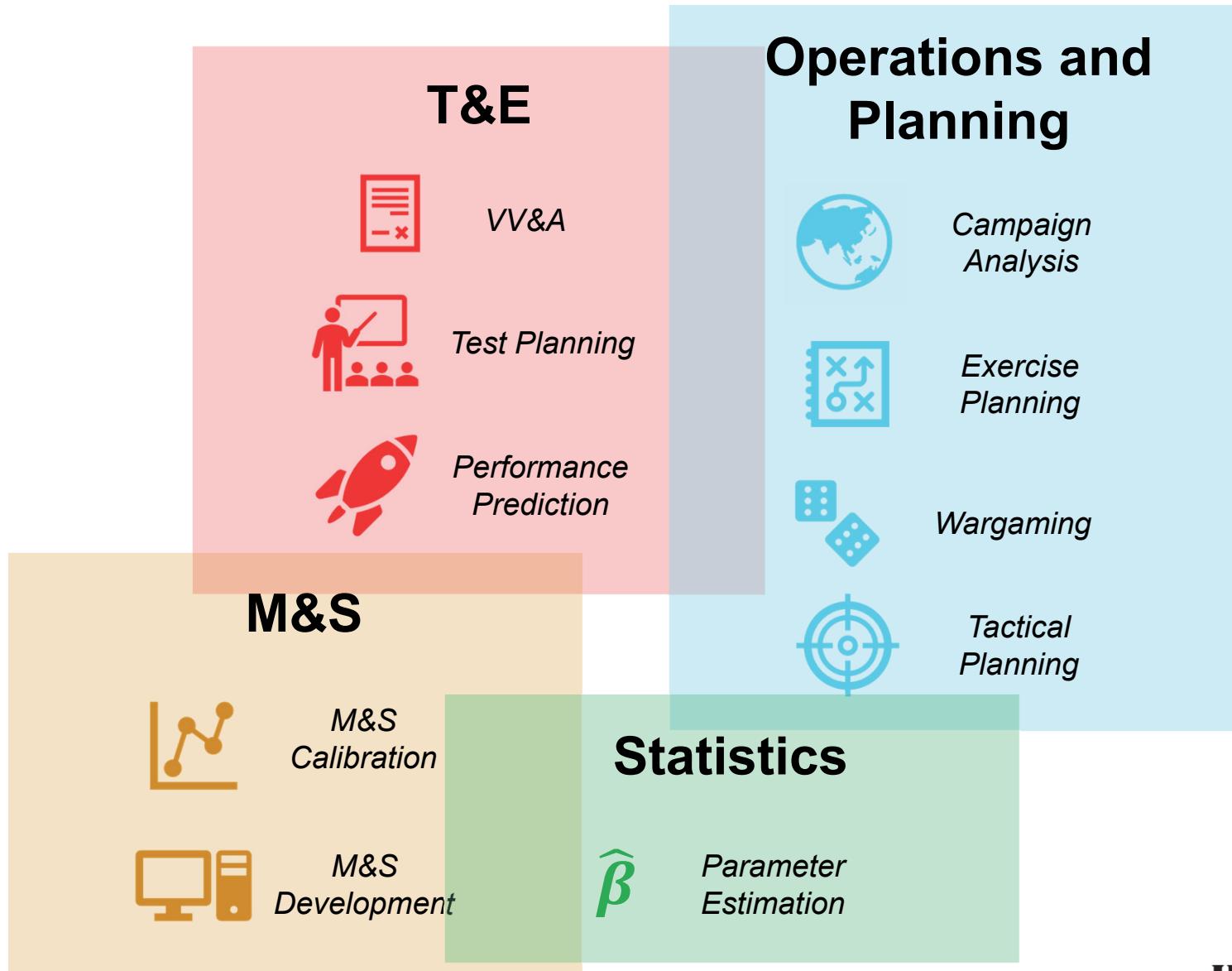
Curtis Miller

April 26, 2022

**Institute for Defense Analyses**  
730 East Glebe Road • Alexandria, Virginia 22305

<https://dataworks.testscience.org/>

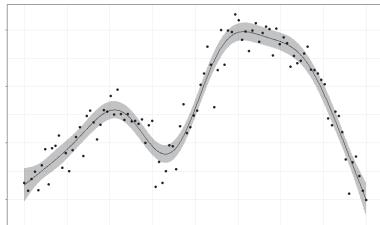
# A statistical surrogate is a useful product in and of itself



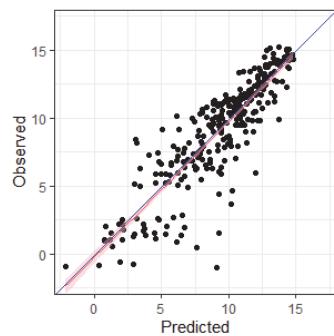
M&S – Modeling and Simulation; T&E – Test and Evaluation; VV&A – Verification, Validation, and Accreditation

# Statistical surrogates must allow for relevant assessments of M&S prediction quality

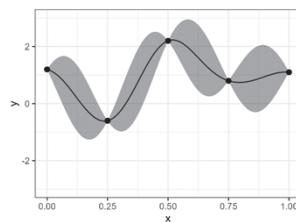
*Statistical surrogates should...*



*... discover unanticipated trends between factors and response variables*



*... predict observed responses well*



*... quantify uncertainty in M&S outputs*

# Statistical surrogates must allow for relevant assessments of M&S prediction quality

*Statistical surrogates should...*

THRESHOLD:



DATA:

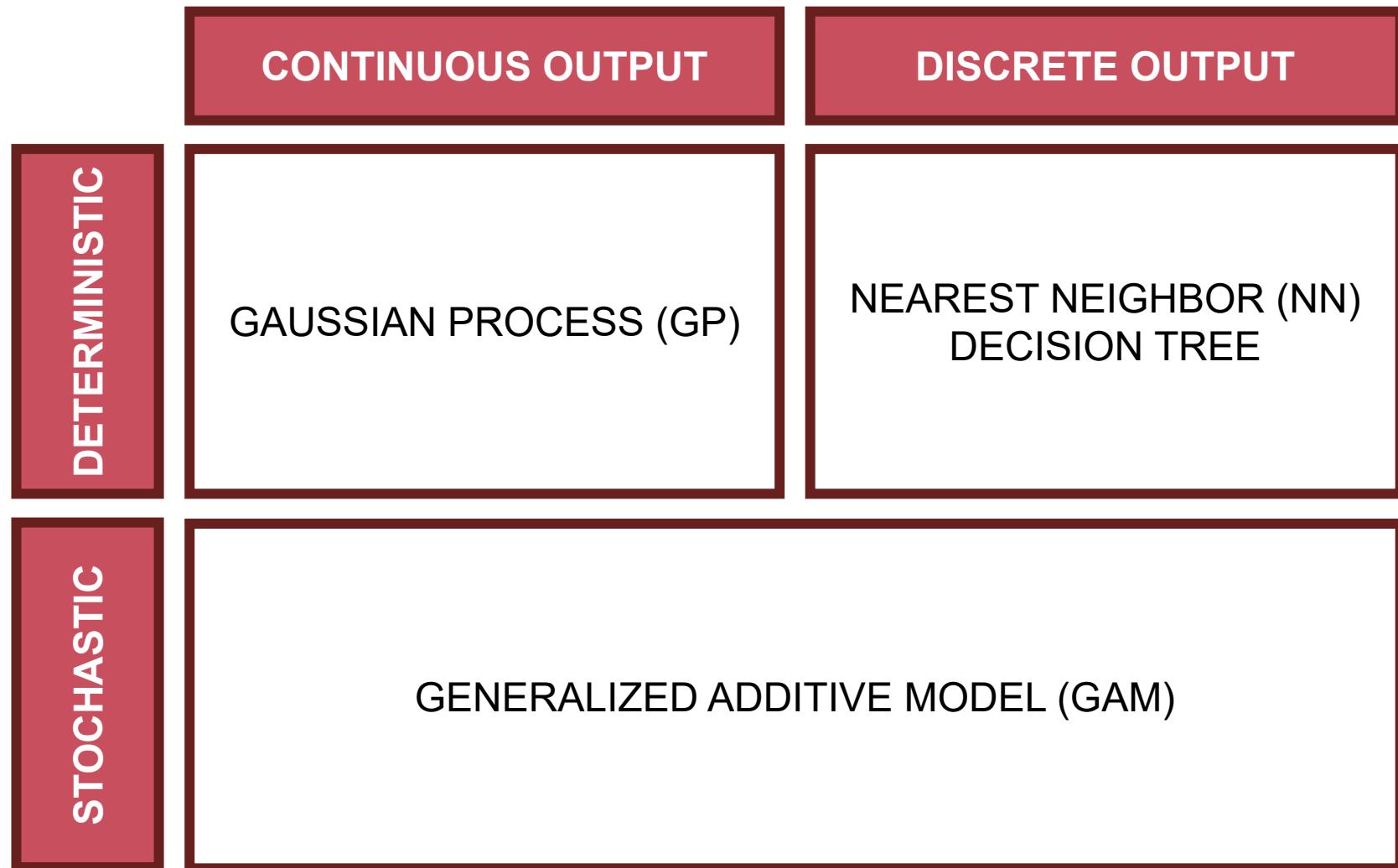


*... allow comparing real-world outcomes to M&S predictions*



*... allow expert judgement on prediction quality*

# We recommend different statistical surrogate procedures based on the M&S output



# We demonstrate methods using a simple numerical ODE solver

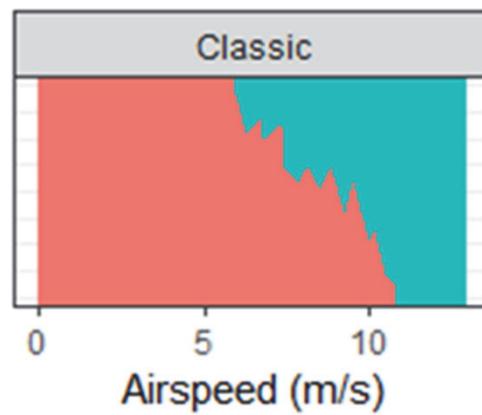
$$\dot{V} = -C_D (\rho V^2 / 2) S/m - g \sin(\gamma)$$

$$\dot{\gamma} = (C_L (\rho V^2 / 2) S/m - g \cos(\gamma)) / V$$

$$\dot{h} = V \sin(\gamma)$$

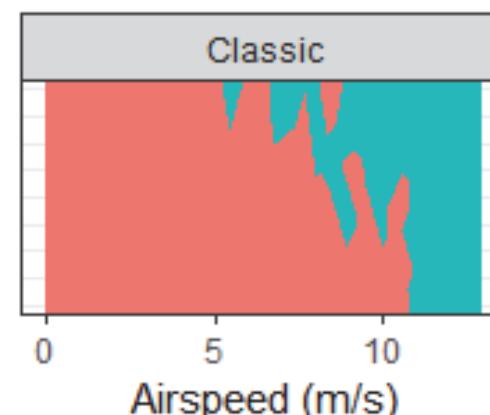
$$\dot{r} = V \cos(\gamma)$$

Randomize initial conditions  
for random output (i.e.,  
variance in toss)



Loop    FALSE    TRUE

DETERMINISTIC

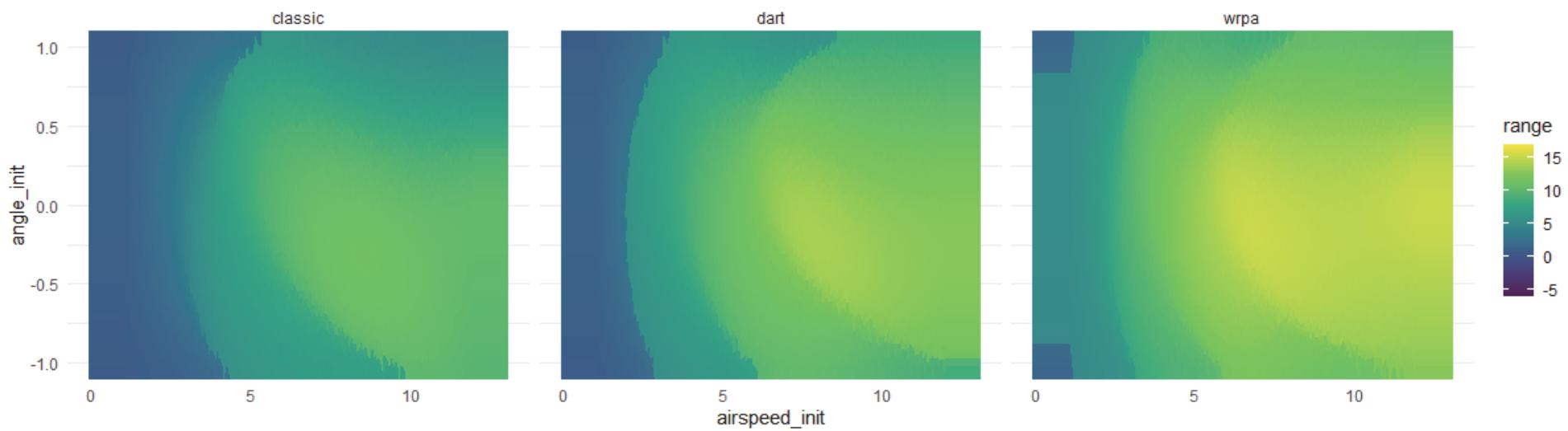


Loop    FALSE    TRUE

STOCHASTIC

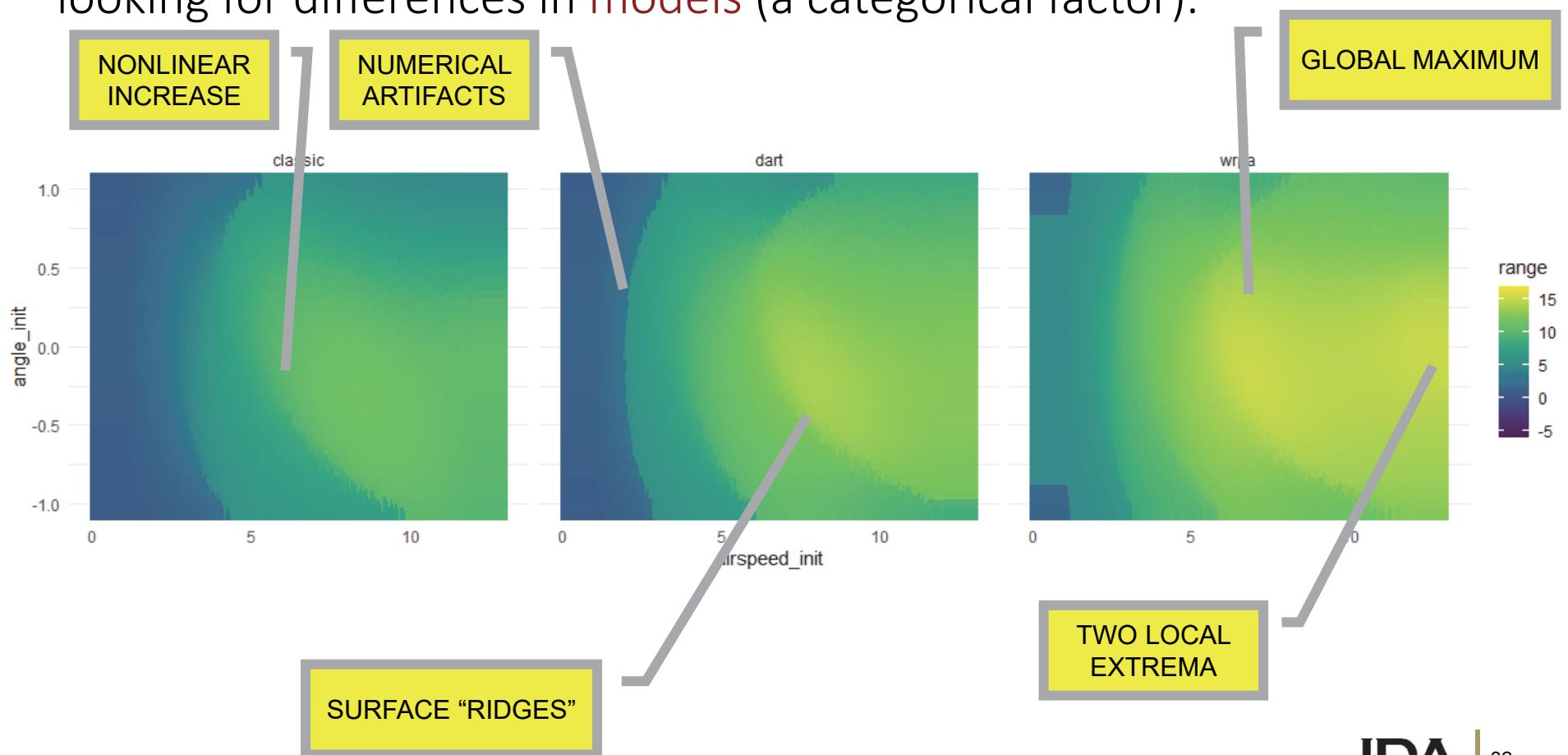
# Model, Airspeed, and Angle Study

Consider a study of the effects of initial **airspeed** and **angle** of flight (continuous factors) on flight terminal **range** (continuous), while also looking for differences in **models** (a categorical factor).



# Model, Airspeed, and Angle Study

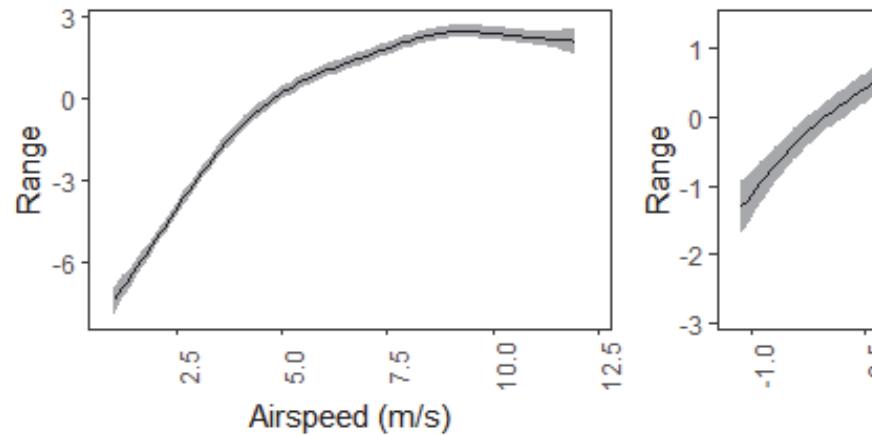
Consider a study of the effects of initial **airspeed** and **angle** of flight (continuous factors) on flight terminal **range** (continuous), while also looking for differences in **models** (a categorical factor).



WRPA – World Record Paper Airplane

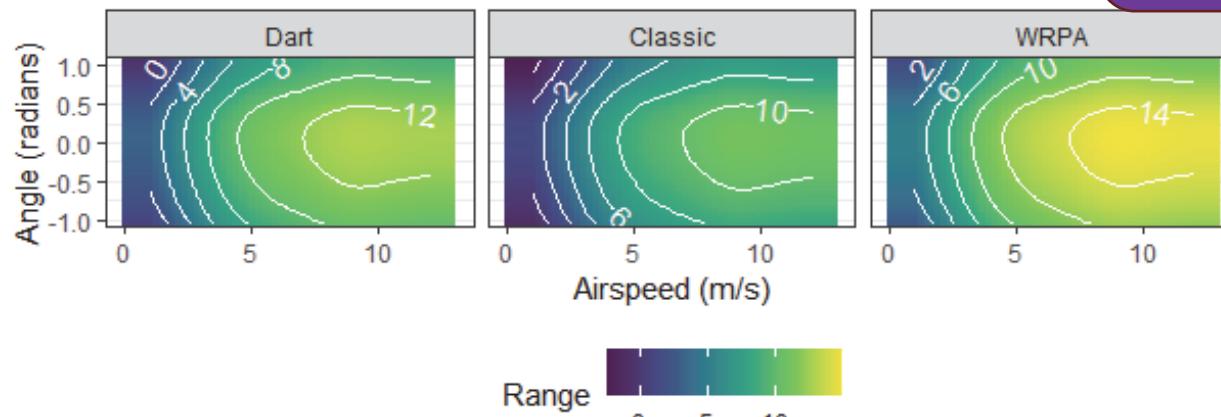
# GAMs allow for qualitative assessment of simulation system performance

$$\text{range}_i = \beta_0 + \beta_{\text{cl}} \text{classic}_i + \beta_{\text{wr}} \text{wrpa}_i + f_{\text{as}}(\text{airspeed}_i) + f_{\text{an}}(\text{angle}_i) + \epsilon_i$$

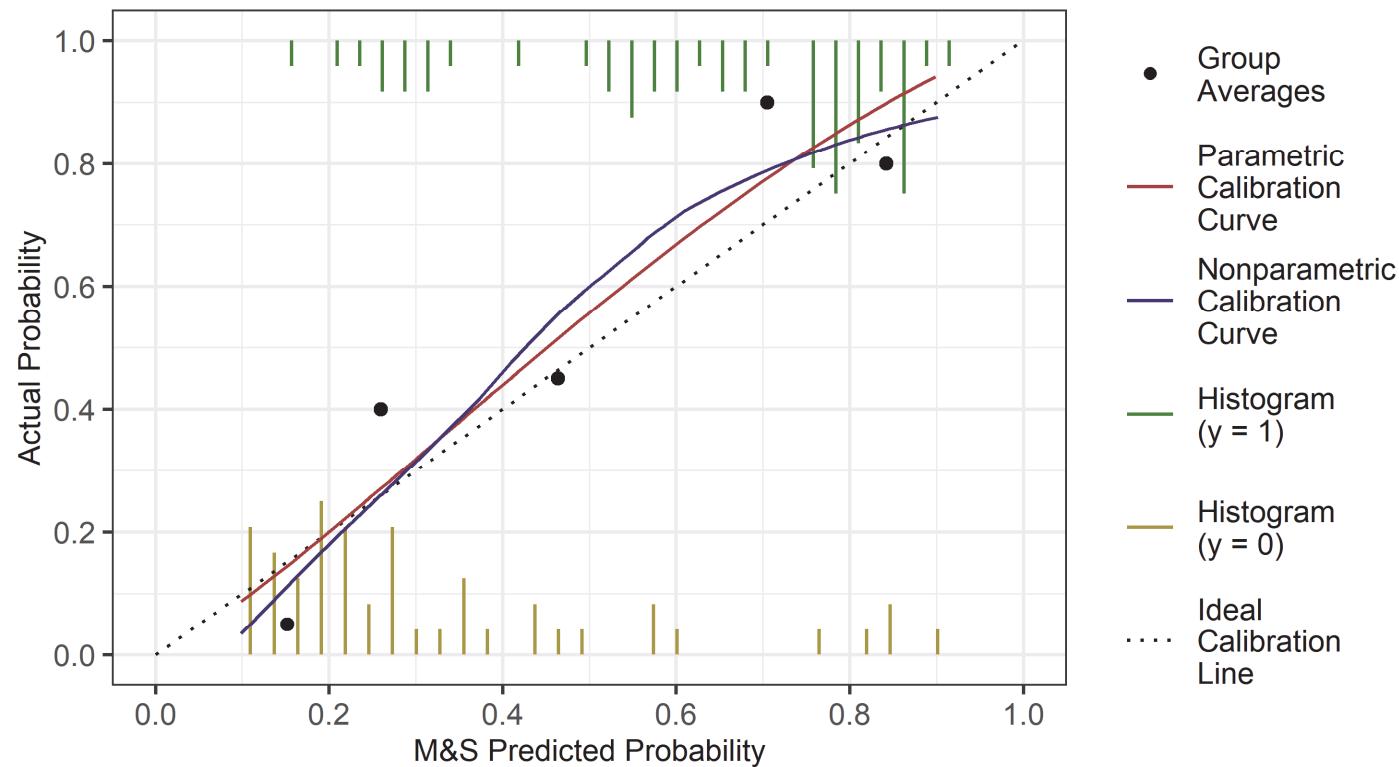


Average Range:  
Dart: 9.3 m  
Classic: 7.4 m  
WRPA: 11.3 m

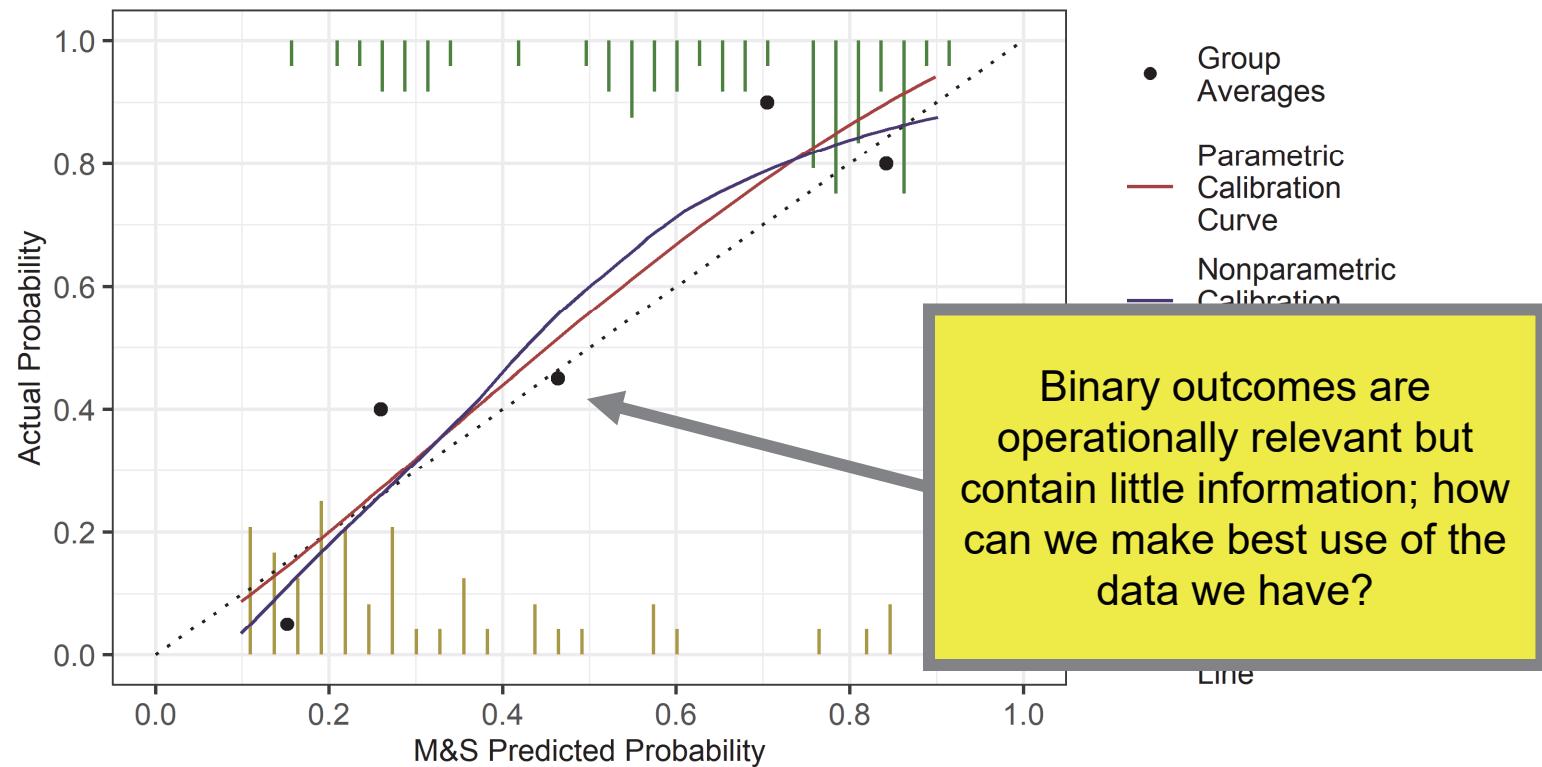
IS THIS  
RIGHT?



# How can statistical surrogates allow for M&S validation with small sample sizes?



# How can statistical surrogates allow for M&S validation with small sample sizes?



# How should statistical testing incorporate statistical surrogates?

$H_0$ : Sim=Live

$H_A$ :  $H_0$  is false

$$W = \frac{(\bar{Y}_{\text{sim}} - \bar{Y}_{\text{live}})^2}{\text{Var}(\bar{Y}_{\text{sim}} - \bar{Y}_{\text{live}})}$$

**THRESHOLD:**



**DATA:**



Reject if  $W > C_\alpha$ .

# How should statistical testing incorporate statistical surrogates?

$H_0$ : Sim=Live

$H_A$ :  $H_0$  is false

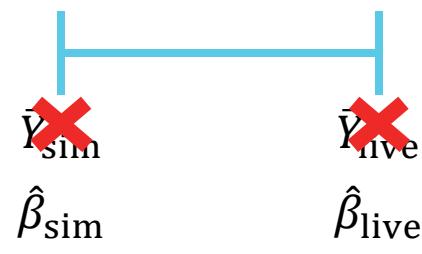
$$W = \frac{(\bar{Y}_{\text{sim}} - \bar{Y}_{\text{live}})^2}{\text{Var}(\bar{Y}_{\text{sim}} - \bar{Y}_{\text{live}})}$$

$$W = (\hat{\beta}_{\text{sim}} - \hat{\beta}_{\text{live}})^T V^{-1} (\hat{\beta}_{\text{sim}} - \hat{\beta}_{\text{live}})$$

THRESHOLD:



DATA:



$\hat{\beta}_{\text{sim}}$

$\hat{\beta}_{\text{live}}$

Reject if  $W > C_\alpha$ .

# How should statistical testing incorporate statistical surrogates?

$H_0$ : Sim=Live

$H_A$ :  $H_0$  is false

$$W = \frac{(\bar{Y}_{\text{sim}} - \bar{Y}_{\text{live}})^2}{\text{Var}(\bar{Y}_{\text{sim}} - \bar{Y}_{\text{live}})}$$

$$W = (\hat{\beta}_{\text{sim}} - \hat{\beta}_{\text{live}})^T V^{-1} (\hat{\beta}_{\text{sim}} - \hat{\beta}_{\text{live}})$$

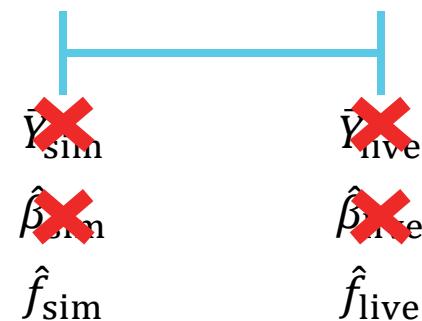
$$W = \|\hat{f}_{\text{sim}} - \hat{f}_{\text{live}}\|_{V^{-1}}^2$$

Reject if  $W > C_\alpha$ .

**THRESHOLD:**



**DATA:**



# How should statistical testing incorporate statistical surrogates?

$H_0$ : Sim=Live

$H_A$ :  $H_0$  is false

$$W = \frac{(\bar{Y}_{\text{sim}} - \bar{Y}_{\text{live}})^2}{\text{Var}(\bar{Y}_{\text{sim}} - \bar{Y}_{\text{live}})}$$

$$W = (\hat{\beta}_{\text{sim}} - \hat{\beta}_{\text{live}})^T V^{-1} (\hat{\beta}_{\text{sim}} - \hat{\beta}_{\text{live}})$$

$$W = \|\hat{f}_{\text{sim}} - \hat{f}_{\text{live}}\|_{V^{-1}}^2$$

**THRESHOLD:**



**DATA:**



Should these be  
- Regression model  
coefficients?  
- Response functions (e.g.,  
GAM smooth)?  
- Average response at select  
factor combinations?

Reject if  $W > C_\alpha$ .

How will we overcome the challenges of modeling and simulation validation?

# M&S validation faces many challenges that must be overcome

*Operational relevance*

*Continually changing  
M&S systems*

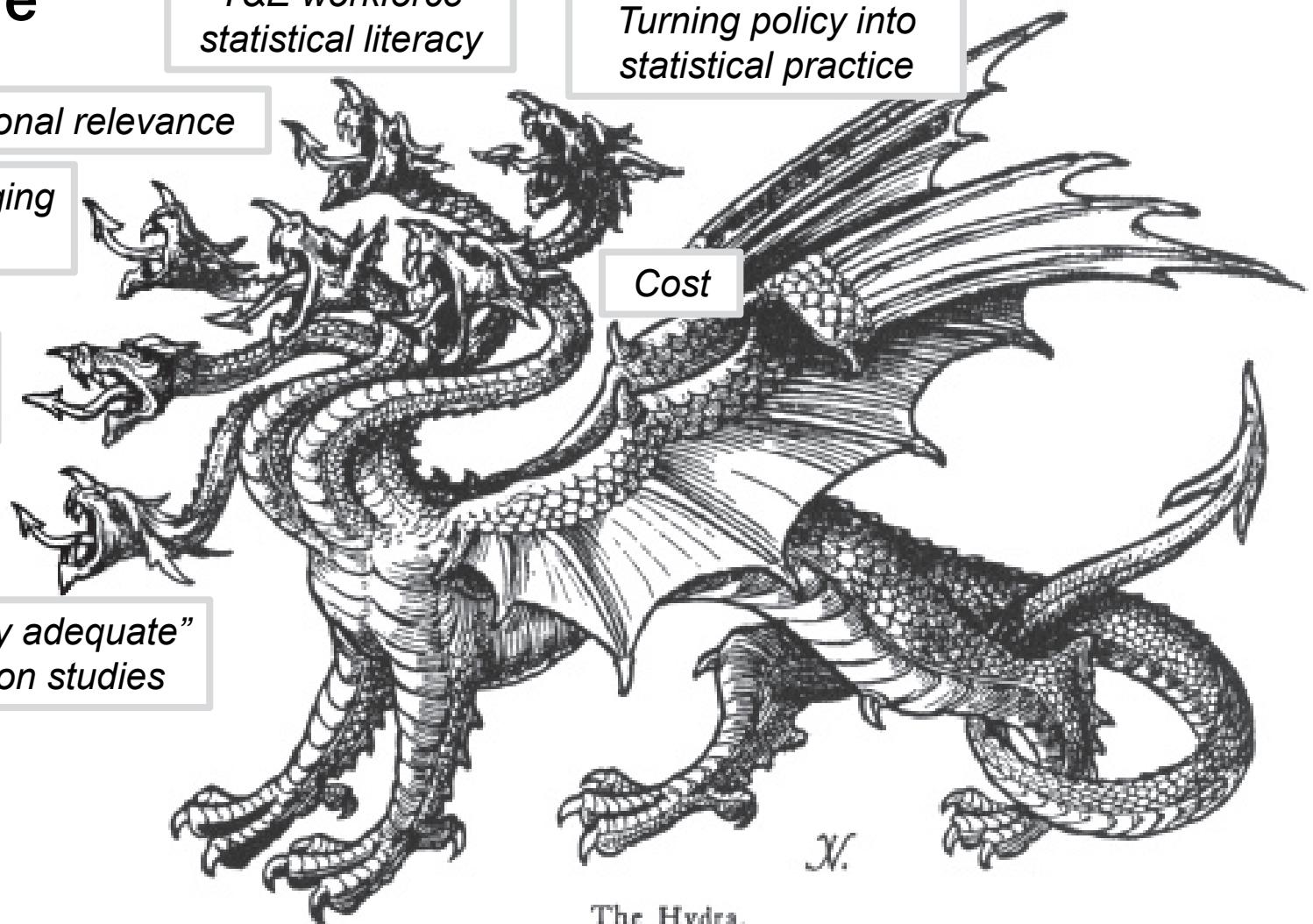
*Small real world  
samples*

*"Minimally adequate"  
simulation studies*

*T&E workforce  
statistical literacy*

*Turning policy into  
statistical practice*

*Cost*



The Hydra.

M&S – Modeling and Simulation; T&E – Test and Evaluation

# M&S validation faces many challenges that must be overcome

*Operational relevance*

*Continually changing  
M&S systems*

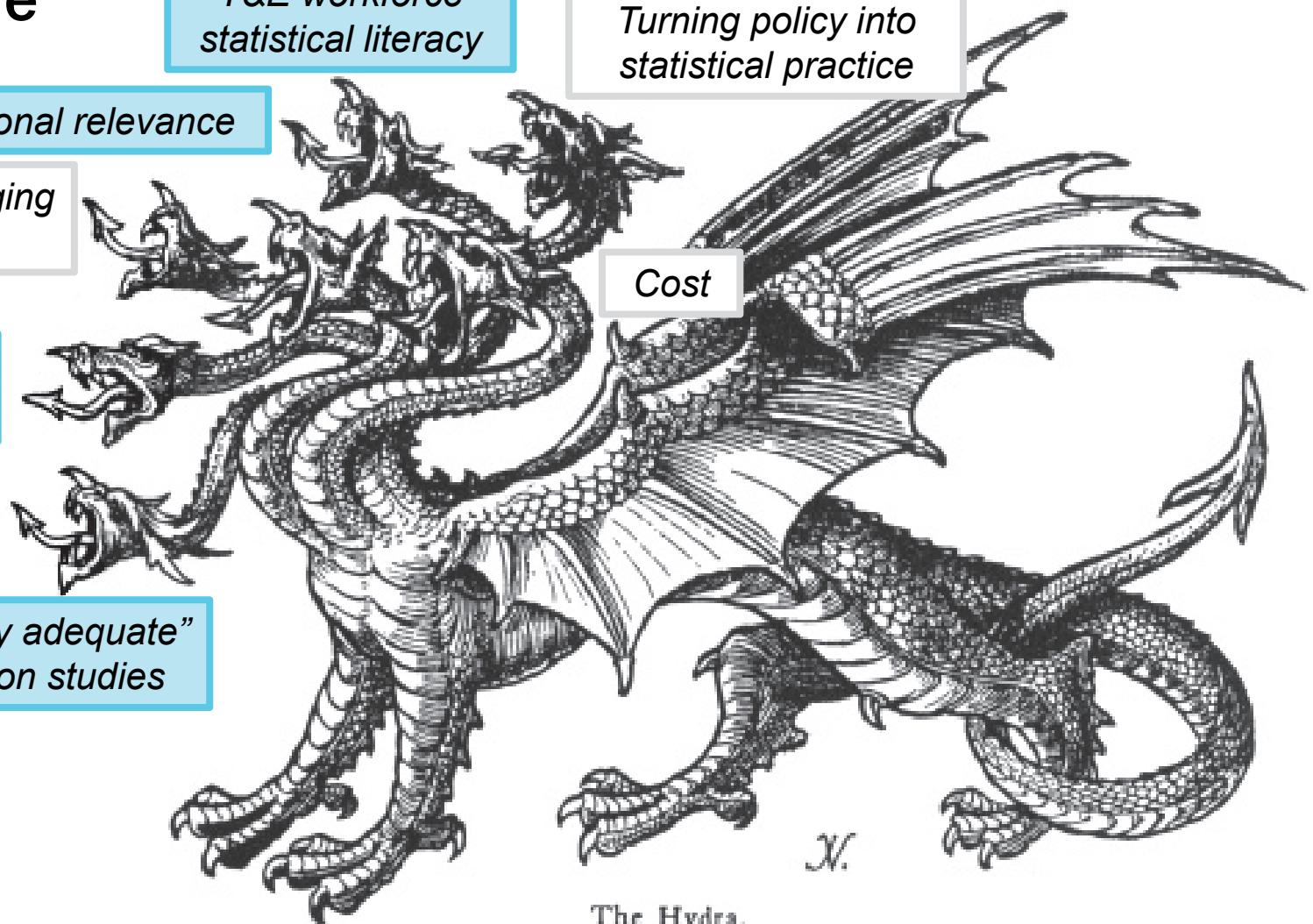
*Small real world  
samples*

*“Minimally adequate”  
simulation studies*

*T&E workforce  
statistical literacy*

*Turning policy into  
statistical practice*

*Cost*



The Hydra.

M&S – Modeling and Simulation; T&E – Test and Evaluation

# M&S validation faces many challenges that must be overcome

*Operational relevance*

*Continually changing  
M&S systems*

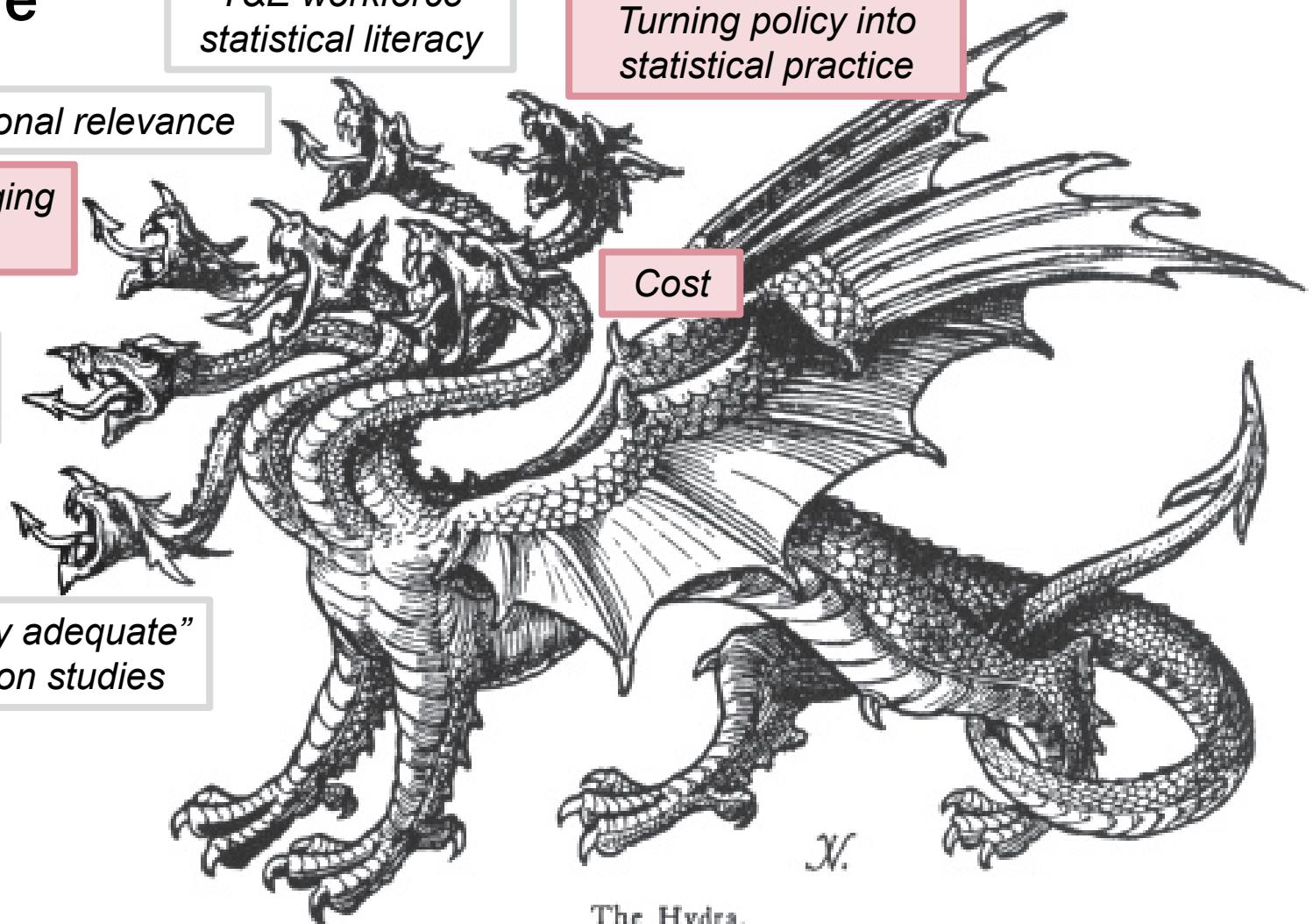
*Small real world  
samples*

*“Minimally adequate”  
simulation studies*

*T&E workforce  
statistical literacy*

*Turning policy into  
statistical practice*

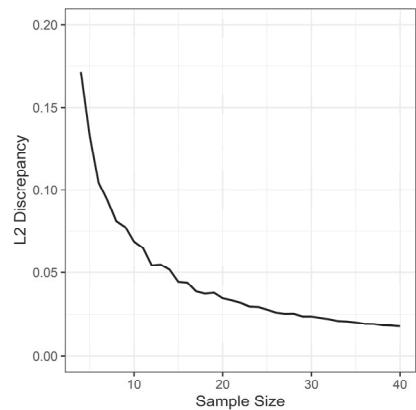
*Cost*



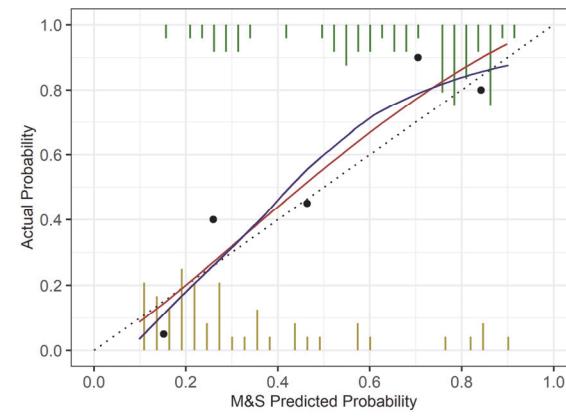
The Hydra.

M&S – Modeling and Simulation; T&E – Test and Evaluation

# We want statisticians to address...



*... how many observations to collect from M&S*



*... how to best handle binary responses*

$$W = \|\hat{f}_{\text{sim}} - \hat{f}_{\text{live}}\|_{V^{-1}}^2$$

*... how to make statistical decisions with surrogate models*

**REPORT DOCUMENTATION PAGE**

Form Approved  
OMB No. 0704-0188

The public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing the burden, to Department of Defense, Washington Headquarters Services, Directorate for Information Operations and Reports (0704-0188), 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to any penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.

**PLEASE DO NOT RETURN YOUR FORM TO THE ABOVE ADDRESS.**

1. REPORT DATE (DD-MM-YYYY)			2. REPORT TYPE		3. DATES COVERED (From - To)	
4. TITLE AND SUBTITLE			<p>5a. CONTRACT NUMBER</p> <p>5b. GRANT NUMBER</p> <p>5c. PROGRAM ELEMENT NUMBER</p>			
6. AUTHOR(S)			<p>5d. PROJECT NUMBER</p> <p>5e. TASK NUMBER</p> <p>5f. WORK UNIT NUMBER</p>			
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)					8. PERFORMING ORGANIZATION REPORT NUMBER	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)					10. SPONSOR/MONITOR'S ACRONYM(S)	
					11. SPONSOR/MONITOR'S REPORT NUMBER(S)	
12. DISTRIBUTION/AVAILABILITY STATEMENT						
13. SUPPLEMENTARY NOTES						
14. ABSTRACT						
15. SUBJECT TERMS						
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON	
a. REPORT	b. ABSTRACT	c. THIS PAGE			19b. TELEPHONE NUMBER (Include area code)	