INSTITUTE FOR DEFENSE ANALYSES

DATAWorks 2024: A Preview of Functional Data Analysis for Modeling and Simulation Validation

Rebecca M. Medlin, Project Leader

Curtis G. Miller

March 2024

IDA Product ID 3001829

INSTITUTE FOR DEFENSE ANALYSES
730 East Glebe Road
Alexandria, Virginia 22305

The Institute for Defense Analyses is a nonprofit corporation that operates three Federally Funded Research and Development Centers. Its mission is to answer the most challenging U.S. security and science policy questions with objective analysis, leveraging extraordinary scientific, technical, and analytic expertise.

About This Publication
This work was conducted by the Institute for Defense Analyses (IDA) under contract HQ0034-19-D-0001, Task C9082, "CRP Statistics and Data Science Working Group." The views, opinions, and findings should not be construed as representing the official position of either the Department of Defense or the sponsoring organization.

Acknowledgments
The IDA Technical Review Committee was chaired by Dr. V. Bram Lillard and consisted of Dr. John Haman, Dr. Kelly Avery, Dr. Keyla Pagan-Rivera, Dr. Logan Ausman, Dr. Luis Aguirre, and Dr. Nikolai Lipscomb from the Operational Evaluation Division.

For more information:
Dr. Rebecca M. Medlin, Project Leader
rmedlin@ida.org • 703-845-6731

Dr. V. Bram Lillard, Director, Operational Evaluation Division
vlillard@ida.org • (703) 845-2230

Rigorous Analysis | Trusted Expertise | Service to the Nation

INSTITUTE FOR DEFENSE ANALYSES

IDA Product ID 3001829

DATAWorks 2024: A Preview of Functional Data Analysis for Modeling and Simulation Validation

Rebecca M. Medlin, Project Leader

Curtis G. Miller

Executive Summary

In operational test and evaluation, a single observation (such as a radar track on an aircraft) is oftentimes more complex than a simple number. It is a multi-dimensional time series of information, and a single run might contain thousands of data points. To validate radar signal emulators, testers are advantaged to use functional data analysis (FDA), which appropriately considers the time-correlated, multi-dimensional structure of radar track data.

Many analysts validate radar track data by creating plots of radar tracks seen in simulation, a plot of the track observed in a live test, and judging visually whether the live track appears typical. While plotting radar tracks provides useful information, modeling and simulation (M&S) validation should not end at judgements about plots. The resulting judgements are subjective, feature no statistical error control, may be difficult to read, and the human eye may not be able to detect atypical behavior except for obvious situations. FDA may provide the rigorous framework that plot reading lacks.

We demonstrate the utility of FDA by analyzing a notional data set of radar tracks, featuring 20 live scenarios and 100 M&S replicates per scenario.

The basic FDA methodologies we propose include:

- computing mean tracks and confidence intervals for radar tracks to characterize what the mean track may be,
- performing a Fisher combined probability test on radar tracks to determine whether tracks collected from live testing came from the same distribution as the simulated tracks, and
- using functional principal component analysis to make functional data resemble simpler multivariate data, giving us access to more statistical tools.

We describe how FDA differs from other analyses. Ultimately, we advocate for the FDA approach in validation studies of radar track data because it provides a more wholistic, thoughtful, and applicable validation.

This presentation previews FDA. We expect FDA to be useful in contexts outside validation of M&S radar tracks. The field of FDA is also larger than the two methods shown here, though the methods we discuss both could be used as

an analysis and serve as key steps to moving into more advanced methods.

Ramsay and Silverman's *Functional Data Analysis* and Horváth and Kokoszka's *Inference for Functional Data with Applications* provide more information on applying FDA methods.

We hope that analysts will consider further how to apply FDA methods to test data.

A Preview of Functional Data Analysis for Modeling and Simulation Validation

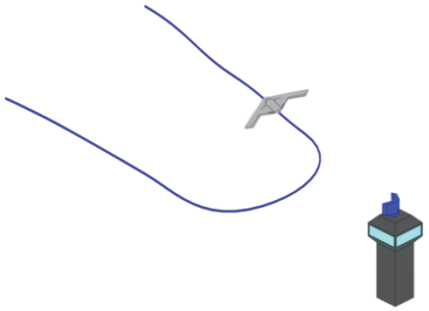
Curtis Miller

April 17, 2024

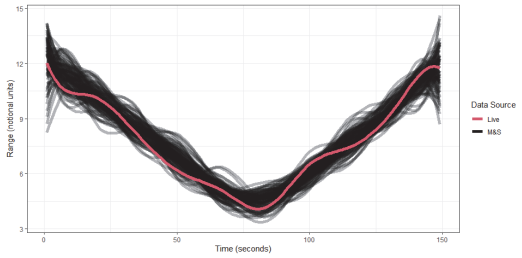
Institute for Defense Analyses
730 East Glebe Road ● Alexandria, Virginia 22305

Studying whole radar tracks could improve modeling and simulation validation assessments

Modeling and simulation validation may involve comparing live data to simulation outputs

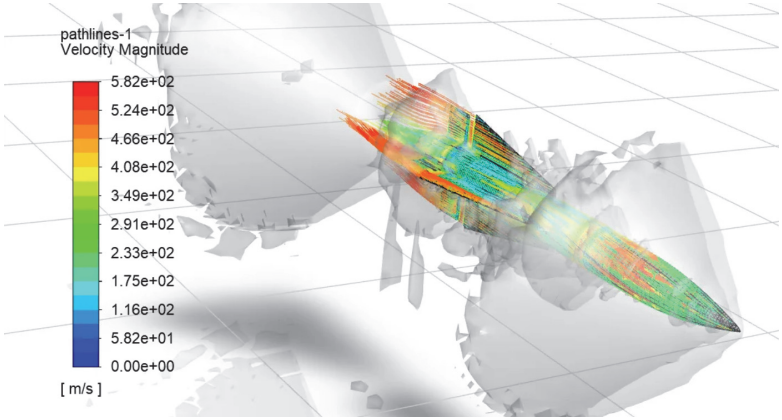


Data sets like radar tracks should be assessed as a whole



Functional data methods can use whole tracks to allow for modeling and simulation assessments

Weapon system modeling and simulation estimates performance with better statistical precision if the M&S is trustworthy



Digital Simulation

(By Bean_from_accounts, *Low-quality steady-state CFD simulation of an AIM-54 missile flying at Mach 1.5*, 2021, Reddit, https://www.reddit.com/r/dcsworld/comments/mqmvy3/lowquality_steadystate_cfd_simulation_of_an_aim54)

Hardware in the Loop

(From Elliot Bartis, *A Validation Case Study: The Environment Centric Weapons Analysis Facility (ECWAF)*, 2020, Institute for Defense Analyses, https://testscience.org/wp-content/uploads/formidable/13/D-12081-A-Validation-Case-Study.pptx)

M&S: Modeling and Simulation

Weapon system modeling and simulation estimates performance with better statistical precision if the M&S is trustworthy

HOW SHOULD WE STATISTICALLY COMPARE M&S OUTPUTS TO LIVE DATA?

Digital Simulation

(By Bean_from_accounts, *Low-quality steady-state CFD simulation of an AIM-54 missile flying at Mach 1.5*, 2021, Reddit, https://www.reddit.com/r/dcsworld/comments/mqmvy3/lowquality_steadystate_cfd_simulation_of_an_aim54)

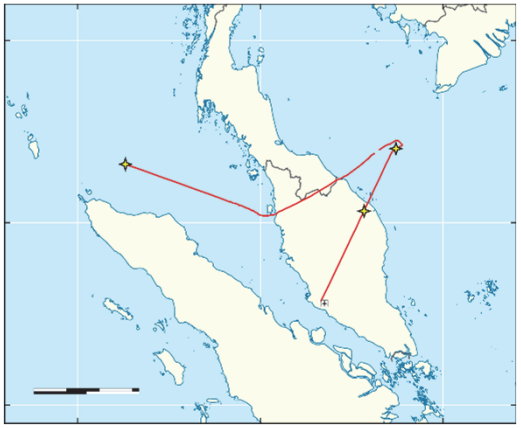
Hardware in the Loop

(From Elliot Bartis, *A Validation Case Study: The Environment Centric Weapons Analysis Facility (ECWAF)*, 2020, Institute for Defense Analyses, https://testscience.org/wp-content/uploads/formidable/13/D-12081-A-Validation-Case-Study.pptx)

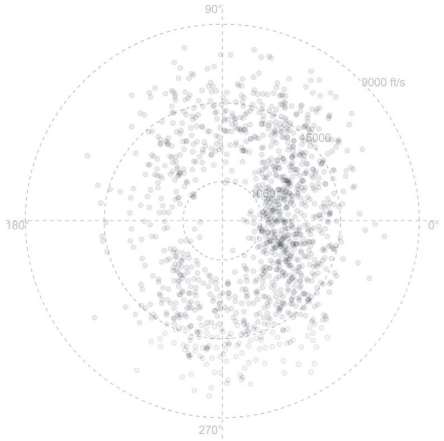
M&S: Modeling and Simulation

Test and evaluation involves data sets that can be data rich even with few trials

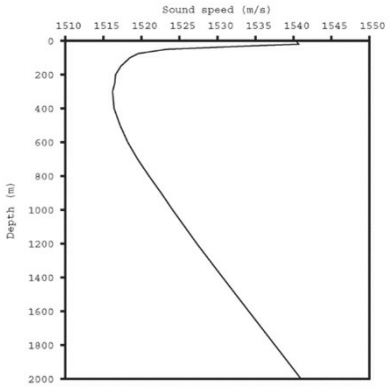
Radar Tracks



Flight Paths



Blast Fragment Distribution

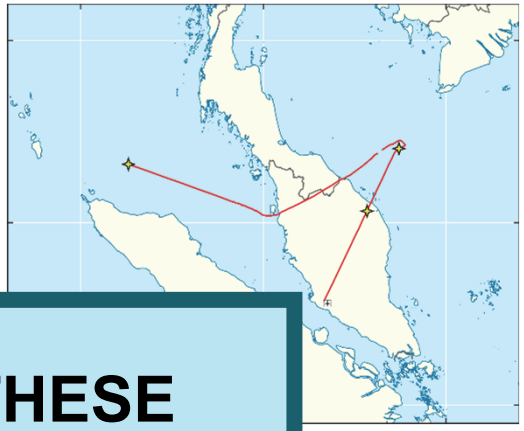


Sound Velocity Profile

(Radar tracks from National Archives, *A Close-Up View of One of the Radar Consoles Used for Tracking Aircraft During the Air-to-Air Combat Training Exercise William Tell '82,* October 9, 1982, https://catalog.archives.gov/id/6367394 ; flight paths by Andrew Heneen, *The Known Flight Path of Malaysia Airlines Flight 370*, December 12, 2014, Wikimedia, https://upload.wikimedia.org/wikipedia/commons/d/d2/MH370_flight_path_blank.png; blast fragment distribution from Mark Couch et al., *DATAWorks 2021: Warhead Arena Analysis Advancements*, April, 2021, Institute for Defense Analyses, https://www.ida.org/-/media/feature/publications/d/da/dataworks-2021-warhead-arena-analysis-advancements/d-11038.ashx; sound velocity profile by Jprockett66, *Sound Speed Profile*, October 13, 2012, Wikipedia, https://en.wikipedia.org/wiki/Sound_speed_profile.)

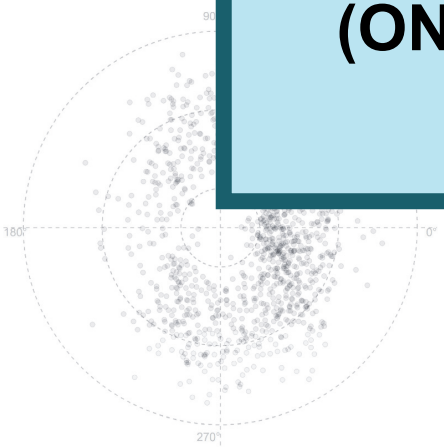
Test and evaluation involves data sets that can be data rich even with few trials

Radar T

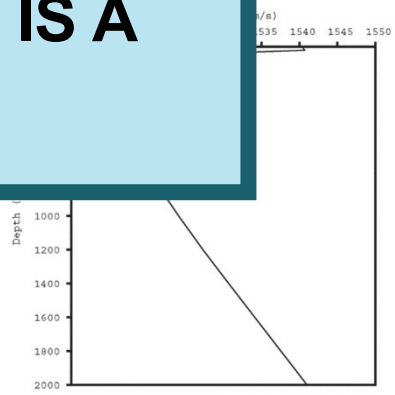


aths

WE WILL CALL THESE FUNCTIONAL DATA SETS (ONE "OUTCOME" IS A FUNCTION)



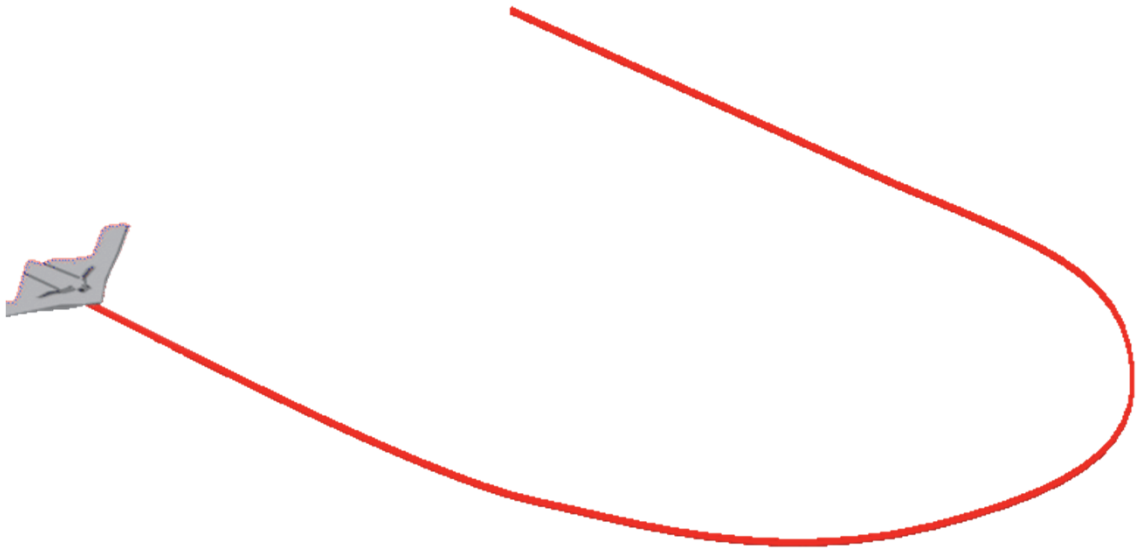
Blast Fragment Distribution

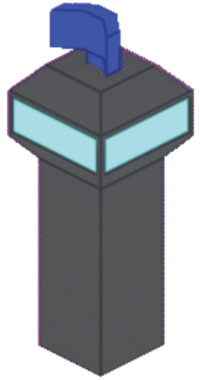


Sound Velocity Profile

(Radar tracks from National Archives, *A Close-Up View of One of the Radar Consoles Used for Tracking Aircraft During the Air-to-Air Combat Training Exercise William Tell '82,* October 9, 1982, https://catalog.archives.gov/id/6367394 ; flight paths by Andrew Heneen, *The Known Flight Path of Malaysia Airlines Flight 370*, December 12, 2014, Wikimedia, https://upload.wikimedia.org/wikipedia/commons/d/d2/MH370_flight_path_blank.png; blast fragment distribution from Mark Couch et al., *DATAWorks 2021: Warhead Arena Analysis Advancements*, April, 2021, Institute for Defense Analyses, https://www.ida.org/-/media/feature/publications/d/da/dataworks-2021-warhead-arena-analysis-advancements/d-11038.ashx; sound velocity profile by Jprockett66, *Sound Speed Profile*, October 13, 2012, Wikipedia, https://en.wikipedia.org/wiki/Sound_speed_profile.)

Our running example will consider modeling the radar track of a bomber's flight

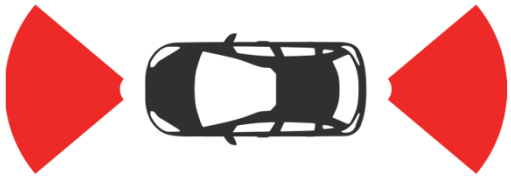




All data in this presentation is notional

Different radars provide different information about the objects they track

Velocity



Range

Two-Dimensional Motion

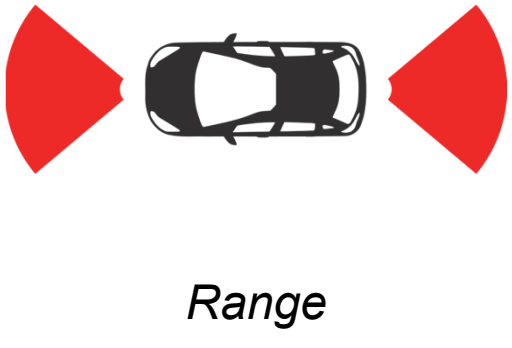
Three-Dimensional Motion

(First image by Fábio Pozzebom, *Microdigicam Laser Radar Gun in Use in Brazil*, February 20, 2007, https://en.wikipedia.org/wiki/Radar_speed_gun#/media/File:Radarvelocidade20022007.jpg; third image by Amada44, *Marine Radar*, 2011, Wikipedia, https://en.wikipedia.org/wiki/Marine_radar#/media/File:Rotating_marine_radar_-_rotating_waveguide_antenna.gif; fourth image by Christopher Smith, *Patriot Radar*, April 25, 2019, 10th Army Air and Missile Defense Command, Mielec, https://www.dvidshub.net/image/7251462/patriot-radar)

Different radars provide different information about the objects they track

Vel

Demonstrated today



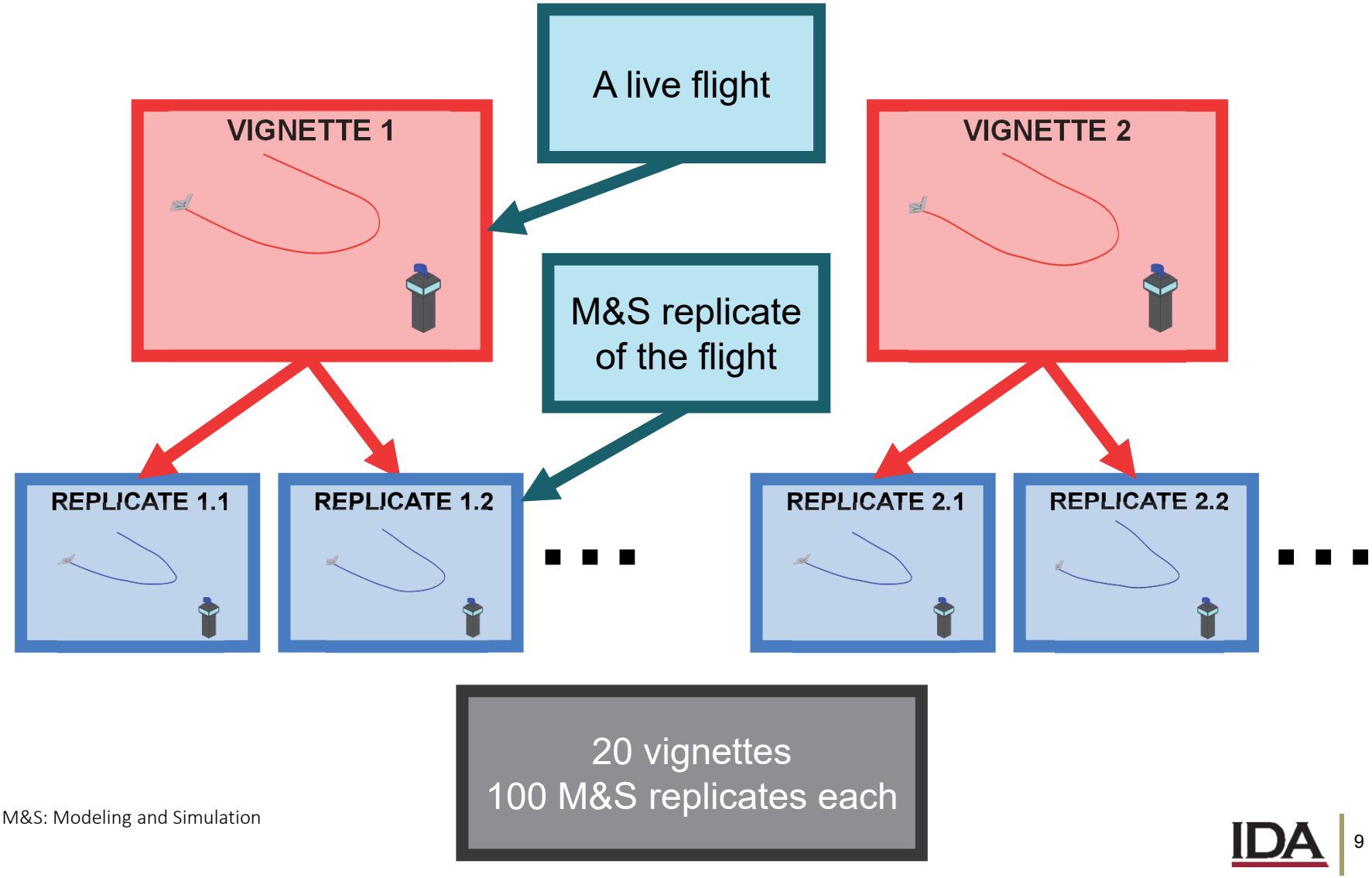
Range

Two-Dimensional Motion

Three-Dimensional Motion

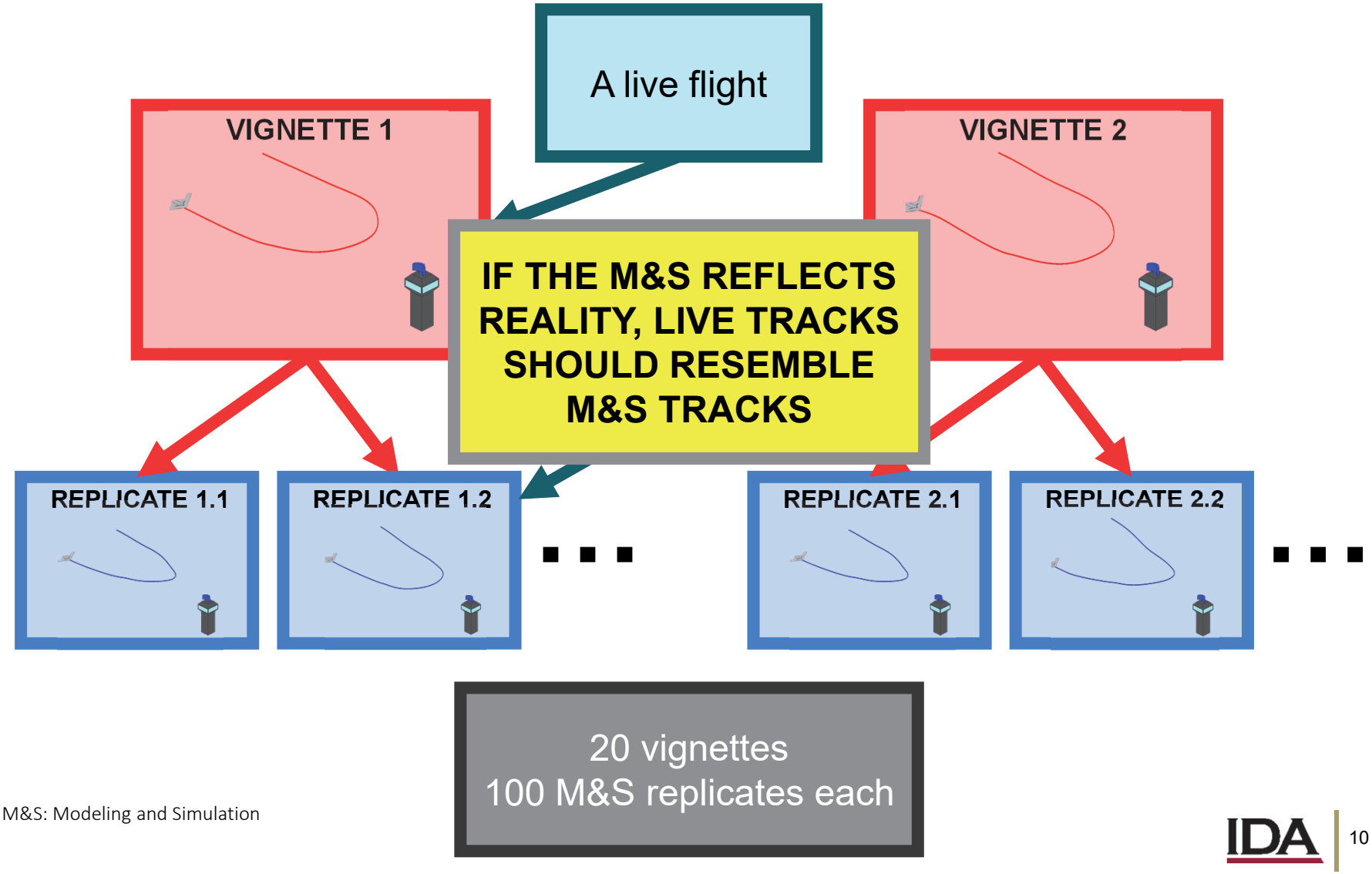
(First image by Fábio Pozzebom, *Microdigicam Laser Radar Gun in Use in Brazil*, February 20, 2007, Wikipedia, https://en.wikipedia.org/wiki/Radar_speed_gun#/media/File:Radarvelocidade20022007.jpg; third image by Amada44, *Marine Radar*, 2011, Wikipedia, https://en.wikipedia.org/wiki/Marine_radar#/media/File:Rotating_marine_radar_-_rotating_waveguide_antenna.gif; fourth image by Christopher Smith, *Patriot Radar*, April 25, 2019, 10th Army Air and Missile Defense Command, Mielec, https://www.dvidshub.net/image/7251462/patriot-radar)

In our demonstration, 20 live range-only radar tracks are replicated with 100 M&S tracks each



M&S: Modeling and Simulation

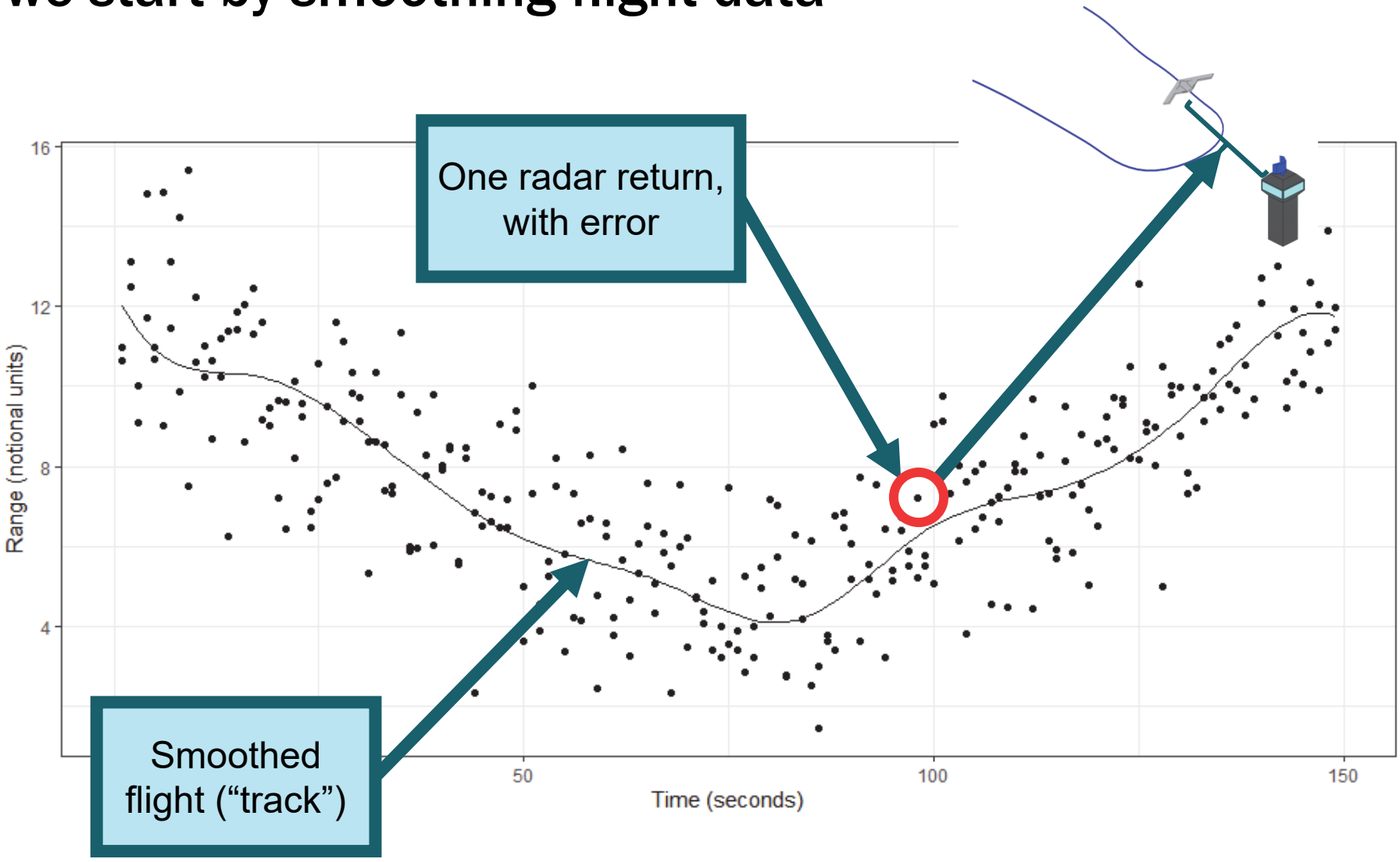
In our demonstration, 20 live range-only radar tracks are replicated with 100 M&S tracks each



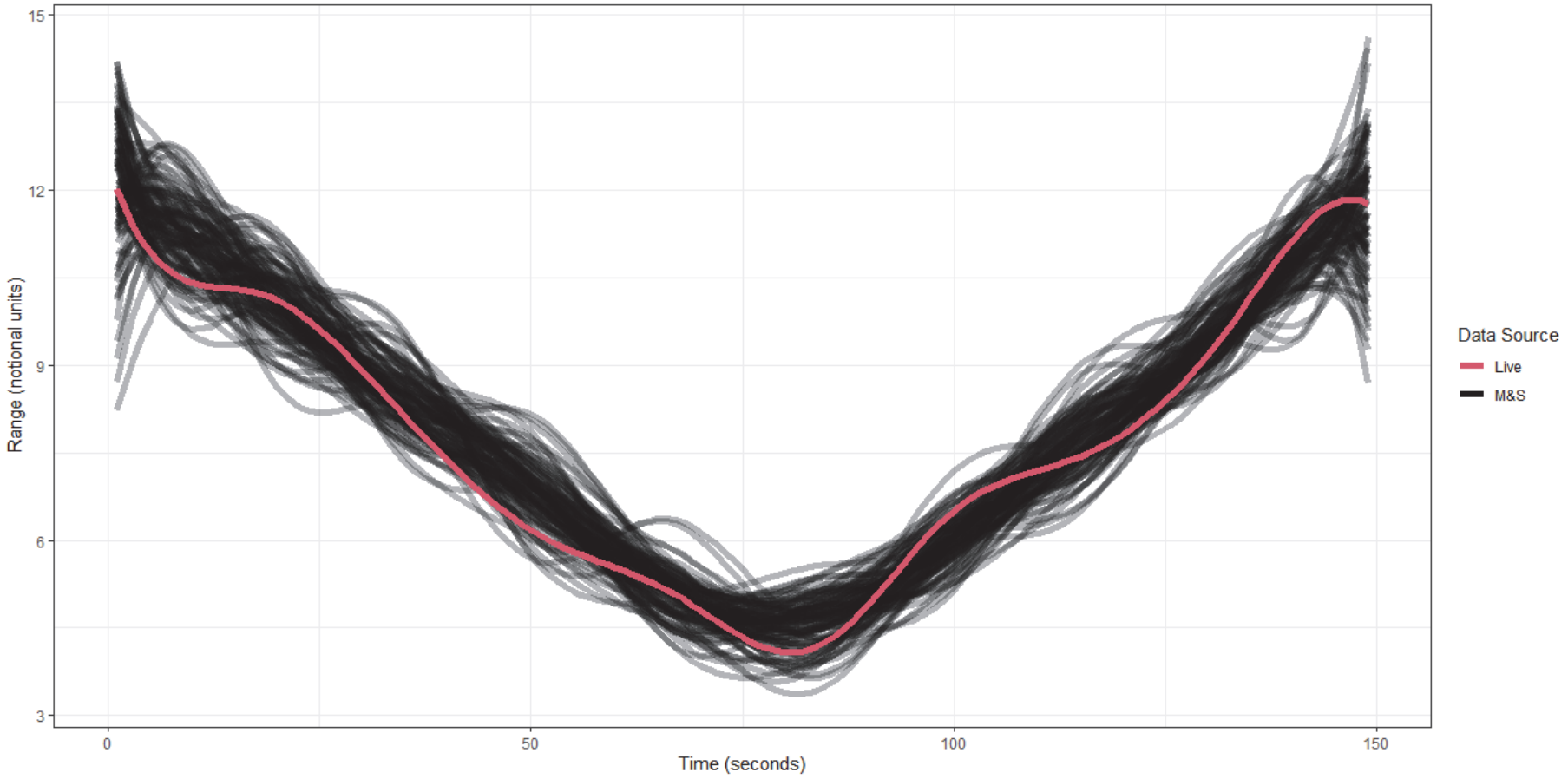
M&S: Modeling and Simulation

DEMONSTRATION

Our data consist of range to object over time per flight; we start by smoothing flight data

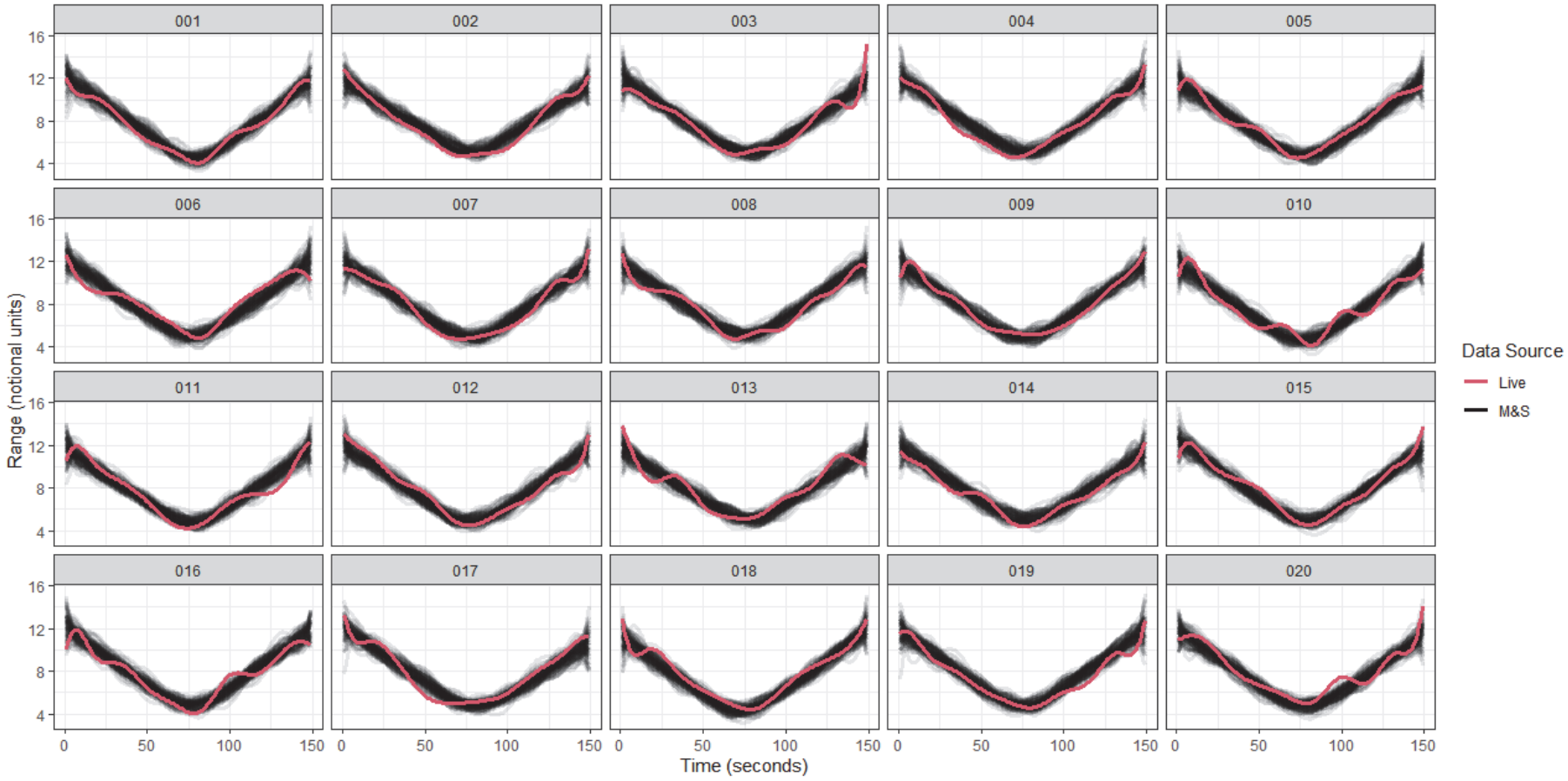


For a single vignette, we obtain one live radar track and 100 M&S replication radar tracks

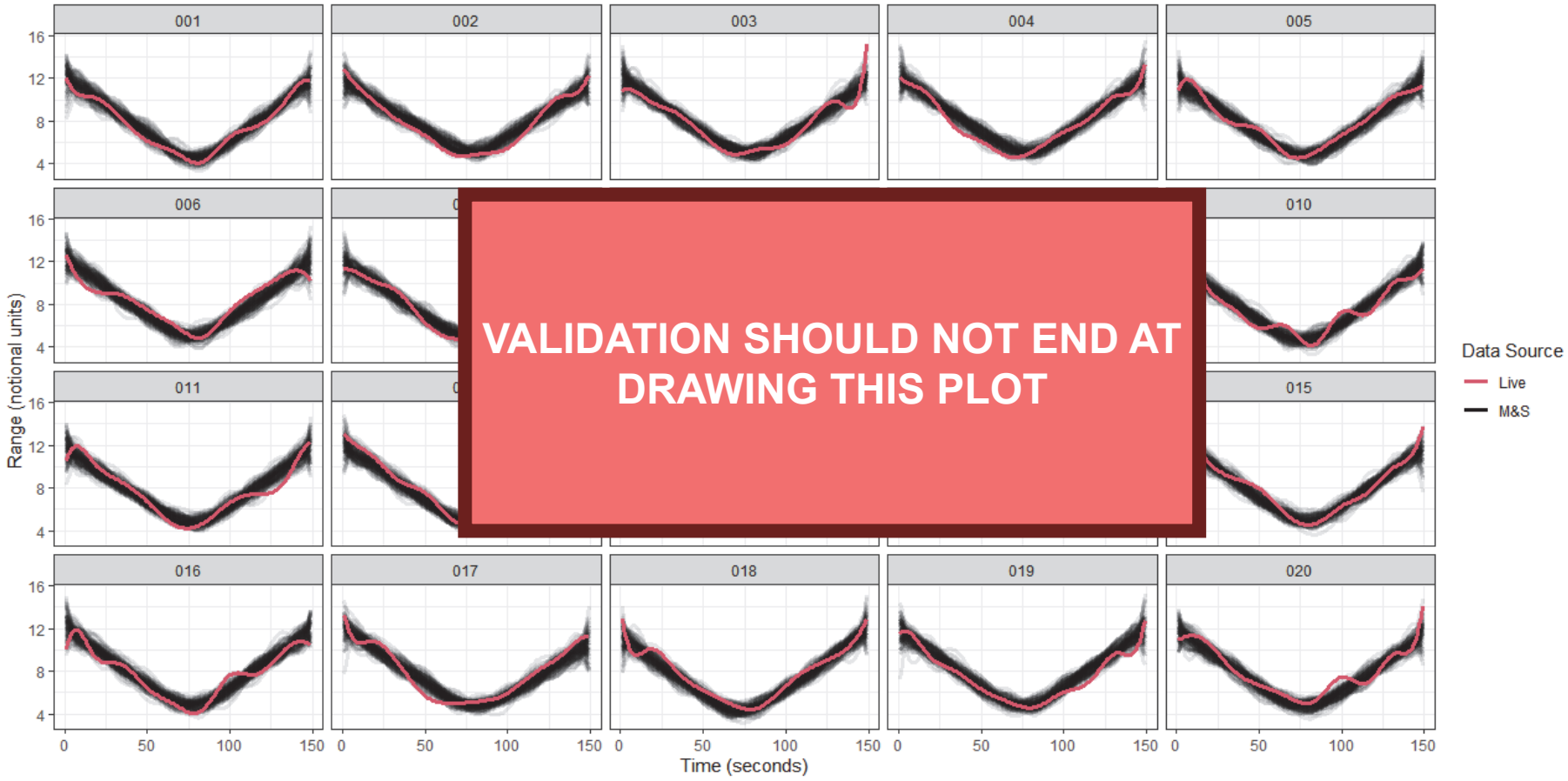


M&S: Modeling and Simulation

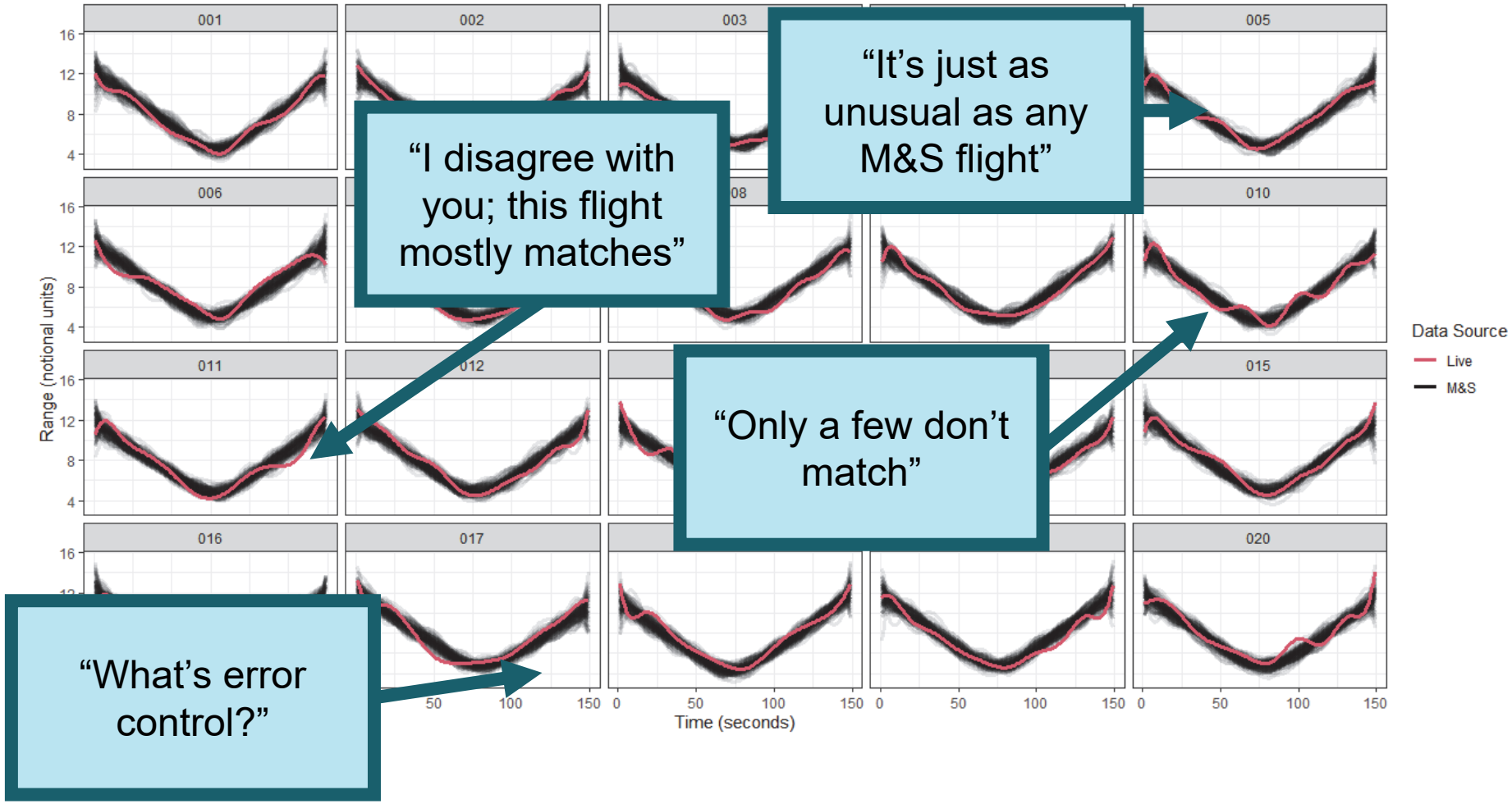
We obtain smoothed flight data sets for each live and simulation track in each vignette



We obtain smoothed flight data sets for each live and simulation track in each vignette



Validation based only on this plot is insufficient



We will compare the live track to the mean track per vignette, assuming the M&S agrees with live data

SAMPLE MEAN FLIGHT PATH

$$\overline{X}_j(t) = \frac{1}{n_j} \sum_{i=1}^{n_j} X_{ij}(t)$$

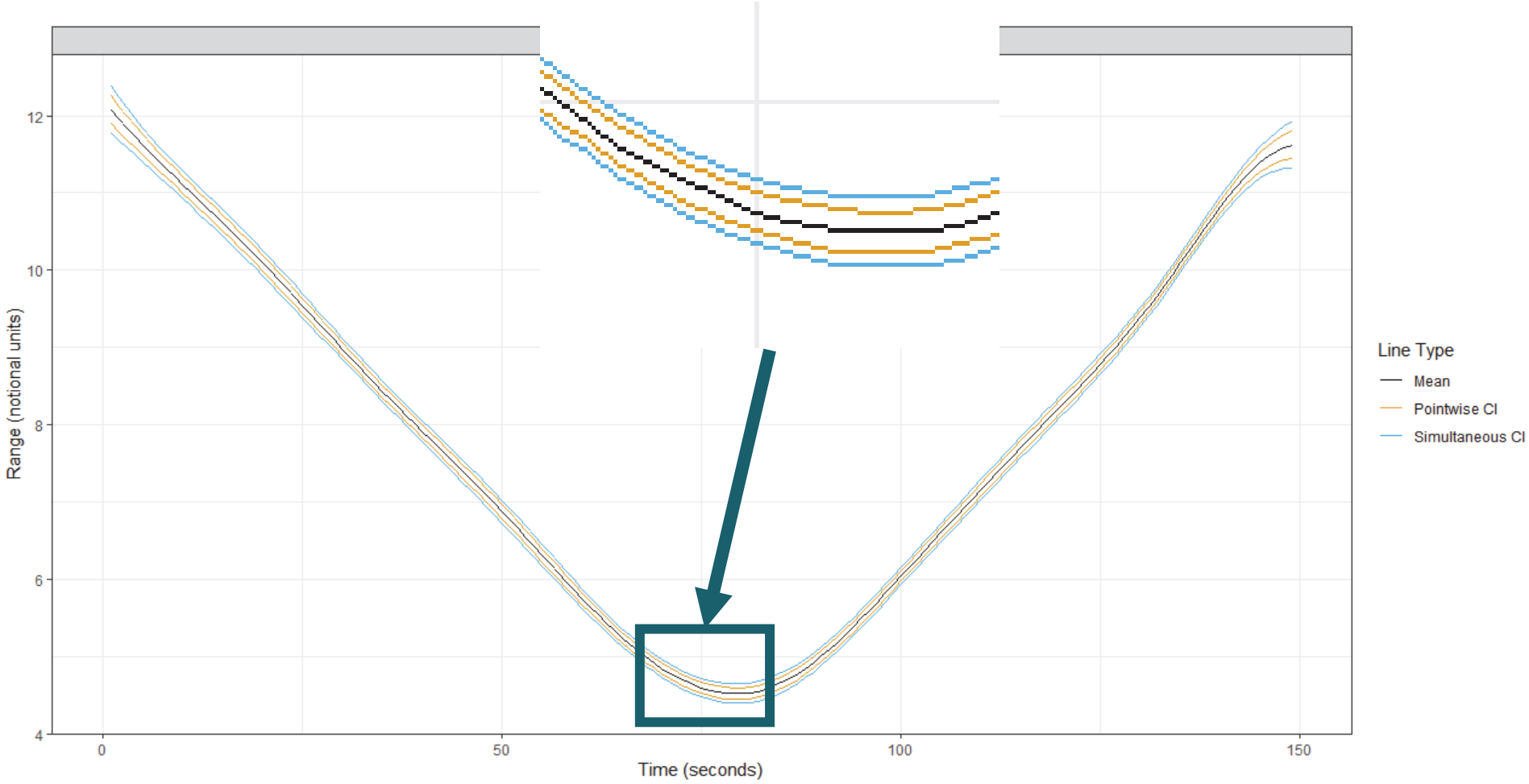
Mean position at time t for vignette j

n flights for vignette j

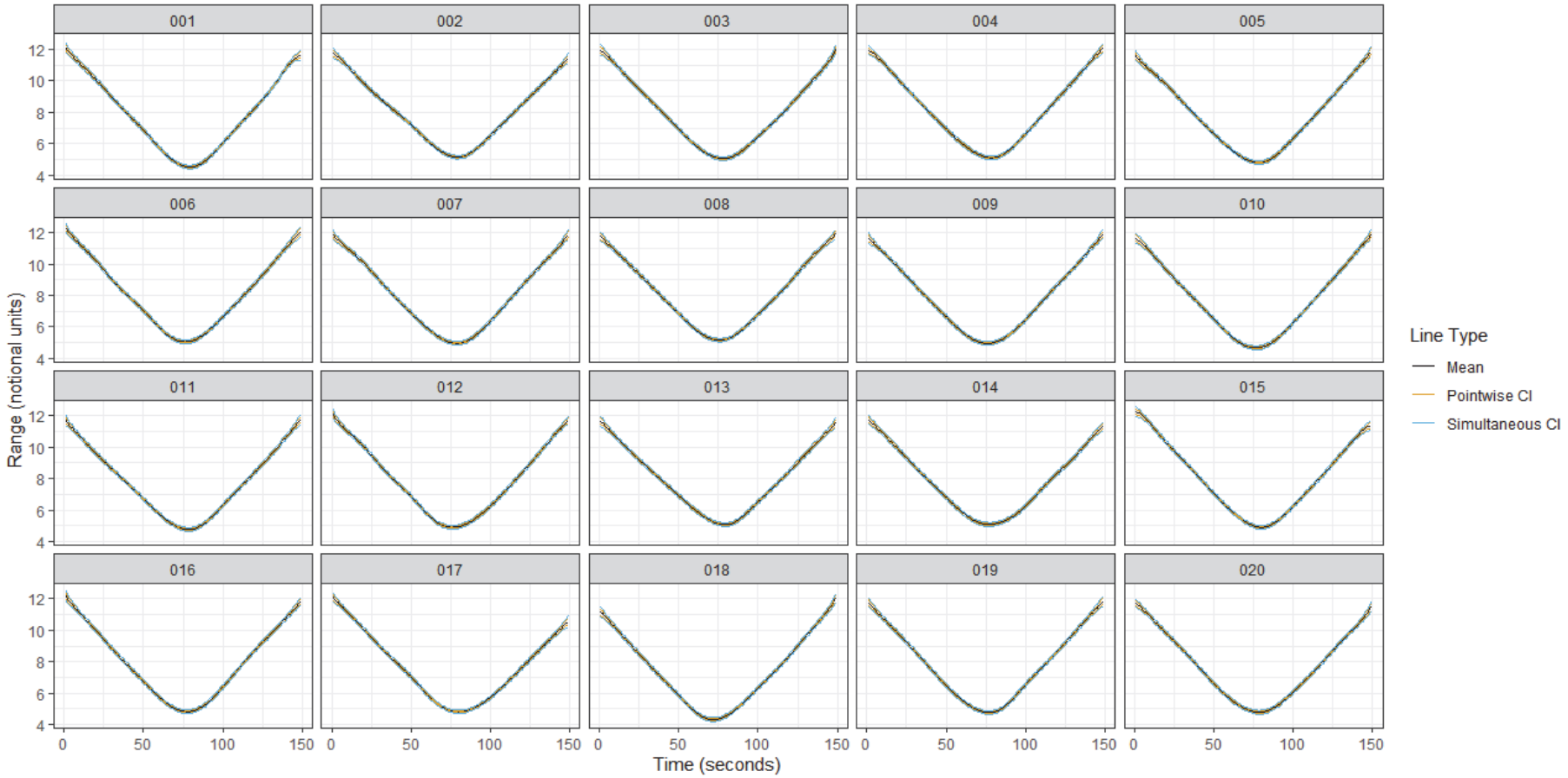
Position for flight i at time t for vignette j

M&S: Modeling and Simulation

Analysts should construct confidence intervals appropriate for functional data



We compute mean tracks for each vignette for later use



We will examine how different the flights tend to be from the mean flight

DISTANCE BETWEEN FUNCTIONS

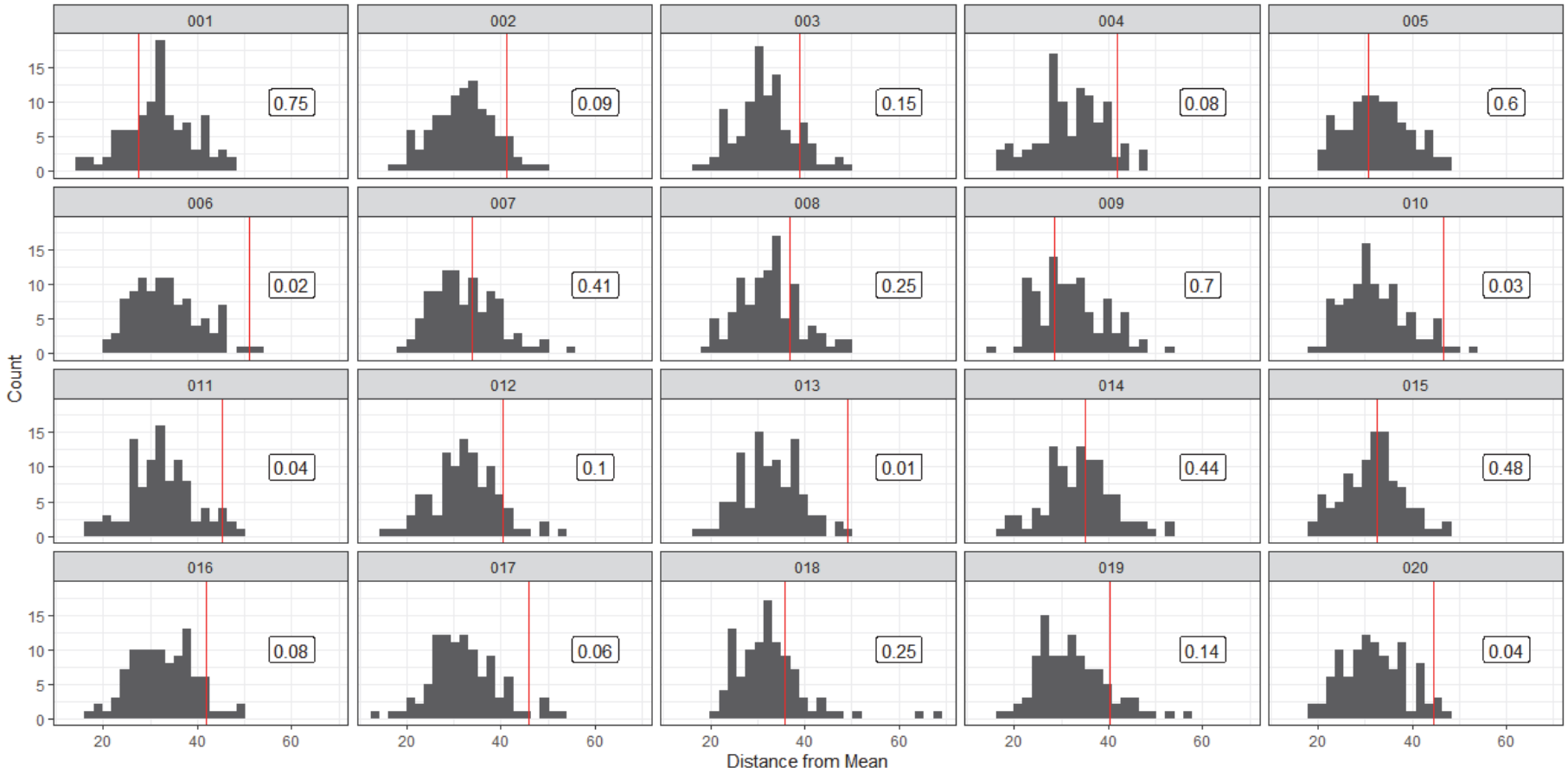
$$\|f - g\| = \sqrt{\int_0^T \big(f(t) - g(t)\big)^2 dt}$$

Substitute with mean flight $\overline{X}_j(t)$

Substitute with individual flight $X_{ij}(t)$

M&S: Modeling and Simulation

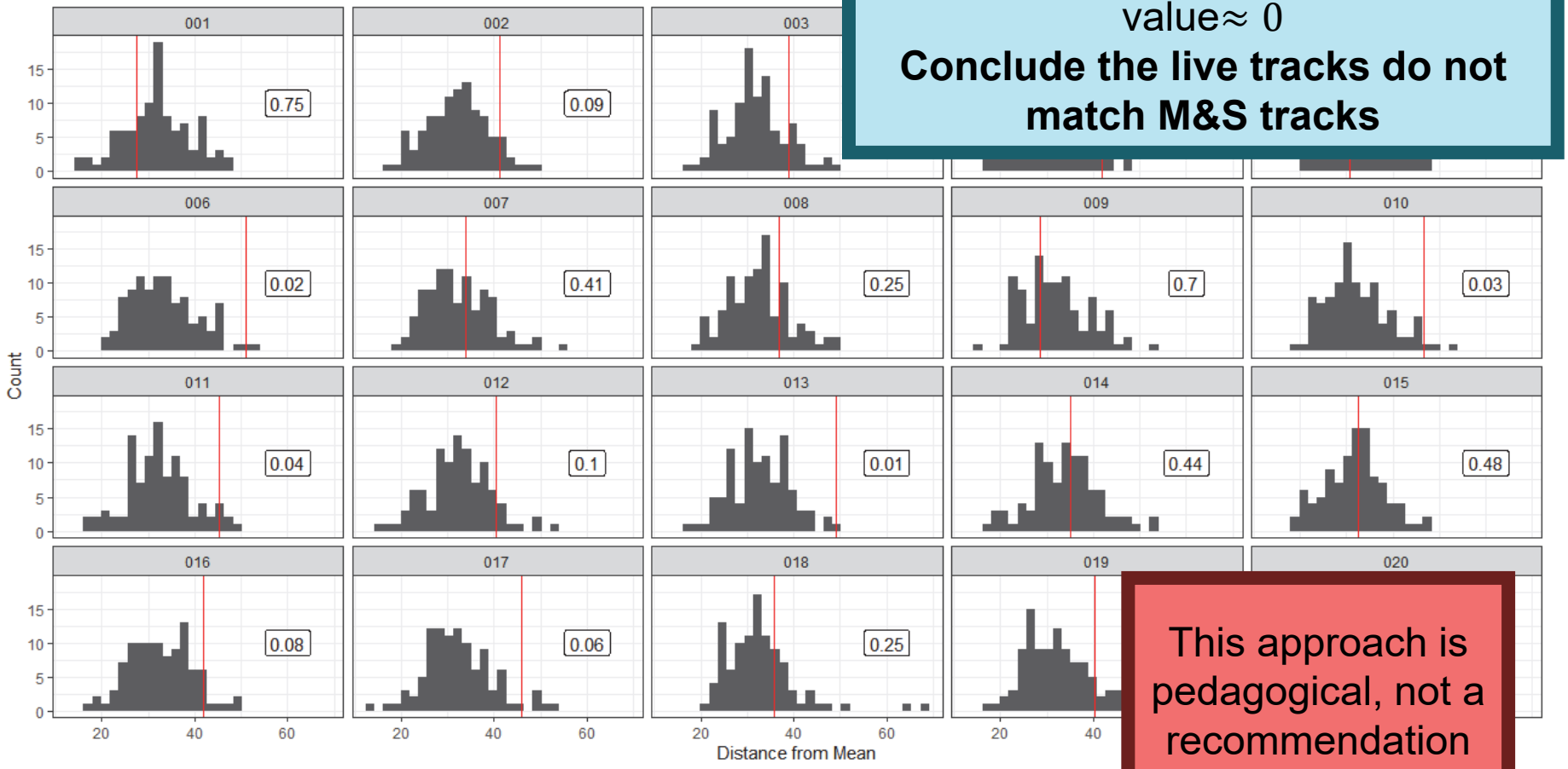
The live tracks tend to be distant from the mean relative to the simulated tracks



The live tracks tend to be distant from the mean relative to the simulated tracks

Fisher Combined Probability Test p-value≈ 0

Conclude the live tracks do not match M&S tracks

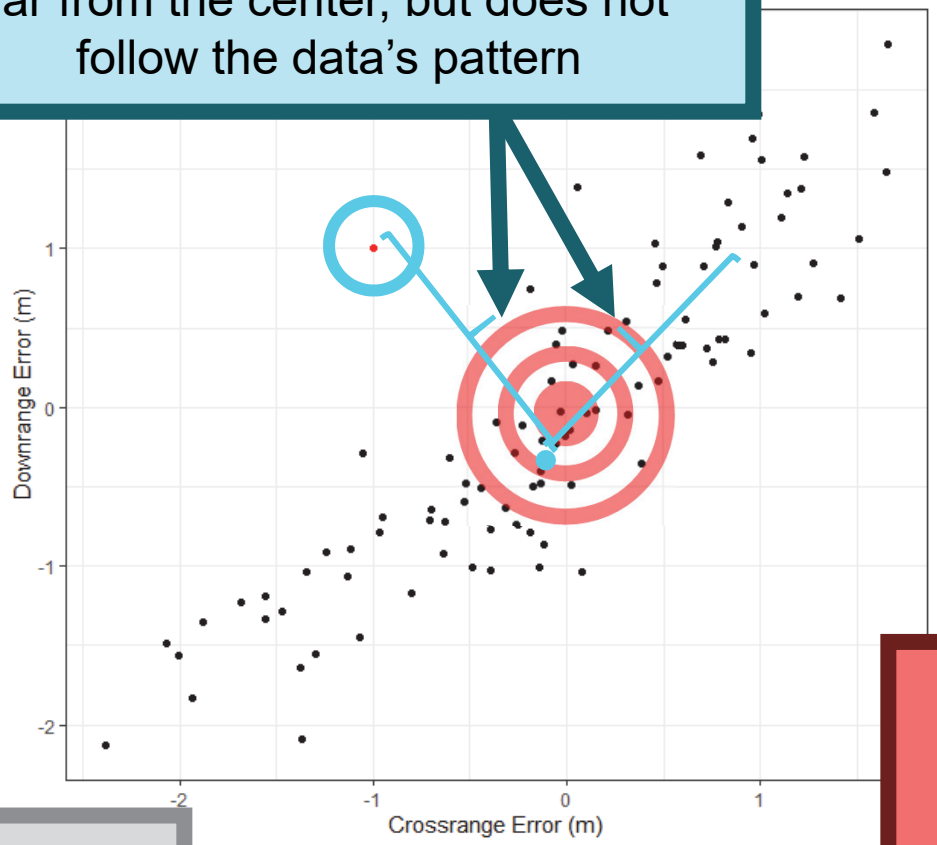


This approach is pedagogical, not a recommendation

M&S: Modeling and Simulation

Looking at distance alone would not reveal outliers in general for data sets with two variables

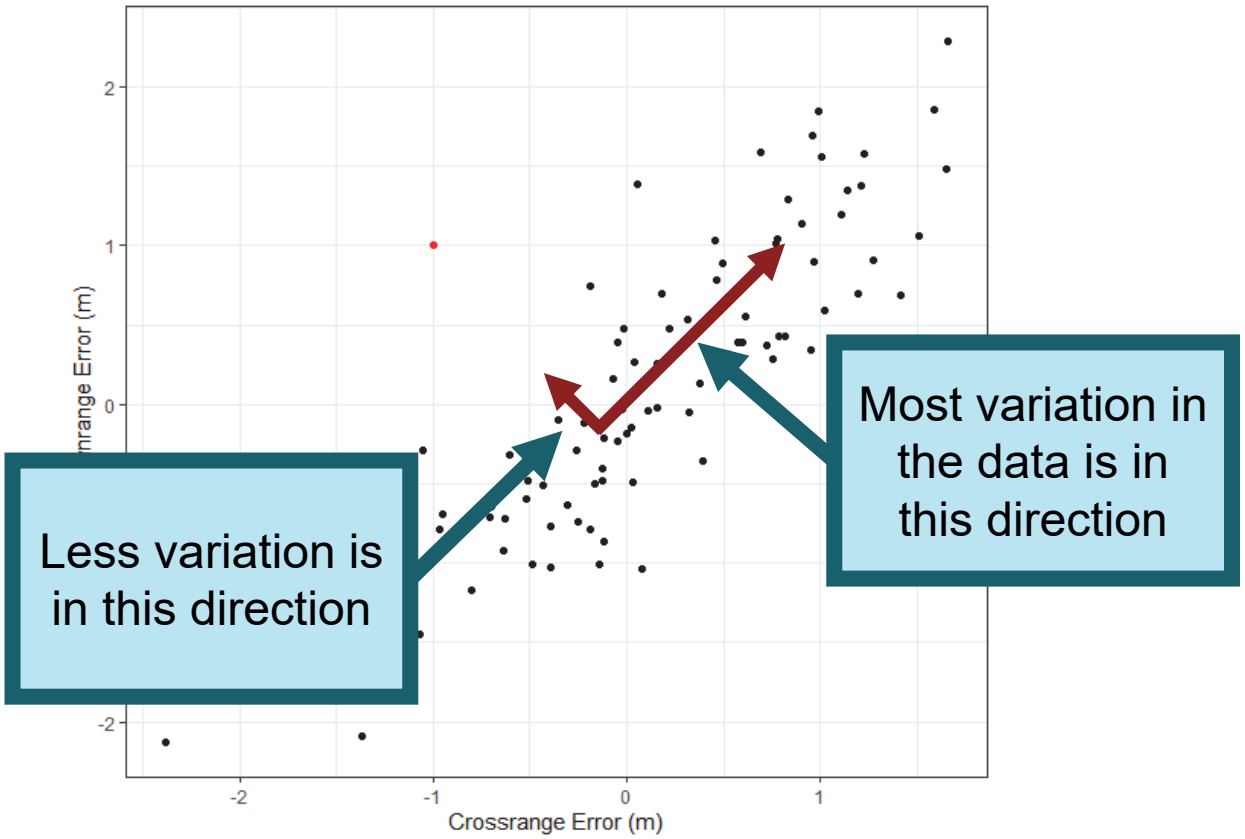
The red observation is not unusually far from the center, but does not follow the data's pattern



Distance alone does not show how unusual the red observation is

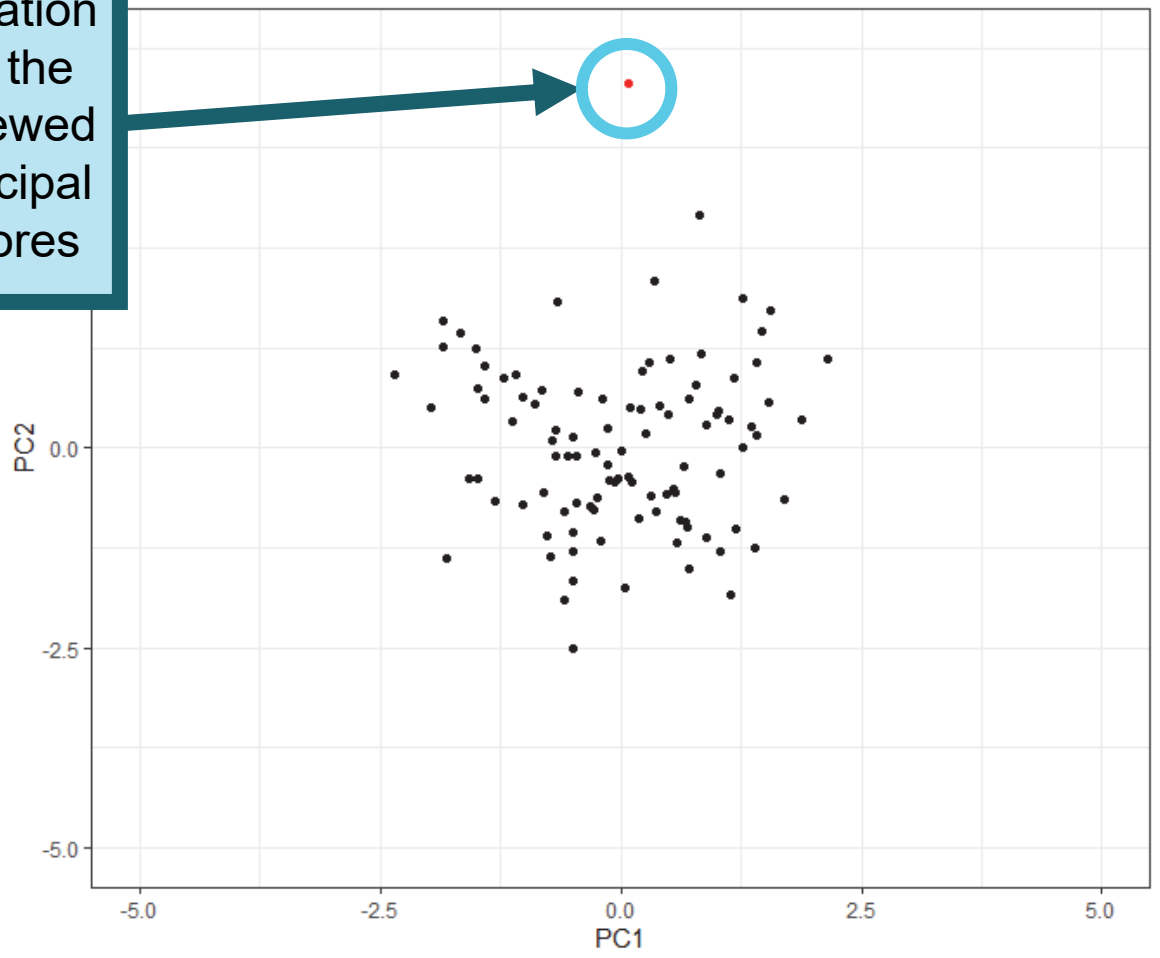
This example resembles two-dimensional miss distance assessments

Principal component analysis identifies correlations in the data and variation in key directions



Accounting for correlation and variation in the data set can better help reveal outliers

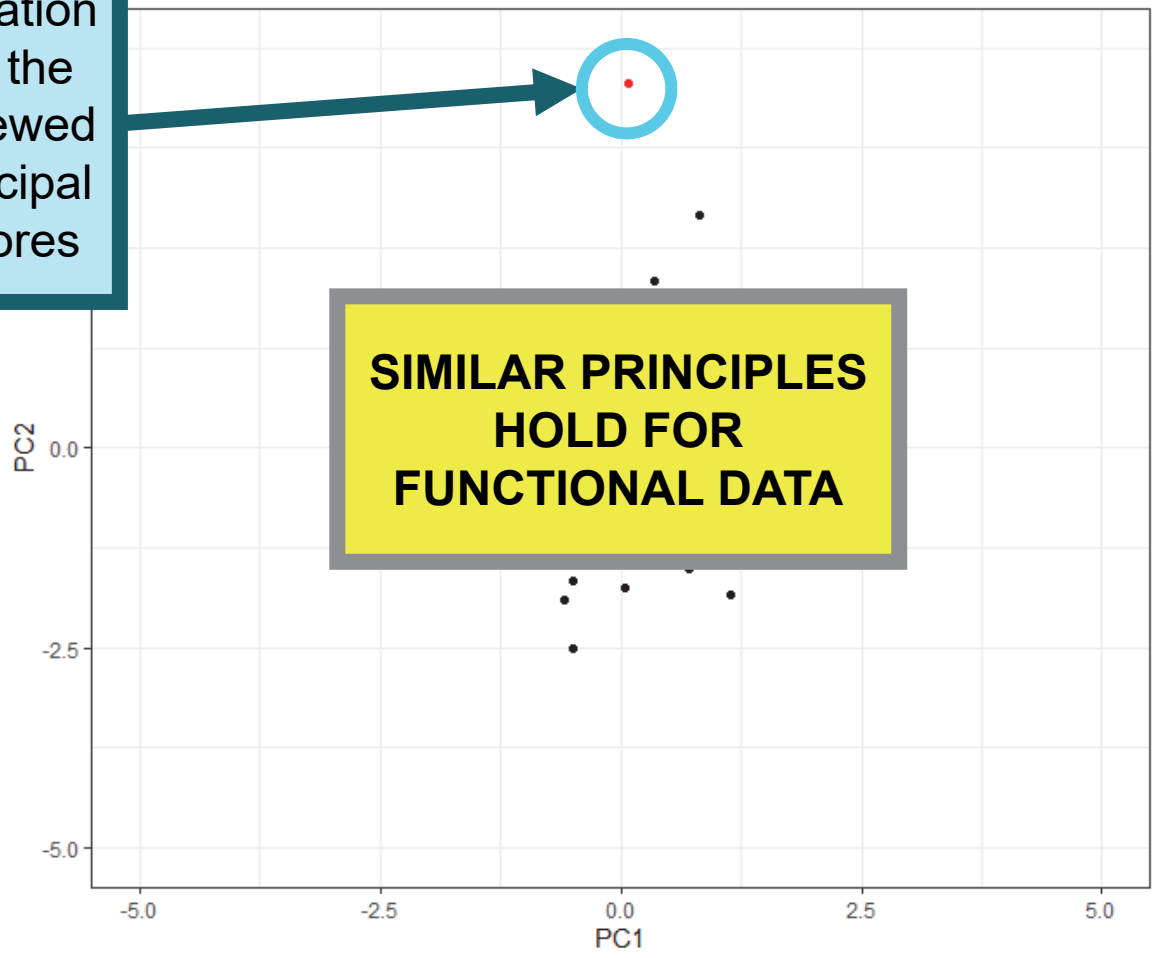
The red observation is distant from the others when viewed in terms of principal component scores



PC: Principal Component

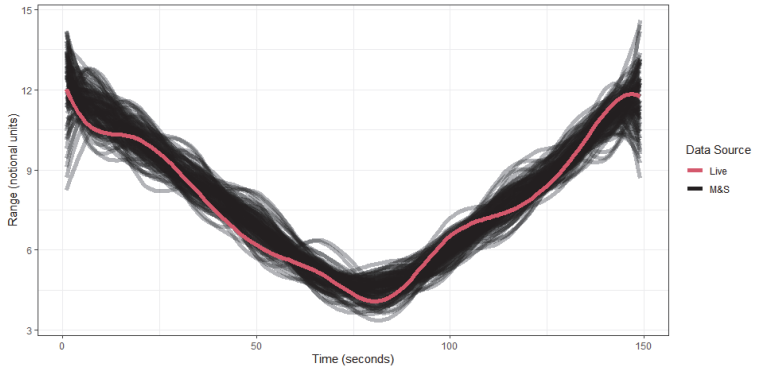
Accounting for correlation and variation in the data set can better help reveal outliers

The red observation is distant from the others when viewed in terms of principal component scores



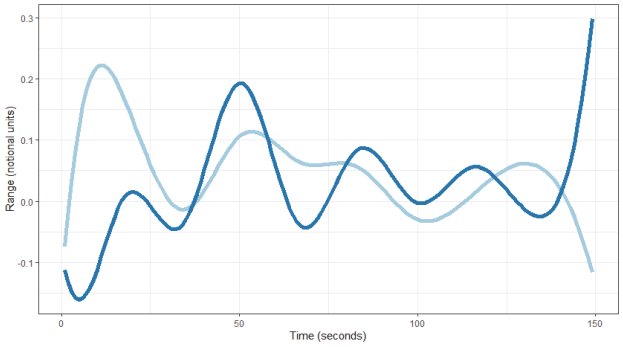
PC: Principal Component

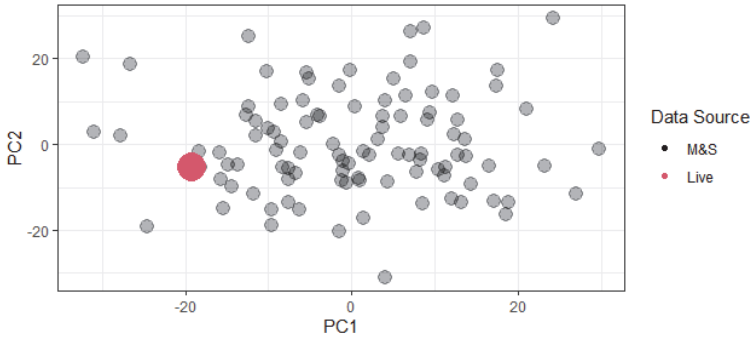
Functional principal component analysis manages systematic correlation in functional data



KARHUNEN-LOÉVE EXPANSION

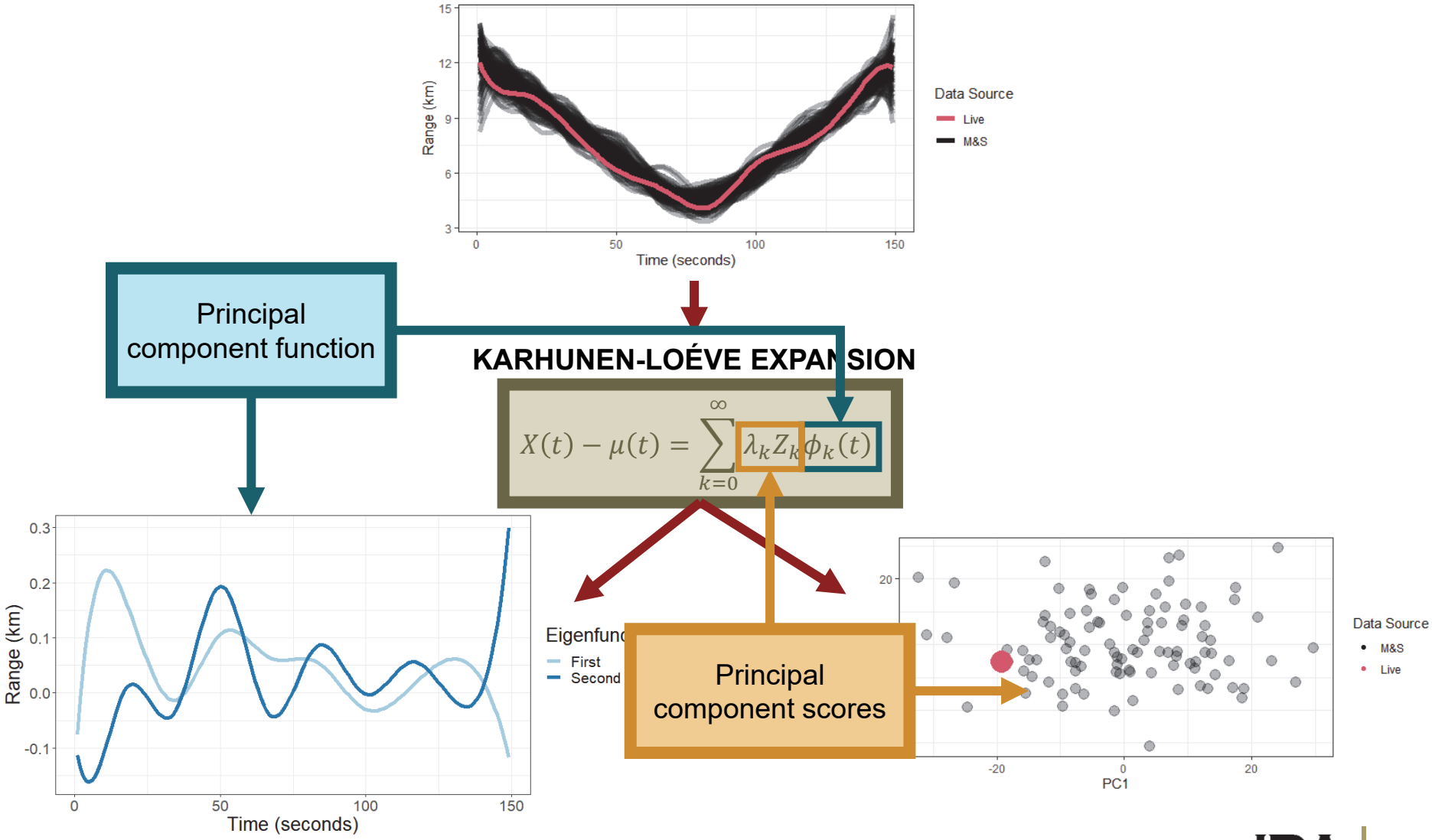
$$X(t) - \mu(t) = \sum_{k=0}^{\infty} \lambda_k Z_k \phi_k(t)$$





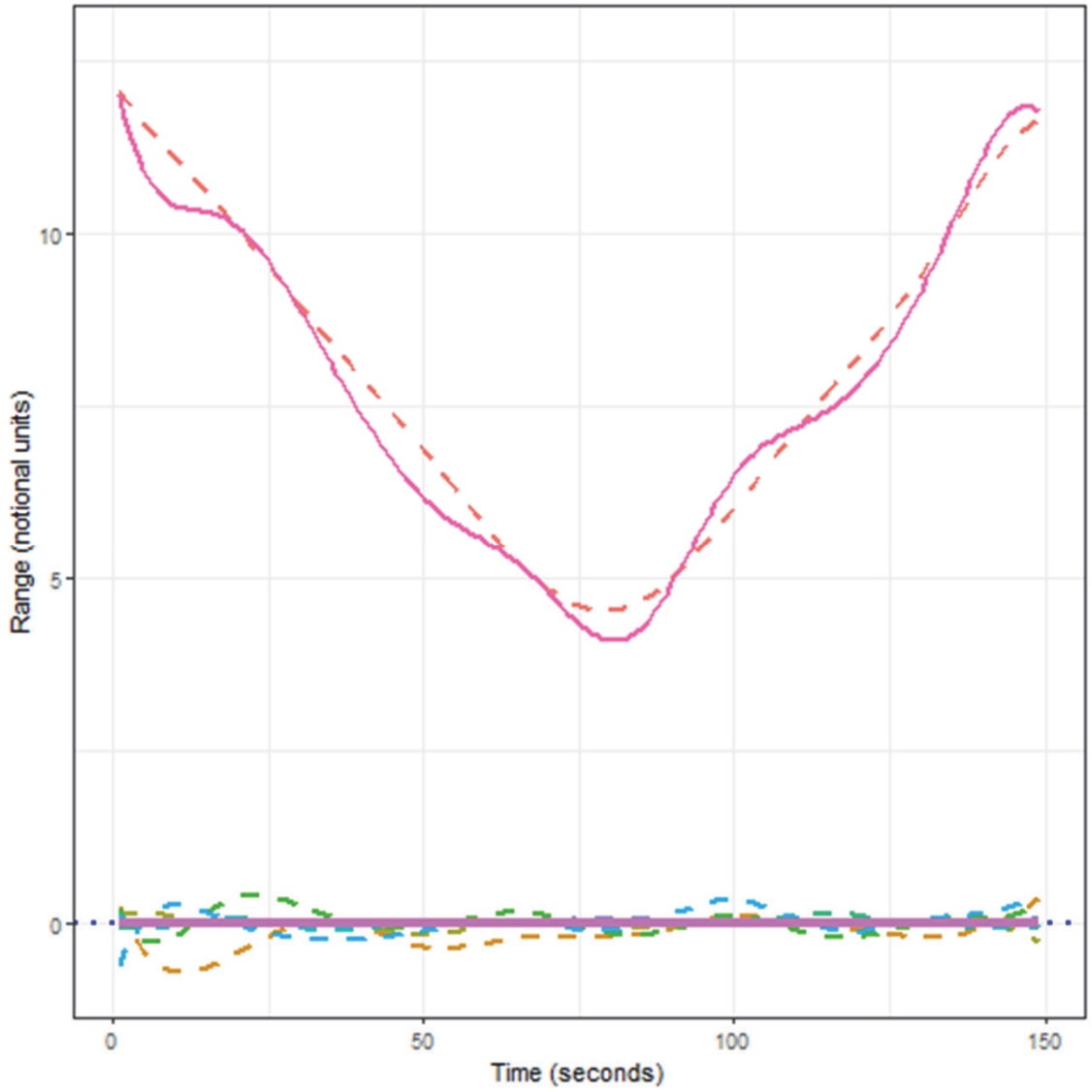
M&S: Modeling and Simulation; PC: Principal Component

Functional principal component analysis manages systematic correlation in functional data

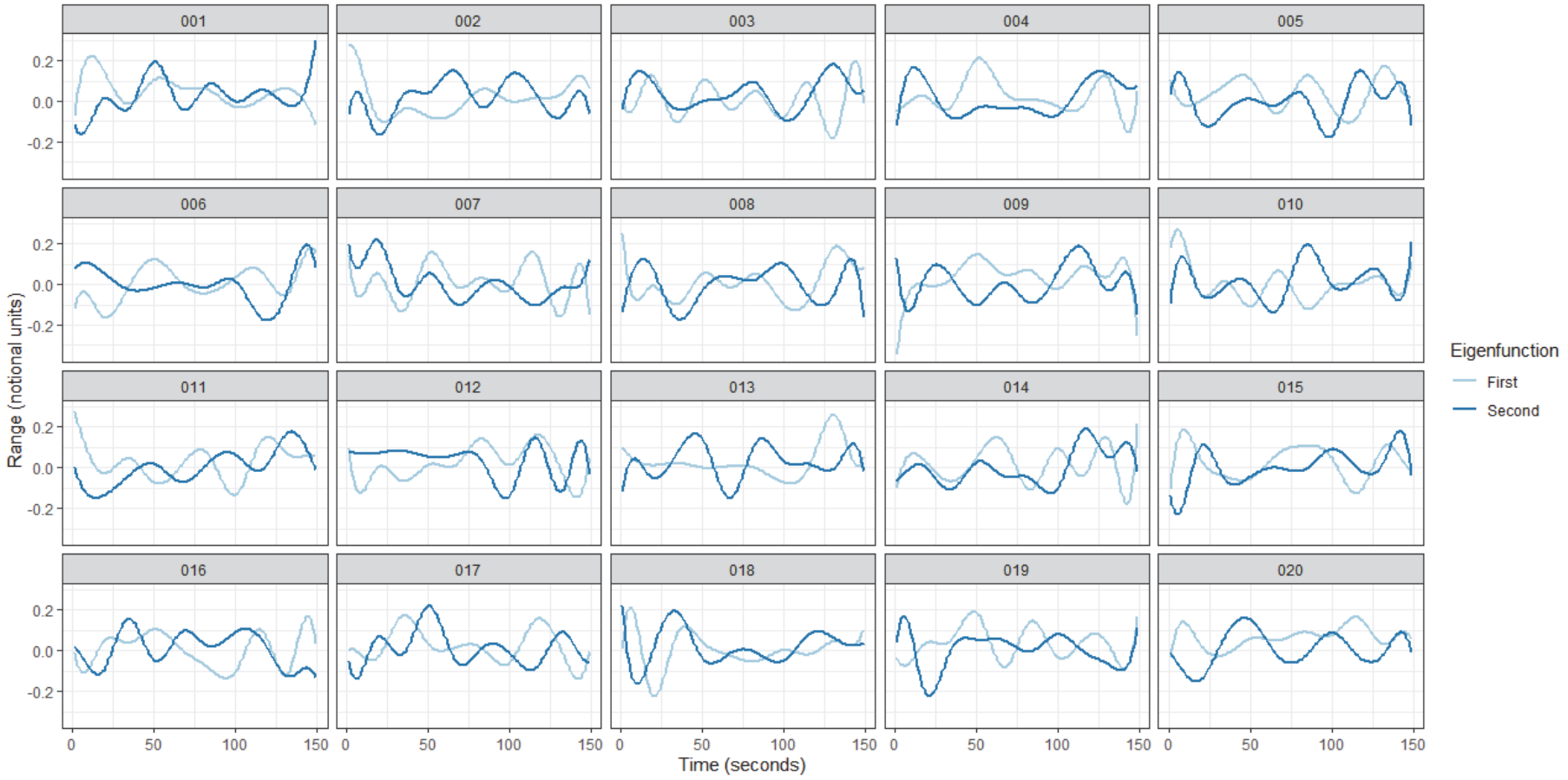


M&S: Modeling and Simulation; PC: Principal Component

Using more principal components allows for better approximations of functions in the data set

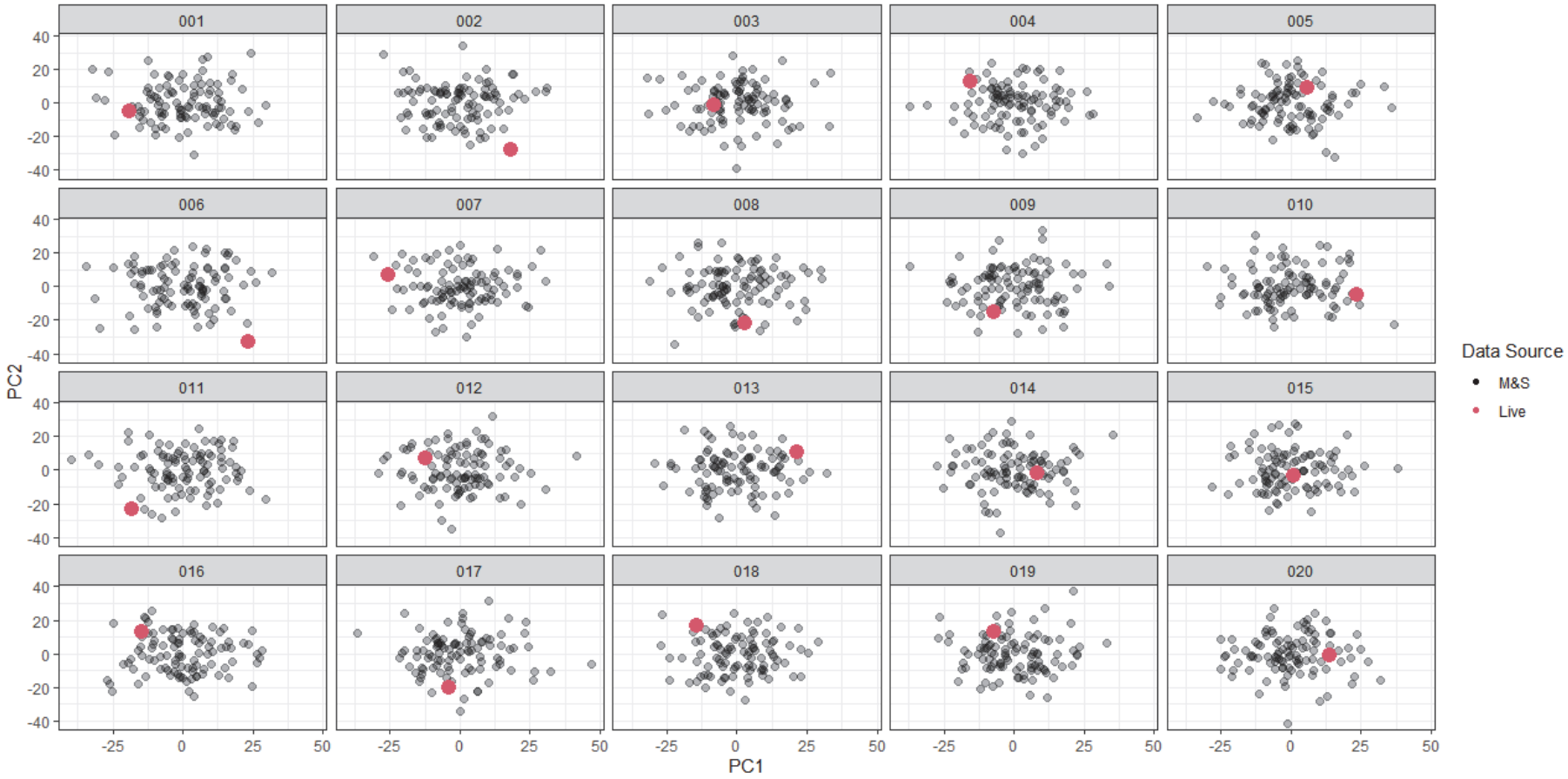


We can perform FPCA for each vignette



FPCA: Functional Principal Component Analysis

Then we look at the FPCA scores for each function in each vignette

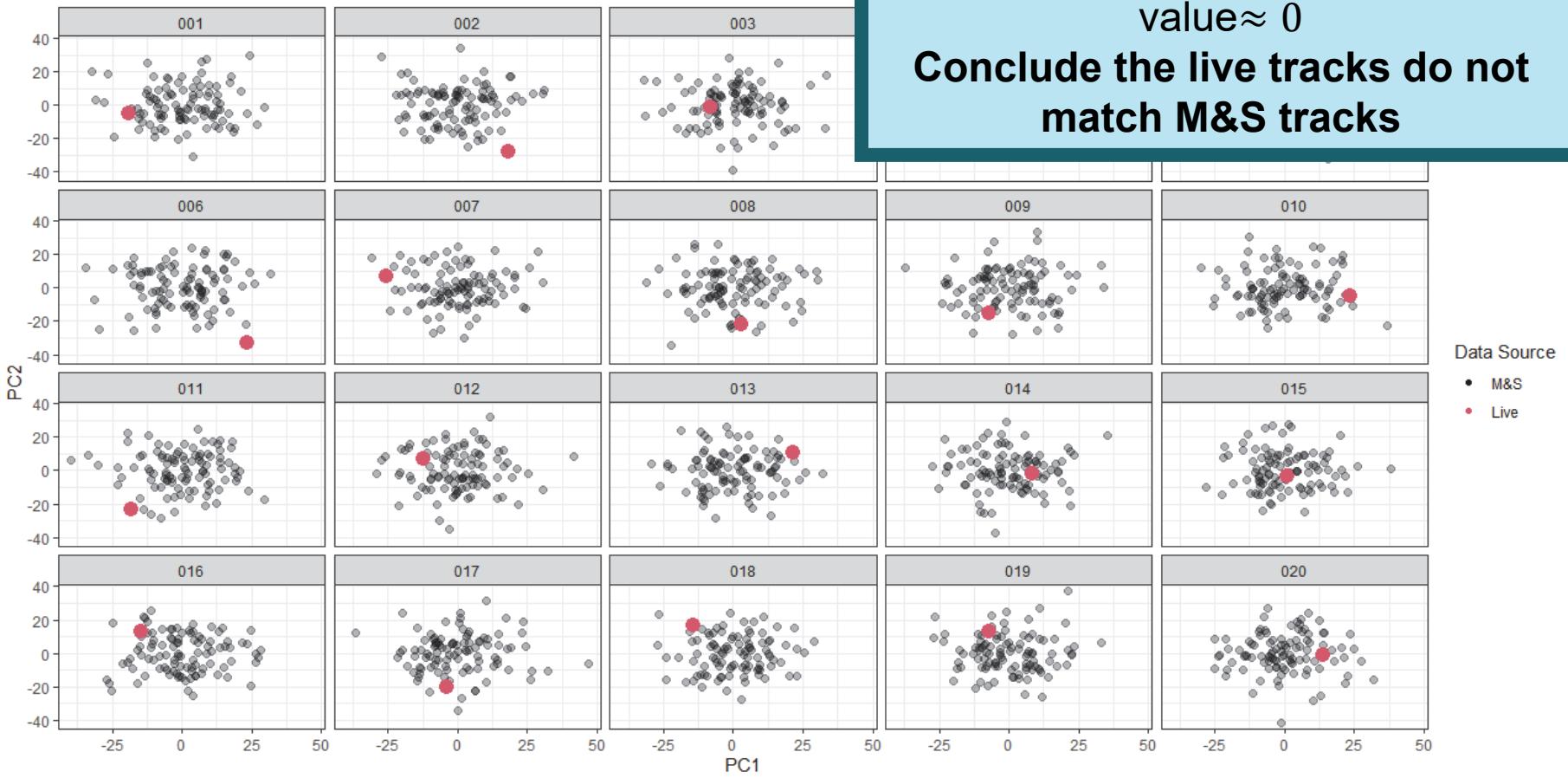


FPCA: Functional Principal Component Analysis; M&S: Modeling and Simulation; PC: Principal Component

FPCA scores can be used as a basis for a Fisher combined probability test

Fisher Combined Probability Test p-value≈ 0

Conclude the live tracks do not match M&S tracks



FPCA: Functional Principal Component Analysis; M&S: Modeling and Simulation; PC: Principal Component

WHERE NEXT?

Functional data methods can assist M&S validation decisions involving radar tracks, helping many programs

Enhanced Sea-Sparrow Missile

Rolling Airframe Missile

SPY-6

MIM-104 Patriot

E-3 Airborne Warning and Control System

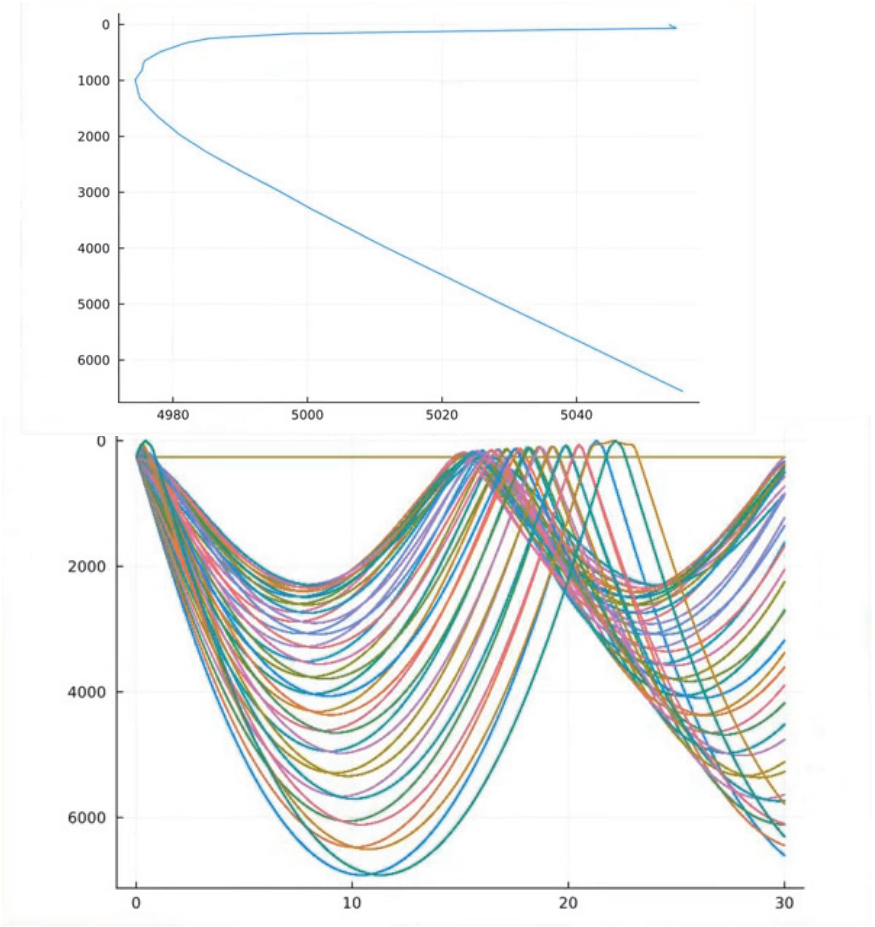
F-35

(Rolling airframe missile by Gary Granger Jr., *RIM-116 Rolling Airframe Missile*, 2013, Wikipedia, https://en.wikipedia.org/wiki/RIM-116_Rolling_Airframe_Missile#/media/File:USS_New_Orleans_(LPD-18)_launches_RIM-116_missile_2013.jpg; enhanced sea-sparrow missile by Matthew J. Haran, *RIM-162 ESSM*, July 23, 2010, Wikipedia, https://en.wikipedia.org/wiki/RIM-162_ESSM#/media/File:RIM-162_launched_from_USS_Carl_Vinson_(CVN-70)_July_2010.jpg; MIM-104 Patriot by Ra Boe, *Airday-Nordholz 2013*, 2013, Wikipedia, https://en.wikipedia.org/wiki/MIM-104_Patriot#/media/File:Airday-Nordholz_2013_by-RaBoe_106.jpg; SPY-6 by Seiko Okano, *AN/SPY-6*, January, 2017, PEO IWS, https://www.navsea.navy.mil/Portals/103/Documents/Exhibits/SNA-AboveWaterSensors.pdf; E-3 airborne warning and control system from Director, Operational Test and Evaluation, *E-3 Airborne Warning and Control System (AWACS)*, 2012, https://www.dote.osd.mil/Portals/97/pub/reports/FY2012/af/2012e-3awacs.pdf?ver=2019-08-22-111755-067; F-35 by Donald R. Allen, *Aerial Refueling of F-35 Lightning II Joint Strike Fighter at Eglin AFB, Fla.*, May 15, 2013, Defense Visual Information Distribution Service, https://en.wikipedia.org/wiki/Lockheed_Martin_F-35_Lightning_II#/media/File:F-35A_flight_(cropped).jpg)

M&S: Modeling and Simulation; SPY: water-born surveillance and control radar

Functional data methods could assist in many other areas relevant to operational and live fire testing

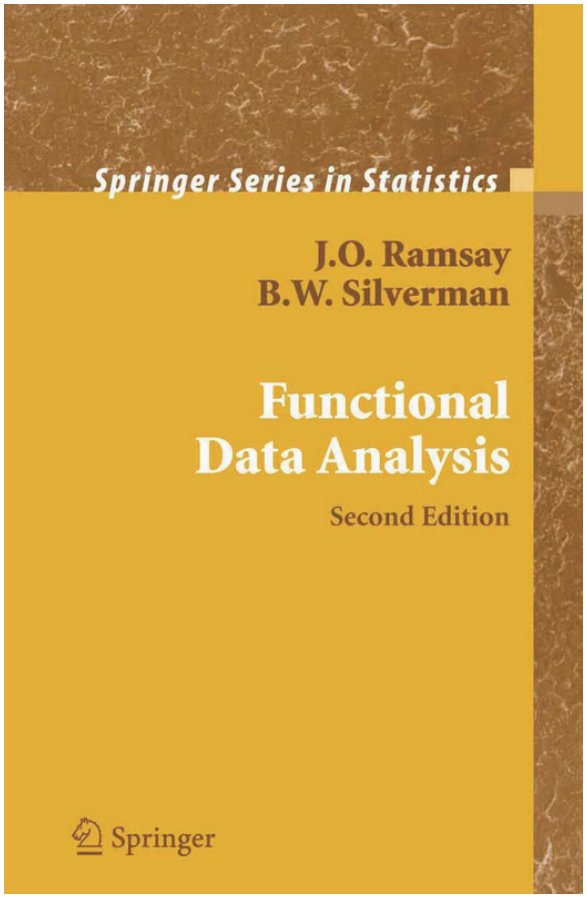
Arena Testing for Blast Fragmentation

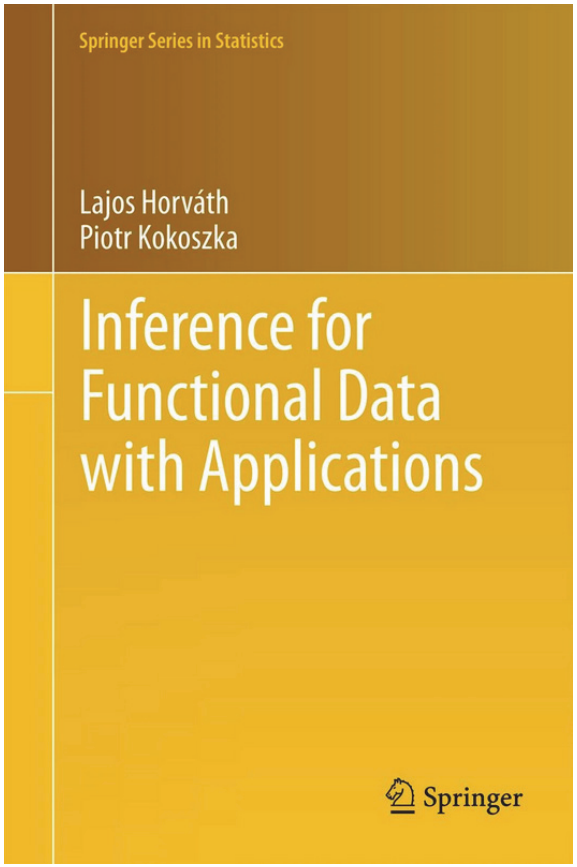


Underwater Sound Propagation

(Arena test image by Leonard_G, *Arena Firing of Continuous-Rod Warhead, 1972 at Naval Air Weapons Station China Lake*, October 21, 2024, Wikipedia, China Lake, https://en.wikipedia.org/wiki/Continuous-rod_warhead#/media/File:CRWarheadTest.jpg; Curtis Miller, *ASuW Wargame Tutorial and Design Notes*, March 18, 2024, YouTube, video, https://www.youtube.com/watch?v=Wq7-l8As7Zc)

Functional data analysis is an active area of research with great references





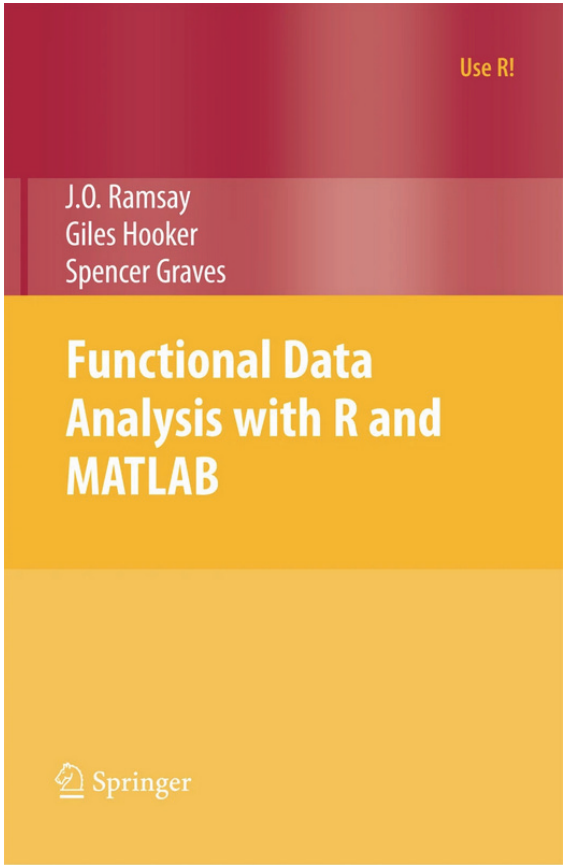


Image References

Amada44. “Marine Radar.” 2011. Video Recording. Wikipedia. https://en.wikipedia.org/wiki/Marine_radar#/media/File:Rotating_marine_radar_-_rotating_waveguide_antenna.gif.

Andrew Heneen. “The Known Flight Path of Malaysia Airlines Flight 370.” December 12, 2014. Image. Wikimedia. https://upload.wikimedia.org/wikipedia/commons/d/d2/MH370_flight_path_blank.png.

Bean_from_accounts. “Low-quality steady-state CFD simulation of an AIM-54 missile flying at Mach 1.5.” 2021. Image. Reddit. https://www.reddit.com/r/dcsworld/comments/mqmvy3/lowquality_steadystate_cfd_simulation_of_an_aim54/

Christopher Smith. *Patriot Radar.* April 25, 2019. Photograph. 10th Army Air and Missile Defense Command. Mielec. https://www.dvidshub.net/image/7251462/patriot-radar.

Curtis Miller. *ASuW Wargame Tutorial and Design Notes*. March 18, 2024. Video. YouTube. https://www.youtube.com/watch?v=Wq7-l8As7Zc.

Director, Operational Test and Evaluation. “E-3 Airborne Warning and Control System (AWACS).” 2012. https://www.dote.osd.mil/Portals/97/pub/reports/FY2012/af/2012e-3awacs.pdf?ver=2019-08-22-111755-067

Donald R. Allen. *Aerial Refueling of F-35 Lightning II Joint Strike Fighter at Eglin AFB, Fla.* May 15, 2013. Defense Visual Information Distribution Service. https://en.wikipedia.org/wiki/Lockheed_Martin_F-35_Lightning_II#/media/File:F-35A_flight_(cropped).jpg.

Elliot Bartis. *A Validation Case Study: The Environment Centric Weapons Analysis Facility (ECWAF).* 2020. Institute for Defense Analyses. Alexandria. https://testscience.org/wp-content/uploads/formidable/13/D-12081-A-Validation-Case-Study.pptx.

Fábio Pozzebom. “Microdigicam Laser Radar Gun in Use in Brazil.” February 20, 2007. Photograph. Wikipedia. https://en.wikipedia.org/wiki/Radar_speed_gun#/media/File:Radarvelocidade20022007.jpg.

Gary Granger Jr. “RIM-116 Rolling Airframe Missile.” 2013. Wikipedia. https://en.wikipedia.org/wiki/RIM-116_Rolling_Airframe_Missile#/media/File:USS_New_Orleans_(LPD-18)_launches_RIM-116_missile_2013.jpg

Leonard_G. “Arena Firing of Continuous-Rod Warhead, 1972 at Naval Air Weapons Station China Lake.” October 21, 2024. Wikipedia. https://en.wikipedia.org/wiki/Continuous-rod_warhead#/media/File:CRWarheadTest.jpg

Matthew J. Haran. “RIM-162 ESSM.” July 23, 2010. Wikipedia. https://en.wikipedia.org/wiki/RIM-162_ESSM#/media/File:RIM-162_launched_from_USS_Carl_Vinson_(CVN-70)_July_2010.jpg.

Mark Couch, Thomas Johnson, John Haman, Kerry Wazl, Heather Wojton, Thomas Hatch-Aguilar, and David Higdon. *DATAWorks 2021: Warhead Arena Analysis Advancements.* April, 2021. Presentation. Institute for Defense Analyses. Alexandria. https://www.ida.org/-/media/feature/publications/d/da/dataworks-2021-warhead-arena-analysis-advancements/d-11038.ashx.

National Archives. “A Close-Up View of One of the Radar Consoles Used for Tracking Aircraft During the Air-to-Air Combat Training Exercise William Tell ’82.” October 9, 1982. Photograph. https://catalog.archives.gov/id/6367394.

Ra Boe. “Airday-Nordholz 2013.” 2013. Wikipedia. https://en.wikipedia.org/wiki/MIM-104_Patriot#/media/File:Airday-Nordholz_2013_by-RaBoe_106.jpg

Seiko Okano. *AN/SPY-6*. January, 2017. Program Executive Office Integrated Warfare Systems. https://www.navsea.navy.mil/Portals/103/Documents/Exhibits/SNA-AboveWaterSensors.pdf.

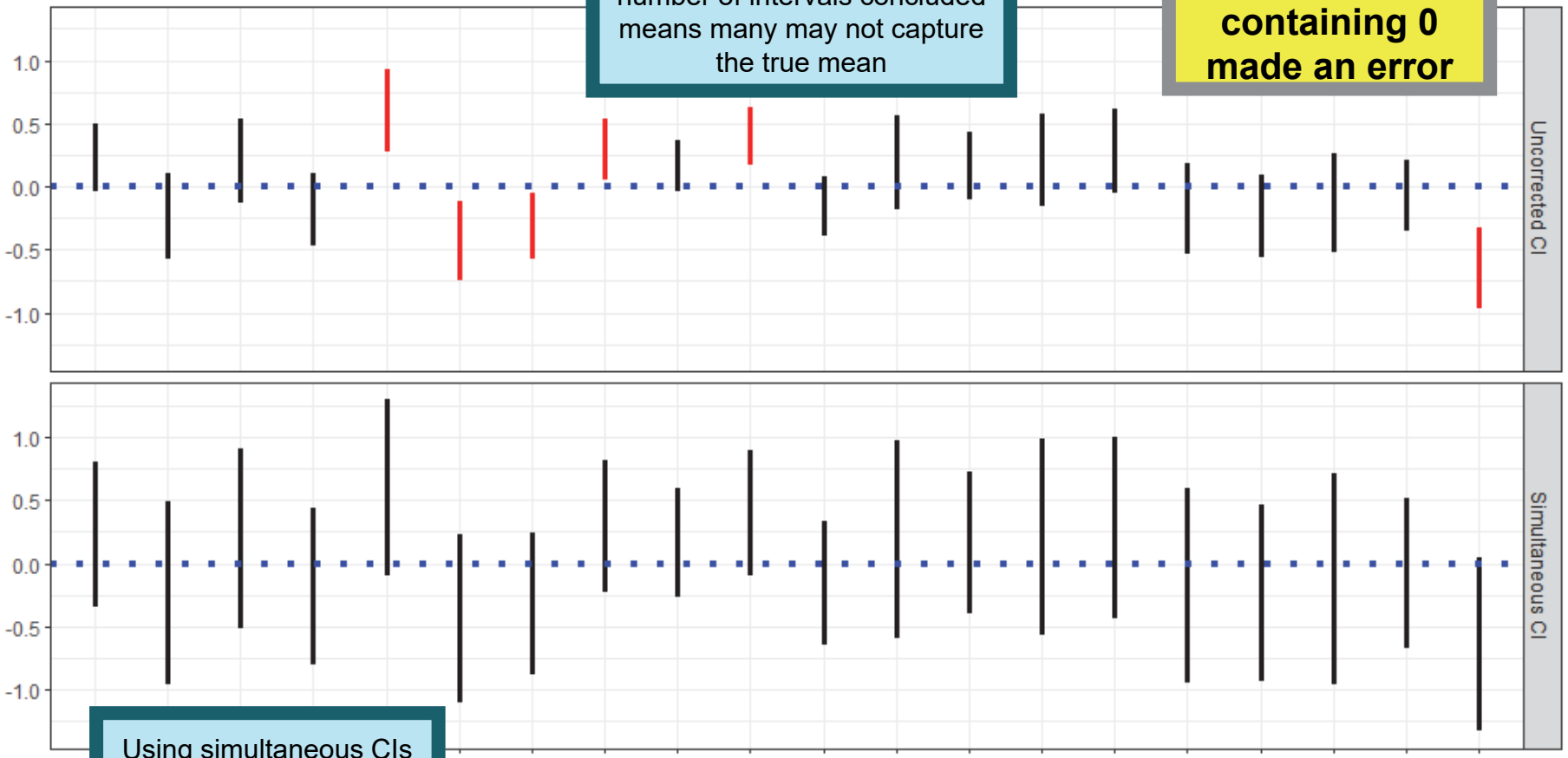
Wikipedia. “Sound Speed Profile.” October 13, 2012. Article. https://en.wikipedia.org/wiki/Sound_speed_profile.

BACKUP

Confidence interval guarantees hold for single intervals unless corrected

Computing lots of 80% CIs without correcting for the number of intervals concluded means many may not capture the true mean

Any interval not containing 0 made an error

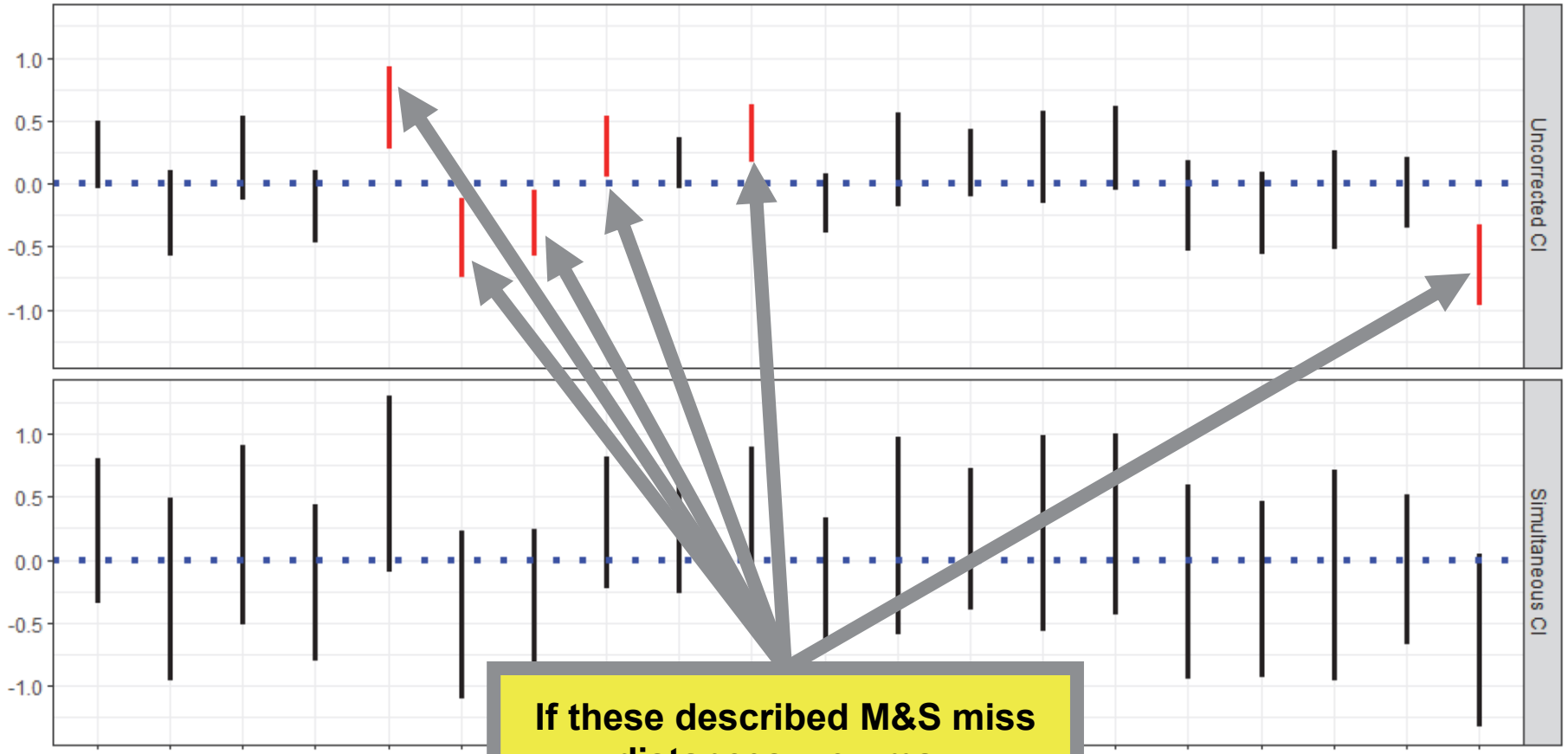


Using simultaneous CIs means there's an 80% probability ALL intervals capture their true means

CIs in the top row are t CIs with confidence level 80%. CIs in the bottom row are 80% simultaneous CIs using the SMM method.

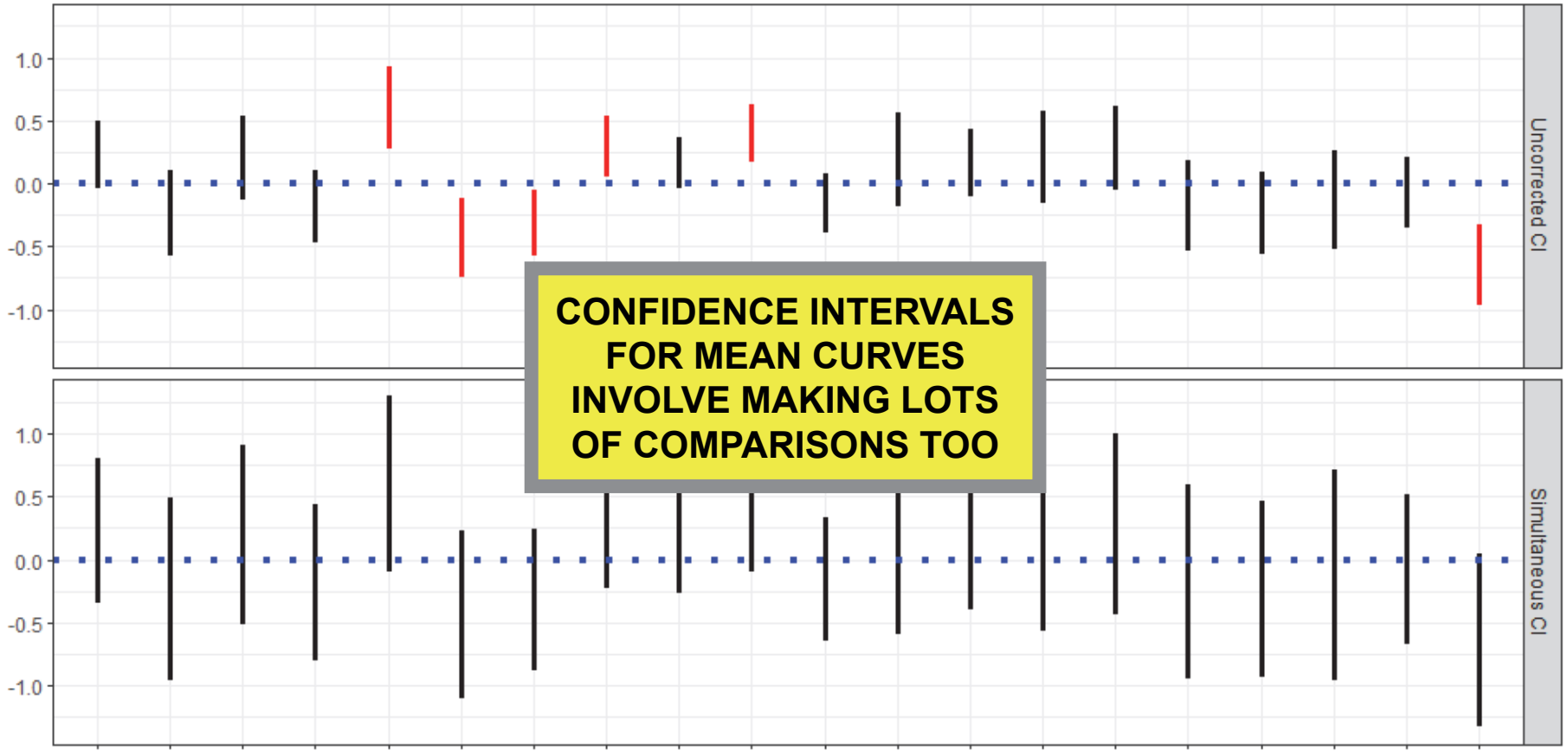
CI: Confidence Interval; SMM: Studentized Maximum Modulus

Confidence interval guarantees hold for single intervals unless corrected



CI: Confidence Interval; M&S: Modeling and Simulation

Confidence interval guarantees hold for single intervals unless corrected



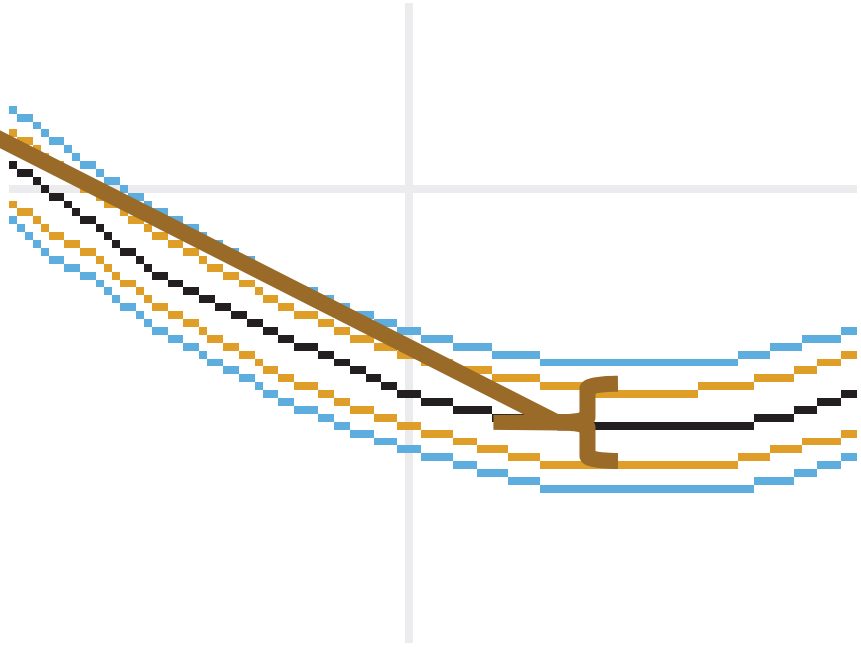
CI: Confidence Interval

Pointwise confidence intervals may be too narrow for the types of inference we wish to make

POINTWISE CONFIDENCE INTERVAL (GREEN)

$$\text{For all } t,\ P\big(L(t) \le \mu(t) \le U(t)\big) = 0.95$$

Error is controlled only at each point; **global error is uncontrolled**



This would be the interval you would compute using methods recommended from an introductory statistics textbook

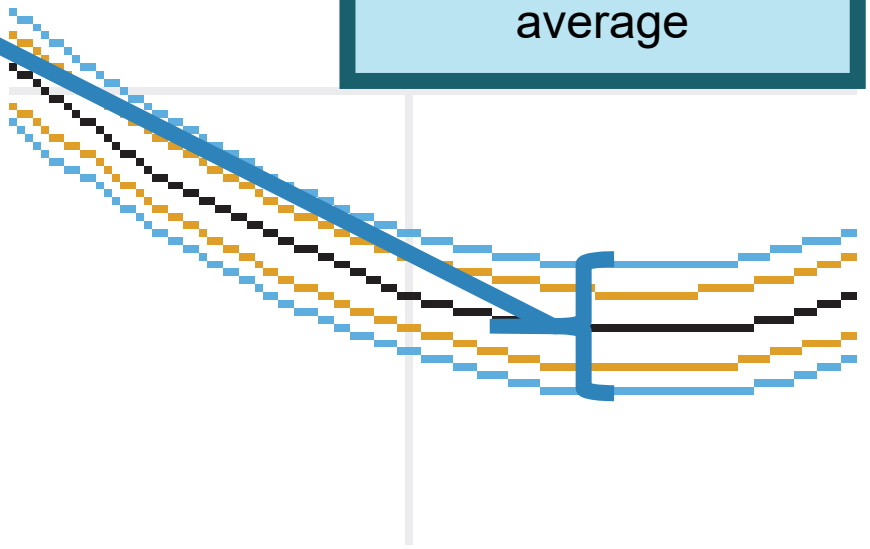
Simultaneous confidence intervals are wider but more robust

SIMULTANEOUS CONFIDENCE INTERVAL (BLUE)

$$P(L(t) \leq \mu(t) \leq U(t) \text{ for all } t) = 0.95$$

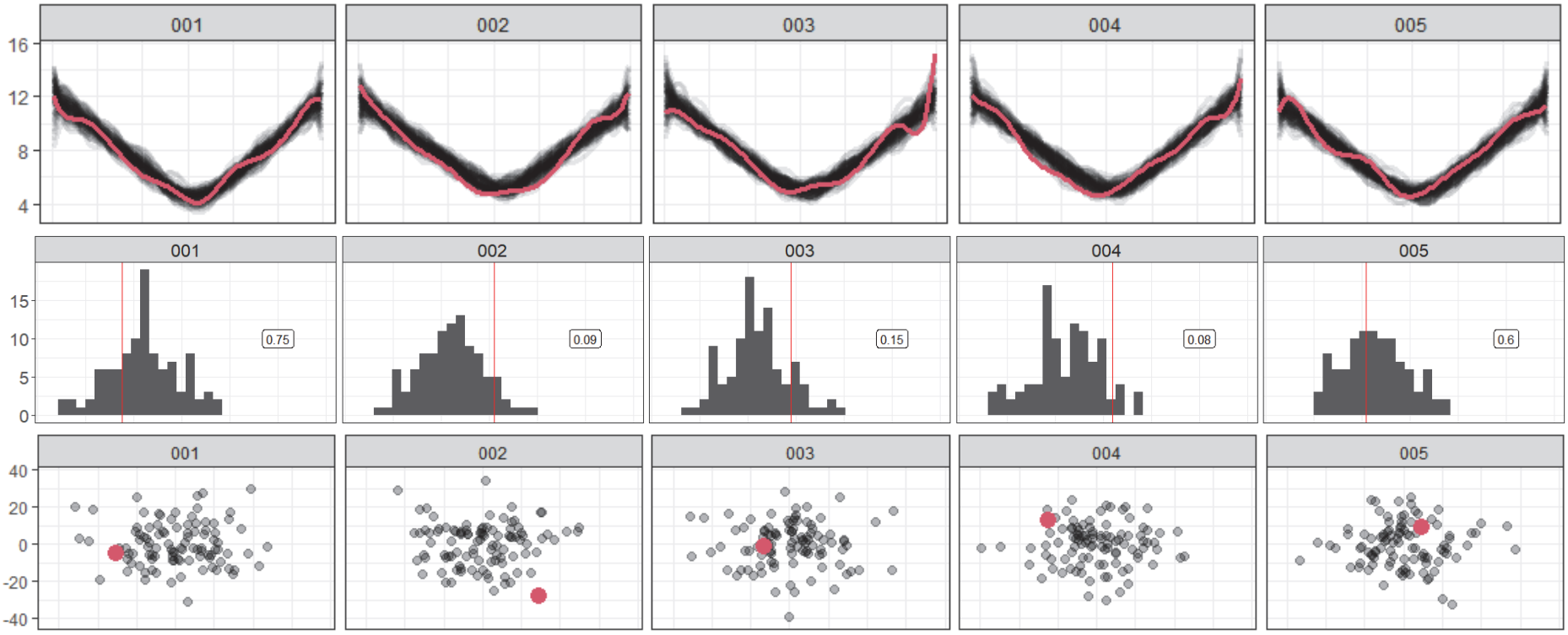
In our example, simultaneous confidence intervals are 66% wider on average

Global error is controlled



Simultaneous intervals here are computed using methods described in Degras (2017): “Simultaneous confidence bands for the mean of functional data”

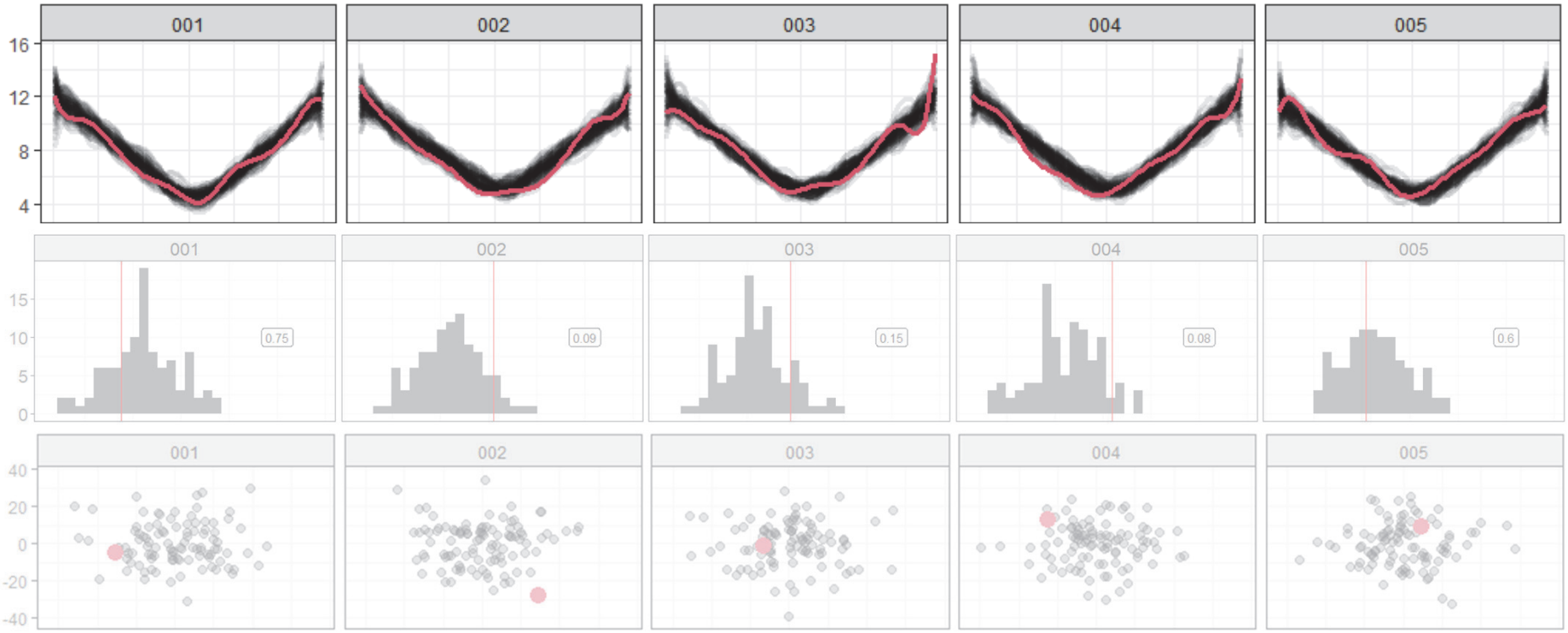
Even if we restricted this analysis to the first five tracks, we still would have rejected the null hypothesis easily…



Fisher Combined Probability Test p-value≈ 0
Conclude the live tracks do not match M&S tracks

M&S: Modeling and Simulation

… But not if you were only looking at the top plot



REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. REPORT DATE *(DD-MM-YYYY)*	2. REPORT TYPE	3. DATES COVERED *(From - To)*
XX-03-2024	Final	

4. TITLE AND SUBTITLE

DATAWorks 2024: A preview of functional data analysis for modeling and simulation validation

5a. CONTRACT NUMBER

HQ0034-19-D-0001

5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)

Curtis G Miller

5d. PROJECT NUMBER

C9082

5e. TASK NUMBER

C9082

5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

Institute for Defense Analyses
730 East Glebe Road
Alexandria, Virginia 22305

8. PERFORMING ORGANIZATION REPORT NUMBER

3001829

3001829

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

10. SPONSOR/MONITOR'S ACRONYM(S)

CRP

11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT

Approved for public release: distribution is unlimited.

13. SUPPLEMENTARY NOTES

Project Leader: Rebecca M Medlin

14. ABSTRACT

Modeling and simulation (M&S) validation for operational testing often involves comparing live data with simulation outputs. Statistical methods known as functional data analysis (FDA) provides techniques for analyzing large data sets ("large" meaning that a single trial has a lot of information associated with it), such as radar tracks. We preview how FDA methods could assist M&S validation by providing statistical tools handling these large data sets. This may facilitate analyses that make use of more of the data available and thus allows for better detection of differences between M&S predictions and live test results. We demonstrate some fundamental FDA approaches with a notional example of live and simulated radar tracks of a bomber's flight.

15. SUBJECT TERMS

functional data analysis, hypothesis testing, confidence intervals, simultaneous confidence intervals, Fisher's combined probability test, modeling and simulation validation, radar tracks, functional principal component analysis

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT	b. ABSTRACT	c. THIS PAGE			Rebecca M Medlin
Unclassified	Unclassified	Unclassified	SAR	44	19b. TELEPHONE NUMBER *(Include area code)* 703-845-6731