INSTITUTE FOR DEFENSE ANALYSES

DATAWorks 2020: Ballistic Miss Distance Poster

Rebecca M. Medlin, Project Leader

Thomas H. Johnson
Breeana G. Anderson
Heather M. Wojton
John T. Haman

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INSTITUTE FOR DEFENSE ANALYSES
4850 Mark Center Drive
Alexandria, Virginia 22311-1882

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For more information:
Rebecca M. Medlin, Project Leader
rmedlin@ida.org • 703-845-6731

Robert R. Soule, Director, Operational Evaluation Division
rsoule@ida.org • (703) 845-2482

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Executive Summary

The attached poster was developed for the 2020 DATAWorks. It is now intended for DATAWorks 2021. This study compares two methods for creating CEP prediction intervals: a frequentist method using restricted maximum likelihood (REML) and a Kenward-Roger's statistic, and a Bayesian regression approach. It will be presented during the poster session.

Circular prediction regions are used in ballistic testing to express the uncertainty in shot accuracy. We compare two modeling approaches for estimating circular prediction regions for the miss distance of a ballistic projectile. The miss distance response variable is bivariate normal and has a mean and variance that can change with one or more experimental factors. The first approach fits a heteroscedastic linear model using restricted maximum likelihood, and uses the Kenward-Roger statistic to estimate circular prediction regions. The second approach fits an analogous Bayesian model with unrestricted likelihood modifications, and computes circular prediction regions by sampling from the posterior predictive distribution. The two approaches are applied to an example problem, and are compared using simulation.

Tom Johnson, John Haman, Heather Wojton, Breeana Anderson

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Problem Statement

Circular Error Probable (CEP) is a common metric used to model two-dimensional miss distances. CEP is effective at modeling one group of data, but cannot be used to analyze the designed experiments common in DoD operational testing.

Objective

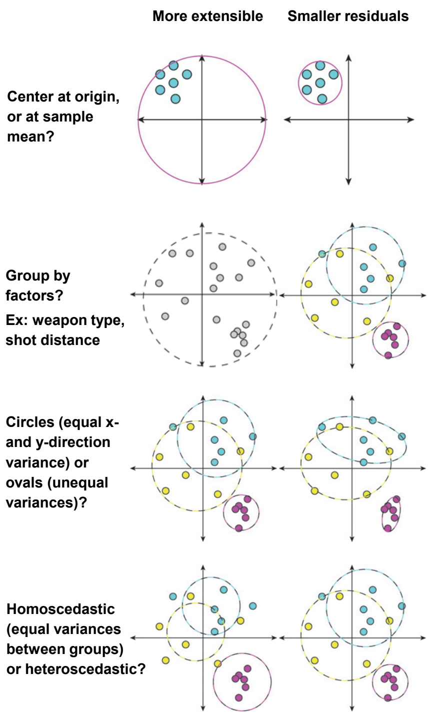
To compare the ability of two statistical methods for creating circular prediction intervals:

- A frequentist method using restricted maximum likelihood (REML) and a Kenward-Roger's statistic
- A Bayesian regression approach

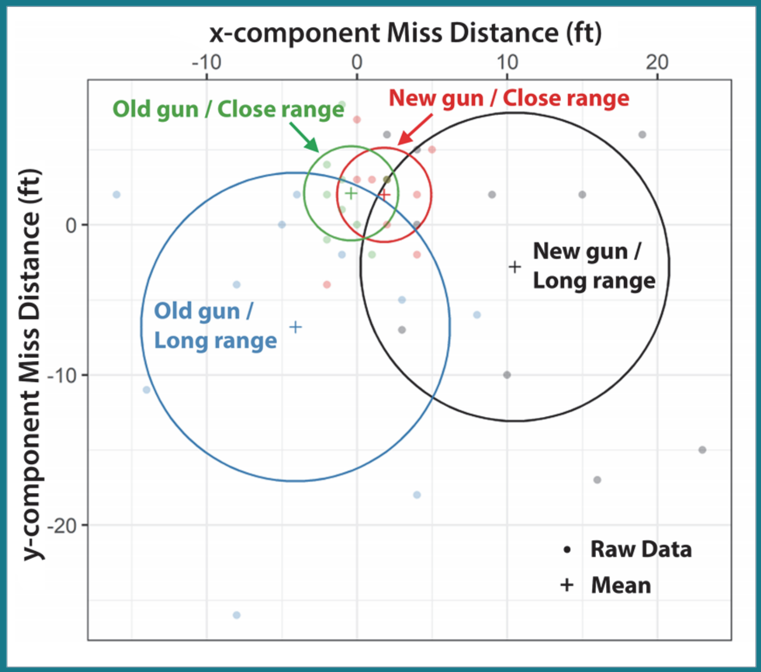
These methods can be used to create models that make more efficient use of test data.

CEP Modeling Choices

The goal is to create a model that best describes the data and that can also be used to describe trends or predict future results:



By leveraging statistics, researchers can produce better models of ballistic miss distances than traditional CEP



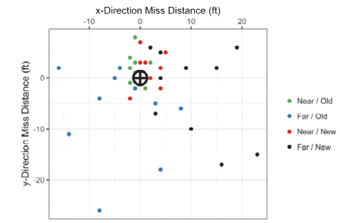
Bayesian and frequentist techniques use data more efficiently than CEP, and unlike CEP, can be used to create inferential or predictive models

Motivating Example

To compare prediction interval methods, we used notional data from an experiment comparing the accuracy of handguns under varying conditions:

2 Weapon Types: Old and New

2 Firing Distances: Near and Far



Comparing Prediction Intervals

Model similarities:	Frequentist	Bayesian
Circular Uncertainty Interval	Prediction	Prediction
Univariate or bivariate RV?	Bivariate	Bivariate
Centered at mean or origin?	Mean	Mean
Cartesian or polar coords?	Cartesian	Cartesian
Grouped or Un-grouped?	Grouped by factors	Grouped by factors
Bias or precision?	Fixed effects for mean	Fixed effects for mean
Constant size circles?	Fixed effects for variance	Fixed effects for variance
Unified Model?	Yes	Yes

Model differences:	Frequentist	Bayesian
Probabilities are:	Long-term frequencies	Degrees of belief
Inference based on:	Sampling distribution	Posterior distribution
Parameters are:	Fixed (but unknown)	Random
Intervals/Regions are:	Random	Fixed
Modeling Goal:	Maximize likelihood (typically)	Estimate *entire* posterior distribution

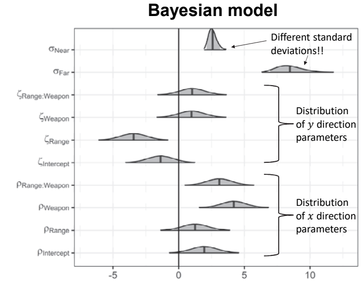
Comparing Model Outputs

Frequentist model

Term	$\hat{\rho}$ (horiz.)	$\hat{\zeta}$ (vert.)	KR F-stat	Approx df	p-value
Intercept	1.95	-1.38			
Range	1.25	-3.42	7.11	47.76	~0
Weapon	4.20	0.97	9.95	47.76	~0
Range by weapon	3.10	1.02	5.70	47.76	0.01

- Frequentist method allows for convenient ANOVA-like table
- Algorithm is ad-hoc to the model – that is, it is difficult to change or extend the model
- Computation is fast, especially with small sample sizes

Bayesian model



- Typically slower computation
- Model is easily extended
- Distributions for all model parameters
- Model comparison more challenging

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14. ABSTRACT

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