INSTITUTE FOR DEFENSE ANALYSES

D-Optimal as an Alternative to Full Factorial Designs: A Case Study

Breeana G. Anderson, Project Leader
Heather M. Wojton, Project Leader

Keyla Pagan-Rivera

April 2019

Approved for public release. Distribution is unlimited.

IDA Document NS D-10580

Log: H 2019-000161

INSTITUTE FOR DEFENSE ANALYSES
4850 Mark Center Drive
Alexandria, Virginia 22311-1882

IDA

The Institute for Defense Analyses is a non-profit corporation that operates three federally funded research and development centers to provide objective analyses of national security issues, particularly those requiring scientific and technical expertise, and conduct related research on other national challenges.

About This Publication

This work was conducted by the Institute for Defense Analyses (IDA) under contract HQ0034-14-D-0001, Task BD-9-2299(18), "Chemical Biological Warfare (CBW) Defense," and BD-9-2299(90), "Test Science Applications," for the Office of the Director, Operational Test and Evaluation. The views, opinions, and findings should not be construed as representing the official position of either the Department of Defense or the sponsoring organization.

Acknowledgments

The review was conducted by Mr. Robert R. Soule, Director, Operational Evaluation Division, and Heather Wojton and Rebecca Medlin from the Operational Evaluation Division.

For more information:

Breanna G. Anderson, Project Leader
banderso@ida.org • (703) 845-6967

Robert R. Soule, Director, Operational Evaluation Division
rsoule@ida.org • (703) 845-2482

Copyright Notice

INSTITUTE FOR DEFENSE ANALYSES

IDA Document NS D-10580

D-Optimal as an Alternative to Full Factorial Designs: A Case Study

Breeana G. Anderson, Project Leader
Heather M. Wojton, Project Leader

Keyla Pagan-Rivera

Executive Summary

In planning an experiment, testers rely on experimental design to determine how to cover the operational space in an efficient way. Understanding which factors can affect a detector's performance can influence military tactics, techniques and procedures, and improve a commander's situational awareness when making decisions in an operational environment. A full factorial design ensures that every possible combination of levels across factors is tested, whereas a D-optimal design minimizes the parameter estimate variability by maximizing $det[X'X]$. The D-optimal design can also account for the irregularities or constraints in the data.

Testing military systems includes evaluating the effectiveness of detectors (chemical or metal detector, radars, etc.) under different circumstances. Sometimes the operational space includes disallowed combinations. That is, some levels of one factor cannot be tested at the same time as some levels from another factor. While a tester could try to use a full factorial design for this type of situation, a D-optimal design might be a better alternative.

This document shows how the use of a D-optimal design could have reduced the amount of data needed when testing the effectiveness of a detector. We used an adequate D-optimal design to obtain a subset of the original dataset. In order to assess how the operational environment can affect the detector's performance, we used this subset of the data to fit frequentist and Bayesian logistic regression models. The results were similar to the ones obtained when fitting the models to the full dataset.

D-Optimal as an Alternative to Full Factorial Designs: A Case Study

Keyla Pagán-Rivera

Institute for Defense Analyses

Summary

- Test goal was to determine an instrument's probability of detecting a target under different circumstances
- Data were collected using a full factorial design and Bayesian methods to determine the concentration level
- Future testing could use a D-optimal design, which might help reduce the number of runs

Data*

- 26% failed to detect vs 74% that detected the target
- Nine different types of solutions (S1, S2, …, S9)
- 41 different organisms (O1, O2, …, O41)
- Multiple solution-organism disallowed combinations
- Wide range of concentration levels

Concentration levels summary statistics					
Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
0.003	8	27	4,778	524	26,8918

Original design (snapshot)

	S2	S1	S7	S3	S4	S8	S6	S5	S9
O6	32	32	X	X	X	X	X	31	X
O10	X	23	X	19	X	14	X	21	X
O12	20	32	X	18	15	17	22	20	20
O20	19	X	21	19	18	X	23	19	X
O24	23	X	X	X	22	X	17	21	X
O31	6	X	X	X	X	X	X	X	X
O33	189	X	X	X	X	X	X	X	X
O41	21	21	X	X	X	X	X	4	X

- Full factorial design
- Two categorical factors (organism and solution) and one continuous factor (concentration; not shown)
- Binary outcome (detection: yes/no)
- Some scenarios have more runs than others (e.g., S2 and O31 has six runs, whereas S2 and O33 has 189 runs)
- Some solution-organism combinations only include high concentrations
- Power = 0.99

D-optimal design (snapshot)

	S2	S1	S7	S3	S4	S8	S6	S5	S9
O6	19	21	X	X	X	X	X	18	X
O10	X	13	X	24	X	25	X	12	X
O12	X	2	X	15	14	19	14	X	19
O20	8	X	37	17	14	X	14	8	X
O24	13	X	X	X	22	X	23	13	X
O31	54	X	X	X	X	X	X	X	X
O33	54	X	X	X	X	X	X	X	X
O41	18	21	X	X	X	X	X	19	X

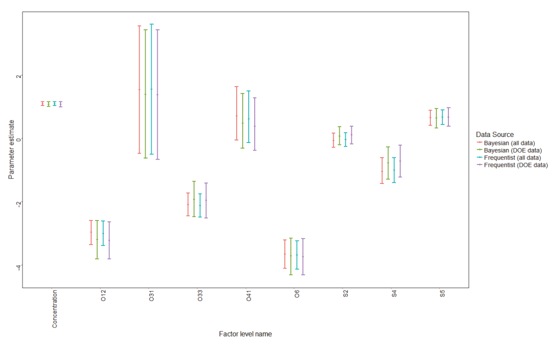
- Same factors and levels as the previous design (table shows organism and solution, not concentration)
- Total number of runs needed is 13% less than the original design
- More balanced design (e.g., O31 and O33 have the same number of runs)
- Power = 0.96

D-optimal design with available data (snapshot)

	S2	S1	S7	S3	S4	S8	S6	S5	S9
O6	19	21	X	X	X	X	X	18	X
O10	X	13	X	14	X	14	X	12	X
O12	X	2	X	15	12	17	11	X	19
O20	8	0	21	16	14	0	14	8	0
O24	13	X	X	X	20	X	17	13	X
O31	6	X	X	X	X	X	X	X	X
O33	54	X	X	X	X	X	X	X	X
O41	16	19	X	X	X	X	X	4	X

- Same factors and levels as previous tables
- Using the original data set and the D-optimal design, we obtained a subset of the data
- For some levels (e.g., O33) we had the runs the D-optimal design required, but for other levels (e.g., O31) we did not have the number of runs the D-optimal design required
- Sometimes we had enough runs for a specific solution and organism combination, but not enough high/low concentration levels within that combination (e.g., O41)
- Acceptable design with fewer runs (65% of the original data)
- Power = 0.77

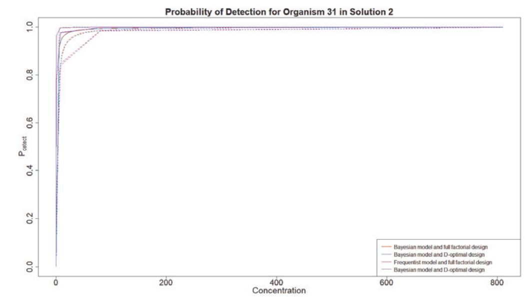
Parameter estimates and intervals

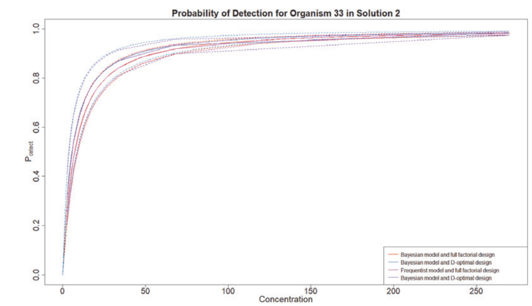


- We fitted Bayesian and frequentist logistic models to the original data (from the full factorial design) and to the subset of the data (from the D-optimal design with available data)
- The resulting parameter estimates and credible and confidence intervals are shown in the above figure
- Results from the frequentist model are similar to the results using the Bayesian model
- Results when we had almost all the data required by the D-optimal design (e.g., O41, S5) were similar to the original design (full factorial)
- Levels that needed more runs (e.g., O31) had wider intervals

**Concentration levels have been scaled to protect proprietary information*

Detection curves examples*





- Figures show the detection curves obtained using the results from the four models fitted: frequentist model using the full factorial design, frequentist model using the D-optimal design, and two Bayesian models using those designs
- Top figure shows the detection curves obtained from the models' results when we did not have enough data collected
- Bottom figure shows the detection curves obtained from the models' results when we had all the runs needed for O33 and almost all the runs needed for S2 (according to the D-optimal design)
- Each figure shows similar detection curves regardless of the model (frequentist/Bayesian) or design used (full factorial/D-optimal)

Conclusions

- It is important to determine which data points to collect in order to obtain as much information as possible with each run
- The use of a D-optimal design is a good alternative to a full factorial design, because it produces similar results while saving resources
- Regardless of the experimental design used to collect the data, Bayesian and frequentist methods produce similar results when analyzing these data

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. REPORT DATE *(DD-MM-YYYY)* 03-04-2019	**2. REPORT TYPE** OED Draft	**3. DATES COVERED** *(From - To)*

4. TITLE ANDSUBTITLE

D-Optimal as an Alternative to Full Factorial Designs: A Case Study

5a. CONTRACT NUMBER
HQ0034-14-D-0001

5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)

Keyla Pagan-Rivera (OED)

5d. PROJECT NUMBER
BD-9-2299(18); BD-9-2299(90)

5e. TASK NUMBER
229918; 229990

5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

Institute for Defense Analyses
4850 Mark Center Drive
Alexandria, Virginia 22311-1882

8. PERFORMING ORGANIZATION REPORT NUMBER

D-10580-NS

H 2019-000161

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)

Director, Operational Test and Evaluation
1700 Defense Pentagon
Washington, DC 20301-1700

10. SPONSOR/MONITOR'S ACRONYM(S)

11. SPONSOR/MONITOR'S REPORT NUMBER

12. DISTRIBUTION / AVAILABILITY STATEMENT
Approved for public release. Distribution is unlimited.

13. SUPPLEMENTARY NOTES
Project Leader: Keyla Pagan-Rivera

14. ABSTRACT
The use of Bayesian statistics and experimental design as tools to scope testing and analyze data related to defense has increased in recent years. Planning a test using experimental design will allow testers to cover the operational space while maximizing the information obtained from each run. Understanding which factors can affect a detector's performance can influence military tactics, techniques and procedures, and improve a commander's situational awareness when making decisions in an operational environment. This presentation will explain how a D-optimal experimental design could be an option for planning a test when the number of runs is limited but an adequate test is desired. Additionally, it will describe how the results of a Bayesian multiple logistic model could be used to show in what way the operational environment can affect the detector's performance.

15. SUBJECT TERMS
Bayesian; Design of Experiments (DOE); Logistic Regression

16. SECURITY CLASSIFICATION OF:			**17. LIMITATION OF ABSTRACT**	**18. NUMBER OF PAGES**	**19a. NAME OF RESPONSIBLE PERSON** Breeana Anderson (OED)
a. REPORT Unclassified	**b. ABSTRACT** Unclassified	**c. THIS PAGE** Unclassified	Unlimited		**19b. TELEPHONE NUMBER** *(include area code)* (703) 845-6967

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI std. Z39.18