Quantum computing for finance

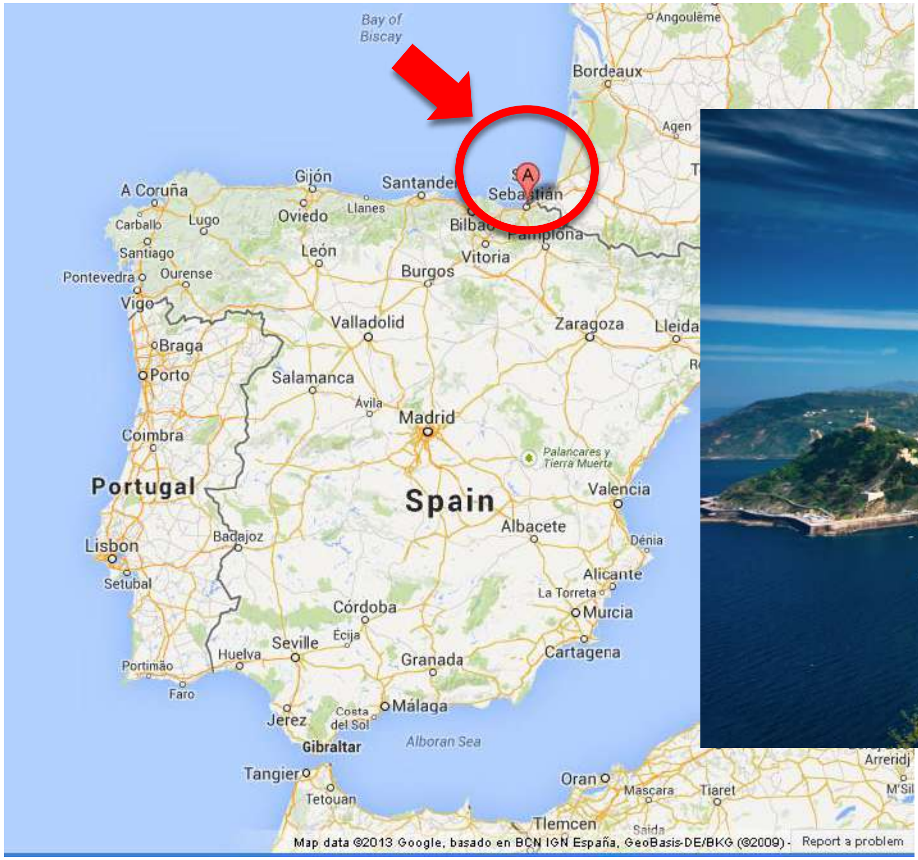
Román Orús

Donostia International Physics Center (DIPC)
Multiverse Computing

September 3rd 2020

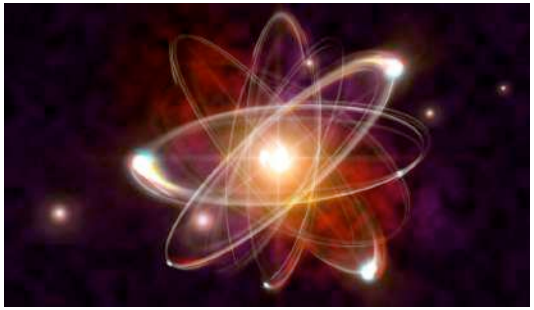
Quantum @ Donostia – San Sebastián

MULTIVERSE
COMPUTING



What is quantum computing?

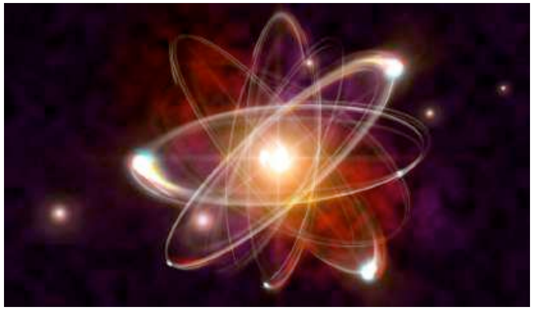
What is quantum computing?



Use **quantum mechanics**
to do calculations...

...that otherwise are **intractable**.

What is quantum computing?



Use **quantum mechanics** to do calculations...

...that otherwise are **intractable**.

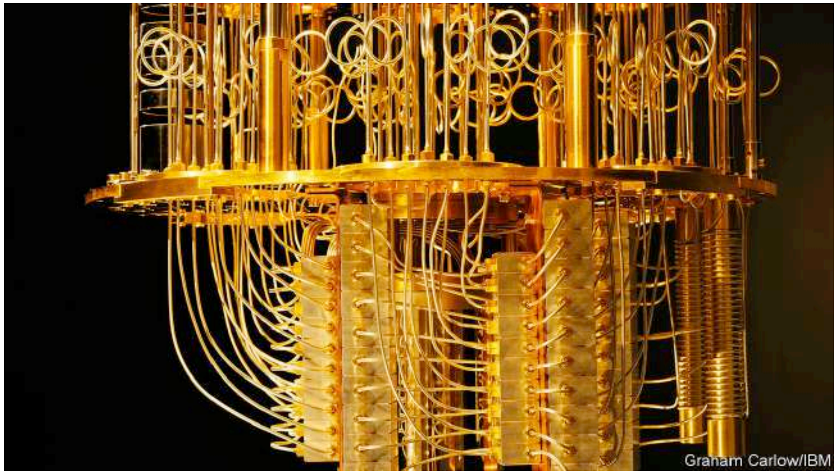
- **Quantum annealing** *(energy minimization)*

D-Wave The Quantum Computing Company™

- **Quantum gates** *(quantum circuits)*

IBM Google rigetti

(not exhaustive)



Quantum processors improving **every day**

A nice example

Traffic flow optimization with a quantum annealer

The Quantum Computing Company™

Optimization of routes to the airport for 10,000 taxis in Beijing

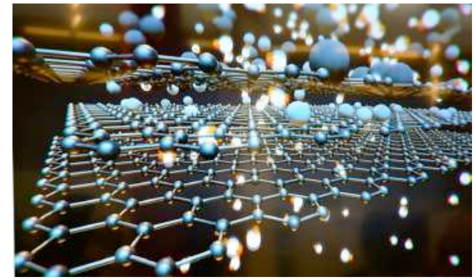
Predict taxi demand from GPS data in Barcelona

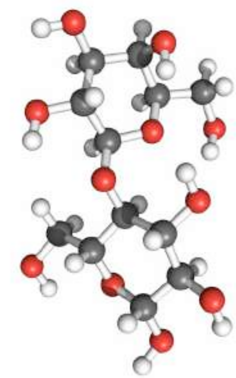
F. Neukart et al, arXiv:1708.01625

More cool examples: https://www.youtube.com/watch?v=NTnu1UiFXVo

Quantum computer use-cases

- Material science
- Finance
- Chemistry
- Optimization
- Machine learning
- ... and whatever you can imagine, and much more





Quantum computer use-cases

- Material science
- **Finance**
- Chemistry
- Optimization
- Machine learning
- ... and whatever you can imagine, and much more

Quantum computer use-cases

- Material science
- **Finance**
- Chemistry
- Optimization
- Machine learning
- ... and whatever you can imagine, and much more

WHO?

- Banks
- Central banks
- Finance departments
- Rating agencies
- Regulators
- Tax offices
- ...

Why quantum computing for finance?

Why quantum computing for finance?

Why quantum computing for finance?

Wrong motivation!!! ☺

Why quantum computing for finance?

Because it’s full of hard problems!

Motivation

* **Finance** deals with the uncertainty in the future behavior of an asset, and the prices and returns (profits or losses) it may have in the future.

→ Complex system

→ Strongly correlated

→ Difficult to predict

Motivation

* **Finance** deals with the uncertainty in the future behavior of an asset, and the prices and returns (profits or losses) it may have in the future.

→ Complex system

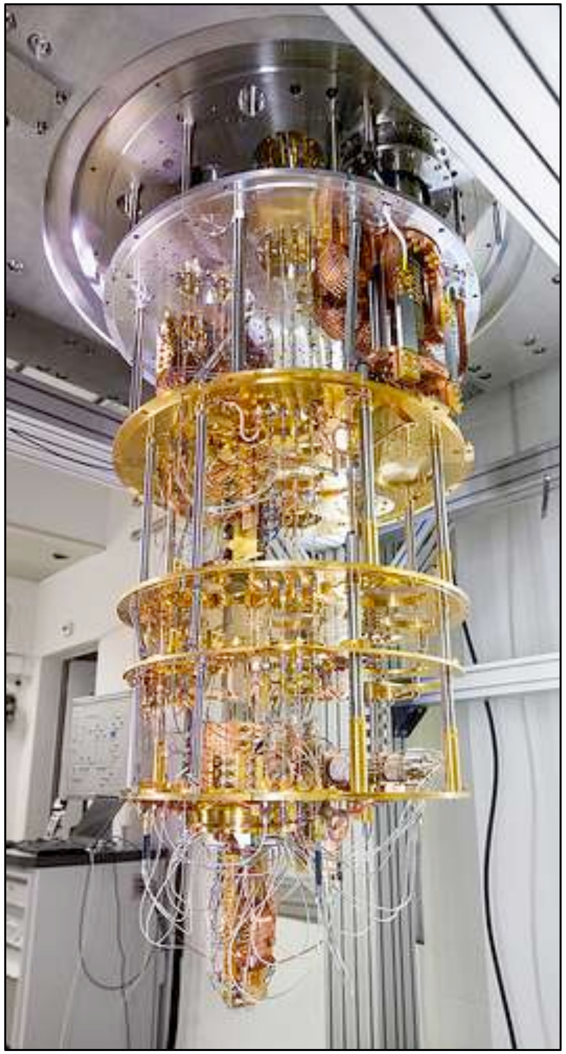
→ Strongly correlated

→ Difficult to predict

* **Strongly mathematical**: optimization problems, Monte Carlo sampling, stochastic differential equations, machine learning...

quantitative finance (“quants“)

Motivation

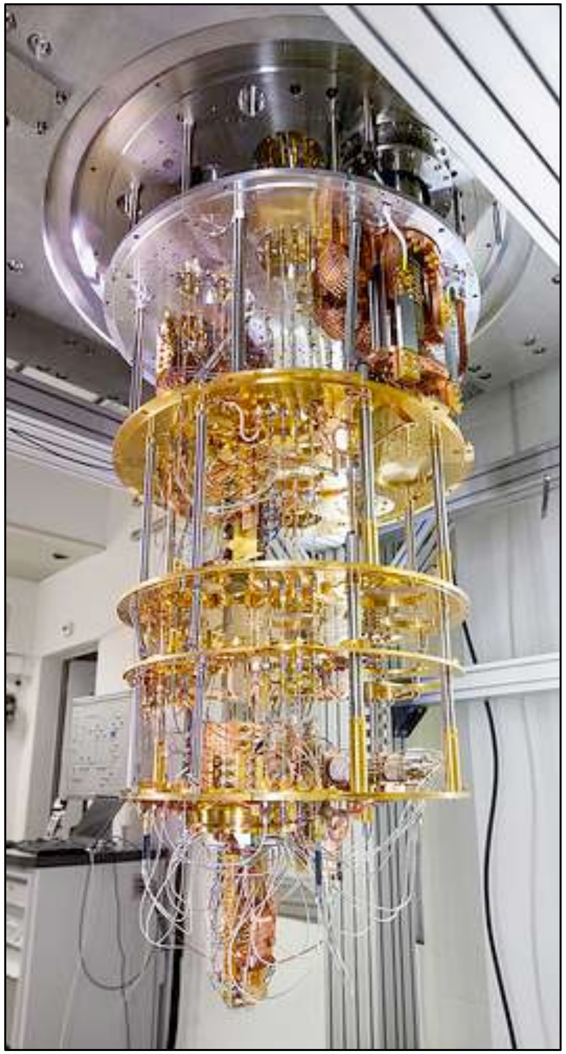


* There must be applications of quantum computing to financial problems.

IBM D-Wave The Quantum Computing Company™

Google rigetti

Motivation

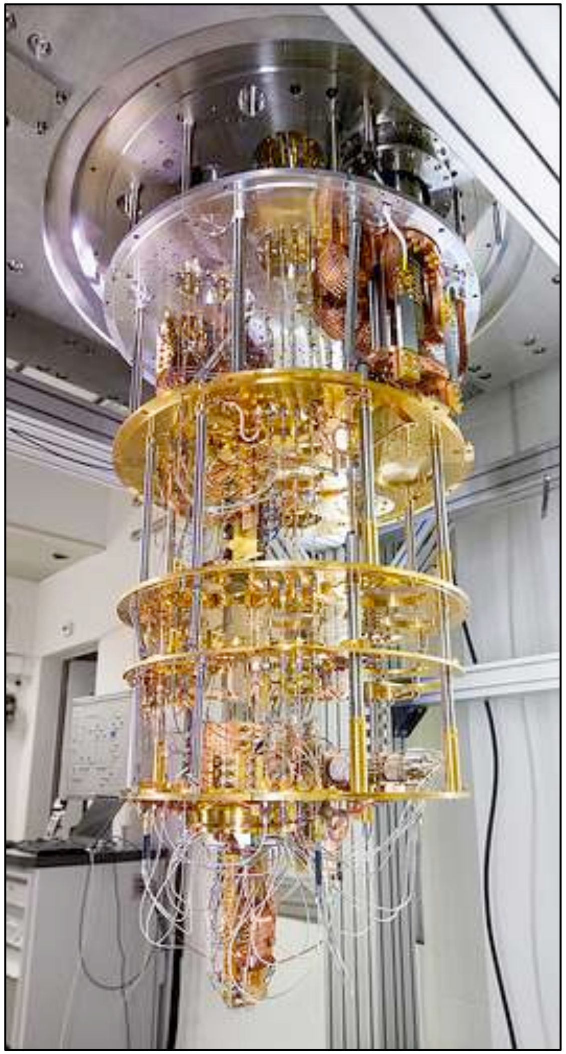


IBM D:WAVE
The Quantum Computing Company™
Google rigetti

* There must be applications of quantum computing to financial problems.

* Using less time = earning more money

Motivation



IBM D:WAVE The Quantum Computing Company™

Google rigetti

* There must be applications of quantum computing to financial problems.

* Using less time = earning more money

* Particularly relevant with the advent of quantum technologies!

Google AI Blog

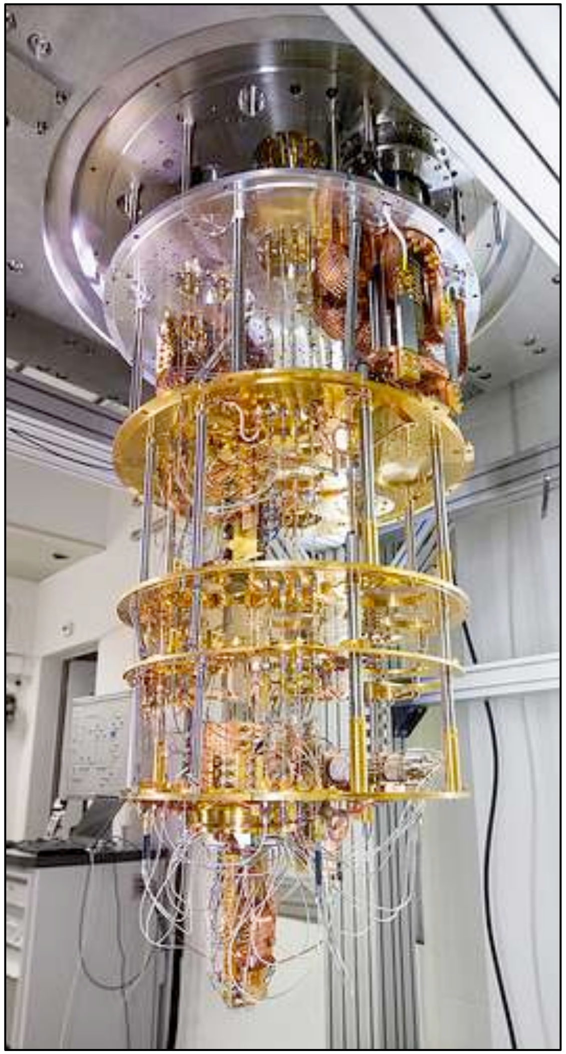
The latest news from Google AI

Quantum Supremacy Using a Programmable Superconducting Processor

Wednesday, October 23, 2019

Posted by John Martinis, Chief Scientist Quantum Hardware and Sergio Boixo, Chief Scientist Quantum Computing Theory, Google AI Quantum

Motivation



* There must be applications of quantum computing to financial problems.

* Using less time = earning more money

* Particularly relevant with the advent of quantum technologies!

Google AI Blog

The latest news from Google AI

Quantum Supremacy Using a Programmable Superconducting Processor

Wednesday, October 23, 2019

Posted by John Martinis, Chief Scientist Quantum Hardware and Sergio Boixo, Chief Scientist Quantum Computing Theory, Google AI Quantum

* **Technological, practical & sociological relevance.**

Motivation

Question	Broad approach

Motivation

Question	Broad approach
Which assets should be included in an optimum portfolio, and how should one change its composition according to the market?	Optimization models

Motivation

Question	Broad approach
Which assets should be included in an optimum portfolio, and how should one change its composition according to the market?	Optimization models
How to detect opportunities in the different assets in the market, and take profit by trading them?	Machine learning

Motivation

Question	Broad approach
Which assets should be included in an optimum portfolio, and how should one change its composition according to the market?	Optimization models
How to detect opportunities in the different assets in the market, and take profit by trading them?	Machine learning
How to estimate the risk of a portfolio, a company, or even the whole financial system?	Monte Carlo

Motivation

Question	Broad approach
Which assets should be included in an optimum portfolio, and how should one change its composition according to the market?	Optimization models **Quantum optimization**
How to detect opportunities in the different assets in the market, and take profit by trading them?	Machine learning **Quantum machine learning**
How to estimate the risk of a portfolio, a company, or even the whole financial system?	Monte Carlo **Quantum amplitude estimation**

Quantum optimization

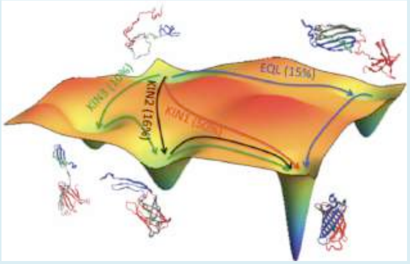
- Quantum Annealing
- Quantum Approximate Optimization Algorithm (QAOA)
- Variational Quantum Eigensolver (VQE)

Quantum optimization

We need tools to solve **optimization problems**
(i.e., finding the best solution, or at least a good one,
from many possible options)

In physics: **energy minimization problem**

e.g., protein folding

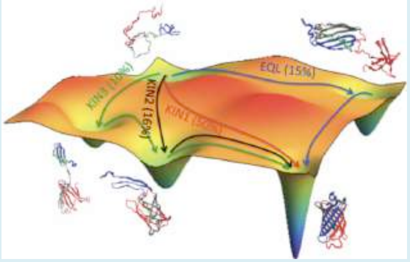


Quantum optimization

We need tools to solve **optimization problems** (i.e., finding the best solution, or at least a good one, from many possible options)

In physics: **energy minimization problem**

e.g., protein folding



Toy example: $x_1, x_2, x_3 = 0, 1$ (bits); Which configurations satisfy $x_1 + x_2 + x_3 = 1$?

(simple 3-bit instance of Exact Cover problem, NP-Complete)

Solutions: (0,0,1), (0,1,0), (1,0,0)

As an optimization problem: find the minimum of

$$f(x_1, x_2, x_3) = (x_1 + x_2 + x_3 - 1)^2$$

$$= 2x_1x_2 + 2x_2x_3 + 2x_1x_3 - x_1 - x_2 - x_3 + 1$$

QUBO formula (QUadratic Binary Optimization)

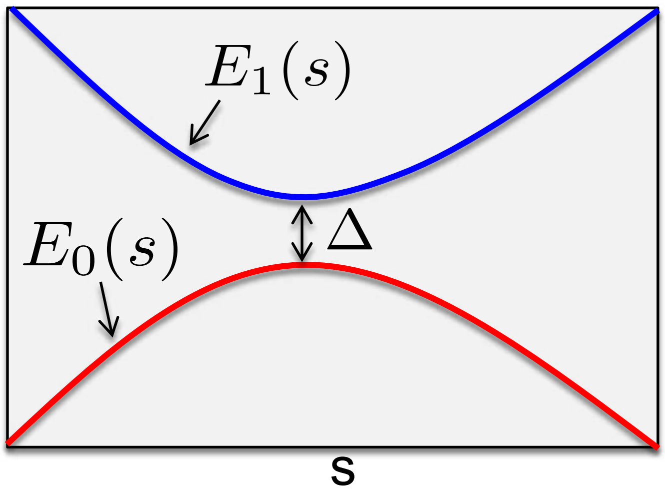
Quantum annealing $\hat{x} = (1 + \hat{\sigma}^z)/2$

"Non-ideal" implementation of adiabatic quantum computation

$$H(s(t)) = s(t)H_0 + (1 - s(t))H_P$$

$$s(0) = 1; s(T) = 0$$

Usually: $s(t) = \left(1 - \frac{t}{T}\right)$ $\quad T = O\left(\frac{1}{\Delta^2}\right)$



$$H_0 = \sum_i \sigma_i^x \qquad H_P = \sum_i h_i \sigma_i^z + \sum_{i,j} J_{ij} \sigma_i^z \sigma_j^z$$

(not quantum universal!)

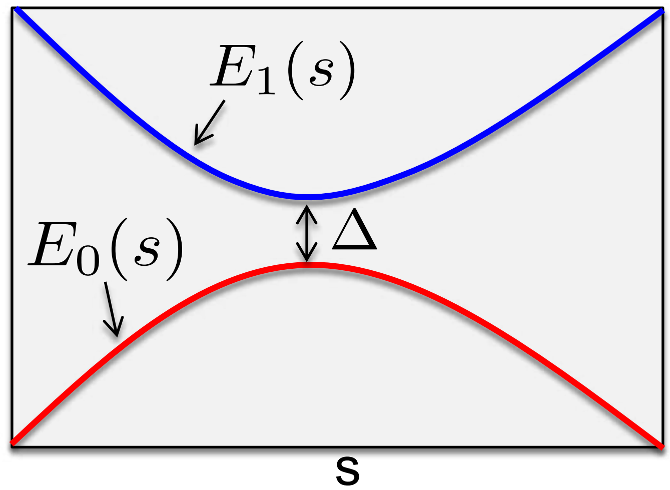
Quantum annealing $\hat{x} = (1 + \hat{\sigma}^z)/2$

"Non-ideal" implementation of adiabatic quantum computation

$$H(s(t)) = s(t)H_0 + (1 - s(t))H_P$$

$$s(0) = 1; s(T) = 0$$

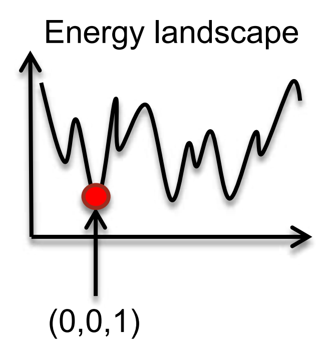
Usually: $s(t) = \left(1 - \frac{t}{T}\right)$ $\quad T = O\left(\frac{1}{\Delta^2}\right)$



$$H_0 = \sum_i \sigma_i^x \qquad H_P = \sum_i h_i \sigma_i^z + \sum_{i,j} J_{ij} \sigma_i^z \sigma_j^z$$

(not quantum universal!)

Repeat process many times, **sample outcomes**, and choose the best found solution (non-ideal conditions imply solution may not be optimal)

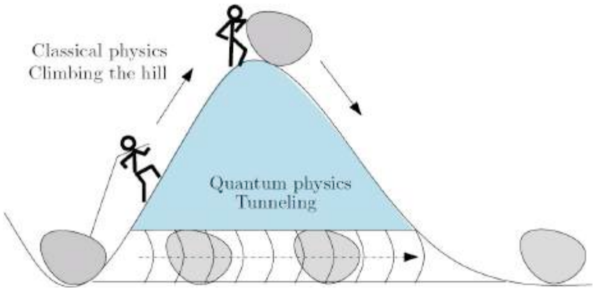


Why faster than classical?

Why faster than classical?

Because of quantum tunnelling

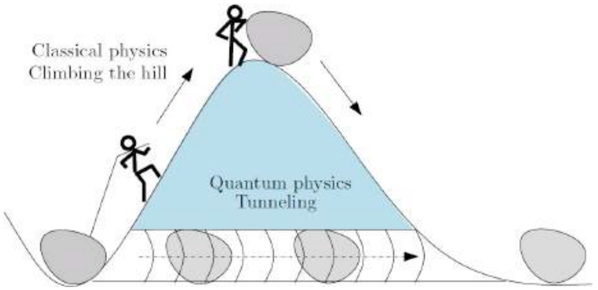
(among other things)



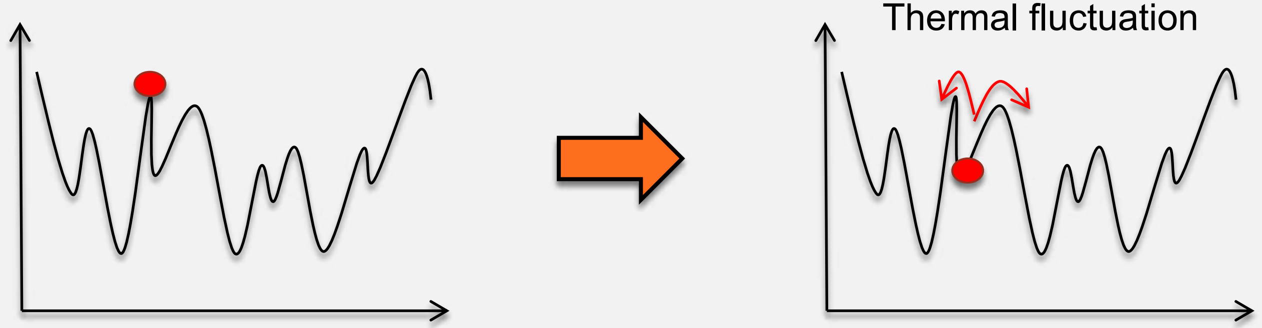
Why faster than classical?

Because of quantum tunnelling

(among other things)



Classical annealing

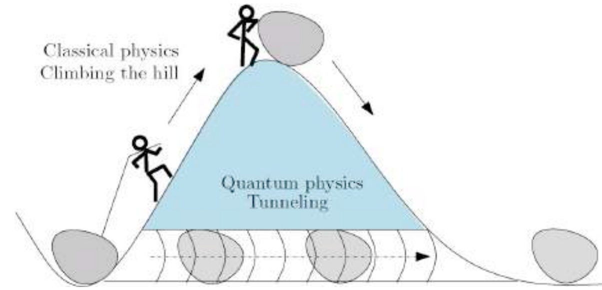


Slow to get out

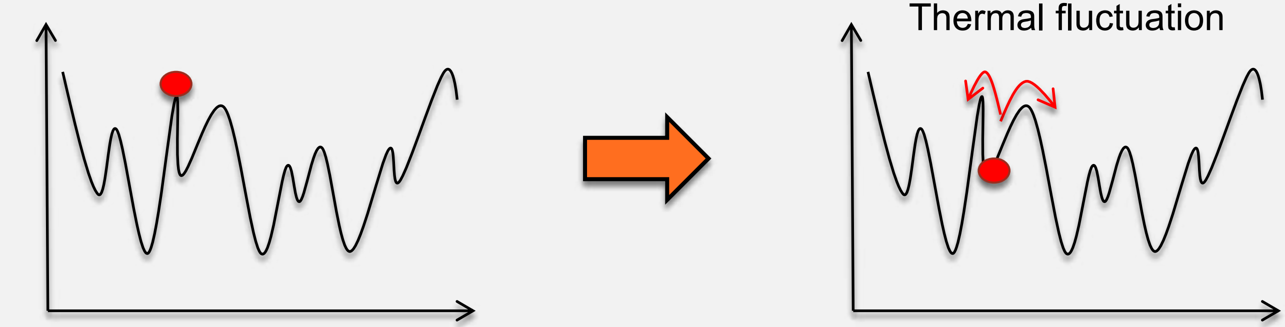
Why faster than classical?

Because of quantum tunnelling

(among other things)



Classical annealing

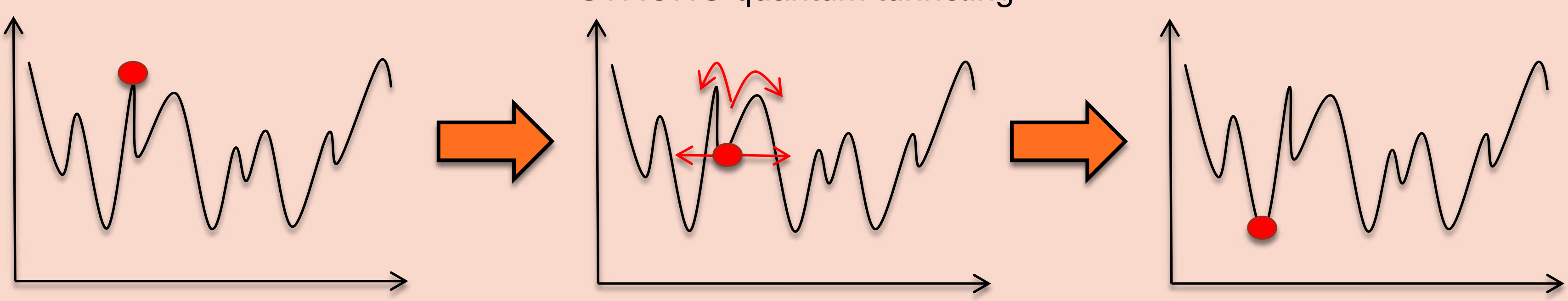


Slow to get out

Quantum annealing

Weak thermal fluctuation AND
STRONG quantum tunneling

Faster to get out



Quant. Approx. Opt. Algorithm (QAOA)

E. Farhi, J. Goldstone, S. Gutmann, arXiv:1411.4028

“QUA-WA“

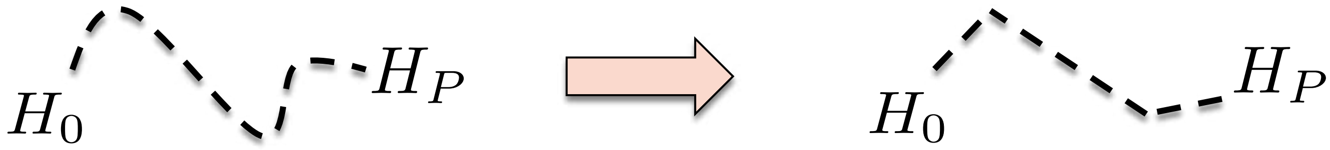
“Break“ adiabatic evolution into discrete optimized steps

Quant. Approx. Opt. Algorithm (QAOA)

E. Farhi, J. Goldstone, S. Gutmann, arXiv:1411.4028

“QUA-WA“

“Break“ adiabatic evolution into discrete optimized steps

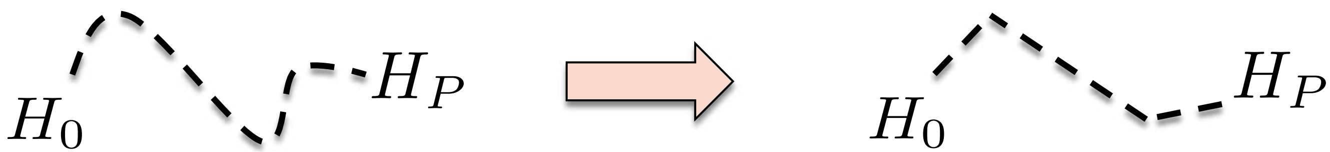


Quant. Approx. Opt. Algorithm (QAOA)

E. Farhi, J. Goldstone, S. Gutmann, arXiv:1411.4028

“QUA-WA“

“Break“ adiabatic evolution into discrete optimized steps



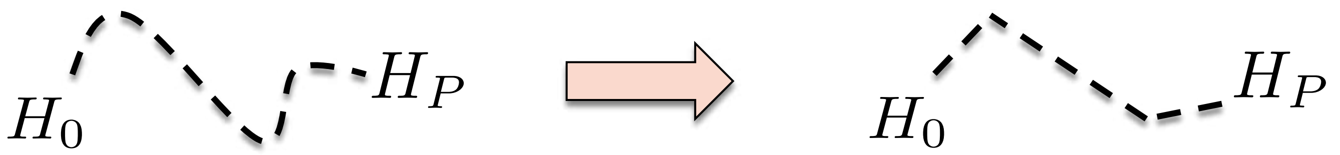
$$U(t) \Rightarrow \prod_{i=1}^{m} \left(e^{-iH_0\alpha_i} e^{-iH_P\beta_i}\right)$$

Quant. Approx. Opt. Algorithm (QAOA)

E. Farhi, J. Goldstone, S. Gutmann, arXiv:1411.4028

“QUA-WA“

“Break“ adiabatic evolution into discrete optimized steps



$$U(t) \Rightarrow \prod_{i=1}^{m} \left(e^{-iH_0\alpha_i} e^{-iH_P\beta_i}\right)$$

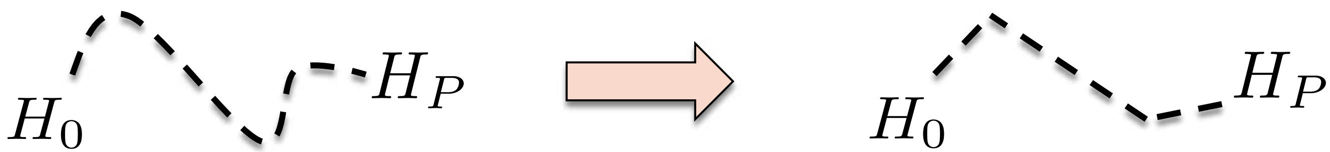
Choose m, sample the energy form the outcome of the quantum circuit, use it to **optimize over alphas and betas**, and repeat until convergence (i.e., optimize the discrete path in Hamiltonian space)

Quant. Approx. Opt. Algorithm (QAOA)

E. Farhi, J. Goldstone, S. Gutmann, arXiv:1411.4028

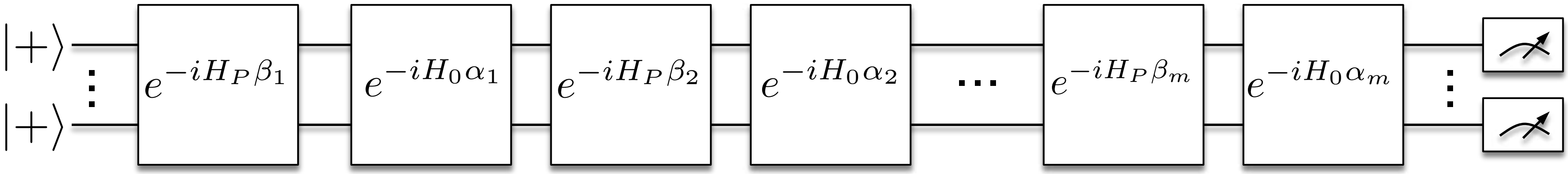
“QUA-WA“

“Break“ adiabatic evolution into discrete optimized steps



$$U(t) \Rightarrow \prod_{i=1}^{m} \left(e^{-iH_0\alpha_i} e^{-iH_P\beta_i} \right)$$

Choose m, sample the energy form the outcome of the quantum circuit, use it to **optimize over alphas and betas**, and repeat until convergence (i.e., optimize the discrete path in Hamiltonian space)

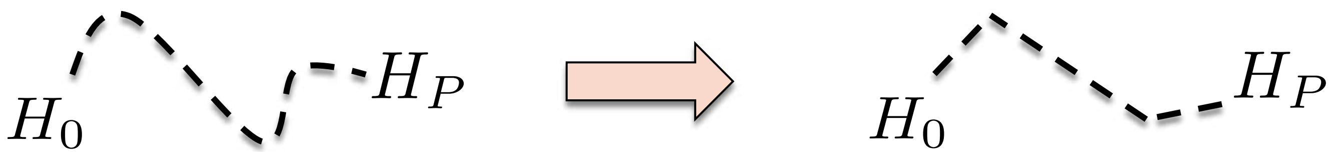


Quant. Approx. Opt. Algorithm (QAOA)

E. Farhi, J. Goldstone, S. Gutmann, arXiv:1411.4028

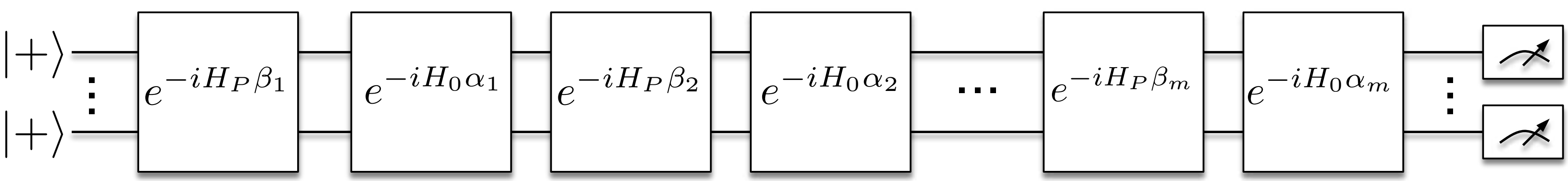
“QUA-WA“

“Break“ adiabatic evolution into discrete optimized steps



$$U(t) \Rightarrow \prod_{i=1}^{m} \left(e^{-iH_0\alpha_i} e^{-iH_P\beta_i} \right)$$

Choose m, sample the energy form the outcome of the quantum circuit, use it to **optimize over alphas and betas**, and repeat until convergence (i.e., optimize the discrete path in Hamiltonian space)



- **Good:** it has the correct structure of entanglement in the variational quantum circuit
- **Bad:** the quantum circuit may be difficult to implement

Variational Quantum Eigensolver (VQE)

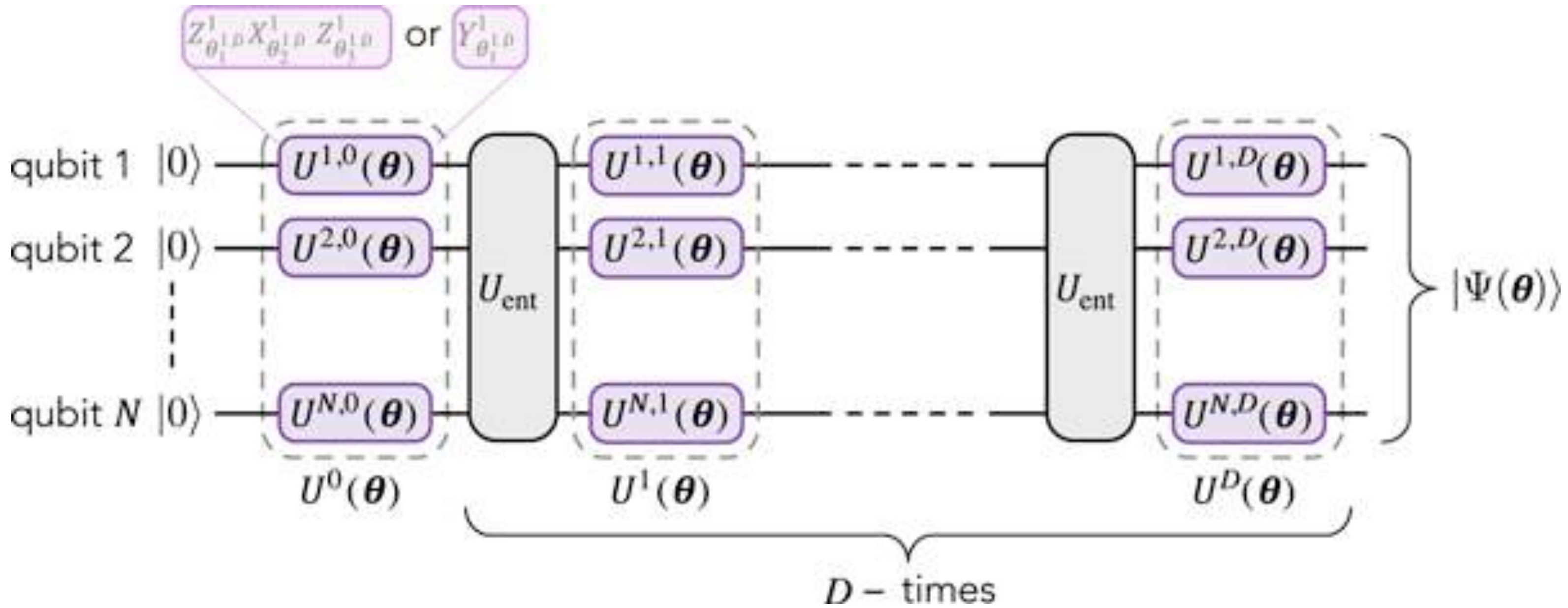
A. Peruzo et at., Nat. Comm. 5 (2014).

Similar to QAOA, but this time we choose the circuit

Variational Quantum Eigensolver (VQE)

A. Peruzo et at., Nat. Comm. 5 (2014).

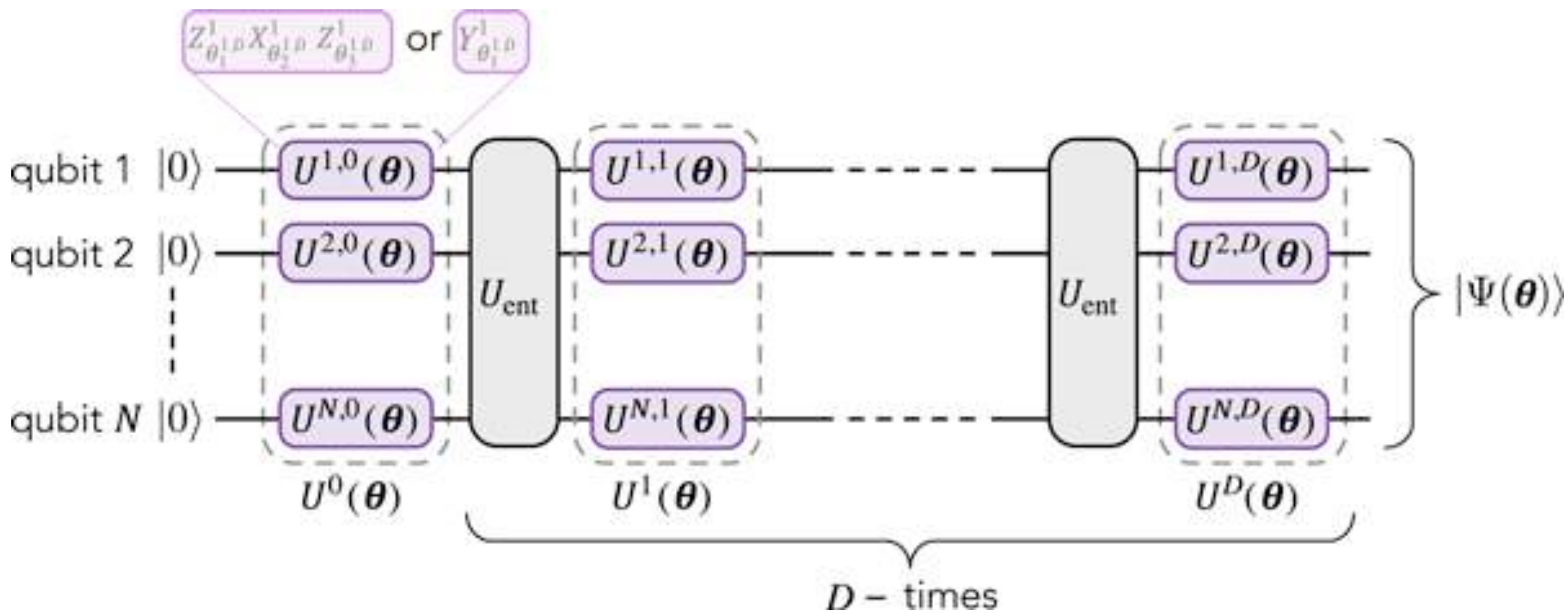
Similar to QAOA, but this time we choose the circuit



Variational Quantum Eigensolver (VQE)

A. Peruzo et at., Nat. Comm. 5 (2014).

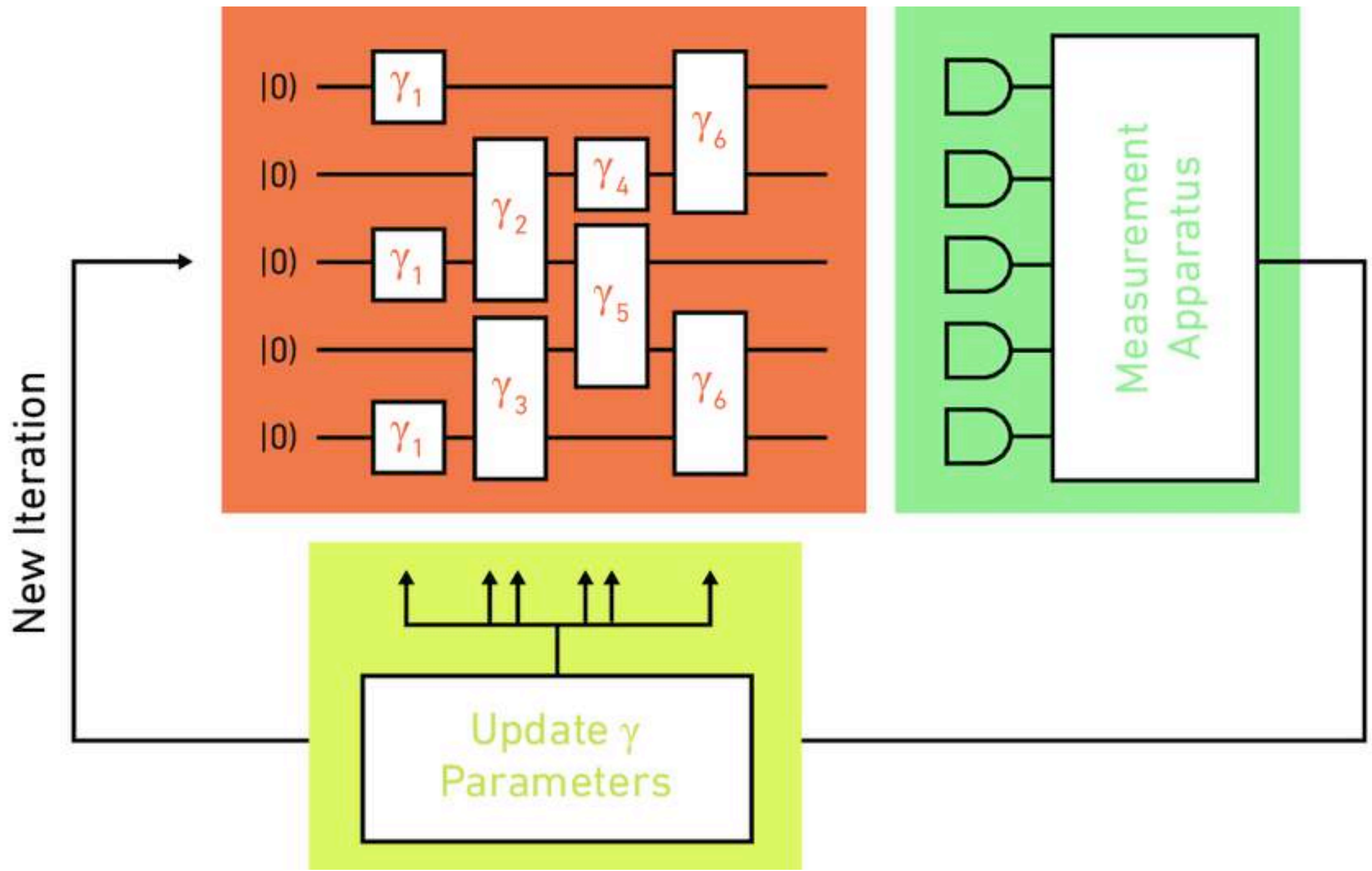
Similar to QAOA, but this time we choose the circuit



- **Good**: we choose the circuit, therefore we can control very well its implementation

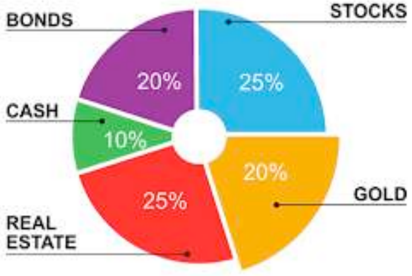
- **Bad:** we choose the circuit, therefore the entanglement structure of the ansatz may not be the one of the problem

QAOA & VQE: hybrid heuristic algorithms



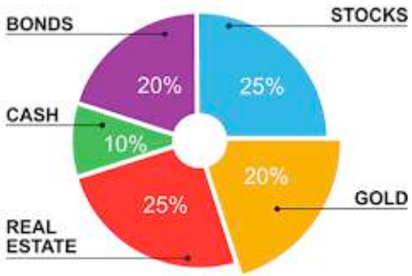
Four examples in finance

1) Dynamic portfolio optimization



* **Problem:** find optimal trajectory in the portfolio space, taking into account transaction costs and market impact

1) Dynamic portfolio optimization



* **Problem:** find optimal trajectory in the portfolio space, taking into account transaction costs and market impact

holdings　　risk aversion

$$\omega = \sum_{t=1}^{T}\left(\mu_t^T\omega_t - \frac{\gamma}{2}\omega_t^T\Sigma_t\omega_t - \underbrace{\Delta\omega_t^T\Lambda_t\Delta\omega_t + \Delta\omega_t^T\Lambda_t'\omega_t}_{\text{transaction costs}}\right)$$

return　　forecast returns　　forecast covariance tensor

Constraints: $\sum_{n=1}^{N}\omega_{nt} = K \quad \forall t, \qquad \omega_{nt} \leq K' \quad \forall t, n$

G. Rosenberg et al, IEEE Journal of Selected Topics in Signal Processing 10, 1053 (2016)
See also P. Rebentrost, S. Lloyd, arXiv:1811.03975

MULTIVERSE
COMPUTING

BBVA

Dynamic Portfolio Optimization with Real Datasets Using Quantum Processors and Quantum-Inspired Tensor Networks

Samuel Mugel,[1] Carlos Kuchkovsky,[2] Escolástico Sánchez,[2] Samuel Fernández-Lorenzo,[2] Jorge Luis-Hita,[2] Enrique Lizaso,[3] and Román Orús[3,4,5]

[1]Multiverse Computing, Banting Institute, 100 College Street, ONRamp Suite 150, Toronto, ON M5G 1L5 Canada
[2]BBVA Research & Patents, Calle Sauceda 28, 28050 Madrid, Spain
[3]Multiverse Computing, Paseo de Miramón 170, E-20014 San Sebastián, Spain
[4]Donostia International Physics Center, Paseo Manuel de Lardizabal 4, E-20018 San Sebastián, Spain
[5]Ikerbasque Foundation for Science, Maria Diaz de Haro 3, E-48013 Bilbao, Spain

In this paper we tackle the problem of dynamic portfolio optimization, i.e., determining the optimal trading trajectory for an investment portfolio of assets over a period of time, taking into account transaction costs and other possible constraints. This problem, well-known to be NP-Hard, is central to quantitative finance. After a detailed introduction to the problem, we implement a number of quantum and quantum-inspired algorithms on different hardware platforms to solve its discrete formulation using real data from daily prices over 8 years of 52 assets, and do a detailed comparison of the obtained Sharpe ratios, profits and computing times. In particular, we implement classical solvers (Gekko, exhaustive), D-Wave Hybrid quantum annealing, two different approaches based on Variational Quantum Eigensolvers on IBM-Q (one of them brand-new and tailored to the problem), and for the first time in this context also a quantum-inspired optimizer based on Tensor Networks. In order to fit the data into each specific hardware platform, we also consider doing a preprocessing based on clustering of assets. From our comparison, we conclude that D-Wave Hybrid and Tensor Networks are able to handle the largest systems, where we do calculations up to 1272 fully-connected qubits for demonstrative purposes. Finally, we also discuss how to mathematically implement other possible real-life constraints, as well as several ideas to further improve the performance of the studied methods.

arXiv:2007.00017, first implementation with real data up to 52 assets and 8 years on D-Wave, VQE, and Tensor Networks (quantum-inspired)

Hybrid quantum-classical optimization for financial index tracking

Samuel Fernández-Lorenzo
BBVA Client Solutions Research & Patents, Calle Sauceda 28, 28050 Madrid, Spain.
E-mail: samuel.fernandez.lorenzo.contractor@bbva.com

Diego Porras
Instituto de Física Fundamental, IFF-CSIC, Calle Serrano 113b, 28006 Madrid, Spain.

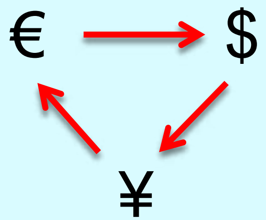
Juan José García-Ripoll
Instituto de Física Fundamental, IFF-CSIC, Calle Serrano 113b, 28006 Madrid, Spain.

Abstract. Tracking a financial index boils down to replicating its trajectory of returns for a well-defined time span by investing in a weighted subset of the securities included in the benchmark. Picking the optimal combination of assets becomes a challenging NP-hard problem even for moderately large indices consisting of dozens or hundreds of assets, thereby requiring heuristic methods to find approximate solutions. Hybrid quantum-classical optimization with variational gate-based quantum circuits arises as a plausible method to improve performance of current schemes. In this work we introduce a heuristic pruning algorithm to find weighted combinations of assets subject to cardinality constraints. We further consider different strategies to respect such constraints and compare the performance of relevant quantum ansätze and classical optimizers through numerical simulations.

CSIC
CONSEJO SUPERIOR DE INVESTIGACIONES CIENTÍFICAS

arXiv:2008.12050, another nice paper by good friends

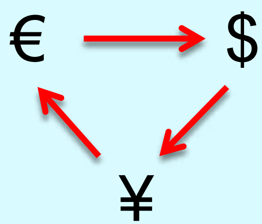
2) Finding arbitrage opportunities



And make profit!

* **Problem:** given a network of assets and prices, find cycles that provide a positive return without incurring in risks (NP-Hard)

2) Finding arbitrage opportunities



And make profit!

* **Problem:** given a network of assets and prices, find cycles that provide a positive return without incurring in risks (NP-Hard)

conversion rates (non-symmetric)

$$\omega = \sum_{(i,j)\in E} x_{ij} \log c_{ij} - M_1 \underbrace{\sum_{i\in V} \left(\sum_{j,(i,j)\in E} x_{ij} - \sum_{j,(j,i)\in E} x_{ji} \right)^2}_{\text{flow constraint (cycle)}} - M_2 \underbrace{\sum_{i\in V} \sum_{j,(i,j)\in E} x_{ij} \left(\sum_{j,(i,j)\in E} x_{ij} - 1 \right)}_{\text{only once through an asset}}$$

boolean variables $x_{ij} = 1$ if {i,j} belongs to the cycle

Implemented on D-Wave 2X for a small network of 5 assets

G. Rosenberg, 1QBit Whitepaper (2016)

3) Feature selection in credit scoring

* **Problem:** which data on past credit applicants provides information on the creditworthiness of new applicants

3) Feature selection in credit scoring

* **Problem:** which data on past credit applicants provides information on the creditworthiness of new applicants

Matrix U:
columns = features of past credit applicant (age, etc)
rows = numerical values

Vector V: record of past credit decisions

boolean variable = 1 if selected feature

$$\omega = -\left(\alpha \sum_{j=1}^{n} x_j |\rho_{V_j}| - (1-\alpha) \sum_{j=1}^{n} \sum_{k \neq j}^{n} x_j x_k |\rho_{jk}| \right)$$

control parameter

correlation between column j of U and V (influence on outcome)

correlation between columns j,k of U (mutual independence)

Proof of principle with 1QBit SDK

A. Milne, M. Rounds, P. Goddard, 1QBit Whitepaper (2017)

4) Financial crash prediction

* **Problem:** given a financial network in equilibrium, if there is a tiny change in the prices of assets, could there be a massive failure of institutions? (NP-Hard)

4) Financial crash prediction

* **Problem:** given a financial network in equilibrium, if there is a tiny change in the prices of assets, could there be a massive failure of institutions? (NP-Hard)

self-ownership asset prices Failure term (non-linear)

$$\left(\vec{v} - \tilde{\mathbf{C}}(\mathbb{I} - \mathbf{C})^{-1}\left(\mathbf{D}\vec{p} - \vec{b}(\vec{v}, \vec{p})\right)\right)^2 \geq 0$$

market values cross-holdings asset ownership

Equality ➔ financial equilibrium

Variational problem!

4) Financial crash prediction

* **Problem:** given a financial network in equilibrium, if there is a tiny change in the prices of assets, could there be a massive failure of institutions? (NP-Hard)

self-ownership asset prices Failure term (non-linear)

$$\left(\vec{v} - \tilde{\mathbf{C}}(\mathbb{I} - \mathbf{C})^{-1}\left(\mathbf{D}\vec{p} - \vec{b}(\vec{v}, \vec{p})\right)\right)^2 \geq 0$$

market values cross-holdings asset ownership

Equality ➔ financial equilibrium

Variational problem!

It's equivalent to finding the ground state of a spin-1/2 system with 2-body interactions

$$H_P = \sum_i h_i \sigma_i^z + \sum_{i,j} J_{ij} \sigma_i^z \sigma_j^z$$

n = number of institutions
2q+1 = bits to describe market values
r = degree of non-linearity

$$N_{qubits} = n(2q+1) + O\left(r\left(\frac{enq}{r}\right)^{2r}\right)$$

Physically:
financial crash ~ 1st order QPT

RO, S. Mugel, E. Lizaso, arXiv:1810.07690 (to appear in PRA)

Procedure *(technical)*

1) Bit expansion of variables $v_i \approx \sum_{\alpha=-q}^{q} x_{i,\alpha} 2^{\alpha}$

Procedure *(technical)*

1) Bit expansion of variables $v_i \approx \sum_{\alpha=-q}^{q} x_{i,\alpha} 2^{\alpha}$

2) Polynomial expansion of failure $b_i(v_i, \vec{p}) \approx \beta_i(\vec{p}) \mathrm{Poly}_r(v_i - v_i^c)$

Procedure *(technical)*

1) Bit expansion of variables $v_i \approx \sum_{\alpha=-q}^{q} x_{i,\alpha} 2^{\alpha}$

2) Polynomial expansion of failure $b_i(v_i, \vec{p}) \approx \beta_i(\vec{p})\mathrm{Poly}_r(v_i - v_i^c)$

3) Promote to quantum spin Hamiltonian $\hat{x} = (1 + \hat{\sigma}^z)/2$

Procedure *(technical)*

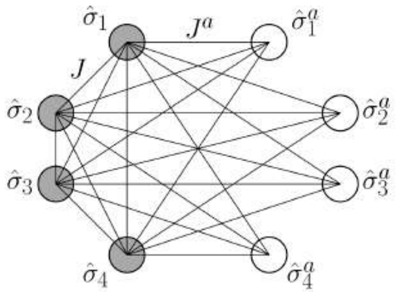
1) Bit expansion of variables $v_i \approx \sum_{\alpha=-q}^{q} x_{i,\alpha} 2^{\alpha}$

2) Polynomial expansion of failure $b_i(v_i, \vec{p}) \approx \beta_i(\vec{p}) \mathrm{Poly}_r(v_i - v_i^c)$

3) Promote to quantum spin Hamiltonian $\hat{x} = (1 + \hat{\sigma}^z)/2$

4) Reduce to 2-spin interactions using ancillas

$$N_{qubits} = n(2q+1) + O\left(r\left(\frac{enq}{r}\right)^{2r}\right)$$



Procedure *(technical)*

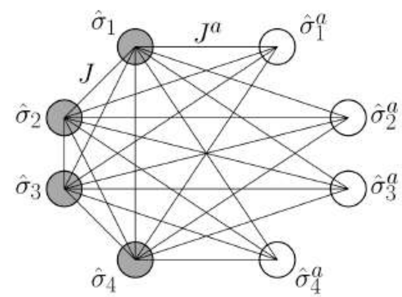
1) Bit expansion of variables $v_i \approx \sum_{\alpha=-q}^{q} x_{i,\alpha} 2^{\alpha}$

2) Polynomial expansion of failure $b_i(v_i, \vec{p}) \approx \beta_i(\vec{p}) \mathrm{Poly}_r(v_i - v_i^c)$

3) Promote to quantum spin Hamiltonian $\hat{x} = (1 + \hat{\sigma}^z)/2$

4) Reduce to 2-spin interactions using ancillas

$$N_{qubits} = n(2q+1) + O\left(r\left(\frac{enq}{r}\right)^{2r}\right)$$



$$H_P = \sum_i h_i \sigma_i^z + \sum_{i,j} J_{ij} \sigma_i^z \sigma_j^z$$

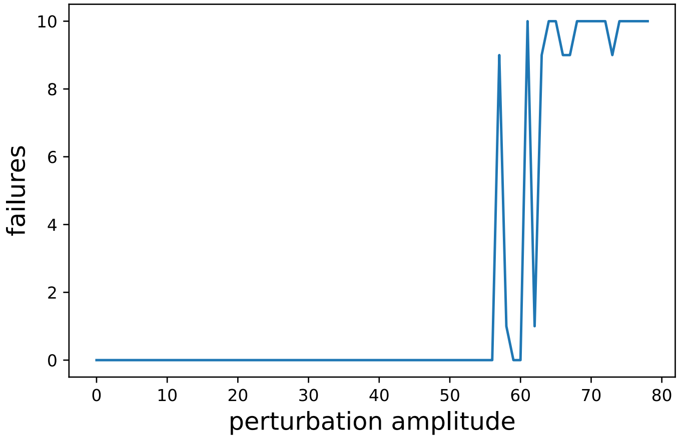
Feed this to the quantum annealer

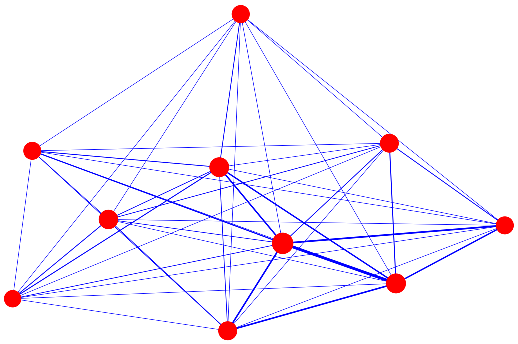
Ground state = financial equilibrium

market

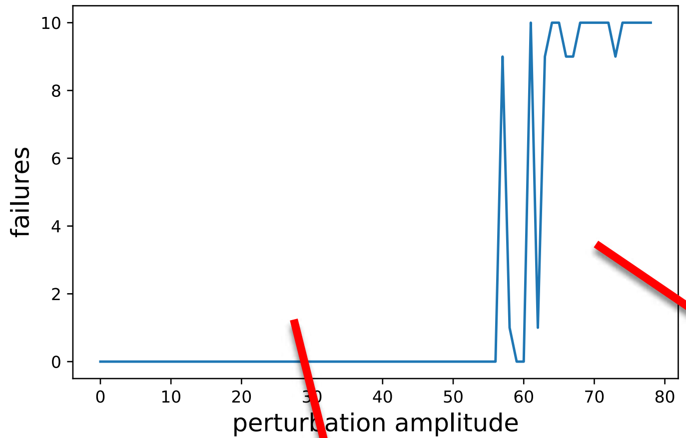
magnetite

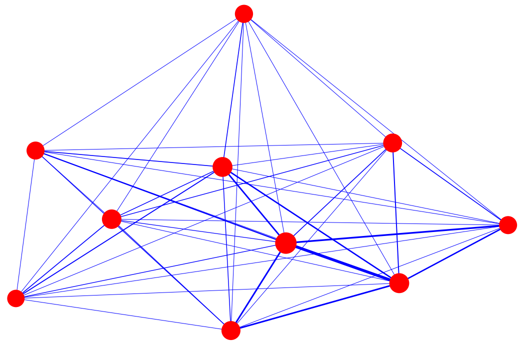
“Magnetic phase” → “Financial phase”





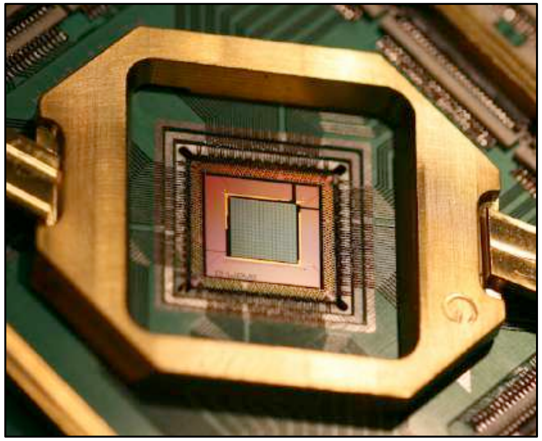
"Magnetic phase" → "Financial phase"

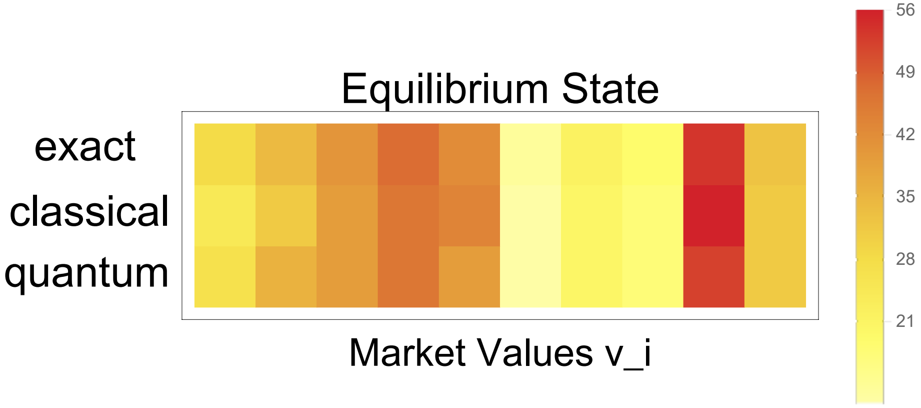


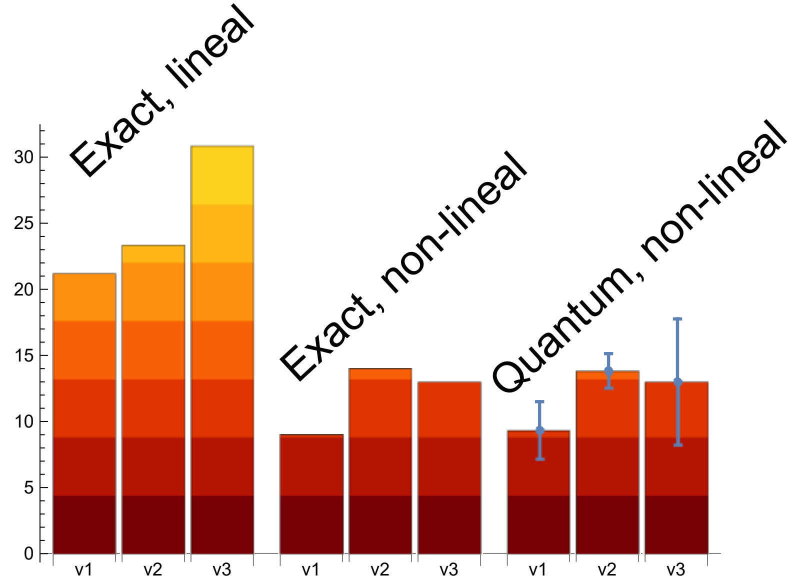


Economic equilibrium with D-Wave

2048 *incoherent* qubits







Y. Ding et al, arXiv:1904.05808

Quantum machine learning

Summary of quantum speed-ups

Method	Speedup	AA	HHL	Adiabatic	QRAM
Bayesian Inference [107, 108]	$O(\sqrt{N})$	Y	Y	N	N
Online Perceptron [109]	$O(\sqrt{N})$	Y	N	N	optional
Least squares fitting [9]	$O(\log N^{(*)})$	Y	Y	N	Y
Classical BM [20]	$O(\sqrt{N})$	Y/N	optional/N	N/Y	optional
Quantum BM [22, 62]	$O(\log N^{(*)})$	optional/N	N	N/Y	N
Quantum PCA [11]	$O(\log N^{(*)})$	N	Y	N	optional
Quantum SVM [13]	$O(\log N^{(*)})$	N	Y	N	Y
Quantum reinforcement learning [30]	$O(\sqrt{N})$	Y	N	N	N

J. Biamonte et al, Nature 549, 195 (2017)

* Usually need a universal quantum computer

* More challenging than quantum annealers

* Some topics still under investigation (input/output, optimal number of gates...)

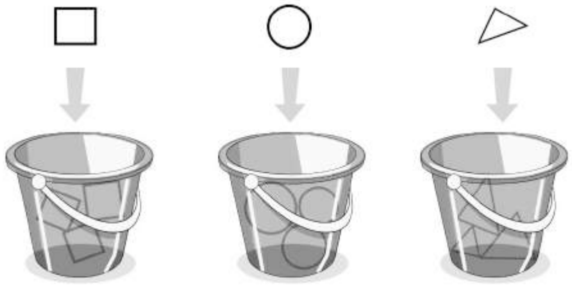
Not going into all details here...

Some examples

1) Data classification

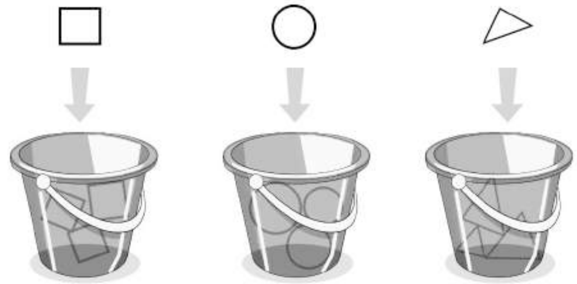
* **Problem:** classify data in different subsets according to features (supervised, unsupervised).

* **Example:** credit scoring in finance (high-risk / low-risk)



1) Data classification

* **Problem:** classify data in different subsets according to features (supervised, unsupervised).



* **Example:** credit scoring in finance (high-risk / low-risk)

→ **Quantum classifiers:** assign N-dim vectors to a cluster of M states in O(log(MN)) time (classical O(poly(MN)). Also, classify M vectors into k clusters in O(k log(kMN)) time.

S. Lloyd, M. Mohseni, P. Rebentrost, arXiv:1307.0411

→ **Quantum support vector machines:** classify a vector into one of two classes given M training data points. Time O(log(MN)) for training and classification (classical O(poly(M,n)).

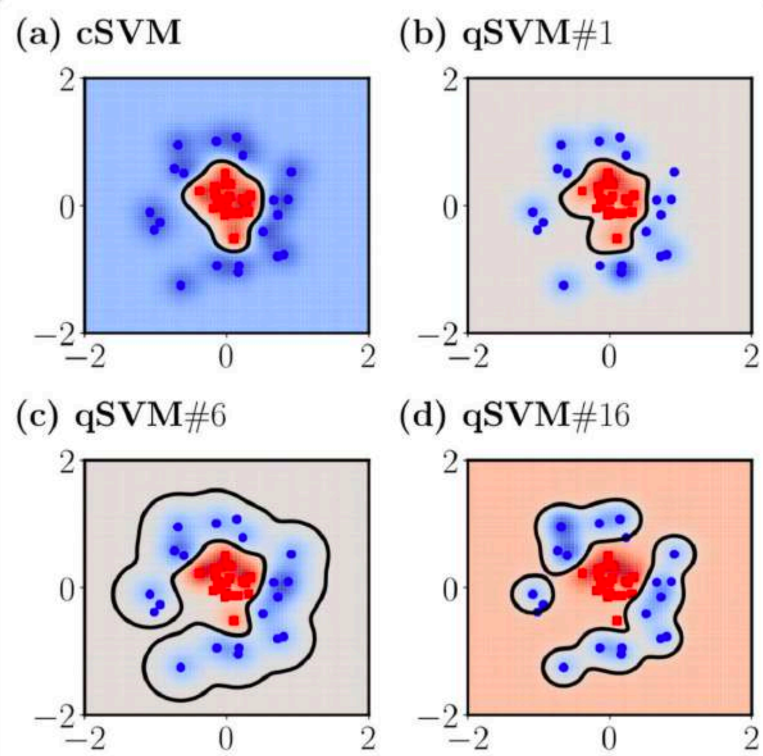
P. Rebentrost, M. Mohseni, S. Lloyd, PRL113, 1 (2014)

qSVM with D-Wave

Juanjo Garcia Ripoll
@jjgarciaripoll

Support Vector Machines in the D-Wave quantum annealer "[qSVM] produces different solutions that often generalizes better to unseen data than the single global minimum of an SVM trained on a conventional computer, especially [with] limited training data."
arxiv.org/pdf/1906.06283...

Traducir Tweet



2) Regression

* **Problem:** given a set of past data points, predict plausible future behavior

* **Examples:** how many umbrellas will I sell next week? Will NASDAQ go up or down?

2) Regression

* **Problem:** given a set of past data points, predict plausible future behavior

* **Examples:** how many umbrellas will I sell next week? Will NASDAQ go up or down?

→ Optimal fit parameters obtained by minimizing least-squares error between training data and values predicted by the model

→ Implies inverse of training data matrix: computationally costly

2) Regression

* **Problem:** given a set of past data points, predict plausible future behavior

* **Examples:** how many umbrellas will I sell next week? Will NASDAQ go up or down?

→ Optimal fit parameters obtained by minimizing least-squares error between training data and values predicted by the model

→ Implies inverse of training data matrix: computationally costly

→ **Quantum algorithm for linear systems of equations:** quantum O(poly(log(N),k) vs classical O(N $k^{1/2}$) for N x N matrix with condition number k.

A. W. Harrow, A. Hassidim, S. Lloyd, PRL 103, 150502 (2009)

3) Principal Component Analysis

* **Input:** data vector $\vec{v}_j$, e.g., stock prices between times (t_j, t_{j+1})

$$C \equiv \sum_j \vec{v}_j \vec{v}_j^T$$ “covariance“ matrix (encodes correlations)

* **Problem:** find dominant eigenvalues and eigenvectors of C (~ SVD!). These can be used to predict future trends

(in finance, 3 dominant eigenvectors: “shift, twist & butterfly”)

3) Principal Component Analysis

* **Input:** data vector $\vec{v}_j$, e.g., stock prices between times (t_j, t_{j+1})

$$C \equiv \sum_j \vec{v}_j \vec{v}_j^T$$ "covariance" matrix (encodes correlations)

* **Problem:** find dominant eigenvalues and eigenvectors of C (~ SVD!). These can be used to predict future trends

(in finance, 3 dominant eigenvectors: "shift, twist & butterfly")

→ Classical computational cost: $O(N^2)$ for N x N matrix

3) Principal Component Analysis

* **Input:** data vector $\vec{v}_j$, e.g., stock prices between times (t_j, t_{j+1})

$$C = \sum_j \vec{v}_j \vec{v}_j^T$$ "covariance" matrix (encodes correlations)

* **Problem:** find dominant eigenvalues and eigenvectors of C (~ SVD!). These can be used to predict future trends

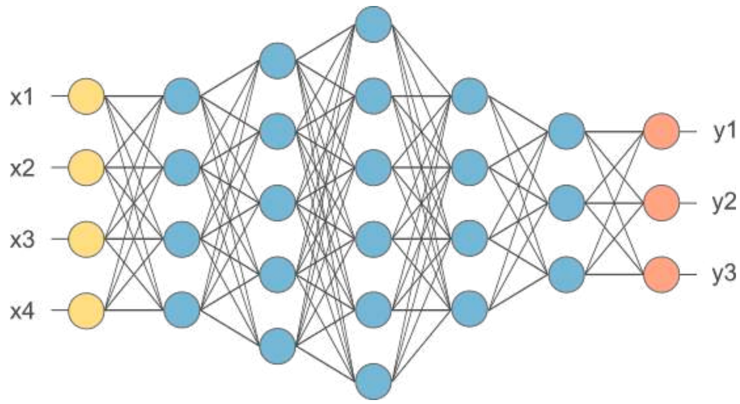
(in finance, 3 dominant eigenvectors: "shift, twist & butterfly")

→ Classical computational cost: O(N^2) for N x N matrix

→ **Quantum-PCA algorithm:** O($(\log(N))^2$) cost. It maps the correlation matrix to a density matrix ρ, and uses tricks of quantum tomography to estimate the eigenvectors with largest eigenvalues (principal components).

S. Lloyd, M. Mohseni, P. Rebentrost, Nat. Phys. 10, 631 (2014).

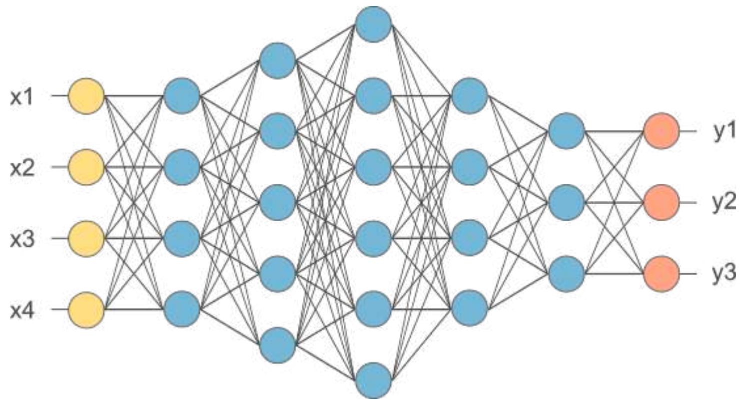
4) Neural networks



Extremely successful in analyzing credit risk and predicting markets

Can be quantumly improved in several generic ways...

4) Neural networks



Extremely successful in analyzing credit risk and predicting markets

Can be quantumly improved in several generic ways...

→ Training of the neural network with a **quantum annealer**. Already done with Boltzmann machines on a D-Wave.

M. Benedetti et al, PRA 94, 1 (2016).

→ Training by gradient descent → using **quantum-PCA** one would have a exponential speed-up.

→ **New quantum neural networks** (quantum perceptrons, quantum hidden Markov models). Still under investigation.

N. Wiebe, A. Kapoor, K. M. Svore, Adv. Neur. Inform. Proc. Syst. 29, 3999 (2016).
A. Monras, A. Beige, K. Wiesner, App. Math. Comp. Sci. 3, 93 (2010).

Quantum amplitude estimation

...or “Grover + Shor“

The quantum algorithm (brief)

The quantum algorithm (brief)

$$A|0\rangle_{n+1} = \sqrt{1-a}|\psi_0\rangle_n|0\rangle + \sqrt{a}|\psi_1\rangle_n|1\rangle$$

* **Problem:** estimate “a“ (prob last qubit = 1)

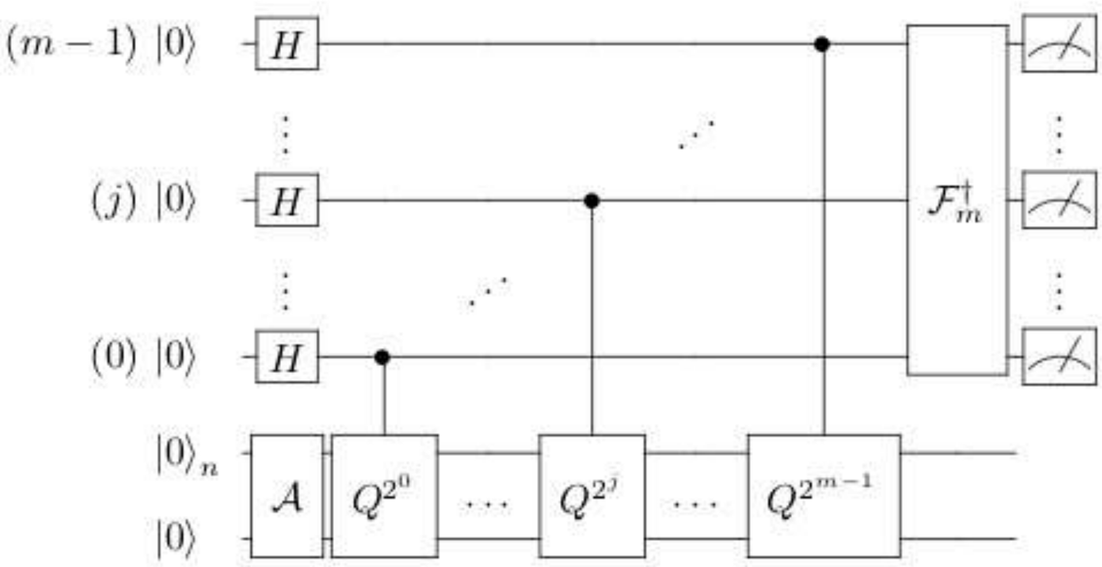
The quantum algorithm (brief)

$$A|0\rangle_{n+1} = \sqrt{1-a}|\psi_0\rangle_n|0\rangle + \sqrt{a}|\psi_1\rangle_n|1\rangle$$

* **Problem:** estimate "a" (prob last qubit = 1)

G. Brassard et al, QCQI 305, 53 (2002)

Define $Q = A(\mathbb{I} - 2|0\rangle_{n+1}\langle 0|_{n+1})A^\dagger$



$$y \in \{0, \ldots, 2^m - 1\}$$

$$\tilde{a} = \sin^2(y\pi/2^m) \in [0,1]$$

$$|a - \tilde{a}| = O(2^{-m})$$

$$prob \geq 8/\pi^2$$

quadratic quantum speed-up wrt classical

Phase estimation algorithm (Shor!) applied to a Grover Kernel

Beating Monte Carlo with QAE

It is possible to estimate the mean of a probability distribution quadratically faster using QAE (wrt Monte Carlo)

A. Montanaro, Proc. Roy. Soc. A: Math. Phys. Eng. Sci. 471, 20150301 (2015)

Beating Monte Carlo with QAE

It is possible to estimate the mean of a probability distribution quadratically faster using QAE (wrt Monte Carlo)

A. Montanaro, Proc. Roy. Soc. A: Math. Phys. Eng. Sci. 471, 20150301 (2015)

$$M|0\rangle_n = \sum_{i=0}^{N-1} \sqrt{p_i}|i\rangle_n$$

Beating Monte Carlo with QAE

It is possible to estimate the mean of a probability distribution quadratically faster using QAE (wrt Monte Carlo)

A. Montanaro, Proc. Roy. Soc. A: Math. Phys. Eng. Sci. 471, 20150301 (2015)

$$M|0\rangle_n = \sum_{i=0}^{N-1} \sqrt{p_i}|i\rangle_n$$

$$F : |i\rangle_n|0\rangle \rightarrow |i\rangle_n \left(\sqrt{1-f(i)}|0\rangle + \sqrt{f(i)}|1\rangle\right)$$

Beating Monte Carlo with QAE

It is possible to estimate the mean of a probability distribution quadratically faster using QAE (wrt Monte Carlo)

A. Montanaro, Proc. Roy. Soc. A: Math. Phys. Eng. Sci. 471, 20150301 (2015)

$$M|0\rangle_n = \sum_{i=0}^{N-1} \sqrt{p_i}|i\rangle_n$$

$$F : |i\rangle_n|0\rangle \to |i\rangle_n \left(\sqrt{1-f(i)}|0\rangle + \sqrt{f(i)}|1\rangle\right)$$

$$F|\psi\rangle_n|0\rangle = \sum_{i=0}^{N-1} \sqrt{1-f(i)}\sqrt{p_i}|i\rangle_n|0\rangle + \sum_{i=0}^{N-1} \sqrt{f(i)}\sqrt{p_i}|i\rangle_n|1\rangle$$

Beating Monte Carlo with QAE

It is possible to estimate the mean of a probability distribution quadratically faster using QAE (wrt Monte Carlo)

A. Montanaro, Proc. Roy. Soc. A: Math. Phys. Eng. Sci. 471, 20150301 (2015)

$$M|0\rangle_n = \sum_{i=0}^{N-1} \sqrt{p_i}|i\rangle_n$$

$$F : |i\rangle_n|0\rangle \to |i\rangle_n \left(\sqrt{1-f(i)}|0\rangle + \sqrt{f(i)}|1\rangle\right)$$

$$F|\psi\rangle_n|0\rangle = \sum_{i=0}^{N-1} \sqrt{1-f(i)}\sqrt{p_i}|i\rangle_n|0\rangle + \sum_{i=0}^{N-1} \sqrt{f(i)}\sqrt{p_i}|i\rangle_n|1\rangle$$

Use QAE to estimate probability of last qubit = 1 (A = FM)

$$prob = \sum_{i=0}^{N-1} p_i f(i) = \mathbb{E}[f(X)] \begin{cases} f(i) = i/(N-1) \to \mathbb{E}[X] \\ f(i) = i^2/(N-1)^2 \to \mathbb{E}[X^2] \end{cases}$$

(Can be generalized to many different scenarios)

Two possible financial applications

1) Pricing of financial derivatives

P. Rebentrost, B. Gupt, T. R. Bromley, PRA 98, 022321 (2018)

In finance, a derivative is a contract that derives its value from the performance of an underlying entity.

1) Pricing of financial derivatives

P. Rebentrost, B. Gupt, T. R. Bromley, PRA 98, 022321 (2018)

In finance, a derivative is a contract that derives its value from the performance of an underlying entity.

→ Classical approaches:

(i) Black-Scholes-Merton model (stochastic differential equation)

(ii) Monte Carlo → sampling paths of asset dynamics. **Can be improved via QAE.**

1) Pricing of financial derivatives

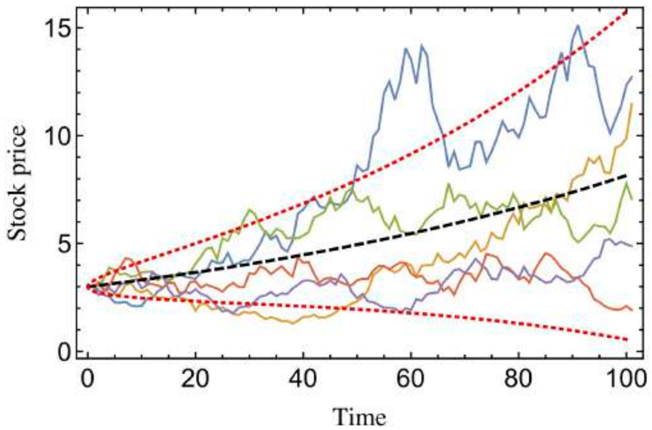
P. Rebentrost, B. Gupt, T. R. Bromley, PRA 98, 022321 (2018)

In finance, a derivative is a contract that derives its value from the performance of an underlying entity.

→ Classical approaches:

(i) Black-Scholes-Merton model (stochastic differential equation)

(ii) Monte Carlo → sampling paths of asset dynamics. **Can be improved via QAE.**



Probability distribution over paths in time

European and Asian options with QAE

P. Rebentrost, B. Gupt, T. R. Bromley, PRA 98, 022321 (2018)

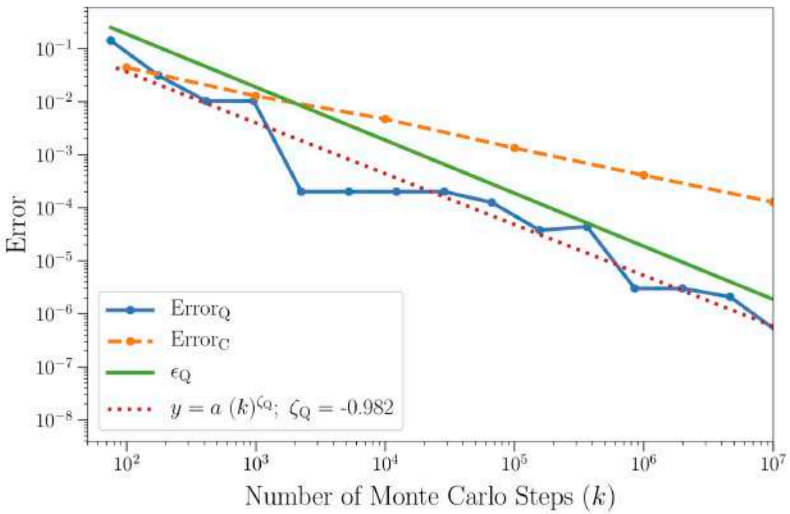


FIG. 3. Scaling of the error in classical and quantum MC methods (defined in Eq. (57)) plotted against number of MC steps for a European call option in log-log scale with $S_0 = \$100$, $K = \$50$, $r = 0.05$, $\sigma = 0.2$, $T = 1$, and $D = 24$. Subscripts C and Q denote the errors from classical MC and quantum phase estimation, respectively. Evidently, the error for the quantum algorithm (with a fitted slope of $\zeta_{\mathrm{Q}} = -0.982$) scales almost quadratically faster than the classical MC method (which has $\zeta_{\mathrm{C}} = -0.5$). The theoretical upper bound on the error in quantum algorithm is shown by the solid green curve, which corresponds to $\zeta_{\mathrm{Q}} = -1$.

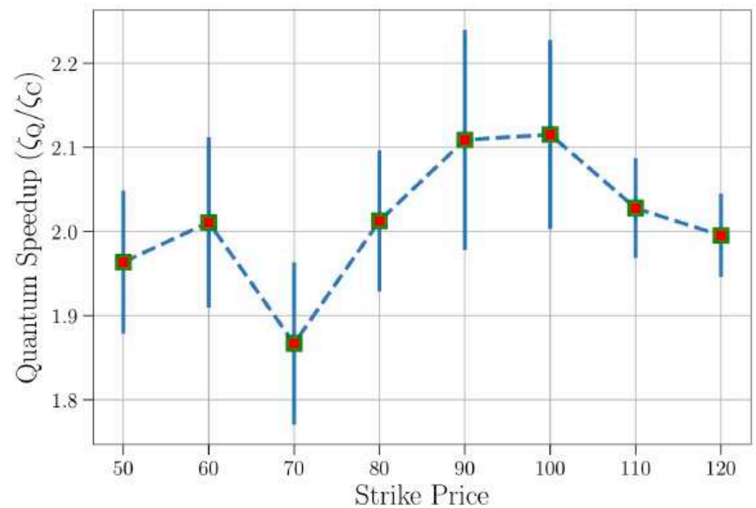


FIG. 4. Ratio of the quantum to classical scaling exponents. The quantum scaling is obtained by fitting the simulations results to the power law in Eq. (58), while the classical scaling exponent is taken to be -0.5. Results are plotted with varying strike price K (in dollars) and fixing other parameters to be the same as in Fig. 3. An almost quadratic speed up is obtained for all chosen values of K.

Quadratic speedup

Another option: use qPCA!

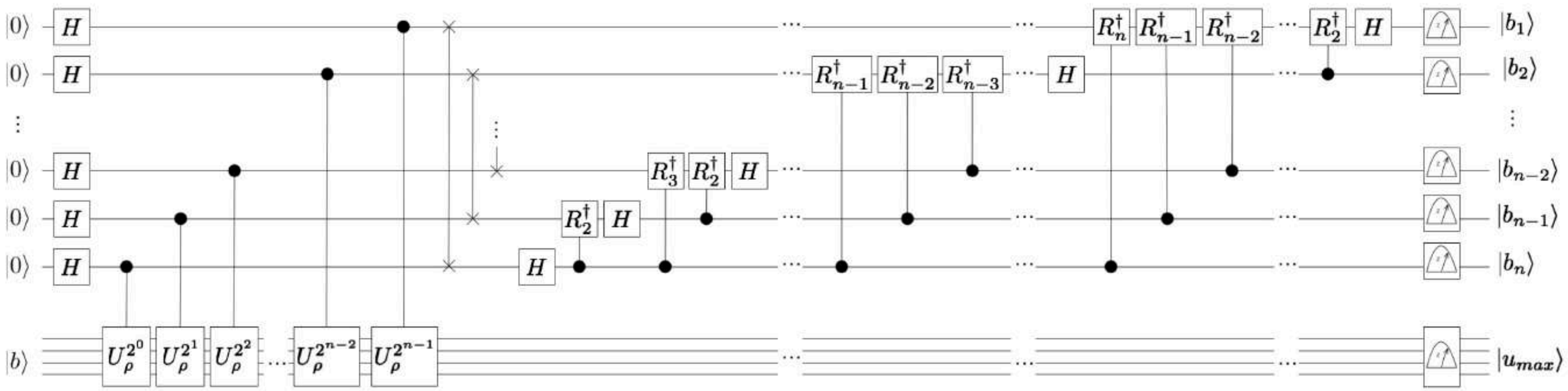
A. Martin et al, arXiv:1904.05803

Estimate largest eigenvalues of time-correlation matrix via Quantum Phase Estimation

Another option: use qPCA!

A. Martin et al, arXiv:1904.05803

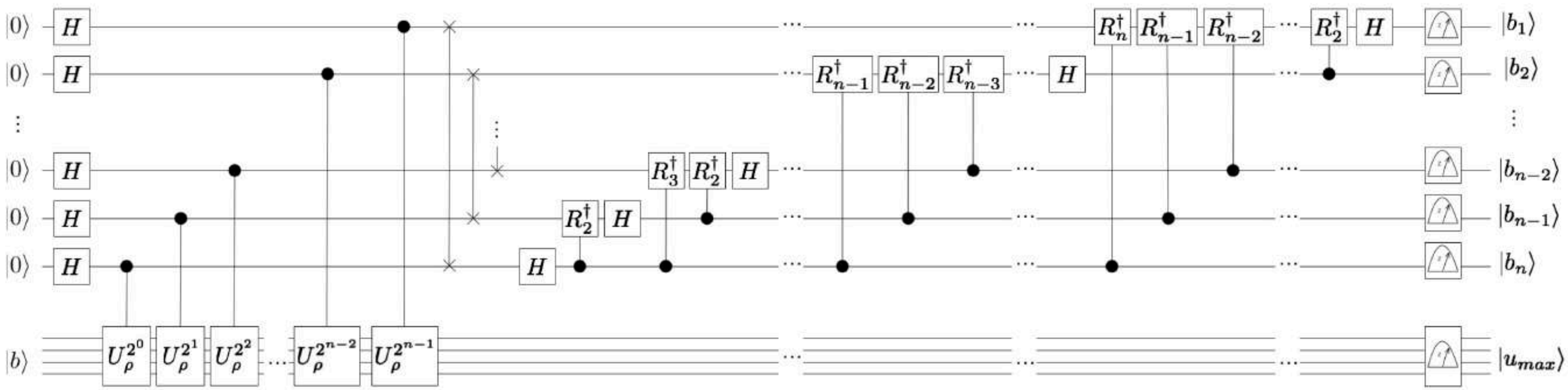
Estimate largest eigenvalues of time-correlation matrix via Quantum Phase Estimation



Another option: use qPCA!

A. Martin et al, arXiv:1904.05803

Estimate largest eigenvalues of time-correlation matrix via Quantum Phase Estimation



$U_\rho = exp(it\rho)$

$$\rho_2 = \frac{\sigma_2}{\mathrm{tr}(\sigma_2)} = \begin{pmatrix} 0.6407 & 0.3288 \\ 0.3288 & 0.3593 \end{pmatrix} \qquad \begin{aligned} |u_{\max}\rangle = {} & [(0.87 \pm \delta) - i(0.10 \pm \delta)]\,|0\rangle \\ & + ((0.47 \pm \delta) + i(0.10 \pm \delta))\,|1\rangle \end{aligned}$$

3x3 some error
4x4 quite some error

Limitation of the 5-qubit IBMQX2

2) Risk analysis

S. Woerner, D. J. Egger, arXiv:1806.06893

E.g., Value at Risk: $VaR_{\alpha}(x) = min\ x \mid Pr(X \leq x) \geq (1-\alpha)$

2) Risk analysis

S. Woerner, D. J. Egger, arXiv:1806.06893

E.g., Value at Risk: $VaR_\alpha(x) = min\ x \mid Pr(X \leq x) \geq (1-\alpha)$

→ Measures the distribution of losses of a portfolio, a company...
Computed by Monte Carlo. **Also quadratic improvement via QAE.**

2) Risk analysis

S. Woerner, D. J. Egger, arXiv:1806.06893

E.g., Value at Risk: $VaR_\alpha(x) = min\ x \mid Pr(X \leq x) \geq (1 - \alpha)$

→ Measures the distribution of losses of a portfolio, a company...
Computed by Monte Carlo. **Also quadratic improvement via QAE.**

$$F : |i\rangle_n |0\rangle \rightarrow |i\rangle_n \left(\sqrt{1 - f(i)} |0\rangle + \sqrt{f(i)} |1\rangle \right)$$

$$f_l(i) = 1 \text{ if } i \leq l, \text{ and } 0 \text{ otherwise}$$

$$l \in \{0, \ldots, N-1\}$$

2) Risk analysis

S. Woerner, D. J. Egger, arXiv:1806.06893

E.g., Value at Risk: $VaR_\alpha(x) = min\ x \mid Pr(X \leq x) \geq (1-\alpha)$

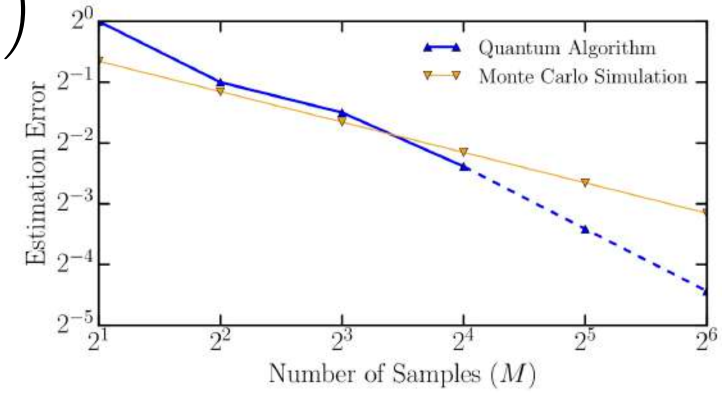
→ Measures the distribution of losses of a portfolio, a company...
Computed by Monte Carlo. **Also quadratic improvement via QAE.**

2-asset portfolio, on 5-qubit IBMQX2

$$F : |i\rangle_n|0\rangle \rightarrow |i\rangle_n\left(\sqrt{1-f(i)}|0\rangle + \sqrt{f(i)}|1\rangle\right)$$

$$f_l(i) = 1 \text{ if } i \leq l, \text{ and } 0 \text{ otherwise}$$

$$l \in \{0, \ldots, N-1\}$$



Quadratic speedup

OUTLOOK

1) Quantum algorithms can be applied to financial problems

2) At least three main trends:

(i) Quantum optimization
(ii) Quantum machine learning
(iii) Quantum amplitude estimation

3) Quantum computers will play a key role in quantitative finance

4) Small-scale quantum processors also imply speed-ups that may be relevant for practical applications
V. Dunjko, Y. Ge, J. I. Cirac, PRL 121, 250501 (2018)

Orús Quantum Stuff @Donostia

ikerbasque
Basque Foundation for Science

MULTIVERSE
COMPUTING

tensor networks!

pintxos!

entanglement!

many-body systems!

sailing!

surfing!

quantum technologies!

michelin stars!

txakoli!

quantum algorithms!

Applications welcomed