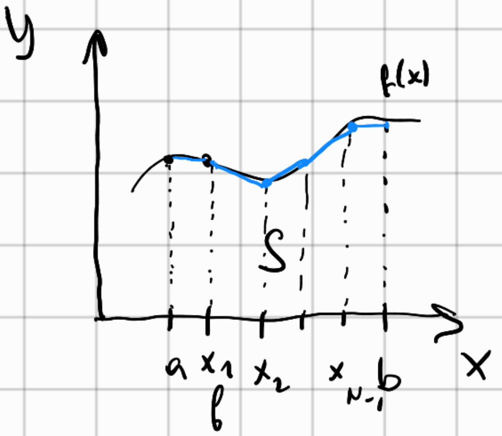
Lecture 11

Trapezoid's method.



$x \in [a, b]$

$$h = x_{i+1} - x_i = \frac{b-a}{N}$$

$$x_i = a + i \cdot h$$

$$S = \int_a^b f(x)\,dx = \sum_{i=0}^{N-1} \int_{x_i}^{x_{i+1}} f(x)\,dx \approx \sum_{i=0}^{N-1} \int_{x_i}^{x_{i+1}} s_i(x)\,dx \approx$$

$s_i(x)$ – spline of the first order.

$$\approx \sum_{i=0}^{N-1} \int_{x_i}^{x_{i+1}} \left[\frac{f(x_{i+1}) - f(x_i)}{x_{i+1} - x_i} (x - x_i) + f(x_i) \right] dx \approx$$

$$\approx \sum_{i=0}^{N-1} \frac{f(x_{i+1}) - f(x_i)}{h} \cdot \frac{(x - x_i)^2}{2} + f(x_i)\, x \Bigg|_{x_i}^{x_{i+1}} \approx$$

$$\approx \sum_{i=0}^{N-1} \frac{f(x_{i+1}) - f(x_i)}{\cancel{h}} \left(\frac{h^{\cancel{2}}}{2} - 0 \right) + f(x_i) \cdot h \approx$$

$$\approx \sum_{i=0}^{N-1} \frac{f(x_{i+1}) - f(x_i)}{2} \cdot h + f(x_i) \cdot h \approx$$

$$\approx \sum_{i=0}^{N-1} \frac{f(x_{i+1}) + f(x_i)}{2} \cdot h$$ – Trapezoid's formula

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$$\Delta = \left| \int_a^b f(x)\,dx - \sum_{i=0}^{N-1} \frac{f(x_{i+1}) + f(x_i)}{2} \cdot h \right| \approx$$

$$\Delta = \left| \int_a f(x)\,dx - \sum_{i=0} \frac{f(x_{i+1}) + f(x_i)}{2} \cdot h \right|$$

$$= \left| \sum_{i=0}^{N-1} \int_{x_i}^{x_{i+1}} \left[f(x) - \left(\frac{f(x_{i+1}) + f(x_i)}{2} \right) \right] dx \right| =$$

$$f(x) \approx f(x_i + x - x_i) \approx f(x_i) + f'(x_i)(x - x_i) +$$

$$+ f''(\xi_i) \frac{(x - x_i)^2}{2}, \quad \xi_i \in [x_i, x_{i+1}].$$

$$f(x_{i+1}) = f(x_i + h) \approx f(x_i) + f'(x_i) \cdot h + f''(\mu_i) \frac{h^2}{2},$$

$$= \left| \sum_{i=0}^{N-1} \int_{x_i}^{x_{i+1}} \left[\cancel{f(x_i)} + f'(x_i)(x - x_i) + f''(\xi_i) \frac{(x - x_i)^2}{2} - \right.\right.$$

$$\left.\left. - \frac{1}{2}\left(\cancel{f(x_i)} + f'(x_i) h + f''(\mu_i) \frac{h^2}{2} \right) + \cancel{f(x_i)} \right) \right] dx \right| =$$

$$= \left| \sum_{i=0}^{N-1} \left[\int_{x_i}^{x_{i+1}} f'(x_i) \left(x - x_i - \frac{h}{2} \right) dx + \int_{x_i}^{x_{i+1}} \left(f''(\xi_i) \frac{(x - x_i)^2}{2} - \right.\right.\right.$$

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$$\left.\left.\left. - f''(\mu_i) \frac{h^2}{4} \right) dx \right] \right| =$$

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1 $$\int_{x_i}^{x_{i+1}} f'(x_i) \left(x - x_i - \frac{h}{2} \right) dx = f'(x_i) \left. \frac{\left(x - x_i - \frac{h}{2} \right)^2}{2} \right|_{x_i}^{x_{i+1}} =$$

$$= \frac{f'(x_i)}{2} \left(\frac{h^2}{4} - \frac{h^2}{4} \right) = 0;$$

2 $$\int_{x_i}^{x_{i+1}} \left(f''(\xi_i) \frac{(x - x_i)^2}{2} - f''(\mu_i) \frac{h^2}{4} \right) dx =$$

$$= \left(f''(\xi_i) \frac{(x-x_i)^3}{6} - f''(\eta_i) \frac{h^2}{4} x \right) \Bigg|_{x_i}^{x_{i+1}} =$$

$$= f''(\xi_i) \frac{h^3}{6} - f''(\eta_i) \frac{h^3}{4}$$

$$= \left| \sum_{i=0}^{N-1} f''(\xi_i) \frac{h^3}{6} - f''(\eta_i) \frac{h^3}{4} \right| \leq$$

$$\leq \sum_{i=0}^{N-1} \left| f''(\xi_i) \frac{h^3}{6} - f''(\eta_i) \frac{h^3}{4} \right| \leq$$

$$\leq \left| f''(\xi_i) \frac{h^3}{6} - f''(\eta_i) \frac{h^3}{4} \right| \cdot N \leq$$

$$M_2 = \sup_{x \in [a,b]} f''(x)$$

$$\leq \left(M_2 \frac{h^3}{6} - M_2 \frac{h^3}{4} \right) \cdot N \leq M_2 N \left(-\frac{h^3}{12} \right) \leq$$

$$\leq M_2 N \frac{h^2}{12} \cdot \frac{(b-a)}{N} \leq M_2 \frac{h^2}{12} (b-a).$$

Lab 9. Find an integral $f(x)$ on $x \in [a, b]$.

1) Find using rectangle's formula ①

2) Using trapezoid's formula ②

3) $\Delta = \left| \int_a^b f(x)\,dx - \sum^{N-1} \ldots \right| \quad k=1,2$

5) $S_K := \int_a f(x)\,dx = \sum_{i=0} \ldots \Big|,\ k \ldots$