Lecture 2 (Cont-n)

Determinant and Inverse matrix computation.

I. Determinant

det (A)

1. $\det(I) = 1$
2. Row swapping : $\det * (-1)$
3. Multiplying a row by a scalar c, multiplies the determinant by c.
4. Row addition does not change the determinant.
3. Singular matrices: a matrix is singular (non-invertible if and only if its determinant is zero).

$$Ax = 0$$

$$\Delta = \det(A) = a_{11} \cdot a_{22}^{(1)} \cdot \ldots \cdot a_{nn}^{(n-1)}$$

1) Provide Gauss elimination till lower triangle is equal to zero.
2) multiply diagonal elements

Ex.

$$\Delta = \begin{vmatrix} 2,0 & 1,0 & -0,1 & 1,0 \\ 0,4 & 0,5 & 4,0 & -8,5 \\ 0,3 & -1,0 & 1,0 & 5,2 \\ 1,0 & 0,2 & 2,5 & -1,0 \end{vmatrix}$$

$$\Delta = 2,0 \cdot 0,3 \cdot 16,425 \cdot (-1,73) = -16,99$$

II. Inverse matrix. A^{-1}

$$A \cdot A^{-1} = A^{-1} \cdot A = I.$$

$$I = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} x_{11} & x_{12} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & x_{2n} \\ \dots & \dots & \dots & \dots \\ x_{n1} & x_{n2} & \dots & x_{nn} \end{pmatrix}$$

$$A \cdot A^{-1} = I.$$

$$Ax = b$$

$$\left[\begin{array}{cccc|cccc} a_{11} & a_{12} & \cdots & a_{1n} & 1 & 0 & \cdots & 0 \\ a_{21} & a_{22} & \cdots & a_{2n} & 0 & 1 & \cdots & 0 \\ \cdots & & & \cdots & & & \cdots & \\ a_{n1} & a_{n2} & \cdots & a_{nn} & 0 & 0 & \cdots & 1 \end{array}\right]$$

$$I \qquad\qquad A^{-1}$$

Ex.

$$A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ 1 & 0 & 0 \end{pmatrix} \qquad A^{-1} = ?$$

Gauss-Jordan Elimination.

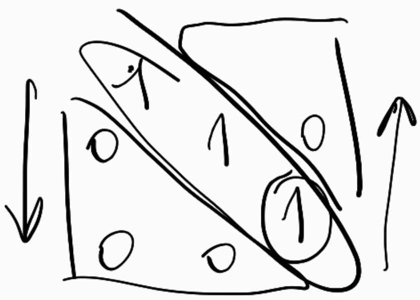
$$\left[\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 1 & 3 & 2 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \sim \begin{matrix} [1]/2 \\ [2]/3 \end{matrix} \left[\begin{array}{ccc|ccc} 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{3} & 1 & \frac{2}{3} & 0 & \frac{1}{3} & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\sim \begin{matrix} [2] - [1]\cdot\frac{1}{3} \\ [3] - [1] \end{matrix} \left[\begin{array}{ccc|ccc} 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & \frac{5}{6} & \frac{1}{3} & -\frac{1}{6} & \frac{1}{3} & 0 \\ 0 & -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & 0 & 1 \end{array}\right] \sim$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & \frac{2}{5} & \frac{2}{5} & -\frac{1}{5} & 0 \\ 0 & \frac{5}{6} & \frac{1}{3} & -\frac{1}{6} & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{5} & -\frac{1}{5} & \frac{1}{5} & \frac{5}{6} \end{array}\right] \sim [1] - \frac{2}{5}\cdot[3]\cdot 5$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{3}{5} & -\frac{2}{5} & -\frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{5} & \frac{2}{5} & -\frac{4}{5} \\ 0 & 0 & 1 & -1 & 1 & 4 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} \frac{3}{5} & -\frac{2}{5} & -\frac{1}{2} \\ \frac{1}{5} & \frac{2}{5} & -\frac{4}{5} \\ -1 & 1 & 4 \end{bmatrix}$$



Lab 2.2 and 2.3.

Find Δ and A^{-1}

Report: 1 file
1) Code
2) output
3) solution by hand.