In [ ]:
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Nurshanov Dias it3-2208
comp math 8 lab
Cubic spline
```

In [9]:
```python
# 1 Code implementation (your own, no existing method for interpolation).
import math
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt

# import matplotlib.pyplot as plt

f = np.tan

# Linear spline
def linear_spline(x_vals, y_vals, x):
    for j in range(len(x_vals) - 1):
        x0, x1 = x_vals[j], x_vals[j + 1]
        y0, y1 = y_vals[j], y_vals[j + 1]
        if x0 <= x <= x1:
            return y0 + (y1 - y0) * (x - x0) / (x1 - x0)
    return None


# Quadratic spline
def quadratic_spline(x_vals, y_vals, x):
    for j in range(1, len(x_vals) - 1):
        x0, x1, x2 = x_vals[j - 1], x_vals[j], x_vals[j + 1]
        y0, y1, y2 = y_vals[j - 1], y_vals[j], y_vals[j + 1]
        if x0 <= x <= x2:
            a = y0 * (x - x1) * (x - x2) / ((x0 - x1) * (x0 - x2))
            b = y1 * (x - x0) * (x - x2) / ((x1 - x0) * (x1 - x2))
            c = y2 * (x - x0) * (x - x1) / ((x2 - x0) * (x2 - x1))
            return a + b + c
    return None


# Cubic spline
def cubic_spline(x_vals, y_vals, x):
    for j in range(len(x_vals) - 3):
        x0, x1, x2, x3 = x_vals[j], x_vals[j + 1], x_vals[j + 2], x_vals[j + 3]
        y0, y1, y2, y3 = y_vals[j], y_vals[j + 1], y_vals[j + 2], y_vals[j + 3]
        if x0 <= x <= x3:
            a = y0 * (x - x1) * (x - x2) * (x - x3) / ((x0 - x1) * (x0 - x2) * (x0 - x3))
            b = y1 * (x - x0) * (x - x2) * (x - x3) / ((x1 - x0) * (x1 - x2) * (x1 - x3))
            c = y2 * (x - x0) * (x - x1) * (x - x3) / ((x2 - x0) * (x2 - x1) * (x2 - x3))
            d = y3 * (x - x0) * (x - x1) * (x - x2) / ((x3 - x0) * (x3 - x1) * (x3 - x2))
            return a + b + c + d
    return None

x_vals = np.linspace(0, math.pi / 2 - 0.1, 10)
y_vals = [f(x) for x in x_vals]

x_points = np.linspace(0, math.pi / 2 - 0.1, 100)
f_values = [f(x) for x in x_points]
s1_values = [linear_spline(x_vals, y_vals, x) for x in x_points]
s2_values = [quadratic_spline(x_vals, y_vals, x) for x in x_points]
s3_values = [cubic_spline(x_vals, y_vals, x) for x in x_points]

delta_s1 = np.abs(np.array(f_values) - np.array(s1_values))
delta_s2 = np.abs(np.array(f_values) - np.array(s2_values))
delta_s3 = np.abs(np.array(f_values) - np.array(s3_values))

data = {
    'x': x_points,
    'f(x)': f_values,
    's1(x)': s1_values,
    's2(x)': s2_values,
    's3(x)': s3_values,
    'delta_s1': delta_s1,
    'delta_s2': delta_s2,
    'delta_s3': delta_s3
}
```

In [10]:
```python
# 2 output of the code in form of the table
table = pd.DataFrame(data)
table
```

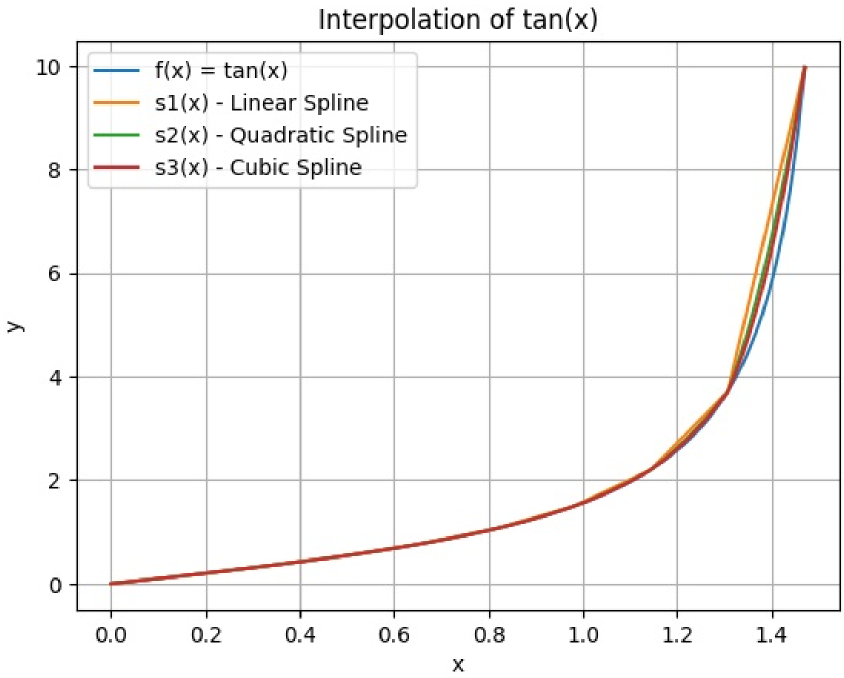
Out[10]:

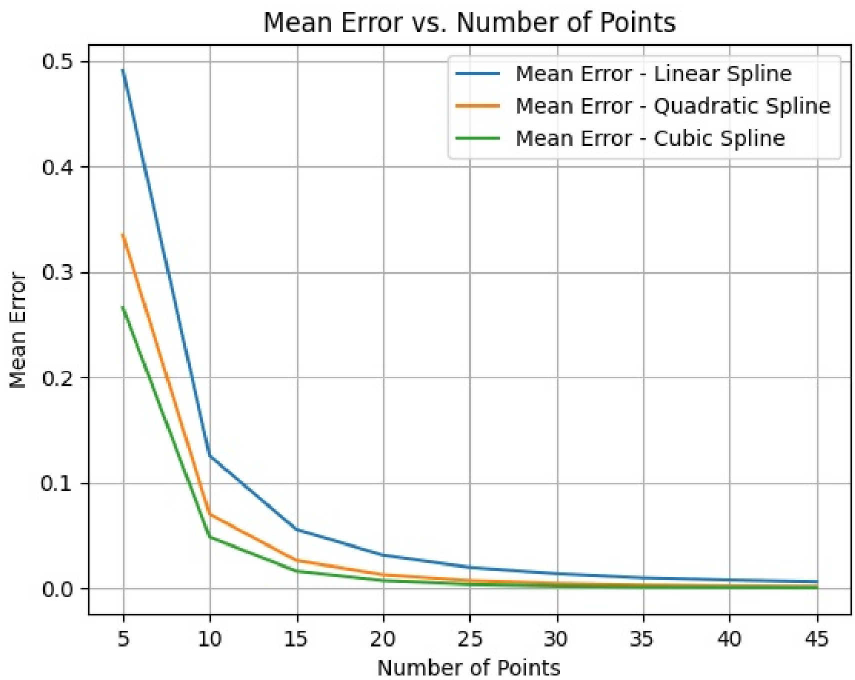
	x	f(x)	s1(x)	s2(x)	s3(x)	delta_s1	delta_s2	delta_s3
0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
1	0.014857	0.014858	0.014990	0.014609	0.014909	0.000133	0.000248	0.000051
2	0.029713	0.029722	0.029980	0.029295	0.029809	0.000259	0.000427	0.000087
3	0.044570	0.044599	0.044971	0.044057	0.044707	0.000372	0.000543	0.000108
4	0.059426	0.059496	0.059961	0.058894	0.059614	0.000465	0.000602	0.000118
...	...	...	...	...	...	...	...	...
95	1.411370	6.219266	7.690765	7.141244	6.892194	1.471499	0.921979	0.672929
96	1.426227	6.868827	8.259735	7.788717	7.563386	1.390908	0.919890	0.694559
97	1.441083	7.666038	8.828705	8.475441	8.297549	1.162667	0.809404	0.631511
98	1.455940	8.668195	9.397675	9.201417	9.097646	0.729480	0.533222	0.429451
99	1.470796	9.966644	9.966644	9.966644	9.966644	0.000000	0.000000	0.000000

100 rows × 8 columns

In [11]:

```
"""
3
Graphs:
    a  Graph of distribution of real function f(x) and spline of the first order,
    second order  and the cubic spline (s_1(x), s_2(x) and s_3(x)) (on one graph)


    b  Dependence of mean average error value on number of points used in your domain.
"""
plt.plot(x_points, f_values, label='f(x) = tan(x)')
plt.plot(x_points, s1_values, label='s1(x) - Linear Spline')
plt.plot(x_points, s2_values, label='s2(x) - Quadratic Spline')
plt.plot(x_points, s3_values, label='s3(x) - Cubic Spline')
plt.legend()
plt.title('Interpolation of tan(x)')
plt.xlabel('x')
plt.ylabel('y')
plt.grid(True)
plt.show()

points_counts = range(5, 50, 5)
mean_errors_s1 = []
mean_errors_s2 = []
mean_errors_s3 = []

for n in points_counts:
    x_vals = np.linspace(0, math.pi / 2 - 0.1, n)
    y_vals = [f(x) for x in x_vals]
    s1_values = [linear_spline(x_vals, y_vals, x) for x in x_points]
    s2_values = [quadratic_spline(x_vals, y_vals, x) for x in x_points]
    s3_values = [cubic_spline(x_vals, y_vals, x) for x in x_points]
    mean_errors_s1.append(np.mean(np.abs(np.array(f_values) - np.array(s1_values))))
    mean_errors_s2.append(np.mean(np.abs(np.array(f_values) - np.array(s2_values))))
    mean_errors_s3.append(np.mean(np.abs(np.array(f_values) - np.array(s3_values))))

plt.plot(points_counts, mean_errors_s1, label='Mean Error - Linear Spline')
plt.plot(points_counts, mean_errors_s2, label='Mean Error - Quadratic Spline')
plt.plot(points_counts, mean_errors_s3, label='Mean Error - Cubic Spline')
plt.legend()
plt.title('Mean Error vs. Number of Points')
plt.xlabel('Number of Points')
plt.ylabel('Mean Error')
plt.grid(True)
plt.show()
```





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