Lecture 5

Iterative methods for solving SLAE.

I. Jacobi method (method of simple iteration)

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2 \\ \dots \\ a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n \end{cases} \qquad (1)$$

$$Ax = b$$

$$\begin{cases} x_1^{(1)} = \frac{1}{a_{11}}\left(b_1 - a_{12}x_2^{(0)} - \dots - a_{1n}x_n^{(0)}\right) \\ x_2^{(1)} = \frac{1}{a_{22}}\left(b_2 - a_{21}x_1^{(0)} - \dots - a_{2n}x_n^{(0)}\right) \\ \dots \\ x_n^{(1)} = \frac{1}{a_{nn}}\left(b_n - a_{n1}x_1^{(0)} - \dots - a_{n-1}x_{n-1}^{(0)}\right) \end{cases} \qquad (2)$$

$$x^{(0)} = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix} \sim \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix}$$

$$x_i^{(k+1)} = \frac{1}{a_{ii}}\left(b_i - \sum_{j \neq i} a_{ij}\, x_j^{(k)}\right)$$

tolerance $\varepsilon = 0{,}001$

Algorithm :

1. Start with initial guess $x^{(0)}$
2. For each iteration k, update each variable x_i by Jacobi method
3. Repeat untill $|x^{(k+1)} - x^{(k)}| < \varepsilon$

Advantages :

- Simple to implement
- Parallelizable : each equation can be solved independently.

Disadvatages :

- Slow convergence
- Initial guess.

II. Gauss - Seidel method.

$$\begin{cases} x_1^{(k+1)} = \dfrac{1}{a_{11}}\left(b_1 - a_{12}x_2^{(k)} - a_{13}x_3^{(k)}\right) \\ x_2^{(k+1)} = \dfrac{1}{a_{22}}\left(b_2 - a_{21}x_1^{(k+1)} - a_{23}x_3^{(k)}\right) \\ x_3^{(k+1)} = \dfrac{1}{a_{33}}\left(b_3 - a_{31}x_1^{(k+1)} - a_{32}x_2^{(k+1)}\right) \end{cases}$$

$$x_i^{(k+1)} = \frac{1}{a_{ii}}\left(b_i - \sum_{j=1}^{i-1} a_{ij}x_j^{(k+1)} - \sum_{j=i+1}^{n} a_{ij}x_j^{(k)}\right)$$

Algorithm :

1. Start with initial guess $x^{(0)}$
2. Update x_i^{k+1} using $x_i^{(k)}$
3. Repeat $|x_i^{k+1} - x_i^{k}| < \varepsilon$

Advantages:
- Faster converges
- Memory efficient

Disadvantages:
- Not always convergent

Lab. 4.

1. Solve SLAE by I and II.
2. Compare each other (which is faster?)
3. Compare results with Gauss method.
4. Solution by hand.