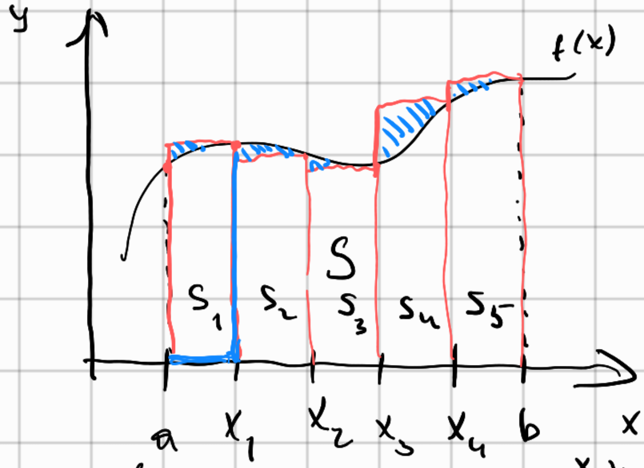
Lecture 10.

Rectangle's method.



$$S = \int_a^b f(x)\,dx \quad , x \in [a, b]$$

$$\Delta x = \frac{b-a}{N}$$

N - number of point

$$x_i = a + i \cdot h$$

$$\int_a^b f(x)\,dx = \sum_{i=0}^{N-1} \int_{x_i}^{x_{i+1}} f(x)dx$$

$$S = \sum_{i=0}^{N-1} S_i = \sum_{i=0}^{N-1} f(x_{i+1}) \cdot \Delta x$$ - left rectangle method.

$$\int_a^b f(x)\,dx \approx \sum_{i=1}^{N} f(x_i)\,\Delta x$$ - right rectangle method

$$\int_a^b f(x)\,dx \approx \sum_{i=0}^{N-1} f(x_{i+\frac{1}{2}})\,\Delta x$$ - middle rectangle method.

$$\Delta = \left| \int_a^b f(x)\,dx - \sum_{i=0}^{N-1} f(x_{i+\frac{1}{2}})\,\Delta x \right| =$$

$$= \left| \sum_{i=0}^{N-1} \int_{x_i}^{x_{i+1}} f(x)dx - \sum_{i=0}^{N-1} f(x_{i+\frac{1}{2}})\,\Delta x \right| =$$

$$= \left| \sum_{i=0}^{N-1} \left[\int_{x_i}^{x_{i+1}} \left(f(x) - f(x_{i+\frac{1}{2}}) \right) dx \right] \right| \ominus$$

$$f(x) \approx f(x_{i+\frac{1}{2}}) + f'(x_{i+\frac{1}{2}})(x - x_{i+\frac{1}{2}}) + f''(\xi_i)\frac{(x - x_{i+\frac{1}{2}})^2}{2}$$

$$\xi_i \in [x_i, x_{i+1}]$$

$$= \left| \sum_{i=0}^{N-1} \left[\int_{x_i}^{x_{i+1}} \left(\cancel{f(x_{i+\frac{1}{2}})} + f'(x_{i+\frac{1}{2}})(x - x_{i+\frac{1}{2}}) + f''(\xi_i) \frac{(x - x_{i+\frac{1}{2}})^2}{2} - \cancel{f(x_{i+\frac{1}{2}})} \right) dx \right] \right| =$$

$$\int_{x_i}^{x_{i+1}} f'(x_{i+\frac{1}{2}})(x - x_{i+\frac{1}{2}})\, dx = f'(x_{i+\frac{1}{2}}) \frac{(x - x_{i+\frac{1}{2}})^2}{2} \Bigg|_{x_i}^{x_{i+1}} =$$

$$= \frac{f'(x_{i+\frac{1}{2}})}{2} \left(\left(\frac{\Delta x}{2}\right)^2 - \left(-\frac{\Delta x}{2}\right)^2 \right) = 0\,;$$

$$\int_{x_i}^{x_{i+1}} f''(\xi_i) \frac{(x - x_{i+\frac{1}{2}})^2}{2}\, dx = \frac{f''(\xi_i)}{2} \frac{(x - x_{i+\frac{1}{2}})^3}{3} \Bigg|_{x_i}^{x_{i+1}} =$$

$$= \frac{f''(\xi_i)}{6} \left(\left(\frac{\Delta x}{2}\right)^3 - \left(-\frac{\Delta x}{2}\right)^3 \right) = f''(\xi_i) \frac{(\Delta x)^3}{24}$$

$$= \left| \sum_{i=0}^{N-1} f''(\xi_i) \frac{(\Delta x)^3}{24} \right| \leq \sum_{i=0}^{N-1} \left| f''(\xi_i) \frac{(\Delta x)^3}{24} \right| \leq$$

$$\left| \sum_i a_i \right| \leq \sum_i |a_i| \quad ; \quad M_2 = \sup_{x \in [a,b]} f''(x)$$

$$\leq M_2 \frac{(\Delta x)^3}{24} \cdot N = M_2 \frac{(\Delta x)^2}{24} (b-a)$$