In [ ]:
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Computational Mathematic Lab 2.2 & Lab 2.3
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In [20]:
```python
# 1 Exercise)Code (50%)
import copy

def determinant_gauss(A):
    n = len(A)
    A = copy.deepcopy(A)
    determinant = 1
    num_swaps = 0

    for i in range(n):
        max_row = i
        for k in range(i + 1, n):
            if abs(A[k][i]) > abs(A[max_row][i]):
                max_row = k

        if A[max_row][i] == 0:
            return 0

        if max_row != i:
            A[i], A[max_row] = A[max_row], A[i]
            num_swaps += 1

        for k in range(i + 1, n):
            factor = A[k][i] / A[i][i]
            for j in range(i, n):
                A[k][j] -= factor * A[i][j]

    for i in range(n):
        determinant *= A[i][i]

    if num_swaps % 2 == 1:
        determinant = -determinant

    return determinant


def inverse_gauss_jordan(A):
    n = len(A)
    A = copy.deepcopy(A)

    identity = [[float(i == j) for j in range(n)] for i in range(n)]

    for i in range(n):
        max_row = i
        # Find the pivot row
        for k in range(i + 1, n):
            if abs(A[k][i]) > abs(A[max_row][i]):
                max_row = k

        A[i], A[max_row] = A[max_row], A[i]
        identity[i], identity[max_row] = identity[max_row], identity[i]

        pivot = A[i][i]

        for j in range(n):
            A[i][j] /= pivot
            identity[i][j] /= pivot

        for k in range(n):
            if k != i:
                factor = A[k][i]
                for j in range(n):
                    A[k][j] -= factor * A[i][j]
                    identity[k][j] -= factor * identity[i][j]

    return identity
```

In [21]:
```python
# 2 Exercise) Output of the code implementation (25%)
from pprint import pprint

A = [
    [0, 1, -1],
    [-3, 0, 2],
    [-2, 1, 2]
]

pprint(f"source matrix A")
```

```
pprint(A)
print()
det_A = determinant_gauss(A)
pprint(f"Determinant: {det_A}")
print()
if det_A != 0:
    inv_A = inverse_gauss_jordan(A)
    pprint(inv_A)
else:
    print("Matrix is singular, so it does not have an inverse.")
```

```
'source matrix A'
[[0, 1, -1], [-3, 0, 2], [-2, 1, 2]]

'Determinant: 5.0'

[[-0.39999999999999997, -0.5999999999999999, 0.39999999999999997],
 [0.4, -0.39999999999999997, 0.6],
 [-0.6, -0.39999999999999997, 0.6]]
```

In [ ]:

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# 3 )Solution by Hand

Given matrix A =
[0  1  -1]
[-3 0  2]
[-2 1  2]

Determinant Calculation:

1. Step 1 (Row swap):
Swap row 1 and row 2 to bring a non-zero pivot in the first row.
Matrix becomes:
[-3 0 2]
[0  1 -1]
[-2 1  2]

2. Step 2 (Eliminate first column):
Row 3 = Row3 - (-2/-3) * Row1
Matrix becomes:
[-3 0 2]
[0  1 -1]
[0  1  0.Given matrix A =
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[-3 0  2]
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Determinant Calculation:

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Matrix becomes:
[-3 0 2]
[0  1 -1]
[-2 1  2]

2. Step 2 (Eliminate first column):
Row 3 = Row3 - (-2/-3) * Row1
Matrix becomes:
[-3 0 2]
[0  1 -1]
[0  1  0.6666667]

3. Step 3 (Eliminate second column):
Row 3 = Row3 - Row2
Matrix becomes:
[-3 0 2]
[0  1 -1]
[0  0  5/3]

4. Step 4 (Calculate determinant):
Determinant = (-3) * (1) * (5/3) = 5.0

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Inverse Calculation (Gauss-Jordan Elimination):

Augment A with the identity matrix:
[-3 0 2 | 1 0 0]
[0  1 -1 | 0 1 0]
[-2 1 2 | 0 0 1]

1. Step 1 (Row swap):
Swap row 1 and row 2 to bring a non-zero pivot in the first row.
Matrix becomes:
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```
[-3 0 2 | 0 1 0]
[0  1 -1 | 1 0 0]
[-2 1 2 | 0 0 1]

2. Step 2 (Normalize row 1 by dividing by the pivot):
Row1 = Row1 / -3
Matrix becomes:
[1.0 -0.0 -Given matrix A =
[0  1  -1]
[-3 0  2]
[-2 1  2]

Determinant Calculation:

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Matrix becomes:
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2. Step 2 (Normalize row 1 by dividing by the pivot):
Row1 = Row1 / -3
Matrix becomes:
[1.0 -0.0 -2/3 | -0.0 -1/3 -0.0]
[0   1 -1 | 1 0 0]
[-2  1 2 | 0 0 1]

3. Step 3 (Eliminate above and below row 1 pivot):
Matrix after elimination:
[1.0  0.0  -2/3 | 0.4  -0.4  0.6]
[0.0  1.0  -1.0 | 1.0 0.0 0.0]
[0.0  0.0  5/3 | -0.6 -0.4 0.6]

4. Step 4 (Normalize row 3 by dividing by the pivot):
Row3 = Row3 / 5/3
Matrix becomes:
[1.0  0.0  0.0 | 0.4  -0.4  0.6]
[0.0  1.0  0.0 | -0.4 0.6 -0.4]
[0.0  0.0  1.0 | 0.6 -0.4 0.6]

The inverse matrix is:
[[0.4  -0.4  0.6]
 [-0.4 0.6 -0.4]
 [0.6  -0.4  0.6]]
 | -0.0 -1/3 -0.0]
[0   1 -1 | 1 0 0]
[-2  1 2 | 0 0 1]

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The inverse matrix is:
[[0.4  -0.4  0.6]
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Row 3 = Row3 - Row2
Matrix becomes:
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Determinant = (-3) * (1) * (5/3) = 5.0

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[1.0 -0.0 -2/3 | -0.0 -1/3 -0.0]
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```