In [ ]:
```
from sys import implementation

Computational Mathematic Lab 3. Methods of square roots
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In [1]:
```
#1) implementation of the method
def cholesky_decomposition(A):
    n = len(A)
    L = [[0.0] * n for _ in range(n)]

    for i in range(n):
        for k in range(i + 1):
            temp_sum = sum(L[i][j] * L[k][j] for j in range(k))
            if i == k:  # Diagonal elements
                L[i][k] = (A[i][i] - temp_sum) ** 0.5
            else:
                L[i][k] = (A[i][k] - temp_sum) / L[k][k]
    return L

def solve_cholesky(L, b):
    n = len(b)
    # Forward substitution to solve L*y = b
    y = [0.0] * n
    for i in range(n):
        y[i] = (b[i] - sum(L[i][j] * y[j] for j in range(i))) / L[i][i]

    # Backward substitution to solve L^T*x = y
    x = [0.0] * n
    for i in range(n-1, -1, -1):
        x[i] = (y[i] - sum(L[j][i] * x[j] for j in range(i + 1, n))) / L[i][i]
    return x
```

In [2]:
```
#2) output of the program
A = [
    [4, 12, -16],
    [12, 37, -43],
    [-16, -43, 98]
]

b = [1, 2, 3]

# Cholesky
L = cholesky_decomposition(A)
x_cholesky = solve_cholesky(L, b)


print("Solution using Cholesky Decomposition:", x_cholesky)
```

```
Solution using Cholesky Decomposition: [28.583333333333332, -7.666666666666666, 1.3333333333333333]
```

In [ ]:
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#3) solution by hand

Cholesky Decomposition Manual Solution

System: 4x + 12y - 16z = 1
12x + 37y - 43z = 2
-16x - 43y + 98z = 3

Matrix A and Vector b: A =
[4, 12, -16]
[12, 37, -43]
[-16, -43, 98]

b = [1, 2, 3]

Decompose A into L and L^T: A = L * L^T =
[l11, 0, 0]
[l21, l22, 0]
[l31, l32, l33] *
[l11, l21, l31]
[0, l22, l32]
[0, 0, l33]

Calculate l11: l11^2 = 4 => l11 = 2

Calculate l21: l21 * l11 = 12 => l21 = 6

Calculate l31: l31 * l11 = -16 => l31 = -8

Calculate l22: l21^2 + l22^2 = 37 => 6^2 + l22^2 = 37
36 + l22^2 = 37 => l22^2 = 1
```

```
l22 = 1

Calculate l32: l31 * l21 + l32 * l22 = -43 => (-8 * 6) + l32 * 1 = -43
-48 + l32 = -43
l32 = 5

Calculate l33: l31^2 + l32^2 + l33^2 = 98
(-8)^2 + 5^2 + l33^2 = 98
64 + 25 + l33^2 = 98
l33^2 = 9 => l33 = 3

L Matrix: L =
[2, 0, 0]
[6, 1, 0]
[-8, 5, 3]

Solve Ly = b: Forward substitution:

y1 = b1 / l11 => y1 = 1 / 2 = 0.5
y2 = (b2 - l21 * y1) / l22 => y2 = (2 - 6 * 0.5) / 1 = -1
y3 = (b3 - l31 * y1 - l32 * y2) / l33
y3 = (3 - (-8 * 0.5) - (5 * -1)) / 3
y3 = (3 + 4 + 5) / 3 = 4

y = [0.5, -1, 4]

Solve L^T x = y: Backward substitution:

x3 = y3 / l33 => x3 = 4 / 3 = 1.33
x2 = (y2 - l32 * x3) / l22 => x2 = (-1 - 5 * 1.33) / 1 = -7.67
x1 = (y1 - l21 * x2 - l31 * x3) / l11
x1 = (0.5 - 6 * (-7.67) - (-8 * 1.33)) / 2
x1 = (0.5 + 46.02 + 10.64) / 2 = 28.58

x = [28.58, -7.67, 1.33]
```