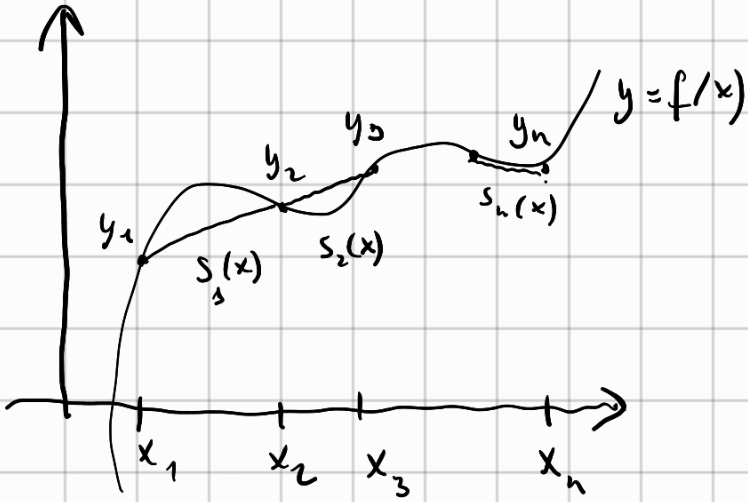
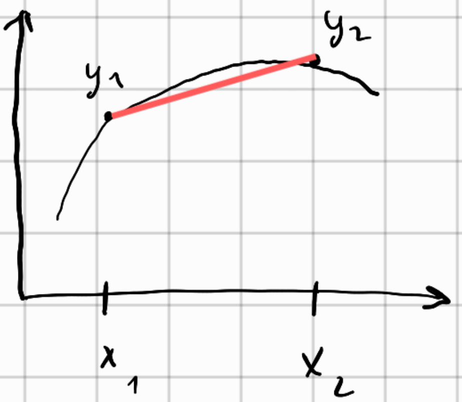
Lecture 7

Interpolation. Spline of the first order.

$y = f(x)$

x	x_1	x_2	$\dots$	x_n
y	y_1	y_2	$\dots$	y_n





$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$y = \frac{(y_2 - y_1)(x - x_1)}{x_2 - x_1} + y_1$$

$x \in [a, b]$, $\Delta = \frac{b-a}{N}$

$x_i = a + i\Delta x$

$$\boxed{S_i(x) = \frac{(f(x_{i+1}) - f(x_i))(x - x_i)}{x_{i+1} - x_i} + f(x_i)}$$ – spline of the 1^{st} order.

$\tilde{x} = x_i + \frac{h}{k}$, $k > 1$; $x_{i+1} = x_i + h$; $h = x_{i+1} - x_i$

$$S_i(\tilde{x}) = \frac{(f(x_{i+1}) - f(x_i))(\tilde{x} - x_i)}{x_{i+1} - x_i} + f(x_i) =$$

$$= \frac{(f(x_{i+1}) - f(x_i))\frac{\cancel{h}}{k}}{\cancel{h}} + f(x_i) = \boxed{\frac{f(x_{i+1}) - f(x_i)}{k} + f(x_i)}$$

$f(x) = \sin x \quad , x \in [\underset{a}{0}, \underset{b}{1}]$

$N, \; h, \; k, \; x_i, \; f(x_i)$

$s, \; f(\bar{x}), \; \Delta = |s(\bar{x}) - f(\bar{x})|$

Graphical representation of $f(x)$ and $s(x)$

Dependence of error (Δ) and number of points (N).

Lab 6.

1) Code
2) Output

x	$f(x)$	$s(x)$	Δ

3) Graphs

Order of approximation.

$$f(x) = f(a) + f'(a)\cdot(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \dots$$

$$\dots + \frac{f^{(n)}(a)}{n!}(x-a)^n + \dots,$$

$$\Delta = |f(\tilde{x}) - s_i(\tilde{x})| = \left| f(\tilde{x}) - \left(\frac{f(x_{i+1}) - f(x_i)}{k} + \right.\right.$$

$$\left.\left. + f(x_i)\right)\right| =$$

$$f(\tilde{x}) = f\left(x_i + \frac{h}{k}\right) \approx f(x_i) + f'(x_i)\frac{h}{k} + \frac{f''(x_i)\left(\frac{h}{k}\right)^2}{2} +$$

$$\xi_i \in (x_i, x_{i+1}) \qquad + \frac{f'''(\xi_i)\left(\frac{h}{k}\right)^3}{3!} \quad \Big/ \, O\left(\left(\frac{h}{k}\right)^3\right)$$

$$f(x_{i+1}) = f(x_i + h) \approx f(x_i) + f'(x_i)h + f''(x_i)\frac{h^2}{2} +$$

$$+ \frac{f'''(\mu_i)h^3}{6}$$

$$\circledcirc \left| \cancel{f(x_i)} + \cancel{f'(x_i)\frac{h}{k}} + \frac{f''(x_i)\left(\frac{h}{k}\right)^2}{2} + \frac{f'''(\xi_i)\left(\frac{h}{k}\right)^3}{6} - \right.$$

$$- \left(\cancel{f(x_i)} + \cancel{f'(x_i)h} + f''(x_i)\frac{h^2}{2} + \frac{f'''(\mu_i)h^3}{6} - \cancel{f(x_i)}\right)\cdot\frac{1}{k} -$$

$$\left. - \cancel{f(x_i)} \right| = \left| \frac{f''(x_i)}{2}\left(\frac{h}{k}\right)^2 - f''(x_i)\frac{h^2}{2k} + \frac{f'''(\xi_i)\left(\frac{h}{k}\right)^3}{6} - \right.$$

$$\left. - f'''(\mu_i)\frac{h^3}{6k} \right| = \left| f''(x_i)\cdot\frac{h^2}{2k}\left(\frac{1}{k} - 1\right) + \right.$$

$$\left. + \frac{h^3}{6k}\left(\underbrace{f'''(\xi_i)\,\frac{1}{k^2}}_{} - f'''(\mu_i)\right)\right| \leq M_2 \frac{h^2}{2k}\left(\frac{1}{k} - 1\right)$$

$$\sup_{x \in [a,b]} |f''(x)| = M_2$$

second order of approvoximation