Lecture 2.

Gauss method.

System of linear algebraic equations (SLAE).

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1, \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2, \\ \text{-----------------} \\ a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n = b_n \end{cases} \qquad (1)$$

$$Ax = b$$

$$A = (a_{ij}) = \begin{pmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ - & - & - & - \\ a_{n1} & a_{n2} & \ldots & a_{nn} \end{pmatrix}$$

$$x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \qquad b = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}$$

$$\det A = \begin{vmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ - & - & - & - \\ a_{n1} & a_{n2} & \ldots & a_{nn} \end{vmatrix} \neq 0$$

I. Direct step

$[1]/a_{11}$ $\qquad x_1 + c_{12}x_2 + \ldots + c_{1n}x_n = d_1$

$$\left(\begin{array}{cccc|c} 1 & c_{12} & \cdots & c_{1n} & d_1 \\ 0 & f_{22} & \cdots & f_{2n} & d_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & f_{n2} & \cdots & f_{nn} & d_n \end{array}\right) \begin{array}{l} [2]-[1]\cdot a_{21} \\ \vdots \\ [n]-[1]\cdot a_{n1} \end{array} \sim [2]/f_{22}$$

$$\sim\left(\begin{array}{cccc|c} 1 & c_{12} & \cdots & c_{1n} & d_1 \\ 0 & 1 & \cdots & e_{2n} & k_2 \\ 0 & - & - & - & \\ 0 & 0 & \cdots & e_{nn} & k_n \end{array}\right) \begin{array}{l} [3]-[2]\cdot f_{32} \\ \vdots \\ [n]-[2]\cdot f_{n2} \\ [n]/e_{nn} \end{array}$$

$$b_{1j}=\frac{a_{1j}}{a_{11}}, \quad (j=2,\dots,n)$$

$$a_{ij}=a_{ij}-a_{i1}\cdot b_{1j} \quad (i=2,\dots,n-1;\ j=2,\dots,n)$$

$$\left(\begin{array}{cccc|c} 1 & - & - & - & b_1 \\ 0 & 1 & & & b_2 \\ 0 & 0 & 1 & - & \vdots \\ 0 & 0 & 0 & 1 & b_n \end{array}\right)$$

II. Inverse step.

$$x_n = b_n$$

$$x_{n-1} = b_{n-1} - x_n \cdot a_{n-1\,n}$$

$$x_3 = b_3 - a_{32}x_2 - a_{31}\cdot x_1$$

Ex.

$$\left(\begin{array}{ccc|c} 2 & -1 & 5 & 10 \\ 1 & 1 & -3 & -2 \\ 2 & 4 & 1 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -0,5 & 2,5 & 5 \\ 0 & 1,5 & -5,5 & -7 \\ 0 & 6 & -4 & -9 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|c} 1 & -0,5 & 2,5 & 5 \\ 0 & 1 & \frac{-5,5}{1,5} = k & \frac{-7}{1,5} = l \\ 0 & 0 & \underline{-4-k\cdot 6} & \underline{-9-6\cdot l} \end{array}\right)$$

$$x_3 = \frac{-9-6\cdot l}{-4-6k} \qquad x_1 = 5 - 2,5\cdot x_3 + 0,5\cdot x_2$$

$$x_2 = l - k\cdot x_3$$

Laboratory work #2.

Implement Gauss method for SLAE.

Submit a report

1) Code (no supplementary libraries)
2) Output of the code
3) Solution by hand