

Computational mathematics

Computation. Error

$$\Delta = \underset{\substack{| \\ \text{true}}}{a} - \underset{\substack{| \\ \text{computed}}}{a_0} \quad - \text{absolute error}$$

$$\delta = \frac{\Delta}{|a|} \quad (a \neq 0) \quad - \text{relative error}$$

Horner's scheme.

$$P(x) = a_0 x^n + a_1 x^{n-1} + \dots + a_n \quad (1)$$

$a_0, \dots, a_n = \text{constant coefficients.}$

$$\begin{aligned} x &= \xi \\ P(\xi) &= a_0 \xi^n + a_1 \xi^{n-1} + \dots + a_n = \\ &= (\dots(((a_0 \xi + a_1) \xi + a_2) \xi + \dots + a_n)) \end{aligned}$$

$$\left. \begin{array}{ll} b_0 = a_0 \\ c_1 = b_0 \xi & b_1 = a_1 + c_1 \\ c_2 = b_1 \xi & b_2 = a_2 + c_2 \\ \vdots & \vdots \\ c_n = b_n \xi & b_n = a_n + c_n \end{array} \right\} \quad (2)$$
$$b_n = P(\xi)$$

$$c_k = b_{k-1} \xi, \quad b_k = a_k + c_k \quad (k = 1, 2, \dots, n)$$

$$\begin{array}{r}
 a_0 \quad a_1 \quad a_2 \quad \dots \quad a_n \quad | \quad \xi \\
 + \quad b_0 \xi \quad b_1 \xi \quad \dots \quad b_{n-1} \xi \\
 \hline
 b_0 = a_0 \quad b_1 \quad b_2 \quad \dots \quad b_n = P(\xi)
 \end{array}$$

Example 1. $P(x) = 7x^5 + 3x^4 + 2x^3 + x^2 + 4x - 1$
 $x = 2$

Solution.

$$\begin{array}{r}
 7 \quad 3 \quad 2 \quad 1 \quad 4 \quad -1 \quad | \quad 2 \\
 14 \quad 34 \quad 72 \quad 146 \quad 300 \\
 \hline
 7 \quad 17 \quad 36 \quad 73 \quad 150 \quad 299 = P(2)
 \end{array}$$

$$P(2) = 299.$$

Input : n, a_k, ξ

Output : $P(\xi)$

Laboratory work 1.

Report #1: word/pdf
(docx)

File. $\begin{cases} 1. \text{ Code} \\ 2. \text{ Output} \\ 3. \text{ Solution by hand} \end{cases}$