Teaching Principle Project

Introduction

A famous question I hear students ask their teachers is, "Will this test be cumulative?" When students ask this question, they want to know if the material they should study includes material on previous exams. Students probably ask this question because they want to avoid studying for material that they don't think they need, which may stem from the misconception that the content they learn is disconnected and is not relevant across units. In fact, the material in any mathematics curriculum should be interconnected and coherent (NCTM, 2000), so I would argue, "All of mathematics is cumulative."

To help students understand this philosophy, prior knowledge needs to be activated. I am interested in the techniques teachers use to require that students use their prior knowledge to learn a new concept. My research question is, *How often do teachers ask students questions that call for retrieval of prior knowledge, and what questions do they ask?*

Literature Review

The NCTM (2000) outlines the Teaching Principle in three general aspects. Firstly, teachers should understand three components: what students know, what students need to learn, and pedagogical strategies for effective teaching. The second of these, what students need to learn, is the mathematical content in the curriculum. Teachers should have a deep understanding of this "subject-matter knowledge," which will enrich their students' learning experience. The third component that teachers should understand is also known as "pedagogical knowledge," which consists of teaching strategies not particular to the math classroom but to all classrooms (Even, 1993). Even also states that these two components are not mutually exclusive, and she calls their overlap "pedagogical content knowledge"—knowing how to effectively represent subject matter and knowing the common pitfalls when learning it (1993). The last two aspects of the Teaching Principle requires "a challenging and supportive classroom learning environment [and] continually seeking improvement" (NCTM, 2000, pp. 18–19).

This study focuses on the first component of the first aspect: that teachers should know what their students know. "Effective teachers know how to ask questions and plan lessons that reveal students' prior knowledge; they can then design experiences and lessons that respond to, and build on, this knowledge" (NCTM, 2000, p. 18). Only then will students will be able to connect new concepts to previous ones and learn in a more meaningful way.

Stillman (2000) further elaborates on the different classes of, and the importance of, prior knowledge in the way students approach a task. Prior knowledge of the "academic" variety consists of knowledge gained through outside academic experiences, such as content learned in another class or study. "Encyclopaedic prior knowledge" includes general facts and trivia of the world, and "episodic prior knowledge" is that which is gained by a learner from personal experiences outside of an academic setting.

Sloyer (2004) offers a strategy he calls the "extension-reduction strategy." This is a pedagogical strategy in which a teacher will present a problem that requires his or her students to use their prior knowledge to construct new knowledge. The goal of the strategy is to get the students to reduce the new problem down to a simpler and better understood problem (e.g., finding the area of a polygon by subdividing it into triangles). The teacher's role is to help and guide the students in activating their prior knowledge. Even though the students may *have* this prior knowledge, they may not always know how to *use* it productively. The example that Sloyer gives is a problem in which students try to find the volume of a segment of a cone. The prior knowledge here was that of finding a part of a whole. The value of the desired part is found by taking the value of the whole (whether that be a region's area, a finite series, a solid's volume, etc.) and subtracting the "extra" amount.

Hare's (2004) example is a bit more complex. In this study, students learned implicit differentiation by reinforcing and expanding their concept of a function. Students had varying misconceptions of the definition of "function" and thus could not take the implicit derivative of an equation properly. Through guided questions and activities, the students were not only able to use their prior knowledge but also improve on it, while at the same time gain new knowledge.

Methods

I collected data in four total periods. Each period took place in an Algebra II classroom. The first two—periods 6 and 7—were on one day and took place between 12 P.M. and 2 P.M., and the other two—periods 7 and 8—took place on another day one week later, between 11 A.M. and 12:15 P.M. On my first day, the classes were practicing simplifying rational expressions and solving rational equations. The next week they were covering geometric sequences and series. The first day of data collection was during the normal bell schedule, so each period I observed was exactly 48 minutes long. On the second day of my data collection the students had a 2 1/2 hour early dismissal, so the periods I observed were an average of 31 minutes in length.

My method of data collection was simple. I observed my mentor teacher, Mr. Noble, searching his discourse for questions that would require students to recall information from a previous module or course. I created a numbered list (a tally) of Mr. Noble's such questions, quoting him if possible, but paraphrasing otherwise. If Mr. Noble repeated a question for the purpose of requiring students to recall previous information, I counted it again; however I did not count questions that Mr. Noble repeated due to misinterpretation/miscommunication by the student(s) or other unrelated reasons (e.g., repeating a question because the student(s) were not paying attention). In addition to my tally, I wrote annotations to myself indicating other aspects of the question, such as the concepts or objects to which any pronouns referred, or the correct answer to the question. The sole reason for my annotation was to help myself remember what Mr. Noble meant when I will have looked back on my data collection notes.

After collecting data, I counted the tally for each class period. Calculating the number of questions Mr. Noble asked per minute would not provide interpretable data, so I instead decided to calculate the reciprocal: the number of minutes between each of Mr. Noble's questions, to which I will refer as "inverse frequency." I will use this term instead of the conventional term *period* to avoid confusion with the meaning of class period. Since I did not record the times at which Mr. Noble asked such questions, I was limited in only calculating *average* inverse frequency of Mr. Noble's questions (per period, the total number of minutes divided by the total number of questions). Thus it is difficult to obtain an accurate representation of how often Mr. Noble asked these questions, because it is likely that these incidents were unevenly distributed, e.g., he may have asked more questions toward the beginning of the class period. Mr. Noble's average inverse frequency is expressed in minutes per question, and is accurate to the nearest tenth. Aside from Mr. Noble's average inverse frequency per class period, I used a weighted average calculation to find Mr. Noble's average inverse frequency across all the class periods I observed. In this case, the "weight" is the proportion of the class's time to the total observation time across four classes, which is 158 minutes. **Figure 3** is the calculation.

Results

Of the periods in which I collected data, Mr. Noble's questions had wide ranges across many dimensions. Regarding processes: he asked questions that required that students remember which process to perform in a given situation, he asked students if they knew how to perform some processes, and he asked questions that guided students through a particular process.

Mr. Noble's other questions revolved around content. By definition, this content existed in a previous unit or lesson, so the students were expected to have known the answers. Mr. Noble's content questions activated knowledge of the following topics: rules involving nested exponents, rules involving multiplying powers with equal bases, factoring quadratic expressions, adding and multiplying rational expressions, finding the least common multiple of polynomial expressions, how to calculate terms and differences in arithmetic sequences, and general 'axioms' of mathematics.

There was a total of 158 minutes in all the four classes I observed. Considering a weighted scale, Mr. Noble's inverse frequency across all four classes was a rate of about 15.32 minutes per question activating prior knowledge (**Figure 3**). A detailed list of Mr. Noble's questions can be found in **Figure 2**. Direct quotations are enclosed in quotation marks and my own annotations are enclosed in square brackets. **Figure 1** shows the specifications of each class period I observed, and Mr. Noble's average inverse frequency, per class period, of asking questions that activate prior knowledge.

Conclusions

Almost all of Mr. Noble's questions requiring prior knowledge learned in an academic setting. His questions spanned across both content and process, but those which are traditionally learned in the classroom. Stillman (2000) says there are two other types of prior knowledge from which students could benefit if they relate it to an in-class, mathematical setting. By posing problems and letting students investigate how to solve them, they might offer unforeseen ideas to the teacher and each other because each students' episodic prior knowledge is different. All of mathematics is cumulative. The more often students can relate to prior mathematical knowledge from the same classroom, other classrooms, and outside of the classroom, the more meaningful their learning experience will be.

Further Research

Because this study was on the frequency and content of teachers' questions aiming to activate students' prior knowledge, it would have been difficult to collect data that focuses on the students. One of the dimensions upon which this study did not touch was the directness of the questions. For example, some of Mr. Noble's questions were directed to the entire class, and other questions were directed to an individual student. On a different scale, some questions were rhetorical in nature while others required a specific answer. The desire to analyze these scopes, and others, is one source of motivation for future studies on the Teaching Principle.

The smaller portion of this study analyzed how often teachers ask questions that activate prior knowledge. As I stated before, I found the average inverse frequency of my mentor teacher's questions. A future study might record times at which teachers ask such questions to get a more accurate representation of the distribution of questions.

As an afterthought: One more small modification can be made to this study, centered on what teachers *say* to students (not necessarily *ask* them) that will cause them to retrieve prior knowledge.

Appendix

Label	Date	Class Period Number	Course	Time of Day	Average Inverse Frequency (minutes per question)
A	29 Mar 2011	6	Algebra II	12:04–12:52 PM	6.9
B	29 Mar 2011	7	Algebra II	12:57–1:45 PM	16.0
C	05 Apr 2011	7	Algebra II	11:03–11:33 AM	10.0
D	05 Apr 2011	8	Algebra II	11:38 AM–12:10 PM	32.0

Figure 1. Details of observation periods.

Period A

1. What does $(t^1)^2$ equal? [t^3 or t^2?]
2. What does s^3s^2 equal? [s^5 or s^6?]
3. “Do you remember how to factor [quadratic trinomials]?”
4. “It’s adding now, so what do we need?” [About adding fractions; expected answer: a common denominator]
5. “Can we factor a difference of squares? Do you [all] know how to do it?”
6. “What does this one have that that one needs?” [About factors of denominators of addends]
7. “What do we do with rational equations?” [Desired answer: multiply both sides by the least common multiple of the denominators (a.k.a. the least common denominator)]

Period B

1. “What’s the first thing we do in these types of problems?” [Problems with rational expressions] “Factor everything!”
2. “What do we do when dividing fractions?” [Desired answer: flip and multiply]
3. Repeat of #2.

Period C

1. “How do we get to the next number in an arithmetic sequence?”
2. “How do we find the common difference in an arithmetic sequence?”
3. “What can r not be?” [Regarding the formula below]

$$s_n = \frac{a_1(1-r^n)}{1-r}$$

Period D

1. “How do we get to the next number [of an arithmetic sequence]?”

Figure 2. Annotated tally of Mr. Noble’s questions to students.

$$\left(\frac{48}{158}\right)\left(\frac{48}{7}\right)+\left(\frac{48}{158}\right)\left(\frac{48}{3}\right)+\left(\frac{30}{158}\right)\left(\frac{30}{3}\right)+\left(\frac{32}{158}\right)\left(\frac{32}{1}\right)\approx 15.32$$

Figure 3. Mr. Noble’s weighted average inverse frequency.

References

Even, R. (1993). Subject-matter knowledge and pedagogical content knowledge: Prospective secondary teachers and the function concept. *Journal for Research in Mathematics Education, 24*(2), 94–116.

Hare, A. & Phillippy, D. (2004). Building mathematical maturity in calculus: Teaching implicit differentiation through a review of functions. *Mathematics Teacher, 98*(1), 6–12.

The National Council of Teachers of Mathematics [NCTM]. (2000). *Principles and standards for school mathematics*. Reston, VA: NCTM.

Sloyer, C. W. (2004). The extension-reduction strategy: Activating prior knowledge. *Mathematics Teacher, 98*(1), 48–50.

Stillman, G. (2000). Impact of prior knowledge of task content on approaches to applications tasks. *Journal of Mathematical Behavior, 19*, 333–361.