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Teaching Philosophy

Introduction

As a pre-service mathematics educator, I have been in the process of developing a philosophy about what my future classroom will be like. This paper is a concrete culmination of my views and motives.

First, I have uncovered my own personal beliefs about mathematics, which include the nature of math itself, what goals I have set for students learning it, and how these goals tie in to my beliefs. Second, I have constructed ideas about how to achieve these goals and how to make my classroom an environment in which students will become successful. Third, I have chosen accredited research and referenced scholarly evidence to support my approaches and values.

Note that this paper is a reflection about my current beliefs and ideas. Throughout my career, I plan to expose myself to more research on mathematics education and change my teaching philosophy appropriately. In this way, I take on the role of "teacher as researcher."

Values

Epistemology of Mathematics

Humans are creatures of organization. We feel comfortable when we can apply labels to things and put them in boxes and classify them together so that we can better understand them. Specifically, we do this when we create a *mathematical model* for a phenomenon.

For instance in high school kinematics, when students are exploring the acceleration, velocity, and position of a falling object, air resistance and change in radius are ignored. This is because certain assumptions have been made to simplify the model to make the problem easier to solve. (Namely, the model disregards the (usually) negligible effect of air resistance on the object, and that as the object falls, the distance to the center of Earth negligibly changes thus changing the acceleration due to gravity.) Enderton (1977) lists other examples. When astronomers study extremely large distances, a theory they present is that space might not be Euclidean. The simple models of space that we use only apply to applicable distances, so the Euclidean model suffices. In addition, physicists are open to the idea that extremely small distances might be quantized: Space itself might be discrete rather than continuous. However the real number line is a model that ignores this negligible possibility. Our general notions about Euclidean space do not hold for extremely large and extremely small distances. I conclude that Euclidean space is an approximation of our world, not the other way around.

When encountering real-world phenomena, we create mathematical models to simplify and categorize them so we can explain and predict them more easily. When presented with a problem, we develop an abstract mathematical model, use pure mathematics to manipulate the model and obtain a mathematical solution, and then interpret the mathematical solution to provide a real-world solution. Thus it is my opinion that we *construct* mathematics in order to solve problems of the real world. Math is man-made. If we discover something new about our world, our mathematical model—or even our mathematics—has to change. Mankind's progression of science, technology, and religion throughout history supports this view.

Goals for Teaching and Learning

Why should students learn math? The reasons are many, but I have three foremost goals for secondary school students: (1) learn to become mathematical problem solvers; (2) learn to reason mathematically; and (3) learn to value mathematics.

A common question teachers get from students is, "Why is this important?" For many topics in secondary school math courses, the unfortunate answer is that the content itself is probably not important. Adults in the real world rarely use math above an eighth grade level (depending on profession, of course). Thus I would argue that it is the processes learned in math classes, and not necessarily the content, that are valuable. The NCTM (2000) lists and describes five process standards that are important not only in an academic setting in every grade K-12, but are also useful in real life: Problem Solving, Reasoning and Proof, Communication, Connections, and Representation. The NCTM urges that these processes be nurtured and developed in math courses, and in other subject areas as well, I might add.

Of the five process standards recommended by the NCTM, my philosophy focuses on two: Problem Solving and Reasoning and Proof. Although the other three are just as important, I believe Problem Solving and Reasoning and Proof are the processes that pertain mostly to courses in mathematics, and these processes are directly related to the first two of my foremost goals for students learning math.

Approaches and Evidence

Problem Solving

For those like me who take Earnest's (1989) "problem solving" (or rather, *constructivist*) view, students need to use and expand math to solve more complex problems. As the problems of students' lives become increasingly difficult, more advanced math is required to solve them. One of NCTM's (2000) four major recommendations for instructional programs is to enable students to build new mathematical knowledge through problem solving. "To meet new challenges in work, school, and life, students will have to adapt and extend whatever mathematics they know. Doing so effectively lies at the heart of problem solving" (p. 334). Solving a problem is defined as "engaging in a task for which the solution method is not known in advance" (p. 52). The solution method, as well as the solution itself, must be unknown. Once the algorithm for completing a task is known, the problem is no longer a 'problem' even if the answer is still unknown. Teachers must be careful not to allow their assigned tasks to fall into the Low-Level Cognitive Demand ratings, as defined in Stein, et al. (2000). These tasks require complex thinking without the necessity of an algorithm, require students to monitor and regulate their own processes and actions, and require knowledge of relevant mathematical content as well as knowledge of problem-solving strategies. Most often, students need to analyze these types of problems before they commence. Posing problems with multiple methods and solutions will keep students engaged and their minds dynamically active, and in addition will force students to observe their own processes and methods.

Ludholz (1990) says teachers must move from the descriptive language of students to the technical language of mathematics, which will help students develop an explicit mathematical model for a problem. I have personally observed that one way to do this is to pose more word problems in class. Even though most students struggle with word problems (which may be due to their inexposure to word problems in the first place), they force students to translate between the languages of English and Math; the more they do this, the more they will think about their world as a mathematical model, which enhances their ability to problem solve.

Another strategy to implement good problem-solving is to have students create problems for each other. Being able to analyze the problem by deciding which data is required or unnecessary helps students develop a problem-solving orientation. Asking students hypothetical questions is a strategy I use to keep their minds in a problem-solving mode. In my personal experience as a tutor, I can get students to 'think outside the box' by forcing them to view the problem from a different perspective.

Students need to recognize that not all problems can be solved quickly and directly. Making it clear to students that problems often have more than one correct method to finding a solution (and maybe even more than one solution) will raise self-confidence in students. One of the properties of tasks with multiple techniques is that not only must students be able to identify the most efficient—or most desired—technique to use, but students must know *how* to use the technique, and they must also be able to successfully carry out that technique (Schoenfeld, 1979). The more students are exposed to these problems, the more they will be accustomed to these three requirements.

Lastly, NCTM states that posing good problem solving tasks requires careful planning to foster knowledge. Good problem solving tasks will give students the opportunity to expand on their mathematical knowledge and problem-solving tactics. Problem solving should be integrated into every mathematics curricula and not as a separate content area, like Schoenfeld's (1979) problem solving course taught to undergraduates in mathematics.

Reasoning and Proof

Formal proofs are a subject to which I had been keen ever since my introduction to them. Although researchers agree that secondary school students are not yet ready for "formal" proofs, reasoning and argumentation is to be emphasized across all mathematics curricula (NCTM, 2000). This process "helps students better understand meaning and the concepts involved, and ... is a natural process in cognition (Germain-McCarthy, 1999; Hanna, 1983; Ko & Knuth, 2009; O'Daffer & Thornquist 1993; Rips, 1994)" (Harvey, 2010, p. 3). When attempting to develop and solve abstract models based on real-world problems, mathematical reasoning is a requirement. Specifically when reasoning about a given problem in a mathematics course, relevant mathematical knowledge is required in order to produce a valid argument. When students must decide whether to prove or disprove a conjecture, they need sufficient background on the topic.

In a traditional classroom, whatever the teacher says is always considered true, and students take it for granted. There is no need to question the statement or decide its validity. To foster students' reasoning and argumentation skills however, teachers should ask students whether the statements they say are correct. As long as a student can provide a counterexample (if the statement is false) or informally explain why it always works (if the statement is true), then it will make more sense in his or her mind. I have stated (2010) that getting students to develop a habit of noticing patterns and anomalies are one way to foster this processes. "Patterns help students to form conjectures, and teachers should encourage their students to test the conjectures they make. Anomalies (counterexamples) disprove false conjectures and force students to reconsider their thinking" (p. 12).

Students' role in the classroom need to be more active and need to be regularly engaged in discussion so that they can make and test more conjectures. When students are confident about their own reasoning skills, they are more likely to challenge arguments made in a resource, by a classmate, or even by a teacher. "Teachers ... should strive to create a climate of discussing, questioning, and listening in their classes" (NCTM, 2000, p. 346). This emphasizes a separate importance of reasoning in the classroom: communication. Ko and Knuth (2009) agree, "proof serves as a means to communicate thoughts with learners in the mathematics community" (p. 69).

When a student provides a solution (either to a problem or to a small step), they must be asked "why." I ask my tutees why they carry out incorrect techniques or produce incorrect answers for two reasons: to gather more information about what they are thinking and to possibly provide a remedy; and to give them a hint that what they did was wrong. However, I would recommend also asking "why" to correct processes and solutions so that students won't form a habit of becoming suspicious whenever they are asked. When students can provide reasoning to their methods, even if their methods are correct, they will gain a better understanding and a bigger picture of the problem, and will become more experienced at providing explanations, a necessary component of argumentation.

The Value of Mathematics

The third of my foremost goals is to get students to develop an appreciation for math. In my experience, one of the most prominent reasons students say they "don't like math" is because they feel it is boring. I want my classroom to be engaging and interactive for my students. I will not adhere to the traditional lecture style, but rather a style of conversation. Students are motivated in an environment where collaboration and communication is prominent. Teaching is a two-way path. I would like to think of my classroom as a place where teacher and students learn from each other, and where information flows in two directions.

One way to increase engagement is the use of technology. My purpose as teacher is to help students investigate and understand relationships among objects, and to *complexify* this investigation so that the students can *simplify* it. Rather than making decisions about what and how to investigate, my job is to guide students to make those decisions, and reaffirm that they're making the right (or wrong) ones. Students would get a more beneficial and intriguing experience if they could investigate and discover things for themselves (Ball & Stacey, 2005). Constructing lessons of Type 2 (McGraw & Grant, 2005) can achieve this process. Type 2 lessons incorporate more opportunities for students' decision-making, encourage reasoning and reflection (fulfilling my second goal) and involves less time on following directions, but more time on investigation and development of students' own ideas.

I think discovery is an important step in the learning process. Students will remember and own what they've found much easier than what they've been told. In addition, discovery helps students make connections to other topics, other courses, and to their lives outside of school. Part of implementing this philosophy in my classroom involves letting students obtain their own information. I will try to get my classroom students to ask themselves questions such as, "How can we go about solving this problem?" or, "What do we need to do to get the answer?" This kind of thinking promotes non-algorithmic processes and operational understanding (Kokol-Voljc).

It is not enough for a classroom to be engaging. I also want my classroom environment to be collaborative where students frequently work in groups and hold discussions. Social interaction will be fostered and students will be able to explain things to each other in their own language. "Interacting with others offers opportunities for exchanging and reflecting on ideas" (NCTM, 2000, p. 348). Omrod (2009) adds that students seem to remember experiences better when they communicate with others. When students are exposed to things from multiple points of view and senses (e.g. visually, aurally, tangibly), they experience a more diverse and enriching learning environment.

Conclusion

In summary, my views on the nature of mathematics and how humans create and use it to solve problems of the real world drive my goals for students in secondary school. I believe that the processes of learning to become mathematical problem solvers, to reason mathematically, and to hold an appreciation for mathematics are more important than the content prescribed in the mathematics curricula, but I see the content as a tool to foster these processes. To carry out these goals, the teacher must condition the student’s mind by posing the right problems, asking the right questions, and creating a stimulating and motivating learning environment.

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