Tuning and Temperaments

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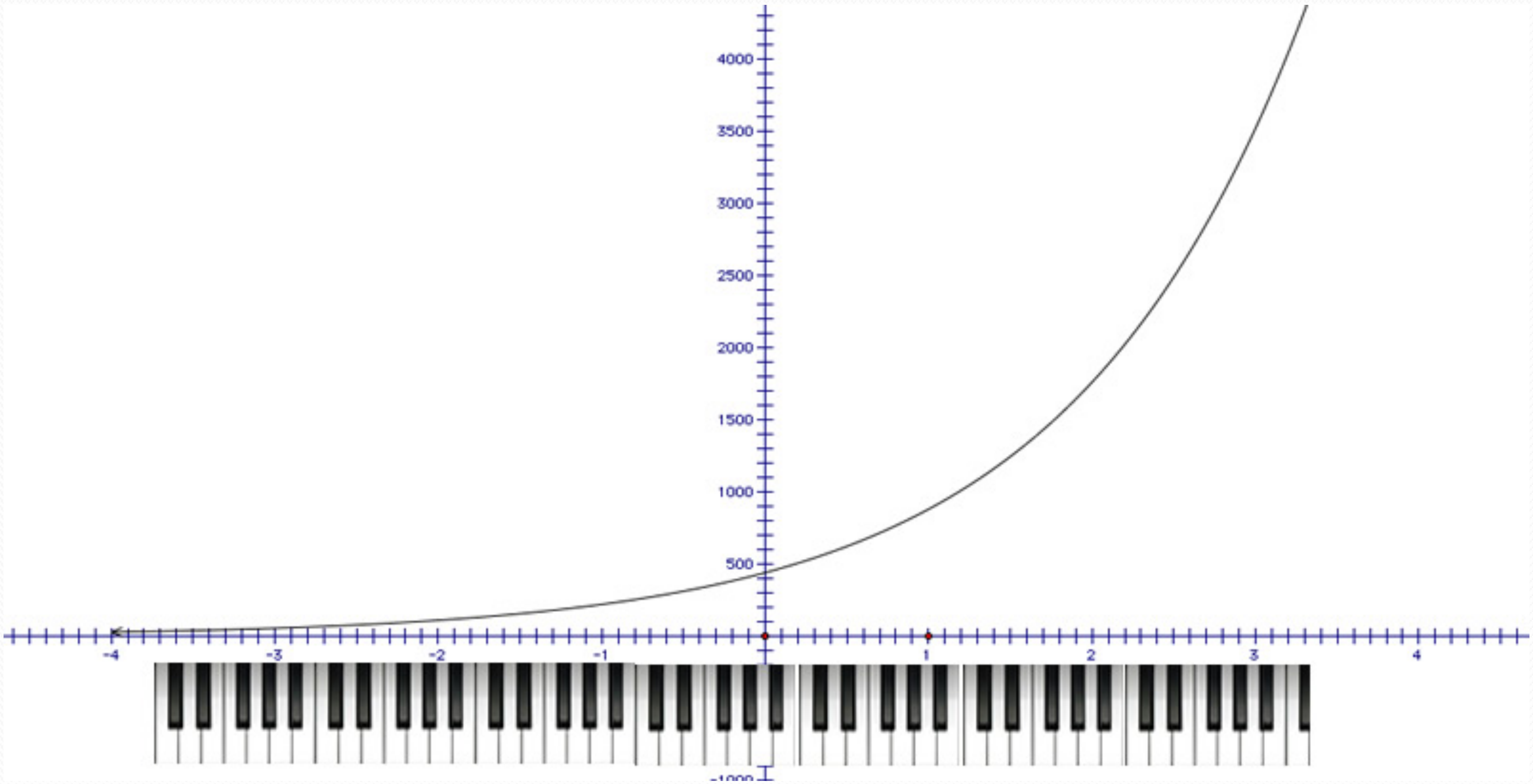
Overview

frequency and pitch

consonance versus dissonance

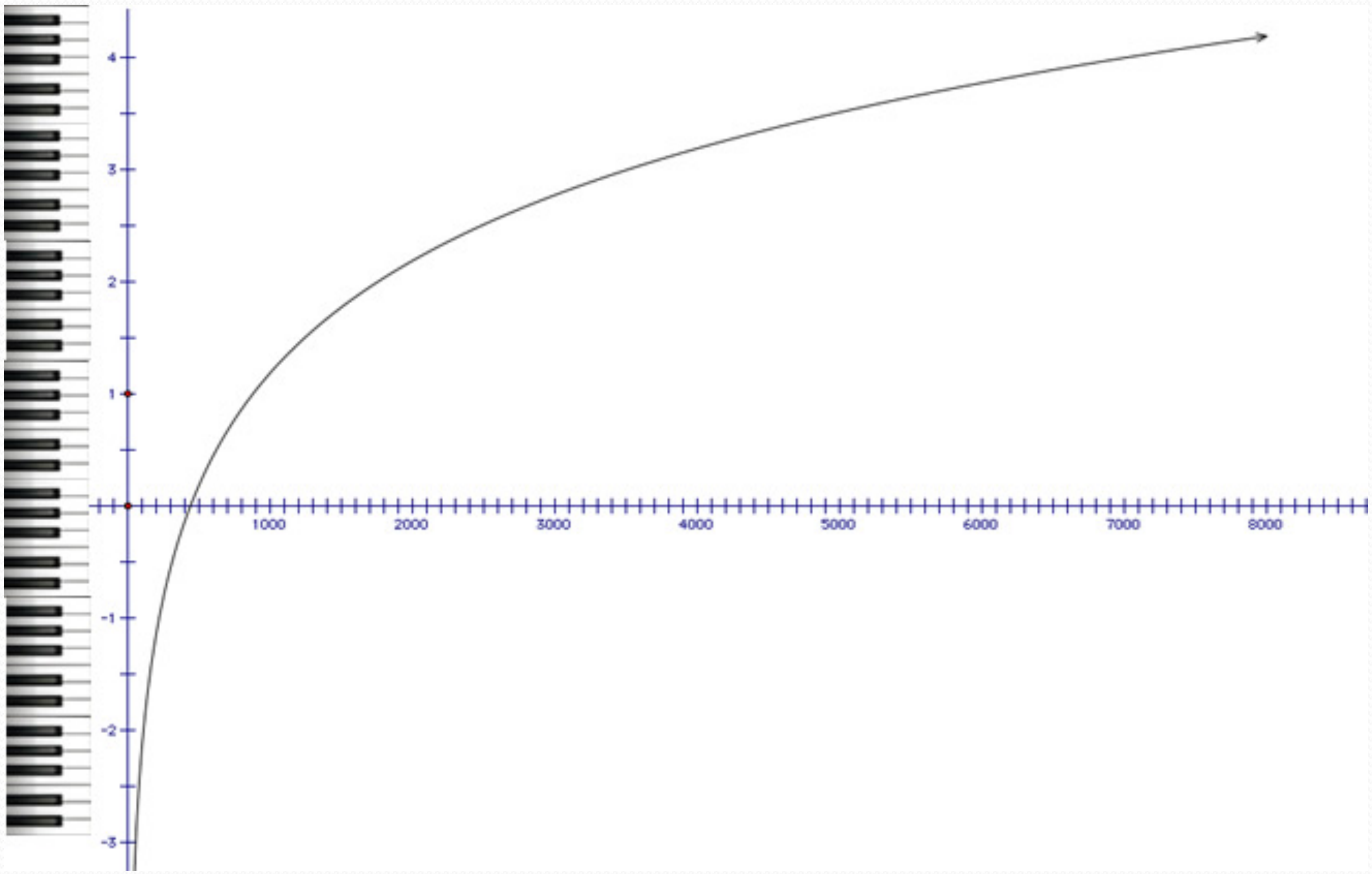
temperament scales

Frequency vs. Pitch



chromatic scale

Pitch vs. Frequency



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Harmonics

- every sound (except a pure tone) has harmonics
- integer multiples of a fundamental frequency (Hz)
- the combination of these harmonics makes the timbre
  - we can identify an instrument's sound by its timbre

220	440	660	880	1320	1100	1540	1760	…
A	A	E	A	C#	E	(G)	A	…
do	do	sol	do	mi	sol	(te)	do	…
1	P8	P5	P4	M3	m3	(m3)	(M2)	…

Consonance

- consonance:
  - subjective
  - varies by opinion
  - no measurable unit
- frequencies of smaller integer ratios are more consonant than those with larger integer ratios
  - Two notes, 200 Hz and 100 Hz have a ratio of 2:1. They will be more consonant than 199:100 even though both ratios have approximately the same value.
- because overtones ‘ride’ on top of each other

Overlapping Overtones

- the greater the number of overtones two fundamentals share, the more consonant they will sound
- we can attribute this to Pythagoras (585-500 BCE)

Perfect Octave								
220	440	660	880	1100	1320	1540	1760	...
440	880	1320	1760	2200	2640	3080	3520	...

Perfect Fifth									
220	440	660	880	1100	1320	1540	1760	1980	...
330	660	990	1320	1650	1980	2310	2640	2970	...

Scales and Systems

Pythagorean
Just
Mean-Tone
Equal

Pythagorean Scale

- Pythagoras found that P8 and P5 were ‘consonant’
- created a scale derived from only octaves and fifths with the following rules:
  - factor of 2 = P8
  - factor of 3 = P12
  - multiply to increase pitch, divide to decrease pitch
- used the first three harmonics (first two overtones)

Pythagorean Scale

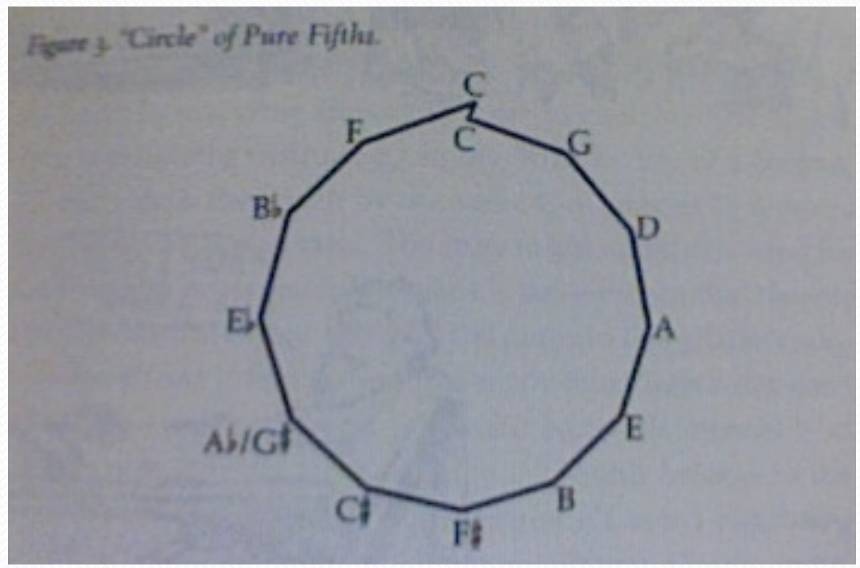
- Pythagoras went around the circle of fifths to derive every note in the major (and twelve-tone) scale
  - multiply 3:2 by a frequency to obtain a P5 above
- this created a problem with the circle of fifths:
- add 12 fifths and then subtract 7 octaves:
  - a bit sharp $\frac{\left(\frac{3}{2}\right)^{12}}{2^7} \approx 1.014$
- subtract 12 fifths and then add 7 octaves:
  - a bit flat $\frac{2^7}{\left(\frac{3}{2}\right)^{12}} \approx 0.987$

click

fa	do	sol	re	la	mi	ti
4:3	1	3:2	9:8	27:16	81:64	243:128
1.333	1	1.5	1.125	1.688	1.266	1.898

‘Spiral’ of Fifths

- The farther we deviate from the fundamental, the more out of tune we become.
- This makes it nearly impossible to play in other keys without playing out of tune.
- The small error is known as a ‘comma.’



Just Intonation

- goal: more consonant sound, smaller integer ratios
- uses first 16 harmonics to derive scale degrees
  - do: 1st harmonic
  - sol: 3rd harmonic dropped 1 octave
  - mi: 5th harmonic dropped 2 octaves
  - re: 9th harmonic dropped 3 octaves
  - ti: 15th harmonic dropped 3 octaves

fa	do	sol	re	la	mi	ti
4:3	1	3:2	9:8	27:16	81:64	243:128
1.333	1	1.5	1.125	1.688	1.266	1.898

do	sol	mi	re	ti	fa	la
1	3:2	5:4	9:8	15:8	4:3	5:3
1	1.5	1.25	1.125	1.875	1.333	1.666

What about
fa and la?

Just Intonation

Scarlatti K380 Just Intonation center C

Scarlatti K380 Just Intonation center C#

- fa – taken from Pythagoras – 4:3
- la – subtract P5 from the 5th harmonic and then subtract a P8 – 5:3

do	sol	mi	re	ti	fa	la
1	3:2	5:4	9:8	15:8	4:3	5:3
1	1.5	1.25	1.125	1.875	1.333	1.666

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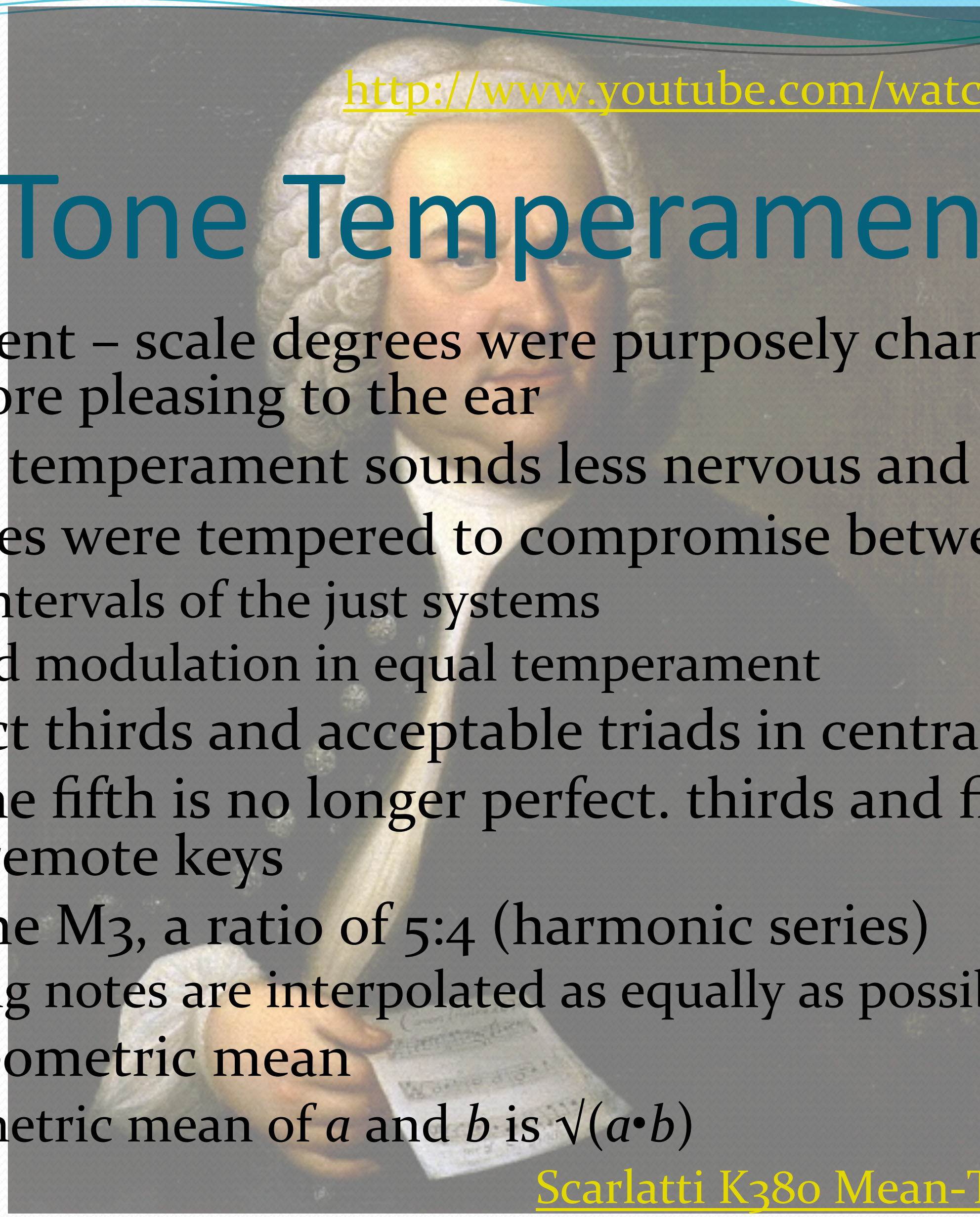
- Non-diatonic pitches (the black keys in C Major) are extremely out of tune since they do not exist in the harmonic series, or they exist but are out of tune or too high
  - e.g. minor 7th or augmented 4th

compare Pythagorean and Just

http://www.youtube.com/watch?v=teVlrYJGKAE

Mean-Tone Temperament



- Temperament – scale degrees were purposely changed to become more pleasing to the ear
- mean-tone temperament sounds less nervous and less agitating
- certain notes were tempered to compromise between:
  - perfect intervals of the just systems
  - unlimited modulation in equal temperament
- goal: perfect thirds and acceptable triads in central keys
- sacrifice: the fifth is no longer perfect. thirds and fifths very out of tune in remote keys
- based on the M3, a ratio of 5:4 (harmonic series)
  - remaining notes are interpolated as equally as possible
- 'mean' = geometric mean
  - the geometric mean of *a* and *b* is $\sqrt{(a \cdot b)}$

Scarlatti K380 Mean-Tone center C

Scarlatti K380 Mean-Tone center C#

Equal Temperament

- Cents – We assign 1200 cents per octave such that there are 100 cents per semitone. But just how many Hz are in 100 cents?
- Since pitch is logarithmic with frequency, we can't add a constant value of Hz to obtain the next semitone, because this would change depending on where we are on the keyboard. We have to multiply by some *s*. (Marin Mersenne, 1588-1648)

Scarlatti K380 E.T.

$$v \cdot s^{12} = 2v$$

$$s^{12} = 2$$

$$s = \sqrt[12]{2} = 2^{\frac{1}{12}} \approx 1.059$$

Equal Temperament

- One can play in tune in any key because all keys are treated the same.
- Semitones are equidistant from each other (frequency-ratio wise).
- E.T. thirds are about 14 cents sharper than ‘perfect’ (harmonic) thirds.
- Feeling of nervousness/agitation compared to just and mean-tone scales
- Motivation for serial music and chromaticism in the 20th Century.

Mean-Tone and Equal Temperaments

do	re	mi	fa	sol	la	ti
1	$\sqrt{5}:2$	$5:4$	$2:5^{1/4}$	$5^{1/4}:1$	$5^{3/4}:2$	$5^{5/4}:4$

do	re	mi	fa	sol	la	ti
1	$2^{2/12}$	$2^{4/12}$	$2^{5/12}$	$2^{7/12}$	$2^{9/12}$	$2^{11/12}$

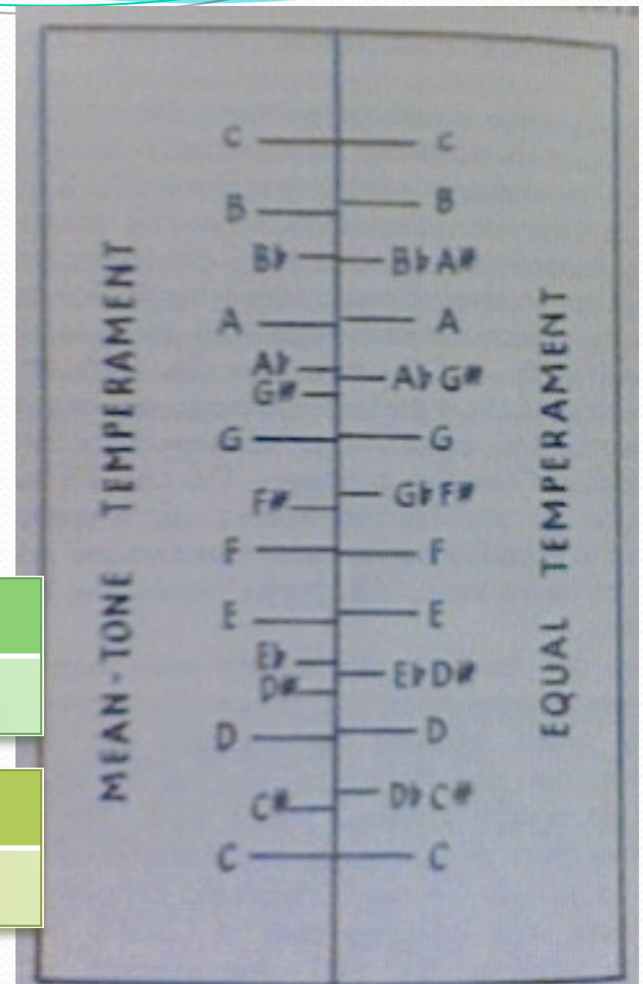


Fig. 7

E.T. and Just Intonation

Pop Quiz!