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The Life and Works of Marin Mersenne (1588-1648)

Marin Mersenne was born on September 8, 1588 to a poor family in France. He was enrolled at the Jesuit College of La Flèche, during which time he had became acquaintances with René Descartes. At the age of 23, he joined the Minim Friars, which was a religious order of the Roman Catholic Church founded in the fifteenth century in Italy. He became a full priest in Paris in 1613. Soon afterward, he began to teach theology and philosophy, and met other famous mathematicians such as Descartes, Pascal, Fermat, and Galileo. He had taken an interest in music theory, of which was the subject of two of six books he wrote. The other four books concerned theology, philosophy of science and mathematics, and natural science and mathematics.

The first book, *Quaestiones Celeberrimae in Genesim*, was written in 1623 and its primary concern was that of theology. It can be thought of as a commentary on the Book of Genesis of the Bible, but it also attacks against the magical arts, pantheistic views, astrology, and other sacrilegious concepts.

Mersenne's contribution to music theory was influenced by his work with combinatorics. He considered counting problems in permutations, arrangements (ordered selections), and combinations (unordered selections). One of his most famous achievements is the calculations of the factorials of the first sixty-four integers, which is the most any human has ever done by hand. He also discovered a relation between additive number theory and permutations with repetitions, by listing all thirty partitions of nine in order to find all thirty types of repetitions of nine notes.

In the subject of arrangements, he calculated all the values of the permutation $P(22,n)$ with $n = \{1,2,3,\ldots,22\}$. Lastly, Mersenne invented the rule that calculates combinations: $C(n,r) = \dfrac{P(n,r)}{P(p,r)}$, or in modern notation, $\dbinom{n}{r}$, and discovered what was later known as Pascal's Fifth Consequence, that $\dbinom{n}{r} = \dbinom{n}{n-r}$, although he had not formulated a proof of this truth.

Mersenne's other contributions to music theory include his "laws of stretched strings," which state that the frequency v of a stretched string is 1) inversely proportional to the string's length l, 2) directly proportional to the square root of its tension T, and 3) inversely proportional to the square root of its linear density ρ. Mathematically he developed the equation $v = \dfrac{n}{2l}\sqrt{\dfrac{T}{\rho}}$ where n is the string's *n*th harmonic. In addition to the physics of strings, he discovered the relationship between pitch and frequency, independent of Galileo's same discovery. He confirmed Pythagoras' discovery that doubling a frequency will increase the pitch by one octave and that multiplying a frequency by $\frac{3}{2}$ will increase the pitch by one fifth, and thus made use of string vibrations and pulses as the basis of consonance and harmony. He also invented two tuning scales for the spinet, two for the lute, and one improved meantone temperament scale for all instruments. Mersenne's meantone temperament differed slightly from the Pythagorean scale, whose frequencies of pitch were based solely on octaves and fifths. Although the Pythagorean scale was perfect to the harmonics of a fundamental frequency, it was hardly just, meaning that it was particular to only a few keys. If a musician wanted to play a piece in a key that deviated from the system's fundamental, the piece would sound out of tune. Therefore the just systems took precedence, to adjust for other tonal centers. He introduced the value $\sqrt[12]{2}$, the factor by which the frequencies of two semitones differ, in the modern Equal-Tempered scale. Not only was it more accurate than the tuning system of the time, but also this ratio could be constructed with a straightedge and compass, and Plato would have approved. It was Mersenne's aptitude in the physics of sound that deemed him the title, the "father of acoustics."

Mersenne's second book, *Harmonie Universelle*, was intended to be a treatise that would contain all those things that a "perfect musician" should know, including descriptions of instruments, different scale systems, and compositional techniques on the musical side, and on the physical side, it included explanations of the physical nature of sound, vibrations and frequencies of strings and air columns, and the mechanics of movement. He demonstrated that mechanics of sound implied harmony, but in doing so decided that harmony itself was embedded in the mechanics of sound. His treatise on harmony illustrates how consonances can be found everywhere. He believed that harmony was an essential part of the Creation of the universe. This principle is reminiscent of Pythagoras's idea of the "music of the spheres."

Mersenne was as fluent in number theory as he was in music theory. He developed a concept of "perfect numbers," which were integers n such that the sum of all divisors of n, or $\sigma(n)$, is equal to twice n. In other words, the sum of the all of the aliquot divisors (divisors of n less than n) of n would add up to n. Consequently, no perfect number can be prime. Mathematicians have previously conjectured, incorrectly, that 1) the nth perfect number P_n has exactly n digits, and 2) all perfect numbers that are even terminate alternately in the digits 6 or 8. It can be shown, however, that all even perfect numbers do end in either 6 or 8, but do not alternate in that manner. Currently, no odd perfect numbers have been found, and we do not yet know whether there is a finite or infinite number of perfect numbers.

Mersenne also thought all numbers of the form $2^n - 1$ were prime, which is incorrect, but we honor that assertion by calling these integers Mersenne numbers. The Mersenne numbers that so happen to be prime are called Mersenne primes.

Pierre de Fermat was one of Mersenne's frequent correspondents. In his letters to Mersenne, he made conjectures of number theory, including the hypothesis that every positive integer could be expressed as a sum of at most three triangular numbers, which was proved later by Carl Gauss. Fermat also conjectured that all numbers of the form $2^{2^n} + 1$ are prime, but Euler disproved this hypothesis by counterexample: $2^{2^5} + 1 = 641 \cdot 6700417$.

Another frequent correspondent and friend of Mersenne was René Descartes. They wrote many letters to each other on topics of algebra and geometry, including Pascal's mystic hexagon theorem. Blaise Pascal was the son of one of the first members of Mersenne's circle, Étienne Pascal. Blaise, although a learner of Mersenne, exceeded him in many aspects. Mersenne had one idea to test the effects of atmospheric pressure on a column of mercury, but Pascal had already conducted the experiment before Mersenne could carry out his plan. Pascal showed that the height of the mercury was proportional to altitude. Another experiment of physics of interest to Mersenne was the determination of the acceleration of a falling object, in comparison to the swing of a pendulum. He measured the length of a pendulum whose period is two seconds (one swing per second), and also determined that large swings have a longer period than small swings. This finding was contrary to Galileo's hypothesis that all swings of different lengths, *ceteris paribus*, would have the same period and thus sweep over the same time interval.

Mersenne had a great number of correspondents during his lifetime. After his death, letters to 78 different colleagues were found, and these letters were enough to fill a scientific journal. Perhaps Mersenne's greatest contribution to science was the organization of frequent meetings of scientists, mathematicians, and philosophers. His time was before that of the scientific journal, and he found it deplorable that scholars had no regular place or time to meet. Thus he founded an “academy” or gathering to exchange information on scientific and technological topics of the day. Essentially, it was an informal forum for exchange of news of advancement, questions and answers, and technological discoveries and experimental results. It was this network that helped him become popular with the important thinkers of the world, and an important thinker of history.

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