Final Synthesis

Introduction

At the beginning of the semester, I had read overviews of NCTM's (2000) six Principles and summarized them. The assignment I was given was to choose two that 'stood out' and reflect on why I considered them more important. After a semester's worth of research, discussion, and observation, I feel my knowledge and familiarity of these Principles has increased dramatically, and I am in a better position to reevaluate my views on them.

When one takes a moment to think about the field of education, one realizes its underlying motivation and purpose. To examine what qualities make an effective teacher, we must examine the reason for teaching in the first place. There may be many different individual and personal reasons to teach, but all teachers have one thing in common. As many sources (NCTM, 2000; McGraw & Grant, 2005; Niess, Lee, & Kajder, 2008) will say, learning should be student centered. Thus teachers teach because they want students to learn. Teaching exists *because of* learning.

Teaching and Learning with the Other Four Principles

Assessment in Teaching and Learning

I believe the only Principle used equally by both teachers and learners is the Assessment Principle. Granted, the other three (Equity, Technology, and Curriculum), are used by both teachers and learners, but there is a peculiar balance between teachers' use and learners' use of Assessment. Teachers use assessment in two ways: summative assessment helps teachers understand what learners know, and formative assessment helps teachers make decisions on what or how to teach (NCTM, 2000). On the other side, learners use assessment in a fascinatingly symmetric manner. Countering the summative aspect, learners use assessment to identify what content they *perceive* they know and do not know. The counterpart of the formative aspect is how learners use assessment to make decisions on what or how to study. Basically, while teachers are making instructional decisions, students are making decisions about study habits.

Equity in Assessment

Notice that on the formative side, learners only *perceive* what they know and need to learn, and this perception may or may not be accurate. The way teachers provide feedback to students is an issue with the Equity Principle. Students make judgements on what they are 'good at' based on their teacher's expectations and remarks. The way a student interprets feedback will affect his or her self-efficacy (Morge, 2007).

Teachers' expectations for students is directly linked to assessment. When a teacher assesses what his or her students know and need to know, the students' performance rating is based on the teacher's expectations of them. This is the definition of a grading scale. To the point: The idea of expectations being high, but still relative to each student, conflicts with the notion that assessment should be unbiased. This incompatibility was a major topic of discussion in class, and there is no straightforward solution. The best we as educators can do is to address it and raise awareness.

NCTM suggests that to provide an equitable classroom environment, accommodations need to be made to provide every student with the opportunity to learn mathematics effectively. That is because learning styles and paces are different for each student. At the same time though, it would not be fair to give Sally a higher score than Amir—if they performed exactly the same on a test—just because the teacher's expectations for Amir are higher (for whatever reason that might be). Moreover, differences in expectation need to be justified, and even the right to *hold* differences in expectation is controversial. Regardless, assessment needs to be unbiased, and that includes formal and informal assessment.

The data from my Equity project showed that my cooperating teacher calls on 38% more males than expected when leading class discussion. While this is a form of informal assessment and thus seems an unimportant datum, this method is nevertheless inequitable because Mr. Noble is providing significantly less opportunity for females to participate in discussion (this is likely an unconscious effort). As a possible consequence, females in Mr. Noble's class may feel it is unimportant to participate or have an opinion in a mathematics class, or possibly that they feel he is not calling on them for a reason: that they are not as proficient as he expects them to be. On the converse, it is equally likely that *males* feel the same way: that Mr. Noble calls on them because he feels they are performing poorly and thus need to participate more to learn more. The number of possible interpretations of this data is endless. The reality is Mr. Noble's expectations based on gender affect the bias of his assessment method.

Learning in Assessment

NCTM's vision includes students as "autonomous learners" (p. 21). Teachers should want their students to be able to monitor and actively change their own progress, identify and set their own goals, and detect their own proficiency. Teachers do not only need to relay information, but they also need to help students develop effective study skills required in higher education. Therefore, student's views and beliefs about how good they are at mathematics (or a particular subject within mathematics) should not be entirely dependent on the teacher's view of that student's proficiency. Young learners are not always able to determine how well they know a topic (Ormrod, 2009). Teachers should help students identify with what they themselves are confident.

Teaching in Assessment

There is an overwhelming variety of assessment available to teachers and learners. Examples of such types range from traditional tests and quizzes to journals, projects, portfolios, artwork, group tasks, and questions with no or more than one correct answer (Bailey & Chen, 2005; Brown-Herbst.1999; Goetz, 2005). Teachers should use a wide variety of assessment to give students more opportunities to show off their full potential and to gain an accurate depiction of students' achievements (NCTM, 2000). Moreover, students will receive feedback from more than one type of source, which helps diversify their beliefs of their abilities in the classroom.

Kennedy (1999) takes a radical, yet becomingly popular, view on how assessment should be used. So far, I have discussed assessment as a tool that helps teachers and learners obtain information. However, assessment can also be used as a learning situation *per se*. Teachers often use assessment to give hints to students or provide *new* information to them. Learners may often challenge and question this information or reflect back on their own knowledge, engaging in reasoning and sense making. Open-ended assessment requires students to develop their own solutions and provide 'proof' of correctness (Goetz, 2005).

Creative assessments should test general abilities rather than computational skills. Even without open-ended assessment, students should be tested on conceptual understanding rather than procedural understanding. While the act of understanding is important, the act of overcoming an obstacle is equally important. Thus assessments like these not only serve as a learning situation because of the manner in which they are constructed, but also in the manner they are presented. They provide intellectual concentration and emotional tension, which are beneficial supplements to the process of learning (Kokol-Voljc).

Technology in Assessment

Kennedy's (1999) and Kokol-Voljc's suggestions for assessment are particularly useful in the Technology Principle. Assessments that involve technology are fragile though, because teachers must be careful not to test students' proficiency with the tool, but rather the concept involved. Otherwise students may pick up on the notion that they are being tested on, for example, how to use the calculator rather than on how to represent a constant rate of change.

Technology in Learning

With the implementation of technology, the focus of a lesson changes from algorithmic learning to operational learning. That is, the knowledge of how to perform a sequence of steps (following directions) is not as valued as the understanding of the concept behind those steps and the decision-making involved when choosing which sequence of steps to perform (Kokol-Voljc). For students to build understanding and learn with sense-making, the learning process should play an active role in the learner (Niess, Lee, & Kajder, 2008). Students need to discover and investigate mathematical objects on their own and derive their own rules, formulas, and properties. Phillip and Vincent (2003) clarify that when students take ownership of their learning, they willl remember what they have learned easier and longer (compared to when the content is delivered to them). Technology should be a tool that elicits this type of learning.

My cooperating teacher's classroom has a SMART Board, an interactive whiteboard that carries the advantages of a computer, a projector screen, and a touchpad. Depending on how it is used, the SMART Board can be an extremely effective tool in the teaching and learning processes. Students are able to investigate dynamic changes in parameters and form and test conjectures. The SMART Board and other technology should eliminate the tedious burdens that would normally be present without it. For example, instead of graphing eight circles with differing center and radii on a worksheet, the SMART Board is able to graph one circle with a changeable center and radius. Students are able to develop the understanding of relationships between the properties of circles and their equation in (h,k)-form without focusing too much on the process of drawing the graphs. McGehee and Griffith (2004) provide more examples like this one. They conclude that with advances in technology, students can spend less time with procedure and more time 'playing around' and developing conceptual understanding.

This type of 'play' also elicits reasoning and sense making. When learners notice patterns, they generate the desire to find out if that pattern is truly a pattern or a coincidence. Thus they will attempt to (informally) prove that it is true, or attempt to find a counterexample. Not all learners take this initiative however, so teachers must explicitly draw out this process by asking hypothetical questions such as, "What would happen if…?" or, "Is that always true?"

Technology in Teaching

Kokol-Voljc states two goals for teaching that should be fulfilled with the use of technology: to develop theory of mathematical concepts, and to use mathematical concepts in real-world models and applications. The first goal, illustrated above, is centered on student learning. The second goal is from the teacher's perspective. Teachers should make the curriculum relevant and interesting, and highlight its importance in the real world. They should make clear how the mathematical content and processes students learn will prepare them for their future (NCTM, 2000). When implementing technology in lessons and assessment, teachers should consider how the technology models real-world situations, and decide if a parameter or solution is reasonable. This corresponds to NCTM's Number and Operations Content Standard. Students should be able to make accurate estimations and decide if given data in a problem and their answers are 'reasonable.' Thus teachers should be careful to make sure the problems they assign and the lessons they provide have realistic data.

For instance, one of the problems I came across on a test in Mr. Noble's Geometry class involved finding the surface area of a square pyramid given the length of a base edge and one of the side edges. Calculating the surface area was not a problem, but if one student had been curious enough to find its volume, it would have been impossible! The dimensions of the pyramid were given such that the height was an imaginary number.

Technology and Assessment in Curriculum

NCTM stresses the interaction between the Technology and Curriculum principles: technology has a major influence on the curriculum. As time passes, technology develops at an increasing rate, ergo judgements change about facts and procedures that are essential. Neiss, Lee, and Kajder (2008) overview the history of the teaching of mathematics over the past century. In 1896, eighth grade students were expected to correctly compute the cube root of 14,742.488 (p. 38). In contrast, children in today's generation would quickly jump to the calculator without thinking twice. As technology increases, educators and policymakers are continually adjusting their decisions on what procedures and concepts are valued, which in turn influences the curriculum.

With changes in curriculum come changes in assessment. Assessment should reflect instruction, and *vice versa*. As a class we decided that both assessment and instruction are subsets of each other; that is, no item in an assessment should be left out of instruction, and all items of instruction should potentially be assessed in some way.

Curriculum in Teaching

As a class, we also agreed that the mathematics curriculum should be coherent in three dimensions. In a vertical manner, curriculum should be interconnected within a classroom. From section to section and from chapter to chapter, students should be made aware, explicitly, the connections between concepts. Ormrod (2009) elaborates, "information in long-term memory is interconnected and organized" (p. 28). Knowledge is not to be thought of as a grab bag of facts, but rather like a 'connect-the-dots' drawing. Only when the connections are there can you see the bigger picture.

The second dimension of curricular coherence, represented by horizontal width, is coherence across time, that is, across multiple math classes. The traditional sequence of math classes in my observation high school seems like a discrete set of courses through which students progress, without any real logical order other than complexity. Rather, the secondary school curriculum should draw on the prior knowledge of students from previous classes. There are uncountable connections between geometry and algebra, however as a high school student I could not understand how a second-degree polynomial (quadratic) formula, a focus and directrix, and a cross section of a cone were related—all three form parabolas. It wasn't until my senior year in college that these connections were made explicit. Teachers in secondary schools should collaborate with each other and plan their lessons that draw on previous knowledge and set up new knowledge for subsequent years.

The third and last type of coherence we discussed is the depth component, which describes coherence across content areas of different subjects. This coherence is probably the hardest to perceive, because many students do not see connections between their classes. Similar to math teachers across grade levels, teachers within grade levels should collaborate and discuss connections among their subjects. For example, math teachers covering statistics and science teachers covering measurement and 'significant figures' can work together to develop an activity that includes both classes.

Conclusion

Although each of the NCTM's six Principles are important, I find that Learning and Teaching are the ones that stand out the most, and those two principles are integrated seamlessly with the other four. The foundation of Education begins with Learning, and is followed by Teaching, and the other principles stem out from there. This paper illustrates the interactions of these Principles. The overall theme is of Learning and Teaching, but the true detailed discussion revolves around Assessment, Equity, Technology, and Curriculum and the roles they play in and out of Learning and Teaching.

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