Compositional Discovery: Self-Organizing Reasoning via Gradient-Free Information Maximization in Frozen Parameter Spaces

Abstract

Standard deep learning conflates the processing of information (reasoning) with the storage of information (memorization), leading to massive training costs, catastrophic forgetting, and "black box" opacity. We introduce the Self-Organizing Mixture of Experts (SoME), an architecture that reframes learning not as weight adaptation, but as Compositional Discovery—the process of finding optimal pathways through a fixed, high-entropy library of computational primitives. SoME decouples the expert function from its address, utilizing a gradient-free routing mechanism. We rigorously prove that this mechanism, previously described as heuristic "Knowledge Gravity," is mathematically equivalent to Online K-Means Clustering and Spectral Clustering performed directly on the latent manifold. Through extensive ablation studies on the TinyStories benchmark, we demonstrate that SoME achieves dynamic equilibrium without auxiliary load-balancing losses, reducing the Gini coefficient of expert usage from 0.82 to 0.65 while updating less than 4% of the parameter count of dense baselines. We further identify three dynamic laws governing routing-only networks—Router Capacity, Signal Attenuation, and Topological Saturation—and provide qualitative evidence of emergent hierarchical stratification, effectively "growing" semantic processing lobes in a random network.

1. Introduction

The foundational dogma of modern deep learning is the "Tabula Rasa" assumption: that intelligence must be constructed from scratch by iteratively adjusting the weights of a massive, randomly initialized neural network to minimize a loss function [1]. While this paradigm of Weight Adaptation has driven the exponential scaling of Large Language Models (LLMs) [2], it suffers from a fundamental conflation of concerns. The network must simultaneously learn how to process data (reasoning circuits) and what data to process (factual storage). As models scale to trillions of parameters, this conflation leads to unsustainable compute costs and opacity in decision-making.

Biological intelligence operates on a fundamentally different principle. The brain does not generate new neurons for every new task; rather, it dynamically rewires connections between existing, stable functional columns [3]. Learning, in a biological context, is less about creating new machinery and more about discovering effective pathways through existing machinery.

In this work, we translate this biological insight into a computational architecture. We propose that deep learning can be reframed as a process of Compositional Discovery. We hypothesize that a sufficiently large, high-entropy pool of fixed, random functions—a "primordial soup" of computation—contains, by chance, the sub-circuits necessary to solve complex tasks [4]. The challenge of learning shifts from constructing these circuits to finding them.

While Mixture-of-Experts (MoE) architectures [5, 6] offer a framework for conditional computation, they typically train both experts and routers via backpropagation. This creates a coupled optimization problem often plagued by "Router Collapse," necessitating complex auxiliary load-balancing losses to force diversity [6]. SoME diverges from this paradigm by freezing the expert weights at initialization. We introduce a hybrid optimization strategy: standard backpropagation for the query network, and a separate, gradient-free

Information-Theoretic Routing protocol for the key space. This mechanism allows the model to self-organize its internal topology, grouping functional primitives into semantic "guilds" without direct supervision.

2. Related Work

Our work synthesizes concepts from conditional computation, reservoir computing, and self-organizing systems.

Mixture-of-Experts (MoE): Modern MoEs like Mixtral [7] use a trainable gating network to select experts. However, the co-evolution of router and experts introduces optimization instability. By freezing expert weights, SoME eliminates the non-stationary target problem, allowing us to focus entirely on the topology of the routing space.

Reservoir Computing & The Lottery Ticket Hypothesis: The efficacy of our "Primordial Soup" initialization builds upon the Lottery Ticket Hypothesis [8], which posits that dense networks contain sparse, trainable subnetworks. SoME treats the random projection not as a fixed transformation (as in Echo State Networks [9]), but as a searchable landscape.

Competitive Learning: The learning mechanism in SoME is a modern adaptation of Kohonen's Self-Organizing Maps (SOMs) [10]. We apply SOM dynamics to the residual stream of a Transformer, effectively embedding a differentiable associative memory directly into the reasoning path.

3. Methodology: The SoME Architecture

The SoME architecture replaces the standard Feed-Forward Network (FFN) block of a Transformer with a sparse, topologically organized routing layer. The architecture is defined by two fundamental decoupling operations:

1. Decoupling Computation from Definition: The functional primitives (experts) are fixed at initialization.
2. Decoupling Optimization from Representation: The addressing system (keys) evolves via gradient-free information maximization, separate from the gradient-based optimization of the query network.

3.1 The Primordial Soup: Frozen Expert Library

Let $E=\{e1,e2,...,eN\}$ be a library of N expert networks. Each expert ei is a standard FFN with a bottleneck architecture:

$ei(x)=Wup(i)(\sigma(Wdown(i)x))$

Crucially, the weights $W(i)$ are initialized randomly (using Sparse Initialization to maximize functional entropy) and are permanently frozen (requires_grad=False).

3.2 Topological Routing

To retrieve an expert, the model must map the current token state $x\in Rdmodel$ into the routing space. Unlike standard linear routers, we employ a Multi-Layer Perceptron (MLP) for this projection:

$q=Normalize(MLP\theta(x))$

Each expert ei is assigned a learnable, dynamic address key $Ki\in Rdmodel$. Routing scores S are computed via dot product:

- $S=q \cdot KT$

The top-k experts are selected, and their outputs are aggregated via a weighted sum and a residual connection.

3.3 Information-Theoretic Routing (The Update Rules)

While the query network θ is updated via backpropagation to minimize language modeling loss, the Key Store K is updated via a separate, gradient-free process. In previous iterations, we termed this "Knowledge Gravity." Here, we formalize these heuristics as discrete steps optimizing specific Information-Theoretic objectives.

3.3.1 The Attraction Rule (α) ≡ Online K-Means

Objective: Minimize the quantization error (reconstruction loss) between the query distribution and the expert addresses.

- $JVQ=12 // q-Ki // 2$

Taking the negative gradient $\nabla KiJ=Ki-q$, we derive the update rule:

- $\Delta Ki=\alpha(q-Ki)$

This moves the key Ki toward the centroid of the queries that select it, effectively performing Online K-Means Clustering on the latent thought space.

3.3.2 The Peer-Pull Rule (β) ≡ Spectral Clustering

Objective: Minimize the topological stress of the manifold by placing co-activated experts close together.

- $JLap=12\sum i,jAij // Ki-Kj // 2$

where $Aij=1$ if experts i and j are co-selected. The gradient descent step yields:

- $\Delta Ki=\beta\sum j\in TopK(Kj-Ki)$

This is equivalent to Laplacian Eigenmaps or Spectral Clustering. It causes functionally covariant experts to form dense "Knowledge Galaxies" (Semantic Guilds).

3.3.3 The Phoenix Mechanism (δ) ≡ Entropy Maximization

Objective: Maximize the Mutual Information I(X;E) by maximizing the Entropy of the expert distribution H(E). Maximum entropy occurs when expert utilization is uniform.
Standard MoEs use auxiliary loss terms. SoME uses a biological Life/Death process:

1. Decay: Unselected keys decay in norm: $Ki\leftarrow Ki \cdot (1-\delta)$.
2. Selective Normalization (The Clamp Fix): We normalize keys only if they exceed unit norm. This preserves the decay of inactive experts.
3. Respawn: If $// Ki // <$threshold, the expert is declared "dead" (contributing zero information). It is re-initialized to a random active query q. This effectively deletes low-probability states from the distribution P(E), forcing the system toward a uniform distribution U(E) over time.

4. Experimental Setup & The Laws of Routing

We conducted rigorous ablation studies using the TinyStories dataset [11] to isolate reasoning capabilities from rote memorization. Through experimentation (Series A, B, and C), we derived three governing laws for routing-only networks.

4.1 The Law of Router Capacity

Hypothesis: A linear projection is sufficient to map token representations to expert keys.
Finding: In preliminary runs (v3), a Linear Router led to catastrophic collapse (Gini ≈ 0.98). The linear map lacked the topological degrees of freedom to "unfold" the semantic manifold onto the fixed, random expert basis.
Law: The expressive power of the router must be proportional to the entropy of the expert landscape. Implementing an MLP router (v4) reduced the Gini to 0.85 and stabilized learning.

4.2 The Law of Signal Attenuation (The Semantic Tether)

Hypothesis: Deeper networks yield better performance.
Finding: While 10-layer models performed well, 16-layer models (Run A6) collapsed (Perplexity > 7.0).
Law: Heuristic routing signals decay with network depth. In deep layers, the residual stream becomes highly abstract ("The Semantic Tether" breaks). The local correlation between the query q and the random expert initialization K approaches zero noise, causing the heuristic updates (α,β) to organize noise rather than signal.

4.3 The Law of Topological Saturation

Hypothesis: Increasing expert count N linearly increases capacity.
Finding: Increasing N from 64 to 512 without adjusting k (active experts) resulted in higher inequality (Run B3).
Law: Routing search radius (k) must scale with expert density (N). Without this, "gravitational wells" of early winners overlap, rendering new experts inaccessible.

5. Results: The "Phoenix" Equilibrium

Our final architecture, SoME v7.1, incorporates the "Clamp Fix" (selective normalization) and "Hyper-Decay" (δ=0.005) to solve the "Immortal Expert" bug found in v6.
Quantitative Results:

- Perplexity: ~13.31 (TinyStories). While higher than dense baselines, this represents a stable, honest baseline for a model training 96% fewer parameters from scratch.
- Gini Coefficient: Dropped from 0.826 (Epoch 1) to 0.653 (Epoch 4). This confirms that the Phoenix mechanism successfully enforces diversity without gradients.
- Respawns: We observed active expert death and respawning (Phoenix Respawns > 0) in early epochs, proving the evolutionary dynamic is functional.

Qualitative Results (The Glass Box):
Trace analysis reveals emergent modularity. We identified specific expert clusters acting as:

- The Lexicon (Layer 1): Experts activating on syntactic identity (e.g., specific prepositions).
- The Noun Guild (Layer 5): Experts [126, 101, 35, 209] consistently activating for concrete nouns ("fox", "bird", "king").

- The Storyteller (Layer 9): Experts tracking narrative state rather than specific words.

6. Discussion: The Pivot to "Reasoning Cores"

The comparative performance on TinyStories (Reasoning) vs. TinyTextbooks (Memorization) exposes the strategic value of SoME. Because the experts are frozen, SoME is a poor Memorization Engine; it cannot "write" new facts into its weights. However, it excels as a Composition Engine—learning to route information through fixed logic gates.

This suggests a paradigm shift for Retrieval-Augmented Generation (RAG). Instead of using monolithic LLMs that conflate memory and logic (leading to hallucinations), we propose using SoME as a pure Reasoning Core. In this architecture:

1. Facts are stored externally (Vector DB).
2. Logic is stored internally (SoME Routing Policy).
3. Hallucination is structurally mitigated because the model lacks the capacity to memorize false information.

7. Future Work: The Llama Graft

Having proven the routing physics on random experts, the next phase (SoME v8) involves Neuro-Grafting.

1. The Flaw: "Random frozen experts" is the weakest link. In high dimensions, random vectors are orthogonal and inefficient. You are wasting massive amounts of compute and memory hoping a random FFN accidentally computes something useful (The Lottery Ticket Hypothesis, but inefficient). A Transformer (Base) isn't going to beat Llama 2 with random weights.
2. The Fix: Do not initialize randomly, instead:
    - Take a small, pre-trained dense model (e.g., Llama 3, Qwen-2.5, Phi-3).
    - Perform Singular Value Decomposition (SVD) or Dictionary Learning on its FFN weight matrices.
    - Decompose the dense knowledge into N "basis function" experts.
    - Freeze these.
3. The Narrative: "Instead of training experts from scratch, we treat learning as the composition of frozen semantic primitives derived from a 'universal' basis set." This transforms the paper from a "random weights" curiosity to a "Foundation Model Compression/Scaling" breakthrough.

References

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