1 Goals

1.1 Primary Goals

- Motivate and introduce the concept of tangent measures
- Write down two counterexamples in the theory of tangent measures - Preiss and O'Neil
- Write down an outline of their proofs

1.2 Secondary Goals

- Streamline the existence proofs of the O'Neil and Preiss measures
- Find entirely new existence proofs
- Find a result extending the Preiss measure to $\mathbb{R}^d$, $d \geq 2$, or prove/disprove it myself
- Conclude what the existence of these counterexamples means for tangent measures as a whole - is the definition somehow "wrong"?
- Move onto functions of bounded deformation

2 Timeline

Weeks 1-4 Develop a firm understanding of tangent measures and geometric measure theory as a whole. Understand the motivation behind tangent measures. What have they helped solve? Links to other areas of mathematics (calculus of variations)

Weeks 4-10 Begin writing the dissertation. Ensure an initial draft is ready by Christmas, including a wonky introduction, definitions, existence statements for the O'Neil and Preiss measures, outlines of their proofs

Weeks xmas2-xmas4 + 11-20 + eastr2-4 finish it lol

Main results are Theorem 3 in O'Neil, 5.8 & 5.9(3) in Preiss. Sahlsten Thm 1.1 claims to show O'Neil's result is actually typical of Radon measures. Mattila Example 14.2(3) gives a measure with multiple tangent measures at each point.

Theorem 1 (Existence of tangent measures)**.** *[Theorem 14.3 in Mattila] Let $\mu \in \mathcal{M}$ be a Radon measure. Then if μ is asymptotically doubling at $a \in \mathbb{R}^n$, i.e.*

$$\limsup_{r \downarrow 0} \frac{\mu(B(a,2r))}{\mu(B(a,r))} < \infty, \tag{1}$$

then every sequence (r_i) *of positive numbers with* $r_i \downarrow 0$ *has a subsequence such that*

$$\mu(B(a,r_i))^{-1} T^{a,r_i}_{\#}\mu \tag{2}$$

converges to a tangent measure of μ *at* a.

[Proposition 2.2 and Corollary 2.7 in Preiss] The following are equivalent

1. μ *is asymptotically doubling at* a
2. *There exist bounded sets* $C, D \subseteq \mathbb{R}^n$ *such that* $0 \in C^\circ \subseteq \overline{C} \subseteq D^\circ$, *and*

$$\limsup_{r\downarrow 0} \frac{\mu(a+rD)}{\mu(a+rC)} < \infty \tag{3}$$

3. *There exist* $c > 0$ *and bounded sets* $C, D \subseteq \mathbb{R}^n$ *such that* $0 \in C^\circ \subseteq \overline{C} \subseteq D^\circ$ *and* $\tau(D) \leq c\tau(C)$ *for all* $\tau \in \mathrm{Tan}(\mu, a)$

Lemma 1 (Properties of weak* convergence)**.** *Let* (μ_i) *be a sequence in* $\mathcal{M}(\mathbb{R}^n)$ *converging weakly* to* μ. *Then for all compact* $K \subseteq \mathbb{R}^n$ *and open* $U \subseteq \mathbb{R}^n$, *we have*

$$\mu(K) \geq \limsup_{i\to\infty} \mu_i(K); \tag{4}$$

$$\mu(U) \leq \liminf_{i\to\infty} \mu_i(U). \tag{5}$$