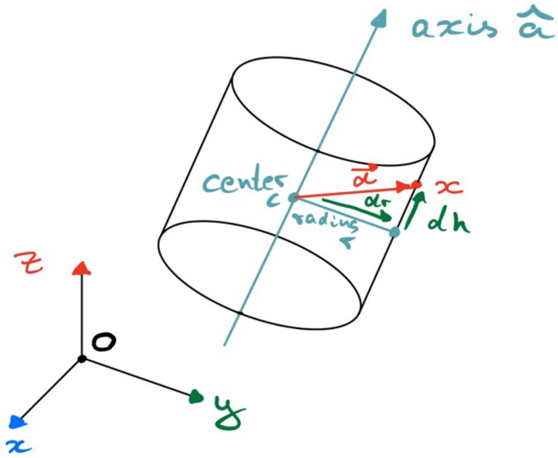
Antoine ROGER, Baptiste COPROS, Benoît MORAWIEC

‘’Derive the expression for a Ray-Cylinder intersection’’

First thing first, in order to find the expression for a ray-cylinder intersection we need to find an implicit parametrization for the cylinder with the following parameters :

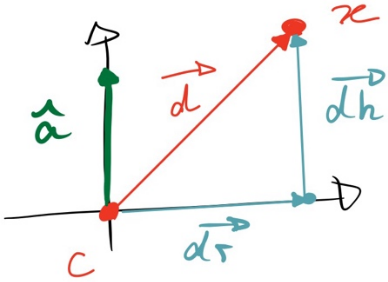
- center **c**
- radius **r**
- axis **a**
- height *h*



For an infinite cylinder, the points on the cylinder surface are points such that the radial distance **dr** from point to axis **a** is of length radius R.

Using the distance vector **d** from a point x to the center c of the cylinder

$$\vec{d} = x - c$$



We can compute the radial distance **dr** and the height distance **dh** as follow :

$$\vec{dh} = \langle \hat{a}, \vec{d} \rangle \cdot \hat{a}$$
$$\Rightarrow dh = \langle \hat{a}, \vec{d} \rangle$$
$$\vec{dr} = \vec{d} - \vec{dh}$$
$$= \vec{d} - \langle \hat{a}, \vec{d} \rangle \hat{a}$$
$$\Rightarrow dr = \langle \vec{dr}, \vec{dr} \rangle$$

A point x is on the infinite cylinder if it’s radial distance with the axis is such that :

$$dr = \text{radius } R$$

For a finite cylinder, we still need to verify that the height distance from center is less than h/2 :

$$dh \leq h/2$$

This leads to the following system of equation to determine if a point x is on the cylinder of height h :

$$(1) \quad dr(x) = \text{radius } r$$
$$(2) \quad dh(x) \leq h/2$$

Now let’s consider the ray-cylinder intersection problem. The ray equation is as follow :

$$ray(t) = o + t \cdot \vec{d}$$

To search for ray-cylinder intersection we first try to find a t such that the point on the ray will be solution of the equation 1 :

$$d_r(ray(t)) - r = 0$$

$$\Rightarrow \| ray(t) - \vec{a}(ray(t) \cdot \vec{a}) \| = 0$$

$$\| O_{ray} + t\vec{d} - \vec{a}((O_{ray} + t\vec{d}) \cdot \vec{a})) = 0$$

If there are points satisfying this equation, we pass to the second test, otherwhise there cannot be a cylinder-ray intersection.

The second test consists of looking if one of the potential points find in the previous step, has an height distance to center smaller than h/2. If yes, we found the one, otherwhise there cannot be an intersection.