Script Video Asdos

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1. Nyatakan bilangan kompleks berikut ke dalam bentuk kutub dan gambarlah dalam bidang kompleks:

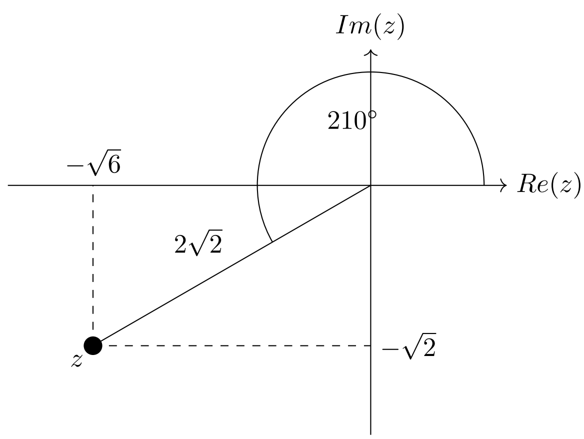
   (e) $z = -\sqrt{6} - \sqrt{2}i$
   Penyelesaian. Modulus dari z adalah

   $$r = |-\sqrt{6} - \sqrt{2}i| = \sqrt{(-\sqrt{6})^2 + (-\sqrt{2})^2} = \sqrt{6+2} = 2\sqrt{2}$$

   θ atau Argumen dari z adalah

   $$\tan^{-1}\left(\frac{b}{a}\right) = \tan^{-1}\left(\frac{-\sqrt{2}}{-\sqrt{6}}\right) = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \left\{\frac{\pi}{6}, \frac{7\pi}{6}\right\}$$

   Karena a dan b keduanya negatif maka $Arg(z)$ atau θ adalah $\frac{7\pi}{6}$.
   $\therefore$ Bentuk kutubnya adalah $z = 2\sqrt{2}\left(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}\right)$

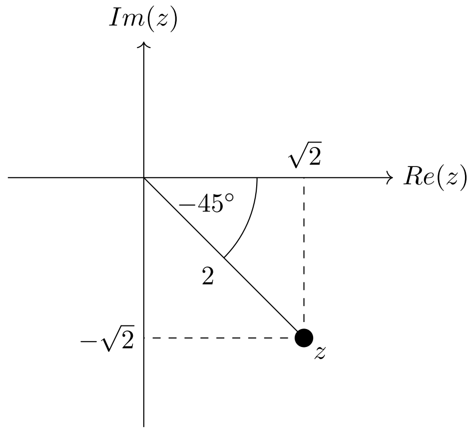


2. Nyatakan bilangan kompleks berikut ke dalam bentuk $z = a + bi$ dan gambarlah dalam bidang kompleks:

   (f) $2e^{-\frac{\pi}{4}i}$
   Penyelesaian. Ingat bahwa $\boxed{re^{\theta i} = r\text{cis}(\theta)}$

   $$2e^{-\frac{\pi}{4}i} = 2\text{cis}\left(-\frac{\pi}{4}\right) = 2\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right) = 2\left(\cos\left(\frac{\pi}{4}\right) - i\sin\left(\frac{\pi}{4}\right)\right) = 2\left(\frac{1}{2}\sqrt{2} - \frac{1}{2}\sqrt{2}i\right)$$

   $\therefore 2e^{-\frac{\pi}{4}i} = \sqrt{2} - \sqrt{2}i$



3. Diberikan $z_1 = 2\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$ dan $z_2 = 3\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$.

   (a) $z_1 z_2$
   Penyelesaian. *Teorema De Moivre* $\boxed{z_1 z_2 = r_1 r_2(\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2))}$

   $$z_1 z_2 = 2\cdot 3\left(\cos\left(\frac{\pi}{4}+\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{4}+\frac{\pi}{6}\right)\right) = 6\left(\cos\left(\frac{5\pi}{12}\right) + i\sin\left(\frac{5\pi}{12}\right)\right)$$

   (b) $\frac{z_1}{z_2}$
   Penyelesaian. *Teorema De Moivre* $\boxed{\frac{z_1}{z_2} = \frac{r_1}{r_2}(\cos(\theta_1-\theta_2) + i\sin(\theta_1-\theta_2))}$

   $$\frac{z_1}{z_2} = \frac{2}{3}\left(\cos\left(\frac{\pi}{4}-\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{4}-\frac{\pi}{6}\right)\right) = \frac{2}{3}\left(\cos\left(\frac{\pi}{12}\right) + i\sin\left(\frac{\pi}{12}\right)\right)$$

4. Hitunglah

   (e) $\left(\frac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^{10}$
   Penyelesaian. Ubah kedalam bentuk kutub $\boxed{1+\sqrt{3}i = 2\text{cis}\left(\frac{\pi}{3}\right)}$ dan $\boxed{1-\sqrt{3}i = 2\text{cis}\left(-\frac{\pi}{3}\right)}$. Sehinggan didapatkan

   $$\left(\frac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^{10} = \left(\frac{2\text{cis}\left(\frac{\pi}{3}\right)}{2\text{cis}\left(-\frac{\pi}{3}\right)}\right)^{10} = \left(\text{cis}\left(\frac{\pi}{3} - \left(-\frac{\pi}{3}\right)\right)\right)^{10} = \left(\text{cis}\left(\frac{2\pi}{3}\right)\right)^{10} = \text{cis}\left(\frac{20\pi}{3}\right) = \text{cis}\left(\frac{2\pi}{3} + 6\pi\right)$$

   $$= \text{cis}\left(\frac{2\pi}{3}\right) = \cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right) = \frac{1}{2} + \frac{1}{2}\sqrt{3}i$$

   (f) $\left(\frac{\sqrt{3}-i}{\sqrt{3}+i}\right)^4 \left(\frac{1-i}{1+i}\right)^5$
   Penyelesaian. Dengan cara yang sama seperti 4e, Didapatkan

   $$\left(\frac{\sqrt{3}-i}{\sqrt{3}+i}\right)^4 \left(\frac{1-i}{1+i}\right)^5 = \left(\frac{2\text{cis}\left(\frac{11\pi}{6}\right)}{2\text{cis}\left(\frac{\pi}{6}\right)}\right)^4 \left(\frac{2\text{cis}\left(\frac{7\pi}{4}\right)}{2\text{cis}\left(\frac{\pi}{4}\right)}\right)^5 = \left(\text{cis}\left(\frac{5\pi}{3}\right)\right)^4 \left(\text{cis}\left(\frac{3\pi}{2}\right)\right)^5 = \text{cis}\left(\frac{20\pi}{3}\right)\text{cis}\left(\frac{15\pi}{2}\right)$$

   $$= \text{cis}\left(\frac{2\pi}{3}\right)\text{cis}\left(\frac{3\pi}{2}\right) = \text{cis}\left(\frac{2\pi}{3}+\frac{3\pi}{2}\right) = \text{cis}\left(\frac{5\pi}{6}\right) = \cos\left(\frac{5\pi}{6}\right) + i\sin\left(\frac{5\pi}{6}\right) = -\frac{1}{2}\sqrt{3} + \frac{1}{2}i$$

7. Carilah semua bilangan kompleks z yang memenuhi persamaan berikut:

   (c) $z^2 + (-2+i)z + 3 - i = 0$

   Penyelesaian. Dengan menggunakan rumus abc didapatkan:

$$z = \frac{2-i \pm \sqrt{(2-i)^2 - 4(1)(3-i)}}{2(1)} = \frac{2-i \pm \sqrt{3-4i-12+4i}}{2} = \frac{2-i \pm \sqrt{-9}}{2} = \frac{2-i \pm 3i}{2}$$

$$\therefore z_1 = 1+i \quad \vee \quad z_2 = 1-2i$$

8. Dapatkan semua nilai dari

$$z = (2-2i)^{\frac{3}{5}}$$

Penyelesaian. Ubah kedalam bentuk kutub

$$(2-2i)^{\frac{3}{5}} = \left(2\sqrt{2}\text{cis}\left(-\frac{\pi}{4}\right)\right)^{\frac{3}{5}} = \left(2\sqrt{2}\text{cis}\left(2k\pi - \frac{\pi}{4}\right)\right)^{\frac{3}{5}} = (2^{\frac{3}{2}})^{\frac{3}{5}}\,\text{cis}\left(\frac{8k\pi - \pi}{4} \cdot \frac{3}{5}\right) = 2^{\frac{9}{10}}\,\text{cis}\left(\frac{(8k-1)3\pi}{20}\right)$$

Sehingga didapatkan

$$k = 1 \Longrightarrow z_1 = 2^{\frac{9}{10}}\,\text{cis}\left(\frac{(8(1)-1)3\pi}{20}\right) = 2^{\frac{9}{10}}\,\text{cis}\left(\frac{21\pi}{20}\right)$$

$$k = 2 \Longrightarrow z_2 = 2^{\frac{9}{10}}\,\text{cis}\left(\frac{(8(2)-1)3\pi}{20}\right) = 2^{\frac{9}{10}}\,\text{cis}\left(\frac{9\pi}{4}\right)$$

$$k = 3 \Longrightarrow z_3 = 2^{\frac{9}{10}}\,\text{cis}\left(\frac{(8(3)-1)3\pi}{20}\right) = 2^{\frac{9}{10}}\,\text{cis}\left(\frac{69\pi}{20}\right) = 2^{\frac{9}{10}}\,\text{cis}\left(\frac{29\pi}{20}\right)$$

$$k = 4 \Longrightarrow z_4 = 2^{\frac{9}{10}}\,\text{cis}\left(\frac{(8(4)-1)3\pi}{20}\right) = 2^{\frac{9}{10}}\,\text{cis}\left(\frac{93\pi}{20}\right) = 2^{\frac{9}{10}}\,\text{cis}\left(\frac{13\pi}{20}\right)$$

$$k = 5 \Longrightarrow z_5 = 2^{\frac{9}{10}}\,\text{cis}\left(\frac{(8(5)-1)3\pi}{20}\right) = 2^{\frac{9}{10}}\,\text{cis}\left(\frac{117\pi}{20}\right) = 2^{\frac{9}{10}}\,\text{cis}\left(\frac{37\pi}{20}\right)$$