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2. Buktikan $\Delta \sin(a+bx) = 2\sin\left(\frac{b}{2}\right)\cos\left(a+\frac{b}{2}+bx\right)$
Jawab:

$$\begin{aligned}\Delta \sin(a+bx) &= \sin(a+b(x+1)) - \sin(a+bx)\\ &= \sin(a+bx+b) - \sin(a+bx)\end{aligned}$$

Ingat $\sin(x)-\sin(y) = 2\sin\left(\frac{x-y}{2}\right)\cos\left(\frac{x+y}{2}\right)$, Sehingga

$$\begin{aligned}\sin(a+bx+b)-\sin(a+bx) &= 2\sin\left(\frac{a+bx+b-(a+bx)}{2}\right)\cos\left(\frac{a+bx+b+(a+bx)}{2}\right)\\ &= 2\sin\left(\frac{b}{2}\right)\cos\left(a+bx+\frac{b}{2}\right) \blacksquare\end{aligned}$$

9. Hubungan antara tekanan uap(P) dan temperatur(T) diberikan oleh tabel berikut:

T:	361	367	378	387	399
P:	154,9	167	191	212,5	244,2

Dengan rumus interpolasi Lagrange dapatkan tekanannya pada saat temperaturnya $372,1°$.
Jawab:
Diketahui:

$T_0 = 361$ $T_1 = 367$ $T_2 = 378$ $T_3 = 387$ $T_4 = 399$
$P_0 = 154,9$ $P_1 = 167$ $P_2 = 191$ $P_3 = 212,5$ $P_4 = 244,2$

$P(T)$ dinyatakan sebagai

$$\begin{aligned}P(T) =&\frac{(T-T_1)(T-T_2)(T-T_3)(T-T_4)}{(T_0-T_1)(T_0-T_2)(T_0-T_3)(T_0-T_4)}P_0 + \frac{(T-T_1)(T-T_2)(T-T_3)(T-T_4)}{(T_1-T_0)(T_1-T_2)(T_1-T_3)(T_1-T_4)}P_1\\ &+\frac{(T-T_1)(T-T_2)(T-T_3)(T-T_4)}{(T_2-T_0)(T_2-T_1)(T_2-T_3)(T_2-T_4)}P_2 + \frac{(T-T_1)(T-T_2)(T-T_3)(T-T_4)}{(T_3-T_0)(T_3-T_1)(T_3-T_2)(T_3-T_4)}P_3\\ &+\frac{(T-T_1)(T-T_2)(T-T_3)(T-T_4)}{(T_4-T_0)(T_4-T_1)(T_4-T_2)(T_4-T_3)}P_4\end{aligned}$$

Sehingga untuk $P(372,1)$ didapatkan

$$\begin{aligned}P(372,1) =&\frac{(372,1-367)(372,1-378)(372,1-387)(372,1-244,2)}{(361-367)(361-378)(361-387)(361-244,2)}(154,9)\\ &+\frac{(372,1-367)(372,1-378)(372,1-387)(372,1-244,2)}{(367-361)(367-378)(367-387)(367-244,2)}(167)\\ &+\frac{(372,1-367)(372,1-378)(372,1-387)(372,1-244,2)}{(378-361)(378-367)(378-387)(378-244,2)}(191)\\ &+\frac{(372,1-367)(372,1-378)(372,1-387)(372,1-244,2)}{(387-361)(387-367)(387-378)(387-244,2)}(212)\\ &+\frac{(372,1-367)(372,1-378)(372,1-387)(372,1-244,2)}{(244,2-361)(244,2-367)(244,2-378)(244,2-387)}(244,2)\\ =&177,4\end{aligned}$$

2.(b) $1^4 + 2^4 + 3^4 + 4^4 + ... + n^4$
Jawab:

$$x^4 = Ax^{(4)} + Bx^{(3)} + Cx^{(2)} + Dx^{(1)} + E$$
$$x^4 = Ax(x-1)(x-2)(x-3) + Bx(x-1)(x-2) + Cx(x-1) + Dx + E$$

Didapatkan $E-0; D=1; C=7; B=6; A=1$.

$$x^4 = x^{(4)} + 6x^{(3)} + 7x^{(2)} + x^{(1)}$$

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$$\sum_{1}^{n} x^{(4)} + 6x^{(3)} + 7x^{(2)} + x^{(1)} = \Delta^{-1}\, x^{(4)} + 6x^{(3)} + 7x^{(2)} + x^{(1)} \Big|_{0}^{n+1}$$
$$= \frac{1}{5}x^{(5)} + \frac{3}{2}x^{(4)} + \frac{7}{3}x^{(3)} + \frac{1}{2}x^{(2)} \Big|_{0}^{n+1}$$
$$= \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

3. $2 \cdot 5 \cdot 8 + 5 \cdot 8 \cdot 11 + 8 \cdot 11 \cdot 14 + ... + 20 \cdot 23 \cdot 26.$
Jawab:
Suku umum $U_x = (3x-1)(3x+2)(3x+5) = (3x+5)^{(3)}$.

$$\sum_{1}^{7}(3x+5)^{(3)} = \Delta^{-1}(3x+5)^{(3)}\Big|_{1}^{7+1}$$
$$= \frac{(3x+5)^{(4)}}{3(4)}\Big|_{1}^{8}$$
$$= \frac{(3x+5)(3x+2)(3x-1)(3x-4)}{12}\Big|_{1}^{8}$$
$$= \frac{(29)(26)(23)(20)}{12} - \frac{(8)(5)(2)(-1)}{12}$$
$$= \frac{346840+80}{12}$$
$$= \frac{346920}{12}$$
$$= 28910$$

15. $\frac{1}{1\cdot 4} + \frac{1}{4\cdot 7} + \frac{1}{7\cdot 10} + ...$ s/d suku ke-n.
Jawab:
Suku umum $U_x = \frac{1}{(3x-2)(3x+1)} = \frac{1}{(3x+1)^{(2)}} = \frac{1}{(3(x+2)+1-6)^{(2)}} = (3x-5)^{(-2)}$.

$$\sum_{1}^{n}(3x-5)^{(-2)} = \Delta^{-1}(3x-5)^{(-1-1)}\Big|_{0}^{n+1}$$
$$= -\frac{(3x-5)^{(-1)}}{3(1)}\Big|_{0}^{n+1}$$
$$= -\frac{1}{3}\cdot\frac{1}{3(x+1)-5}\Big|_{0}^{n+1}$$
$$= -\frac{1}{3}\cdot\frac{1}{3x-2}\Big|_{0}^{n+1}$$
$$= \frac{1}{9n+3} + \frac{1}{6}$$
$$= \frac{n+1}{6n+2}$$