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1. Cari  $P_4(x)$  dan  $P_5(x)$ .

**Jawab:**

Rumus rekursif Legendre adalah  $P_{n+1}(x) = \frac{2n+1}{n+1}xP_n(x) - \frac{n}{n+1}P_{n-1}(x)$ . Dengan diketahui  $P_2(x) = \frac{1}{2}(3x^2 - 1)$  dan  $P_3(x) = \frac{1}{2}(5x^3 - 3x)$ .

$$\begin{aligned}
 \Rightarrow P_4(x) &= \frac{2(3)+1}{3+1}xP_3(x) - \frac{3}{3+1}P_2(x) \\
 &= \frac{7}{4}x \left( \frac{1}{2}(5x^3 - 3x) \right) - \frac{3}{4} \left( \frac{1}{2}(3x^2 - 1) \right) \\
 &= \frac{7}{8}(5x^4 - 3x^2) - \frac{3}{8}(3x^2 - 1) \\
 &= \frac{35}{8}x^4 - \frac{21}{8}x^2 - \frac{9}{8}x^2 + \frac{3}{8} \\
 &= \frac{35}{8}x^4 - \frac{30}{8}x^2 + \frac{3}{8} \\
 &= \frac{1}{8}(35x^4 - 30x^2 + 3) \\
 \Rightarrow P_5(x) &= \frac{2(4)+1}{4+1}xP_4(x) - \frac{4}{4+1}P_3(x) \\
 &= \frac{9}{5}x \left( \frac{1}{8}(35x^4 - 30x^2 + 3) \right) - \frac{4}{5} \left( \frac{1}{2}(5x^3 - 3x) \right) \\
 &= \frac{9}{40}(35x^5 - 30x^3 + 3x) - \frac{4}{5} \left( \frac{1}{2}(5x^3 - 3x) \right) \\
 &= \frac{1}{8}(63x^5 - 70x^3 + 15x)
 \end{aligned}$$

2. Buktikan  $(1+x^2)P'_n(x) = n[P_{n-1}(x) - xP_n(x)]$ .

**Jawab:**

Diketahui sifat 12 yaitu

$$\begin{aligned}
 nP_n(x) &= xP'_n(x) - P'_{n-1}(x) \\
 nxP_n(x) &= x^2P'_n(x) - xP'_{n-1}(x) \dots (1)
 \end{aligned}$$

Disisi lain

$$\begin{aligned}
 (n+1)P_n(x) &= P'_{n+1}(x) - xP'_n(x) \\
 nP_{n-1}(x) &= P'_n(x) - xP'_{n-1}(x) \\
 xP_{n-1}(x) &= P'_n(x) - P_{n-1}(x) \dots (2)
 \end{aligned}$$

Substitusi persamaan (2) ke persamaan (1) didapat

$$n[P_{n-1}(x) - xP_n(x)] = nxP_n(x) = (1-x^2)P'_n(x)$$

3. Kontruksi polinomial Legendre dari  $f(x) = x^4 + 2x^3 + 2x^2 - x + 3$ .

**Jawab:**

- Koefisien  $x^4$ .

$$\begin{aligned}
 \frac{8}{35}P_4(x) &= \frac{8}{35} \left( \frac{1}{8}(35x^4 - 30x^2 + 3) \right) \\
 &= x^4 - \frac{30}{35}x^2 + \frac{3}{35} = x^4 - \frac{6}{7}x^2 + \frac{3}{35}
 \end{aligned}$$

- Koefisien  $2x^3$ .

$$\begin{aligned}\frac{4}{5}P_3(x) &= \frac{4}{5} \left( \frac{1}{2}(5x^3 - 3x) \right) \\ &= 2x^3 - \frac{6}{5}x\end{aligned}$$

- Koefisien  $2x^2$ . Pertimbangkan Koefisien  $x^2$  pada  $P_4(x)$ .

$$-\frac{6}{7}x^2 + \frac{3k}{2}x^2 = 2x^2 \Rightarrow k = \frac{40}{21}$$

- Koefisien  $-x$ .

$$-\frac{6}{5}x + kx = -x \Rightarrow k = \frac{1}{5}$$

- Konstanta 3

$$\frac{3}{35} - \frac{20}{21}k = 3 \Rightarrow k = \frac{58}{15}$$

$$\therefore f(x) = \frac{8}{35}P_4(x) + \frac{4}{5}P_3(x) - \frac{40}{21}P_4(x) + \frac{1}{5}P_3(x) + \frac{58}{15}P_4(x)$$

4.