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1. Buatlah grup permutasi yang isomorphisma dengan grup $\mathbb{Z}_8$. **Jawab**:

$$\phi_0 = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{pmatrix} = \begin{pmatrix} 1 \end{pmatrix}$$

$$\phi_1 = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{pmatrix}$$

$$\phi_2 = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 3 & 4 & 5 & 6 & 7 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 2 & 4 & 6 \end{pmatrix}\begin{pmatrix} 1 & 3 & 5 & 7 \end{pmatrix}$$

$$\phi_3 = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 4 & 5 & 6 & 7 & 0 & 1 & 2 \end{pmatrix} = \begin{pmatrix} 0 & 3 & 6 & 1 & 4 & 7 & 2 & 5 \end{pmatrix}$$

$$\phi_4 = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 & 0 & 1 & 2 & 3 \end{pmatrix} = \begin{pmatrix} 0 & 4 \end{pmatrix}\begin{pmatrix} 1 & 5 \end{pmatrix}\begin{pmatrix} 2 & 6 \end{pmatrix}\begin{pmatrix} 3 & 7 \end{pmatrix}$$

$$\phi_5 = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 5 & 6 & 7 & 0 & 1 & 2 & 3 & 4 \end{pmatrix} = \begin{pmatrix} 0 & 5 & 2 & 7 & 4 & 1 & 6 & 3 \end{pmatrix}$$

$$\phi_6 = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 6 & 7 & 0 & 1 & 2 & 3 & 4 & 5 \end{pmatrix} = \begin{pmatrix} 0 & 6 & 4 & 2 \end{pmatrix}\begin{pmatrix} 1 & 7 & 5 & 3 \end{pmatrix}$$

$$\phi_7 = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 7 & 0 & 1 & 2 & 3 & 4 & 5 & 6 \end{pmatrix} = \begin{pmatrix} 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 \end{pmatrix}$$

$\therefore \overline{\mathbb{Z}_8} = \{\phi_0, \phi_1, \phi_2, \phi_3, \phi_4, \phi_5, \phi_6, \phi_7\}$

2. Buatlah grup permutasi yang isomorphisma dengan grup $\mathbb{U}(8)$.
Jawab:

$$\phi_1 = \begin{pmatrix} 1 & 3 & 5 & 7 \\ 1 & 3 & 5 & 7 \end{pmatrix} = \begin{pmatrix} 1 \end{pmatrix}$$

$$\phi_3 = \begin{pmatrix} 1 & 3 & 5 & 7 \\ 3 & 1 & 7 & 5 \end{pmatrix} = \begin{pmatrix} 1 & 3 \end{pmatrix}\begin{pmatrix} 5 & 7 \end{pmatrix}$$

$$\phi_5 = \begin{pmatrix} 1 & 3 & 5 & 7 \\ 5 & 7 & 1 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 5 \end{pmatrix}\begin{pmatrix} 3 & 7 \end{pmatrix}$$

$$\phi_7 = \begin{pmatrix} 1 & 3 & 5 & 7 \\ 7 & 5 & 3 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 7 \end{pmatrix}\begin{pmatrix} 3 & 5 \end{pmatrix}$$

$\therefore \overline{\mathbb{U}(8)} = \{\phi_1, \phi_3, \phi_5, \phi_7\}$

3. Buatlah grup permutasi yang isomorphisma dengan grup $G = A, B, C, D$,dimana

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, C = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, D = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

Catatan: Buatlah domain untuk permutasinya A, B, C, D.

Jawab:

$$\phi_A = \begin{pmatrix} \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} & \begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix} & \begin{bmatrix}-1 & 0\\ 0 & 1\end{bmatrix} & \begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix} \\ \\ \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} & \begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix} & \begin{bmatrix}-1 & 0\\ 0 & 1\end{bmatrix} & \begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix} \end{pmatrix} = \begin{pmatrix}A\end{pmatrix}$$

$$\phi_B = \begin{pmatrix} \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} & \begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix} & \begin{bmatrix}-1 & 0\\ 0 & 1\end{bmatrix} & \begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix} \\ \\ \begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix} & \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} & \begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix} & \begin{bmatrix}-1 & 0\\ 0 & 1\end{bmatrix} \end{pmatrix} = \begin{pmatrix}A & B\end{pmatrix}\begin{pmatrix}C & D\end{pmatrix}$$

$$\phi_C = \begin{pmatrix} \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} & \begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix} & \begin{bmatrix}-1 & 0\\ 0 & 1\end{bmatrix} & \begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix} \\ \\ \begin{bmatrix}-1 & 0\\ 0 & 1\end{bmatrix} & \begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix} & \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} & \begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix} \end{pmatrix} = \begin{pmatrix}A & C\end{pmatrix}\begin{pmatrix}B & D\end{pmatrix}$$

$$\phi_D = \begin{pmatrix} \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} & \begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix} & \begin{bmatrix}-1 & 0\\ 0 & 1\end{bmatrix} & \begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix} \\ \\ \begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix} & \begin{bmatrix}-1 & 0\\ 0 & 1\end{bmatrix} & \begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix} & \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \end{pmatrix} = \begin{pmatrix}A & D\end{pmatrix}\begin{pmatrix}B & C\end{pmatrix}$$

$\therefore \overline{G} = \{\phi_A, \phi_B, \phi_C, \phi_D\}$

4. Misalkan $\phi : \mathbb{Z}_{12} \to \mathbb{Z}_3$ adalah suatu homomorpisma dengan
$$\ker(\phi) = \{[0]_{12}, [3]_{12}, [6]_{12}, [9]_{12}\}$$
dan $\phi([4]_{12}) = [2]_3$. Dapatkan semua $x \in \mathbb{Z}_{12}$ yang memenuhi $\phi(x) = [1]_3$, dan tunjukkan bahwa himpunan $\{x \in \mathbb{Z}_{12} \mid \phi(x) = [1]_3\}$ suatu koset dari $\ker(\phi)$ dalam $\mathbb{Z}_{12}$.

Jawab:

Diketahui bahwa $\phi([4]_{12}) = [2]_3$ dan $\ker(\phi) = \{[0]_{12}, [3]_{12}, [6]_{12}, [9]_{12}\}$. Dengan sifat homomorpisma grup didapatkan

- $\phi([7]_{12}) = \phi([4]_{12} + [3]_{12}) = \phi([4]_{12}) + \phi([3]_{12}) = [2]_3 + [0]_3 = [2]_3$
- $\phi([10]_{12}) = \phi([4]_{12} + [6]_{12}) = \phi([4]_{12}) + \phi([6]_{12}) = [2]_3 + [0]_3 = [2]_3$
- $\phi([1]_{12}) = \phi([4]_{12} + [9]_{12}) = \phi([4]_{12}) + \phi([9]_{12}) = [2]_3 + [0]_3 = [2]_3$

Sehingga kita bisa dapatkan himpunan $\{x \in \mathbb{Z}_{12} \mid \phi(x) = [1]_3\}$

- $\phi([2]_{12}) = \phi([1]_{12} + [1]_{12}) = \phi([1]_{12}) + \phi([1]_{12}) = [2]_3 + [2]_3 = [1]_3$

- $\phi([5]_{12}) = \phi([4]_{12} + [1]_{12}) = \phi([4]_{12}) + \phi([1]_{12}) = [2]_3 + [2]_3 = [1]_3$
- $\phi([8]_{12}) = \phi([7]_{12} + [1]_{12}) = \phi([7]_{12}) + \phi([1]_{12}) = [2]_3 + [2]_3 = [1]_3$
- $\phi([11]_{12}) = \phi([10]_{12} + [1]_{12}) = \phi([10]_{12}) + \phi([1]_{12}) = [2]_3 + [2]_3 = [1]_3$

Perhatikan bahwa hal diatas tidak lain adalah salah satu koset dari $\ker(\phi)$ dalam $\mathbb{Z}_{12}$, yaitu $\ker(\phi)_{[2]_{12}}$. **(Terbukti)**

5. Diberikan suatu grup $G, a \in G$ dan $\phi : \mathbb{Z} \to G$ adalah homomorpisma yang diberikan oleh $\phi(n) = a^n, \forall n \in \mathbb{Z}$. Uraikan semua $\ker(\phi)$ yang mungkin.
   Jawab:
   $$\begin{aligned}\ker(\phi) &= \{n \in \mathbb{Z} \mid \phi(n) = e_G\} \\ &= \{n \in \mathbb{Z} \mid a^n = e_G\} \\ &= \{n \in \mathbb{Z} \mid (a^m)^k = e_G \text{ dengan } |a| = m\} \\ &= \{k|a| \mid k \in \mathbb{Z} \text{ dan } a \in G\}\end{aligned}$$

6. Diberikan $f : \mathbb{U}(10) \to \mathbb{Z}_4$ homomorphisma grup dimana dimana $f([3]_{10}) = [3]_4$.
   (a) Tentukan $f([x]_{10})$ untuk setiap $[x]_{10} \in \mathbb{U}(10)$.
   Jawab:
   $f([1]_{10}) = [0]_4$ *Sifat Homgrup*
   $f([3]_{10}) = [3]_4$
   $f([7]_{10}) = f([3 \cdot 3 \cdot 3]_{10}) = [3]_4 + [3]_4 + [3]_4 = [1]_4$
   $f([9]_{10}) = f([3 \cdot 3]_{10}) = [3]_4 + [3]_4 = [2]_4$

   (b) Tentukan $\ker(f)$.
   Jawab:
   Dapat dilihat pada nomor 6a bahwa hanya ada satu anggota $\ker(f)$ yaitu $\{[1]_{10}\}$.

   (c) Apakah f isomorphisma?
   Jawab:
   f akan isomorphisma jika dan hanya jika f homomorphisma yang pemetaannya bijektif.

   Adit. bahwa f injektif dan surjektif.
   f bersifat injektif karena
   - $[0]_4 = [0]_4 \Longrightarrow [1]_{10} = [1]_{10}$
   - $[1]_4 = [1]_4 \Longrightarrow [7]_{10} = [7]_{10}$
   - $[2]_4 = [2]_4 \Longrightarrow [9]_{10} = [9]_{10}$

- $[3]_4 = [1]_4 \Longrightarrow [3]_{10} = [3]_{10}$

f bersifat surjektif karena

- $[0]_4 = f([1]_{10})$
- $[1]_4 = f([7]_{10})$
- $[2]_4 = f([9]_{10})$
- $[3]_4 = f([3]_{10})$

$\therefore$ f merupakan isomorphisma.