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12. Misalkan $r = x\vec{i} + y\vec{j} + z\vec{k}$ dan $r = |r|$ periksalah kebenaran persamaan berikut ini

(a) $\nabla \cdot r = 3$

Jawab:

$$\nabla r = \left(\frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k} \right)\left(x\vec{i} + y\vec{j} + z\vec{k} \right)$$

$$= 1 + 1 + 1 = 3 \ \blacksquare$$

(b) $\nabla^2 r^3 = 12r$

Jawab:

$$\nabla^2 r^3 = \left(\frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k} \right)^2 \left(\sqrt{x^2 + y^2 + z^2} \right)^3$$

$$= \left(\frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k} \right) \left(\frac{\partial \left(x^2 + y^2 + z^2\right)^{3/2}}{\partial x}\vec{i} + \frac{\partial \left(x^2 + y^2 + z^2\right)^{3/2}}{\partial y}\vec{j} + \frac{\partial \left(x^2 + y^2 + z^2\right)^{3/2}}{\partial z}\vec{k} \right)$$

$$= \left(\frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k} \right) \left(3x \left(x^2 + y^2 + z^2\right)^{1/2}\vec{i} + 3y \left(x^2 + y^2 + z^2\right)^{1/2}\vec{j} + 3z \left(x^2 + y^2 + z^2\right)^{1/2}\vec{k} \right)$$

$$= \left(\frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k} \right) \left(3x\vec{i} + 3y\vec{j} + 3z\vec{k} \right)\sqrt{x^2 + y^2 + z^2}$$

$$= \left[(3 + 3 + 3)\sqrt{x^2 + y^2 + z^2} \right] + \left[\frac{3x^2}{\sqrt{x^2 + y^2 + z^2}}\vec{i} + \frac{3y^2}{\sqrt{x^2 + y^2 + z^2}}\vec{j} + \frac{3z^2}{\sqrt{x^2 + y^2 + z^2}}\vec{k} \right]$$

$$= [9r] + \frac{1}{r}\left(3x^2\vec{i} + 3y^2\vec{j} + 3z^2\vec{k} \right)$$

$$= 9r + \frac{3r^2}{r} = 12r \ \blacksquare$$

(c) $\nabla \cdot r\, r = 4r$

Jawab:

$$\nabla \cdot r\, r =$$

(d) $\nabla r = r/r$

(e) $\nabla \left(\dfrac{1}{r} \right) = -r/r^3$

(f) $\nabla \times r = 0$

(g) $\nabla \ln r = r/r^2$

(h) $\nabla r\, f(r) = 3f(r) + |r|\dfrac{df}{dr}$