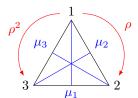# Tugas Aljabar I

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1. (a) Tentukan semua elemen dari $D3$.

$$\rho_0 = \begin{pmatrix} 1 \end{pmatrix}$$
$$\rho = \begin{pmatrix} 1 & 2 & 3 \end{pmatrix}$$
$$\rho^2 = \begin{pmatrix} 1 & 3 & 2 \end{pmatrix}$$
$$\mu_1 = \mu = \begin{pmatrix} 2 & 3 \end{pmatrix}$$
$$\mu_2 = \rho\mu = \begin{pmatrix} 1 & 2 \end{pmatrix}$$
$$\mu_3 = \rho^2\mu = \begin{pmatrix} 1 & 3 \end{pmatrix}$$
$$D_3 = \{\rho_0, \rho, \rho^2, \mu_1, \mu_2, \mu_3\}$$

(b) Buatlah tabel dari $D3$.

$\circ$	ρ_0	ρ	ρ^2	μ_1	μ_2	μ_3
ρ_0	ρ_0	ρ	ρ^2	μ_1	μ_2	μ_3
ρ	ρ	ρ^2	ρ_0	μ_2	μ_3	μ_1
ρ^2	ρ^2	ρ_0	ρ	μ_3	μ_1	μ_2
μ_1	μ_1	μ_3	μ_2	ρ_0	ρ^2	ρ
μ_2	μ_2	μ_1	μ_3	ρ	ρ_0	ρ^2
μ_3	μ_3	μ_2	μ_1	ρ^2	ρ	ρ_0

Tabel komposisi

(c) Dari tabel tentukan $(\rho\mu)^{-1}$ dan $(\rho^2\mu)^{-1}$.

$$(\rho\mu)^{-1} = (\mu_2)^{-1} = \mu_2 = \rho\mu$$
$$(\rho^2\mu)^{-1} = (\mu_3)^{-1} = \mu_3 = \rho^2\mu$$

2. $(D_5, \circ)$ grup dehidral.

$f, g, h, i \in D_4$

$f = \rho\mu$

$g = \rho^3$

$h = \rho^2\mu$

$i = \rho^3\mu$

(i) Tentukan k dimana

    ⓐ $f \circ g = \rho^k\mu$

$$(\rho\mu)\rho^3 = \rho^k\mu$$
$$(\mu\rho^3)\rho^3 = \rho^k\mu$$
$$\mu(\rho^3\rho^3) = \rho^k\mu$$
$$\mu\rho^2 = \rho^k\mu$$
$$\rho^2\mu = \rho^k\mu$$
$$\therefore k = 2$$

    ⓑ $g \circ f = \rho^k\mu$

$$\rho^3(\rho\mu) = \rho^k\mu$$
$$\rho_0\mu = \rho^k\mu$$
$$\therefore k = 0$$

    ⓒ $h \circ i = \rho^k\mu$

$$(\rho^2\mu)(\rho^3\mu) = \rho^k\mu$$
$$\rho^2(\mu\rho^3)\mu = \rho^k\mu$$
$$\rho^2(\rho\mu)\mu = \rho^k\mu$$
$$(\rho^2\rho)(\mu\mu) = \rho^k\mu$$
$$(\rho^3)(\mu\mu)\mu^{-1} = \rho^k\mu\mu^{-1}$$
$$\rho^3\mu = \rho^k$$

$\therefore$ tidak ada k yang memenuhi

    ⓓ $i \circ h = \rho^k\mu$

$$(\rho^3\mu)(\rho^2\mu) = \rho^k\mu$$
$$\rho^3(\mu\rho^2)\mu = \rho^k\mu$$
$$\rho^3(\rho^2\mu)\mu = \rho^k\mu$$
$$(\rho^3\rho^2)(\mu\mu) = \rho^k\mu$$
$$(\rho)(\mu\mu)\mu^{-1} = \rho^k\mu\mu^{-1}$$
$$\rho\mu = \rho^k$$

$$\therefore \text{tidak ada } k \text{ yang memenuhi}$$

(ii) Tentukan h^{-1}, g^{-1}

$$
\begin{aligned}
h^{-1} &= (\rho^2 \mu)^{-1} \\
&= (\mu)^{-1} (\rho^2)^{-1} \\
&= \mu \rho^2 \\
&= \rho^2 \mu \\
g^{-1} &= (\rho^3)^{-1} \\
&= \rho
\end{aligned}
$$

3. $f, g \in S_7$ dimana

$$
f = \begin{pmatrix} 1 & 3 & 4 \end{pmatrix} \begin{pmatrix} 2 & 5 & 7 & 6 \end{pmatrix}
$$
$$
g = \begin{pmatrix} 2 & 3 & 5 \end{pmatrix} \begin{pmatrix} 1 & 4 & 7 \end{pmatrix}
$$

Nyatakan hasil berikut dalam komposisi sikel yang saling asing.

ⓐ $f \circ g$

$$
\begin{pmatrix} 1 & 3 & 4 \end{pmatrix} \begin{pmatrix} 2 & 5 & 7 & 6 \end{pmatrix} \circ \begin{pmatrix} 2 & 3 & 5 \end{pmatrix} \begin{pmatrix} 1 & 4 & 7 \end{pmatrix}
$$
$$
= \begin{pmatrix} 1 \end{pmatrix} \begin{pmatrix} 2 & 4 & 6 \end{pmatrix} \begin{pmatrix} 3 & 7 \end{pmatrix} \begin{pmatrix} 5 \end{pmatrix}
$$
$$
= \begin{pmatrix} 2 & 4 & 6 \end{pmatrix} \begin{pmatrix} 3 & 7 \end{pmatrix}
$$

ⓑ $g \circ f$

$$
\begin{pmatrix} 2 & 3 & 5 \end{pmatrix} \begin{pmatrix} 1 & 4 & 7 \end{pmatrix} \circ \begin{pmatrix} 1 & 3 & 4 \end{pmatrix} \begin{pmatrix} 2 & 5 & 7 & 6 \end{pmatrix}
$$
$$
= \begin{pmatrix} 1 & 5 \end{pmatrix} \begin{pmatrix} 2 \end{pmatrix} \begin{pmatrix} 3 & 7 & 6 \end{pmatrix} \begin{pmatrix} 4 \end{pmatrix}
$$
$$
= \begin{pmatrix} 1 & 5 \end{pmatrix} \begin{pmatrix} 3 & 7 & 6 \end{pmatrix}
$$

ⓒ Apakah $f \circ g = g \circ f$?

Tidak, karena fakta bahwa sikel-sikel dalam f dan g tidak saling asing, sehingga dapat dilihat dari ⓐ dan ⓑ bahwa $f \circ g \neq g \circ f$.