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Deret Fourier

$f(x)$ fungsi periodik pada interval $-L \leq x \leq L$ dimana $f(x+2L) = f(x)$. Sehingga $f(x)$ dapat ditulis sebagai berikut

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right) \right]$$

dimana

$$a_0 = \frac{1}{L} \int_{-L}^{L} f(x)\, dx \tag{1}$$

$$a_n = \frac{1}{L} \int_{-L}^{L} f(x) \cos\left(\frac{n\pi x}{L}\right)\, dx \tag{2}$$

$$b_n = \frac{1}{L} \int_{-L}^{L} f(x) \sin\left(\frac{n\pi x}{L}\right)\, dx \tag{3}$$

1(g). $f(x) = \begin{cases} 0, & -\pi \leq x \leq 0 \\ x^2, & 0 < x \leq \pi \end{cases}, \ f(x+2\pi) = f(x)$

Jawab:

$$\begin{aligned} a_0 &= \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\, dx \\ &= \frac{1}{\pi} \left[\int_{-\pi}^{0} f(x)\, dx + \int_{0}^{\pi} f(x)\, dx \right] \\ &= \frac{1}{\pi} \left[0 + \int_{0}^{\pi} x^2\, dx \right] \\ &= \frac{1}{\pi} \left[\frac{1}{3} x^3 \right]_0^{\pi} = \frac{\pi^2}{3} \end{aligned}$$

$$\begin{aligned} a_n &= \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos\left(\frac{n\cancel{\pi} x}{\cancel{\pi}}\right)\, dx \\ &= \frac{1}{\pi} \left[\int_{-\pi}^{0} f(x) \cos(nx)\, dx + \int_{0}^{\pi} f(x) \cos(nx)\, dx \right] \\ &= \frac{1}{\pi} \left[0 + \int_{0}^{\pi} x^2 \cos(nx)\, dx \right] \\ &= \frac{1}{\pi} \left[\frac{x^2}{n} \sin(nx) + \frac{2x}{n^2} \cos(nx) - \frac{2}{n^3} \sin(nx) \right]_0^{\pi} \\ &= \frac{1}{n^3 \pi} \left[\underbrace{n^2 x^2 \sin(nx)}_{0} + 2nx \cos(nx) - \underbrace{2 \sin(nx)}_{0} \right]_0^{\pi} \\ &= \frac{1}{n^3 \pi} \left[2n\pi \cos(n\pi) - 0 \right] \\ &= \frac{2}{n^2} \cos(n\pi) = \begin{cases} - \frac{2}{n^2}, & n \text{ ganjil} \\ \frac{2}{n^2}, & n \text{ genap} \end{cases} \end{aligned}$$

$$
\begin{aligned}
b_n &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin\left(\frac{n\cancel{\pi}x}{\cancel{\pi}}\right)\,dx\\
&= \frac{1}{\pi}\left[\int_{-\pi}^{0} f(x)\sin(nx)\,dx + \int_{0}^{\pi} f(x)\sin(nx)\,dx\right]\\
&= \frac{1}{\pi}\left[0 + \int_{0}^{\pi} x^2\sin(nx)\,dx\right]\\
&= \frac{1}{\pi}\left[-\frac{x^2}{n}\cos(nx) + \frac{2x}{n^2}\sin(nx) + \frac{2}{n^3}\cos(nx)\right]_0^{\pi}\\
&= \frac{1}{n^3\pi}\left[-n^2x^2\cos(nx) + \underbrace{2nx\sin(nx)}_{0} + 2\cos(nx)\right]_0^{\pi}\\
&= \frac{1}{n^3\pi}\left[-n^2\pi^2\cos(n\pi) + 0 + 2\cos(n\pi) - (-0+0+2)\right]\\
&= \frac{1}{n^3\pi}\left[-n^2\pi^2\cos(n\pi) + 2\cos(n\pi) - 2\right]\\
&= \begin{cases} \frac{\pi}{n} - \frac{4}{n^3\pi}, & n \text{ ganjil}\\ \frac{\pi}{n}, & n \text{ genap}\end{cases}
\end{aligned}
$$

Sehingga deret fourier-nya adalah

$$
\begin{aligned}
f(x) = \frac{\pi^2}{6} &+ \sum_{n=1}^{\infty}\left[-\frac{2}{(2n-1)^2}\cos((2n-1)x) + \left(\frac{\pi}{2n-1} - \frac{4}{(2n-1)^3\pi}\right)\sin((2n-1)x)\right]\\
&+ \sum_{m=1}^{\infty}\left[\frac{1}{2m^2}\cos(2mx) + \frac{\pi}{2m}\sin(2mx)\right]
\end{aligned}
$$

1(l). $f(x) = \begin{cases} 2-x, & 0 \le x \le 4\\ x-6, & 4 < x \le 8\end{cases}$, $f(x+8) = f(x)$

Jawab:

$$
\begin{aligned}
a_0 &= \frac{1}{4}\int_0^8 f(x)\,dx\\
&= \frac{1}{4}\left[\int_0^4 f(x)\,dx + \int_4^8 f(x)\,dx\right]\\
&= \frac{1}{4}\left[\int_0^4 2-x\,dx + \int_4^8 x-6\,dx\right]\\
&= \frac{1}{4}\left[2x - \frac{1}{2}x^2\Big|_0^4 + \frac{1}{2}x^2 - 6x\Big|_4^8\right]\\
&= \frac{1}{4}\left[(8-8-0) + (-16+16)\right] = 0
\end{aligned}
$$

$$
\begin{aligned}
a_n &= \frac{1}{4}\int_0^8 f(x)\cos\left(\frac{n\pi x}{4}\right)\,dx \\
&= \frac{1}{4}\left[\int_0^4 (2-x)\cos\left(\frac{n\pi x}{4}\right)\,dx + \int_4^8 (x-6)\cos\left(\frac{n\pi x}{4}\right)\,dx\right] \\
&= \frac{1}{4}\left[\underbrace{\int_0^4 2\cos\left(\frac{n\pi x}{4}\right)\,dx}_{0} - \int_0^4 x\cos\left(\frac{n\pi x}{4}\right)\,dx + \int_4^8 x\cos\left(\frac{n\pi x}{4}\right)\,dx - \underbrace{\int_4^8 6\cos\left(\frac{n\pi x}{4}\right)\,dx}_{0}\right] \\
&= \frac{1}{4}\left[\int_4^8 x\cos\left(\frac{n\pi x}{4}\right)\,dx - \int_0^4 x\cos\left(\frac{n\pi x}{4}\right)\,dx\right] \\
&= \frac{1}{4}\left[\left.\underbrace{\frac{4x}{n\pi}\sin\left(\frac{n\pi x}{4}\right)}_{0} + \frac{16}{n^2\pi^2}\cos\left(\frac{n\pi x}{4}\right)\right|_4^8 + \left.\underbrace{\frac{4x}{n\pi}\sin\left(\frac{n\pi x}{4}\right)}_{0} + \frac{16}{n^2\pi^2}\cos\left(\frac{n\pi x}{4}\right)\right|_4^0\right] \\
&= \frac{1}{4}\left[\left.\frac{16}{n^2\pi^2}\cos\left(\frac{n\pi x}{4}\right)\right|_4^8 + \left.\frac{16}{n^2\pi^2}\cos\left(\frac{n\pi x}{4}\right)\right|_4^0\right] \\
&= \frac{4}{n^2\pi^2}\left[\left.\cos\left(\frac{n\pi x}{4}\right)\right|_4^8 + \left.\cos\left(\frac{n\pi x}{4}\right)\right|_4^0\right] \\
&= \frac{4}{n^2\pi^2}\left[\underbrace{\cos(2n\pi)}_{1} - \cos(n\pi) + \cos(0) - \cos(n\pi)\right] \\
&= \frac{4}{n^2\pi^2}\left[2 - 2\cos(n\pi)\right] = \begin{cases} \frac{16}{n^2\pi^2} & , \quad n \text{ ganjil} \\ 0 & , \quad n \text{ genap} \end{cases}
\end{aligned}
$$

$$
\begin{aligned}
b_n &= \frac{1}{4}\int_0^8 f(x)\sin\left(\frac{n\pi x}{4}\right)\,dx\\
&= \frac{1}{4}\left[\int_0^4 (2-x)\sin\left(\frac{n\pi x}{4}\right)\,dx + \int_4^8 (x-6)\sin\left(\frac{n\pi x}{4}\right)\,dx\right]\\
&= \frac{1}{4}\left[\int_0^4 2\sin\left(\frac{n\pi x}{4}\right)\,dx - \int_0^4 x\sin\left(\frac{n\pi x}{4}\right)\,dx + \int_4^8 x\sin\left(\frac{n\pi x}{4}\right)\,dx - \int_4^8 6\sin\left(\frac{n\pi x}{4}\right)\,dx\right]\\
&= \frac{1}{4}\left[-\frac{8}{n\pi}\cos\left(\frac{n\pi x}{4}\right)\Big|_0^4 + \frac{24}{n\pi}\cos\left(\frac{n\pi x}{4}\right)\Big|_4^8 - \int_0^4 x\sin\left(\frac{n\pi x}{4}\right)\,dx + \int_4^8 x\sin\left(\frac{n\pi x}{4}\right)\,dx\right]\\
&= \frac{1}{4}\left[\frac{16}{n\pi} + \frac{48}{n\pi} - \int_0^4 x\sin\left(\frac{n\pi x}{4}\right)\,dx + \int_4^8 x\sin\left(\frac{n\pi x}{4}\right)\,dx\right]\\
&= \frac{1}{4}\left[\frac{64}{n\pi} - \left(-\frac{4x}{n\pi}\cos\left(\frac{n\pi x}{4}\right) + \underbrace{\frac{16}{n^2\pi^2}\sin\left(\frac{n\pi x}{4}\right)}_{0}\Bigg|_0^4\right) - \frac{4x}{n\pi}\cos\left(\frac{n\pi x}{4}\right) + \underbrace{\frac{16}{n^2\pi^2}\sin\left(\frac{n\pi x}{4}\right)}_{0}\Bigg|_4^8\right]\\
&= \frac{1}{4}\left[\frac{64}{n\pi} + \frac{4x}{n\pi}\cos\left(\frac{n\pi x}{4}\right)\Big|_0^4 - \frac{4x}{n\pi}\cos\left(\frac{n\pi x}{4}\right)\Big|_4^8\right]\\
&= \frac{1}{4}\left[\frac{64}{n\pi} + \frac{16}{n\pi}\cos(n\pi) - 0 - \frac{32}{n\pi} + \frac{16}{n\pi}\cos(n\pi)\right]\\
&= \frac{1}{4}\left[\frac{32}{n\pi} + \frac{32}{n\pi}\cos(n\pi)\right]\\
&= \frac{8}{n\pi} + \frac{8}{n\pi}\cos(n\pi) = \begin{cases} 0 & , \quad n \text{ ganjil} \\ \frac{16}{n\pi} & , \quad n \text{ genap} \end{cases}
\end{aligned}
$$

Sehingga deret fourier-nya adalah

$$
\begin{aligned}
f(x) &= \sum_{n=1}^{\infty} \frac{16}{(2n-1)\pi^2}\cos\left(\frac{(2n-1)\pi x}{4}\right) + \sum_{n=1}^{\infty} \frac{16}{2n\pi}\sin\left(\frac{n\pi x}{2}\right)\\
&= \frac{16}{\pi^2}\sum_{n=1}^{\infty} \frac{1}{2n-1}\cos\left(\frac{(2n-1)\pi x}{4}\right) + \frac{8}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(\frac{n\pi x}{2}\right)
\end{aligned}
$$

2. Hasil dari penderetan menurut Fourier $f(x) = \begin{cases} 0, & -\pi \leq x \leq 0 \\ 1, & 0 < x \leq \pi \end{cases}$, $f(x+2\pi) = f(x)$ untuk, $0 < x < \pi$ adalah

$$
f(x) = \frac{1}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\sin(2n-1)x}{2n-1}
$$

untuk nilai x berapakah, sehingga dapat ditunjukkan nilai deret berikut ini adalah benar

$$
\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots
$$

Jawab:

$$\boxed{x = \frac{\pi}{2}}$$

$$\begin{aligned}
f\left(\frac{\pi}{2}\right) &= \frac{1}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\sin\left(\frac{(2n-1)\pi}{2}\right)}{2n-1} \\
1 &= \frac{1}{2} + \frac{2}{\pi}\left(\sin\left(\frac{\pi}{2}\right) + \frac{1}{3}\sin\left(\frac{3\pi}{2}\right) + \frac{1}{5}\sin\left(\frac{5\pi}{2}\right) + \dots\right) \\
\frac{1}{2} &= \frac{2}{\pi}\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots\right) \\
\frac{\pi}{4} &= 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots \quad \blacksquare
\end{aligned}$$

(+). $f(x) = \begin{cases} \cos x, & 0 \le x \le \pi \\ 0, & \pi < x \le 2\pi \end{cases}, \ f(x + 2\pi) = f(x)$

Jawab:

$$\begin{aligned}
a_0 &= \frac{1}{\pi}\int_0^{2\pi} f(x)\,dx \\
&= \frac{1}{\pi}\left[\int_0^{\pi} f(x)\,dx + \int_{\pi}^{2\pi} f(x)\,dx\right] \\
&= \frac{1}{\pi}\left[\int_0^{\pi} \cos(x)\,dx + 0\right] = 0
\end{aligned}$$

$$\begin{aligned}
a_n &= \frac{1}{\pi}\int_0^{2\pi} f(x)\cos\left(\frac{n\cancel{\pi}x}{\cancel{\pi}}\right)\,dx \\
&= \frac{1}{\pi}\left[\int_0^{\pi} f(x)\cos(nx)\,dx + \int_{\pi}^{2\pi} f(x)\cos(nx)\,dx\right] \\
&= \frac{1}{\pi}\left[\int_0^{\pi} \cos(x)\cos(nx)\,dx + 0\right] \\
&= \frac{1}{\pi}\int_0^{\pi} \cos(x)\cos(nx)\,dx = \begin{cases} \frac{1}{2}, & n = 1 \\ 0, & n \text{ yang lain} \end{cases}
\end{aligned}$$

$$
\begin{aligned}
b_n &= \frac{1}{\pi}\int_0^{2\pi} f(x)\sin\left(\frac{n\cancel{\pi}x}{\cancel{\pi}}\right)\,dx \\
&= \frac{1}{\pi}\left[\int_0^{\pi} f(x)\sin(nx)\,dx + \int_{\pi}^{2\pi} f(x)\sin(nx)\,dx\right] \\
&= \frac{1}{\pi}\left[\int_0^{\pi} \cos(x)\sin(nx)\,dx + 0\right] \\
&= \frac{1}{2\pi}\left[\int_0^{\pi} \sin(nx+x)+\sin(nx-x)\,dx\right] \\
&= \frac{1}{2\pi}\left[\int_0^{\pi} \sin((n+1)x)+\sin((n-1)x)\,dx\right] \\
&= \frac{1}{2\pi}\left[\frac{-\cos((n+1)x)}{n+1} - \frac{\cos((n-1)x)}{n-1}\right]_0^{\pi} \\
&= \frac{1}{2\pi}\left[-\frac{\cos((n+1)\pi)}{n+1} - \frac{\cos((n-1)\pi)}{n-1} + \frac{1}{n+1} + \frac{1}{n-1}\right] \\
&= \frac{1}{2\pi}\left[-\frac{(n-1)\cos((n+1)\pi)-(n+1)\cos((n-1)\pi)+2n}{n^2-1}\right] \\
&= \begin{cases} 0 & , \quad n \text{ ganjil} \\ \frac{2n}{(n^2-1)\pi} & , \quad n \text{ genap} \end{cases}
\end{aligned}
$$

Sehingga deret fourier-nya adalah

$$f(x) = \frac{1}{2}\cos(x) + \sum_{n=1}^{\infty} \frac{4n}{(4n^2-1)\pi}\sin(2nx)$$

6. Diberikan fungsi $f(x) = e^{-x}$

   (a) Dapatkan deret fourier dari $f(x)$ untuk $-\pi \leq x \leq \pi$.
   Jawab:

$$
\begin{aligned}
a_0 &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx \\
&= \frac{1}{\pi}\int_{-\pi}^{\pi} e^{-x}\,dx \\
&= \frac{1}{\pi}\left[-e^{-x}\right]_{-\pi}^{\pi} \\
&= \frac{1}{\pi}\left[-e^{-\pi}+e^{\pi}\right] \\
&= \frac{1}{\pi}\left[e^{\pi}-e^{-\pi}\right]
\end{aligned}
$$

$$
\begin{aligned}
a_n &= \frac{1}{\pi}\int_{-\pi}^{\pi} e^{-x}\cos\left(\frac{n\cancel{\pi}x}{\cancel{\pi}}\right)\,dx \\
&= \frac{1}{\pi}\int_{-\pi}^{\pi} e^{-x}\cos\left(nx\right)\,dx \\
&= \frac{1}{\pi}\left[\frac{ne^{-x}\sin(nx) - e^{-x}\cos(nx)}{n^2+1}\right]_{-\pi}^{\pi} \\
&= \frac{1}{(n^2+1)\pi}\left[ne^{-x}\sin(nx) - e^{-x}\cos(nx)\right]_{-\pi}^{\pi} \\
&= \frac{1}{(n^2+1)\pi}\left[ne^{-\pi}\sin(n\pi) - e^{-\pi}\cos(n\pi) - (ne^{\pi}\sin(-n\pi) - e^{\pi}\cos(-n\pi))\right] \\
&= \frac{1}{(n^2+1)\pi}\left[0 - e^{-\pi}\cos(n\pi) - (0 - e^{\pi}\cos(-n\pi))\right] \\
&= \frac{1}{(n^2+1)\pi}\left[e^{\pi}\cos(n\pi) - e^{-\pi}\cos(n\pi)\right] \\
&= \frac{e^{\pi} - e^{-\pi}}{(n^2+1)\pi}(-1)^n
\end{aligned}
$$

$$
\begin{aligned}
b_n &= \frac{1}{\pi}\int_{-\pi}^{\pi} e^{-x}\sin\left(\frac{n\cancel{\pi}x}{\cancel{\pi}}\right)\,dx \\
&= \frac{1}{\pi}\int_{-\pi}^{\pi} e^{-x}\sin\left(nx\right)\,dx \\
&= \frac{1}{\pi}\left[\frac{-ne^{-x}\cos(nx) - e^{-x}\sin(nx)}{n^2+1}\right]_{-\pi}^{\pi} \\
&= \frac{1}{(n^2+1)\pi}\left[-ne^{-x}\cos(nx) - e^{-x}\sin(nx)\right]_{-\pi}^{\pi} \\
&= \frac{1}{(n^2+1)\pi}\left[-ne^{-\pi}\cos(n\pi) - e^{-\pi}\sin(n\pi) - (-ne^{\pi}\cos(n-\pi) - e^{\pi}\sin(n-\pi))\right] \\
&= \frac{1}{(n^2+1)\pi}\left[-ne^{-\pi}\cos(n\pi) - 0 - (-ne^{\pi}\cos(-n\pi) - 0)\right] \\
&= \frac{1}{(n^2+1)\pi}\left[ne^{\pi}\cos(n\pi) - ne^{-\pi}\cos(n\pi)\right] \\
&= \frac{e^{\pi} - e^{-\pi}}{(n^2+1)\pi}(-1)^n n
\end{aligned}
$$

$\therefore$ Deret fouriernya adalah

$$
f(x) = \frac{e^{\pi} - e^{-\pi}}{2\pi} + \frac{e^{\pi} - e^{-\pi}}{\pi}\sum_{n=1}^{\infty}\left(\frac{(-1)^n}{n^2+1}\right)\cos(nx) + n\sin(nx)
$$

(b) Ekspansikan $f(x)$ ke dalam deret cosinus fourier untuk $0 \le x \le \pi$.

$$
\begin{aligned}
a_0 &= \frac{2}{\pi}\int_{0}^{\pi} e^{-x}\,dx \\
&= \frac{2}{\pi}\left[-e^{-x}\right]_{0}^{\pi} \\
&= \frac{2}{\pi}\left[-e^{-\pi} + e^{0}\right] \\
&= \frac{2(1 - e^{-\pi})}{\pi}
\end{aligned}
$$

$$
\begin{aligned}
a_n &= \frac{2}{\pi}\int_0^{\pi} e^{-x}\cos\left(\frac{n\cancel{\pi}x}{\frac{\cancel{\pi}}{2}}\right)\,dx \\
&= \frac{2}{\pi}\int_0^{\pi} e^{-x}\cos\left(2nx\right)\,dx \\
&= \frac{2}{\pi}\left[\frac{2ne^{-x}\sin(2nx)-e^{-x}\cos(2nx)}{4n^2+1}\right]_0^{\pi} \\
&= \frac{2}{(4n^2+1)\pi}\left[2ne^{-x}\sin(2nx)-e^{-x}\cos(2nx)\right]_0^{\pi} \\
&= \frac{2}{(4n^2+1)\pi}\left[2ne^{-\pi}\sin(2n\pi)-e^{-\pi}\cos(2n\pi)-(2ne^{0}\sin(0)-e^{0}\cos(0))\right] \\
&= \frac{2}{(4n^2+1)\pi}\left[0-e^{-\pi}\cos(2n\pi)-(0-1)\right] \\
&= \frac{2}{(4n^2+1)\pi}\left[1-e^{-\pi}\cos(2n\pi)\right] \\
&= \frac{2}{(4n^2+1)\pi}\left[1-e^{-\pi}\right] \\
&= \frac{2(1-e^{-\pi})}{(4n^2+1)\pi}
\end{aligned}
$$

$\therefore$ Deret fouriernya adalah

$$f(x) = \frac{1-e^{-\pi}}{\pi} + \frac{2(1-e^{-\pi})}{\pi}\sum_{n=1}^{\infty}\frac{1}{4n^2+1}\cos(2nx)$$

(c) Ekspansikan $f(x)$ ke dalam deret sinus fourier untuk $0 \le x < \pi$.

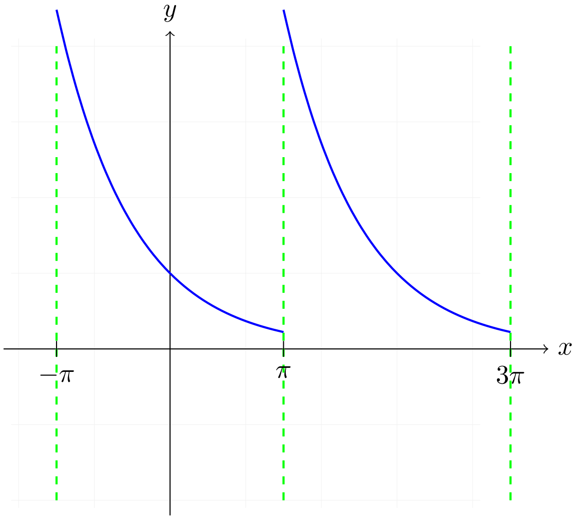
$$
\begin{aligned}
b_n &= \frac{2}{\pi}\int_0^{\pi} e^{-x}\sin\left(\frac{n\cancel{\pi}x}{\frac{\cancel{\pi}}{2}}\right)\,dx \\
&= \frac{2}{\pi}\int_0^{\pi} e^{-x}\sin\left(2nx\right)\,dx \\
&= \frac{2}{\pi}\left[\frac{-2ne^{-x}\cos(2nx)-e^{-x}\sin(2nx)}{4n^2+1}\right]_0^{\pi} \\
&= \frac{2}{(4n^2+1)\pi}\left[-2ne^{-x}\cos(2nx)-e^{-x}\sin(2nx)\right]_0^{\pi} \\
&= \frac{2}{(4n^2+1)\pi}\left[-2ne^{-\pi}\cos(2n\pi)-e^{-\pi}\sin(2n\pi)-(-2ne^{0}\cos(0)-e^{0}\sin(0))\right] \\
&= \frac{2}{(4n^2+1)\pi}\left[-2ne^{-\pi}-0-(-2n-0)\right] \\
&= \frac{2}{(4n^2+1)\pi}\left[2n-2ne^{-\pi}\right] \\
&= \frac{4n(1-e^{-\pi})}{(4n^2+1)\pi}
\end{aligned}
$$

$\therefore$ Deret fouriernya adalah

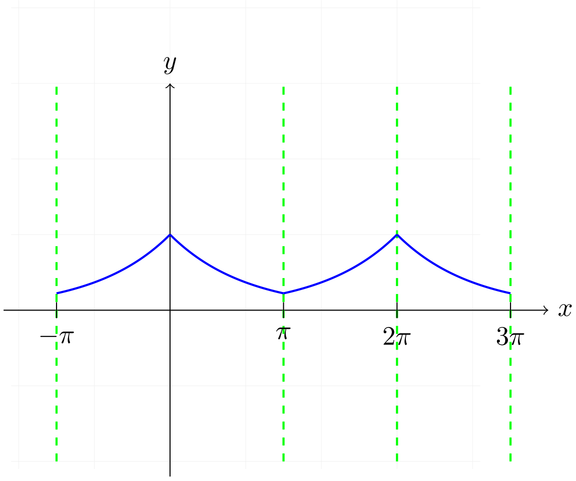
$$f(x) = \frac{4(1-e^{-\pi})}{\pi}\sum_{n=1}^{\infty}\frac{n}{4n^2+1}\sin(2nx)$$

(d) Gambar fungsi (a), (b), dan (c) untuk 2 gelombang.

(a) $f(x)$ untuk $-\pi \le x \le \pi$.



(b) Deret cosinus fourier untuk $0 \le x \le \pi$.



(c) Deret sinus fourier untuk $0 \le x \le \pi$.