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3. Kembang $f(x) = \begin{cases} 2x+1 & ; \quad 0 < x \leq 1 \\ 0 & ; \quad -1 \leq x < 0 \end{cases}$ pada $a_n P_n(x)$.

Jawab:

$$\boxed{a_n = \frac{2n+1}{2} \int_{-1}^{1} f(x) P_0(x)\ dx}$$

$$\begin{aligned}
a_0 &= \frac{1}{2} \int_{-1}^{1} f(x) P_0(x) dx \\
&= \frac{1}{2} \int_{-1}^{0} 0\, dx + \frac{1}{2} \int_{0}^{1} (2x+1)\, dx \\
&= \frac{1}{2} \left[x^2 + x\right]_0^1 = 1 \\
a_1 &= \frac{3}{2} \int_{-1}^{1} f(x) P_1(x) dx \\
&= \frac{3}{2} \int_{-1}^{0} 0\, dx + \frac{3}{2} \int_{0}^{1} (2x+1)x\, dx \\
&= \frac{3}{2} \left[\frac{2}{3}x^3 + \frac{1}{2}x^2\right]_0^1 = \frac{3}{2}\left[\frac{7}{6}\right] = \frac{7}{4} \\
a_2 &= \frac{5}{2} \int_{-1}^{1} f(x) P_2(x) dx \\
&= \frac{5}{2} \int_{-1}^{0} 0\, dx + \frac{5}{2} \int_{0}^{1} (2x+1)\left(\frac{3}{2}x^2 - \frac{1}{2}\right) dx \\
&= \frac{5}{2} \int_{0}^{1} (6x^3 + 3x^2 - 2x - 1)\, dx \\
&= \frac{5}{2} \left[\frac{3}{2}x^4 + x^3 - x^2 - x\right]_0^1 = \frac{5}{2}\left[\frac{1}{2} + \frac{1}{2}\right] = \frac{5}{8} \\
a_3 &= \frac{7}{2} \int_{-1}^{1} f(x) P_3(x) dx \\
&= \frac{7}{2} \int_{-1}^{0} 0\, dx + \frac{7}{2} \int_{0}^{1} (2x+1)\left(\frac{5}{2}x^3 - \frac{3}{2}x\right) dx \\
&= \frac{7}{4} \int_{0}^{1} (10x^4 + 5x^3 - 6x^2 - 3x)\, dx \\
&= \frac{7}{4} \left[2x^5 + \frac{5}{4}x^4 - 2x^3 - \frac{3}{2}x^2\right]_0^1 = \frac{7}{4}\left[2 + \frac{5}{4} - 2 - \frac{3}{2}\right] = -\frac{7}{8}
\end{aligned}$$

$$\therefore\ f(x) = \frac{1}{2}P_0(x) + \frac{7}{4}P_1(x) + \frac{5}{8}P_2(x) - \frac{7}{8}P_3(x)$$

4. Nyatakan $J_{\frac{5}{2}}(x) = \sqrt{\frac{2}{\pi x}} \left(\frac{3 - x^2}{x} \sin x - \frac{3}{x} \cos x\right)$.

Jawab:

$$
\begin{aligned}
\frac{2n}{x} J_n(x) &= J_{n-1}(x) + J_{n+1}(x) \\
\frac{3}{x} J_{\frac{3}{2}}(x) &= J_{\frac{1}{2}}(x) + J_{\frac{5}{2}}(x) \\
J_{\frac{5}{2}}(x) &= \frac{3}{x} J_{\frac{3}{2}}(x) - J_{\frac{1}{2}}(x) \\
&= \frac{3}{x} \left(\sqrt{\frac{2}{\pi x}} \left(\frac{\sin x}{x} - \cos x \right) \right) - \sqrt{\frac{2}{\pi x}} \sin x \\
&= \sqrt{\frac{2}{\pi x}} \left(\frac{3 - x^2}{x} \sin x - \frac{3}{x} \cos x \right)
\end{aligned}
$$