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1. $U_{n+2} - 3U_{n+1} + 2U_n = 0$
Jawab:

$$U_{n+2} - 3U_{n+1} + 2U_n = 0$$
$$E^2U_n - 3EU_n + 2U_n = 0$$
$$(E^2 - 3E + 2)U_n = 0$$

Subtitusi $U_n = p^n$, didapatkan persamaan

$$p^2 - 3p + 2 = 0$$
$$(p-2)(p-1) = 0$$
$$p = 2 \vee p = 1$$

$\therefore$ PUPB: $U_n = C_1 + C_2(2^n)$

5. $y_{n+2} - 7y_{n+1} + 12y_n = 0$
Jawab:

$$y_{n+2} - 7y_{n+1} + 12y_n = 0$$
$$E^2y_n - 7Ey_n + 12y_n = 0$$
$$(E^2 - 7E + 12)y_n = 0$$

Subtitusi $y_n = p^n$, didapatkan persamaan

$$p^2 - 7p + 12 = 0$$
$$(p-3)(p-4) = 0$$
$$p = 3 \vee p = 4$$

$\therefore$ PUPB: $y_n = C_1(3^n) + C_2(4^n)$

7. $\Delta^2 y_n - 7\Delta y_n + 12y_n = 0$
Jawab:

$$\Delta^2 y_n - 7\Delta y_n + 12y_n = 0 = 0$$
$$(\Delta^2 - 7\Delta + 12)y_n = 0$$
$$(\Delta - 3)(\Delta - 4)y_n = 0$$

Subtitusi $y_n = p^n$ dan $\Delta = E - 1$, didapatkan persamaan

$$(p-4)(p-5) = 0$$
$$p = 4 \vee p = 5$$

$\therefore$ PUPB: $y_n = C_1(4^n) + C_2(5^n)$

15. $y_{n+2} - 3y_{n+1} + 2y_n = 2^n$
Jawab:

$$y_c = C_1 + C_2(2^n)$$
$$y_p = \frac{1}{(E-2)(E-1)}2^n = \frac{1}{E-2}\frac{2^n}{2-1} = \frac{1}{E-2}2^n = 2^n\frac{1}{2E-2}1$$
$$= 2^n\frac{1}{2(\Delta+1)-2}1 = 2^{n-1}\frac{1}{\Delta}1 = n2^{n-1}$$

$\therefore$ PUPB: $y_n = y_c + y_p = C_1 + C_2(2^n) + n2^{n-1}$

22. $(\Delta^2 + \Delta + 1)U_n = n^2$

Jawab:

Solusi komplementer:

$$y_c : \Delta^2 + \Delta + 1 = (E-1)^2 + (E-1) + 1 = E^2 - E + 1$$
$$\Rightarrow p^2 - p + 1 = 0$$
$$\Rightarrow p = \frac{1 \pm \sqrt{1^2 - 4(1)(1)}}{2(1)} = \frac{1 \pm \sqrt{-3}}{2} = \frac{1 \pm \sqrt{3}i}{2}$$

Karena akar bernilai kompleks dengan $r = |p| = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = 1$ dan $\theta = \tan^{-1}\left(\frac{\sqrt{3}/2}{1/2}\right) = \tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$, diperoleh

$$y_c = r^n(C_1 \cos(n\theta) + C_2 \sin(n\theta))$$
$$y_c = \boxed{C_1 \cos\left(\frac{n\pi}{3}\right) + C_2 \sin\left(\frac{n\pi}{3}\right)}$$

Solusi partikulir:

$$\begin{aligned} y_p = \frac{1}{\Delta^2 + \Delta + 1} n^2 &= (1 - \Delta + \Delta^3 - \Delta^4 + ...)(n^{(2)} + n^{(1)}) \\ &= (1 - \Delta)(n^{(2)} + n^{(1)}) \\ &= n^{(2)} + n^{(1)} - \Delta(n^{(2)} + n^{(1)}) \\ &= n^2 - (2n^{(1)} + 1) \\ &= \boxed{n^2 - 2n - 1} \end{aligned}$$

$\therefore$ PUPB: $y_n = y_p + y_c = \boxed{n^2 - 2n - 1 + C_1 \cos\left(\frac{n\pi}{3}\right) + C_2 \sin\left(\frac{n\pi}{3}\right)}$

4. $$f_n = \{2, 3^2, 3^2, 4^2, ...\}$$

Jawab:

$U_n = n^2, n = 1, 2, 3, ..$

$$Z\{n^2\} = \sum_0^\infty (n+1)^2 z^{-n} =$$