

Nama	: Teosofi Hidayah Agung
NRP	: 5002221132

1. Tunjukkan bahwa:

(a) $\nabla \times (r^2 \vec{r}) = 0$

Jawab:

$$\begin{aligned}
 \nabla \times (r^2 \vec{r}) &= \nabla \times (r^2 x \vec{i} + r^2 y \vec{j} + r^2 z \vec{k}) \\
 &= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ r^2 x & r^2 y & r^2 z \end{vmatrix} \\
 &= \left(\frac{\partial}{\partial y}(r^2 z) - \frac{\partial}{\partial z}(r^2 y) \right) \vec{i} - \left(\frac{\partial}{\partial x}(r^2 z) - \frac{\partial}{\partial z}(r^2 x) \right) \vec{j} + \left(\frac{\partial}{\partial x}(r^2 y) - \frac{\partial}{\partial y}(r^2 x) \right) \vec{k} \\
 &= (0 - 0) \vec{i} - (0 - 0) \vec{j} + (0 - 0) \vec{k} \\
 &= 0 \vec{i} + 0 \vec{j} + 0 \vec{k} \\
 &= 0 \blacksquare
 \end{aligned}$$

(b) $\nabla \cdot \vec{r} f(r) = 3f(r) + |r| \frac{df}{dr}$

Jawab:

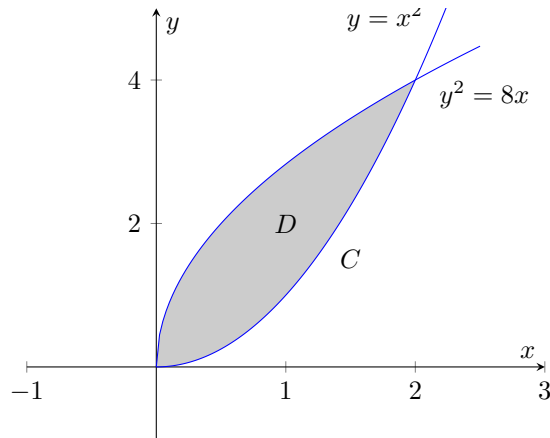
$$\begin{aligned}
 \nabla \cdot \vec{r} f(r) &= (\nabla \cdot \vec{r}) f(r) + \vec{r} \cdot (\nabla f(r)) \\
 &= 3f(r) + \vec{r} \cdot \left(\frac{\partial f}{\partial x} \vec{i} + \frac{\partial f}{\partial y} \vec{j} + \frac{\partial f}{\partial z} \vec{k} \right) \\
 &= 3f(r) + \vec{r} \cdot \left(\frac{\partial f}{\partial r} \frac{\partial r}{\partial x} \vec{i} + \frac{\partial f}{\partial r} \frac{\partial r}{\partial y} \vec{j} + \frac{\partial f}{\partial r} \frac{\partial r}{\partial z} \vec{k} \right) \\
 &= 3f(r) + \vec{r} \cdot \left(\frac{\partial r}{\partial x} \vec{i} + \frac{\partial r}{\partial y} \vec{j} + \frac{\partial r}{\partial z} \vec{k} \right) \frac{\partial f}{\partial r} \\
 &= 3f(r) + \vec{r} \cdot (\nabla r) \frac{\partial f}{\partial r} \\
 &= 3f(r) + \vec{r} \cdot \left(\frac{\vec{r}}{r} \right) \frac{\partial f}{\partial r} \\
 &= 3f(r) + |r| \frac{df}{dr} \blacksquare
 \end{aligned}$$

2. Diberikan integral berikut $\oint_C (2xy - x^2)dx + (x + y^2)dy$ jika C adalah kurva tertutup yang dibatasi oleh $y = x^2$, dan $8x = y^2$.

(a) Hitung integral tersebut dengan menggunakan teorema Green.

Jawab:

Daerah D yang dibatasi kurva C adalah sebagai berikut:



Sehingga nilai integralnya dengan menggunakan teorema Green adalah

$$\begin{aligned}
 \oint_C (2xy - x^2)dx + (x + y^2)dy &= \iint_D \left(\frac{\partial}{\partial x}(x + y^2) - \frac{\partial}{\partial y}(2xy - x^2) \right) dA \\
 &= \iint_D (1 - 2x) dA \\
 &= \int_0^2 \int_{2\sqrt{2}\sqrt{x}}^{x^2} (1 - 2x) dy dx \\
 &= \int_0^2 [y - 2xy]_{2\sqrt{2}\sqrt{x}}^{x^2} dx \\
 &= \int_0^2 [x^2 - 2x(x^2) - 2\sqrt{2}\sqrt{x} + 4x\sqrt{2}\sqrt{x}] dx \\
 &= \int_0^2 [x^2 - 2x^3 - 2\sqrt{2}\sqrt{x} + 4\sqrt{2}x\sqrt{x}] dx \\
 &= \int_0^2 [x^2 - 2x^3 - 2\sqrt{2}\sqrt{x} + 4\sqrt{2}x^{3/2}] dx \\
 &= \left[\frac{x^3}{3} - \frac{x^4}{2} - \frac{4\sqrt{2}}{3}x^{3/2} + \frac{8\sqrt{2}}{5}x^{5/2} \right]_0^2 \\
 &= \frac{8}{3} - 8 - \frac{16}{3} + \frac{64}{5} \\
 &= \frac{32}{15}
 \end{aligned}$$

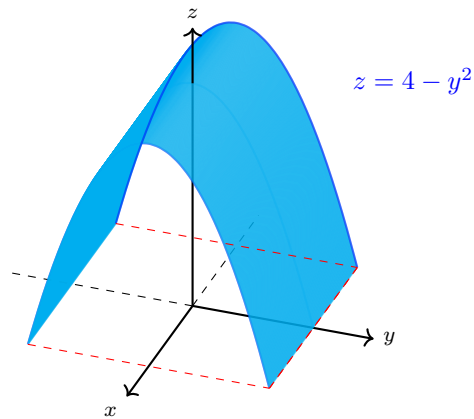
(b) Hitung integral tersebut secara langsung tanpa menggunakan teorema Green.

Jawab:

3. Diberikan vektor gaya $\vec{A} = (2xy + z)\vec{i} + y^2\vec{j} + (x + 3y)\vec{k}$ dan S adalah permukaan yang dibatasi oleh $z = 4 - y^2$, $x = 0$, $x = 2$ dan bidang xy .

(a) Gambarkan geometri batasan permukaan benda.

Jawab:



(b) Hitunglah $\iint_S \vec{A} \cdot \vec{n} \, dS$ dengan menggunakan teorema Gauss. **Jawab:**

$$\begin{aligned}
 \iint_S \vec{A} \cdot \vec{n} \, dS &= \iiint_V \nabla \cdot \vec{A} \, dV \\
 &= \iiint_V \left(\frac{\partial}{\partial x}(2xy + z) + \frac{\partial}{\partial y}(y^2) + \frac{\partial}{\partial z}(x + 3y) \right) dV \\
 &= \iiint_V (2y + 1) dV \\
 &= \int_0^2 \int_0^2 \int_0^{4-y^2} (2y + 1) dz \, dy \, dx
 \end{aligned}$$

(c) Hitunglah $\iint_S \vec{A} \cdot \vec{n} \, dS$ secara langsung tanpa menggunakan teorema Gauss.