Tugas Aljabar I

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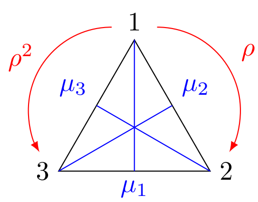


$$\begin{aligned}
\rho_0 &= \begin{pmatrix}1\end{pmatrix} \\
\rho &= \begin{pmatrix}1 & 2 & 3\end{pmatrix} \\
\rho^2 &= \begin{pmatrix}1 & 3 & 2\end{pmatrix} \\
\mu_1 &= \begin{pmatrix}2 & 3\end{pmatrix} \\
\mu_2 &= \begin{pmatrix}1 & 2\end{pmatrix} \\
\mu_3 &= \begin{pmatrix}1 & 3\end{pmatrix} \\
D_3 &= \{\rho_0, \rho_1, \rho_2, \mu_1, \mu_2, \mu_3\}
\end{aligned}$$

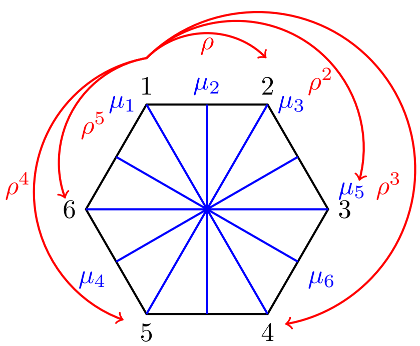
Komposisi:

- $\rho_0 \circ \rho_0 = \begin{pmatrix}1\end{pmatrix} \circ \begin{pmatrix}1\end{pmatrix} = \begin{pmatrix}1\end{pmatrix} = \rho_0$
- $\rho_0 \circ \rho = \begin{pmatrix}1\end{pmatrix} \circ \begin{pmatrix}1 & 2 & 3\end{pmatrix} = \begin{pmatrix}1 & 2 & 3\end{pmatrix} = \rho$
- $\rho_0 \circ \rho^2 = \begin{pmatrix}1\end{pmatrix} \circ \begin{pmatrix}1 & 3 & 2\end{pmatrix} = \begin{pmatrix}1 & 3 & 2\end{pmatrix} = \rho^2$
- $\rho_0 \circ \mu_1 = \begin{pmatrix}1\end{pmatrix} \circ \begin{pmatrix}2 & 3\end{pmatrix} = \begin{pmatrix}2 & 3\end{pmatrix} = \mu_1$
- $\rho_0 \circ \mu_2 = \begin{pmatrix}1\end{pmatrix} \circ \begin{pmatrix}1 & 2\end{pmatrix} = \begin{pmatrix}1 & 2\end{pmatrix} = \mu_2$
- $\rho_0 \circ \mu_3 = \begin{pmatrix}1\end{pmatrix} \circ \begin{pmatrix}1 & 3\end{pmatrix} = \begin{pmatrix}1 & 3\end{pmatrix} = \mu_1$
- $\rho \circ \rho_0 = \begin{pmatrix}1 & 2 & 3\end{pmatrix} \circ \begin{pmatrix}1\end{pmatrix} = \begin{pmatrix}1 & 2 & 3\end{pmatrix} = \rho$
- $\rho \circ \rho = \begin{pmatrix}1 & 2 & 3\end{pmatrix} \circ \begin{pmatrix}1 & 2 & 3\end{pmatrix} = \begin{pmatrix}1 & 3 & 2\end{pmatrix} = \rho^2$
- $\rho \circ \rho^2 = \begin{pmatrix}1 & 2 & 3\end{pmatrix} \circ \begin{pmatrix}1 & 3 & 2\end{pmatrix} = \begin{pmatrix}1\end{pmatrix} = \rho_0$
- $\rho \circ \mu_1 = \begin{pmatrix}1 & 2 & 3\end{pmatrix} \circ \begin{pmatrix}2 & 3\end{pmatrix} = \begin{pmatrix}1 & 2\end{pmatrix} = \mu_2$

- $\rho \circ \mu_2 = \begin{pmatrix} 1 & 2 & 3 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 3 \end{pmatrix} = \mu_3$
- $\rho \circ \mu_3 = \begin{pmatrix} 1 & 2 & 3 \end{pmatrix} \circ \begin{pmatrix} 1 & 3 \end{pmatrix} = \begin{pmatrix} 2 & 3 \end{pmatrix} = \mu_1$
- $\rho^2 \circ \rho_0 = \begin{pmatrix} 1 & 3 & 2 \end{pmatrix} \circ \begin{pmatrix} 1 \end{pmatrix} = \begin{pmatrix} 1 & 3 & 2 \end{pmatrix} = \rho^2$
- $\rho^2 \circ \rho = \begin{pmatrix} 1 & 3 & 2 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 \end{pmatrix} = \begin{pmatrix} 1 \end{pmatrix} = \rho_0$
- $\rho^2 \circ \rho^2 = \begin{pmatrix} 1 & 3 & 2 \end{pmatrix} \circ \begin{pmatrix} 1 & 3 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \end{pmatrix} = \rho$
- $\rho^2 \circ \mu_1 = \begin{pmatrix} 1 & 3 & 2 \end{pmatrix} \circ \begin{pmatrix} 2 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 3 \end{pmatrix} = \mu_3$
- $\rho^2 \circ \mu_2 = \begin{pmatrix} 1 & 3 & 2 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 \end{pmatrix} = \begin{pmatrix} 2 & 3 \end{pmatrix} = \mu_1$
- $\rho^2 \circ \mu_1 = \begin{pmatrix} 1 & 3 & 2 \end{pmatrix} \circ \begin{pmatrix} 1 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 2 \end{pmatrix} = \mu_2$
- $\mu_1 \circ \rho_0 = \begin{pmatrix} 2 & 3 \end{pmatrix} \circ \begin{pmatrix} 1 \end{pmatrix} = \begin{pmatrix} 2 & 3 \end{pmatrix} = \mu_1$
- $\mu_1 \circ \rho = \begin{pmatrix} 2 & 3 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 \end{pmatrix} = \begin{pmatrix} 2 & 3 \end{pmatrix} = \mu_3$
- $\mu_1 \circ \rho^2 = \begin{pmatrix} 2 & 3 \end{pmatrix} \circ \begin{pmatrix} 1 & 3 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 2 \end{pmatrix} = \mu_2$
- $\mu_1 \circ \mu_1 = \begin{pmatrix} 2 & 3 \end{pmatrix} \circ \begin{pmatrix} 2 & 3 \end{pmatrix} = \begin{pmatrix} 1 \end{pmatrix} = \rho_0$
- $\mu_1 \circ \mu_2 = \begin{pmatrix} 2 & 3 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 3 & 2 \end{pmatrix} = \rho^2$
- $\mu_1 \circ \mu_3 = \begin{pmatrix} 2 & 3 \end{pmatrix} \circ \begin{pmatrix} 1 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \end{pmatrix} = \rho$
- $\mu_2 \circ \rho_0 = \begin{pmatrix} 1 & 2 \end{pmatrix} \circ \begin{pmatrix} 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 \end{pmatrix} = \mu_2$
- $\mu_2 \circ \rho = \begin{pmatrix} 1 & 2 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 \end{pmatrix} = \begin{pmatrix} 2 & 3 \end{pmatrix} = \mu_1$
- $\mu_2 \circ \rho^2 = \begin{pmatrix} 1 & 2 \end{pmatrix} \circ \begin{pmatrix} 1 & 3 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 3 \end{pmatrix} = \mu_3$
- $\mu_2 \circ \mu_1 = \begin{pmatrix} 1 & 2 \end{pmatrix} \circ \begin{pmatrix} 2 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \end{pmatrix} = \rho$
- $\mu_2 \circ \mu_2 = \begin{pmatrix} 1 & 2 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 \end{pmatrix} = \begin{pmatrix} 1 \end{pmatrix} = \rho_0$
- $\mu_2 \circ \mu_3 = \begin{pmatrix} 1 & 2 \end{pmatrix} \circ \begin{pmatrix} 1 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 3 & 2 \end{pmatrix} = \rho^2$
- $\mu_3 \circ \rho_0 = \begin{pmatrix} 1 & 3 \end{pmatrix} \circ \begin{pmatrix} 1 \end{pmatrix} = \begin{pmatrix} 1 & 3 \end{pmatrix} = \mu_3$
- $\mu_3 \circ \rho = \begin{pmatrix} 1 & 3 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 2 \end{pmatrix} = \mu_2$
- $\mu_3 \circ \rho^2 = \begin{pmatrix} 1 & 3 \end{pmatrix} \circ \begin{pmatrix} 1 & 3 & 2 \end{pmatrix} = \begin{pmatrix} 2 & 3 \end{pmatrix} = \mu_1$
- $\mu_3 \circ \mu_1 = \begin{pmatrix} 1 & 3 \end{pmatrix} \circ \begin{pmatrix} 2 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 3 & 2 \end{pmatrix} = \rho^2$
- $\mu_3 \circ \mu_2 = \begin{pmatrix} 1 & 3 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \end{pmatrix} = \rho$
- $\mu_3 \circ \mu_3 = \begin{pmatrix} 1 & 3 \end{pmatrix} \circ \begin{pmatrix} 1 & 3 \end{pmatrix} = \begin{pmatrix} 1 \end{pmatrix} = \rho_0$

$\circ$	ρ_0	ρ	ρ^2	μ_1	μ_2	μ_3
ρ_0	ρ_0	ρ	ρ^2	μ_1	μ_2	μ_3
ρ	ρ	ρ^2	ρ_0	μ_2	μ_3	μ_1
ρ^2	ρ^2	ρ_0	ρ	μ_3	μ_1	μ_2
μ_1	μ_1	μ_3	μ_2	ρ_0	ρ^2	ρ
μ_2	μ_2	μ_1	μ_3	ρ	ρ_0	ρ^2
μ_3	μ_3	μ_2	μ_1	ρ^2	ρ	ρ_0

Tabel komposisi



1. Tentukan anggota D_6?

   Jawab:

   - $\rho_0 = \begin{pmatrix}1\end{pmatrix}$
   - $\rho = \begin{pmatrix}1 & 2 & 3 & 4 & 5 & 6\end{pmatrix}$
   - $\rho^2 = \begin{pmatrix}1 & 3 & 5\end{pmatrix}\begin{pmatrix}2 & 4 & 6\end{pmatrix}$
   - $\rho^3 = \begin{pmatrix}1 & 4\end{pmatrix}\begin{pmatrix}2 & 5\end{pmatrix}\begin{pmatrix}3 & 6\end{pmatrix}$
   - $\rho^4 = \begin{pmatrix}1 & 5 & 3\end{pmatrix}\begin{pmatrix}2 & 6 & 4\end{pmatrix}$
   - $\rho^5 = \begin{pmatrix}6 & 5 & 4 & 3 & 2 & 1\end{pmatrix}$
   - $\mu_1 = \begin{pmatrix}2 & 6\end{pmatrix}\begin{pmatrix}3 & 5\end{pmatrix}$
   - $\mu_2 = \begin{pmatrix}1 & 2\end{pmatrix}\begin{pmatrix}3 & 6\end{pmatrix}\begin{pmatrix}4 & 5\end{pmatrix}$
   - $\mu_3 = \begin{pmatrix}1 & 3\end{pmatrix}\begin{pmatrix}4 & 6\end{pmatrix}$
   - $\mu_4 = \begin{pmatrix}2 & 3\end{pmatrix}\begin{pmatrix}1 & 4\end{pmatrix}\begin{pmatrix}5 & 6\end{pmatrix}$
   - $\mu_5 = \begin{pmatrix}1 & 5\end{pmatrix}\begin{pmatrix}2 & 4\end{pmatrix}$
   - $\mu_6 = \begin{pmatrix}3 & 4\end{pmatrix}\begin{pmatrix}2 & 5\end{pmatrix}\begin{pmatrix}1 & 6\end{pmatrix}$

2. Tunjukkan bahwa

   Jawab:

   $$\bullet \rho \circ \mu_1 = \begin{pmatrix}1 & 2 & 3 & 4 & 5 & 6\end{pmatrix} \circ \begin{pmatrix}2 & 6\end{pmatrix}\begin{pmatrix}3 & 5\end{pmatrix}$$
   $$= \begin{pmatrix}1 & 2\end{pmatrix}\begin{pmatrix}3 & 6\end{pmatrix}\begin{pmatrix}4 & 5\end{pmatrix} = \mu_2$$
   $$\bullet \rho^2 \circ \mu_1 = \begin{pmatrix}1 & 3 & 5\end{pmatrix}\begin{pmatrix}2 & 4 & 6\end{pmatrix} \circ \begin{pmatrix}2 & 6\end{pmatrix}\begin{pmatrix}3 & 5\end{pmatrix}$$
   $$= \begin{pmatrix}1 & 3\end{pmatrix}\begin{pmatrix}4 & 6\end{pmatrix} = \mu_3$$
   $$\bullet \rho^3 \circ \mu_1 = \begin{pmatrix}1 & 4\end{pmatrix}\begin{pmatrix}2 & 5\end{pmatrix}\begin{pmatrix}3 & 6\end{pmatrix} \circ \begin{pmatrix}2 & 6\end{pmatrix}\begin{pmatrix}3 & 5\end{pmatrix}$$
   $$= \begin{pmatrix}2 & 3\end{pmatrix}\begin{pmatrix}1 & 4\end{pmatrix}\begin{pmatrix}5 & 6\end{pmatrix} = \mu_4$$
   $$\bullet \rho^4 \circ \mu_1 = \begin{pmatrix}1 & 5 & 3\end{pmatrix}\begin{pmatrix}2 & 6 & 4\end{pmatrix} \circ \begin{pmatrix}2 & 6\end{pmatrix}\begin{pmatrix}3 & 5\end{pmatrix}$$
   $$= \begin{pmatrix}1 & 5\end{pmatrix}\begin{pmatrix}2 & 4\end{pmatrix} = \mu_5$$
   $$\bullet \rho^5 \circ \mu_1 = \begin{pmatrix}6 & 5 & 4 & 3 & 2 & 1\end{pmatrix} \circ \begin{pmatrix}2 & 6\end{pmatrix}\begin{pmatrix}3 & 5\end{pmatrix}$$
   $$= \begin{pmatrix}3 & 4\end{pmatrix}\begin{pmatrix}2 & 5\end{pmatrix}\begin{pmatrix}1 & 6\end{pmatrix} = \mu_6$$

3. Tentukan k sehingga $\rho^2\mu_1 = \mu_1\rho^k$.

   Jawab:
   Dengan sebuah teorema maka didapat $k = 4$, buktinya:
   $$\rho^2\mu_1 = \begin{pmatrix}1 & 3 & 5\end{pmatrix}\begin{pmatrix}2 & 4 & 6\end{pmatrix} \circ \begin{pmatrix}2 & 6\end{pmatrix}\begin{pmatrix}3 & 5\end{pmatrix}$$
   $$= \begin{pmatrix}1 & 3\end{pmatrix}\begin{pmatrix}4 & 6\end{pmatrix}$$
   $$\mu_1\rho^4 = \begin{pmatrix}2 & 6\end{pmatrix}\begin{pmatrix}3 & 5\end{pmatrix} \circ \begin{pmatrix}1 & 5 & 3\end{pmatrix}\begin{pmatrix}2 & 6 & 4\end{pmatrix}$$
   $$= \begin{pmatrix}1 & 3\end{pmatrix}\begin{pmatrix}4 & 6\end{pmatrix}$$
   $\therefore k = 4$ memenuhi persamaan diatas.

4. Tentukan l sehingga $\rho^3\mu_1 = \mu_1\rho^l$.

   Jawab:
   Dengan sebuah teorema maka didapat $l = 3$, buktinya:
   $$\rho^3\mu_1 = \begin{pmatrix}1 & 4\end{pmatrix}\begin{pmatrix}2 & 5\end{pmatrix}\begin{pmatrix}3 & 6\end{pmatrix} \circ \begin{pmatrix}2 & 6\end{pmatrix}\begin{pmatrix}3 & 5\end{pmatrix}$$
   $$= \begin{pmatrix}2 & 3\end{pmatrix}\begin{pmatrix}1 & 4\end{pmatrix}\begin{pmatrix}5 & 6\end{pmatrix}$$
   $$\mu_1\rho^3 = \begin{pmatrix}2 & 6\end{pmatrix}\begin{pmatrix}3 & 5\end{pmatrix} \circ \begin{pmatrix}1 & 4\end{pmatrix}\begin{pmatrix}2 & 5\end{pmatrix}\begin{pmatrix}3 & 6\end{pmatrix}$$
   $$= \begin{pmatrix}2 & 3\end{pmatrix}\begin{pmatrix}1 & 4\end{pmatrix}\begin{pmatrix}5 & 6\end{pmatrix}$$
   $\therefore l = 3$ memenuhi persamaan diatas.

5. Tentukan invers dari setiap elemen pada D_6.

   Jawab:

   - $(\rho_0)^{-1} = \rho_0$

- $(\rho)^{-1} = \rho^5$
- $(\rho^2)^{-1} = \rho^4$
- $(\rho^3)^{-1} = \rho^3$
- $(\rho^4)^{-1} = \rho^2$
- $(\rho^5)^{-1} = \rho$
- $(\mu_1)^{-1} = \mu_1$
- $(\mu_2)^{-1} = \mu_2$
- $(\mu_3)^{-1} = \mu_3$
- $(\mu_4)^{-1} = \mu_4$
- $(\mu_5)^{-1} = \mu_5$
- $(\mu_6)^{-1} = \mu_6$

6. $f, g \in S_7$.
   $f = \begin{pmatrix}1 & 2 & 3\end{pmatrix}\begin{pmatrix}4 & 5\end{pmatrix}$
   $g = \begin{pmatrix}2 & 3 & 1\end{pmatrix}\begin{pmatrix}4 & 5 & 7\end{pmatrix}$

   (a) Buatlah f dan g dalam permutasi
   Jawab:

   $$f = \begin{pmatrix}1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 3 & 1 & 5 & 4 & 6 & 7\end{pmatrix}$$
   $$g = \begin{pmatrix}1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 3 & 1 & 5 & 7 & 6 & 4\end{pmatrix}$$

   (b) Tentukan $f \circ g$ dan $g \circ f$
   Jawab:

   $$\bullet f \circ g = \begin{pmatrix}1 & 2 & 3\end{pmatrix}\begin{pmatrix}4 & 5\end{pmatrix} \circ \begin{pmatrix}2 & 3 & 1\end{pmatrix}\begin{pmatrix}4 & 5 & 7\end{pmatrix}$$
   $$= \begin{pmatrix}1 & 3 & 2\end{pmatrix}\begin{pmatrix}5 & 7\end{pmatrix}$$
   $$\bullet g \circ f = \begin{pmatrix}2 & 3 & 1\end{pmatrix}\begin{pmatrix}4 & 5 & 7\end{pmatrix} \circ \begin{pmatrix}1 & 2 & 3\end{pmatrix}\begin{pmatrix}4 & 5\end{pmatrix}$$
   $$= \begin{pmatrix}1 & 3 & 2\end{pmatrix}\begin{pmatrix}4 & 7\end{pmatrix}$$

   (c) Tentukan f^{-1} dan g^{-1}
   Jawab:

   $$f^{-1} = \begin{pmatrix}3 & 2 & 1\end{pmatrix}\begin{pmatrix}4 & 5\end{pmatrix}$$
   $$g^{-1} = \begin{pmatrix}1 & 3 & 2\end{pmatrix}\begin{pmatrix}7 & 5 & 4\end{pmatrix}$$

Dapat kita cek kembali

$f \circ f^{-1} = \begin{pmatrix} 1 & 2 & 3 \end{pmatrix} \begin{pmatrix} 4 & 5 \end{pmatrix} \circ \begin{pmatrix} 3 & 2 & 1 \end{pmatrix} \begin{pmatrix} 4 & 5 \end{pmatrix} = \begin{pmatrix} 1 \end{pmatrix}$

$f^{-1} \circ f = \begin{pmatrix} 3 & 2 & 1 \end{pmatrix} \begin{pmatrix} 4 & 5 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 \end{pmatrix} \begin{pmatrix} 4 & 5 \end{pmatrix} = \begin{pmatrix} 1 \end{pmatrix}$

$g \circ g^{-1} = \begin{pmatrix} 2 & 3 & 1 \end{pmatrix} \begin{pmatrix} 4 & 5 & 7 \end{pmatrix} \circ \begin{pmatrix} 1 & 3 & 2 \end{pmatrix} \begin{pmatrix} 7 & 5 & 4 \end{pmatrix} = \begin{pmatrix} 1 \end{pmatrix}$

$g^{-1} \circ g = \begin{pmatrix} 1 & 3 & 2 \end{pmatrix} \begin{pmatrix} 7 & 5 & 4 \end{pmatrix} \circ \begin{pmatrix} 2 & 3 & 1 \end{pmatrix} \begin{pmatrix} 4 & 5 & 7 \end{pmatrix} = \begin{pmatrix} 1 \end{pmatrix}$