

Nama	: Teosofi Hidayah Agung
NRP	: 5002221132

1.  $U_{n+2} - 3U_{n+1} + 2U_n = 0$

**Jawab:**

$$U_{n+2} - 3U_{n+1} + 2U_n = 0$$

$$E^2U_n - 3EU_n + 2U_n = 0$$

$$(E^2 - 3E + 2)U_n = 0$$

Substitusi  $U_n = p^n$ , didapatkan persamaan

$$p^2 - 3p + 2 = 0$$

$$(p - 2)(p - 1) = 0$$

$$p = 2 \vee p = 1$$

$$\therefore \text{PUPB: } U_n = C_1 + C_2(2^n)$$

5.  $y_{n+2} - 7y_{n+1} + 12y_n = 0$

**Jawab:**

$$y_{n+2} - 7y_{n+1} + 12y_n = 0$$

$$E^2y_n - 7Ey_n + 12y_n = 0$$

$$(E^2 - 7E + 12)y_n = 0$$

Substitusi  $y_n = p^n$ , didapatkan persamaan

$$p^2 - 7p + 12 = 0$$

$$(p - 3)(p - 4) = 0$$

$$p = 3 \vee p = 4$$

$$\therefore \text{PUPB: } y_n = C_1(3^n) + C_2(4^n)$$

7.  $\Delta^2 y_n - 7\Delta y_n + 12y_n = 0$

**Jawab:**

$$\Delta^2 y_n - 7\Delta y_n + 12y_n = 0 = 0$$

$$(\Delta^2 - 7\Delta + 12)y_n = 0$$

$$(\Delta - 3)(\Delta - 4)y_n = 0$$

Substitusi  $y_n = p^n$  dan  $\Delta = E - 1$ , didapatkan persamaan

$$(p - 4)(p - 5) = 0$$

$$p = 4 \vee p = 5$$

$$\therefore \text{PUPB: } y_n = C_1(4^n) + C_2(5^n)$$

15.  $y_{n+2} - 3y_{n+1} + 2y_n = 2^n$

**Jawab:**

$$y_c = C_1 + C_2(2^n)$$

$$\begin{aligned} y_p &= \frac{1}{(E-2)(E-1)} 2^n = \frac{1}{E-2} \frac{2^n}{2-1} = \frac{1}{E-2} 2^n = 2^n \frac{1}{2E-2} 1 \\ &= 2^n \frac{1}{2(\Delta+1)-2} 1 = 2^{n-1} \frac{1}{\Delta} 1 = n2^{n-1} \end{aligned}$$

$$\therefore \text{PUPB: } y_n = y_c + y_p = C_1 + C_2(2^n) + n2^{n-1}$$

22.  $(\Delta^2 + \Delta + 1)U_n = n^2$

**Jawab:**

Solusi komplementer:

$$y_c : \Delta^2 + \Delta + 1 = (E - 1)^2 + (E - 1) + 1 = E^2 - E + 1$$

$$\Rightarrow p^2 - p + 1 = 0$$

$$\Rightarrow p = \frac{1 \pm \sqrt{1^2 - 4(1)(1)}}{2(1)} = \frac{1 \pm \sqrt{-3}}{2} = \frac{1 \pm \sqrt{3}i}{2}$$

Karena akar bernilai kompleks dengan  $r = |p| = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = 1$  dan  $\theta = \tan^{-1}\left(\frac{\sqrt{3}/2}{1/2}\right) =$

$$\tan^{-1}(\sqrt{3}) = \frac{\pi}{3}, \text{ diperoleh}$$

$$y_c = r^n (C_1 \cos(n\theta) + C_2 \sin(n\theta))$$

$$y_c = \boxed{C_1 \cos\left(\frac{n\pi}{6}\right) + C_2 \sin\left(\frac{n\pi}{6}\right)}$$

Solusi partikular:

$$y_p = \frac{1}{\Delta^2 + \Delta + 1} n^2 = (1 - \Delta + \Delta^3 - \Delta^4 + \dots)(n^{(2)} + n^{(1)})$$

$$= (1 - \Delta)(n^{(2)} + n^{(1)})$$

$$= n^{(2)} + n^{(1)} - \Delta(n^{(2)} + n^{(1)})$$

$$= n^2 - (2n^{(1)} + 1)$$

$$= \boxed{n^2 - 2n - 1}$$

$$\therefore \text{PUPB: } y_n = y_p + y_c = \boxed{n^2 - 2n - 1 + C_1 \cos\left(\frac{n\pi}{6}\right) + C_2 \sin\left(\frac{n\pi}{6}\right)}$$