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### Operator Beda Maju

Misalkan ada fungsi  $f$  yang nilainya  $f(t)$  pada waktu  $t$  dan bernilai  $f(t+1)$  pada waktu  $(t+1)$ , maka beda pertama didefinisikan sebagai berikut:

$$\Delta f(t) = f(t+1) - f(t)$$

2. Buktikan  $\Delta \sin(a+bx) = 2 \sin\left(\frac{b}{2}\right) \cos\left(a + \frac{b}{2} + bx\right)$

**Jawab:**

$$\begin{aligned}\Delta \sin(a+bx) &= \sin(a+b(x+1)) - \sin(a+bx) \\ &= \sin(a+bx+b) - \sin(a+bx)\end{aligned}$$

Ingat  $\sin(x) - \sin(y) = 2 \sin\left(\frac{x-y}{2}\right) \cos\left(\frac{x+y}{2}\right)$ , Sehingga

$$\begin{aligned}\sin(a+bx+b) - \sin(a+bx) &= 2 \sin\left(\frac{a+bx+b-(a+bx)}{2}\right) \cos\left(\frac{a+bx+b+(a+bx)}{2}\right) \\ &= 2 \sin\left(\frac{b}{2}\right) \cos\left(a+bx+\frac{b}{2}\right) \blacksquare\end{aligned}$$

6. Buktikan  $\Delta^n x^{(n)} = n!$

**Jawab:**

Sebelumnya telah dibuktikan bahwa  $\Delta x^{(n)} = nx^{(n-1)}$ . Sehingga didapatkan

$$\begin{aligned}\Delta^2 x^{(n)} &= \Delta(\Delta x^{(n)}) = \Delta nx^{(n-1)} = n\Delta x^{(n-1)} = n(n-1)x^{(n-2)} \\ \Delta^3 x^{(n)} &= \Delta(\Delta^2 x^{(n)}) = \Delta n(n-1)x^{(n-2)} = n(n-1)\Delta x^{(n-2)} = n(n-1)(n-2)x^{(n-3)} \\ &\vdots \\ \Delta^n x^{(n)} &= n(n-1)(n-2)\dots 2 \cdot 1 \cdot x^{(n-n)} \\ &= n!x^{(0)} = n! \blacksquare\end{aligned}$$

Mengubah fungsi polinomial ke polinomial faktorial.

11.  $f(x) = x^3 - x + 1$

**Jawab:**

$$\begin{aligned}x^3 - x + 1 &= Ax^{(3)} + Bx^{(2)} + Cx^{(1)} + D \\ x^3 - x + 1 &= Ax(x-1)(x-2) + Bx(x-1) + Cx + D\end{aligned}$$

$$\begin{aligned}\bullet x=0 &\Rightarrow D=1 \\ \bullet x=1 &\Rightarrow C=0 \\ \bullet x=2 &\Rightarrow B=3 \\ \bullet x=3 &\Rightarrow A=1\end{aligned}$$

$$\therefore x^3 - x + 1 = x^{(3)} + 3x^{(2)} + 1$$

13.  $f(x) = 3x^3 - 7x^2 + 8x - 1$

$$\begin{aligned}3x^3 - 7x^2 + 8x - 1 &= Ax^{(3)} + Bx^{(2)} + Cx^{(1)} + D \\ 3x^3 - 7x^2 + 8x - 1 &= Ax(x-1)(x-2) + Bx(x-1) + Cx + D\end{aligned}$$

- $x = 0 \Rightarrow D = -1$
- $x = 1 \Rightarrow 3 - 7 + 8 - 1 = C - 1 \Rightarrow C = 4$
- $x = 2 \Rightarrow 24 - 28 + 16 - 1 = 2B + 8 - 1 \Rightarrow B = 2$
- $x = -1 \Rightarrow -3 - 7 - 8 - 1 = -6A + 4 - 4 - 1 \Rightarrow A = 3$

$$\therefore 3x^3 - 7x^2 + 8x - 1 = 3x^{(3)} + 2x^{(2)} + 4x^{(1)} - 1$$

15.  $f(x) = x^4 - 2x^3 - x$

$$x^4 - 2x^3 - x = Ax^{(4)} + Bx^{(3)} + Cx^{(2)} + Dx^{(1)}$$

$$x^4 - 2x^3 - x = Ax(x-1)(x-2)(x-3) + Bx(x-1)(x-2) + Cx(x-1) + Dx$$

- $x = 1 \Rightarrow 1 - 2 - 1 = D \Rightarrow D = -2$
- $x = 2 \Rightarrow 16 - 16 - 2 = 2C - 4 \Rightarrow C = 1$
- $x = 3 \Rightarrow 81 - 54 - 3 = 6B + 6 - 6 \Rightarrow B = 4$
- $A = 1$

$$\therefore x^4 - 2x^3 - x = x^{(4)} + 4x^{(3)} + x^{(2)} - 2x^{(1)}$$