1.
$$u_{xx} + x^2 u_{yy} = 0,\ x \neq 0$$
dan bentuk kanonikalnya adalah
$$\omega_{\xi\xi} + \omega_{\eta\eta} = -\frac{1}{2\eta}\omega_\eta \tag{1}$$
Bentuk separable
$$\omega(\xi,\eta) = X(\xi)Y(\eta) \tag{2}$$
Subtitusi (2) ke (1) diperoleh
$$X''(\xi)Y(\eta) + X(\xi)Y''(\eta) = -\frac{1}{2\eta}X(\xi)Y'(\eta) \tag{3}$$
Selanjutnya kita bagi (3) dengan $X(\xi)Y(\eta)$:
$$\frac{X''(\xi)}{X(\xi)} + \frac{Y''(\eta)}{Y(\eta)} = -\frac{1}{2\eta}\frac{Y'(\eta)}{Y(\eta)} \tag{4}$$
dengan pemisahan variabel
$$\frac{X''(\xi)}{X(\xi)} = -\lambda,\ \frac{Y''(\eta)}{Y(\eta)} + \frac{1}{2\eta}\frac{Y'(\eta)}{Y(\eta)} = \lambda \tag{5}$$

- $X''(\xi) + \lambda X(\xi) = 0$
  $X(\xi) = A\cos(\sqrt{\lambda}\xi) + B\sin(\sqrt{\lambda}\xi)$
- $Y''(\eta) + \frac{1}{2\eta}Y'(\eta) + \lambda Y(\eta) = 0 = Y_\eta(\eta)$

$$\omega(\xi,\eta) = \sum_{n=1}^{\infty}\left[A\cos(\sqrt{\lambda}\xi) + B\sin(\sqrt{\lambda}\xi)\right]Y_\eta(\eta) \tag{6}$$
$$\eta = \frac{y + \frac{1}{2}x^2}{2}, \quad u = \frac{y - \frac{1}{2}x^2}{2} \tag{7}$$
Subtitusi (7) ke (6) sehingga solusinya
$$\omega(\xi,\eta) = \sum_{n=1}^{\infty}\left[A\cos\left(\sqrt{\lambda}\left(\frac{y+\frac{1}{2}x^2}{2}\right)\right) + B\sin\left(\sqrt{\lambda}\left(\frac{y+\frac{1}{2}x^2}{2}\right)\right)\right]Y_\eta\left(\frac{y-\frac{1}{2}x^2}{2}\right)$$

2. Bentuk kanonik : $\omega_{\xi\xi} + \omega_{\eta\eta} = \frac{2}{3}\omega_\eta - \frac{2\sqrt{3}}{3}\omega_\eta$
Asumsikan solusinya berbentuk
$$\omega(\xi,\eta) = e^{\alpha\xi+\beta\eta} \tag{8}$$
Turunkan parsial (8) terhadap ξ dan η:
$$\omega_\xi = \alpha e^{\alpha\xi+\beta\eta},\quad \omega_{\xi\xi} = \alpha^2 e^{\alpha\xi+\beta\eta},\quad \omega_\eta = \beta e^{\alpha\xi+\beta\eta},\quad \omega_{\eta\eta} = \beta^2 e^{\alpha\xi+\beta\eta},$$

Subtitusi ke dalam persamaan (8):

$$\alpha^2 e^{\alpha\xi+\beta\eta} + \beta^2 e^{\alpha\xi+\beta\eta} = \frac{2}{3}\alpha e^{\alpha\xi+\beta\eta} - \frac{2\sqrt{3}}{3}\beta e^{\alpha\xi+\beta\eta}$$
$$\implies \alpha^2 + \beta^2 = \frac{2}{3}\alpha - \frac{2\sqrt{3}}{3}\beta$$

Misalkan $\alpha = 1$ maka β diperoleh dari

$$1 + \beta^2 = \frac{2}{3} - \frac{2\sqrt{3}}{3}\beta$$
$$\beta^2 + \frac{2\sqrt{3}}{3}\beta + \frac{1}{3} = 0$$
$$3\beta^2 + 2\sqrt{3}\beta + 1 = 0$$
$$\begin{aligned}\beta &= \frac{-2\sqrt{3} \pm \sqrt{(2\sqrt{3})^2 - 4.3.1}}{2 \cdot 3} \\ &= \frac{-2\sqrt{3} \pm \sqrt{12 - 12}}{6} \\ &= -\frac{\sqrt{3}}{3}\end{aligned}$$

Artinya

$$\omega(\xi,\eta) = e^{\xi - \frac{\sqrt{3}}{3}\eta}$$
$$\omega(\xi,\eta) = f(\xi - \frac{\sqrt{3}}{3}\eta)$$

- $\xi = y - \frac{1}{2}x$
- $\eta = \frac{\sqrt{3}}{3}x$

Oleh karena itu diperoleh

$$\begin{aligned}u(x,y) &= f(\eta - \frac{\sqrt{3}}{3}\xi) \\ &= f\left(y - \frac{1}{2}x - \frac{\sqrt{3}}{3} \cdot \frac{\sqrt{3}}{3}x\right) \\ &= f\left(y - \frac{1}{2}x - \frac{1}{2}x\right) \\ u(x,y) &= f\left(y - x\right)\end{aligned}$$