

Quality Assurance ECSA Project

22539794

Daniel Human

Table of Contents

Introduction.....	4
Part 1: Data Wrangling	5
Part 2: Descriptive Statistics	6
Frequency Plots for Different Descriptive Features:	6
Class Plots:.....	7
Global Plot:	8
Process Capability.....	9
Part 3: Statistical Process Control (SPC):	10
X-Charts for the first 30 samples of the 7 different classes:	10
Technology:	10
Clothing:	10
Household:	11
Luxury:	11
Food:.....	11
Gifts:	12
Sweets:	12
S-Charts for the first 30 samples of the 7 different classes:	13
Technology:	13
Clothing:	13
Household:	14
Luxury:	14
Food:.....	15
Gifts:	15
Sweets:	15
SPC Reports:	16
SPC Values for X charts:.....	16
SPC Values for S charts:	16
X-Charts for the rest of the samples of the 7 different classes:.....	17
Technology:	17
Clothing:	18
Household:	19
Gifts:	20
Luxury:	21
Food:.....	22
Sweets:	23

S-Charts for the rest of the samples of the 7 different classes:	24
Technology:	24
Clothing:	25
Household:	26
Luxury:	27
Food:	28
Gifts:	29
Sweets:	30
Part 4: Optimizing the Delivery Processes:	31
4.1 Control Limit Analysis:	31
X-Chart outliers beyond the outer control limits per class:	31
S-Chart outliers beyond the outer control limits per class:	31
Most Consecutive Samples between $0.4 \cdot \text{Sigma}$ and $-0.3 \cdot \text{Sigma}$ of the Mean per Class:	32
4.2 Likelihood of Type 1 Error:	33
4.3 Delivery Time Cost Optimization:	33
4.4 Likelihood of Type 2 Errors:	34
Part 5: DOE and MANOVA	34
Part 6: Reliability of the service and products	35
6.1 Problem 6 and 7 of chapter 7	35
Problem 6:	35
Problem 7:	35
6.2 Problem 27 of chapter 7	35
6.3 Binomial Probability	36
21 Vehicles:	36
21 Drivers:	37
Vehicles and Drivers:	38
For 22 Vehicles and 21 Drivers:	38
Conclusion	39
References	40

Introduction

In this report the provided sales data is statistically analysed to reveal trends within the data. These trends are beneficial to the company providing the data as alterations can be made to their business model to account for these newfound trends. These changes can increase revenue, lower expense and reduce stress on their outbound lines.

Part 1: Data Wrangling

Before the data can be analysed it needs to be cleaned and organized. The original dataset containing 180,000 entries was split into valid and invalid datasets after removing the “NA” entries and negative prices entries. The valid dataset was then organized by Class, Year, Month, Day and then Price. This valid dataset was then used throughout the rest of the report.

Part 2: Descriptive Statistics

Frequency Plots for Different Descriptive Features:

A bar graph plotting the frequency of “Why Bought” reveals that recommendation is the most effective method of advertising.

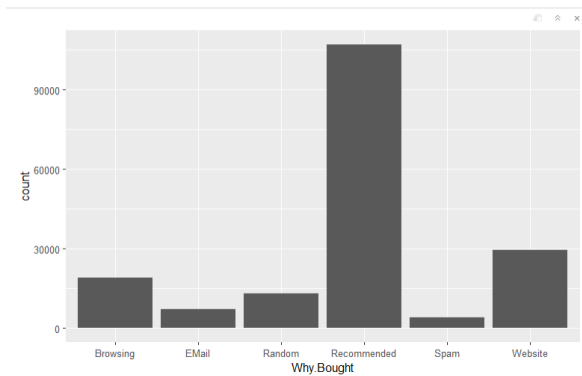


Figure 1: Frequency Plot for "Why.Bought" Feature

A histogram plotting the frequency of “AGE” reveals the age distribution of customers buying the stock. The data is skewed right with the modal age group being around 45.

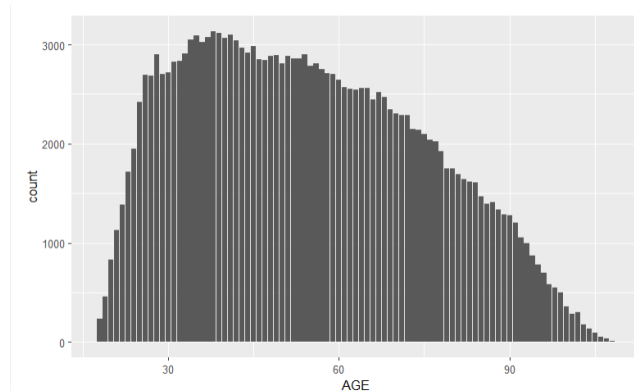


Figure 2: Frequency Plot for "Age" Feature

A bar graph of the frequency of “Class” reveals the most popular product type to be “Gifts” with the second most popular being “Technology”.

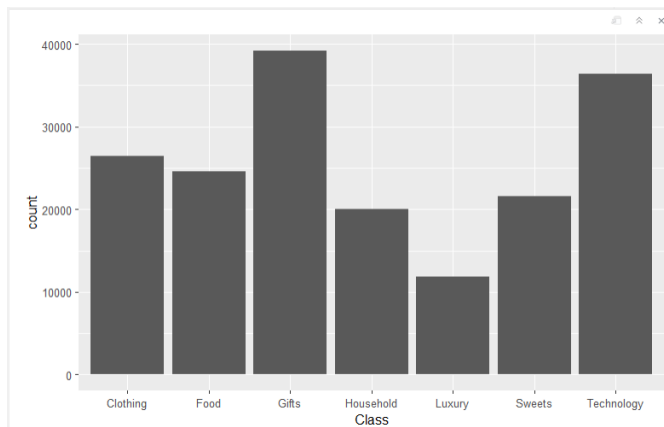


Figure 3: Frequency Plot for "Class" Feature

Class Plots:

A box and whisker plot of “Delivery time” against “Class” reveals which classes have the highest median, IQR, mix and max entries. Household has the longest delivery time mean, max and IQR range. This is expected as household items are larger and as a result will require longer delivery times.

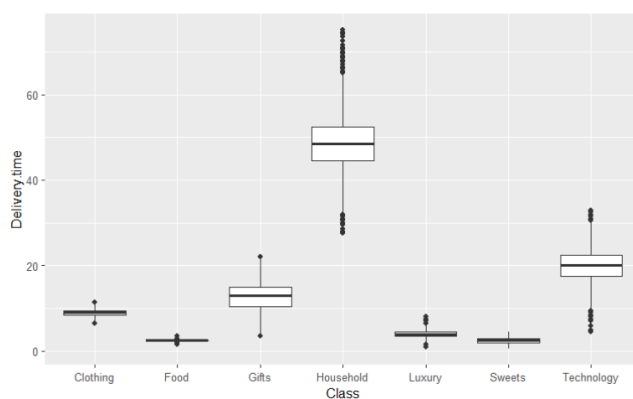


Figure 4: Box and whisker plot showing Delivery time vs Class

A box and whisker plot of “AGE” against “Class” reveals the buyer age spread per class. As expected Technology, Luxury and Clothing had a lower median age of around 45. This is probably because younger people are the most likely to spoil themselves. The youngest across all classes was 18. Interestingly the median age for sweets was high with most of the data distributed towards the younger customers as expected.

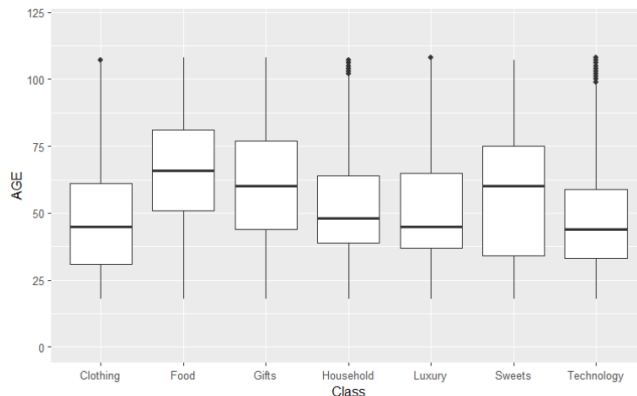
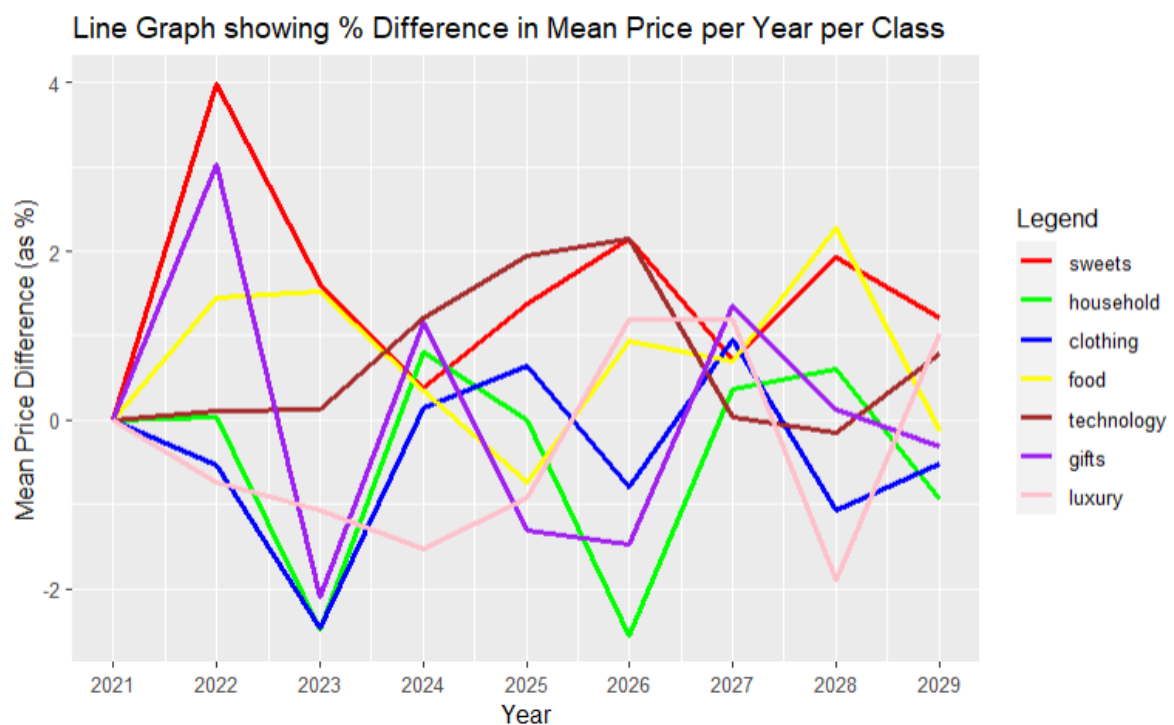


Figure 5: Box and whisker plot showing Age vs Class

Global Plot:

The mean difference in price vs year was plotted for each class. (The difference between the current year and 2021 as a %). This revealed that there was a rise in mean price for gifts and sweets in 2022. There was also a decrease in mean price for clothing, gifts and household in 2023 – all of which then saw a rise again in 2024 and later again in 2027. These trends could be explained by seasonal circumstances like Christmas or New year parties where sweets, gifts, clothing etc are given.



Process Capability

```
USL = 24
LSL = 0
u <- valid_data %>% filter(Class == "Technology")
u <- mean(u$Delivery.time)
sd <- valid_data %>% filter(Class == "Technology")
sd <- sd(sd$Delivery.time)

Cp <- (USL-LSL)/(6*sd)
Cpu <- (USL - u)/(3*sd)
Cpl <- (u-LSL)/(3*sd)
Cpk <- min(Cpu,Cpl)

print("Process Capability")
Cp
Cpu
Cpl
Cpk
...
```

[1] "Process Capability"
[1] 1.142207
[1] 0.3796933
[1] 1.90472
[1] 0.3796933

A LSL of 0 is logical because the “Lower Specification Limit” for delivery times cannot be lower than 0. A CPK value of less than 1 is considered poor and the process is not capable. The company should aim for a CPK > 1.33.

Part 3: Statistical Process Control (SPC):

The first 450 delivery time entries from each class of the “valid” dataset are sampled into groups of 15 (30 groups in total). X-bar and S Control Charts are then made for each of these first 30 samples. These initial samples will be used to calculate the control limits for the rest of the data. If any outliers were present beyond the upper and lower control limits those samples were removed entirely, the charts were reinitialised and the control limits were recalculated.

X-Charts for the first 30 samples of the 7 different classes:

Technology:

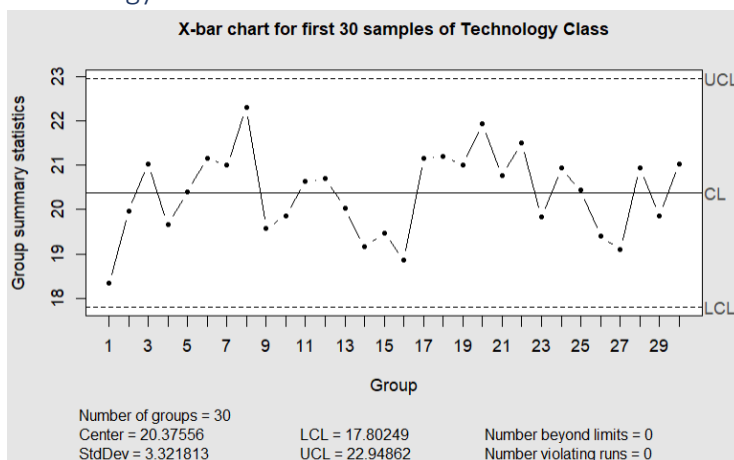


Figure 6: X-Chart of first 30 samples of Technology

Clothing:

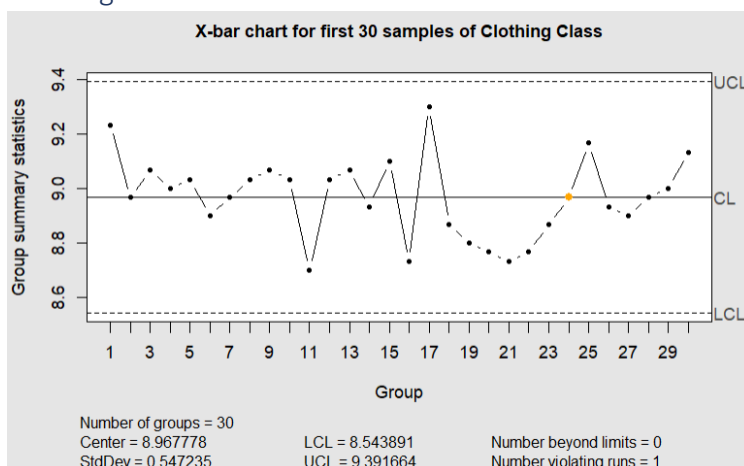


Figure 7: X-Chart of first 30 samples of Clothing

Household:

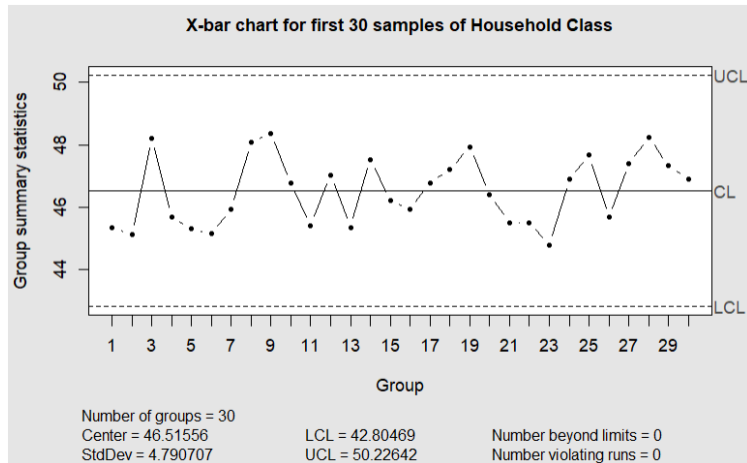


Figure 8: X-Chart of first 30 samples of Household

Luxury:

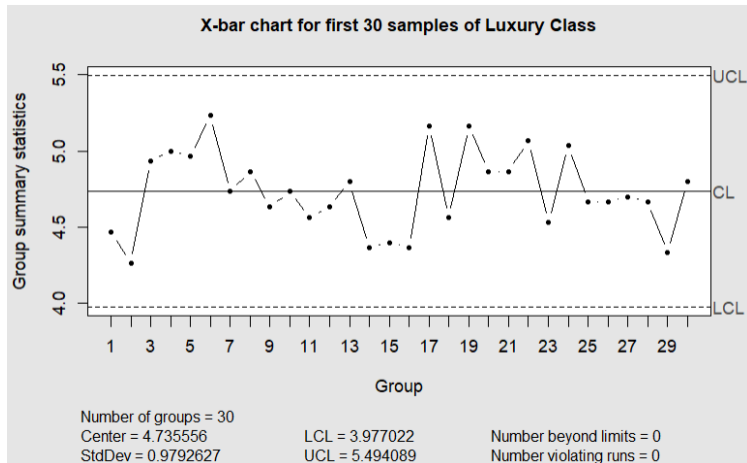


Figure 9: X-Chart of first 30 samples of Luxury

Food:

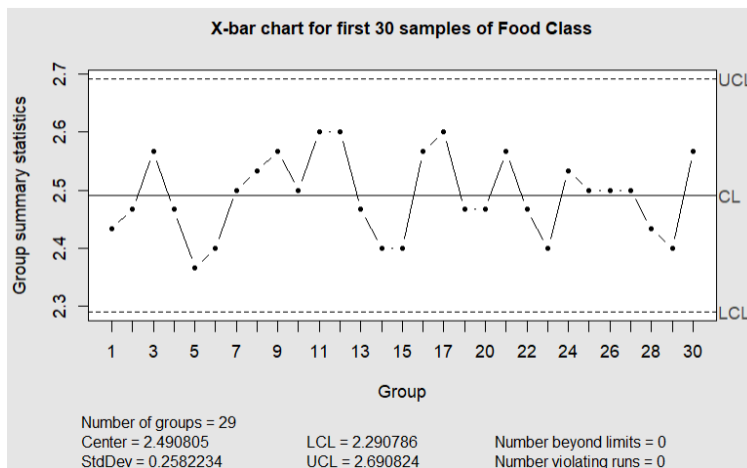


Figure 10: X-Chart of first 30 samples of Food

Gifts:

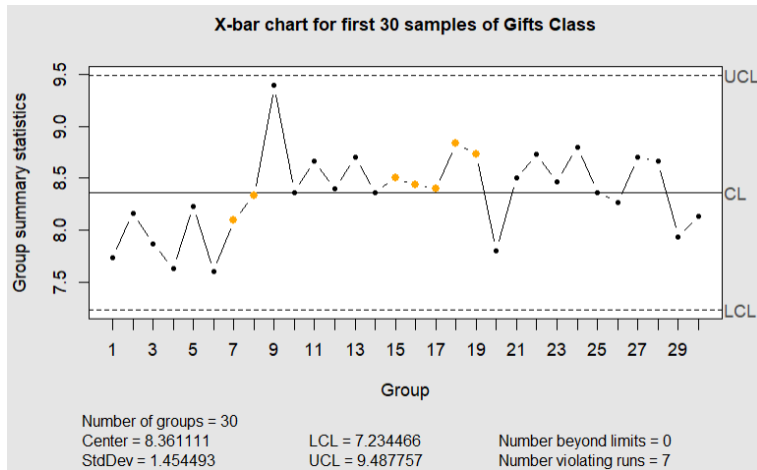


Figure 11: X-Chart of first 30 samples of Gifts

Sweets:

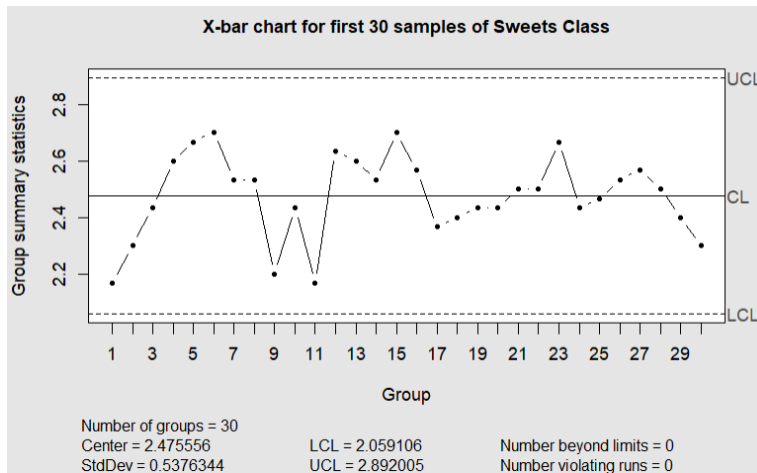


Figure 12: X-Chart of first 30 samples of Sweets

S-Charts for the first 30 samples of the 7 different classes:

Technology:

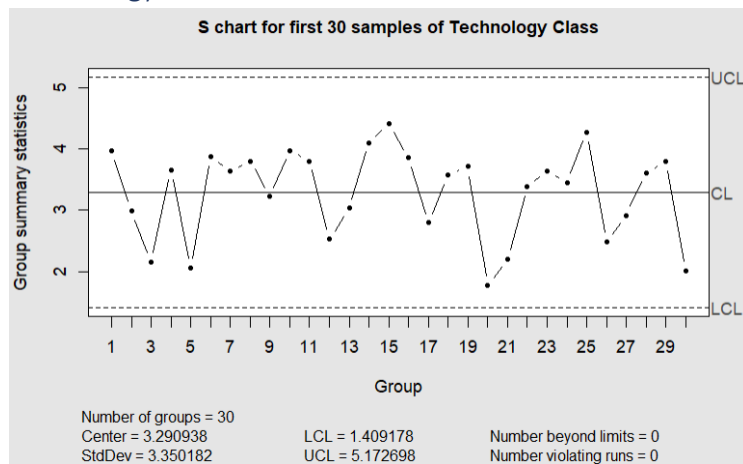


Figure 13: S-Chart of first 30 samples of Technology

Clothing:

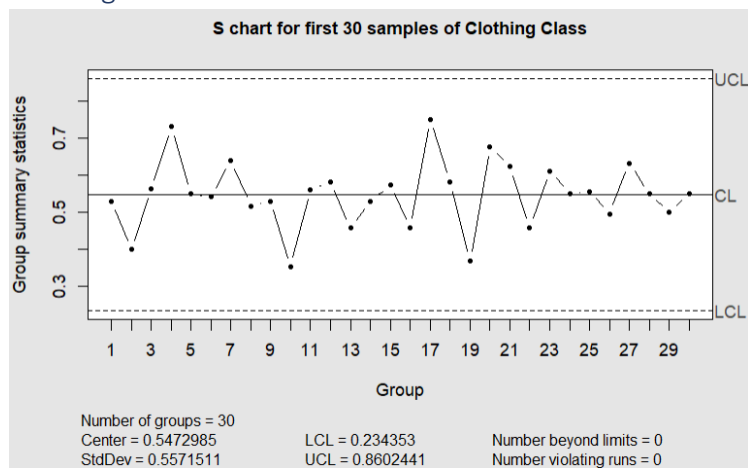


Figure 14: S-Chart of first 30 samples of Clothing

Household:

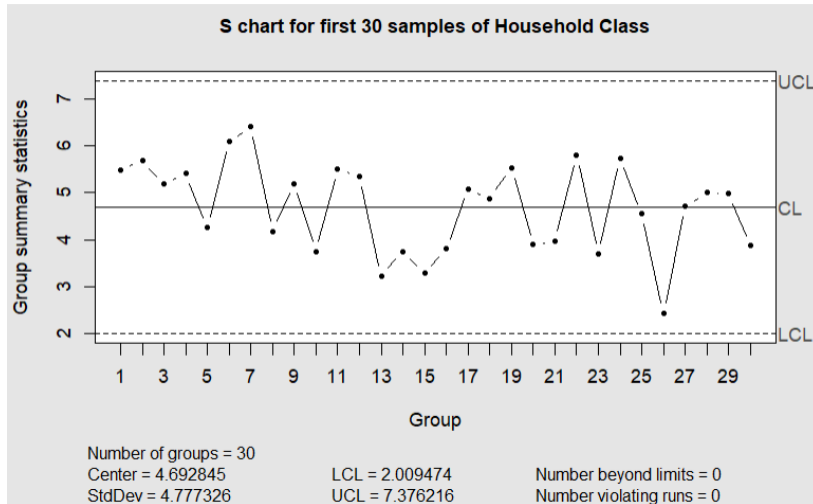


Figure 15: S-Chart of first 30 samples of Household

Luxury:

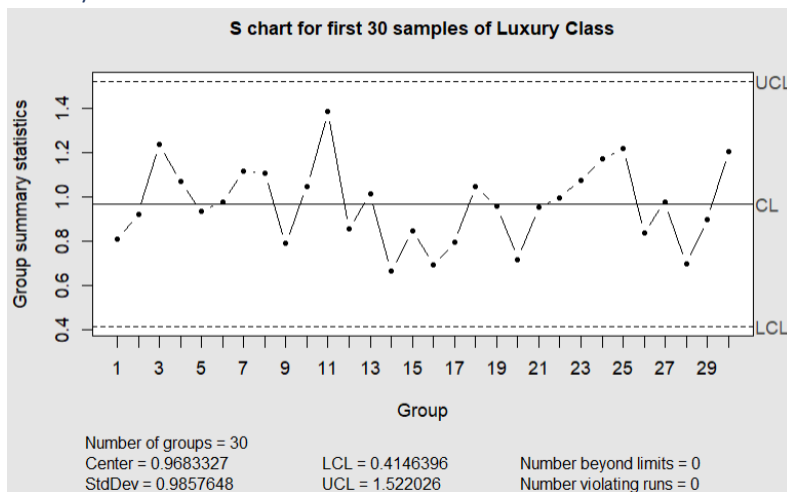


Figure 16: S-Chart of first 30 samples of Luxury

Food:

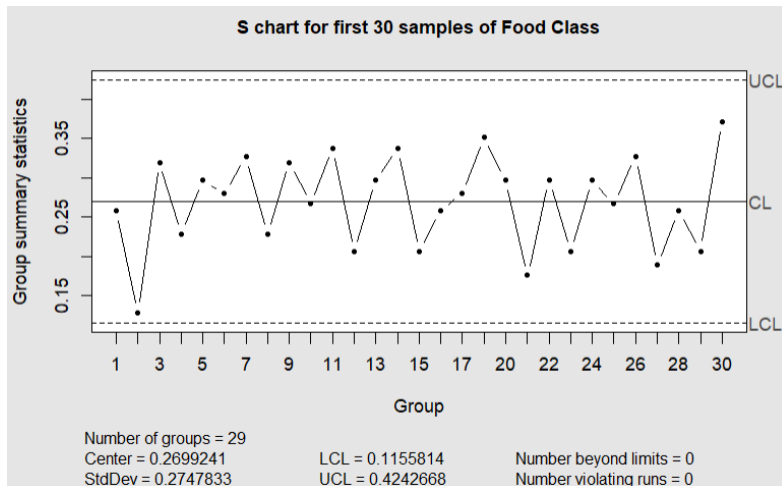


Figure 17: S-Chart of first 30 samples of Food

Gifts:

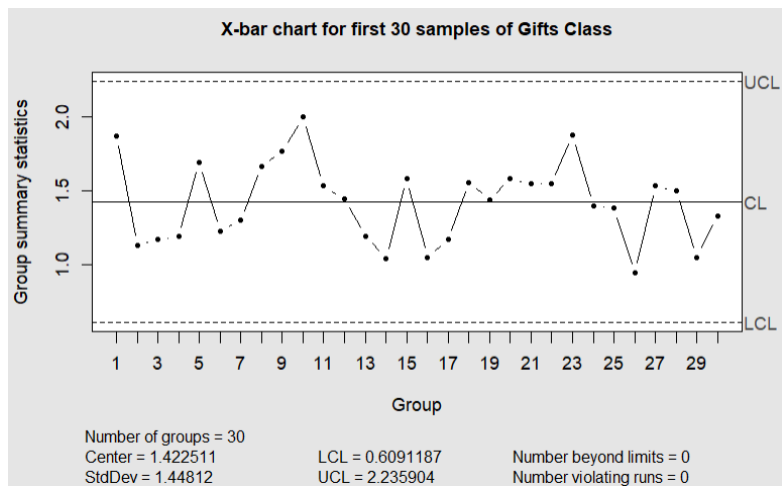


Figure 18: S-Chart of first 30 samples of Gifts

Sweets:

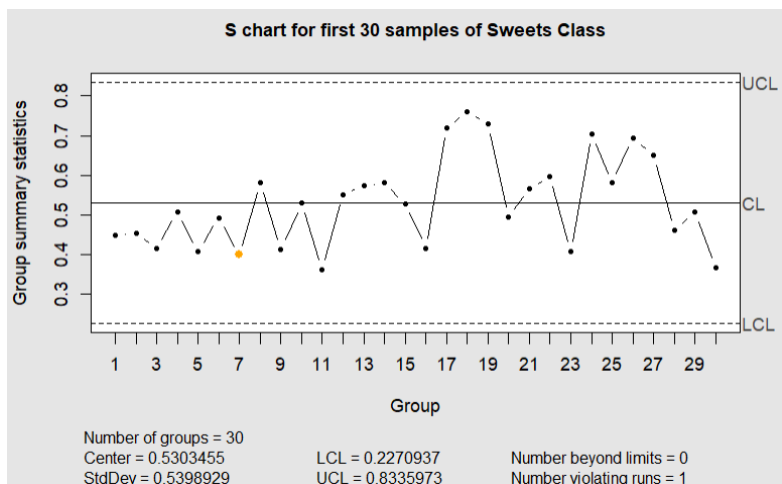


Figure 19: S-Chart of first 30 samples of Sweets

SPC Reports:

The UCL, U2Sigma, U1Sigma, Centre, L1Sigma, L2Sigma and LCL limits were tabulated for each class for both X and S charts.

SPC Values for X charts:

	Class	UCL	U2Sigma	U1Sigma	CL	L1Sigma	L2Sigma	LCL
1	Technology	22.948621	22.090932	21.233244	20.375556	19.517867	18.660179	17.802491
2	Clothing	9.391664	9.250369	9.109073	8.967778	8.826482	8.685187	8.543891
3	Household	50.226421	48.989466	47.752511	46.515556	45.278600	44.041645	42.804690
4	Luxury	5.494089	5.241245	4.988400	4.735556	4.482711	4.229866	3.977022
5	Food	2.690824	2.624151	2.557478	2.490805	2.424132	2.357459	2.290786
6	Gifts	9.487757	9.112208	8.736660	8.361111	7.985563	7.610014	7.234466
7	Sweets	2.892005	2.753189	2.614372	2.475556	2.336739	2.197922	2.059106

Figure 20: X-Table of SPC values

SPC Values for S charts:

	Class	UCL	U2Sigma	U1Sigma	CL	L1Sigma	L2Sigma	LCL
1	Technology	5.1726978	4.5454446	3.9181913	3.2909380	2.6636847	2.0364314	1.4091781
2	Clothing	0.8602441	0.7559289	0.6516137	0.5472985	0.4429834	0.3386682	0.2343530
3	Household	7.3762155	6.4817586	5.5873017	4.6928447	3.7983878	2.9039309	2.0094740
4	Luxury	1.5220258	1.3374615	1.1528971	0.9683327	0.7837684	0.5992040	0.4146396
5	Food	0.4242668	0.3728193	0.3213717	0.2699241	0.2184765	0.1670290	0.1155814
6	Gifts	2.2359039	1.9647730	1.6936421	1.4225113	1.1513804	0.8802495	0.6091187
7	Sweets	0.8335973	0.7325134	0.6314295	0.5303455	0.4292616	0.3281776	0.2270937

Figure 21: S-Table of SPC values

X-Charts for the rest of the samples of the 7 different classes:

Technology:

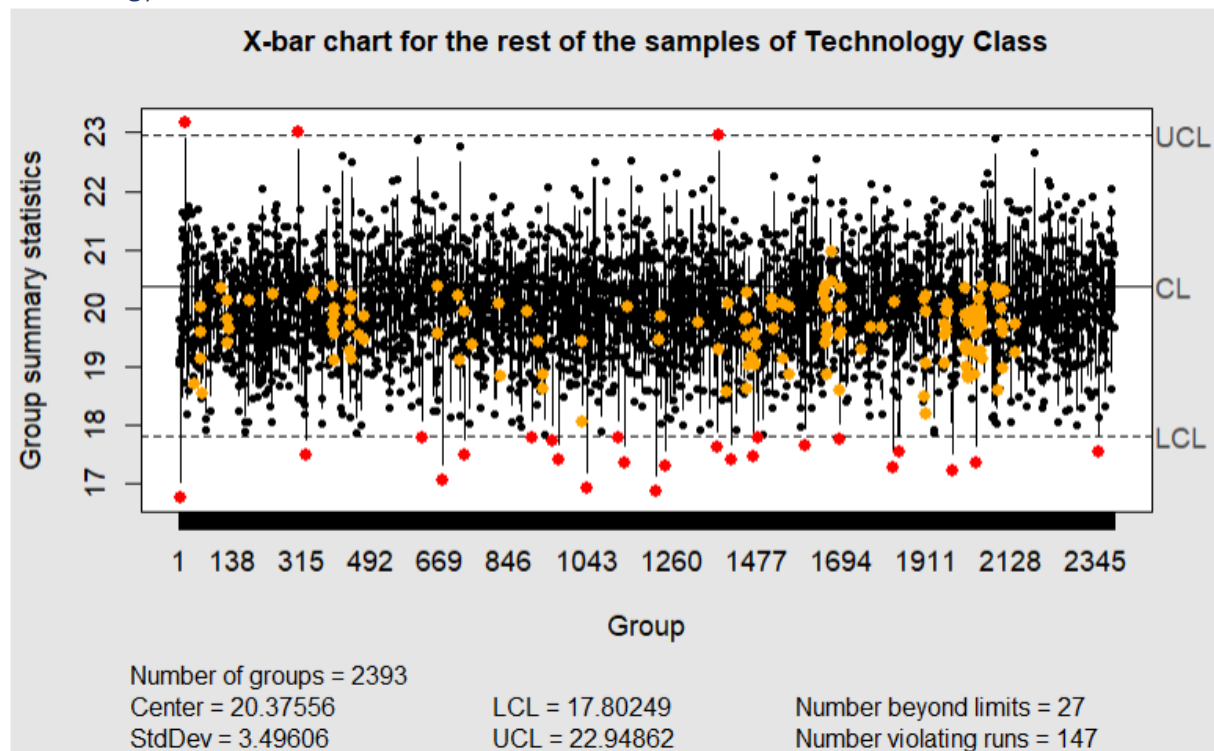


Figure 22: X-Chart of rest of samples of Technology

For the technology class in figure 22 we can see 27 samples violated the Upper and Lower Control Limits (red dots). This indicates that the delivery times for Technology are not 100% in control.

Clothing:

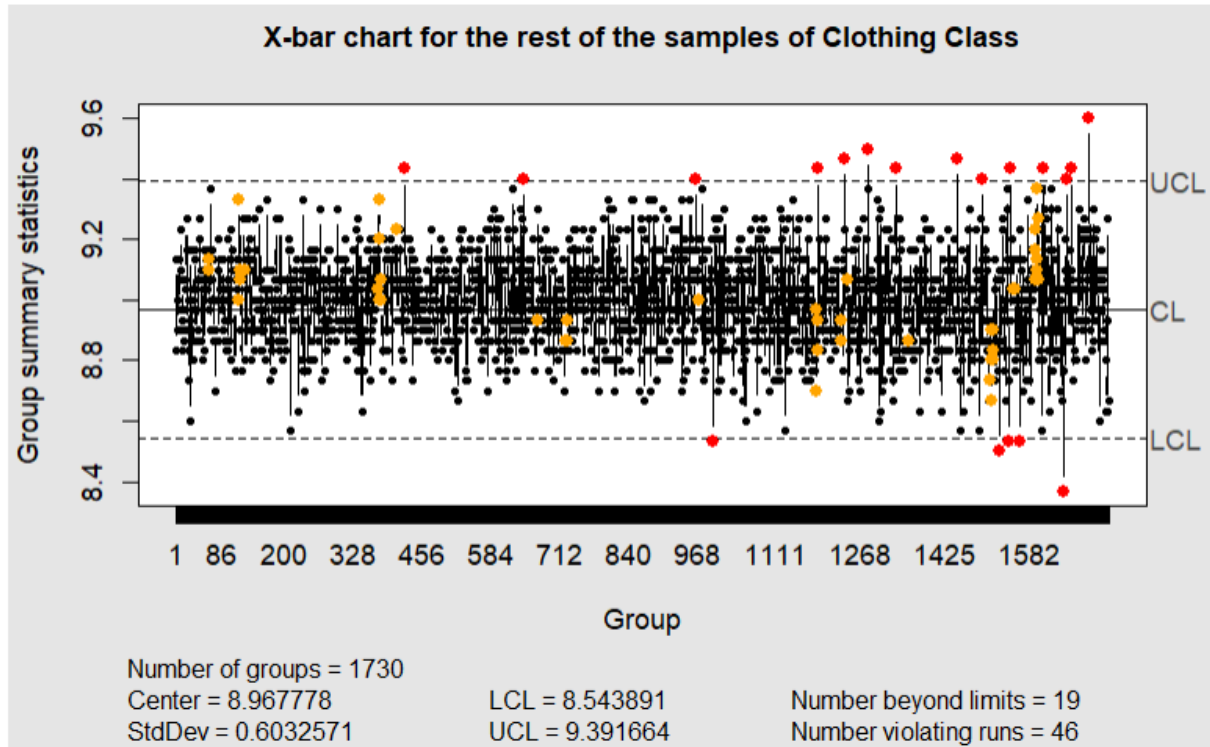


Figure 23: X-Chart of rest of samples of Clothing

For the Clothing class in figure 22 we can see 19 samples violated the Upper and Lower Control Limits (red dots). This indicates that the delivery times for Clothing are not 100% in control.

Household:

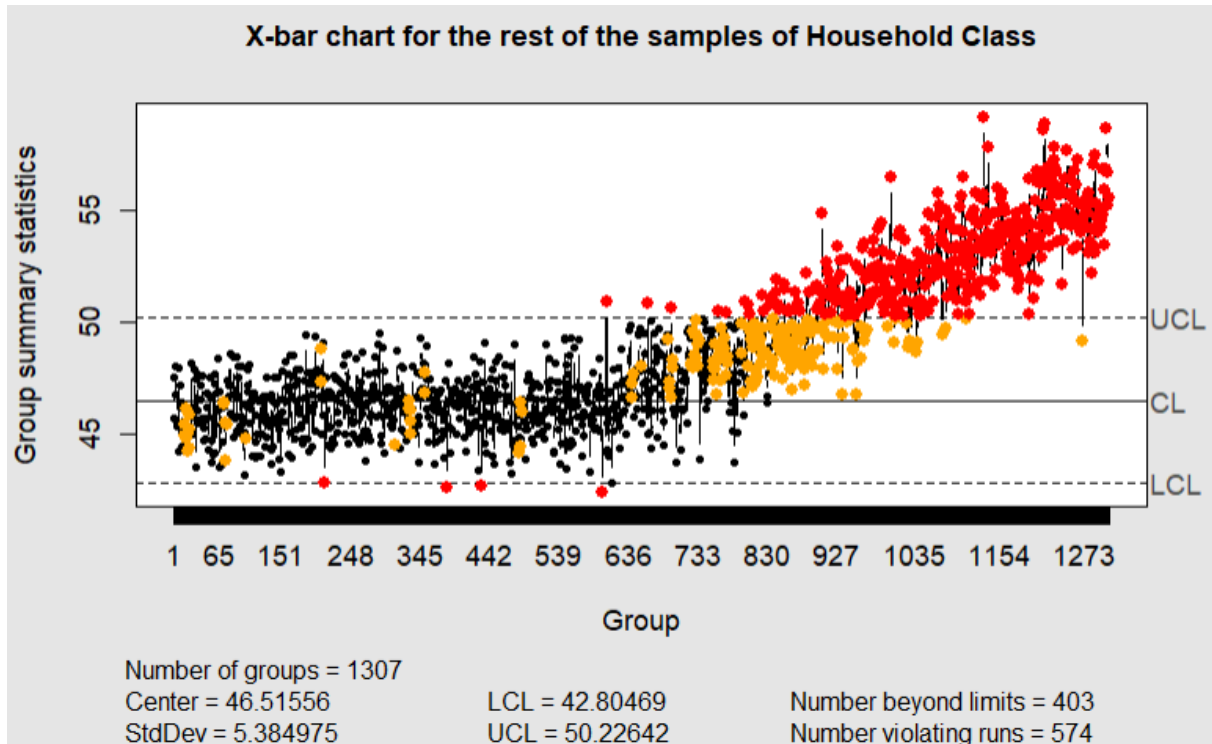


Figure 24: X-Chart of rest of samples of Household

For Household class in figure 24 we can immediately see an out of control scenario from samples 800 onward. This resulted in 403 samples violating the Upper and Lower Control Limits with the majority of outliers lying above the Upper Control Limit. Serious action needs to be taken to correct the outbound process flow for the Household items. This extreme out of control scenario could be due to Household items varying so much in size, shape and weight causing delays with some items.

Gifts:

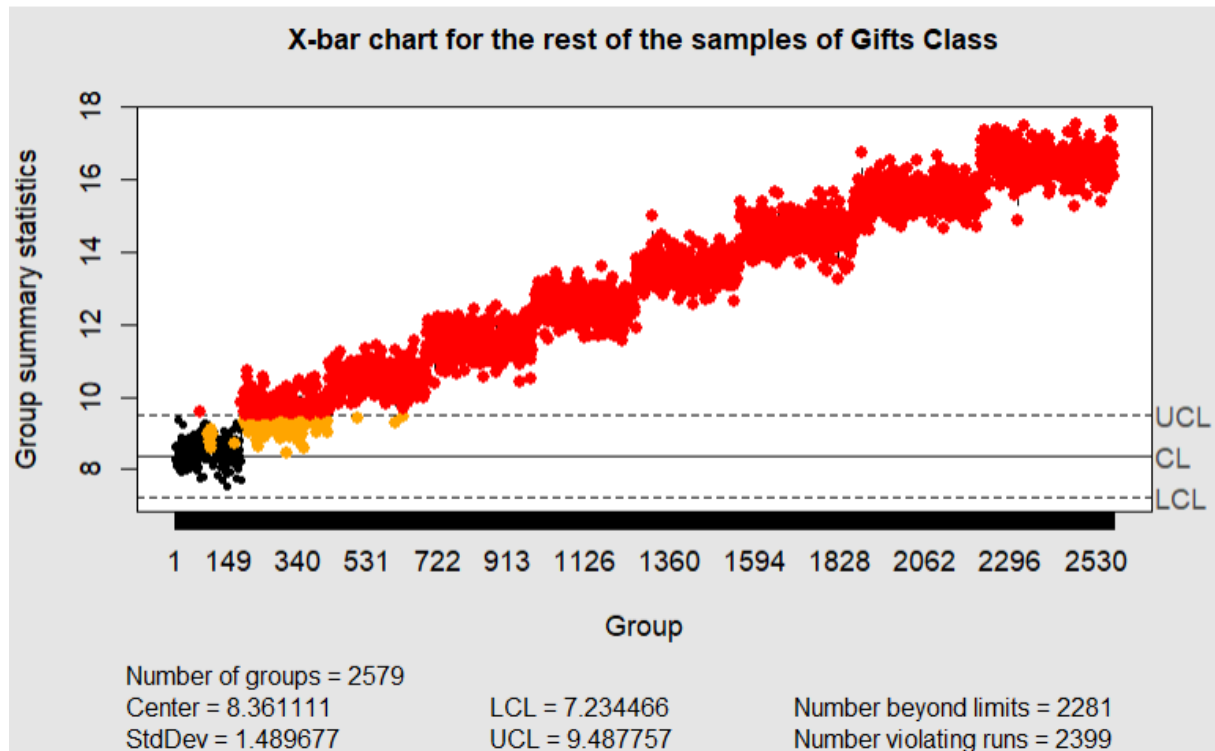


Figure 25: X-Chart of rest of samples of Gifts

For Gifts class in figure 25 we can immediately see an out of control scenario from samples 150 onward. This resulted in 2281 samples violating the Upper Control Limit. Serious action needs to be taken to correct the outbound process flow for the Gifts items. This extreme out of control scenario could be due to certain specific gifts being out of stock from suppliers. As the product isn't standard in the marketplace.

Luxury:

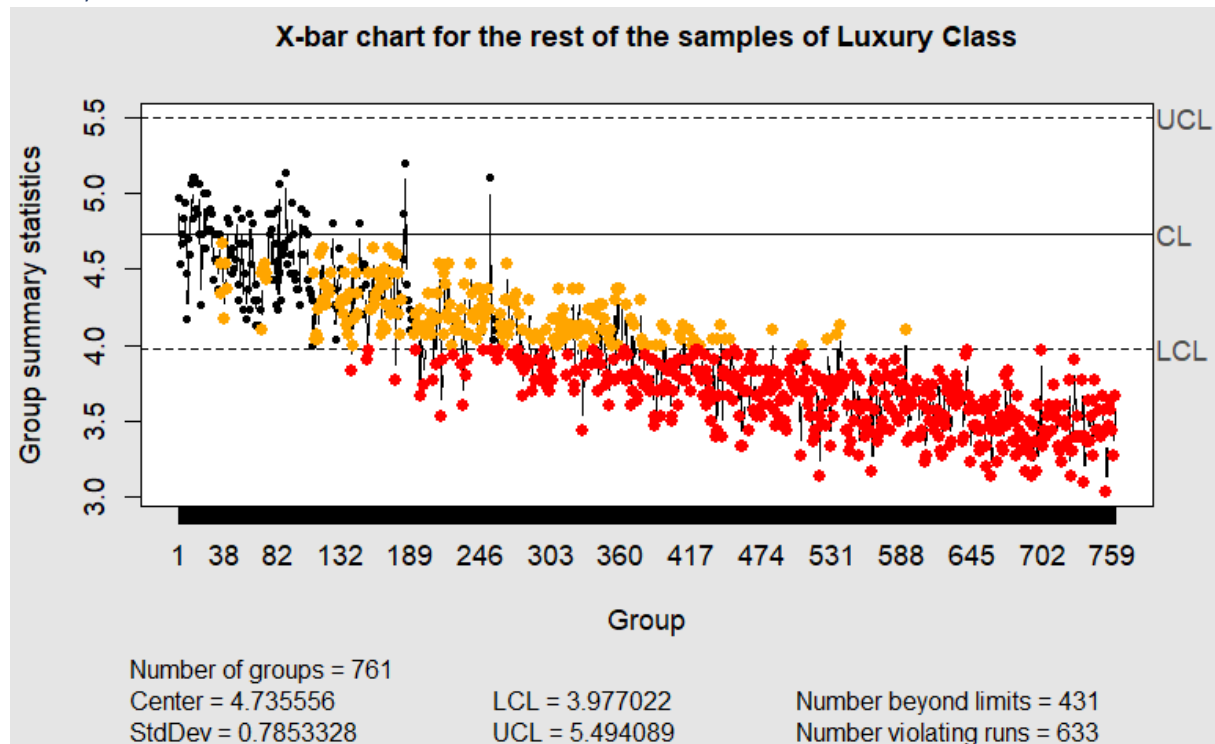


Figure 26: X-Chart of rest of samples of Luxury

For Luxury class in figure 26 we can immediately see an out of control scenario from samples 180 onward. This resulted in 431 samples violating the Lower Control Limit. This isn't too bad as the mean delivery times actually decrease which is beneficial towards the company. This could be due to Luxury items having varying dimensions and towards the end of the dataset more people were buying smaller, more expensive and easy to ship items causing the mean delivery times to drop. Problems could arise, however, if the Control limits are adjusted and then suddenly the mean data rises again to the original limits causing Upper Control Limit violations. This class of items needs to be monitored closely.

Food:

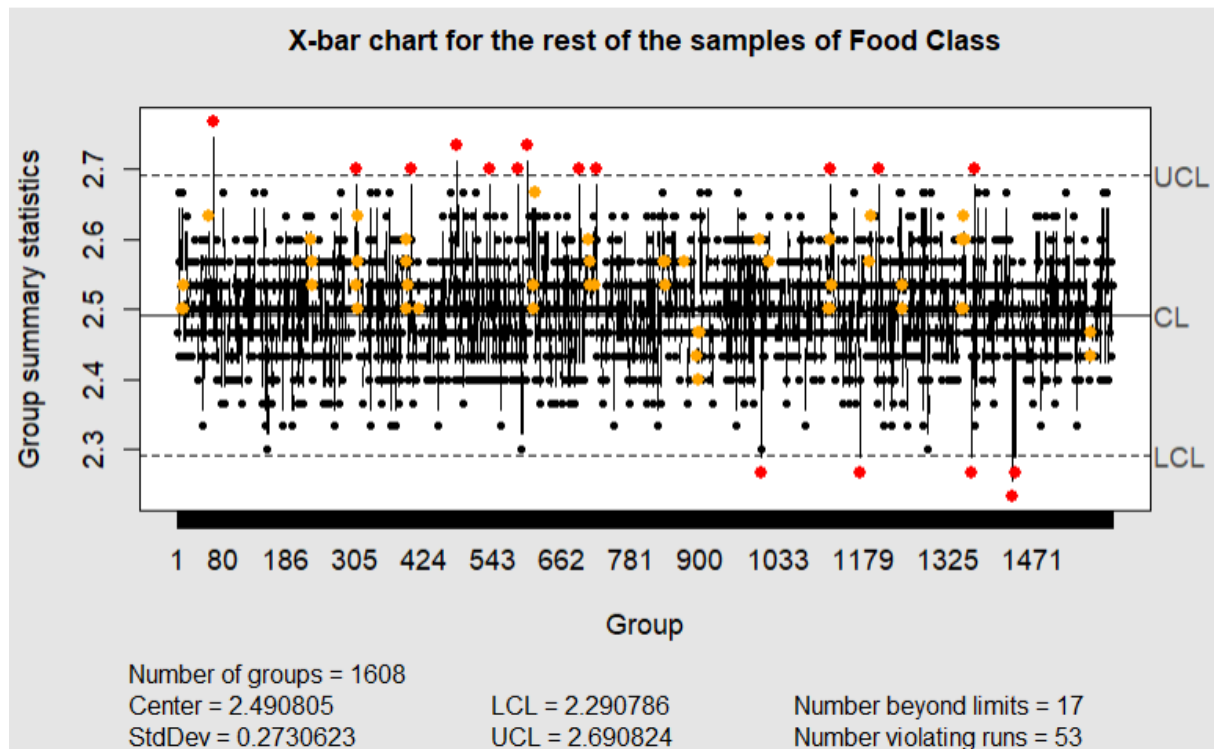


Figure 27: X-Chart of rest of samples of Food

For the Food class in figure 27 we can see 17 samples violated the Upper and Lower Control Limits (red dots). This indicates that the delivery times for Food are not 100% in control.

Sweets:

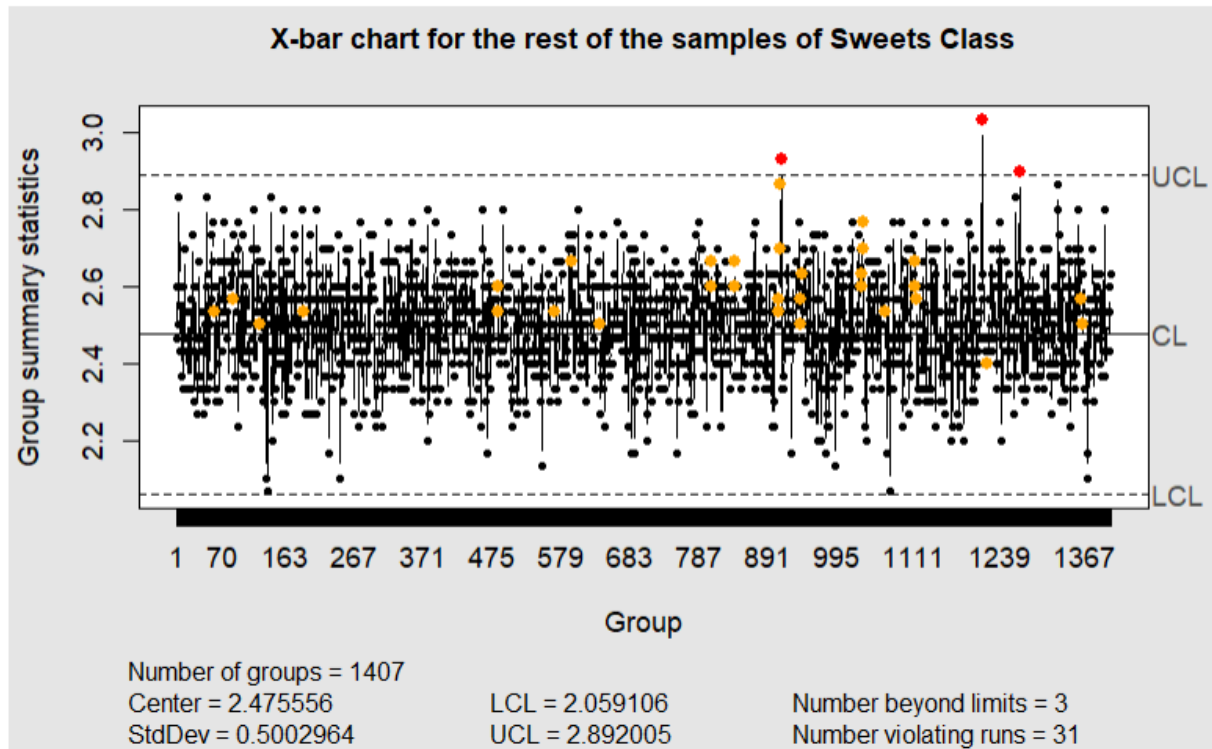


Figure 28: X-Chart of rest of samples of Sweets

For the Sweets class in figure 28 we can see 3 samples violated the Upper Control Limit (red dots). This indicates that the delivery times for Clothing are not 100% in control. This is however, extremely good and is probably due to sweets being easy to ship.

S-Charts for the rest of the samples of the 7 different classes:

Technology:

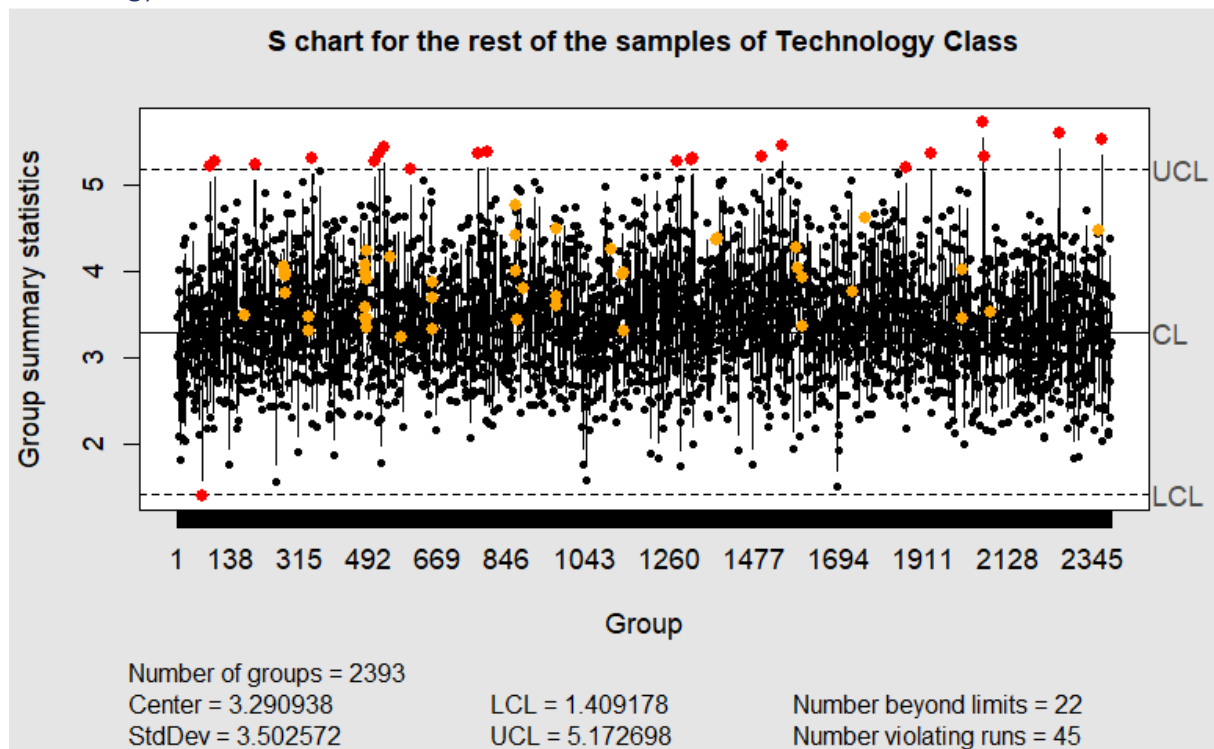


Figure 29: S-Chart of rest of samples of Technology

For the Technology class in figure 29 we can see 22 samples violated the Upper and Lower Control Limits (red dots). This indicates that the delivery times for Technology are not 100% in control. Some variation exists within the delivery times.

Clothing:

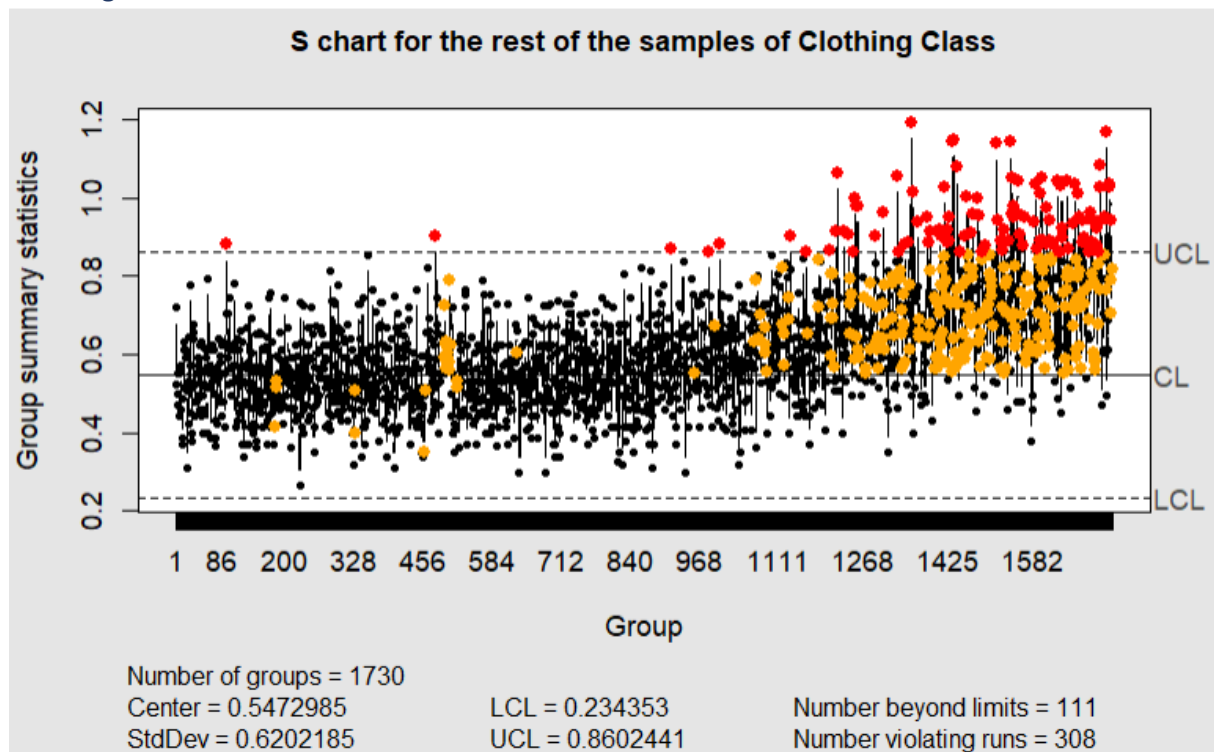


Figure 30: S-Chart of rest of samples of Clothing

For the Clothing class in figure 30 we can see 111 samples violated the Upper Control Limit (red dots). This indicates that the delivery times for Technology are not 100% in control. There was also a reasonable out of control trend from sample 1270 onwards. This indicates high variation within the last samples. This could be due to a fashion trend change causing orders to be made on stock not yet available, thereby varying the delivery times for the end samples. For this reason the Clothing class needs to be monitored closely.

Household:

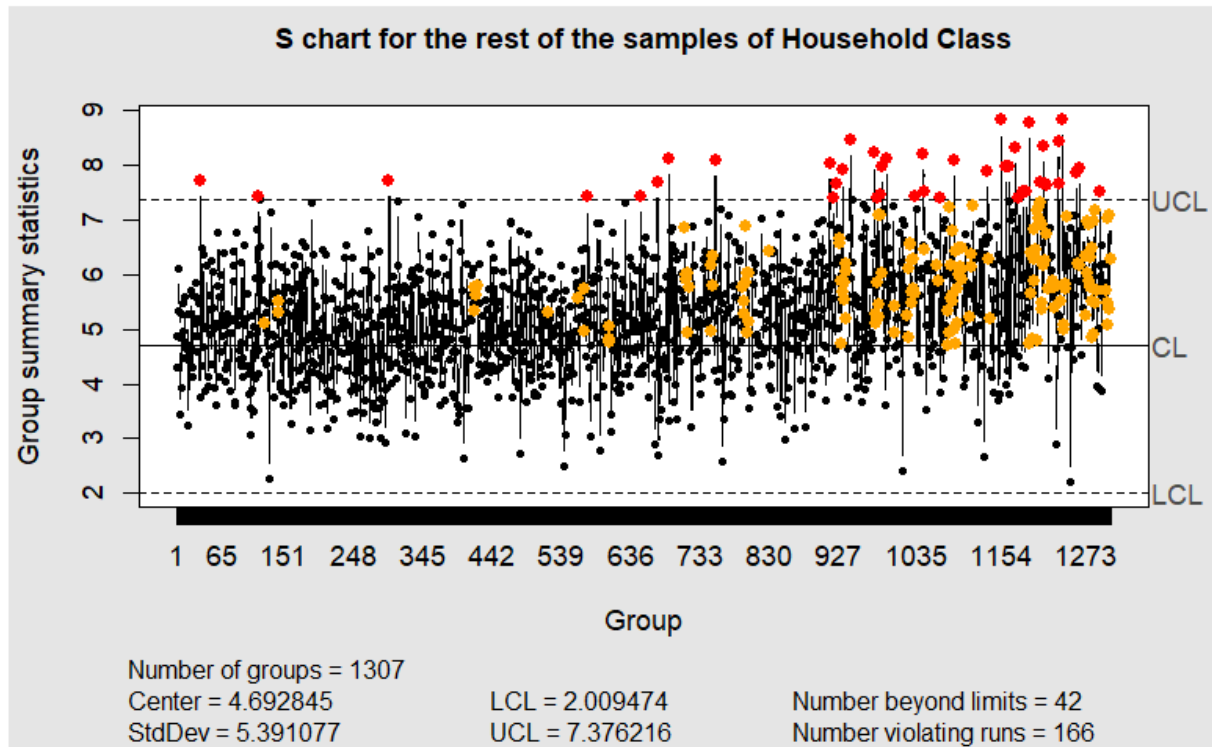


Figure 31: S-Chart of rest of samples of Household

For the Household class in figure 31 we can see 42 samples violated the Upper Control Limit (red dots). This indicates that the delivery times for Technology are not 100% in control. Some variation exists within the delivery times.

Luxury:

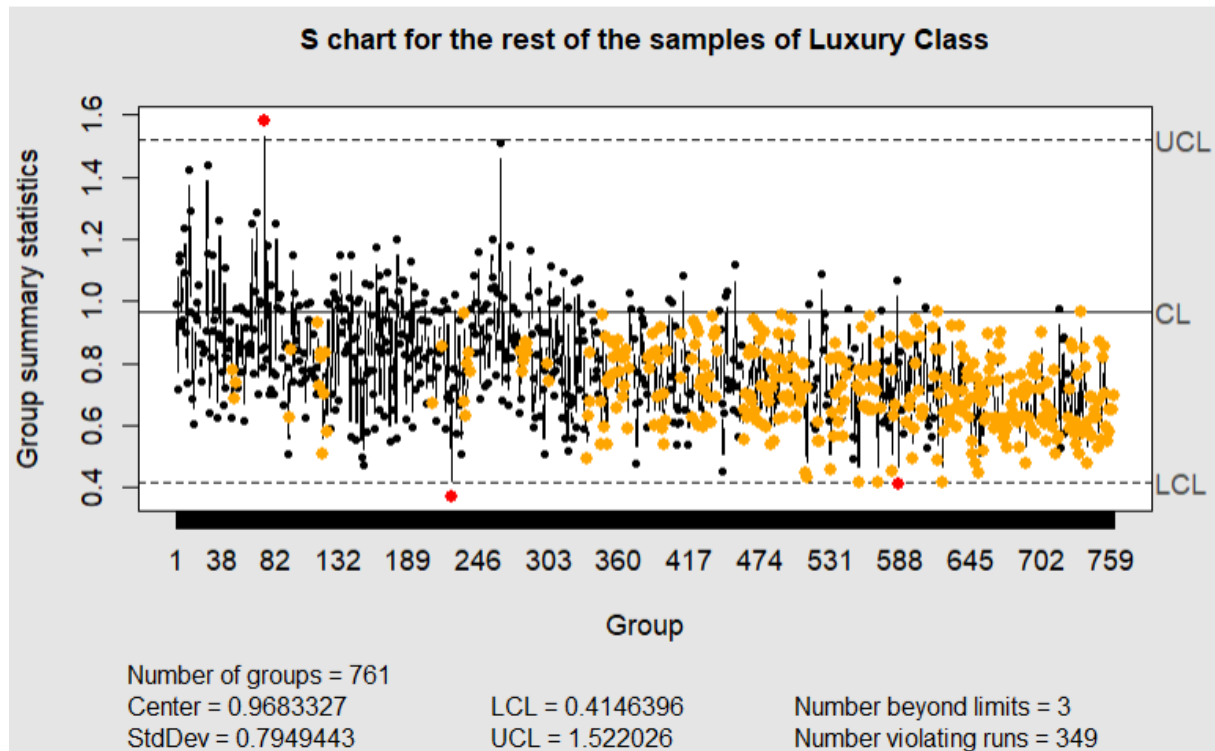


Figure 32: S-Chart of rest of samples of Luxury

For the Luxury class in figure 32 we can see 3 samples violated the Upper and Lower Control Limits (red dots). This indicates that the delivery times for Technology are not 100% in control. Some variation exists within the delivery times. While not many delivery time samples violated the Upper and Lower Control Limits, most of the samples from 360 onwards were below the Centre Line. This could indicate a trend forming.

Food:

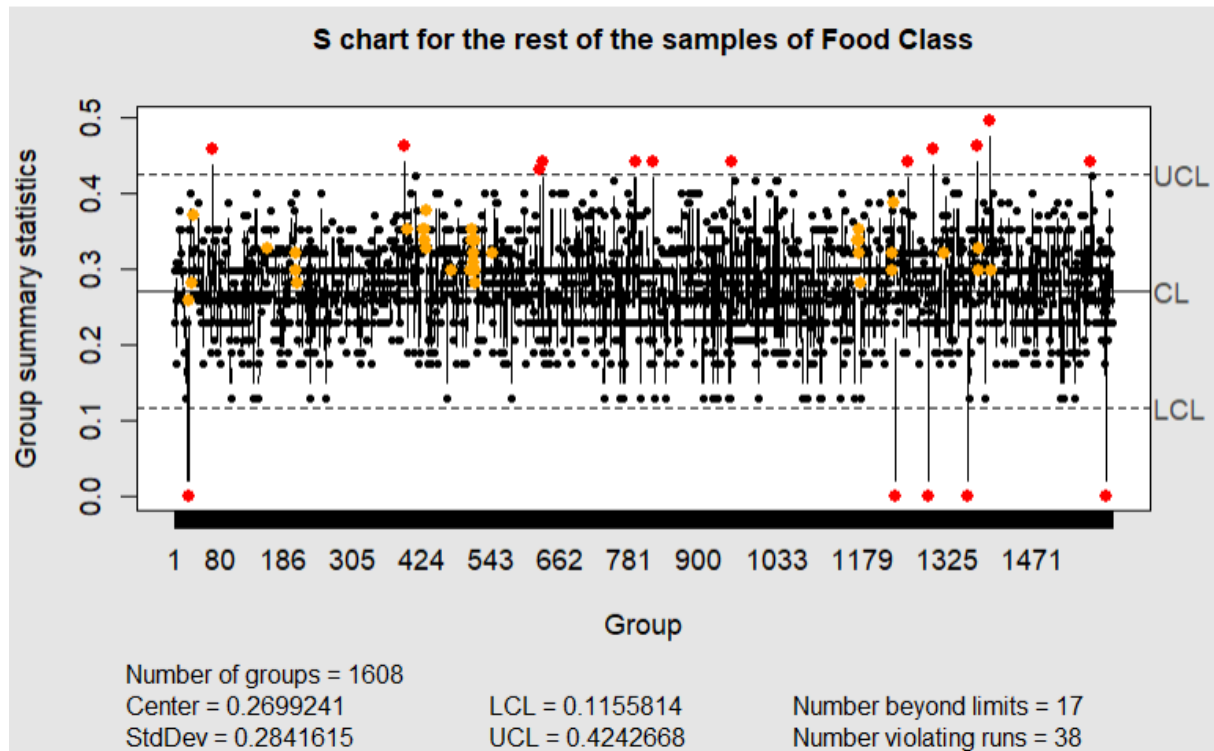


Figure 33: S-Chart of rest of samples of Food

For the Food class in figure 33 we can see 17 samples violated the Upper and Lower Control Limits (red dots). This indicates that the delivery times for Food are not 100% in control. Some variation exists within the delivery times.

Gifts:

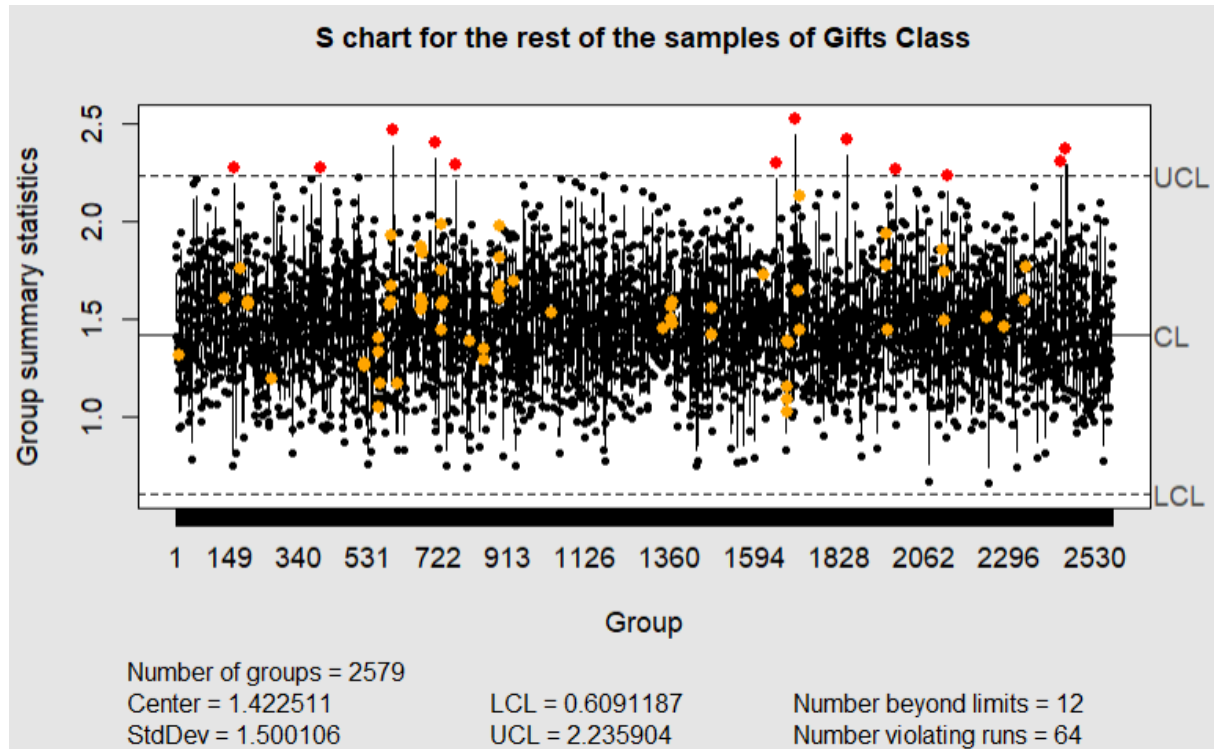


Figure 34: S-Chart of rest of samples of Gifts

For the Gifts class in figure 34 we can see 12 samples violated the Upper Control Limit (red dots). This indicates that the delivery times for Gifts are not 100% in control. Some variation exists within the delivery times.

Sweets:

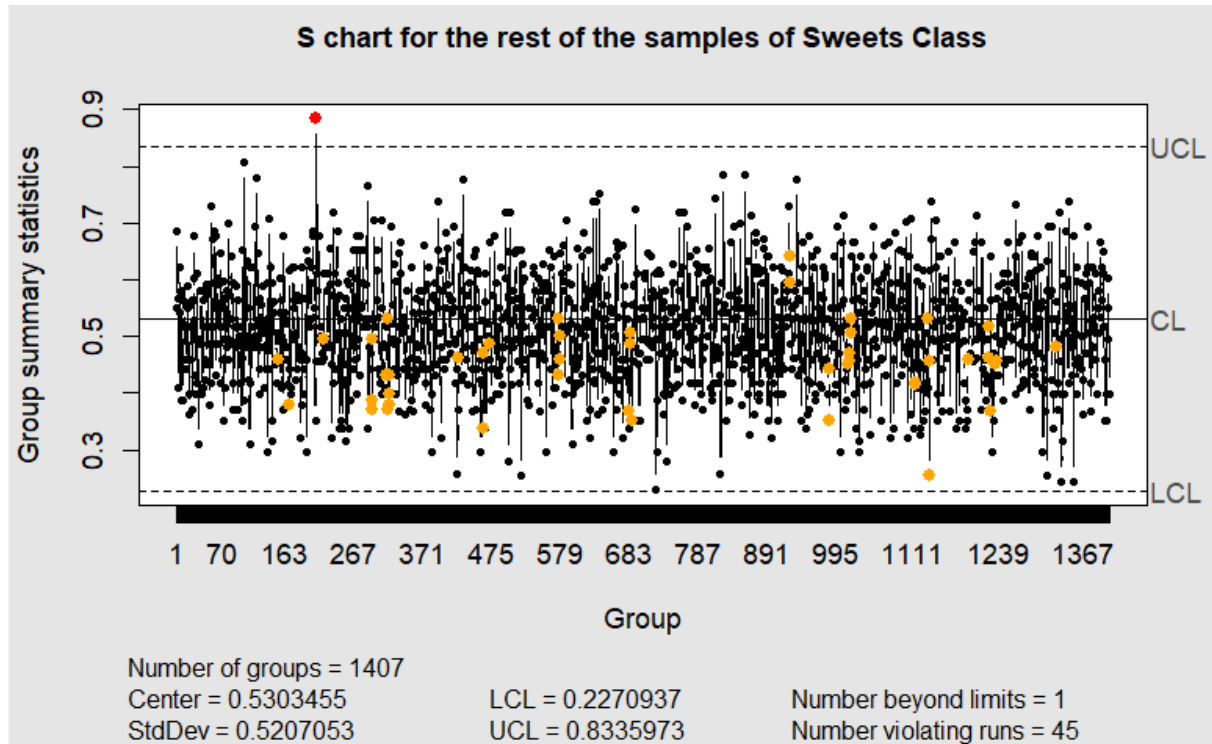


Figure 35: S-Chart of rest of samples of Sweets

For the Sweets class in figure 35 we can see 1 sample violated the Upper Control Limit (red dot). This indicates that the delivery times for Sweets are not 100% in control. Some variation exists within the delivery times.

Part 4: Optimizing the Delivery Processes:

4.1 Control Limit Analysis:

Here, the outliers beyond the UCL and LCL were plotted for both the X and S Charts. If more than 6 outliers were present then only the first 3 and last 3 were tabulated.

X-Chart outliers beyond the outer control limits per class:

	Class	First	Second	Third	Third_Last	Second_Last	Last	Total_number
1	Technology	19	307	1383	1979	2041	2355	27
2	Clothing	425	644	964	1544	1566	1647	19
3	Household	605	663	695	382	431	599	403
4	Luxury	141	154	155	759	760	761	431
5	Food	63	308	402	1367	1437	1440	17
6	Gifts	72	183	188	2577	2578	2579	2281
7	Sweets	912	1213	1270	NA	NA	NA	3

S-Chart outliers beyond the outer control limits per class:

	Class	First	Second	Third	Third_Last	Second_Last	Last	Total_number
1	Technology	87	99	200	2260	2370	65	22
2	Clothing	93	481	916	1724	1726	1727	111
3	Household	34	115	298	1260	1264	1293	42
4	Luxury	73	224	586	NA	NA	NA	3
5	Food	66	395	628	1293	1361	1597	17
6	Gifts	163	400	600	2127	2436	2452	12
7	Sweets	212	NA	NA	NA	NA	NA	1

Most Consecutive Samples between $0.4 \times \text{Sigma}$ and $-0.3 \times \text{Sigma}$ of the Mean per Class:

Class	Max Pattern Length	1 st in Pattern	2 nd in Pattern	3 rd in Pattern	4 th in Pattern	5 th in Pattern
Technology	5	206: 3.144156	207: 3.200818	208: 3.181793	209: 3.217512	210: 3.478848
Clothing	5	209: 0.5606119	210: 0.5232681	211: 0.5300494	212: 0.5498918	213: 0.5936168
Household	2	537: 4.676944	538: 4.828733			
Luxury	0					
Food	0					
Gifts	1	65: 1.39983				
Sweets	0					

Code was run to determine the longest pattern length of samples lying within the range $[0.4 \times \text{Sigma}, -0.3 \times \text{Sigma}]$. Technology and Clothing showed the longest pattern with 5 consecutive samples lying within the range. Interestingly both Technology and Class had their pattern start around the same sample number – 206 for Technology and 209 for Clothing. This could be coincidence or maybe some common factor caused a consistent outbound product flow for those two departments. Maybe after $200 \times 15 + 450 = 3450$ deliveries the two departments hit a consistency streak.

4.2 Likelihood of Type 1 Error:

	when H0 is true	when H1 is true
Do not Reject H0	correct decision $p = 1 - \alpha$	Type II error $p = \beta$
Reject H0	Type I error $p = \alpha$	correct decision $p = 1 - \beta$

Assuming:

Ho = "The process is in control and centred on the centreline calculated using the first 30 samples".

Ha = "The process is not in control and has moved from the centreline or has increased or decreased in variation."

A) For X-Chart:

$$P(z < -3) + P(z > 3) = 0.0026998$$

B) For Consecutive Samples between $0.4 \cdot \sigma$ and $-0.3 \cdot \sigma$

$$P(z < -0.3) + P(z > 0.4) = P(z < -0.3) + (1 - P(z < 0.4)) = 0.6179 + 0.3446 = 0.9625$$

4.3 Delivery Time Cost Optimization:

```
for (n in 1:10){  
  # b increases by 1 hour starting at 0. First cost will be result  
  # of shifting mean by -1. Second by -2...  
  for (n in 1:nrow(tech)){  
    if (tech[n,] - b > 26){  
      cost <- cost + 329*floor((tech[n,]-b-26))  
    }  
    cost <- cost + 2.5*b  
  }  
  cost_frame <- rbind(cost_frame, c(cost,b))  
  cost <- 0  
  b <- b + 1  
}  
cost_frame <- cost_frame[-1,]  
c <- min(cost_frame$cost)  
  
mean(tech$Delivery.time)  
cost_frame$cost[1]  
cost_frame$b[cost_frame$cost == c]  
mean(tech$Delivery.time) - cost_frame$b[cost_frame$cost == c]  
c  
...  
[1] 20.01095  
[1] 633654  
[1] 2  
[1] 18.01095  
[1] 318270
```

Initial Mean Delivery Time = 20.01095

Initial Cost = 633654

Hour reduction = 2

New Mean Delivery Time = 18.01095

New Mean Cost = 318270

4.4 Likelihood of Type 2 Errors:

Given:

The delivery process average moves to 23 hours (unknown to you).

In the type II error, the H_a is true, but we fail to identify this, due to the sample \bar{X} value being between LCL and UCL.

Part 5: DOE and MANOVA

(H_0): There is no significant difference between “Price” and “Delivery.time” per Class.

(H_a): The “Price” and “Delivery.time” differ per Class.

```
##{r}
#The null hypothesis (H0): There is no significant difference between Price and Delivery.time per Class.
#The alternative hypothesis (Ha): The Price and Delivery.time differ per Class.
valid_data
sales.man <- manova(cbind(valid_data$Price, valid_data$Delivery.time) ~ Class, data = valid_data)
summary(sales.man)
summary.aov(sales.man)
##
```

	Df	Pillai	approx F	num Df	den Df	Pr(>F)
Class	6	1.6797	157291	12	359942	< 2.2e-16 ***
Residuals	179971					

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Response 1 :

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
Class	6	5.7168e+13	9.5281e+12	80258	< 2.2e-16 ***
Residuals	179971	2.1366e+13	1.1872e+08		

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Response 2 :

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
Class	6	33458565	5576427	629429	< 2.2e-16 ***
Residuals	179971	1594452	9		

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Figure 36: MANOVA

In figure 36 we can see that the P value for the first and second responses are highly significantly different among features. This indicates that the Price and Delivery time do differ per Class. Therefore the alternate hypothesis is true.

Part 6: Reliability of the service and products

6.1 Problem 6 and 7 of chapter 7

Problem 6:

$$L = k(y-m)^2 = 45$$

$$\text{when } (y-m) = 0.04$$

$$k = 45/(0.04^2) = 28125$$

$$L = 28125(y-0.06)^2$$

Problem 7:

$$k = 35/(0.04^2) = 21875$$

$$L = 21875(y-0.06)^2$$

$$L = 21875 \cdot 0.0272 = 15.95$$

6.2 Problem 27 of chapter 7

```
{r}
RA <- 0.85
RB <- 0.92
RC <- 0.9

#Both A's combined reliability is:
RAA = 1 - (1-RA)^2

#Both B's combined reliability is:
RBB = 1 - (1-RB)^2

#Both C's combined reliability is:
RCC = 1 - (1-RC)^2

#a
RAA*RBB*RCC
#b
(RAA*RBB*RCC-RA*RB*RC)*100
```

```
[1] 0.9615316
[1] 25.77316
```

- a) System reliability = 96%
- b) Improved system reliability = 26%

6.3 Binomial Probability

Vehicles Available /21	Number of Days /1560	Drivers Available /21	Number of Days /1560
20	190	21	22
19	22	20	95
18	3	19	6
17	1	18	1

For reliable service there must be at least 19 vehicles and 19 drivers:

$$f(x) = \binom{n}{x} p^x (1-p)^{n-x} = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

21 Vehicles:

p	n	x	ans								
0.0476	21	1	0.3769		64		0.042	21	0	0.406	633.34
0.0066	21	1	0.1218		1.2586		0.042	21	1	0.3739	583.35
0.0089	21	2	0.0141		0.1963		0.042	21	2	0.164	255.86
0.0243	21	3	0.0122		0.0728		0.042	21	3	0.0456	71.073
0.0197	21	4	0.0006		0.0197		0.042	21	4	0.009	14.028
					0.042						

$$P(21) = ((1560 - 190 - 22 - 3 - 1) / 1560) = 1344 / 1560 = 0.86154 = \binom{21}{0} * p^0 *$$

$$(1 - p)^{(21-0)}$$

$$p = P(0 \text{ Fail}) = 0.0071$$

$$P(20) = (190 / 1560) = 0.12179 = \binom{21}{1} * p^1 * (1 - p)^{21-1}$$

$$p = P(1 \text{ Fail}) = 0.0066$$

$$P(19) = (22 / 1560) = 0.01410 = \binom{21}{2} * p^2 * (1 - p)^{21-2}$$

$$p = P(2 \text{ Fail}) = 0.0089$$

$$P(18) = (3 / 1560) = 0.00192 = \binom{21}{3} * p^3 * (1 - p)^{21-3}$$

$$p = P(3 \text{ Fail}) = 0.0122$$

$$P(17) = (1 / 1560) = 0.0006 = \binom{21}{4} * p^4 * (1 - p)^{21-4}$$

$$p = P(4 \text{ Fail}) = 0.0197$$

$$\text{Weighted P} = \frac{1344(0.0071) + 190(0.0066) + 22(0.0089) + 3(0.0122) + 1 \cdot (0.0006)}{1560} = 0.00707...$$

$$P(0 \text{ Fail}) = \binom{21}{0} * 0.0071^0 * (1 - 0.0071)^{(21-0)} = 0.8617$$

$$P(1 \text{ Fail}) = \binom{21}{1} * 0.0071^1 * (1 - 0.0071)^{(21-1)} = 0.1287$$

$$P(2 \text{ Fail}) = \binom{21}{2} * 0.0071^2 * (1 - 0.0071)^{(21-2)} = 0.0092$$

$$P(3 \text{ Fail}) = \binom{21}{3} * 0.0071^3 * (1 - 0.0071)^{(21-3)} = 0.0004$$

Expected number of days for 0 Fail = $1560 * 0.8617 = 1344.3 = 1344$

Expected number of days for 1 Fail = $1560 * 0.1287 = 200.76 = 200$

Expected number of days for 2 Fail = $1560 * 0.0092 = 14.277 = 14$

Expected number of days for 3 Fail = $1560 * 0.0004 = 0.643 = 0$

$$\text{Expected number of days of reliable delivery} = \frac{1344+200+14}{1560} * 365 = 364$$

21 Drivers:

p	n	x	ans								
0.1837	21	0	0.0141	4.0405	0.0028	21	0	0.9428	1470.8		
0.0031	21	1	0.0609	0.293	0.0028	21	1	0.0556	86.726		
0.0045	21	2	0.0038	0.0268	0.0028	21	2	0.0016	2.4351		
0.0082	21	3	0.0006	0.0082	0.0028	21	3	3E-05	0.0433		
				0.0028							

$$P(21) = 22/1560 = 0.014103 = \binom{21}{0} * p^0 * (1 - p)^{(21-0)}$$

$$p = P(0 \text{ Fail}) = 0.1837$$

$$P(20) = (95/1560) = 0.060897 = \binom{21}{1} * p^1 * (1 - p)^{21-1}$$

$$p = P(1 \text{ Fail}) = 0.0031$$

$$P(19) = (6/1560) = 0.003846 = \binom{21}{2} * p^2 * (1 - p)^{21-2}$$

$$p = P(2 \text{ Fail}) = 0.0045$$

$$P(18) = (1/1560) = 0.000641 = \binom{21}{3} * p^3 * (1 - p)^{21-3}$$

$$p = P(3 \text{ Fail}) = 0.0082$$

$$\text{Weighted P} = \frac{22(0.1837) + 95(0.0031) + 6(0.0045) + 0.0082}{1560} = 0.00280...$$

$$P(0 \text{ Fail}) = \binom{21}{0} * 0.0028^0 * (1 - 0.0028)^{(21-0)} = 0.9428$$

$$P(1 \text{ Fail}) = \binom{21}{1} * 0.0028^1 * (1 - 0.0028)^{(21-1)} = 0.0556$$

$$P(2 \text{ Fail}) = \binom{21}{2} * 0.0028^2 * (1 - 0.0028)^{(21-2)} = 0.0016$$

$$P(3 \text{ Fail}) = \binom{21}{3} * 0.0028^3 * (1 - 0.0028)^{(21-3)} = 0.00003$$

$$\text{Expected number of days for 0 Fail} = 1560 * 0.9428 = 1470.80 = 1470$$

$$\text{Expected number of days for 1 Fail} = 1560 * 0.0556 = 86.73 = 86$$

$$\text{Expected number of days for 2 Fail} = 1560 * 0.0016 = 2.44 = 2$$

$$\text{Expected number of days for 3 Fail} = 1560 * 0.00003 = 0.04 = 0$$

$$\text{Expected number of days of reliable delivery} = \frac{1470+86+2}{1560} * 365 = 364$$

Vehicles and Drivers:

$$\text{Vehicle reliability} = P(0 \text{ Fail}) + P(1 \text{ Fail}) + P(2 \text{ Fail}) = 0.8617 + 0.1287 + 0.0092 = 0.9996$$

$$\text{Driver reliability} = P(0 \text{ Fail}) + P(1 \text{ Fail}) + P(2 \text{ Fail}) = 0.9428 + 0.0556 + 0.0016 = 1$$

$$\text{Total reliability} = 0.9996 * 1 = 0.9996$$

$$\text{Reliable days} = 0.9996 * 365 = 364$$

For 22 Vehicles and 21 Drivers:

Assuming p is the same as above:

$$\sum_{x=0}^4 \binom{22}{x} * 0.0071^x * (1 - 0.0071)^{(22-x)} = 1$$

$$\text{Driver reliability} = P(0 \text{ Fail}) + P(1 \text{ Fail}) + P(2 \text{ Fail}) = 0.9428 + 0.0556 + 0.0016 = 1$$

$$\text{Total reliability} = 1 * 1 = 1$$

$$\text{Reliable days} = 1 * 365 = 365$$

Therefore increasing the vehicles by one but keeping the drivers the same caused a 1 day increase in reliability.

Conclusion

Household, Gifts, Luxury and Clothing X-bar Charts showed out-of-control processes with SPC limit outliers of 403, 2281, 431 and 19 respectively. These outbound shipping departments should be monitored and adjusted accordingly. This will increase the delivery time reliability which will reduce the breakdowns due to fewer unforeseen spikes in delivery time. Since the company's best method of attracting customers is through recommendations it is imperative that the delivery time promise is met. This will increase customer satisfaction which will contribute the most towards business growth.

References

Process Capability Analysis Cp, Cpk, Pp, Ppk – A Guide [Online]. Available:

<https://www.1factory.com/quality-academy/guide-to-process-capability-analysis-cp-cpk-pp-ppk.html> [2022, October 21].

MANOVA Test in R: Multivariate Analysis of Variance [Online]. Available:

<http://www.sthda.com/english/wiki/manova-test-in-r-multivariate-analysis-of-variance> [2022, October 21].

Ted Hensing: What are X bar S Control Charts? [Online]. Available: <https://sixsigmastudyguide.com/x-bar-s-chart/> [2022, October 21].