{EPITECH.}
PARIS GRADUATE SCHOOL OF DIGITAL INNOVATION

Algorithms. Matching

Part II. Preference model.

B9 - Algorithms Matching

M-ALG-102

Introduction

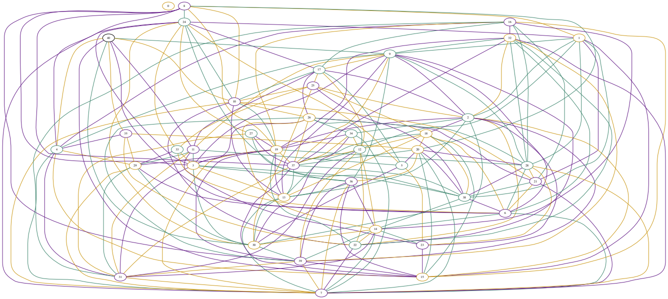

Figure – Graph

Introduction

https ://github.com/nlehir/ALGO2
We will use :

- networkx
- matplotlib
- pandas
- sklearn
- optionnally : ipdb

Compatibility graphs

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- If two nodes were related, they were linked by an edge in the graph.

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- If two nodes were related, they were linked by an edge in the graph.
- Today we are interested in building such graphs directly from the data, we call them **compatibility graphs**.

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Given two nodes in a graph, should there be an edge between them ?

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Compatibility graphs

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Given two nodes in a graph, should there be an edge between them ?
Note : it is not the same problem as the matching problem. In the matching problem, the edges are already defined.
However, once the edges are built, we can apply a matching to it.

Example applications

- Social networks management
- Recommendations

Building compatibility graphs

- We will build graphs first from simple data
- Then from more complex data.

Building a graph from simple data

- We will first build a graph from simple data in the 2D space.

Euclidian distance and compatibility

Consider the following data :

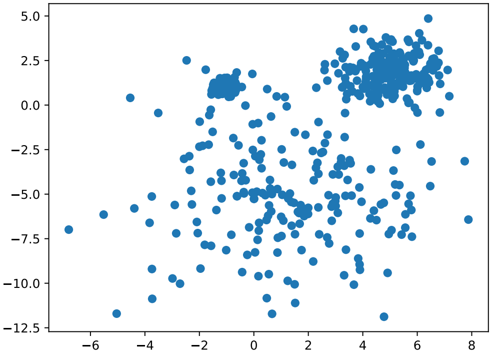

Figure – Data : we would like to define **edge** between some of them

Is this set of edges a good solution ?

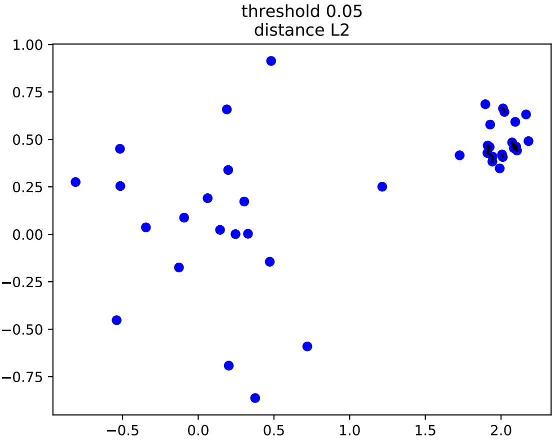

Figure – Some definition of edges

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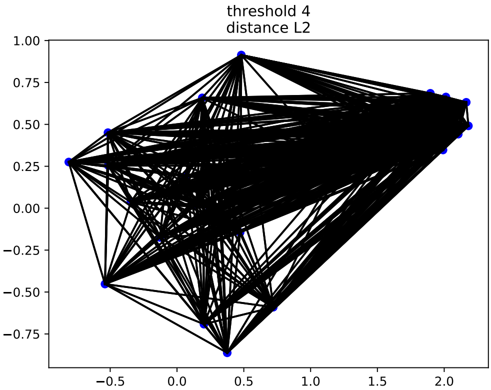


Figure – Some definition of edges

This one looks ok

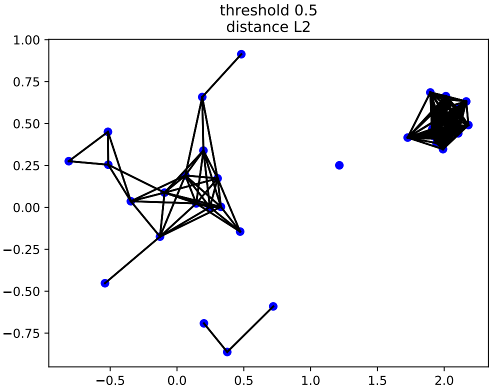
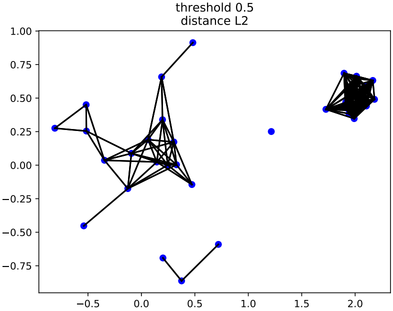


Figure – A proposition of edges

Backboard

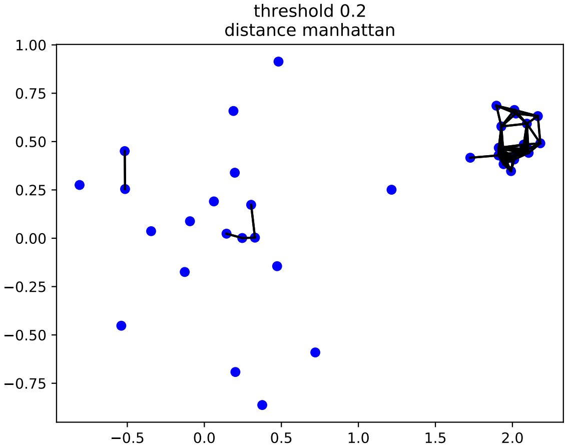
- Euclidian distance and threshold.

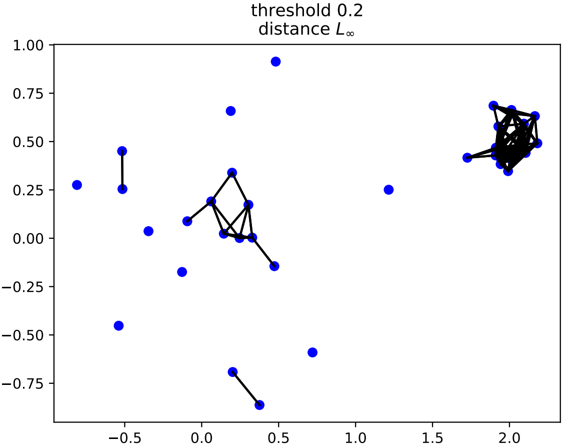
Exercice 1 : Setting a threshold **cd compatibility_simple** and set the threshold used in **build_graph_1.py** to draw relevant edges between the nodes. Feel free to use another dataset !

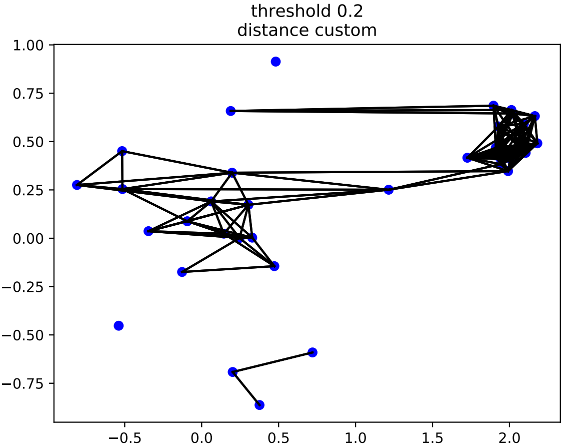


Exercice 2 : Changing the distance

- Assess the impact of changing the distance used. Possible choices :
  - $L1$ distance (Manhattan)
  - $\|\|_\infty$ distance (backboard)
  - custom distance
- use **build_graph_2.py** and edit the distances used at the end of the file.
- Try several values for the threshold.


threshold 0.2
distance manhattan
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−0.5
0.0
0.5
1.0
1.5
2.0


threshold 0.2
distance L_∞
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−0.5
0.0
0.5
1.0
1.5
2.0


threshold 0.2
distance custom
1.00
0.75
0.50
0.25
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−0.5
0.0
0.5
1.0
1.5
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General notion of a distance

- Let us generalize what we experimentally studied.

Examples of distances

$x = (x_1, ..., x_p)$ and $y = (y_1, ..., y_p)$ are p-dimensional **vectors**.

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- L1 : $||x - y||_1 = \sum_{k=1}^{p} |x_k - y_k|$ (Manhattan distance, 1-norm distance)
- weighted L_1 : $\sum_{k=1}^{p} w_k |x_k - y_k|$
- L_∞ : $\max(x_1, \ldots, x_n)$ (infinity norm distance)

Hamming distance

- $\#\{x_i \neq y_i\}$ (Hamming distance)

Hamming distance and edit distance

- $\#\{x_i \neq y_i\}$ (Hamming distance)
- linked to **edit distance** : used to quantify how dissimilar two strings are by counting the number of operations needed to transform one into the other (several variants exist)

General definition of a distance

A **distance** on a set E is an application $d : E \times E \rightarrow \mathbb{R}_+$ that must :

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- **separate the values** : $\forall (x, y) \in E^2, d(x, y) = 0 \Leftrightarrow x = y$
- respect the **triangular inequality** $\forall (x, y, z) \in E^3, d(x, y) \leq d(x, z) + d(z, y)$

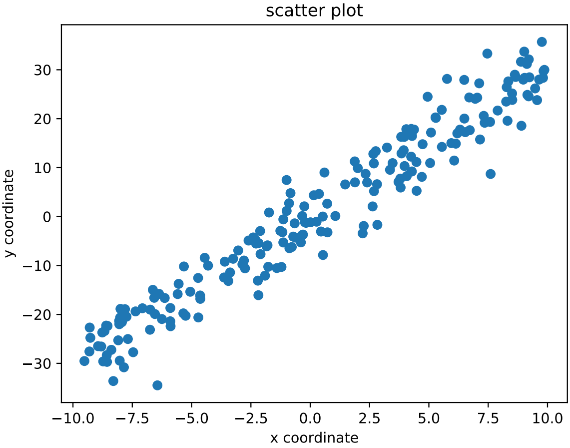
Building compatibility graphs for non geometrical data

- Some data are not geometric
- Some features are not numbers, but could for instance be strings, or categories (categorical data)
- We will use **pandas** process the data from a **csv** file and build a compatibility graph.
- You can use **compatibility_graphs_other_data/build_graph.py** or the notebook.

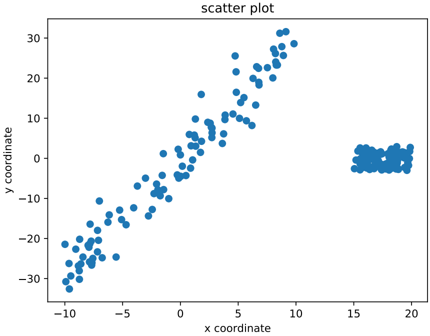
Non geometrical data

Exercice 3 : **Experiment with the data, the threshold in order to build different graphs**

Random variables



Random variables



We want to analyse how the data are **distributed.** For instance the x coordinate, the y coordinate.

Random variables

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- For instance :
  - the result of a dice throw

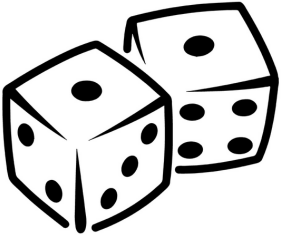

Figure – Dice

Random variables

- A **random variable** is a quantity that can take several values
- For instance :
  - the result of a dice throw
  - waiting time with RATP

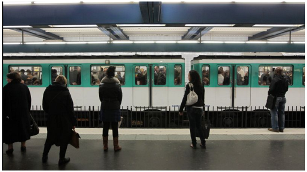

Figure – Some metro station

Random variables

- A **random variable** is a quantity that can take several values
- For instance :
  - the result of a dice throw
  - waiting time with RATP
  - weather

Figure – Weather in November

Random variables

- A **random variable** is a quantity that can take several values
- For instance :
  - the result of a dice throw
  - waiting time with RATP
  - weather
  - number of cars taking the periphérique at the same time

Random variables

What are the differences between these random variables ?

Random variables

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- Some are **continuous**, others **discrete**
- **continuous :**

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- **continuous :** weather, RATP

Random variables

What are the differences between these random variables ?

- Some are **continuous**, others **discrete**
- **continuous** : weather, RATP
- **discrete** : dice (6 possibilities), number of cars (> 10000)

Probability distributions

- A random variable is linked to a **probability distribution**.

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- It quantifies the probability of observing one outcome.

Probability distributions

- A random variable is linked to a **probability distribution**, which is a function P
- It quantifies the probability of observing one outcome.
- For a discrete variable : each possible outcome is associated with a number between 0 and 1

Probability distributions

- For a dice game, the possible outcomes are in the set $\{1, 2, 3, 4, 5, 6\}$
- For a dice game : $P(1) = ?$ $P(2) = ?$ $P(3) = ?$ $P(4) = ?$ $P(5) = ?$ $P(6) = ?$

Probability distributions

- For a dice game, the possible outcomes are in the set $\{1, 2, 3, 4, 5, 6\}$
- For a dice game : $P(1) = \frac{1}{6}$, $P(2) = \frac{1}{6}$, $P(3) = \frac{1}{6}$, $P(4) = \frac{1}{6}$, $P(5) = \frac{1}{6}$, $P(6) = \frac{1}{6}$
- This is called a **uniform distribution**

Probability distributions

- Periphérique :

Probability distributions

- Periphérique : probably a time-dependent very complicated distribution

Continuous variables

- How would you model a continuous variable ? Can you assign a number to a waiting time or a weather ?

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Continuous variables

- How would you model a continuous variable ? Can you assign a number to a waiting time or a weather ?
- One needs to use **probability densities**. Formally, the probably of being between x and $x + dx$ is $p(x)dx$.
- Let's see some examples

Uniform discrete

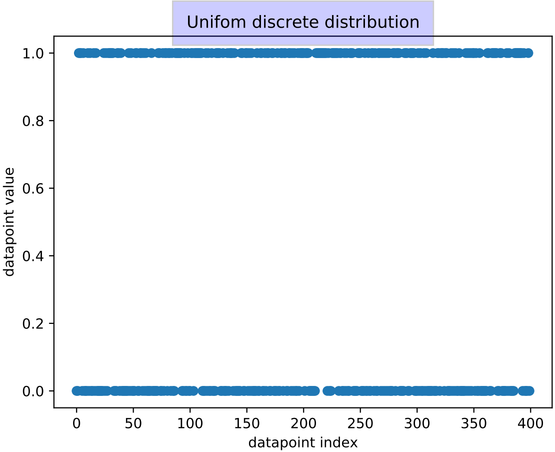

Figure – Uniform discrete distribution with 2 values

Uniform discrete

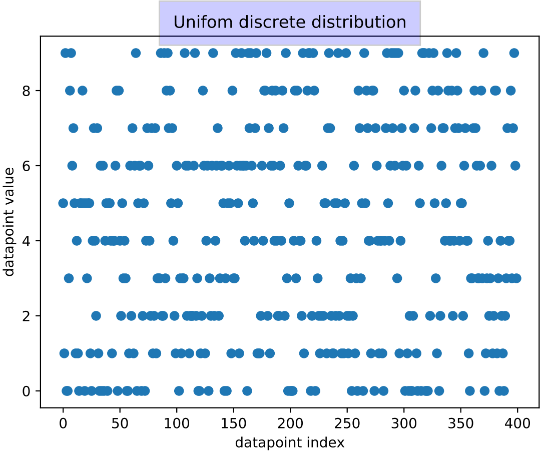

Figure – Uniform discrete distribution with 10 values

Bernoulli

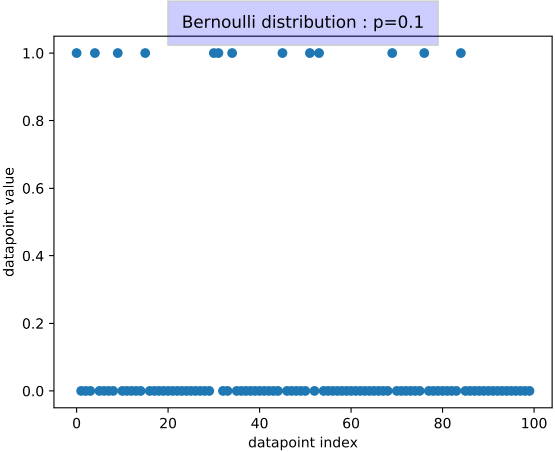

Figure – Bernoulli distribution

Bernoulli p

- With probability p, $X = 1$
- With probability $1 - p$, $X = 0$

Bernoulli

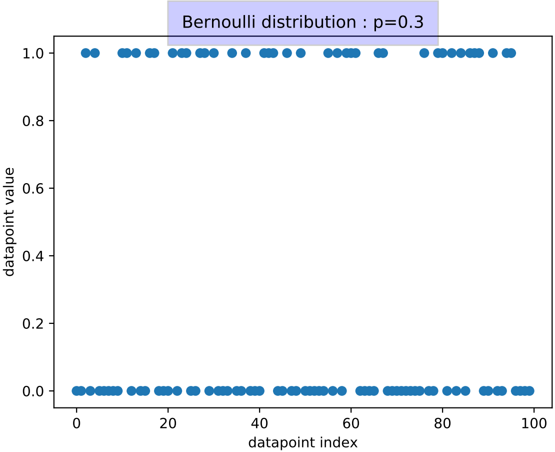

Figure – Bernoulli Distribution

Bernoulli

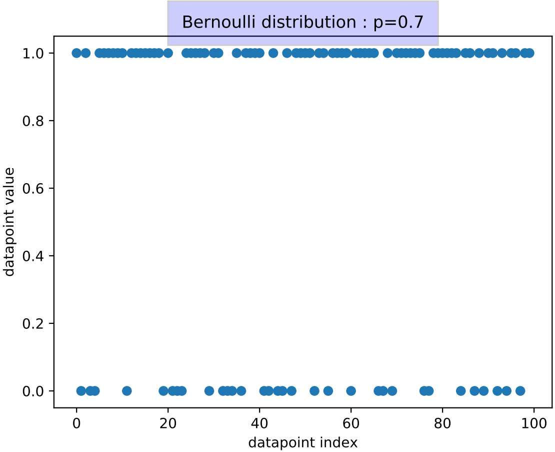

Figure – Bernoulli Distribution

Uniform continuous

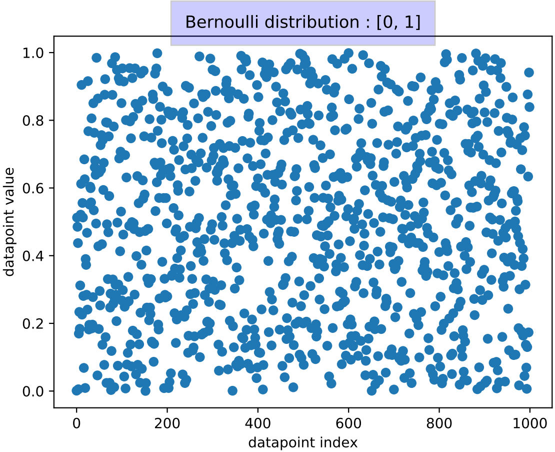

Figure – Uniform continuous distribution

Uniform continuous

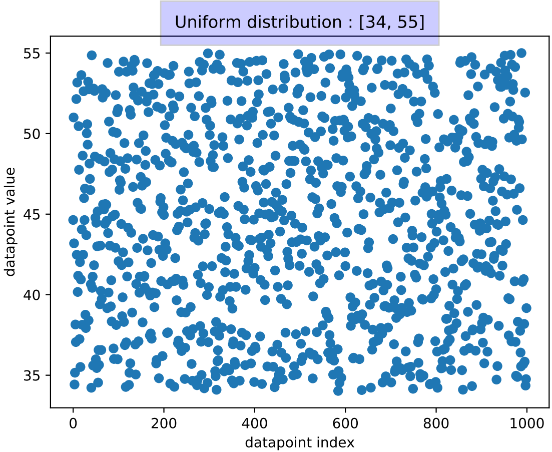

Figure – Uniform continuous distribution

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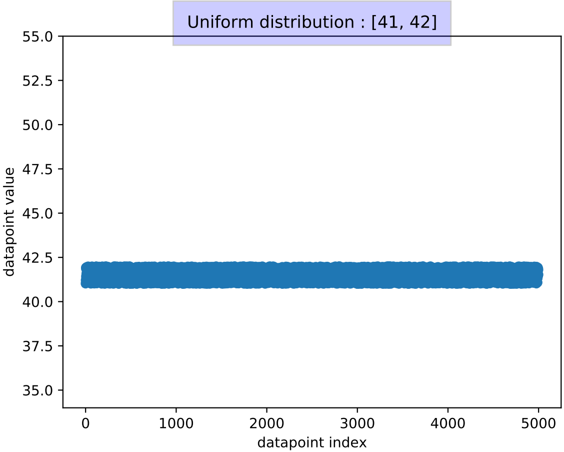

Figure – Uniform continuous distribution

Normal

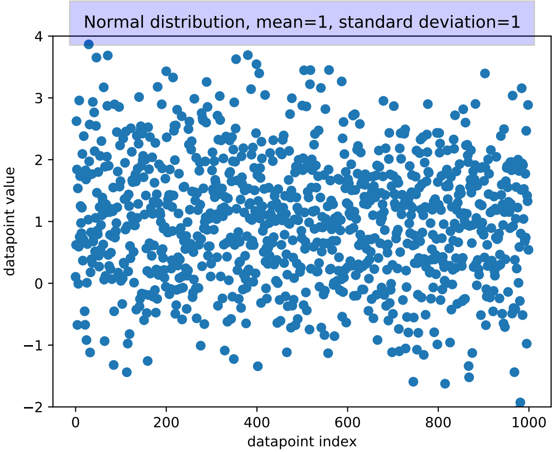

Figure – Normal distribution

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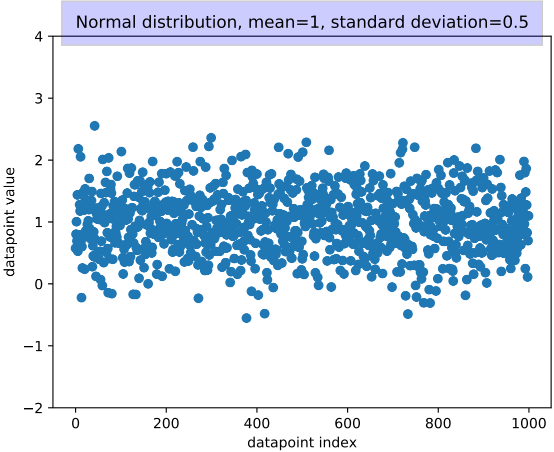

Figure – Normal distribution

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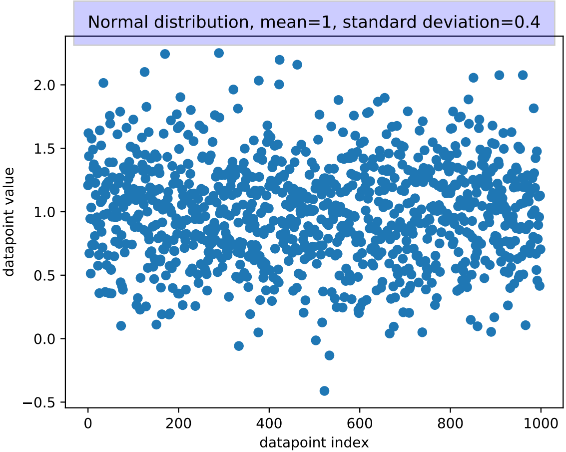

Figure – Normal distribution

White noise

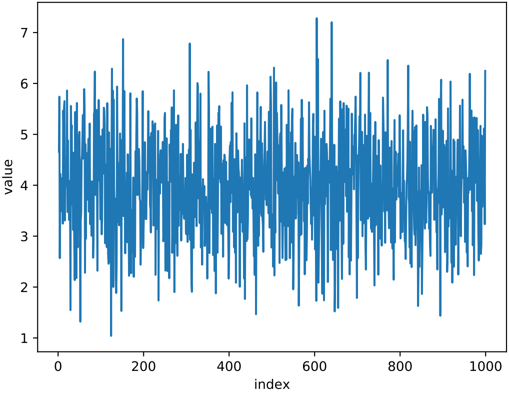

Figure – White noise

Histograms

Is looking at the raw dataset really **informative** ?

Histograms

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It is informative, but often a **histogram** tells more.

Uniform discrete

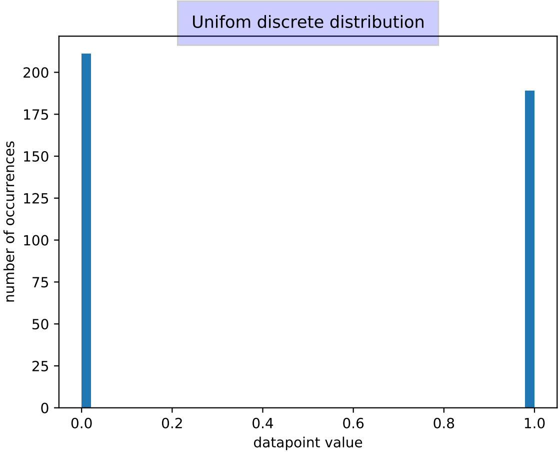

Figure – Historgram 1

Uniform discrete

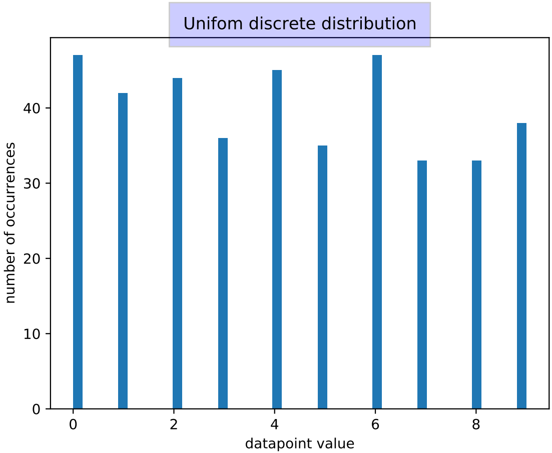

Figure – Historgram 1

Bernoulli

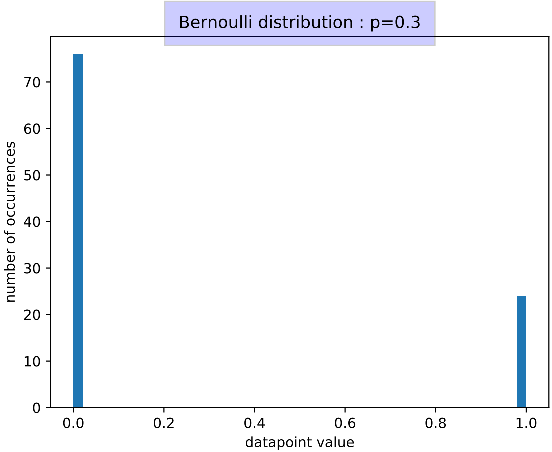

Figure – Historgram 2

Uniform continuous

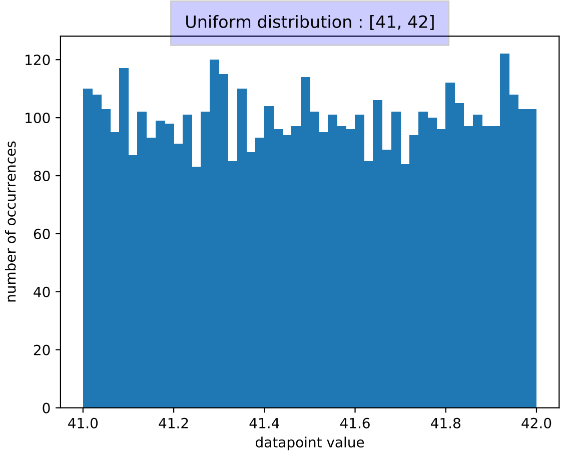

Figure – Historgram 3

Normal

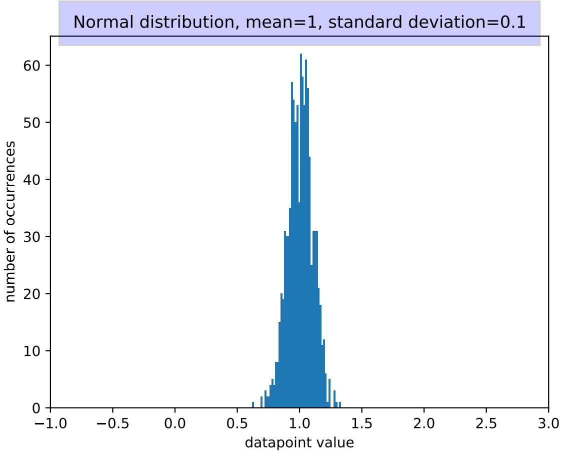

Figure – Historgram 4

Normal

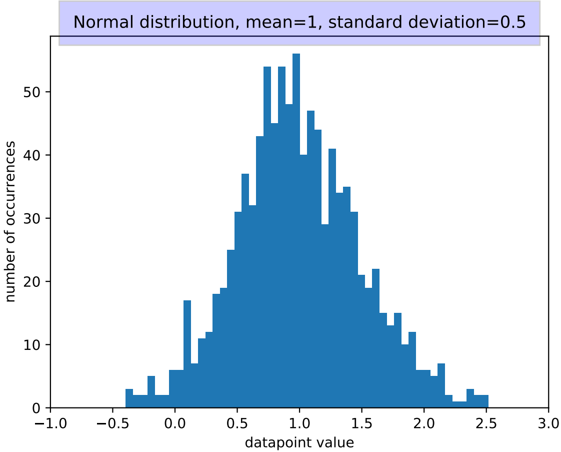

Figure – Historgram 4

Exercice 4 : Analyzing a distribution I put values in the file **mysterious_distro_1.csv**

Exercice 4 : Analyzing a distribution I put values in the file **mysterious_distro_1.csv**
Can you analyze these values in terms of a **distribution** ?
Use **read_myst_1.py** to analyze the distribution (suggestion : change the number of bins used)

Exercice 5 : Analyzing a distribution When you have guessed the kind of distribution it is, you need to finds its **parameters**.

- its mean
- its standard deviation

This is called **fitting** a distribution to a dataset : it's a classical machine learning problem.
To do so, uncomment the last section of the script **read_myst_1.py**

Distribution 1

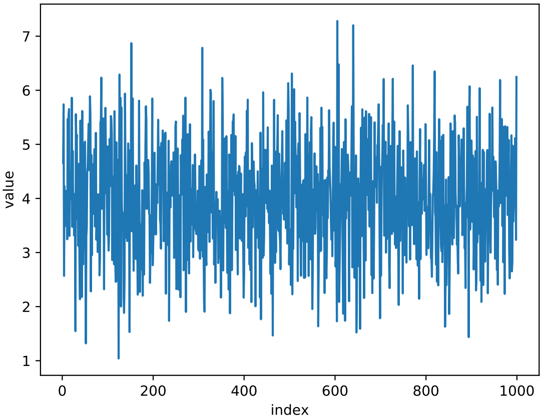

Figure – The data we analyze

histograms

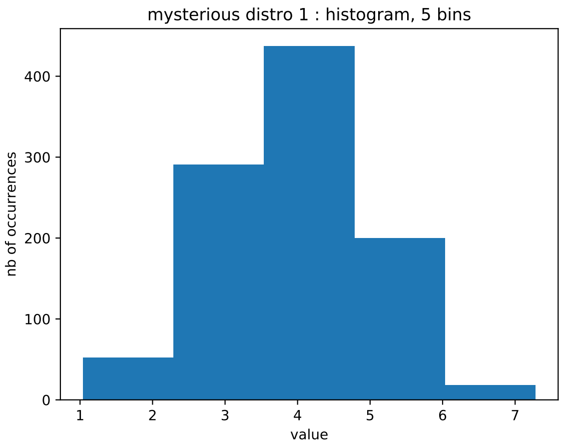

Figure – 5 bins

histograms

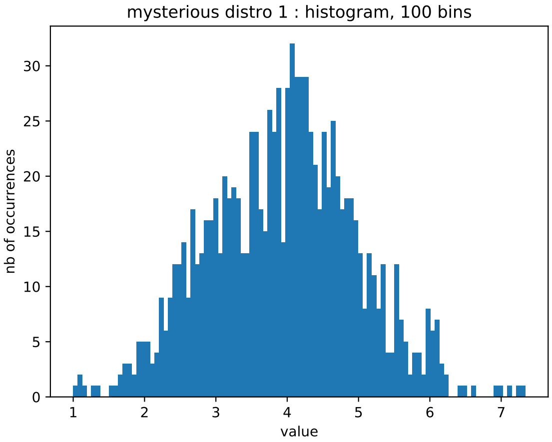

Figure – 100 bins

histograms

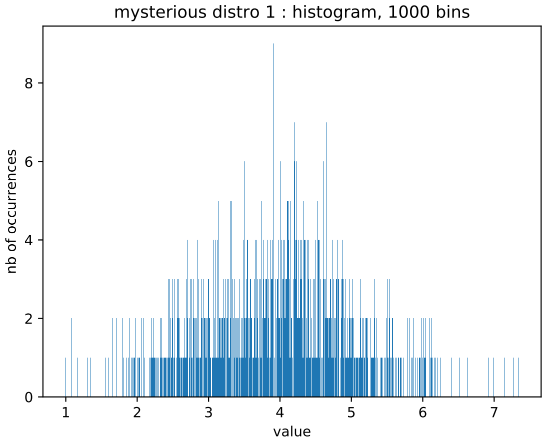

Figure – 1000 bins (too many)

Normal distribution

```python
import csv
import numpy as np

file_name = 'mysterious_distro_1.csv'

mean = 4
std_dev = 1
nb_point = 1000

with open('csv_files/' + file_name, 'w') as csvfile:
    filewriter = csv.writer(csvfile, delimiter=',')
    for point in range(1, nb_point):
        random_variable = np.random.normal(loc=mean, scale=std_dev)
        filewriter.writerow([str(point), str(random_variable)])
```

Figure – **create_normal.py** : Creation of the distribution

Exercice 5 : Second example Let's try to perform the same analysis on the file **mysterious_distro_2.csv** using **read_myst_2.py**.

Second example

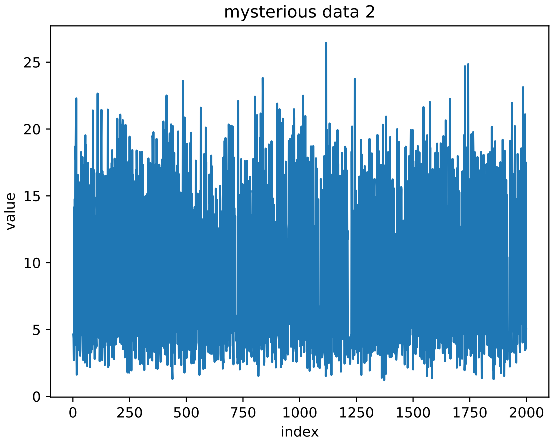

Figure – Second distribution

Multimodal distribution

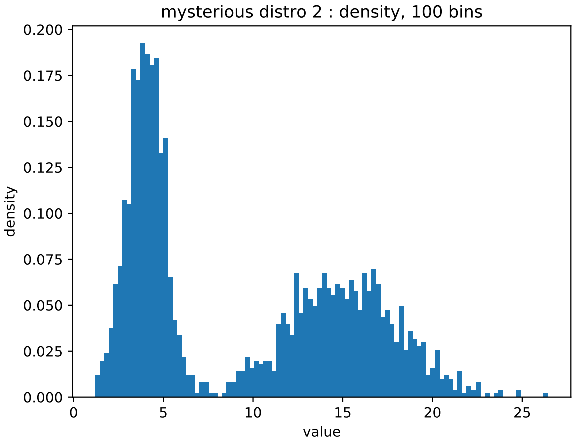

Figure – This distribution has several **modes**

Multimodal distribution

```
mean_1 = 4
std_dev_1 = 1
nb_point_1 = 1000

mean_2 = 15
std_dev_2 = 3
nb_point_2 = 1000

nb_point = nb_point_1 + nb_point_2

with open('csv_files/' + file_name, 'w') as csvfile:
    filewriter = csv.writer(csvfile, delimiter=',')
    for point in range(1, nb_point):
        if random.randint(1, 2) == 1:
            random_variable = np.random.normal(loc=mean_1, scale=std_dev_1)
            filewriter.writerow([str(point), str(random_variable)])
        else:
            random_variable = np.random.normal(loc=mean_2, scale=std_dev_2)
            filewriter.writerow([str(point), str(random_variable)])
```

Figure – **create_bimodal.py** : Generation of multimodal distribution

Fitting

In most cases, it won't be that straightforward to fit a distribution :

Fitting

In most cases, it won't be that straightforward to fit a distribution :

- what distribution do we want to use ?
- even if we know the right shape of the distribution, how to choose the parameters ?

Maximum Likelihood

The **Maximum Likelihood** method is one example method used in Machine Learning.
Say you observe a dataset $(x_1, ..., x_n)$.

Maximum Likelihood

The **Maximum Likelihood** method is one example method used in Machine Learning.
Say you observe a dataset $(x_1, ..., x_n)$.
You first need to choose a **model** (which is the distribution) of your dataset, p.

Maximum Likelihood

The **Maximum Likelihood** method is one example method used in Machine Learning.
Say you observe a dataset $(x_1, ..., x_n)$.
You first need to choose a **model** (which is the distribution) of your dataset, p.
Then, you must optimize the **parameters of this model**, noted θ.

Maximum Likelihood

The **likelihood** (vraisemblance) of your model is

$$L(\theta) = p(x_1, \ldots, x_n|\theta) \tag{1}$$

Maximum Likelihood

The **likelihood** (vraisemblance) of your model is

$$L(\theta) = p(x_1, \dots, x_n | \theta) \tag{2}$$

If $(x_1, \dots, x_n)$ are conditionally independant, then it writes :

$$L(\theta) = \prod_{i=1}^{n} p(x_i | \theta) \tag{3}$$

Maximum Likelihood

The **likelihood** (vraisemblance) of your model is

$$L(\theta) = \prod_{i=1}^{n} p(x_i|\theta) \tag{4}$$

This is the function that you want to **maximise**.

Remark on max-likelihood

Most of the time it's written this way : "minimise $-logL(\theta)$"
Because the log **transforms the product into a sum**, which is easier to **derivate**.

$$-logL(\theta) = -\sum_{i=1}^{n} \log(p(x_i|\theta)) \tag{5}$$

Example 1

Exercice 6 : We observe the data $(1, 0)$. We assume that these data come from a random variable that follows a Bernoulli distribution of parameter p. What is the likelihood of these observations as a function of p ?

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$$L = p(1|p)p(0|p) \tag{6}$$

Example 1

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$$L = p(1|p)p(0|p) \tag{7}$$

For which value of p is this likelihood **maximum** ?

Example 2

Exercice 7 : We observe the data $(2.5, 3.5)$. We assume that these data come from a normal law of parameters μ and σ.
What is the likelihood of (μ, σ) ?

Example 2

Exercice 7 : We observe the data $(2.5, 3.5)$. We assume that these data come from a normal law of parameters μ and σ.

$$L = p(2.5|\mu, \sigma)p(3.5|\mu, \sigma) \tag{8}$$

Example 2

Exercice 7 : We observe the data $(2.5, 3.5)$. We assume that these data come from a normal law of parameters μ and σ.

$$\begin{aligned} L &= p(2.5|\mu,\sigma)p(3.5|\mu,\sigma) \\ &= \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{2.5-\mu}{\sigma}\right)^2} \times \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{3.5-\mu}{\sigma}\right)^2} \end{aligned} \tag{9}$$

Example 2

Exercice 7 : We observe the data $(2.5, 3.5)$. We assume that these data come from a normal law of parameters μ and σ.

$$\begin{aligned} L &= p(2.5|\mu, \sigma)p(3.5|\mu, \sigma) \\ &= \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}(\frac{2.5-\mu}{\sigma})^2} \times \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}(\frac{3.5-\mu}{\sigma})^2} \end{aligned} \tag{10}$$

We wan show that the likelihood is maximum for :

- $\hat{\mu} = \frac{2.5+3.5}{2}$
- $\hat{\sigma}^2 = \frac{(2.5-\hat{\mu})^2+(3.5-\hat{\mu})^2}{2}$

Max Likelihood

In the case of very large datasets, and large numbers of parameters (tens, hundredths, more), most of the time an **analytic solution** is not available.

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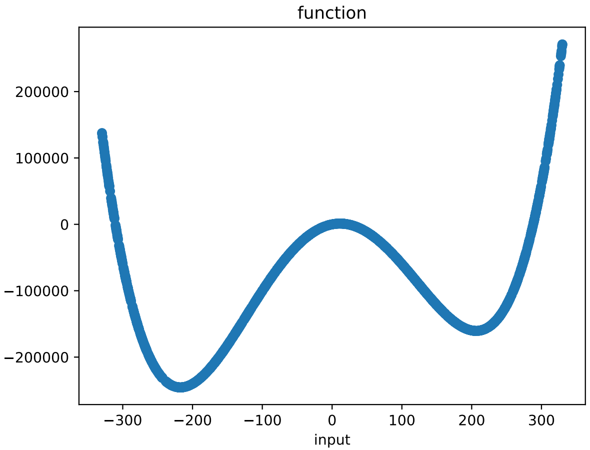
Max Likelihood

In the case of very large datasets, and large numbers of parameters (tens, hundredths, more), most of the time an **analytic solution** is not available. So how can we **maximize** the likelihood ?
Most common method : **gradient descent.**

Notion of optimization



Gradient descent

- In the case a function $f : \mathbb{R} \to \mathbb{R}$, we can study its variations by computing its derivative f', **if it exists**

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Gradient descent

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- If $f'(x) > 0$, the function grows around x.
- If $f'(x) < 0$, the function decreases around x.
- If x is a local extremum, $f'(x) = 0$
- Is the reciprocal true ?

Gradient

- The **gradient** is similar to a derivative but in the case of a function with several inputs, such as (μ, θ).
- Then we store the **partial derivative** with respect to each input in a **vector** called the gradient.

Gradient descent

Consider a function f that has 2 parameters as inputs.

$$\nabla_f(x, y) = (\frac{\delta f}{\delta x}, \frac{\delta f}{\delta y}) \tag{11}$$

We want x to **minimise** f. We perform, until some criteria is satisfied :

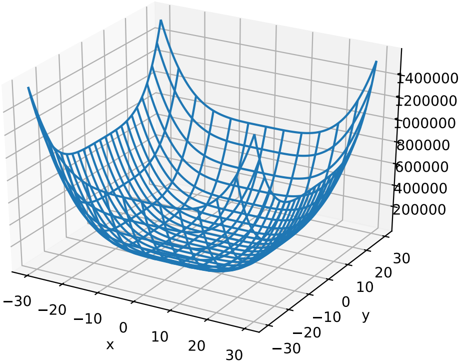
$$x \leftarrow x - \alpha \nabla_f(x) \tag{12}$$

α is a small parameter called the learning rate.

Gradient

Exercice 7 : Implementing the gradient algorithm

We will use the algorithm on two functions.



Gradient

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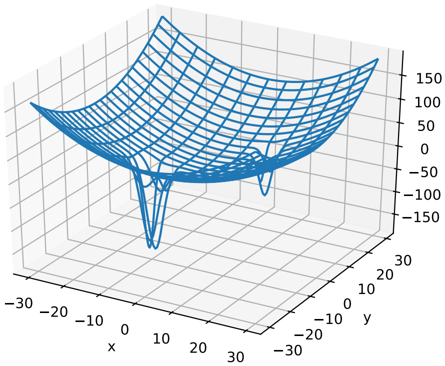
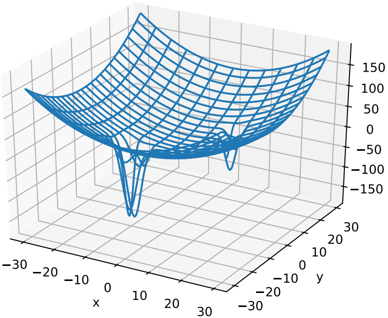


Gradient

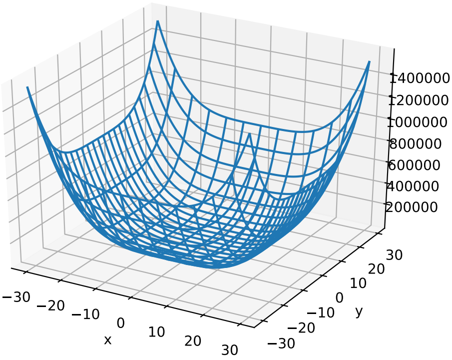
Exercice 7 : **Implementing the gradient algorithm**

cd ./gradient and use the files **gradient.py** and **gradient_2.py** in order to implement the algorithm to find **minima**.



The gradient descent

Experiment with it, try to change all the parameters and to break it again. Is it stable ?



Multidimensional vectors

We can consider data that live in higher dimensional spaces than 2.

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Examples ?

Multidimensional vectors

We can consider data that live in higher dimensional spaces than 2. Examples ?

- images
- sensor that receives **multimodal information**

Correlation

Sometimes the components of a multidimensonial vector $(x_1, ..., x_n)$ are not independent.

Correlation

Sometimes the components of a multidimensonial vector $(x_1, \dots, x_n)$ are not independent.
To study this, we can use the **covariance** of the two components, or the **correlation** which is actually clearer.

Expected value (espérance)

- For a discrete random variable X that takes the values x_i with probability p_i :

$$E(X) = \sum_{i=1}^{n} p_i x_i \tag{13}$$

- For a continuous random variable X with density p(x) :

$$E(X) = \int x p(x) dx \tag{14}$$

Expected value (espérance)

Exercice 7 : Computing an expected value

- For a discrete random variable X that takes the values x_i with probability p_i :

$$E(X) = \sum_{i=1}^{n} p_i x_i \tag{15}$$

- For a continuous random variable X with density p(x) :

$$E(X) = \int xp(x)dx \tag{16}$$

Compute the expected value of the dice game.

Variance

$$var(X) = E\left(\left(X - E(X)\right)^2\right) \tag{17}$$

Variance and Covariance

$$var(X) = E\left((X - E(X))^2\right) \quad (18)$$

$$cov(X, Y) = E\left((X - E(X))(Y - E(Y))\right) \quad (19)$$

Example

Look at the data contained in **mysterious_distro_3.csv**
They contain a random variable with 5 dimensions. Some of these dimensions are correlated.
Think for instance to physics : temperature and pressure, etc. If you have measurements of temperature and pressure, the two would probably be **correlated**.

Correlation

Exercice 8 : Which dimensions of the distribution are correlated ?

Correlation matrix

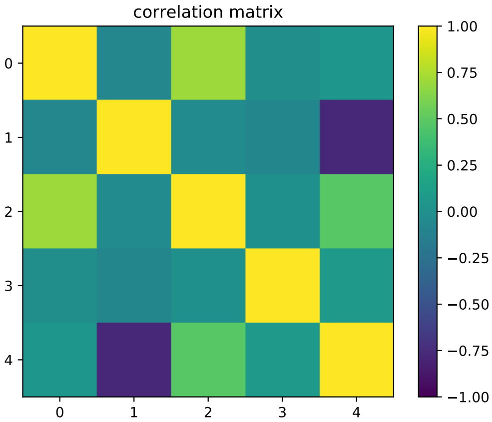

Figure – Correlation matrix for the distribution

Generation of the data

```
mean_1 = 4
std_dev_1 = 1

mean_2 = 15
std_dev_2 = 3

mean_3 = -5
std_dev_3 = 2

mean_noise = 0
noise_std_dev = 1

nb_point = 1000

with open('csv_files/' + file_name, 'w') as csvfile:
    filewriter = csv.writer(csvfile, delimiter=',')
    for point in range(1, nb_point):
        noise = np.random.normal(loc=mean_noise, scale=noise_std_dev)
        random_variable_1 = np.random.normal(loc=mean_1, scale=std_dev_1)
        random_variable_2 = np.random.normal(loc=mean_2, scale=std_dev_2)
        random_variable_3 = random_variable_1 + noise
        random_variable_4 = np.random.normal(loc=mean_3, scale=std_dev_3)
        random_variable_5 = -0.4 * random_variable_2 + noise
        filewriter.writerow([str(point),
                             str(random_variable_1),
                             str(random_variable_2),
                             str(random_variable_3),
                             str(random_variable_4),
                             str(random_variable_5)])
```

Figure – Multidimensional random variable

Removing dimensions

- Sometimes given a question and a dataset, not all dimensions of the data are **relevant**
- It is possible that only one or two of them are sufficient to answer the given question

Removing dimensions

- Sometimes given a question and a dataset, not all dimensions of the data are **relevant**
- It is possible that only one or two of them are sufficient to answer the given question
- We will illustrate this with the titanic dataset and the pandas library

Titanic Dataset

- The dataset contains the list of passengers and several informations on each of them :

PassengerId	Survived	Pclass	Name	Sex	Age	SibSp	Parch	Ticket	Fare	Cabin	Embarked
1	0	3	Braund, Mr. Owen Harris	male	22	1	0	A/5 21171	7.25		S
2	1	1	Cumings, Mrs. John Bradley (Florence Briggs Thayer)	female	38	1	0	PC 17599	71.2833	C85	C
3	1	3	Heikkinen, Miss. Laina	female	26	0	0	STON/O2. 3101282	7.925		S
4	1	1	Futrelle, Mrs. Jacques Heath (Lily May Peel)	female	35	1	0	113803	53.1	C123	S
5	0	3	Allen, Mr. William Henry	male	35	0	0	373450	8.05		S
6	0	3	Moran, Mr. James	male		0	0	330877	8.4583		Q
7	0	1	McCarthy, Mr. Timothy J	male	54	0	0	17463	51.8625	E46	S
8	0	3	Palsson, Master. Gosta Leonard	male	2	3	1	349909	21.075		S
9	1	3	Johnson, Mrs. Oscar W (Elisabeth Vilhelmina Berg)	female	27	0	2	347742	11.1333		S
10	1	2	Nasser, Mrs. Nicholas (Adele Achem)	female	14	1	0	237736	30.0708		C
11	1	3	Sandstrom, Miss. Marguerite Rut	female	4	1	1	PP 9549	16.7	G6	S
12	1	1	Bonnell, Miss. Elizabeth	female	58	0	0	113783	26.55	C103	S
13	0	3	Saundercock, Mr. William Henry	male	20	0	0	A/5. 2151	8.05		S
14	0	3	Andersson, Mr. Anders Johan	male	39	1	5	347082	31.275		S
15	0	3	Vestrom, Miss. Hulda Amanda Adolfina	female	14	0	0	350406	7.8542		S
16	1	2	Hewlett, Mrs. (Mary D Kingcome)	female	55	0	0	248706	16		S
17	0	3	Rice, Master. Eugene	male	2	4	1	382652	29.125		Q
18	1	2	Williams, Mr. Charles Eugene	male		0	0	244373	13		S
19	0	3	Vander Planke, Mrs. Julius (Emelia Maria Vandemoortele)	female	31	1	0	345763	18		S
20	1	3	Masselmani, Mrs. Fatima	female		0	0	2649	7.225		C
21	0	2	Fynney, Mr. Joseph J	male	35	0	0	239865	26		S
22	1	2	Beesley, Mr. Lawrence	male	34	0	0	248698	13	D56	S
23	1	3	McGowan, Miss. Anna "Annie"	female	15	0	0	330923	8.0292		Q
24	1	1	Sloper, Mr. William Thompson	male	28	0	0	113788	35.5	A6	S
25	0	3	Palsson, Miss. Torborg Danira	female	8	3	1	349909	21.075		S
26	1	3	Asplund, Mrs. Carl Oscar (Selma Augusta Emilia Johansson)	female	38	1	5	347077	31.3875		S
27	0	3	Emir, Mr. Farred Chehab	male		0	0	2631	7.225		C
28	0	1	Fortune, Mr. Charles Alexander	male	19	3	2	19950	263	C23 C25 C27	S
29	1	3	O'Dwyer, Miss. Ellen "Nellie"	female		0	0	330959	7.8792		Q
30	0	3	Todoroff, Mr. Lalio	male		0	0	349216	7.8958		S
31	0	1	Uruchurtu, Don. Manuel E	male	40	0	0	PC 17601	27.7208		C
32	1	1	Spencer, Mrs. William Augustus (Marie Eugenie)	female		1	0	PC 17569	146.5208	B78	C
33	1	3	Glynn, Miss. Mary Agatha	female		0	0	335677	7.75		Q
34	0	2	Wheadon, Mr. Edward H	male	66	0	0	C.A. 24579	10.5		S
35	0	1	Meyer, Mr. Edgar Joseph	male	28	1	0	PC 17604	82.1708		C
36	0	1	Holverson, Mr. Alexander Oskar	male	42	1	0	113789	52		S
37	1	3	Mamee, Mr. Hanna	male		0	0	2677	7.2292		C
38	0	3	Cann, Mr. Ernest Charles	male	21	0	0	A./5. 2152	8.05		S
39	0	3	Vander Planke, Miss. Augusta Maria	female	18	2	0	345764	18		S
40	1	3	Nicola-Yarred, Miss. Jamila	female	14	1	0	2651	11.2417		C

Titanic Dataset

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PassengerId	Survived	Pclass	Name	Sex	Age	SibSp	Parch	Ticket
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3	1	3	Heikkinen, Miss. Laina	female	26	0	0	STON/O2. 31
4	1	1	Futrelle, Mrs. Jacques Heath (Lily May Peel)	female	35	1	0	113803
5	0	3	Allen, Mr. William Henry	male	35	0	0	373450
6	0	3	Moran, Mr. James	male		0	0	330877
7	0	1	McCarthy, Mr. Timothy J	male	54	0	0	17463

Figure – Titanic dataset : can be found here `https://www.kaggle.com/c/titanic/data`

Pandas

- The pandas library is used to study large datasets with python
- We will use the **Dataframe** structure to process the titanic dataset
- **cd multivariate_analysis/titanic/** and use the file **pandas_infos.py** to load the dataset to a dataframe and print general information on the dataframe.

Pandas

```
---
all info on passenger 25
PassengerId                                                  26
Survived                                                      1
Pclass                                                        3
Name           Asplund, Mrs. Carl Oscar (Selma Augusta Emilia...
Sex                                                      female
Age                                                          38
SibSp                                                         1
Parch                                                         5
Ticket                                                   347077
Fare                                                    31.3875
Cabin                                                       NaN
Embarked                                                      S
Name: 25, dtype: object
---
age of passenger 25
38.0
```

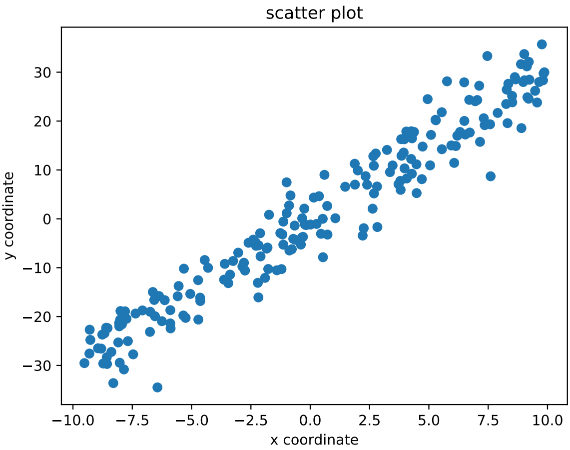
Figure – Some passenger that survived

Exercice 9 : Prediction

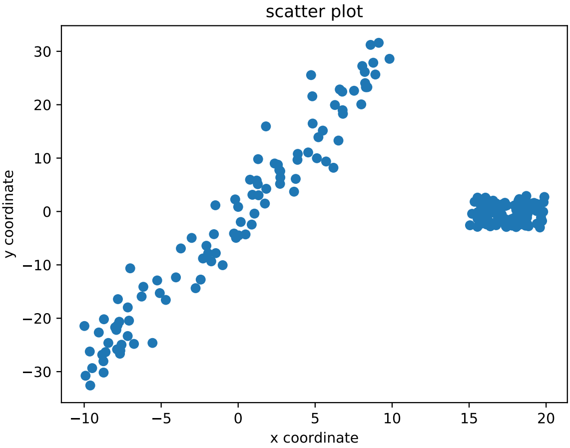
We would like to know if there is a criterion that can help to predict if a passenger survived or not.

Ideally, would would like to remove dimensions that are useless for this problem.

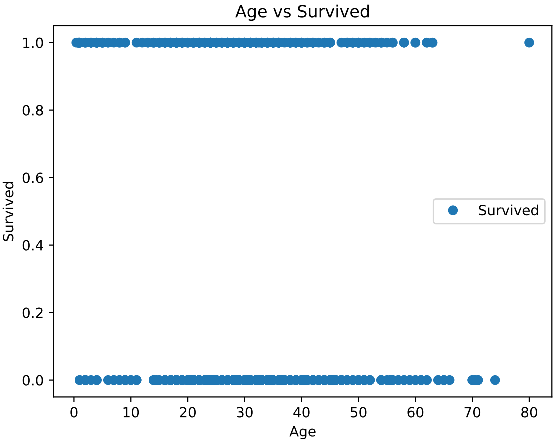
Scatter plot (nuage de points)

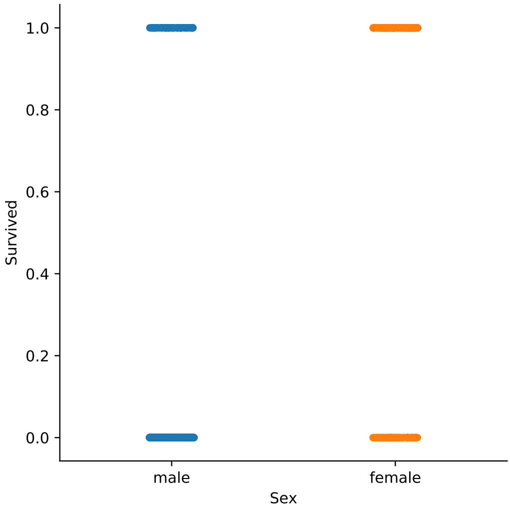


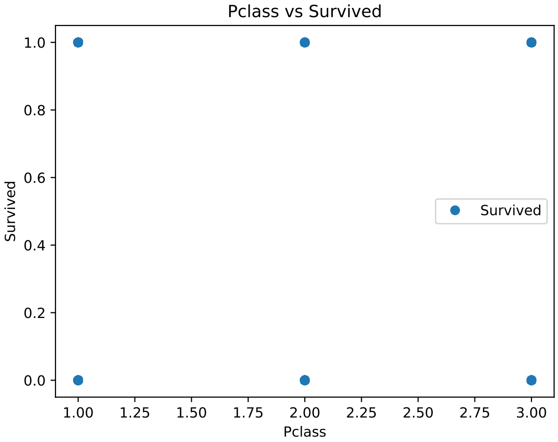
Scatter plot (nuage de points)



Exercice 9 : Prediction
cd multivariate_analysis/titanic/.
Use the file **scatter_titanic.py** to see if one column if sufficient to predict survival. We use the **seaborn** lib.


Age vs Survived
1.0
0.8
0.6
0.4
0.2
0.0
Survived
0
10
20
30
40
50
60
70
80
Age
Survived


1.0
0.8
0.6
0.4
0.2
0.0
Survived
male
female
Sex


Pclass vs Survived
1.0
0.8
0.6
0.4
0.2
0.0
Survived
1.00
1.25
1.50
1.75
2.00
2.25
2.50
2.75
3.00
Pclass
Survived

Exercice 9 : Prediction

How could we plot 3 variables on the same graph ?

Exercice 9 : Prediction

How could we plot 3 variables on the same graph ?

Use the file **scatter_titanic_color.py** in order to color the datapoint in a $2D$ space as a function of survival.

Exercice 9 : Prediction

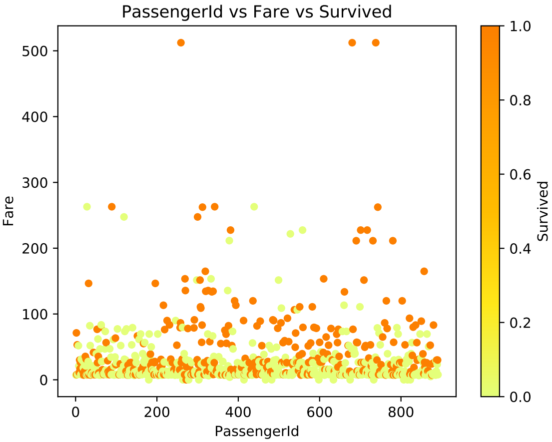


Figure – Not much information

Exercice 9 :

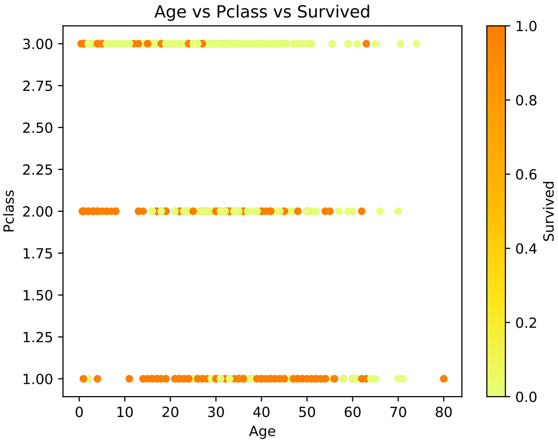


Figure – There seems to be a link between class and survival

Remark : we did not have to use pandas to solve this problem.

Warning

- Some times removing dimensions can lead to a **misinterpretation**

Exercice 10 :

cd multivariate_analysis/causality/.

- Load the dataset **grades.csv** and study it using **process_grades.py**
- What columns are correlated ?

Exercice 10 :

- **cd causality**
- Load the dataset **grades.csv** and study it using **process_grades.py**
- What columns are correlated ?
- Plot the grade as a function of the height.

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- Load the dataset **grades.csv** and study it using **process_grades.py**
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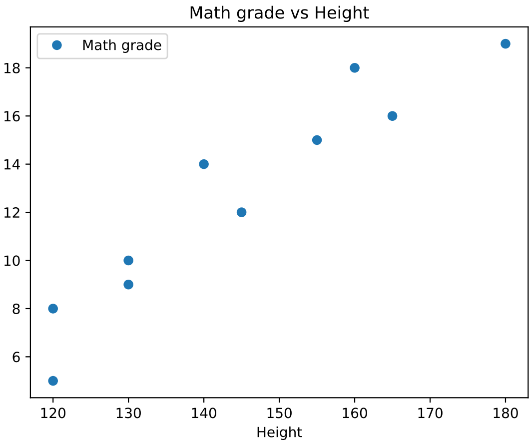
- Plot the grade as a function of the age.

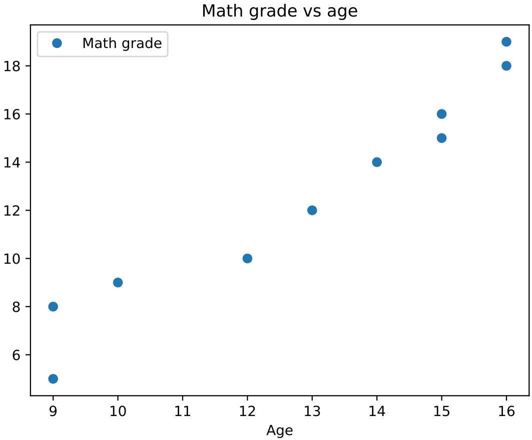
Exercice 10 :

- Load the dataset **grades.csv** and study it using **process_grades.py**
- What columns are correlated ?
- Plot the grade as a function of the height.
- Plot the grade as a function of the age.
- Which one of these plots does not make sense ?

Correlation and causality



Correlation and causality

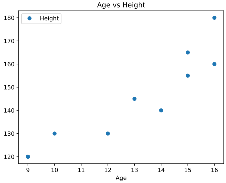


Conclusion

- The height of a student is actually not linked to his/her grade
- If we plot the grade as a function of the height There is a **hidden variable** which is the age of the student
- Correlation is different than causality.

Scatter matrix

- Let us generalize the idea of a scatter plot.



- Given a dataset, we can build all the possible scatter plots and store them in a matrix, called the **scatter matrix**.

Scatter matrix

- Let us generalize the idea of a scatter plot.

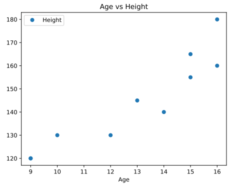


Figure – Exemple scatter plot

- Given a dataset, we can build all the possible scatter plots and store them in a matrix, called the **scatter matrix**.
- What will happen on the diagonal of the matrix ?

Scatter matrix

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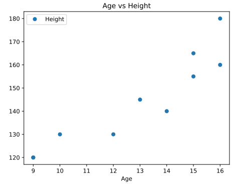

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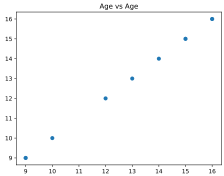


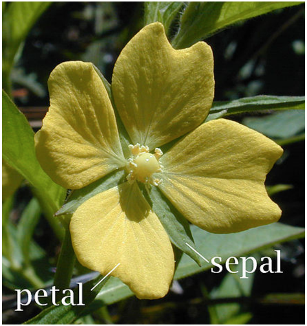
Figure – Variable plotted against itsself : All the points are on the $y = x$ line

Scatter matrix

- Let us generalize the idea of a scatter plot.
- Given a dataset, we can build all the possible scatter plots and store them in a matrix, called the **scatter matrix**.
- On the diagonal, one can plot histograms or the density probability
- The scatter plot can be a good way to start analyzing a dataset when we don't know which variables could be correlated

Iris dataset

- 150 samples of iris flower
- 3 species
- 4 attributes : petal width and length, sepal width and length



Exercice 11 : Iris dataset : scatter matrix

cd multivariate_analysis/titanic/.

- use **iris_scatter_matrix.py** to plot the scatter matrix of the iris dataset with seaborn.
- Is there a variable that can discriminate between the species ?

Exercice 11 : Iris dataset : scatter matrix

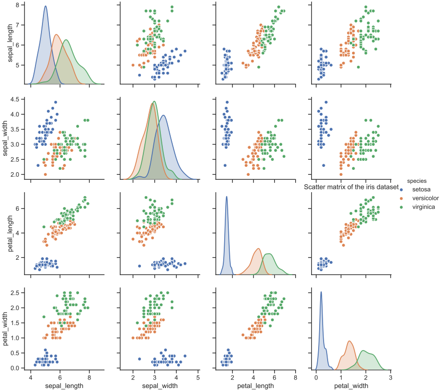


Figure – Plotted in **iris_scatter_matrix.py**

Exercice 11 : Iris dataset : scatter matrix

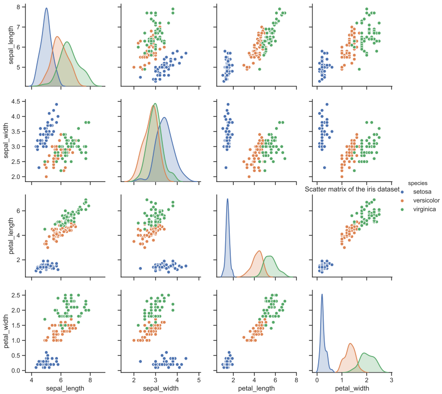


Figure – It seems that **petal width** is a parameter that separates the three species. On the contrary, **sepal width** is not able to do so.

Titanic dataset : scatter matrix

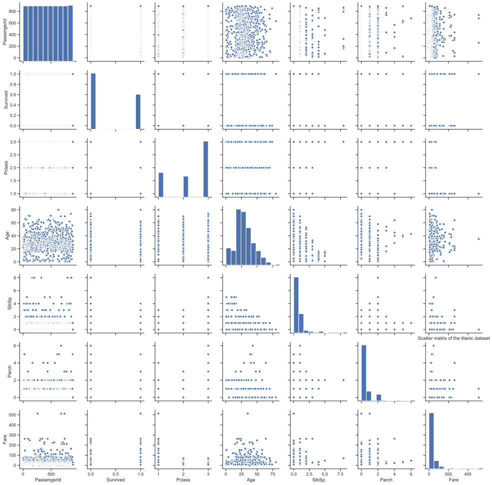
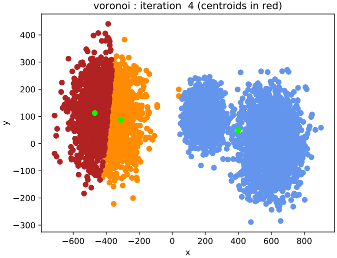


Figure – Titanic dataset scatter matrix

K means clustering

- A famous unsupervised clustering method



-

Figure – K means clustering

Kmeans

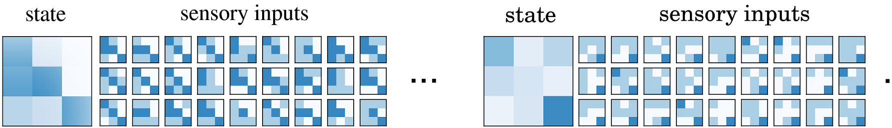


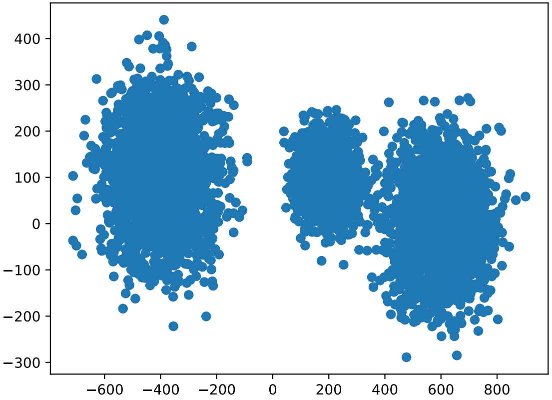
Figure – Other example of kmeans clustering, this time in 9 dimensions [Le Hir et al., 2018]

Kmeans : Expectation Maximisation algorithm

- Classical Machine Learning algorithm (EM)
- Blackboard
- What could be the drawbacks of this algorithm ?

Kmeans clustering

Exercice 12 : Implementing kmeans



Kmeans clustering

Exercice 12 : **Implementing kmeans cd ./kmeans**

- Modify the **k_means.py** file so that it performs the kmeans algorithm.
- There are **two mistake series** :
  - line 64
  - around line 84

  you will need to fix them.

You should obtain something like this :

Kmeans

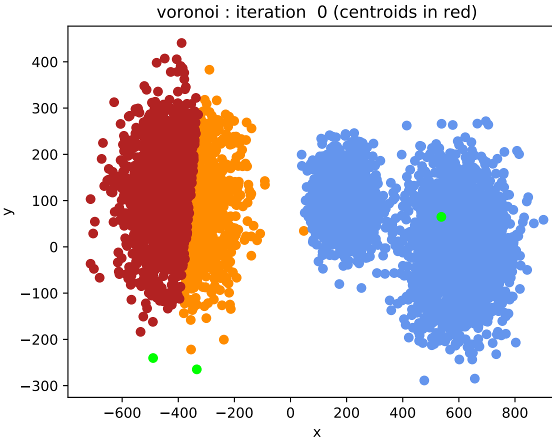


Figure – Voronoi 0th iteration

Kmeans

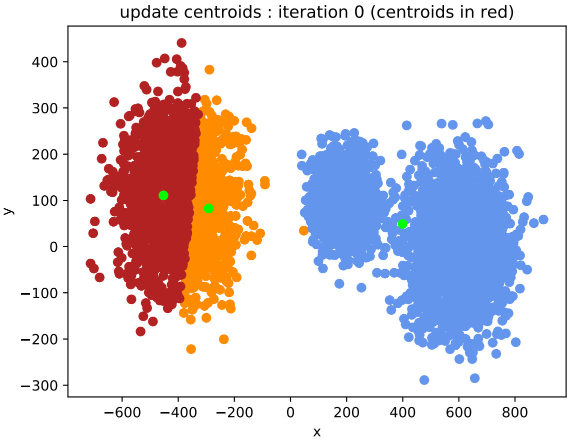


Figure – Centroids 0th iteration

Kmeans

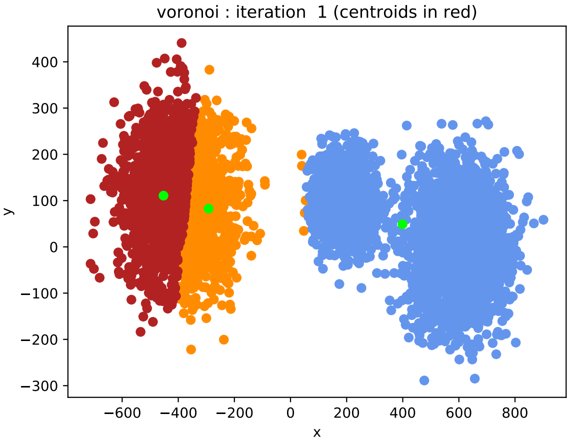


Figure – Voronoi 1st iteration

Kmeans

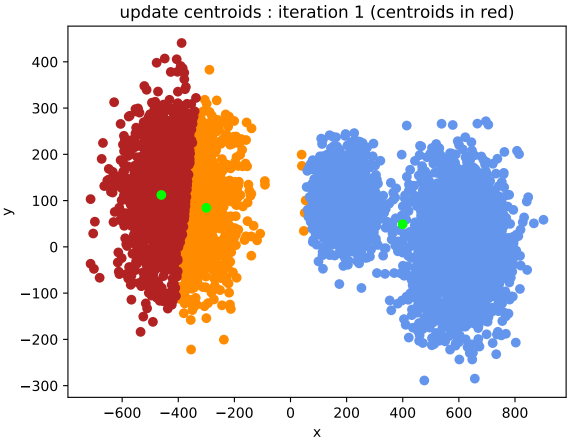


Figure – Centroids 1st iteration

Kmeans and initialization

Note that when launching the algorithm several times, the result may differ.

Sklearn

Exercice 13 : Perform the kmeans algorithm using **sklearn**.

Similarities

- The kmeans were based on a notion of **distance between points**

Similarities

- The kmeans were based on a notion of **distance between points**
- But sometimes you do not have access to a distance between the points (like this morning, when we introduced **dissimilarities**)

Similarities

- The kmeans were based on a notion of **distance between points**
- But sometimes you do not have access to a distance between the points.
- You might need to work with something that is more general, for instance a **similarity**.

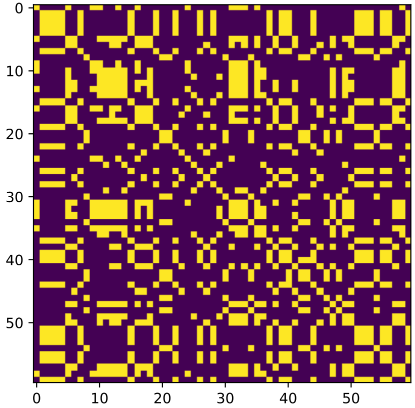
Similarities

- When working with distances, two points that "look the same" should be separated by a **small distance** .
- When working with a similarity, two points that "look the same" should have a **high similarity**.

Example of similarity : adjacency

- An example of similarity is the relationship of **adjacency**.
- If i and j are related by an edge, $S_{ij} = 1$.
- Otherwise $S_{ij} = 0$.

Adjacency matrix



Similarities

Differences between similarities and distances :

- A similarity S is not always symmetrical.

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- Indeed, in a **directed graph**, having a directed edge between i and j does not mean that we have an edge between j and i.

Similarities

Differences between similarities and distances :

- A similarity S is not always symmetrical.
- Indeed, in a **directed graph**, having a directed edge between i and j does not mean that we have an edge between j and i.
- $S_{ij} = 0$ does not mean that $i = j$, it is rather the contrary.

Similarities

- A similarity is a more general notion than a distance. Given a distance between two points, we can deduce a similarity.

Similarities

- A similarity is a more general notion than a distance. Given a similarity between two points, we can deduce a similarity.
- For instance this way, if d_{ij} is the distance betwwen i and j :

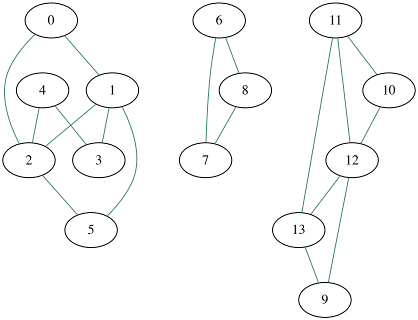
$$S_{ij} = \exp(-d_{ij}) \tag{20}$$

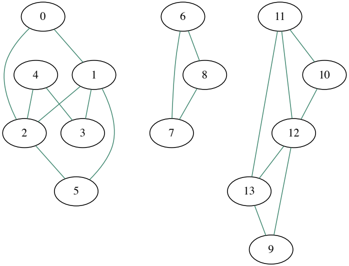
Spectral Clustering

- A clustering method that works with similarities
- It performs a low dimensional embedding of the similarity matrix, followed by a Kmeans

Exercise

We will perform Spectral Clustering on this graph :





Please **cd spectral_clustering/** and use **vanilla_spectral_clustering.py** in order to apply spectral clustering. You first need to input the right **affinity matrix** or **similarity matrix** and then use the **sklearn** library. You also need to **tune the number of clusters. doc** : check the **sklearn** page for Spectral Clustering.

Spectral clustering

Can you guess some drawbacks of the method ?

Spectral clustering

Can you guess some drawbacks of the method ?

- Need to provide the number of clusters.
- Not adapted to a large number of clusters.
- kmeans step : so depends on a random initialization.

Heuristic

- We would like a critetion in order to justify the number of clusters used.

Normalized cut : a measurement of the quality of a clustering

- The **cut of a cluster** is the number of outside connections (connections with other clusters).
- The **degree** of a node is its number of adjacent edges
- The **degree of a cluster** is the sum of the degrees of its nodes.
- The **normalized cut** of a clustering is :

$$NCut(\mathcal{C}) = \sum_{k=1}^{K} \frac{Cut(C_k, V \backslash C_k)}{d_{C_k}} \tag{21}$$

Normalization

- The normalization is useful in order to take the **weight** (degree) of a cluster into account.

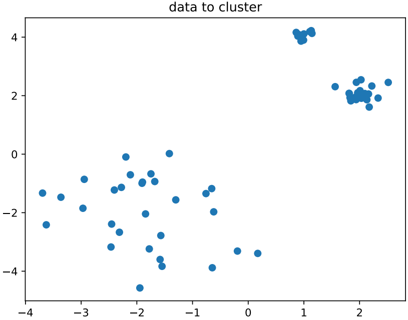
Normalized cut and clustering

Let's see how the normalized cut can help us choose the right number of clusters (backboard).

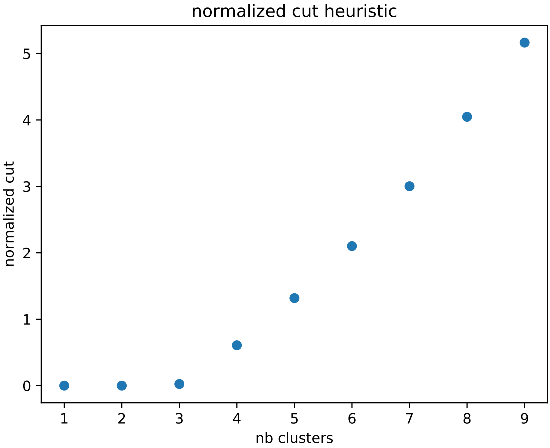
Heuristic

Exercice 14 : Normalized but elbow :

Please use the criterion in the file **normalized_cut.py** in order to guess the relevant number of clusters in order to process the data contained in **data/**. These data are generated by **generate_data.py**.



Normalized cuts



Normalized cuts

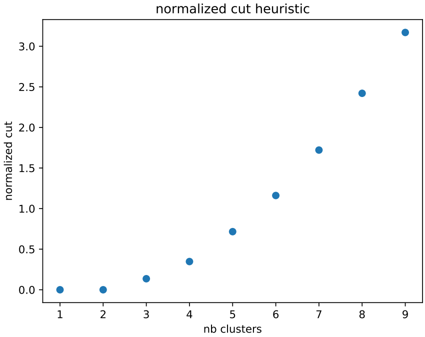
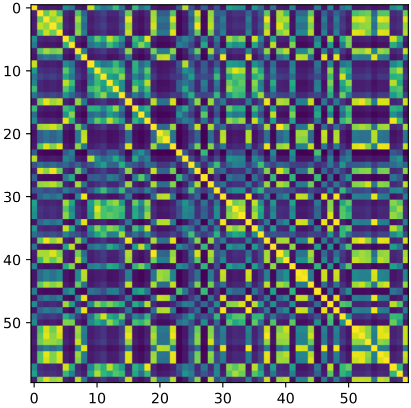


Figure – If the standard deviations in the dataset are larger, it is harder to identify a relevant number of clusters.

Similarity



Example

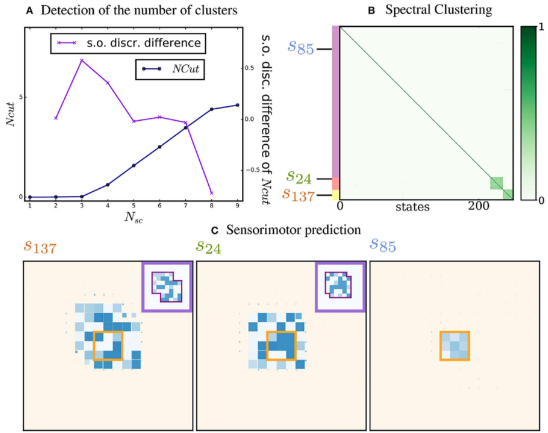


Figure – In a), the elbow method is used to choose the number of clusters. [Le Hir et al., 2018]

Other methods to evaluate the quality of a clustering

- Stability of the result when lauching the algorithm many times
- Separation of the clusters (the mean distance between pairs of centroids is large)
- Ratio inter / intra
- Silhouette coefficient

Other interesting notions

- Agglomerative clustering (CHA : classification Hierarchique Ascendante)
- Xmeans : improvement of k means
- If you know more about probabilities :
  - Latent variables and variational learning
  - Auto Encoders
  - Boltzmann Machines

Project

- Desciption of the project

Questions ?

References

Le Hir, N., Sigaud, O., and Laflaquière, A. (2018).
Identification of Invariant Sensorimotor Structures as a Prerequisite for the Discovery of Objects.
Frontiers in Robotics and AI, 5(June) :1–14.