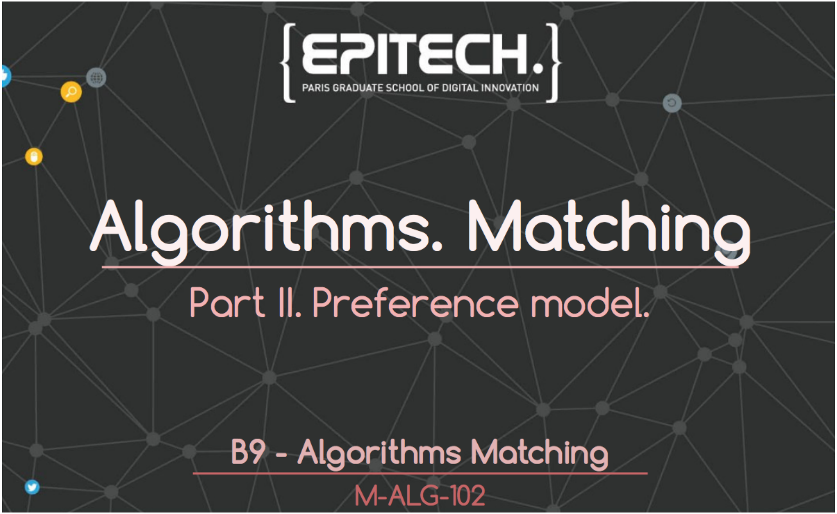

EPITECH.

PARIS GRADUATE SCHOOL OF DIGITAL INNOVATION

Algorithms. Matching

Part II. Preference model.

B9 - Algorithms Matching

M-ALG-102

Introduction

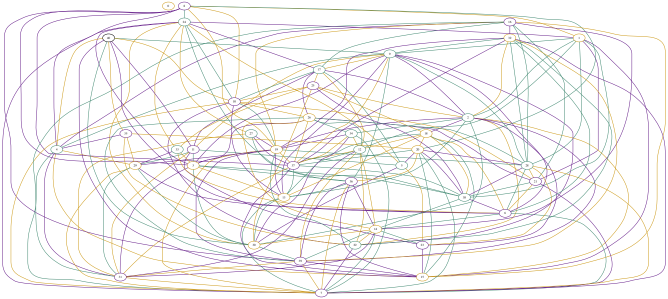

Figure: Graph

Introduction

https://github.com/nlehir/ALGO2
We will need (the installation of these packages should work with pip) :

- matplotlib
- pandas
- sklearn
- optionnally : ipdb

Day 2

- Compatibility graphs
  - Simple geometrical data
  - Complex data
- Probability distributions
  - Reminders on probabilities
  - Analyzing a distribution
  - Optimization and Maximum Likelihood
  - Gradients
- Multivariate analysis and clustering
  - Correlation
  - Kmeans clustering
- Similarities and Spectral Clustering
  - Similarities
  - Spectral Clustering
- Additional considerations and conclusions

Compatibility graphs

- Yesterday we processed graphs describing **relationship between data**
- If two nodes were related, they were linked by an edge in the graph.

Compatibility graphs

- Yesterday we processed graphs describing **relationship between data**
- If two nodes were related, they were linked by an edge in the graph.
- Today we are interested in building such graphs directly from the data, we call them **compatibility graphs**.

Compatibility graphs

We are interested in building **compatibility graphs**.
Given two nodes in a graph, should there be an edge between them ?

Compatibility graphs

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Given two nodes in a graph, should there be an edge between them ?
Note : it is not the same problem as the matching problem. In the matching problem, the edges are already defined.

Compatibility graphs

We are interested in building **compatibility graphs**.
Given two nodes in a graph, should there be an edge between them ?
Note : it is not the same problem as the matching problem. In the matching problem, the edges are already defined.
However, once the edges are built, we can apply a matching to it.

Example applications

- Social networks management
- Recommendations

Building compatibility graphs

- We will build graphs first from simple data
- Then from more complex data.

Building a graph from simple data

- We will first build a graph from simple data in the 2D space.

Euclidian distance and compatibility

Consider the following data :

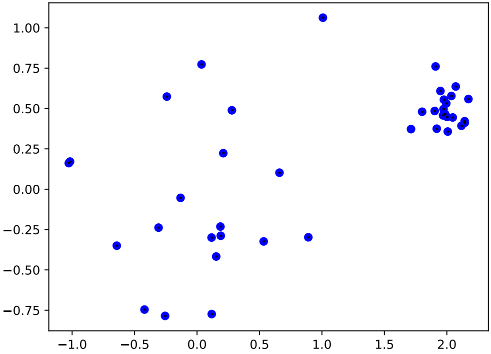


Figure: Data : we would like to define **edge** between some of them

Is this set of edges a good solution ?

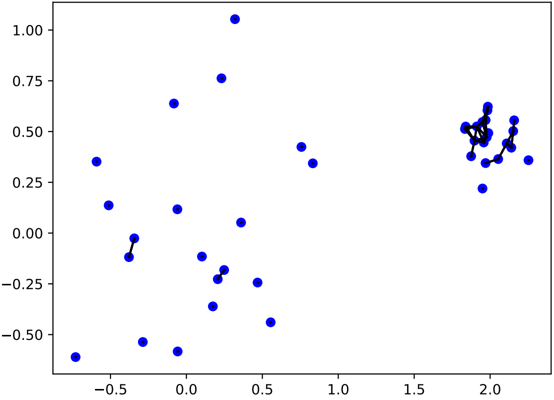


Figure: Some definition of edges

Is this set of edges a good solution ?

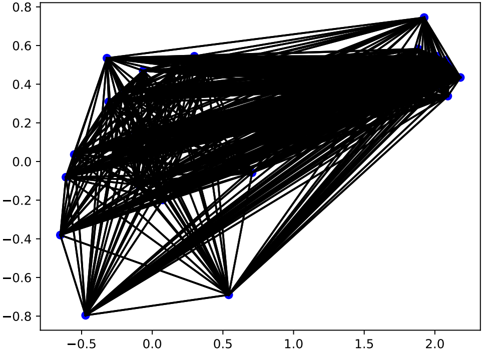
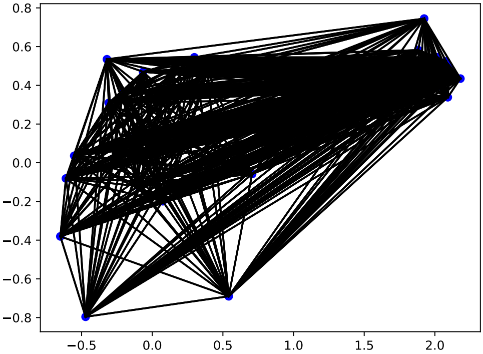


Figure: Some definition of edges

Euclidian distance and compatibility

Here, all we know about the data is their **euclidian distance** :



This one looks ok

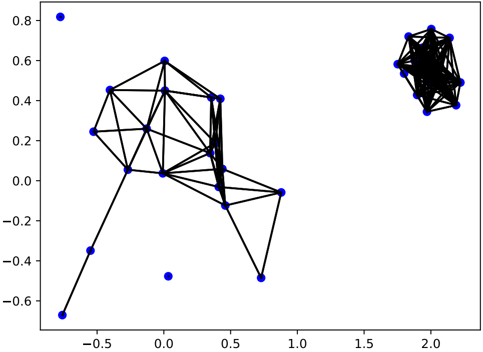


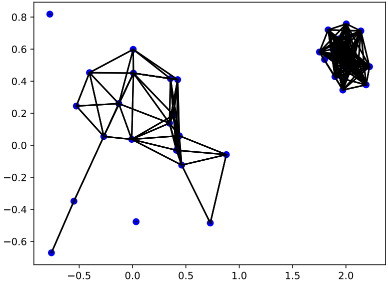
Figure: A proposition of edges

content

Backboard

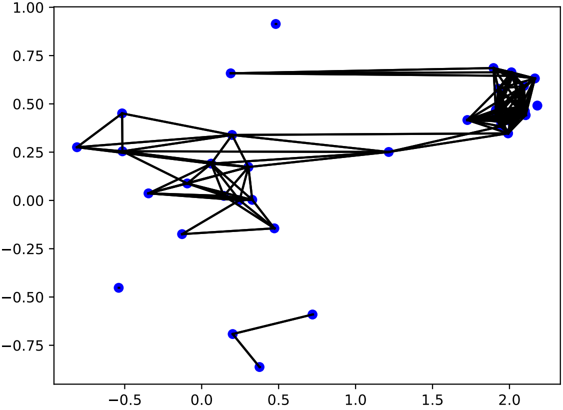
- Euclidian distance and threshold.

Exercice 1 : Setting a threshold **cd compatibility_simple** and set the threshold used in **build_graph.py** to draw relevant edges between the nodes. Feel free to use another dataset !



Exercice 2 : Changing the distance

- Assess the impact of changing the distance used. Possible choices :
  - $L1$ distance (Manhattan)
  - $||||_\infty$ distance (backboard)
  - custom distance
- use **build_graph_other_distance.py** and edit the distances used at the end of the file.
- Try several values for the threshold.


1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−0.5
0.0
0.5
1.0
1.5
2.0

General notion of a distance

- Let us generalize what we experimentally studied.

Examples of distances

$x = (x_1, ..., x_p)$ and $y = (y_1, ..., y_p)$ are p-dimensional **vectors**.

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Examples of distances

$x = (x_1, ..., x_p)$ and $y = (y_1, ..., y_p)$ are p-dimensional **vectors**.

- L2 : $||x - y||_2^2 = \sum_{k=1}^{p} (x_k - y_k)^2$ (Euclidian distance)
- L1 : $||x - y||_1 = \sum_{k=1}^{p} |x_k - y_k|$ (Manhattan distance)

Examples of distances

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- L1 : $||x - y||_1 = \sum_{k=1}^{p} |x_k - y_k|$ (Manhattan distance)
- weighted L1 : $\sum_{k=1}^{p} w_k |x_k - y_k|$

Hamming distance

- $\#\{x_i \neq y_i\}$ (Hamming distance)

Hamming distance and edit distance

- $\#\{x_i \neq y_i\}$ (Hamming distance)
- linked to **edit distance** : used to quantify how dissimilar two strings are by counting the number of operations needed to transform one into the other (several variants exist)

General definition of a distance

A **distance** on a set E is an application $d : E \times E \rightarrow \mathbb{R}_+$ that must :

General definition of a distance

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- be **symmetrical** : $\forall x, y, d(x, y) = d(y, x)$
- **separate the values** : $\forall x, y, d(x, y) = 0 \Leftrightarrow x = y$
- respect the **triangular inequality** $\forall x, y, z, d(x, y) \leq d(x, z) + d(y, z)$

Building compatibility graphs for more complex data

- We will do the same with more complex data:
  - possibly more dimensions
  - possibility categorical variables

Random variables

- A **random variable** is a quantity that can take several values

Random variables

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- For instance :
  - the result of a dice throw

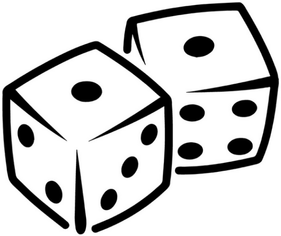

Figure: Dice

Random variables

- A **random variable** is a quantity that can take several values
- For instance :
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  - waiting time with RATP

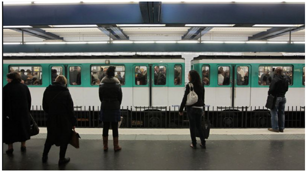

Figure: Some metro station

Random variables

- A **random variable** is a quantity that can take several values
- For instance :
  - the result of a dice throw
  - waiting time with RATP
  - weather

Figure: Weather in November

Random variables

- A **random variable** is a quantity that can take several values
- For instance :
  - the result of a dice throw
  - waiting time with RATP
  - weather
  - number of cars taking the periphrique at the same time

Random variables

What are the differences between these random variables ?

Random variables

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- Some are **continuous**, others **discrete**
- **continuous :**

Random variables

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Random variables

What are the differences between these random variables ?

- Some are **continuous**, others **discrete**
- **continuous :** weather, RATP
- **discrete :** dice (6 possibilities), number of cars (> 10000)

Probability distributions

- A random variable is linked to a **probability distribution**.

Probability distributions

- A random variable is linked to a **probability distribution**.
- It quantifies the probability of observing one outcome.

Probability distributions

- A random variable is linked to a **probability distribution**, which is a function P
- It quantifies the probability of observing one outcome.
- For a discrete variable : each possible outcome is associated with a number between 0 and 1

Probability distributions

- For a dice game, the possible outcomes are in the set $\{1, 2, 3, 4, 5, 6\}$
- For a dice game : $P(1) =?$ $P(2) =?$ $P(3) =?$ $P(4) =?$ $P(5) =?$ $P(6) =?$

Probability distributions

- For a dice game, the possible outcomes are in the set $\{1, 2, 3, 4, 5, 6\}$
- For a dice game : $P(1) = \frac{1}{6}$, $P(2) = \frac{1}{6}$, $P(3) = \frac{1}{6}$, $P(4) = \frac{1}{6}$, $P(5) = \frac{1}{6}$, $P(6) = \frac{1}{6}$
- This is called a **uniform distribution**

Probability distributions

- Periphrique :

Probability distributions

- Periphrique : probably a time-dependent very complicated distribution

Continuous variables

- How would you model a continuous variable ? Can you assign a number to a waiting time or a weather ?

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- One needs to use **probability densities**. Formally, the probably of being between x and $x + dx$ is $p(x)dx$.

Continuous variables

- How would you model a continuous variable ? Can you assign a number to a waiting time or a weather ?
- One needs to use **probability densities**. Formally, the probably of being between x and $x + dx$ is $p(x)dx$.
- Let's see some examples

Uniform discrete

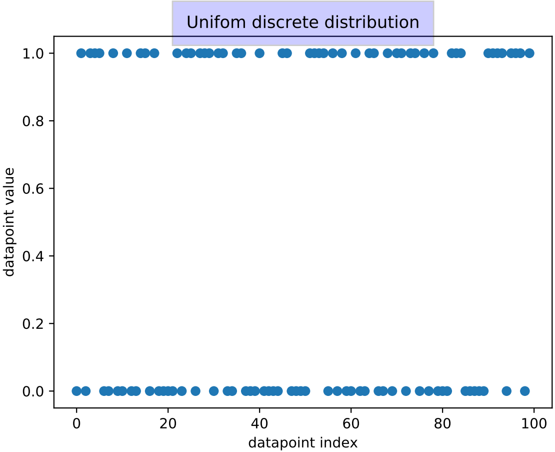


Figure: Uniform discrete distribution

Bernoulli

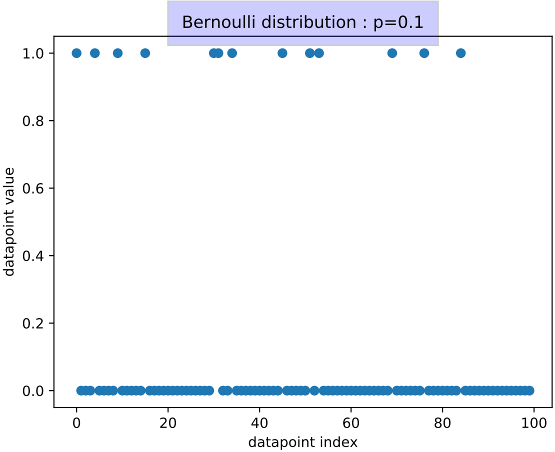


Figure: Bernoulli distribution

Bernoulli p

- With probability p, $X = 1$
- With probability $1 - p$, $X = 0$

Bernoulli

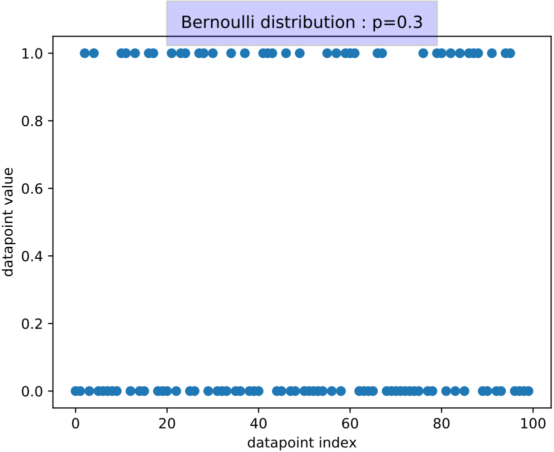


Figure: Bernoulli Distribution

Bernoulli

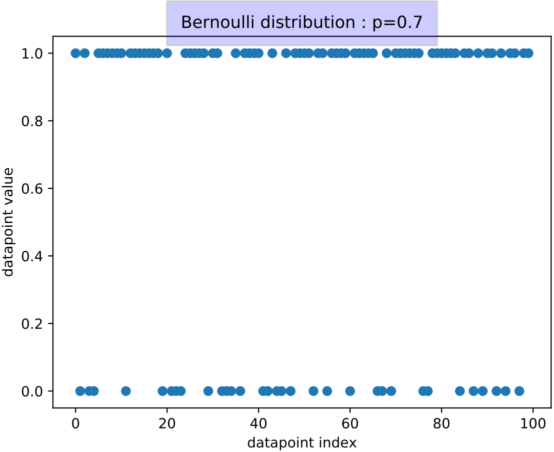


Figure: Bernoulli Distribution

Uniform continuous

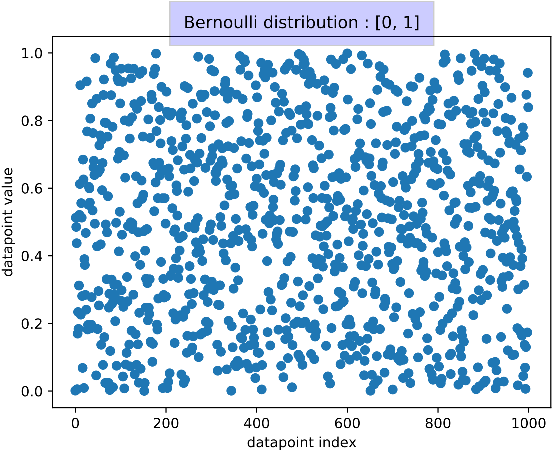


Figure: Uniform continuous distribution

Uniform continuous

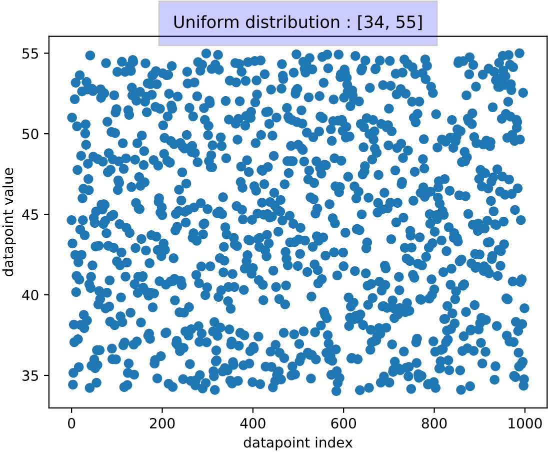


Figure: Uniform continuous distribution

Normal

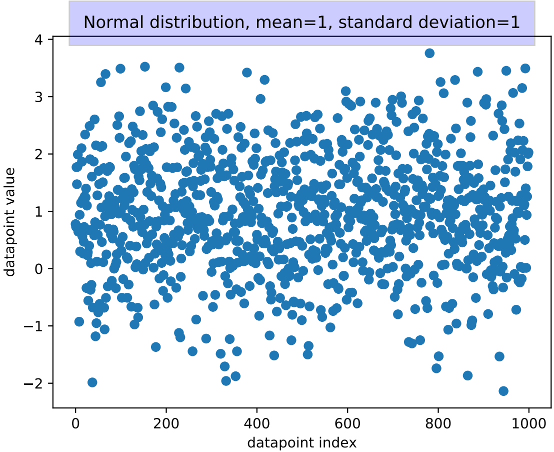


Figure: Normal distribution

Normal

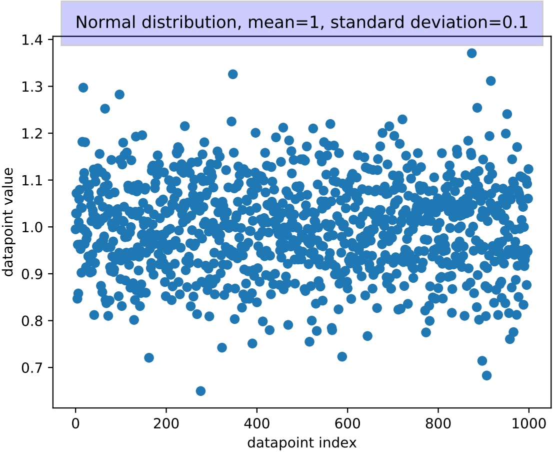


Figure: Normal distribution

Normal

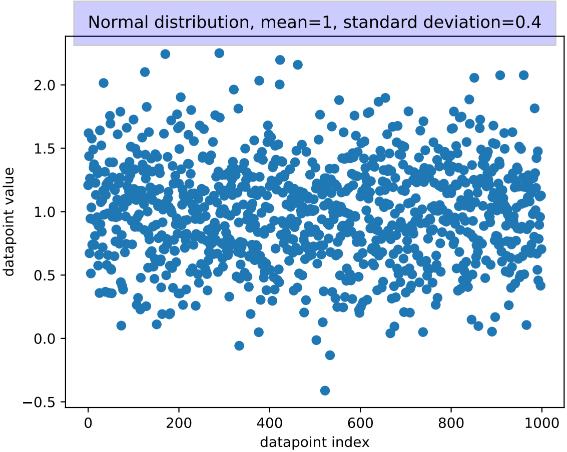


Figure: Normal distribution

White noise

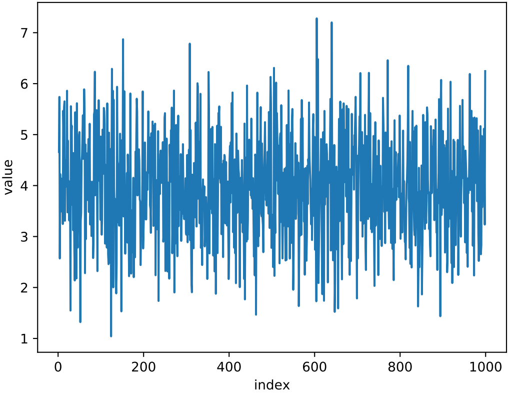


Figure: White noise

Histograms

Is looking at the raw dataset really **informative ?**

Histograms

Is looking at the raw dataset really **informative ?**
It is informative, but often a **histogram** tells more.

Uniform discrete

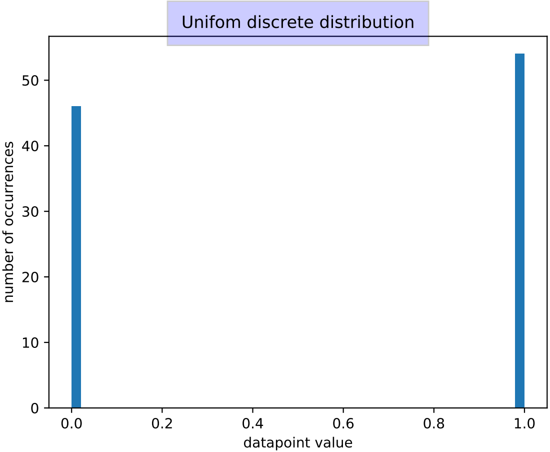


Figure: Historgram 1

Bernoulli

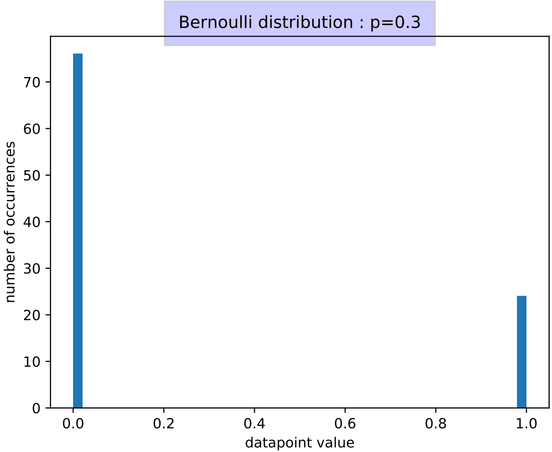


Figure: Historgram 2

Uniform continuous

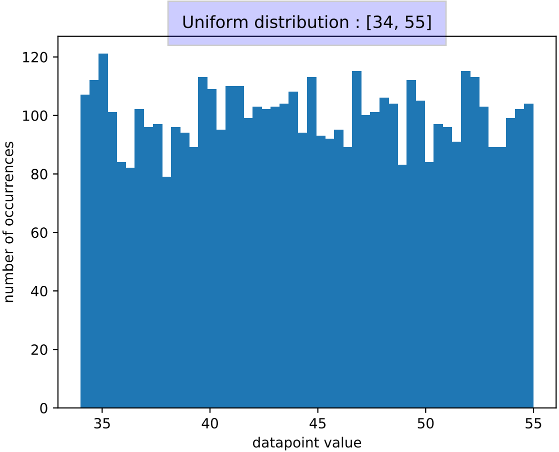


Figure: Historgram 3

Normal

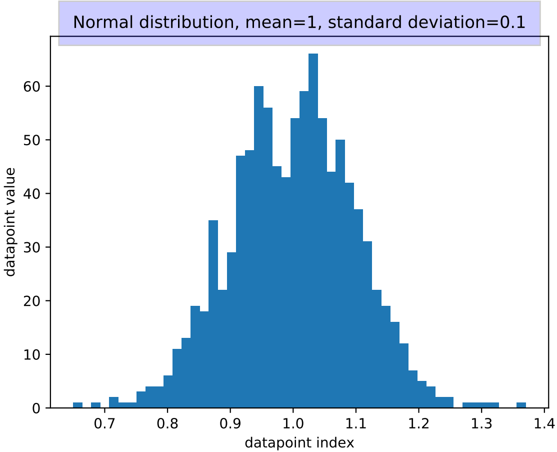


Figure: Historgram 4

Exercice 3 : Analyzing a distribution I put values in the file **mysterious_distro_1.csv**

Exercise

Exercice 4 : Analyzing a distribution I put values in the file **mysterious_distro_1.csv**
Can you analyze these values in terms of a **distribution ?**
Use **read_myst_1.py** to analyze the distribution (suggestion : change the number of bins used)

Exercise

When you have guessed the kind of distribution it is, you need to finds its **parameters**.

- its mean
- its standard deviation

This is called **fitting** a distribution to a dataset : it's a classical machine learning problem.
To do so, uncomment the last section of the script **read_myst_1**

Distribution 1

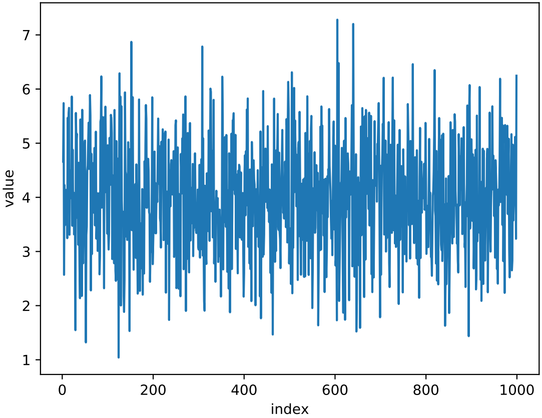


Figure: The data we analyze

histograms

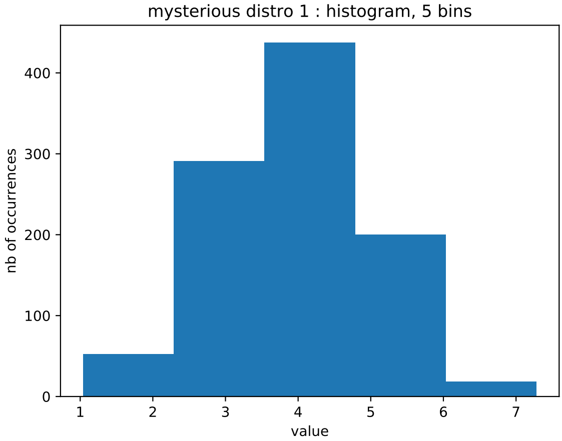


Figure: 5 bins

histograms

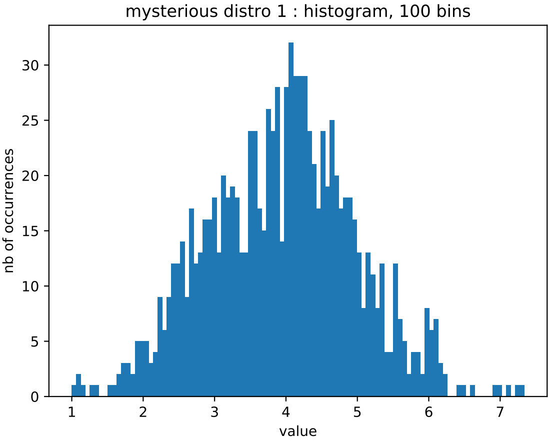


Figure: 100 bins

histograms

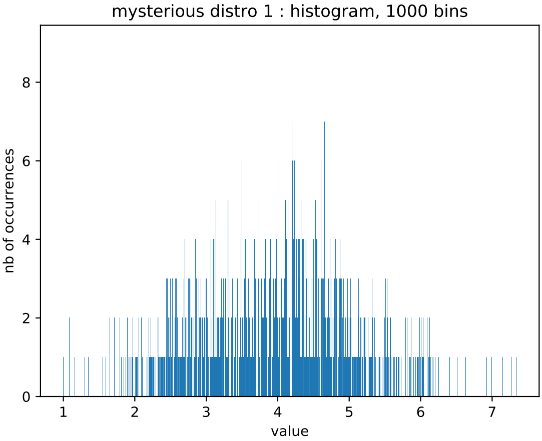
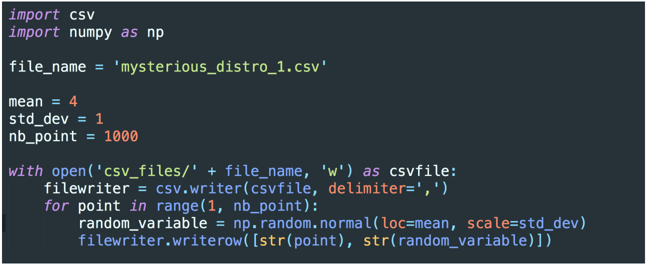


Figure: 1000 bins (too many)

Normal distribution



```python
import csv
import numpy as np

file_name = 'mysterious_distro_1.csv'

mean = 4
std_dev = 1
nb_point = 1000

with open('csv_files/' + file_name, 'w') as csvfile:
    filewriter = csv.writer(csvfile, delimiter=',')
    for point in range(1, nb_point):
        random_variable = np.random.normal(loc=mean, scale=std_dev)
        filewriter.writerow([str(point), str(random_variable)])
```

Figure: **create_normal.py** : Creation of the distribution

Second example

Let's try to perform the same analysis on the file **mysterious_distro_2.csv** using **read_myst_1**.

Second example

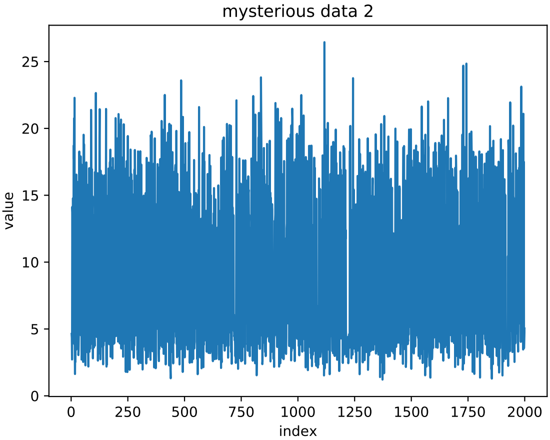


Figure: Second distribution

Multimodal distribution

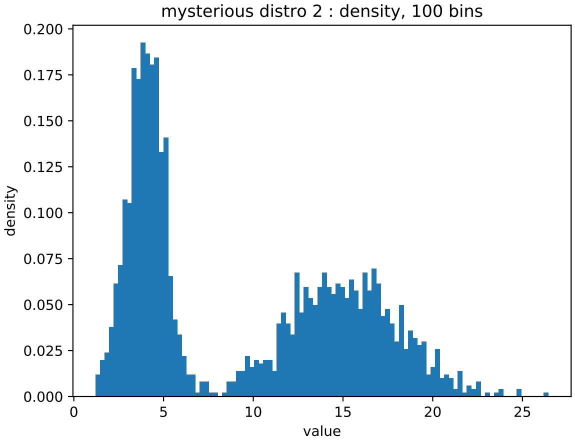
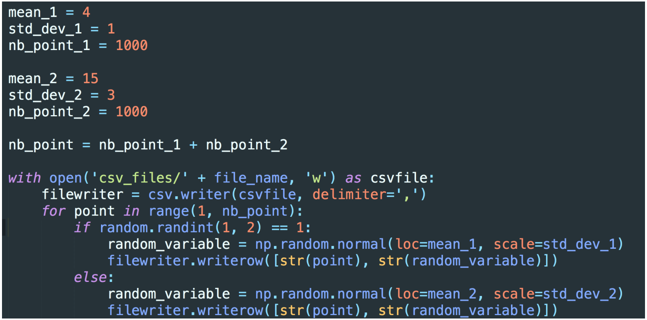


Figure: This distribution has several **modes**

Multimodal distribution



```python
mean_1 = 4
std_dev_1 = 1
nb_point_1 = 1000

mean_2 = 15
std_dev_2 = 3
nb_point_2 = 1000

nb_point = nb_point_1 + nb_point_2

with open('csv_files/' + file_name, 'w') as csvfile:
    filewriter = csv.writer(csvfile, delimiter=',')
    for point in range(1, nb_point):
        if random.randint(1, 2) == 1:
            random_variable = np.random.normal(loc=mean_1, scale=std_dev_1)
            filewriter.writerow([str(point), str(random_variable)])
        else:
            random_variable = np.random.normal(loc=mean_2, scale=std_dev_2)
            filewriter.writerow([str(point), str(random_variable)])
```

Figure: **create_bimodal.py** : Generation of multimodal distribution

Fitting

In most cases, it won't be that straightforward to fit a distribution :

Fitting

In most cases, it won't be that straightforward to fit a distribution :

- what distribution do we want to use ?
- even if we know the right shape of the distribution, how to choose the parameters ?

Maximum Likelihood

The **Maximum Likelihood** method is one example method used in Machine Learning.
Say you have a dataset $(x_1, ..., x_n)$.

Maximum Likelihood

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Say you have a dataset $(x_1, ..., x_n)$.
You first need to choose a **model** (which is the distribution) of your dataset, p.

Maximum Likelihood

The **Maximum Likelihood** method is the one used in Machine Learning.
Say you have a dataset $(x_1, ..., x_n)$.
You first need to choose a **model** (which is the distribution) of your dataset, p.
Then, you must optimize the **parameters of this model**, noted θ.

Maximum Likelihood

The Likelihood of your model is

$$L(\theta) = \prod_{i=1}^{n} p(x_i|\theta) \tag{1}$$

Maximum Likelihood

The Likelihood of your model is

$$L(\theta) = \prod_{i=1}^{n} p(x_i|\theta) \tag{2}$$

This is the function that you want to **maximise**.

Maximum Likelihood

Most of the time it's written this way : "minimise $-logL(\theta)$"
Why ?

Maximum Likelihood

Most of the time it's written this way : "minimise $-logL(\theta)$"
Because the log **transforms the product into a sum**, which is easier to **derivate**.

Maximum Likelihood

$$-logL(\theta) = -\sum_{i=1}^{n} \log(p(x_i|\theta)) \tag{3}$$

Max Likelihood

So how can we minimise $-logL(\theta)$? In the case of very large datasets, and large numbers of parameters (tens, hundredths, more), most of the time an **analytic solution** is not available. So people use **gradient descent**.

The gradient descent

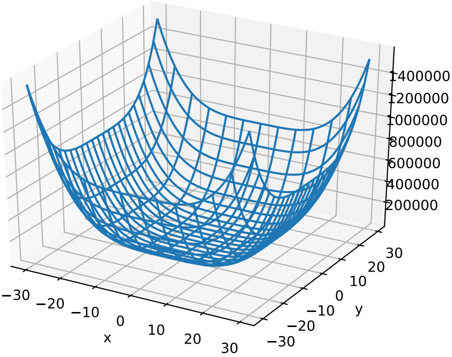
We want x to **minimise** f. We perform, until some criteria is satisfied :

$$x \leftarrow x - \alpha \nabla_f(x) \tag{4}$$

Use the file "gradient_algo.py" and implement the gradient algorithm on a simple example.
I inserted two errors in the code.

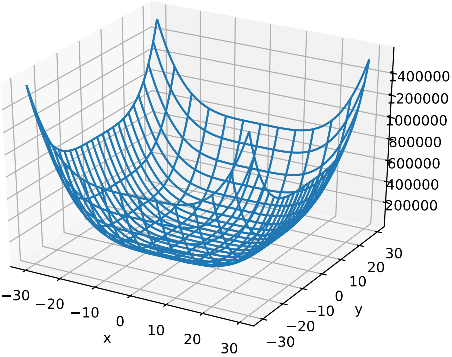
The gradient descent

$$x \leftarrow x - \alpha \nabla_f(x) \qquad (5)$$



The gradient descent

Experiment with it, try to change all the parameters and to break it again. Is it stable ?



Multidimensional vectors

We can consider data that live in higher dimensional spaces than 2.

Multidimensional vectors

We can consider data that live in higher dimensional spaces than 2. Examples ?

Multidimensional vectors

We can consider data that live in higher dimensional spaces than 2. Examples ?

- images
- sensor that receives **multimodal information**

Correlation

Sometimes the components of a multidimensonial vector $(x_1, ..., x_n)$ are not independent.

Correlation

Sometimes the components of a multidimensonial vector $(x_1, ..., x_n)$ are not independent.
To study this, we can use the **covariance** of the two components, or the **correlation** which is actually clearer.

Example

Look at the data contained in **mysterious_distro_3.csv**
They contain a random variable with 5 dimensions. Some of these dimensions are correlated.
Think for instance to physics : temperature and pressure, etc. If you have measurements of temperature and pressure, the two would probably be **correlated**.

Correlation matrix

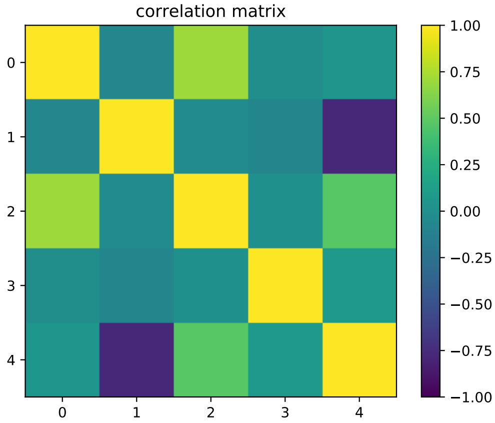
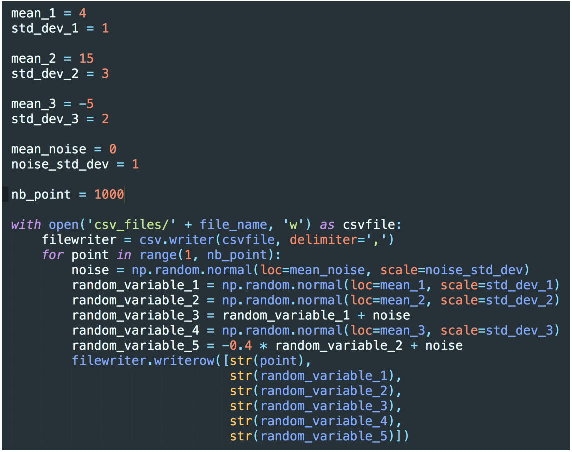


Figure: Correlation matrix for the distribution

Generation of the data



```
mean_1 = 4
std_dev_1 = 1

mean_2 = 15
std_dev_2 = 3

mean_3 = -5
std_dev_3 = 2

mean_noise = 0
noise_std_dev = 1

nb_point = 1000

with open('csv_files/' + file_name, 'w') as csvfile:
    filewriter = csv.writer(csvfile, delimiter=',')
    for point in range(1, nb_point):
        noise = np.random.normal(loc=mean_noise, scale=noise_std_dev)
        random_variable_1 = np.random.normal(loc=mean_1, scale=std_dev_1)
        random_variable_2 = np.random.normal(loc=mean_2, scale=std_dev_2)
        random_variable_3 = random_variable_1 + noise
        random_variable_4 = np.random.normal(loc=mean_3, scale=std_dev_3)
        random_variable_5 = -0.4 * random_variable_2 + noise
        filewriter.writerow([str(point),
                             str(random_variable_1),
                             str(random_variable_2),
                             str(random_variable_3),
                             str(random_variable_4),
                             str(random_variable_5)])
```

Figure: Multidimensional random variable

Kmeans clustering

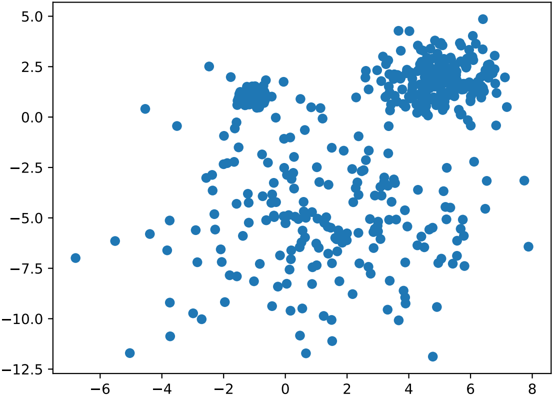


Figure: Data we want to cluster

Kmeans clustering

Modify the **kmeans.py** file so that it performs the kmeans algorithm.
I inserted two errors in the code.
You should obtain something like this:

Kmeans

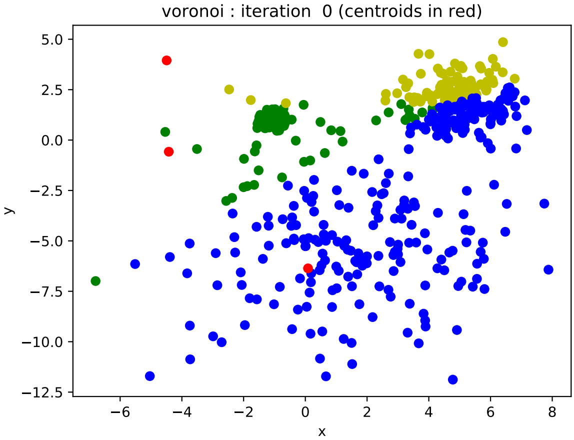


Figure: Voronoi 0th iteration

Kmeans

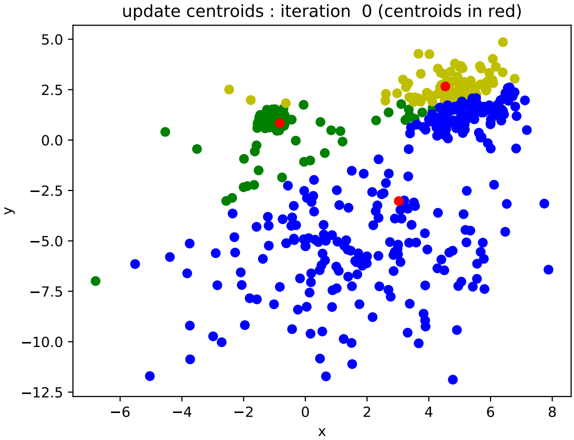


Figure: Centroids 0th iteration

Kmeans

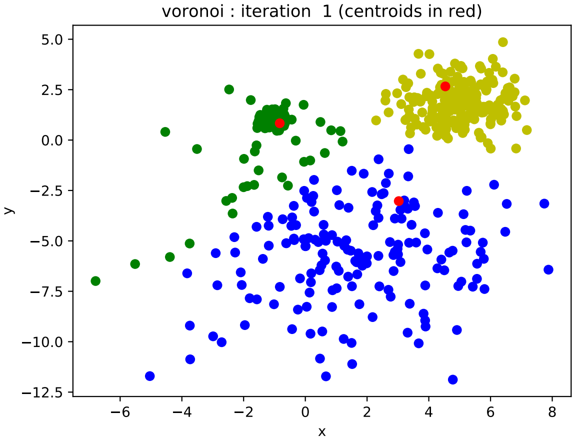


Figure: Voronoi 1st iteration

Kmeans

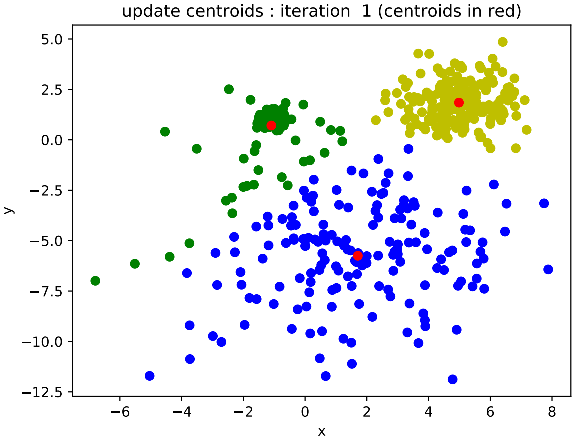


Figure: Centroids 1st iteration

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- But sometimes you do not have access to a distance between the points.
- You might need to work with something that is more general, for instance a **similarity**.

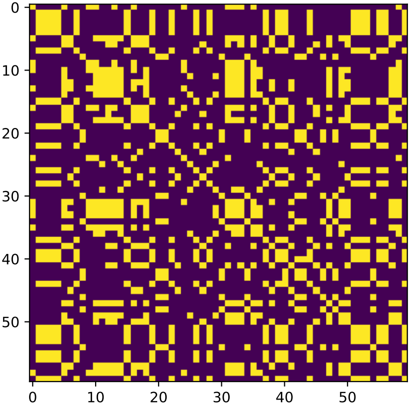
Similarities

- When working with distances, two points that "look the same" should be separated by a **small distance** .
- When working with a similarity, two points that "look the same" should have a **high similarity**.

Example of similarity : adjacency

- An example of similarity is the relationship of **adjacency**.
- If i and j are related by an edge, $S_{ij} = 1$.
- Otherwise $S_{ij} = 0$.

Adjacency matrix



Similarities

Differences between similarities and distances:

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Differences between similarities and distances:

- A similarity S is not always symmetrical.
- Indeed, in a **directed graph**, having a directed edge between i and j does not mean that we have an edge between j and i.
- $S_{ij} = 0$ does not mean that $i = j$, it is rather the contrary.

Similarities

- A similarity is a more general notion than a distance. Given a similarity between two points, we can deduce a similarity.

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- A similarity is a more general notion than a distance. Given a similarity between two points, we can deduce a similarity.
- For instance this way, if d_{ij} is the distance betwwen i and j :

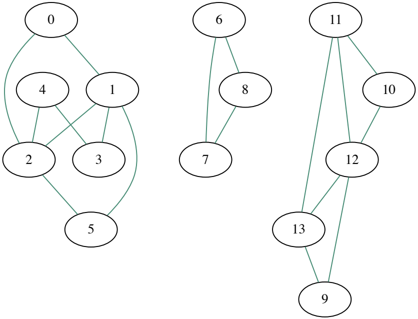
$$S_{ij} = \exp(-d_{ij}) \tag{6}$$

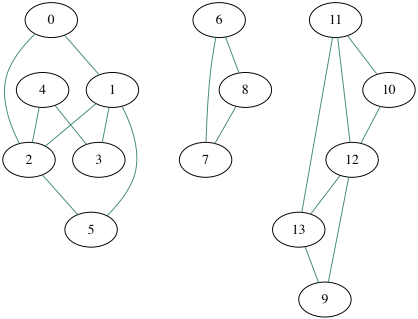
Spectral Clustering

- A clustering method that works with similarities
- It performs a low dimensional embedding of the similarity matrix, followed by a Kmeans

Exercise

We will perform Spectral Clustering on this graph :





Please **cd spectral_clustering** and use **vanilla_spectral_clustering** in order to apply spectral clustering. You first need to input to right **affinity matrix** or **similarity matrix** and then use the **sklearn** library. You also need to **tune the number of clusters.** **doc** : check the sklean page for Spectral Clustering.

Spectral clustering

Can you guess some drawbacks of the method ?

Spectral clustering

Can you guess some drawbacks of the method ?

- Need to provide the number of clusters.
- Not adapted to a large number of clusters.
- kmeans step : so depends on a random initialization.

Heuristic

- We would like a critetion in order to justify the number of clusters used.

Normalized cut : a measurement of the quality of a clustering

- The **cut of a cluster** is the number of outside connections (connections with other clusters).
- The **degree** of a node is its number of adjacent edges
- The **degree of a cluster** is the sum of the degrees of its nodes.
- The **normalized cut** of a clustering is:

$$NCut(\mathcal{C}) = \sum_{k=1}^{K} \frac{Cut(C_k, V \backslash C_k)}{d_{C_k}} \tag{7}$$

Normalization

- The normalization is useful in order to take the **weight** (degree) of a cluster into account.

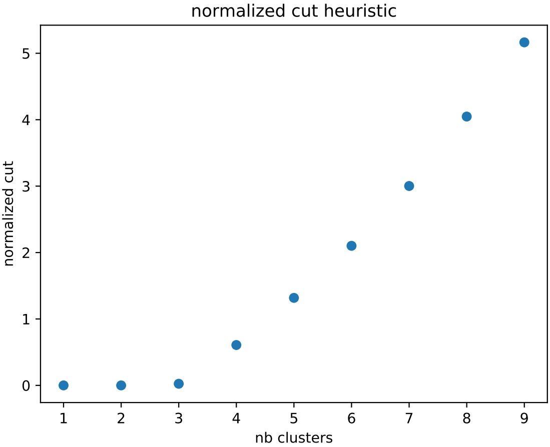
Normalized cut and clustering

Let's see how the normalized cut can help us choose the right number of clusters (backboard).

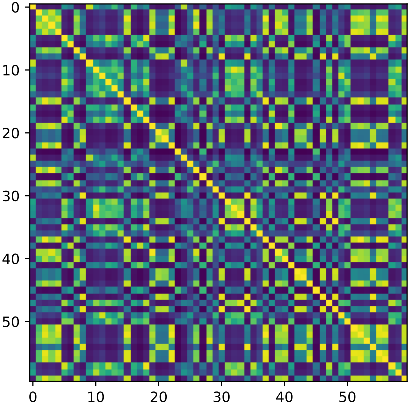
Exercise : normalized cut elbow

Please use the criterion in the file **normalized_cut** in order to guess the relevant number of clusters in order to process the data contained in **data/**

Normalized cuts



Similarity



Example

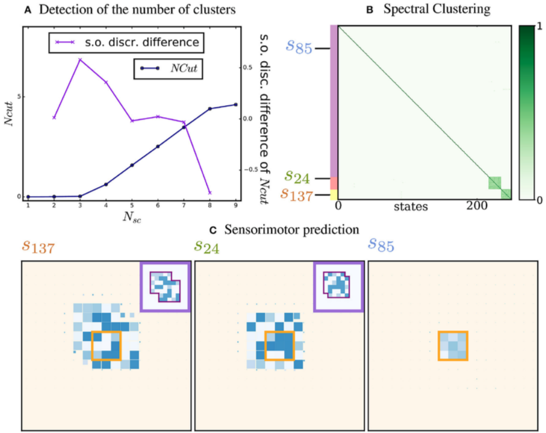


Figure: In a), the elbow method is used to choose the number of clusters. [Le Hir et al., 2018]

Other methods to evaluate the quality of a clustering

- Stability of the result when lauching the algorithm many times
- Separation of the clusters (the mean distance between pairs of centroids is large)
- Ratio inter / intra
- Silhouette coefficient

Other interesting notions

- Agglomerative clustering (CHA : classification Hirarchique Ascendante)
- Xmeans : improvement of k means
- If you know more about probabilities or are curious :
    - Latent variables and variational learning
    - Auto Encoders
    - Boltzmann Machines

Conclusion

Different kinds of problems exist :

- P problems where exact polynomial solutions exist (max matching)
- For other problems :
  - exhaustive search works but is too slow
  - to solve the problem a balance between rapidity and quality must be found.
- Evaluating the quality of a result is not an easy task.

Project

- Desciption of the project

Questions ?

References

Le Hir, N., Sigaud, O., and Laflaquière, A. (2018).
Identification of Invariant Sensorimotor Structures as a Prerequisite for the Discovery of Objects.
Frontiers in Robotics and AI, 5(June):1–14.