

Revision Exercises for Probability

1. Assume A and B are independent events with $P(A) = 0.2$ and $P(B) = 0.3$. Find

(i) $P(A^c \cap B^c)$

(ii) $P(A^c | B)$

- (b) Let random variable X denote the time (in years) it takes to develop a software. Suppose that X has the following probability density function

$$f(x) = \begin{cases} \frac{x}{2} & \text{if } 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

- (i) Compute the probability that it takes more than 6 months to develop the software

- (ii) Compute the expected number of years it takes to develop a software.

2. Let X be a continuous random variable with probability density function

$$f(x) = \begin{cases} c(x^2 - 1) & \text{for } 1 \leq x \leq 3 \\ 0 & \text{elsewhere} \end{cases}$$

where c is a constant

- (a) Find the value of c so that $f(x)$ is a valid pdf
(b) What is the Cumulative Distribution Function (cdf) of X ?
(c) Find $P(X < 2.5 | X > 2)$
(d) Find $E(20X)$
(e) What is the median m of X ?

3(a) (i) Define what it means for two events A and B to be independent (1 mark)

(ii) Given that events A and B are independent such that $P(A) = 0.4$ and $P(B) = 0.6$. Find $P(A^c \cup B^c)$

(b) A problem in statistics is given to three students A, B and C whose chances of solving are $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{2}$ respectively. What is the probability that the problem will be solved by only one student assuming that each student solves independently?

4(a) A R.V X has a pdf defined by;

$$f(x) = \begin{cases} \frac{1}{3}x, & \text{if } 0 \leq x < 1 \\ \frac{1}{3}, & \text{if } 1 < x < 3 \\ \frac{1}{3}(4-x), & \text{if } 3 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find:

(i) The variance of X if the expectation, $E(X) = 2$

(ii) $P(X \leq 2)$

(iii) The cumulative distribution function $F(x)$

(b) A random Variable X has a probability mass function given by;

$$f(x) = c(2x+1), x = 0, 1, 2, 3, 4. \text{ Find;}$$

(i) the value of c

(ii) $P(X = 4)$

(iii) $P(2 \leq X < 4)$

5(a) A discrete random variable X has a probability mass function

$$f(x) = \begin{cases} k(1-x)^2; & x = 0, 1, 2, 3 \\ 0, & \text{otherwise} \end{cases}$$

- (i) Determine the value of k
(ii) Evaluate $\text{Var}(X)$

(b) A continuous random variable X has a probability density function

$$f(x) = \begin{cases} 2(1-x); & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

Evaluate

- (i) $E(X)$
(ii) $P(X \leq \frac{2}{3} \mid X > \frac{1}{3})$
(iii) the median

6(a) The sample data below shows production of a certain product in a Kampala based factory: 98, 99, 99, 100, 100, 100, 101, 101, 102
Find the variance of the sample data

(b) In how many ways can the letters in the word MISSISSIPPI be arranged?

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