UGANDA CHRISTIAN UNIVERSITY

A Centre of Excellence in the Heart of Africa

FACULTY OF ENGINEERING, DESIGN AND TECHNOLOGY

DEPARTMENT OF COMPUTING AND TECHNOLOGY

ADVENT 2024 SEMESTER EXAMINATIONS

PROGRAM: Bachelor of Science in Computer Science

YEAR: III

SEMESTER: 1

COURSE CODE: MTH 3108

COURSE NAME: Numerical Computing and Optimization

EXAMINATION TYPE: Theory

EXAMINATION DATE: December 2024

TIME ALLOWED: 3 HOURS

---

1. The general Uganda Christian University examination guidelines, academic and financial policies apply to this examination. Violating any of the policies by the student automatically makes this examination attempt void, even if you have completed and submitted the answer booklet.
2. Answer all questions.
   (i) Section *A* (40 Marks). Attempt only **TWO** (2) questions from this section; every question carries 20 marks.
   (ii) Section *B* (60 Marks). Attempt only **THREE** (3) questions from this section; every question carries 20 marks.
3. Answers to every question should start on a new paper within the examination answer booklet.
4. Uganda Christian University or its officials reserve the right of admission or disqualification from this exam.

SECTION A (40 *marks*)

Question 1

(a) State the name and defining equations for any five finite difference operators. (5 marks)

(b) Consider the data set

x	0	10	20	30
y	0	0.174	0.347	0.518

(i) Construct a backward difference table. (4 marks)

(ii) Use your results in (b) (i) to find $\mu f_{\frac{3}{2}}$ and $E^2\Delta y_0$. (3 marks)

(c) Prove the finite difference relation, $\Delta\nabla \equiv \Delta - \nabla$. (4 marks)

(d) Suppose a function $f(x) = e^x$, compute $\Delta^2 f(x)$. (4 marks)

Question 2

(a) Consider an integral $S = \displaystyle\int_a^b f(x)\,dx$.

(i) Explain why the numerical method is used to approximate the value S? (2 marks)

(ii) State one limitation of using the numerical method to approximate S. (1 mark)

(b) Use Simpson's $\frac{1}{3}$ rule to approximate the integral $\displaystyle\int_0^4 f(x)dx$, where the values for the function $f(x)$ are in the table

x	0	1	2	3	4
$f(x)$	2	1	2	3	5

(5 Marks)

(c) The velocity of an object as a function of time is given by:

Time (t)	3	5	7	9	11
Velocity ($v(t)$)	12	16	24	15	33

Use the Trapezoidal rule to calculate the distance travelled by the object on $[3, 11]$. (5 marks)

(d) Use the mid point rule with seven ordinates to find an estimate for $\displaystyle\int_0^{12} \ln(x^2+5)\,dx$. Give your answer to three significant figures. (7 marks)

Question 3

(a) Justify the use of Numerical Differentiation. (2 marks)

(b) Given the following data,

x	6.0	6.1	6.2	6.3	6.4
$f(x)$	0.1750	–0.1998	–0.2223	–0.2422	–0.2596

find;

(i) $f'(6.2)$ using a second order method, (4 marks)

(ii) $f'(6.0)$, (4 marks)

(iii) $f''(6.3)$. (4 marks)

(c) Derive the central order method for computing $f''(x)$ and state the expression of the truncation error. (4 marks)

(d) Describe the effect of the size of the step length h in approximating $f'(x)$ using the backward formula. (2 marks)

SECTION B (60 *marks*)

Question 4

(a) Consider the data set

x	15	18	22
y	24	37	25

. Draw a table of divided differences and use it to state Newton's interpolating polynomial. (6 marks)

(b) Let $f(x) = 3^x$ with $x_0 = 0$, $x_1 = 1$, and $x_2 = 2$.

(i) Use Lagrange interpolation to find a polynomial $P_n(x)$ that agrees with $f(x)$ at the given points. (8 marks)

(ii) Show that $\sum_{n=0}^{2} L_k(x) = 1$. (2 marks)

(c) Given the data below,

Time	0	1	2	3	4
Distance	0	6	39	67	100

find the distance traveled when $t = 2.3$. (4 marks)

Question 5

(a) Derive the Newton Raphson's process, $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ for approximating the simple root ξ of a non-linear equation $f(x) = 0$. (4 marks)

(b) Use Newton Raphson's method to approximate $\sqrt{2}$ with $x_0 = 1$. (4 marks)

(c) Use the bisection method up to the fifth iteration to approximate the root of $x^2 = 3$ on $[1, 2]$. (5 marks)

(d) State one advantage and one disadvantage of the Newton Raphson's process as compared to;

(i) Bisection algorithm (2 marks)

(ii) Regular false (2 marks)

(iii) Secant Method (2 marks)

(e) Why could the Bisection method fail when approximating a root of the non-linear equation $x^3 + 4 = 0$, on the interval $[2, 4]$? (1 mark)

Question 6

(a) Consider the linear system,

$$\begin{aligned} 3x + 60y &= 63, \\ 32x + 8y &= 40. \end{aligned}$$

Solve the system using complete pivoting. (4 marks)

(b) Use the LU decomposition to solve the system of linear equations with $A = \begin{pmatrix} 3 & -2 \\ -2 & 6 \end{pmatrix}$ and $b = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$. (4 marks)

(c) By deriving the equation $X^{m+1} = C + GX^m$, describe the difference between Jacobi's method and Gauss-Seidel's method. (6 marks)

(d) Use $A = \begin{pmatrix} 10 & 1 & 1 \\ -1 & 20 & 1 \\ 1 & -2 & 10 \end{pmatrix}$ and $b = \begin{pmatrix} 24 \\ 21 \\ 300 \end{pmatrix}$ to find T and C for the Jacobi iteration scheme and hence, determine $x^{(1)}$ given $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$. (6 marks)

◄ END ►