Assignment 1.

Instructions to candidates:
(i) Attempt ALL questions.

1. (a) Given $f(x) = x^2 + 2$ and $g(x) = \sqrt{3 - x}$. Find $(f \circ g)(x)$.
   (b) Given $f(x) = x^2 + 2$ and $g(x) = \sqrt{3 - x}$. Find $(g \circ f)(1)$.
   (c) Given
$$f(x) = \begin{cases} 2, & x \leq -1 \\ \alpha x, & -1 < x << \leq 1 \\ \beta x^2, & x > 1 \end{cases}$$
   Find α and β such that limit of $f(x)$ exists.
   (d) Given
$$f(x) = \begin{cases} x^2 + kx + 1 & x < 1 \\ x^3 & x \geq 1 \end{cases}$$
   Find the value of k such that $\lim_{x \to 1} f(x)$ exists.
   (e) Given $f(x) = \dfrac{x^2 - 9}{x - 3}$. Find $\lim_{x \to 3} f(x)$.
   (f) Find $\lim_{x \to 0} \dfrac{2 \sin x}{x}$.
   (g) Tom's computer memory capacity decays in such a way that the amount of memory remaining after t days when attacked by a virus, is given by the function $m(t) = 13e^{-0.015t}$ where $m(t)$ is measured in GB.
      (i) Find the memory at time $t = 0$
      (ii) How much of the memory will Tom remain with after 45 days?
      (iii) Is $m(t)$ increasing or decreasing?. Explain.
      (iv) Assuming there is no solution to solve Tom's computer memory decay. Henry seeks to swap his 10GB hard disk with Tom forever. What advise do you have for both Tom and Henry?
   (h) Use the Sandwich theorem to compute $\lim_{x \to \infty} \dfrac{2 - \cos x}{x^2 + 3}$

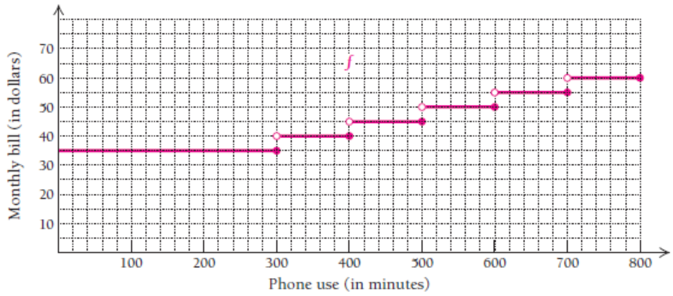
2. (a) Given that
$$\lim_{x \to 2} g(x) = 4, \quad \lim_{x \to 2} f(x) = 5$$
   Find the;
      (i) $\lim_{x \to 2} 5g(x)f(x)$
      (ii) $\lim_{x \to 2} 10g(x)$
      (iii) $\lim_{x \to 2} \left(\dfrac{g(x)}{3g(x) + f(x)} \right)$
   (b) State the formal definition of a limit.
   (c) Use a formal definition of a limit to prove that $\lim_{x \to 2} (3x - 1) = 5$
   (d) Sketch the function
$$y = \begin{cases} x & x \geq 0 \\ -2 & x < 0 \end{cases}$$

(e) Suppose that \$500 is invested at 6% compounded quarterly for t years, given;

$$A(t) = 500\left(1 + \frac{0.06}{4}\right)^{4t} = 100(1.015)^{4t}.$$

where $A(t)$ ia a function of the number of years for which the money is invested. Determine the domain.

(f) Recently, Sprint offered a cellphone calling plan in which the customer's monthly bills could be modeled by the graph below.



(i). Find the range of the function shown.

(ii). Is the function f continuous? Explain.

(g) Stating the rule used, compute

$$\lim_{x\to 0} \frac{(2x+3)^2 - 9}{x}$$

3. (a) State the domain and range of the function $f(x) = \dfrac{1}{\sqrt{x^2+3}}$.

(b) Find the solution to $2(3+2x) - 10 > 7x$?

(c) Find the inverse of the function $f(x) = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$.

(d) Find the solution to $|2x+3| \geq 8$?

(e) Find the solution to $(x-2)(x-3) \geq 0$?

(f) Which of the following sets does not represent a function?

A. $A = \{(-1,1),(4,11),(5,13),(1,5)\}$
B. $A = \{(-1,-1),(-4,-4),(-5,-5),(-10,-10)\}$
C. $A = \{(-1,1),(-1,11),(5,13),(1,5)\}$
D. $A = \{(-2,4),(2,4),(-1,1),(1,1)\}$

(g) Given a function which is represented by the following set $A = \{(-1,1),(4,11),(5,13),(1,5)\}$, which one of the following alternatives represent the range and domain respectively?

A. $\{-1,4,5,1\}$ and $\{1,11,13,5\}$
B. $\{1,11,13,5\}$ and $\{-1,4,5,1\}$
C. Can not be determined.
D. A does not represent a function.

(h) Show whether the function $\dfrac{5x+2}{3}$ is one to one.

(i) Prove that $\sqrt{ab} \leq \dfrac{a+b}{2}$.

(j) Solve the inequality $\dfrac{(x-1)(x+2)}{(x-3)} < 0$.

(k) Show whether the function $\dfrac{x}{x^2-1}$ is either even or odd.

4. (a) All polynomials are continuous at all points in real numbers.

   A. False   B. True

   (b) Which kind of discontinuity does the function $f(x) = \dfrac{\sqrt{x}-2}{x-4}$ have at $x = 4$?

   A. Jumped discontinuity   C. Essential discontinuity
   B. Removable discontinuity   D. None of these.

   (c) Which kind of discontinuity does the function $f(x) = \begin{cases} x^2 & x < 1 \\ 0 & x = 1 \\ 3-x & x > 1 \end{cases}$ have at $x = 1$?

   A. Jumped discontinuity   C. Essential discontinuity
   B. Removable discontinuity   D. None of these.

   (d) Which kind of discontinuity does the function $f(x) = \dfrac{1}{x+1}$ have at $x = -1$?

   A. Jumped discontinuity   C. Essential discontinuity
   B. Removable discontinuity   D. None of these.

   (e) The value of λsuch that $f(x) = \begin{cases} 6-x & x \leq -2 \\ \lambda x^2 & x > -2 \end{cases}$ is continuous at $x = 2$ is;

   (f) Evaluate $\displaystyle\lim_{x\to\infty} \frac{5x^7 - 8x^2 + 2}{3x^6 - x^3 + x^2}$

   (g) Suppose $6x - x^2 \leq f(x) \leq x^2 - 6x + 18$ for all x in real numbers. Find $\displaystyle\lim_{x\to 3} f(x)$.

   (h) Redefine the function $f(x) = \dfrac{\sqrt{x}-2}{x-4}$ such that it is continuous at $x = 4$. Which type of discontinuity is this?

◂ END ☺! ▸