1. **Inference for normal mean and deviation**

a)

The data mean is: 14.768

The data variance is: 2.230

The μ is calculated this way:

posterior_mu = scipy.stats.t.pdf(x, n-1, mean_y, variance_y/n)

n-1 is the degree of freedom

The 95% central interval is: [14.108, 15.427]

We can see that the mean falls in the range of the 95% interval

b)

To calculate the hardness we use this:

posterior_hardness = scipy.stats.t.pdf(x, n-1, mean_y, scale)

where **scale** is calculated using this formula:

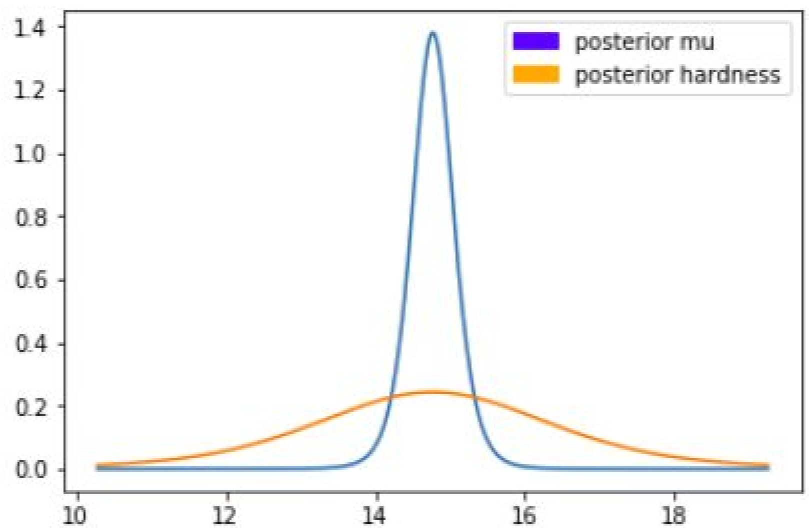
$$\sqrt{1 + \tfrac{1}{n}} * std_y$$

The posterior variance is: 1.584

The 95% central interval for the windshield hardness is:

[11.022, 18.513]

We can say that the hardness of the windshield will be in the interval of [11.022, 18.513] with 95% confidence.



2. Inference for difference between proportions

a)

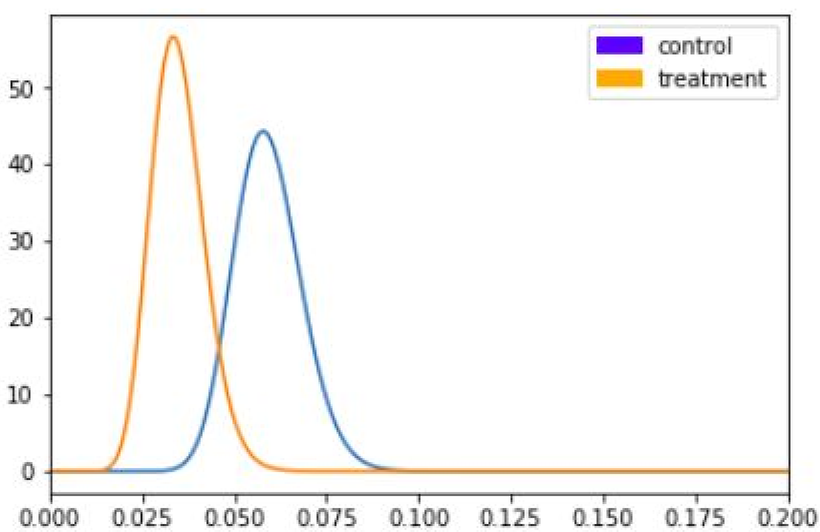
control_number = 674
control_alpha = 39
control_beta = 635

treatment_number = 680
treatment_alpha = 22
treatment_beta = 658

alpha_prior = 1
beta_prior = 1

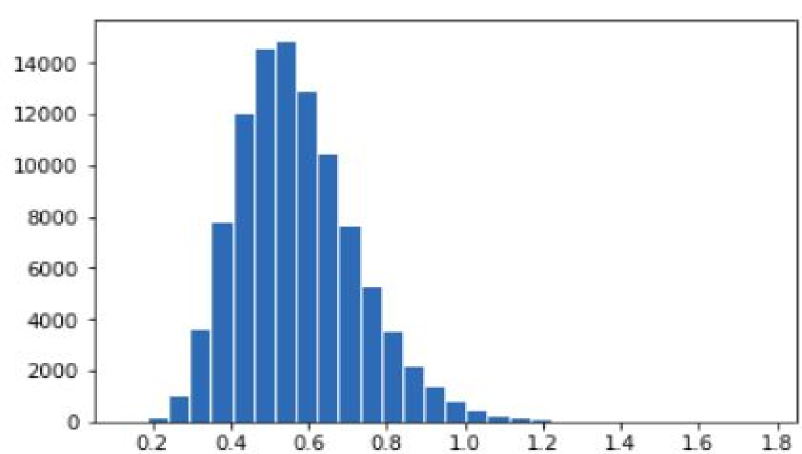
alpha_post_1 = alpha_prior + control_alpha
beta_post_1 = beta_prior + control_beta

alpha_post_2 = alpha_prior + treatment_alpha
beta_post_2 = beta_prior + treatment_beta



The histogram shows the odds ratio, that was calculated with this formula:

$$(p1 / (1 - p1)) / (p0 / (1 - p0))$$



The mean of the control group is: 0.059
The mean of the treatment group is: 0.033

The 95% confidence interval for control group is:
[0.042, 0.078]

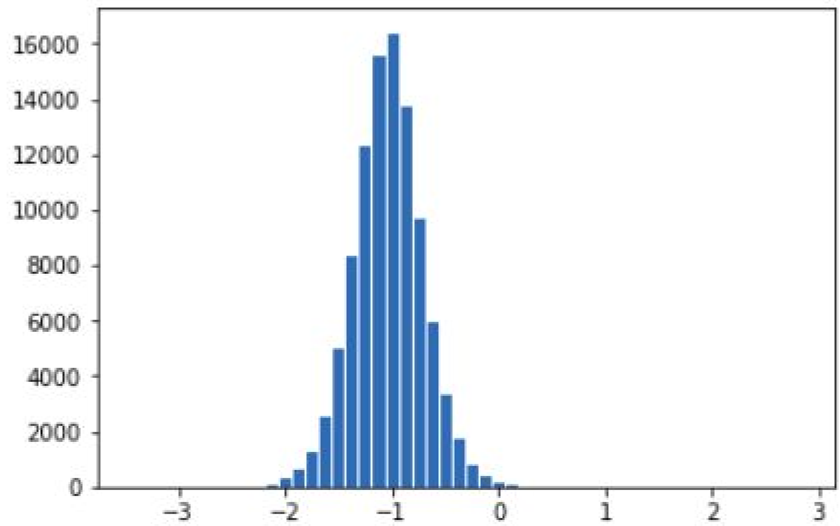
The 95% confidence interval for treatment is:
[0.021, 0.048]

b)
The prior that I chose is an uninformative prior and that prior did not have an impact on the inference, which means that the inference is not that sensitive on the uninformative prior.

3. Inference for difference between normal means

a)
The mean of μ difference is: -1.0391

There is a 95% confidence that the values will be in range:
[-1.7094, -0.3657]



b)

There is 99.6 % chance that the numbers are below 0, which means that it’s highly unlikely that the means are the same.

******* APPENDIX *******

SOURCE CODE

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import matplotlib.patches as mpatches
import math
import scipy.stats
from scipy.stats import beta

# 1.

y = pd.read_csv('windshieldy1.txt', sep='\n')
y = np.array(y)

n = len(y)
mean_y = np.mean(y)
variance_y = scipy.stats.tvar(y)
std_y = np.std(y, ddof=1)

# a)
x = np.arange(mean_y - 3 * math.sqrt(variance_y), mean_y + 3 *
math.sqrt(variance_y), 0.01)
posterior_mu = scipy.stats.t.pdf(x, n-1, mean_y, variance_y/n)
per_25 = scipy.stats.t.ppf(0.025, n-1, mean_y, variance_y/n)
per_975 = scipy.stats.t.ppf(0.975, n-1, mean_y, variance_y/n)

percentile_interval_y = [per_25, per_975]
print('The data average is: ', mean_y)
print('The data variance is: ', variance_y)
print('The 95% central interval is: ', percentile_interval_y)
```

```
plt.plot(x, posterior_mu)
print('We can see that the mean falls in the range of 95% interval')

print('\n')
print('*' * 80)
print('\n')

# b)
scale = math.sqrt(1 + 1/n) * std_y
posterior_hardness = scipy.stats.t.pdf(x, n-1, mean_y, scale)
print('The posterior variance is: ', scale)
per_25_hardness = scipy.stats.t.ppf(0.025, n-1, mean_y, scale)
per_975_hardness = scipy.stats.t.ppf(0.975, n-1, mean_y, scale)
percentile_interval_hardness = [per_25_hardness, per_975_hardness]
print('The 95% central interval for the windshield is: ',
percentile_interval_hardness)

plt.plot(x, posterior_hardness)
# The values should fall into that interval with 95% confidence

blue_patch = mpatches.Patch(color='blue', label='posterior mu')
orange_patch = mpatches.Patch(color='orange', label='posterior hardness')
plt.legend(handles=[blue_patch, orange_patch])
plt.show()




# 2.
# a)

control_number = 674
control_alpha = 39
control_beta = 635
```

```
treatment_number = 680
treatment_alpha = 22
treatment_beta = 658

alpha_prior = 1
beta_prior = 1
control_posterior = (control_alpha+1) / (control_number+1)
treatment_posterior = (treatment_alpha+1) / (treatment_number+1)

alpha_post_1 = alpha_prior + control_alpha
beta_post_1 = beta_prior + control_beta

alpha_post_2 = alpha_prior + treatment_alpha
beta_post_2 = beta_prior + treatment_beta

x = np.arange(0, 1, 0.001)

density_control = beta.pdf(x, alpha_post_1, beta_post_1)
density_treatment = beta.pdf(x, alpha_post_2, beta_post_1)

plt.plot(x, density_control)
plt.plot(x, density_treatment)

blue_patch = mpatches.Patch(color='blue', label='control')
orange_patch = mpatches.Patch(color='orange', label='treatment')
plt.legend(handles=[blue_patch, orange_patch])
plt.axis([0, 0.2, None, None])
plt.show()

p0_random = beta.rvs(alpha_post_1, beta_post_1, size = 100000)
p1_random = beta.rvs(alpha_post_2, beta_post_2, size = 100000)
```

```
odds_ratio = (p1_random / (1 - p1_random)) / (p0_random / (1 -
p0_random))
plt.hist(odds_ratio, bins = 30, ec='white')
plt.show()

control_mean = np.mean(control_number)
control_variance = np.var(control_number)

treatment_mean = np.mean(treatment_number)
treatment_variance = np.var(treatment_number)

control_mean = np.mean(control_posterior)
print('The control mean is: ', control_mean)

per_25_control = np.percentile(p0_random, 2.5)
per_975_control = np.percentile(p0_random, 97.5)
percentile_interval_control = [per_25_control, per_975_control]
print('The 95% confidence interval for control is: ',
percentile_interval_control)


treatment_mean = np.mean(treatment_posterior)
print('The treatment mean is: ', treatment_mean)

per_25_treatment = np.percentile(p1_random, 2.5)
per_975_treatment = np.percentile(p1_random, 97.5)
percentile_interval_treatment = [per_25_treatment, per_975_treatment]
print('The 95% confidence interval for treatment is: ',
percentile_interval_treatment)

# b)
```

```
# 3.
# a)

y_2 = pd.read_csv('windshieldy2.txt', sep='\n')
y_2 = np.array(y_2)

n_2 = len(y_2)
mean_y_2 = np.mean(y_2)
variance_y_2 = scipy.stats.tvar(y_2)
std_y_2 = np.std(y_2, ddof=1)

x_2 = np.arange(mean_y_2 - 3 * math.sqrt(variance_y_2), mean_y_2 + 3 *
math.sqrt(variance_y_2), 0.01)
posterior_mu_2 = scipy.stats.t.pdf(x_2, n-1, mean_y_2, variance_y_2/n_2)

mu_random = scipy.stats.t.rvs(n-1, mean_y, variance_y/n, 100000)
mu_2_random = scipy.stats.t.rvs(n_2-1, mean_y_2, variance_y_2/n_2,
100000)

mu_dif = mu_random - mu_2_random
plt.hist(mu_dif, bins = 50, ec='white')

mean_mu_dif = np.mean(mu_dif)
print('The mean of mu difference is: ', mean_mu_dif)

per_25_mu_dif = np.percentile(mu_dif, 2.5)
per_975_mu_dif = np.percentile(mu_dif, 97.5)
percentile_interval_mu_dif = [per_25_mu_dif, per_975_mu_dif]
print('The 95% confidence interval for mu difference is: ',
percentile_interval_mu_dif)
print('There is a 95% confidence that the values will be in range: ',
percentile_interval_mu_dif)
```

```
# b)
below_0 = scipy.stats.percentileofscore(mu_dif, 0)
print('There is : {0}'.format(below_0) + ' % chance that the numbers are below 0 which means that its highly unlikely that the means are the same')
```