Bayesean Data Analysis - Exercise 8

November 18, 2018

1 Model assessment: LOO_CV for factory data with Stan

1.1 Separate model

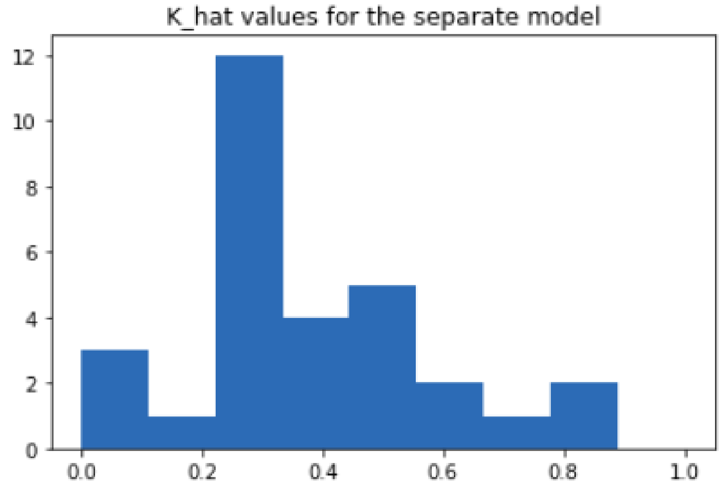
Here is that Stan code for the separate model:

```
data {
    int<lower=0> N; // number of data points
    int<lower=0> K; // number of groups
    int<lower=1,upper=K> x[N]; // group indicator
    vector[N] y; //
}

parameters {
    vector[K] mu;       // group means
    vector<lower=0>[K] sigma;
}

model {
  y ~ normal(mu[x], sigma[x]);
}

generated quantities {
    real ypred;
    vector[N] log_lik;

    ypred = normal_rng(mu[6], sigma[6]);

    for (i in 1:N)
        log_lik[i] = normal_lpdf(y[i] | mu[x[i]], sigma[x[i]]);
}
```

If the PSIS-LOO value is considered to be stable, then lower PSIS-LOO value is considered to be better. The separate model has PSIS-LOO value of -132.133 which is the worst of all the models.

The effective number of parameters is: 9.69, which means that 9 parameters are considered to be effective.

Below is the histogram of the $\hat{k}$ values for the separate model



There are 3 $\hat{k}$ values that are greater than 0.7 which means that the PSIS-LOO value is not completely accurate and better methods should be used for evaluation of those values.

1.2 Hierarchical model

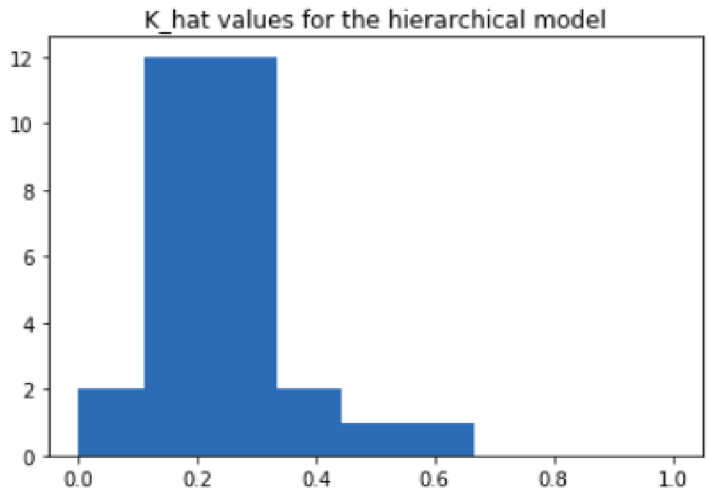
Here is that Stan code for the hierarchical model:

```
data {
    int<lower=0> N; // number of data points
    int<lower=0> K; // number of groups
    int<lower=1,upper=K> x[N]; // group indicator
    vector[N] y; //
}

parameters {
    real mu0;            // prior mean
    real<lower=0> sigma0; // prior std

    vector[K] mu;        // group means
    real<lower=0> sigma;
}

model {
    mu ~ normal(mu0, sigma0);
    y ~ normal(mu[x], sigma);
}

generated quantities {
    real mu7;
    real ypred6;
    vector[N] log_lik;

    mu7 = normal_rng(mu0, sigma0);
    ypred6 = normal_rng(mu[6], sigma);


    for (i in 1:N)
        log_lik[i] = normal_lpdf(y[i] | mu[x[i]], sigma);
}
```

If the PSIS-LOO value is considered to be stable, then lower PSIS-LOO value is considered to be better. The hierarchical model has PSIS-LOO value of -126.758 which is the lowest of all the models and this model can be considered to be **the best**.

The effective number of parameters is: 5.662, which means that 5 parameters are confidered to be effective.

Below is the histogram of the $\hat{k}$ values for the hierarchical model



From the histgram, we can see that all the $\hat{k}$ values are lower than 0.7, which means that the PSIS-LOO value can be considered to be accurate and reliable.

1.3 Pooled model

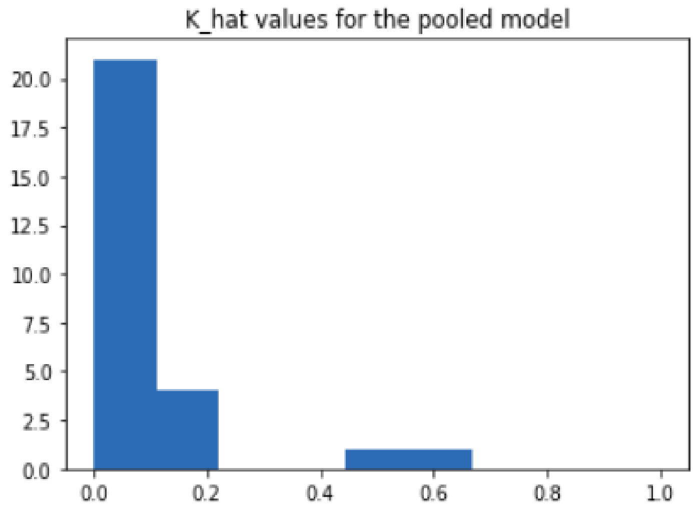
Here is that Stan code for the pooled model:

```
data {
    int<lower=0> N; // number of data points
    vector[N] y;
}

parameters {
    real mu;            // prior mean
    real<lower=0> sigma;
}

model {
  y ~ normal(mu, sigma);
}

generated quantities {
    real ypred;
    vector[N] log_lik;

    ypred = normal_rng(mu, sigma);

    for (i in 1:N)
        log_lik[i] = normal_lpdf(y[i] | mu, sigma);
}
```

If the PSIS-LOO value is considered to be stable, then lower PSIS-LOO value is considered to be better. The pooled model has PSIS-LOO value of -130.995 which is the second best model after the hierarchical.

The effective number of parameters is: 2.04, which means that 2 parameters are considered to be effective.

Below is the histogram of the $\hat{k}$ values for the pooled model



From the histgram, we can see that all the $\hat{k}$ values are lower than 0.7, which means that the PSIS-LOO value can be considered to be accurate and reliable.