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Experiment –04: To perform Image Enhancement in Frequency Domain

Date: _________

1. **Aim:** To perform Image Enhancement (Ideal Low Pass Filter and Butterworth High Pass Filter) in Frequency Domain

2. **Requirements:** Python

3. **Pre-Experiment Exercise**

3.1 Brief Theory

Filtering in the frequency domain is a common image and signal processing technique. It can smooth, sharpen, de-blur, and restore some images. Essentially, filtering is equal to convolving a function with a specific filter function. So one possibility to convolve two functions could be to transform them to the frequency domain, multiply them there and transform them back to spatial domain. The filtering procedure is summarized in Fig. 1.

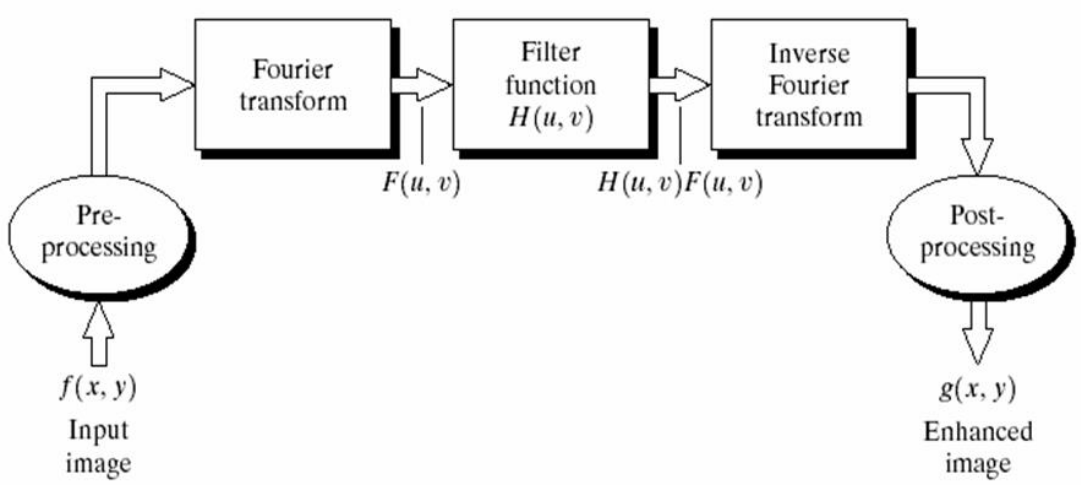

Fig. 1: Basic Steps for filtering in Frequency domain

Basic steps of filtering in the frequency domain:

1. Multiply the input image $f(x, y)$ by $(-1)^{(x+y)}$ to center the transform.
2. Compute $F(u, v)$, the DFT of the input image from (1).
3. Multiply $F(u, v)$ by a *filter* function $H(u, v)$.
4. Compute the inverse DFT of the result in (3).
5. Obtain the real part (better take the magnitude) of the result in (4).
6. Multiply the result in (5) by $(-1)^{(x+y)}$.

The D***iscrete Fourier Transform*** (DFT) is the most important discrete transform, used to perform Fourier analysis in many practical applications. In image processing, the samples can be the values of pixels along a row or column of a raster image. The 2D Discrete Fourier Transform is given by the equation:

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$$F(u,v)=\frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}f(x,y)e^{-j2\pi(ux/M+vy/N)}$$

and its inverse:

$$f(x,y)=\frac{1}{MN}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}F(u,v)e^{j2\pi(ux/M+vy/N)}$$

Ideal Low Pass Filter: The simplest low pass filter is the ideal low pass. It suppresses all frequencies higher than the cut-off frequency D0 and leaves smaller frequencies unchanged.

$$H(u,v)=\begin{cases}1 \text{ if } D(u,v)\le D_0\\ 0 \text{ if } D(u,v)>D_0\end{cases}$$

D0 is called the *cutoff frequency* (nonnegative quantity), and D(u,v) is the distance from point (u,v) to the frequency rectangle.

$$D(u,v)=\sqrt{(u-\frac{M}{2})^2+(v-\frac{N}{2})^2}$$

The drawback of the ideal low pass filter function is a ringing effect that occurs along the edges of the filtered image.

Butterworth High pass filter: The transfer function of Butterworth high pass filter (BHPF) of order n and with *cutoff frequency* D0 is given by

$$H(u,v)=\frac{1}{1+\left[D_0/D(u,v)\right]^{2n}}$$

4. Laboratory Exercise

4.1 Algorithm:

1. Take a gray scale image as an input.
2. Apply basic steps of filtering in frequency domain for ideal low pass filter and Butterworth high pass filter.
3. Display all intermediate (Fourier transform image, Filter function H(u,v), Filtered output) images.

5. Post-Experiment Exercise

5.1 Conclusion:

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5.2 Questions:

1. Apply DFT and IDFT transform on given image :

$$f(x,y) = \begin{matrix} 2 & 2 & 2 & 1 \\ 2 & 4 & 4 & 2 \\ 2 & 4 & 4 & 2 \\ 2 & 2 & 2 & 2 \end{matrix}$$

2. What is the difference between Ideal, Butterworth and Gaussian smoothening filters?