# **Enclosure methods for safe localisation**

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# Formalism State Equation





## State Equation

System defined by state equation

$$\begin{cases} \dot{x} = f(x, u) & (evolution) \\ y = g(x, u) & (observation) \end{cases}$$
 (1)





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## Application

- Dynamic knowledge of the system
- · Control the system
- Simulate the system





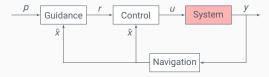


Figure 1: Guidance-Navigation-Control block diagram

## System

System for which state *x* want to be controlled.





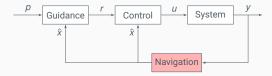


Figure 1: Guidance-Navigation-Control block diagram

#### Navigation

Estimate the state  $\hat{x}$  of the system using its output y.





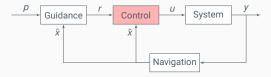


Figure 1: Guidance-Navigation-Control block diagram

Control the system toward the reference r from the guidance using estimated state  $\hat{\chi}$ .





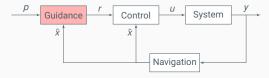


Figure 1: Guidance-Navigation-Control block diagram

#### Guidance

Guide the system toward a desired state using the estimated output  $\hat{x}$  and some input parameters p.





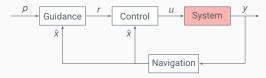


Figure 2: Guidance-Navigation-Control block diagram

# Nonlinear systems

- Approximations in Navigation
- Approximations in Control





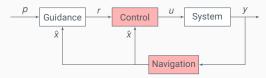


Figure 2: Guidance-Navigation-Control block diagram

#### Nonlinear systems

- · Approximations in Navigation
- · Approximations in Control

#### Set methods

- Deal with nonlinear equations
- Guaranteed results





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# **Navigation** Dead reckoning





## Dead reckoning

- Proprioceptive informations (odometry, inertial, ...)
- Robot achieve dead reckoning





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- Divergent set due to uncertainties

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# <u>K – 1</u>

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- Divergent set due to uncertainties

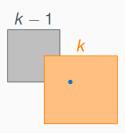




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# **Navigation** Field measurements





#### Field

- Field (magnetic, acoustic, bathymetric, ...)
- Give information about robot possible state
- Can contract the enclosing robot state

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- Mapping known a priori
- Map generated by sensor or simulator

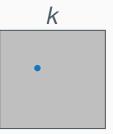




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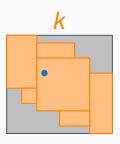




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- Using estimated robot state
- Generate prediction at next time step using enclosing evolution and control function





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- Check the viability of the control
- Danger zones
- · Enclose the next estimated state





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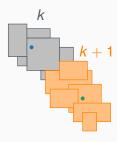






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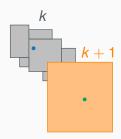






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- Generate control using enclosing control function on estimated state
- Take the barycenter of control as inputs





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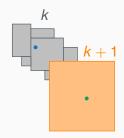
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#### Scope of the example

- Boat simulated with Dubins state equation
- Using simulated bathymetric map as measured field
- Control the boat to follow a Lissajous curve

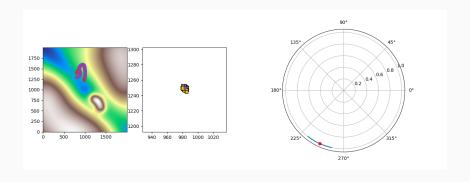
#### **Assumptions**

- State estimation and control only using interval analysis
- Inertial drift set to 0.1 m per seconds
- Bathymetric measurements uncertainty set to 0.1 m

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#### Conclusion

- Works better in highly nonlinear fields
- Possibility to refine next state estimation by enclosing it in a sub-paving
- Still a simulation will be tried in guaranteed robot control experiments.

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