ON SHIP WAVES.

By Sir WILLIAM THOMSON, Kt., F.R.SS. L. and E., LL.D.,
President R.S.E.

Lecture delivered at the Conversazione in the Science and Art Museum, Edinburgh, on Wednesday evening, 3rd August 1887.

"Waves" is a very comprehensive word. It comprehends waves of water, waves of light, waves of sound, and waves of solid matter such as are experienced in earthquakes. It also comprehends much more than these. "Waves" may be defined generally as a progression through matter of a state of motion. The distinction between the progress of matter from one place to another, and the progress of a wave from one place to another through matter, is well illustrated by the very largest examples of waves that we have —largest in one dimension, smallest in another—waves of light, waves which extend from the remotest star, at least a million times as far from us as the sun is. Think of ninty-three million million miles, and think of waves of light coming from stars known to be at as great a distance as that! So much for the distance of propagation or progression of waves of light. But there are two other magnitudes concerned in waves: there is the wave-length, and there is the amount of displacement of a moving particle in the wave. Waves of light consist of vibrations to and fro, perpendicular to the line of progression of the wave. The length of the wave—I shall explain the meaning of "wave-length" presently: it speaks for itself in fact if we look at waves of water, as shown in Fig. 2, Plate 80—the length from crest to crest in waves of light is from one thirty-thousandth to one fifty-thousandth or one sixty-thousandth of an inch; and these waves of light travel through all known space. Waves of sound differ from waves of light in the vibration of the moving particles being along the line of propagation of the wave, instead of perpendicular to it. Waves of water agree more nearly with waves of light than do waves of sound; but waves of water have this great distinction from waves of light and waves of sound, that they are manifested at the surface or termination of the medium or substance whose motion constitutes the wave. It is with waves of water that we are concerned to-night; and of all the beautiful forms of water waves that of Ship Waves is perhaps the most beautiful, if we can compare the beauty of such beautiful things. The subject of ship waves is certainly one of the most interesting in mathematical science. It possesses a special and intense interest, partly from the difficulty of the problem, and partly from the peculiar complexity of the circumstances concerned in the configuration of the waves.

Canal Waves.—I shall not at first speak of that beautiful configuration or wave-pattern, which I am going to describe a little later, seen in the wake of a ship travelling through the open water at sea; but I shall, as included in my special subject of ship waves, refer in the first place to waves in a canal, and to Scott Russell's splendid researches on that subject, made about the year 1834—fifty-three years ago—and communicated by him to the Royal Society of Edinburgh. The diagrams copied from his paper in the Transactions of that Society will serve for a preliminary explanation or illustration of the meaning of the term "wave." I gave a very general and abstract definition; let us now have it in the concrete: a wave of water produced by a boat dragged along a canal. In Plate 80 is reproduced one of Scott Russell's pictures illustrating some of his celebrated experiments. In Fig. 2 is shown a boat in the position that he called behind the wave; and in the rear of the boat is seen a procession of waves. It is this procession of waves that we have to deal with in the first place. We must learn to understand the procession of waves in the rear of the canal boat, before we can follow, or take up the elements of, the more complicated pattern which is seen in the wake of a ship travelling through open water at sea. Scott Russell made a fine discovery in the course of those experiments. He found that it is only when the speed of the boat is less than a certain limit that it leaves that procession of waves in its rear. Now the question that I am going to ask is, how is that procession kept in motion? Does it take power to drag the boat along, and to produce or to maintain that procession of waves? We all know it does take power to drag a boat through a canal; but we do not always think on what part of the phenomena, manifested by the progress of the boat through the canal, the power to drag the boat depends.

I shall ask you for a time to think of water not as it is, but as we can conceive a substance to be—that is, absolutely fluid. In reality water is not perfectly fluid, because it resists change of shape; and non-resistance to change of shape is the definition of a perfect fluid. Is water then a fluid at all? It is a fluid because it permits change of shape; it is a fluid in the same sense that thick oil or treacle is a fluid. Is it only in the same sense? I say yes. Water is no more fluid in the abstract than is treacle or thick oil. Water, oil, and treacle, all resist change of shape. When we attempt to make the change very rapidly, there is a great resistance; but if we make the change very slowly, there is a small resistance. The resistance of these fluids to change of shape is proportionate to the speed of the change: the quicker you change the shape, the greater is the force that is required to make the change. Only give it time, and treacle or oil will settle to its level in a glass or basin, just as water does. No deviation from perfect fluidity, if the question of time does not enter, has ever been discovered in any of these fluids. In the case of all ordinary liquids, anything that looks like liquid and is transparent or clear — or, even if it is not transparent, anything that is commonly called a fluid or liquid—is perfectly liquid in the sense of exerting no permanent resistance to change of shape. The difference between water and a viscous substance, like treacle or oil, is defined merely by taking into account time. Now for some motions of water (as capillary waves), resistance to change of shape, or as we call it viscosity, has a paramount effect; for other cases viscosity has no sensible effect. I may tell you this—I cannot now prove it, for my function this evening is only to explain and bring before you generally some results of mathematical calculation and experimental observation on these subjects—I may tell you that great waves at sea will travel for hours or even for days, showing scarcely any loss of sensible motion—or of energy, if you will allow me so to call it—through viscosity. On the other hand, look at the ripples in a little pond, or in a little pool of fresh rain water lying in the street, which are excited by a puff of wind; the puff of wind is no sooner gone than the ripples begin to subside, and before you can count five or six the water is again perfectly still. The forces concerned in short waves such as ripples, and the forces concerned in long waves such as great ocean waves, are so related to time and to speed that, whereas in the case of short waves the viscosity which exists in water comes to be very potent, in the case of long waves it has but little effect.

Allow me then for a short time to treat water as if it were absolutely free from viscosity—as if it were a perfect fluid; and I shall afterwards endeavour to point out where viscosity comes into play, and causes the results of observation to differ more or less—very greatly in some cases, and very slightly in others—from what we should calculate on the supposition of water being a perfect fluid. If water were a perfect fluid, the velocity of progression of a wave in a canal would be smaller the shorter the wave. That of a "long wave"—whose length from crest to crest is many times the depth of the canal—is equal to the velocity which a body acquires in falling from a height equal to half the depth of the canal. For brevity we might call this height the "speed height"—the height from which a body must fall to acquire a certain speed. Examples: a body falls from a height of 16 feet, and it acquires a velocity of eight times the square root of the height, or 32 feet per second; a body falls from a height of 4 feet, the velocity is therefore only 16 feet per second; and so on. Thus in a canal 8 feet deep the natural velocity of the "long wave" is 16 feet per second, or about 11 miles per hour. If water were a perfect fluid, this would be the state of the case: a boat dragged along a canal at any velocity less than the natural speed of the long wave in the canal would leave a train of waves behind it of so much shorter length that their velocity of propagation would be equal to the velocity of the boat; and it is mathematically proved that the boat would take such a position as is shown in Fig. 2, Plate 80, namely just on the rear slope of the wave. It was not by mathematicians that this was found out; but it was Scott Russell's accurate observation and well devised experiments that first gave us these beautiful conclusions.

To go back: a wave is the progression through matter of a state of motion. The motion cannot take place without the displacement of particles. Vary the definition by saying that a wave is the progression of displacement. Look at a field of corn on a windy day. You see that there is something travelling over it. That something is *not* the ears of corn carried from one side of the field to the other, but *is* the change of colour due to your seeing the sides or lower ends of the ears of corn instead of the tops. A laying down of the stalk is the thing that travels in the wave passing over the corn field. The thing that travels in the wave in Fig. 2, Plate 80, is an elevation of the water at the crest and a depression in the hollow. You might make a wave thus. Place over the surface of the water in a canal a wave-form, made from a piece of paste-board or of plastic material such as gutta-percha that you can mould to any given shape; and take care that the water fills up the wave-form everywhere, leaving no bubbles of air in the upper bends. Now you have a constant displacement of the water from its level. Now take your gutta-percha form, and cause it to move along — drag it along the surface of the canal—and you will thereby produce a wave. That is one of the best and most convenient of mathematical ways of viewing a wave. Imagine a wave generated in that way; calculate what kind of motion can be so generated, and you have not merely the surface motion produced by the force that you applied, but you have the water-motion in the interior. You have the whole essence of the thing discovered, if you can mathematically calculate from a given motion at the surface what is the motion that necessarily follows throughout the interior; and that can be done, and is a part of the elements of the mathematical results which I have to bring before you.

Now to find mathematically the velocity of progression of a free wave, proceed thus. Take your gutta-percha form and hold it stationary on the surface of the water; the water-pressure is less at the crest and greater at the hollow; by the law of hydrostatics, the deeper down you go, the greater is the pressure. Move your form along very rapidly, and a certain result, a centrifugal force, due to the inertia of the flowing water, will now cause the pressure to be greatest at the crest and least at the lowest point of the hollow, Fig. 2, Plate 80. Move it along at exactly the proper speed, and you will cause the pressure to be equal all over the surface of the gutta-percha form. Now have done with it. We only had it in imagination. Having imagined it and got what we wanted out of it, discard it when moving at exactly this proper speed, and then you have a free wave. That is a slight sketch of the mode by which we investigate mathematically the velocity of the free wave. It was by observation that Scott Russell found it out; and then there was a mathematical verification, not of the perfect theoretic kind, but of a kind which showed a wonderful grasp of mind and power of reasoning upon the phenomena that he had observed.

But still the question occurs to everybody who thinks of these things in an engineering way, how does that procession require work to be done to keep it up? or does it require work to be done? May it not be that the work required to drag the boat along the canal has nothing to do with the waves after all? that the formation of the procession of waves once effected leaves nothing more to be desired in the way of work? that the procession once formed will go on of itself, requiring no work to sustain it? Here is the explanation. The procession has an end. The canal may be infinitely long, the time the boat may be going may be as long as you please; but let us think of a beginning—the boat started, the procession begun to form. The next time you make a passage in a steamer, especially in smooth water, look behind the steamer, and you will see a wave or two as the steamer gets into motion. As it goes faster and faster, you will see a wave-pattern spread out; and if you were on shore, or in a boat in the wake of the steamer, you would see that the rear end of the procession of waves follows the steamer at an increasing distance behind. It is an exceedingly complicated phenomenon, and it would take a great deal of study to make out the law of it merely from observation. In a canal the thing is more simple. Scott Russell however did not include this in his work. This was left to Stokes, to Osborne Reynolds, and to Lord Rayleigh. The velocity of progress of a wave is one thing; the velocity of the front of a procession of waves, and of the rear of a procession of waves, is another thing. Stokes made a grand new opening, showing us a vista previously unthought of in dynamical science. As was his manner, he did it merely in an examination question set for the candidates for the Smith prize in the University of Cambridge. I do not remember the year, and I do not know whether any particular candidate answered the question; but this I know, that about two years after the question was put Osborne Reynolds answered it with very good effect indeed. In a contribution to the Plymouth Meeting of the British Association in 1877 (see "Nature" 23 Aug. 1877, pages 343–4), in which he worked out one great branch at all events of the theory thus pointed out by Stokes, Reynolds gave this doctrine of energy that I am going to try to explain; and a few years later Lord Rayleigh took it up and generalised it in the most admirable manner, laying the foundation not only of one part, but of the whole, of the theory of the velocity of groups of waves.

The theory of the velocity of groups of waves, on which is founded the explanation of the wave-making resistance to ships whether in a canal or at sea, I think I have explained in such a way that I hope every one will understand the doctrine in respect to waves in a canal; it is more complex in respect to waves at sea. I shall try to give you something on that part of the subject; but as to the dynamical theory, you will see it clearly in regard to waves in a canal. If this drawing, Fig. 2, Plate 80, were continued backwards far enough, it would show an end to the procession of waves in the rear of the boat; and the distance of that end would depend on the time the boat had been travelling. You will remember that we have hitherto been supposing water to be free from viscosity; but in reality water has enough of viscosity to cause the cessation of the wave procession at a distance corresponding to 50 or 60 or 100 or 1000 wave-lengths in the rear of the ship. In a canal especially viscosity is very effective, because the water has to flow more or less across the bottom and up and down by the banks; so that we have not there nearly the same freedom that we have at sea from the effects of viscosity in respect to waves. The rear of the procession travels forward at half the speed of the ship, if the water be very deep. What do I mean by very deep? I mean a depth equal to at least one wave-length; but it will be nearly the same if the depth be three-quarters of a wave-length. For my present purpose, in which I am not giving results with minute accuracy, we will call very deep any depth more than three-quarters of the wave-length. For instance, if the depth of the water in the canal is anything more than three-quarters of the length from crest to crest of the wave, the rate of progression of the rear of the procession will be half the speed of the boat. Here then is the state of the case. The boat is followed by an ever-lengthening procession of waves; and the work required to drag the boat along in the canal—supposing that the water is free from viscosity—is just equal to the work required to generate the procession of waves lengthening backwards behind the boat at half the speed of the boat. The rear of the procession travels forwards at half the speed of the boat; the procession lengthens backwards relatively to the boat at half the speed of the boat. There is the whole thing; and if you only know how to calculate the energy of a procession of waves, assuming the water free from viscosity, you can calculate the work which must be done to keep a canal boat in motion.

But now note this wonderful result: if the motion of the canal boat be *more rapid* than the most rapid possible wave in the canal (that is, the long wave), it cannot leave behind it a procession of waves—it cannot make waves, properly so called, at all; it can only make a hump or a hillock travelling with the boat, as shown in Fig. 3, Plate 80. What would you say of the work required to move the boat in that case? You may answer that question at once: it would require no work; start it, and it will go on for ever. Everyone understands that a curling stone projected along the ice would go on for ever, were it not for the friction of the ice; and therefore it must not seem so wonderful that a boat started moving through water would also go on for ever, if the water were perfectly fluid: it *would not*, if it is forming an ever-lengthening procession of waves behind it; it *would* go on for ever, if it is *not* forming a procession of waves behind it. The answer then simply is, give the boat a velocity greater than the velocity of propagation of the most rapid wave (the long wave) that the canal can have; and in these circumstances, ideal so far as nullity of viscosity is concerned, it will travel along and continue moving without any work being done upon it. I have said that the velocity of the long wave in a canal is equal to the velocity which a body acquires in falling from a height equal to half the depth of the canal. The term "long wave" I may now further explain as meaning a wave whose length is many times the depth of the water in the canal—50 times the depth will fulfil this condition—the length being always reckoned from crest to crest. Now if the wave-length from crest to crest be 50 or more times the depth of the canal, then the velocity of the wave is that acquired by a body falling through a height equal to half the depth of the canal; if the wave-length be less than that, the velocity can be expressed only by a complex mathematical formula. The results have been calculated; but I need not put them before you, because we are not going to occupy ourselves with them.

The conclusion then at which we have arrived is this: supposing at first the velocity of the boat to be such as to make the waves behind it of wave-length short in comparison with the depth of water in the canal: let the boat go a little faster, and give it time until steady waves are formed behind it; these waves will be of longer wave-length: the greater the speed of the boat, the longer will be the wave-length, until we reach a certain limit; and as the wave-length begins to be equal to the depth, or twice the depth, or three times the depth, we approach a wonderful and critical condition of affairs—we approach the case of constant wave velocity. There will still be a procession of waves behind the boat, but it will be a shorter procession and of higher waves; and this procession will not now lengthen astern at half the speed of the boat, but will lengthen perhaps at a third, or a fourth, or perhaps at a tenth of the speed of the boat. We are approaching the critical condition: the rear of the procession of waves is going forward nearly as fast as the boat. This looks as if we were coming to a diminished resistance; but it is not really so. Though the procession is lengthening less rapidly relatively to the boat than when the speed was smaller, the waves are very much higher; and we approach almost in a tumultuous manner to a certain critical velocity. I will read you presently Scott Russell's words on the subject. Once that crisis has been reached, away the boat goes merrily, leaving no wave behind it, and experiencing no resistance whatever if the water be free from viscosity, but in reality experiencing a very large resistance, because now the viscosity of the water begins to tell largely on the phenomena. I think you will be interested in hearing Scott Russell's own statement of his discovery. I say his discovery, but in reality the discovery was made by a horse, as you will learn. I found almost surprisingly in a mathematical investigation, "On Stationary Waves in Flowing Water," contributed to the *Philosophical Magazine* (Oct. Nov. Dec. 1886 and Jan. 1887), a theoretical confirmation, 49½ years after date, of Scott Russell's brilliant "Experimental Researches into the Laws of Certain Hydrodynamical Phenomena that accompany the Motion of Floating Bodies, and have not previously been reduced into conformity with the known Laws of the Resistance of Fluids."*

These experimental researches led to the Scottish system of fly-boats carrying passengers on the Glasgow and Ardrossan Canal, and between Edinburgh and Glasgow on the Forth and Clyde Canal, at speeds of from eight to thirteen miles an hour, each boat drawn by a horse or pair of horses galloping along the bank. The method originated from the accident of a spirited horse, whose duty it was to drag the boat along at a slow walking speed, taking fright and

* By John Scott Russell, Esq., M.A., F.R.S.E. Read before the Royal Society of Edinburgh, 3 April 1837, and published in the Transactions in 1840.

running off, drawing the boat after him; and it was discovered that, when the speed exceeded the velocity acquired by a body falling through a height equal to half the depth of the canal (and the horse certainly found this), the resistance was less than at lower speeds. Scott Russell's description of how Mr. Houston took advantage for his Company of the horse's discovery is so interesting that I quote it *in extenso*:—

"Canal navigation furnishes at once the most interesting illustrations of the interference of the wave, and most important opportunities for the application of its principles to an improved system of practice.

"It is to the diminished anterior section of displacement, produced by raising a vessel with a sudden impulse to the summit of the progressive wave, that a very great improvement recently introduced into canal transports owes its existence. As far as I am able to learn, the isolated fact was discovered accidentally on the Glasgow and Ardrossan Canal of small dimensions. A spirited horse in the boat of William Houston, Esq., one of the proprietors of the works, took fright and ran off, dragging the boat with it, and it was then observed, to Mr. Houston's astonishment, that the foaming stern surge which used to devastate the banks had ceased, and the vessel was carried on through water comparatively smooth with a resistance very greatly diminished. Mr. Houston had the tact to perceive the mercantile value of this fact to the canal company with which he was connected, and devoted himself to introducing on that canal vessels moving with this high velocity. The result of this improvement was so valuable, in a mercantile point of view, as to bring, from the conveyance of passengers at a high velocity, a large increase of revenue to the canal proprietors. The passengers and luggage are conveyed* in light boats, about

---

* This statement was made to the Royal Society of Edinburgh in 1837, and it appeared in the Transactions in 1840. Almost before the publication in the Transactions the present tense might, alas, have been changed to the past—"passengers *were* conveyed." Is it possible not to regret the old fly-boats between Glasgow and Ardrossan and between Glasgow and Edinburgh, and their beautiful hydrodynamics, when, hurried along on the railway, we catch a

sixty feet long, and six feet wide, made of thin sheet iron, and drawn by a pair of horses. The boat starts at a slow velocity behind the wave, and at a given signal it is by a sudden jerk of the horses drawn up on the top of the wave, where it moves with diminished resistance, at the rate of 7, 8, or 9 miles an hour."

Scott Russell was not satisfied with a mere observation of this kind. He made a magnificent experimental investigation into the circumstances. An experimental station at the Bridge of Hermiston on the Forth and Clyde Canal was arranged for the work, as represented in Fig. 1, Plate 80. The experimental station was so situated that there was a straight run of 1500 feet along the bank, and three pairs of horses are seen galloping along. They seem from the drawing to be galloping on air, but are of course on the towing path; and this remark may be taken as an illustration that, if the horses only galloped fast enough, they could gallop over the water without sinking into it, as they might gallop over a soft clay field. That is a sober fact with regard to the theory of waves; it is only a question of time how far the heavy body will enter into the water, if it is dragged very rapidly over it. This however is a digression. The very ingenious apparatus of Scott Russell's is delineated in Fig. 1. There is a pyramid 75 feet high, supporting a system of pulleys which carry a heavy weight suspended by means of a rope. The horses are dragging one end of this rope, while the other end is fastened to a boat which travels in the opposite direction. It is the old principle applied by Huyghens, and still largely used, in clockwork. Scott Russell employed it to give a constant dragging force to the boat from the necessarily inconstant action of the horses. I need not go into details, but I wish you to see that Scott Russell, in devising these experiments, adopted methods for accurate measurement in order to work out the theory of those results, the general natural history of which he had previously observed.

I will now read certain results from Scott Russell's paper that I think are interesting. The depth of the canal at the experimental

---

glimpse of the Forth and Clyde Canal still used for slow goods traffic; or of some swampy hollows, all that remains of the Ardrossan Canal on which the horse and Mr. Houston and Scott Russell made their discovery?

station was about 4 or 5 feet on an average; it was really $5\frac{1}{2}$ feet in the middle, but a proper average depth must have been about $4\frac{1}{2}$ feet, because Scott Russell found by experiment that the natural speed of the long wave was about 8 British statute miles an hour or 12 feet per second. Here then are some of the results. The "Raith," a boat weighing 10,239 lbs. (nearly 5 tons), took the following forces to drag it along at different speeds:—at 4·72 miles an hour 112 lbs.; at 5·92 miles an hour 261 lbs.; and at 6·19 miles an hour 275 lbs. There is no observation at the critical velocity of about 8 miles an hour. The next is at 9·04 miles an hour, and the force is 250 lbs., as compared with 275 lbs. at 6·19 miles an hour. Then at a higher speed, 10·48 miles an hour, the force required to drag it increases to $268\frac{1}{2}$ lbs. This illustrates that water is not a perfect fluid. It also illustrates the theoretical result in a beautiful and interesting way. If water were a perfect fluid, the forces at the lower speeds would be somewhat less than he has given, perhaps not very much less: at all speeds above 8 miles the force would be nothing; the boat once started, the motion would go on for ever. On the same canal another boat, weighing 12,579 lbs. (nearly 6 tons), gave these still more remarkable results:—at 6·19 miles an hour 250 lbs.; at 7·57 miles an hour 500 lbs.; at 8·52 miles an hour 400 lbs.; and at 9·04 miles an hour only 280 lbs. That is a striking confirmation of the result of the previous observations. Scott Russell says also: "I have seen a vessel in 5 feet water, and drawing only 2 feet, take the ground in the hollow of a wave having a velocity of about 8 miles an hour, whereas at 9 miles an hour the keel was not within 4 feet of the bottom." Again he says: "Two or three years ago, it happened that a large canal in England was closed against general trade by want of water, drought having reduced the depth from 12 to 5 feet. It was now found that the motion of the light boats was rendered more easy than before; the cause is obvious. The velocity of the wave was so much reduced by the diminished depth, that, instead of remaining behind the wave, the vessels rode on its summit." He also makes this interesting statement: "I am also informed by Mr. Smith of Philadelphia, that he distinctly recollects the circumstance of having travelled on the Pennsylvania canal in 1833, when one of the levels was not fully supplied with water, the works having been recently executed, and not being yet perfectly finished. This canal was intended for 5 feet of water, but near Silversford the depth did not exceed 2 feet; and Mr. Smith distinctly recollects having observed to his astonishment, that, on entering this portion, the vessel ceased to ground at the stern, and was drawn along with much greater apparent ease than on the deeper portions of the canal."

Even if one regretted the introduction of railways, do not imagine that it can be set forth on mechanical grounds that traction in a canal can compete for any considerable speeds with traction on a railway. Taking again some of the figures already given, a boat weighing 10,239 lbs. required 112 lbs., or about 1–100th of its weight, to drag it at $4\frac{3}{4}$ miles an hour. So that to drag a boat at that moderate speed took the same force as would be required to drag it on wheels up an incline of 1 in 100, supposing there to be no friction in the wheels on a railway. But at the higher speed of 9 miles an hour, taking advantage of the comparatively smaller force due to having passed the velocity corresponding with the long wave, we have 250 lbs., which divided by 10,239 is about 1 in 40; so that the force required to drag the boat along at the rate of 9 miles an hour was what would be required to drag it on wheels up an incline of 1 in 40. Sad to say, I am afraid the wheels have it in an economical point of view.

Ship Waves at Sea.—I must now call your attention to the most beautiful, the most difficult, and in some respects the most interesting part of my subject, that is, the pattern of waves formed in the rear of a ship at sea, not confined by the two banks of a canal. The whole subject of naval dynamics, including valuable observations and suggestions regarding ship waves, was worked out with wonderful power by William Froude; and the investigations of the father were continued by his son, Edmund Froude, in the Government Experimental Works at Haslar, Gun Creek, Gosport. William Froude commenced his system of nautical experiments in a tank made by himself at Torquay, in Devonshire; first wholly at his own expense for several years, and afterwards with the assistance of the Government he continued those experiments till his death. The Admiralty have taken up the work, and have made for it an experimental establishment in connection with the dockyard of Portsmouth; and now, after the death of William Froude, his son Edmund continues to carry out there his father's ideas, working with a large measure of his father's genius, and, with his father's perseverance and mechanical skill, obtaining results, the practical value of which it is impossible to over-estimate. It is certainly of very great importance indeed to this country, which depends so much on shipbuilding, and the prosperity of which is so much influenced by the success of its shipbuilders, to find the shapes of ships best suited for different kinds of work—ships of war, swift ships for carrying mails and passengers, and goods carriers. I may mention also that one of our great shipbuilding firms on the Clyde, the Dennys, feeling the importance of experiments of this kind, have themselves made a tank for experimental purposes on the same plan as Mr. Froude's tank at Torquay; and Mr. Purvis, who, when a young man, was one of Mr. Froude's assistants, is taking charge of that work. The Dennys are going through, with their own ships, the series of experiments which Mr. Froude found so useful, and which the Admiralty now find so useful, in regard to the design of ships; and as the outcome of all this work a ship can now be confidently designed to go at a certain speed, to carry a certain weight, and to require a certain amount of horse-power from the engine.

The full mathematical theory of ship waves has been exceedingly attractive in one sense, and in another sense it has been somewhat repulsive, because of its great difficulty, for mathematicians who have been engaged in hydrodynamical problems. Following out that principle of Stokes, which was further developed and generalised by Lord Rayleigh, we can see how to work out this theory in a thorough manner. In fact I can now put before you a model, Figs. 10 and 11, Plate 82, constructed from calculations which I have actually made, by following out the lines of theory that I have indicated. I find that the whole pattern of waves is comprised between two straight lines drawn from the bow of the ship and inclined to the wake on its two sides at equal angles of 19° 28′. It is seen in Figs. 9 and 10 that two such lines, drawn from the bow or front shoulders of the ship, include the whole wave-pattern. There is some disturbance in the water abreast of the ship, before coming to these two lines. Theoretically there is a disturbance to an infinite distance ahead and in every direction; but the amount of that disturbance practically is exceedingly small—imperceptible indeed—until you come to these two definite lines. You see the oblique wave-pattern—waves in echelon pattern. The law of that echelon is illustrated by the curves shown in Fig. 9, Plate 82. The algebraical equations of these curves are $x = \sqrt{\frac{1 \pm w}{(3 \pm w)^3}}$ and $\frac{y}{x} = \sqrt{\frac{1}{8}(1 - w^2)}$; where x and y, according to ordinary usage, are measured along, and perpendicular to, the direction of motion from E towards A, and w is an arbitrary variable; by assuming a series of arbitrary values for w, a corresponding series of values for x are found from the first equation, and thence the corresponding values of y from the second. I trace a complete curve thus—ABC and ADC; there is a perfect cusp in each curve at B and D respectively, although it cannot be shown perfectly in the drawing. Another formula, which need not be reproduced here, gives a wave-height for every point of those curves. Take alternate curves for hollows and for crests; and now in clay or plaster of Paris mould a form corresponding with the elevation due to the curve AB, plus the elevation due to the curve BC, adding the two together; thus you get for every point of your curves a certain wave-height. With the assistance of Mr. Maclean and Mr. Niblett the beautiful clay model which is before you (illustrated in Figs. 10 and 11, Plate 82) has been made, and it shows the results of the theory constructed from actual calculation. I will tell you how to construct the angle of 19° 28′ made by each of those two straight lines AB and AD with the direction of motion CA. Draw a circle; produce the diameter from one end to a length equal to the diameter; and from the outer extremity of this projecting line draw two tangents to the circle. Each of those tangent lines makes an angle of 19° 28′ with the produced diameter, that is, with the wake of the ship or with the line of progression of the ship.

A little more as to the law of this diagram, Fig. 9, Plate 82. The echelon waves consist chiefly of the very steep waves at a cusp. The theoretical formula gives infinite height at the cusp; but that is only a theoretical supposition, though it gives an interesting illustration of mathematical "infinity." Blur it, or smooth it down, precisely as an artist does when he designedly blurs a portion of his picture to produce an artistic effect; blur it artistically, correctly, and mathematically, and you get the pattern. It will be impossible to realise that perfectly; but I have endeavoured to do it in the model illustrated in Figs. 10 and 11, necessarily with an enormous exaggeration however, as you will remark. While every other dimension is unchanged, you must suppose each wave to be reduced to about a fifth part of its height shown in this model; thus you will get the steep "steamboat waves," so much enjoyed by the little boys who, regardless of danger, row out their boats to them every day at the Clyde watering places. Theoretically these waves are infinitely steep; practically they are so steep that the boat generally takes in a little water, and is sometimes capsized. There is a distance of perhaps a couple of feet from crest to crest, and the wave is so steep and "lumpy" on the outer border of the echelon that there is frequently broken water there fifty or a hundred yards from the ship. One point of importance in the geometry of this pattern is that each echelon cusp, represented in Fig. 9 at B or D, bisects the angle between the flank line AB or AD and the thwart-ship line BD: the angle in question being $70° 32'$ $(90° - 19° 28')$. An observation of this angle was actually made for me by Mr. Purvis. He observed, from the towing of a sphere instead of a boat (so as to get a more definite point), the angle between the flank line AB and the direction of motion CA, and found it to be $19\frac{1}{4}°$. The theoretical angle is $19° 28'$, and we have therefore in this observation a very admirable and interesting verification of the theory.

The doctrine embodied in the wave-model illustrated in Figs. 10 and 11, Plate 82, may be described in a very general way thus. Think of a ship travelling over water. How is it that it makes the wave? Where was the ship when it gave rise to the wave BCD in Fig. 9? Answer: the portion BCD of the wave-pattern is due to what the ship did to the water when the ship was at E, the point E being at the same distance behind C that the point A is in front of C; when it was at E it was urging the water aside, and the effect of the ship pushing the water aside was to leave a depression. Now suppose the ship to be suddenly annihilated or annulled, what would be the result? The waves would travel out from it, as in the case of a stone thrown into the water. Again suppose the ship to move ten yards forward and then stop, what would be the result? A set of waves travelling forward while the disturbance that the ship made by travelling ten yards remains. Now instead of stopping, let the ship go on its course: the wave disturbance is going *its* course freed from the ship, and travels forward. When the disturbance originated which has now reached any point C, the ship was as far behind that point C as it now is before it. Calculate out the result from the law that the group-velocity is half the wave-velocity—the velocity of a group of waves at sea is half the velocity of the individual waves. Follow the crest of a wave, and you see the wave travelling through the group, if it forms one of a group or procession of waves. Look, quite independently of the ship, at a vast procession of waves, or imagine say fifty waves; look at one of those waves, follow its crest; in imagination fly as a bird over it, keeping above the crest as a bird in soaring does sometimes, and, beginning over the rear of the procession, a hundred yards on either side of the ship's wake, you will find the waves get larger and larger as you go forwards. Then go backwards through the procession, and you will see the waves get smaller and smaller and finally disappear. You have now gone back to the rear of the procession; a small wave increases and travels uniformly forward, and, while the crest of each wave always goes on with the velocity corresponding to the length of the wave, the rear of the procession travels forward at half the speed of the wave: so that every wave is travelling forward through the procession from its rear at a speed which is the same relatively to the rear of the procession as the speed of the rear of the procession relatively to the water. Thus each separate wave is travelling at the ship's speed, which is twice as fast relatively to the water as the rear of the procession of waves is travelling. The wave is the progression of a form; the velocity of a wave is clearly intelligible; the velocity of a procession of waves is still another thing. The penetrating genius of Stokes originated the principle, admirably worked out by Osborne Reynolds and Lord Rayleigh, who have given us this in the shape in which we now have it.

Now I must call your attention to some exceedingly interesting diagrams that I am enabled to show you through the kindness of Mr. W. H. White, director of Naval Construction for the Admiralty, and Mr. Edmund Froude, to whose work I have already referred. Fig. 12, Plate 82, shows a perspective view of echelon waves taken from Mr. William Froude's paper, "Experiments upon the Effect Produced on the Wave-making Resistance of Ships by Length of Parallel Middle Body" (Institution of Naval Architects, vol. xviii 1877, page 77).

The three diagrams from Mr. White, Figs. 6, 7, and 8, Plate 81, show profiles of the thwart-ship waves of various ships at different speeds. Look first at Fig. 6, showing the wave profile for H.M.S. "Curlew" at a speed of nearly 15 knots an hour. Note how the water, after the first elevation, dips down below the still-water line; rises up to a ridge at a distance back from the first nearly but not exactly equal to the wave-length corresponding with the speed; and then falls down again, experiencing various disturbances. From the appearance of the waves raised by ships going at high speeds, we may learn to tell how quickly they are going. The other day, at the departure of the fleet from Spithead after the great naval review, a ship was said to be going at 18 knots, while it was obvious from the waves it made that it was not going more than 12. In Fig. 7 we have wave profiles for another ship at two different speeds. The upper line corresponds to a speed of 18·4 knots; the lower line to a speed of 17 knots. In the first case the water shoots up to its first maximum height close to the bow, sinks to a minimum towards midships, and flows away past the stern slightly above still-water level. In the second case the character of the wave is somewhat similar, but smaller in height; and there is a marked difference at the stern, due to other disturbing causes. In Fig. 8 we have three different speeds for H.M.S. "Orlando" similarly represented.

There is still another very interesting series of diagrams, Figs. 13 to 19, Plates 83 and 84, taken from Mr. Edmund Froude's paper "On the Leading Phenomena of the Wave-making Resistance of Ships," read before the Institution of Naval Architects, 8th April 1881. In Figs. 13 to 17 are shown the waves produced by a torpedo launch at speeds of 9, 12, 15, 18, and 21 knots per hour. We need not here go into the law of wave-length, but I may tell you that it is as the square of the velocity: the wave-length is four times as great for 18 as for 9 knots. Look now at the pattern of the waves in Figs. 9 and 10, Plate 82. Look at the echelon waves and the thwart-ship waves. Mr. Froude had not worked out the theory that has given the curvature of the transverse ridge exactly; but he drew the waves from general observation, and it is wonderful to see how nearly they agree with the theoretical curves, Fig. 9, and the model, Fig. 10.

Velocity and Length of Waves.

Velocity of Wave. Knots per hour.	Length of Wave. Feet.	Velocity of Wave. Knots per hour.	Length of Wave. Feet.
6	19·513	17	156·646
7	26·559	18	175·618
8	34·690	19	195·672
9	43·904	20	216·812
10	54·203	22	262·343
11	65·585	24	312·209
12	78·052	26	366·412
13	91·602	28	424·952
14	106·238	30	487·827
15	121·956	35	663·987
16	138·760	40	867·248

That is a most interesting series of diagrams in Plates 83 and 84, and as a lesson it conveys more than any words of mine.

Here is a table (page 428) giving the length of a free wave; and remember, when once the waves are made and are left by the ship, they are then and thereafter free waves. At 6 knots per hour the wave-length is 19½ feet; at 12 knots it is four times as great. At 10 knots it is 54 feet; at 20 it is four times as much. The greatest speeds in Froude's diagrams give about 240 feet length of wave. Now that is a very critical point with respect to the length of the wave and the speed of the ship. I may tell you that Froude the elder and his son Edmund have made most admirable researches in this subject, and have poured a flood of light on some of the most difficult questions of naval architecture.

Parallel Middle Body.—I should like to say something about the practical question of parallel middle body. When I first remember shipbuilding on the Clyde, and its progress towards its present condition, a very frequent incident was that when a ship was floated it was found to draw too much water forward, in other words to be down by the head. When this happened, the ship was taken out of the water again, and a parallel piece, 10 or 20 or 30 feet long, was put into the middle: a parallel middle body, curved transversely, but with straight lines in the direction of its length. Many a ship was also lengthened with a view to add to its speed. William Froude took up the question of parallel middle body, and the effect of the entrance and run. The entrance is that part of the ship forward, where it enters the water and swells out to the full breadth of the ship; the run is the after part, extending from where the ship begins to narrow to the stern. A ship may consist of entrance, parallel middle body, and run. Froude investigated the question, Is the parallel middle body inserted in a ship an advantage or a disadvantage, in some cases or in all cases? He found it a very complex question. According to the relation of the wave-length to the length of the ship, it produces a good or a bad effect. A ship with a considerable length of parallel middle body shows very curious phenomena regarding the resistance at different speeds. As the speed is raised, the resistance increases; but on a further increase of speed, it seems as if it was beginning to diminish; the resistance never quite diminishes however with increase of speed, but only increases much less rapidly. The curve indicating the relation of the speed to the velocity has a succession of humps or rises, each showing a rapid increase of resistance; between these it becomes almost flat, showing scarcely any increased resistance. Froude has explained that thoroughly by the application of this doctrine of ship waves which I have endeavoured to put before you. When the effect of the entrance or bow, and the effect of the run or stern, are such as to annul or partially to annul each other's influence in the production of waves, then we have a favourable speed for that particular size and shape of ship. On the other hand, when the crest of a wave astern due to the action of the bow agrees with the crest of a wave astern due to that of the stern, then we have an unfavourable speed for that particular size of ship. Thus Froude worked out a splendid theory, according to which, for the speed at which a ship is to go, a certain length of parallel middle body may, if otherwise desired, be an advantage. But on the whole the conclusion was that—if the ship is equally convenient, if it is not too expensive, if it can pass through the lock gates &c., and if all the other practical conditions can be fulfilled, without a parallel body—it is better that the ship should be all entrance and run, according to Newton's form of least resistance: fine lines forward, swelling out to greatest breadth amidships, and tapering finely towards the stern. In other words, the more ship-shape a ship is, the better.

I wish to conclude by offering one suggestion. I must premise that, when I was asked by the Council to give this lecture, I made it a condition that no practical results were to be expected from it. I explained that I could not say one word to enlighten you on practical subjects, and that I could not add one jot or tittle to what had been done by Scott Russell, by Rankine, and by the Froudes, father and son, and by practical men like the Dennys, W. H. White, and others; who have taken up the science and worked it out in practice. But there is one suggestion founded on the doctrine of wave-making, which I venture to offer before I stop. I have not explained how much of the resistance encountered by a ship in motion is due to wave-making, and how much to what is called skin resistance. I can briefly give you a few figures on this point, which have been communicated to me by Mr. Edmund Froude. For a ship A, 300 feet long and $31\frac{1}{2}$ feet beam and 2634 tons displacement, a ship of the ocean mail steamer type, going at 13 knots an hour, the skin resistance is 5·8 tons, and the wave resistance 3·2 tons, making a total of 9 tons. At 14 knots the skin resistance is but little increased, namely 6·6 tons; while the wave resistance is nearly double, namely 6·15 tons. Mark how great, relatively to the skin resistance, is the wave resistance at the moderate speed of 14 knots for a ship of this size and of 2634 tons weight or displacement. In the case of another ship B, 300 feet long and 46·3 feet beam and 3626 tons displacement—a broader and larger ship with no parallel middle body, but with fine lines swelling out gradually—the wave resistance is much more favourable. At 13 knots the skin resistance is rather more than in the case of the other ship, being 6·95 tons as against 5·8 tons; while the wave resistance is only 2·45 tons as against 3·2 tons. At 14 knots there is a very remarkable result in this broader ship with its fine lines, all entrance and run and no parallel middle body:—at 14 knots the skin resistance is 8 tons as against 6·6 tons in ship A, while the wave resistance is only 3·15 tons as compared with 6·15 tons. Another case which I can give you is that of a torpedo boat 125 feet long, weighing 51 tons. At a speed of 20 knots an hour the skin resistance is 1·2 ton, and the wave resistance 1·1 ton; total resistance 2·3 tons. To calculate the horse-power you multiply the speed in knots per hour by $6\frac{2}{3}$, and then multiply the resistance in tons by the product so obtained; and the result for the torpedo boat going at 20 knots an hour is 307 horse-power to overcome a resistance of 2·3 tons or 1–22nd of her weight (51 tons). Again the ship B of 300 feet length, going at 20 knots an hour with an expenditure of 4550 horse-power, experiences a resistance of 34 tons, or about 1–110th of her weight (3626 tons). Thus the energy actually expended in propelling these vessels at 20 knots an hour at sea would be sufficient, if they were supported on frictionless wheels, to drag them at the same speed up railway inclines, of 1 in 22 for the torpedo boat, and 1 in 110 for the ship B.

My suggestion is this, and I offer it with exceedingly little confidence, indeed with much diffidence; but I think it is possibly worth considering. Inasmuch as wave resistance depends almost entirely on action at the surface of the water, and inasmuch as a fish swimming very close to but below the surface makes very little wave disturbance, it seems to me that by giving a great deal of body below the water line we may relatively diminish the wave disturbance very much. To get high speeds of 18 and 20 knots an hour, it is probable that, by swelling out the ship below like the old French ships, instead of having vertical sides—making the breadth of beam say five feet more below the water than at the water line—there may be obtained a large addition to the displacement or carrying-power of the ship, with very little addition to the wave disturbance, and therefore with very little addition to the wave resistance, which is most important at high speeds. I think it may be worth while to consider this in regard to the designs of ships.

In conclusion, I should like to urge you to look at these phenomena for yourselves. Look at the beautiful wave-pattern of capillary waves, which you will find produced by a fishing line hanging vertically from a rod, or from an oar, or from anything carried by a vessel moving slowly through smooth water at speeds of from about ½ knot to 2 knots an hour. Again look at the equally beautiful wave-pattern produced by ships and boats, as illustrated in Plate 82. But you can scarcely see the phenomena more beautifully manifested than by a duck and ducklings. A full-sized duck has a splendidly shaped body for developing a wave-pattern, and going at good speed it produces on the surface of a pond very nearly the exact pattern of ocean waves. A little duckling going as fast as it can, perhaps about a knot an hour, shows very admirably the capillary waves,* differing manifestly from the ocean waves formed in

---

* For information regarding capillary waves, see Scott Russell's Report on Waves (British Association, York, 1844, pages 311–390); also parts III,

the front and at the rear of a larger body moving more rapidly through the open water. I call attention to this, because, having given you perhaps a rather dry statement of scientific facts, if I can say a word that will lead you each to use your eyes in looking at ships, boats, ducks, and ducklings, moving on water at different speeds, and to observe these beautiful phenomena of waves, I think, even were you to remember nothing of this lecture, you would have something to keep in your minds for the rest of your lives.

Sir WILLIAM MUIR, K.C.S.I., proposed a hearty vote of thanks to Sir William Thomson for his most interesting lecture. The test of a lecture of this kind was that it made itself clear to the understanding of the unlearned; and he thought what they had this evening been listening to had opened up a whole vista of remarkable phenomena, and had furnished reasons whereby more could now be understood about waves than had ever been the case before.

The PRESIDENT had great pleasure in seconding the vote of thanks to Sir William Thomson. It was always said that "Britannia rules the waves"; and when it was considered how Great Britain did three-fourths of the carrying trade of the world, he thought it absolutely necessary that she should have ships as perfect as they could be made. All who had enjoyed the privilege of listening to Sir William Thomson's most interesting lecture felt, he was sure, very greatly obliged to him for having devoted so much time to it. Not only had he added to the information that had been acquired on this subject, but he had presented in a condensed form

---

IV, and V of Sir William Thomson's paper, "Hydro-Kinetic Solutions and Observations" (Philosophical Magazine, November 1871).

the results of the long labours of Mr. Scott Russell, Mr. Froude; Lord Rayleigh, and others. On behalf of the Institution of Mechanical Engineers he desired to say that they should be delighted if he would do them the favour of allowing them to publish in the Institution Proceedings the lecture which he had just delivered. All who had listened to the lecture, or who read it on publication, would learn something on the subject.

The vote of thanks was unanimously agreed to.

The President proposed a hearty vote of thanks to the Marquess of Tweeddale for his kindness in remaining a second day in Edinburgh with the Institution, in order to take the chair on the occasion of this evening's lecture.

The vote of thanks was passed unanimously.

The Marquess of Tweeddale thanked the President and Members very warmly for the kind way in which they had welcomed whatever services he had been able to render on this occasion in connection with the Meeting of the Institution in Edinburgh.

SHIP WAVES.

Plate 80.

Fig. 1. *Experimental Apparatus 1835. Scale 1/700th.*

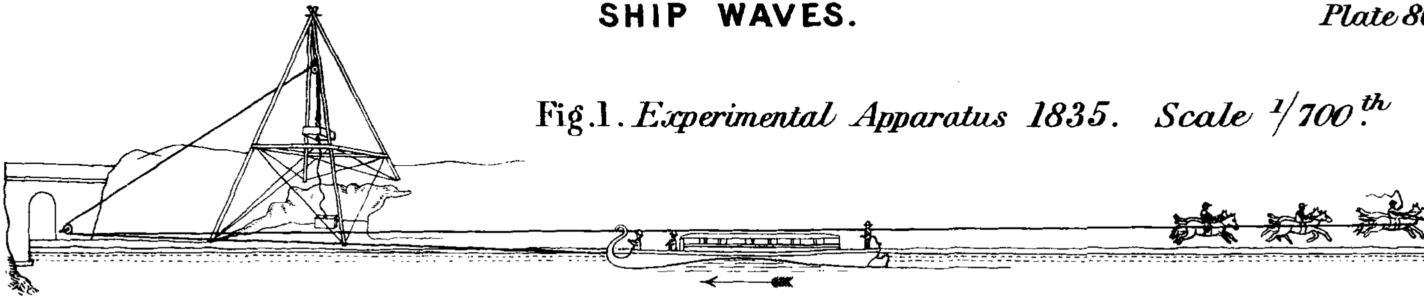


Fig. 2. *Boat behind the wave.*

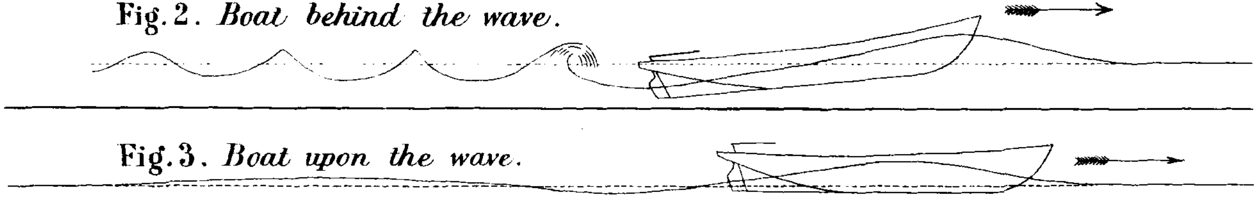


Fig. 3. *Boat upon the wave.*

Section of Waves in wake of 83 feet Launch.

Vertical Scale 1/48th.
4 feet per inch.

Horizontal Scale 1/960th.
80 feet per inch.

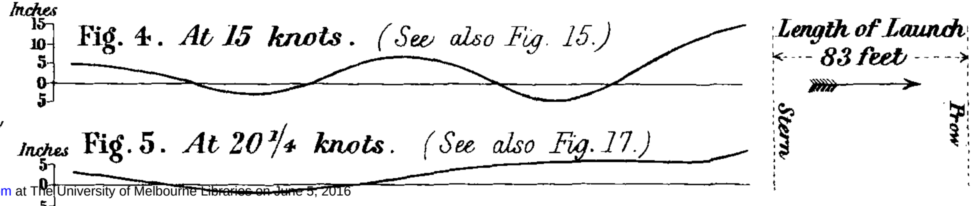


Fig. 4. *At 15 knots. (See also Fig. 15.)*

Fig. 5. *At 20¼ knots. (See also Fig. 17.)*

(Proceedings Inst. M.E. 1887.)

Plate 80.

SHIP WAVES.

Fig. 6. *Wave Profile, H.M.S. "Curlew."*

Length of Ship 195 *feet.*
Breadth 28 *feet.*
Displacement 780 *tons.*

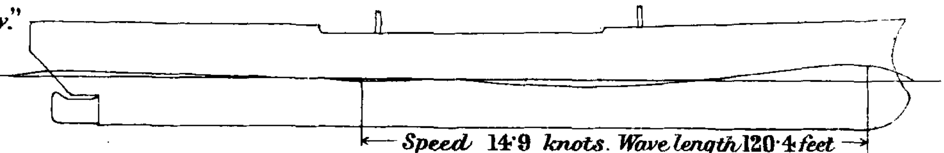


Fig. 7. *Wave Profiles.*

Length of Ship 250 *feet.*
Breadth 37 *feet.*
Displacement 2000 *tons.*

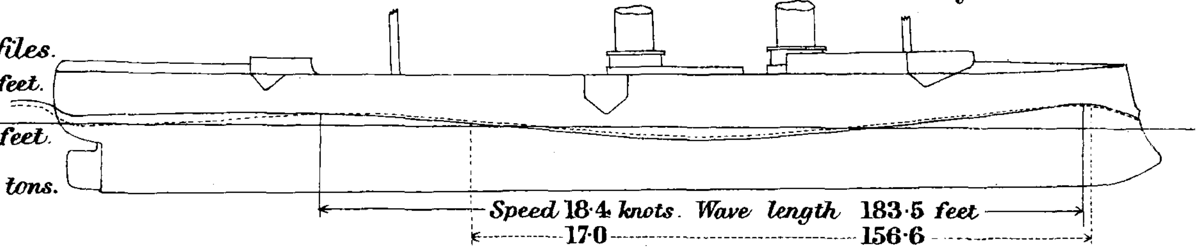
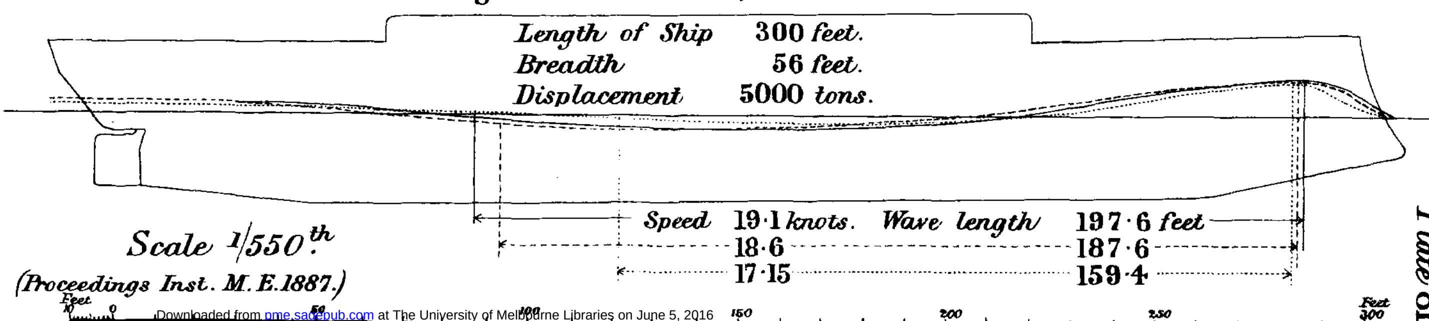


Fig. 8. *Wave Profiles, H.M.S. "Orlando."*



Scale 1/550th.
(Proceedings Inst. M. E. 1887.)

SHIP WAVES.

Wave Pattern.

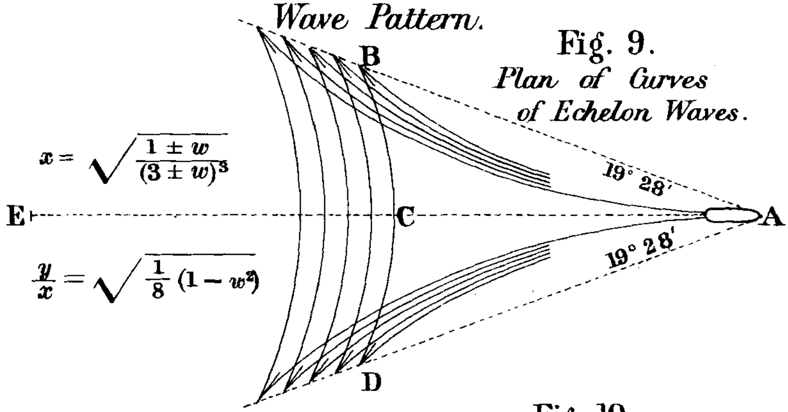


Fig. 9. *Plan of Curves of Echelon Waves.*

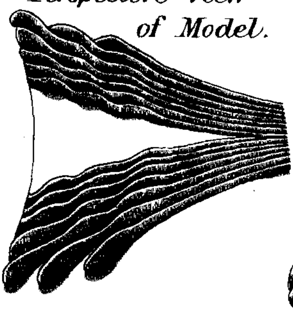


Fig. 11. *Perspective view of Model.*

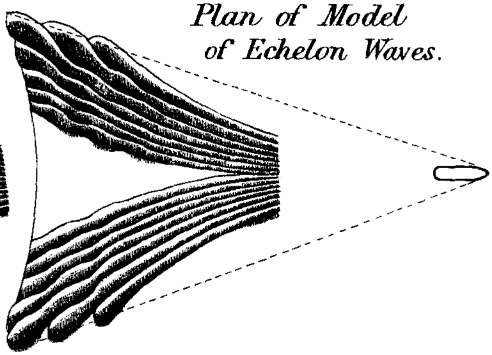
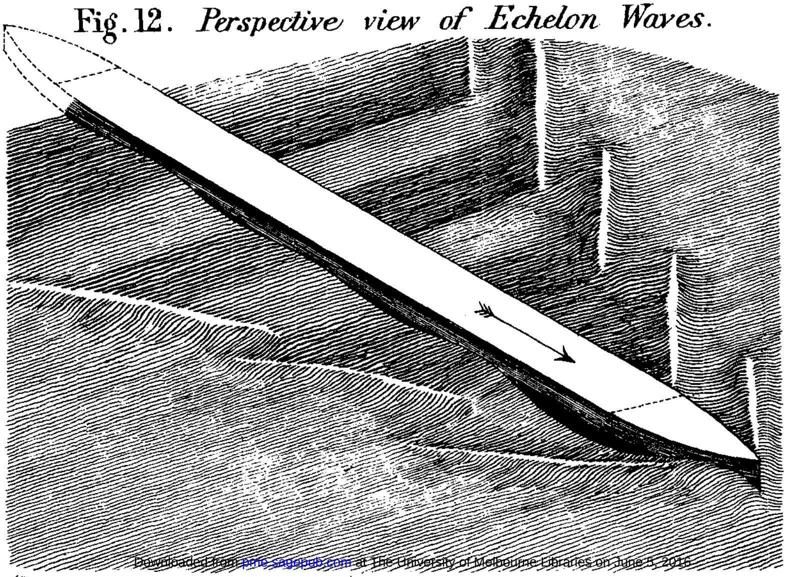


Fig. 10. *Plan of Model of Echelon Waves.*

Fig. 12. *Perspective view of Echelon Waves.*



Plan of Wave System made by 83 feet Launch at various speeds.

The positions of the Wave crests are indicated by the transverse shading; they were accurately measured for a distance of only about two and a half boat's lengths clear of the stern.

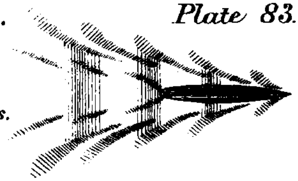


Fig. 13.
At 9 knots.

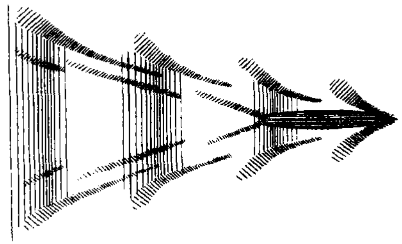


Fig. 14.
At 12 knots.

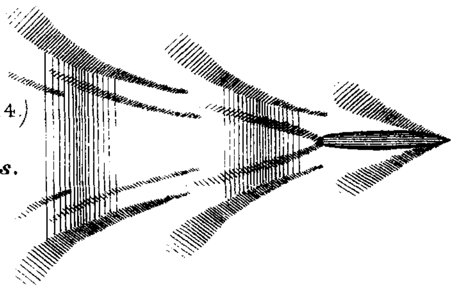


Fig. 15.
(See also Fig. 4.)
At 15 knots.

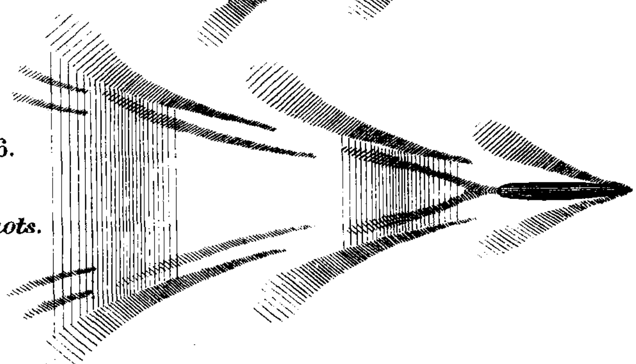


Fig. 16.
At 18 knots.

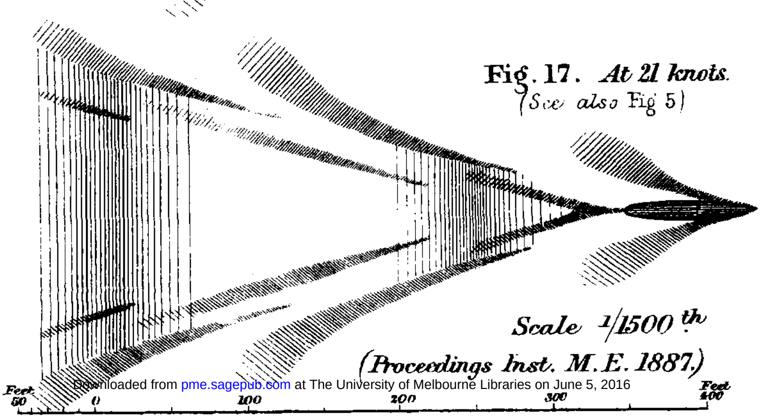


Fig. 17. *At 21 knots.*
(See also Fig 5)

Scale 1/1500 th

(Proceedings Inst. M.E. 1887.)

Plans of Wave Systems made by 83 feet Launch and 333 feet Ship at speed of 18 knots.

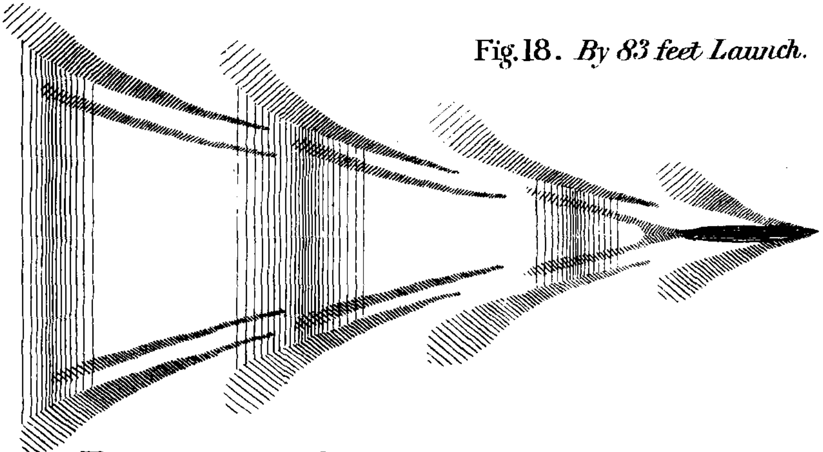


Fig. 18. *By 83 feet Launch.*

The positions of the Wave-crests are indicated by the transverse shading; they were accurately measured for a distance of only about two and a half boat's lengths clear of the stern.

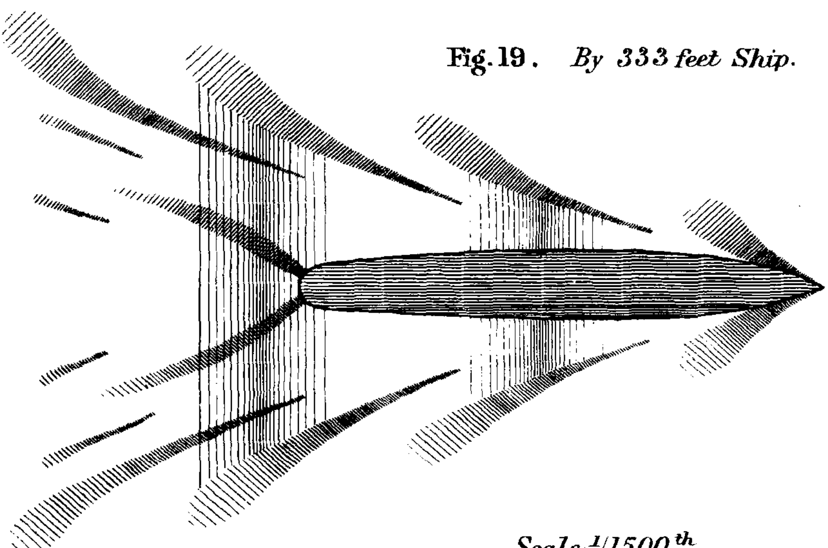


Fig. 19. *By 333 feet Ship.*

Scale 1/1500th.

Feet 50 0 100 200 300 400 Feet.

(Proceedings Inst. M.E. 1887.)