Computational Exercise Two

for PHYS*2330 Electricity and Magnetism I

In this exercise you will apply simple methods of numerical integration to two different problems in E&M: calculating the total electric field for a continuous (one dimensional) distribution of charge, and calculating the work done moving a charge along a specified path through an electric field.

<u>Learning Outcomes:</u>

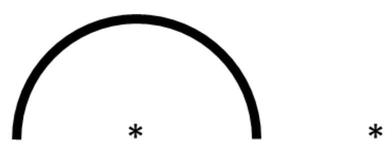
- **Become familiar with two simple methods of numerical integration (Trapezoidal and Simpson's)**
- **Improve coding design skills by writing versatile functions**
- **Compute two commonly occurring types of integrations in electrostatics – integration over a distribution of charge, and line integrals**

Part A (Done in Tutorial):

We will write a python script, "***chargeIntegration.py***" that calculates the electric field at a point in space $\vec{r}_p = (x_p, y_p)$ due to a linear distribution of charge:

- To create a line of charge: We will write a function that creates arrays of x and y coordinates for line segments in the xy-plane (i.e. not just along x-axis). The length of the arrays (N) should be specified as an input, along with initial/final points of the line.
- We will create a separate function that computes the electric field components due to a segment of the line (dE_x, dE_y).
- We will create a third function, ***trapz***, to perform the Trapezoidal Method of Integration.
- For a straight, uniformly charged segment of length 10.0 m, total charge +3.0 C, we will determine the electric field at a distance of 5.0 m from (i) the midpoint of the line, and (ii) beyond one end of the line, at a point along the axis of the line (see below, left). For simplicity, you may position the line along the x-axis, centred at the origin.

Line of uniform charge



Semi-circle of uniform charge

Part B (Graded):

- Another numerical integration method is *Simpson's Rule*. This can easily be found online. Write a function, ***simpson***, and calculate the electric field at the two specified points in Part A.
- Using your code from Part A, calculate the electric field of a uniformly charged semi-circle radius 5.0 m, total charge +3.0 C. Determine the electric field at (i) the centre-position, and (ii) 5.0 m beyond one end of the semi-circle, at shown (above, right).
  - Obtaining an analytical value for $\vec{E}$ at the second position is far from trivial – hopefully you recognize the benefits of performing integrations numerically!
- **QUESTION B1:** Calculate the analytical solution for the electric field at the centre of the half-circular segment. When computing the same integral using trapz and simpsons functions, what minimum value of N is needed for each, so that your computed result is within 0.01% of the analytical value?

Part C (Give it a try! Not for marks):

Next, we will use the integration routines to calculate line integrals for a charged particle moving along a path in the presence of an applied electric field:

$$\int_{\vec{r}_i}^{\vec{r}_f} \vec{E} \cdot \overrightarrow{dl}$$

- Create a function ***extField*** that calculates the electric field due to a distribution of point charges. For this exercise, use the same charge distribution from Computational Exercise 1.
- Keep in mind that the integrand includes a vector dot product.
- Calculate the work done on a +4.0 C charge as it moves from $\vec{r}_i = (-4, -3)$ to $\vec{r}_f = (3,2)$, along a straight-line path.
- Depending on how you have written your functions in Part A, it is possible that much of the code can be used, with minimal modification here. You are encouraged to consider this when designing your functions.
- Save your python script as ***"lineIntegral.py"***.
- **QUESTION C1:** If you had to extend your code allow integration along other paths (e.g. a circular arc from $\vec{r}_i$ to $\vec{r}_f$), what modifications would you need to make to your code to allow for this? Do you feel that you would be able to code this yourself?
- In principle, the program can be extended to allow the charged particle to move along a curved path. This is a challenging task to pursue, especially given the dot product involved! If you are interested in trying this out (or even just giving it some thought) we would be happy to discuss your ideas during tutorials or office hours!

REFLECTION QUESTION: Did your computational work help your understanding of the physics content explored in this exercise?

Assignment Submission:

Your submission will consist of **two parts within Courselink**:

1) Question & Reflection 2330_Ex2: Under the "Quizzes" tab; you will enter your answers to questions and the reflection question within.
    a. Also, if you learned any new python features or made efforts in some area (e.g. plot formatting, algorithms/logic, etc.), tell us about it in the Reflection space!
2) Once (1) has been completed, you will be able to Submit your completed work into the Courselink Dropbox folder titled "2330_Ex2: charge integration". For this assignment, submit the following file:
    - Python scripts: chargeIntegration.py
    - **There are no plot outputs for this exercise. Instead, add comments to the TOP of your python script and include the results of the four integrals (both x and y components) using N=501 segments with the trapezoid/simpson's rules to perform your numerical integrations. Also include the exact results for the three analytically-solvable integrals.**

This exercise will be graded out of 40 marks; 4 marks for your answer to Question B1; 36 marks will be awarded using the rubric posted in Courselink.