Group Members

Tevin Achong - 816000026

Jimmel Greer - 816000045

Course Code: COMP3602

Course Title: Theory of Computing

Assignment: 2

November 21, 2019

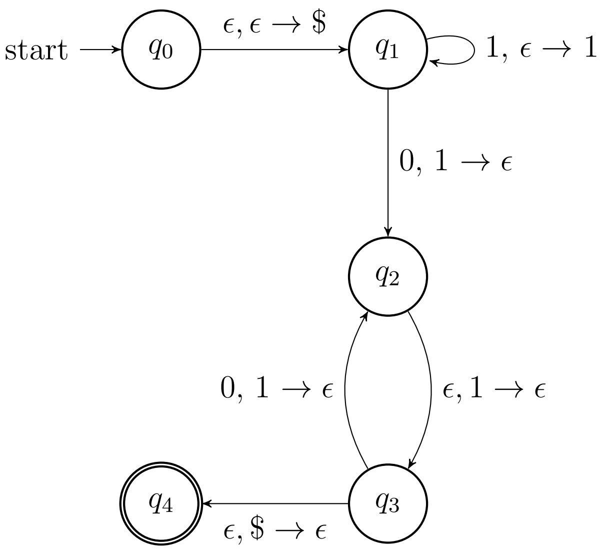
1.

- $L = \{1^n 0^{\frac{n}{2}} \mid n \geq 2$, n is even$\}$
  $G = (\{S\}, \{0, 1\}, P, S)$
  where P is:
  S $\rightarrow$ 11S0 | 110
- $\{+w + w^R + \mid w \in \{0, 1\}^+\}$
  $G = (\{S, W\}, \{+, 0, 1\}, P, S)$
  where P is:
  S $\rightarrow$ $+W+$
  W $\rightarrow$ 1W1 | 0W0 | 1+1 | 0+0

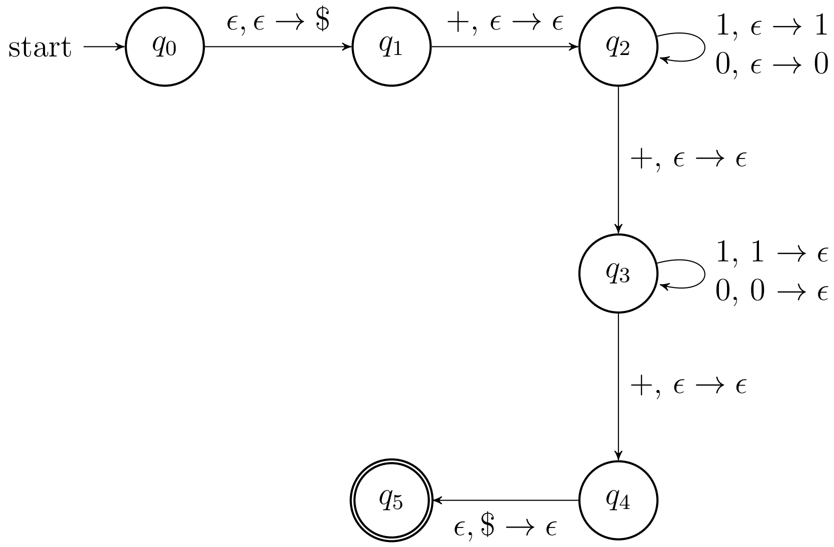
- $\{w + v \mid w, v \in \{0, 1\}^*$, w^R is the prefix of $v\}$
  G = ({S, W, V}, {+, 0, 1, ϵ}, P, S)
  where P is:
  S $\rightarrow$ WV | ϵ + ϵ
  W $\rightarrow$ 1W1 | 0W0 | 1+1 | 0+0
  V $\rightarrow$ 0V | 1V | ϵ

2.

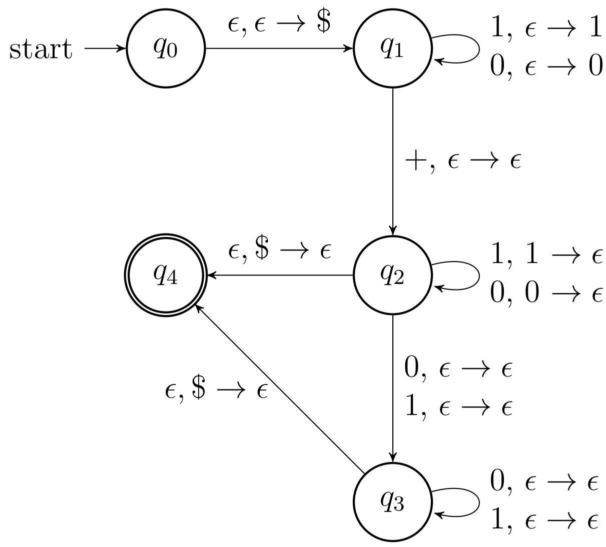
- $\{1^n 0^{\frac{n}{2}} \mid n \geq 2$, n is even$\}$



- $\{+w + w^R + \mid w \in \{0, 1\}^+\}$


start
q_0
$\epsilon, \epsilon \to \$$
q_1
$+, \epsilon \to \epsilon$
q_2
$1, \epsilon \to 1$
$0, \epsilon \to 0$
$+, \epsilon \to \epsilon$
q_3
$1, 1 \to \epsilon$
$0, 0 \to \epsilon$
$+, \epsilon \to \epsilon$
q_4
$\epsilon, \$ \to \epsilon$
q_5

- $\{w + v \mid w, v \in \{0,1\}^*,\ w^R \text{ is the prefix of } v\}$



3. If L_1 and L_2 are any context-free languages, $L_1 \cup L_2$ is also context free.
   Proof by Construction:
   If L_1 and L_2 are context free, then, by definition, there exist grammars $G_1 = (V_1, \Sigma_1, S_1, P_1)$ and $G_2 = (V_2, \Sigma_2, S_2, P_2)$ with $L(G_1) = L_1$ and $L(G_1) = L_1$. We can assume that V_1 and V_2 are disjoint. Now consider the grammar $G = (V, \Sigma, S, P)$, where S is a new nonterminal symbol and
   $V = V_1 \cup V_2 \cup \{S\}$,
   $\Sigma = \Sigma_1 \cup \Sigma_2$, and
   $P = P_1 \cup P_2 \cup \{(S \rightarrow S_1), (S \rightarrow S_2)\}$

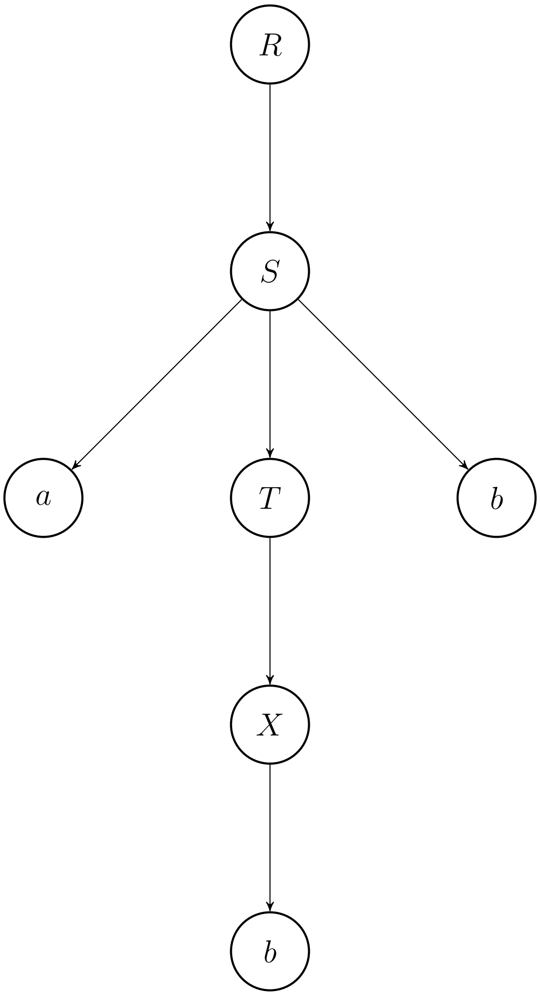
   Clearly $L(G) = L_1 \cup L_2$, so $L_1 \cup L_2$ is a regular language.

4. If L is any context-free language, L^* is also context free.
   Proof by Construction:
   If L is context free then, by definition, there exists a grammar $G = (V, \Sigma, S, P)$ with $L(G) = L$. Now consider the grammar $G' = (V', \Sigma', S', P')$, where S' is a new nonterminal symbol and
   $V' = V \cup \{S'\}$, and
   $P' = P \cup \{(S' \rightarrow SS'), (S' \rightarrow \epsilon)\}$

   Now $L(G') = L^*$, so L^* is a context-free language.

5. (a) **Non-terminals:**
R, S, T, X
   (b) **Terminals:**
a, b, ϵ

(c) **Parse Tree for** abb



(d) **Derivation of** bba**:**
$R \Rightarrow S \Rightarrow bTa \Rightarrow bXa \Rightarrow bba$