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Course Code: COMP3602
Course Title: Theory of Computing
Assignment: 1

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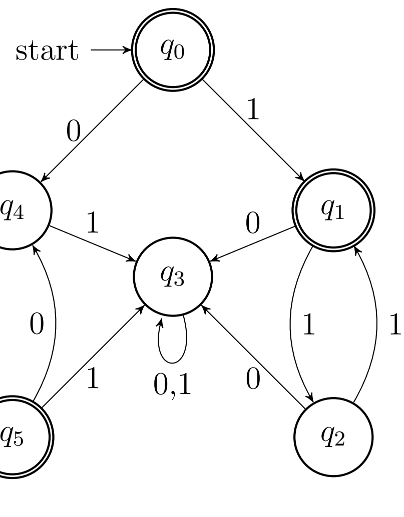
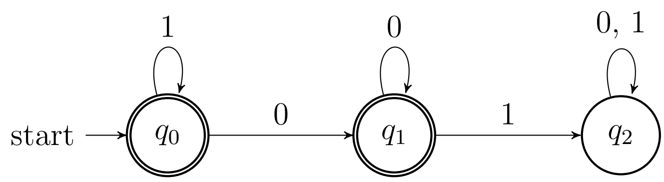
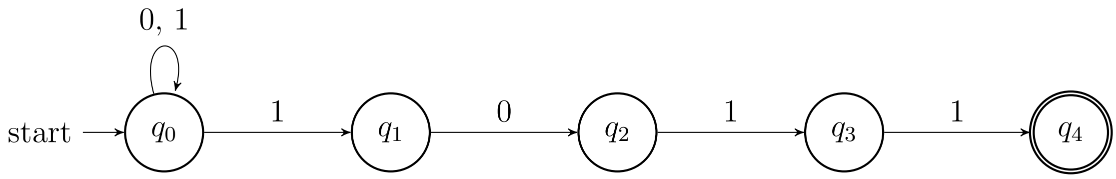
1. (a) $\{0^n \vee 1^m \mid n \text{ is even}, m \text{ is odd}\}$



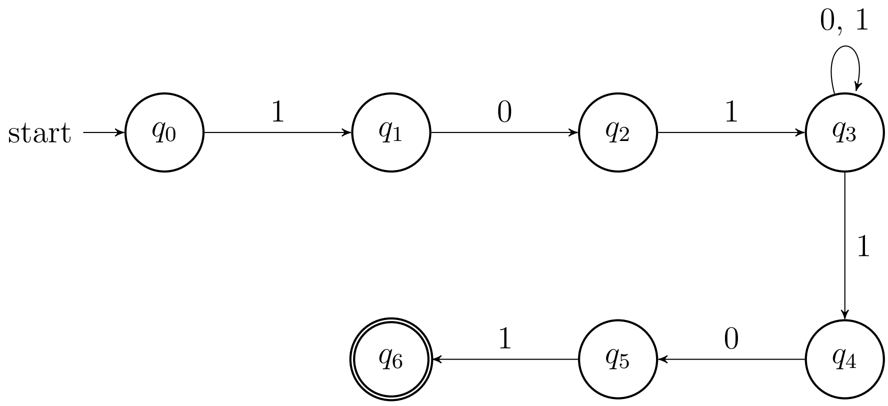
(b) Any string that does not contain the substring 01



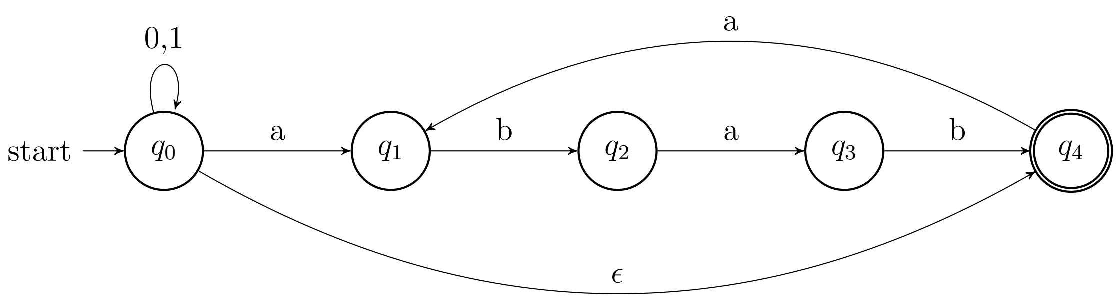
2. (a) Strings ending in 1011



(b) $\{101x101 \mid x \in \Sigma^*\}$



(c) $\{x(ab)^n \mid x \in \Sigma^*\, n \text{ is even}\}$



3. Let us refer to the DFA in Question 1b as M. Then $M = \{\{q_0, q_1, q_2\}, \{0, 1\}, \delta, q_0, \{q_0, q_1\}\}$ where δ is given by:
$\delta(q_0, 0) = q_1$
$\delta(q_0, 1) = q_0$
$\delta(q_1, 0) = q_1$
$\delta(q_1, 1) = q_2$
$\delta(q_2, 0) = q_2$
$\delta(q_2, 1) = q_2$

4. Let us refer to the NFA in Question 2b as N. Then $N = \{\{q_0, q_1, q_2, q_3, q_4, q_5, q_6\}, \{0, 1\}, \delta, q_0, \{q_6\}\}$ where δ is given by:
$\delta(q_0, 0) = \{\}$
$\delta(q_0, 1) = \{q_1\}$
$\delta(q_0, \epsilon) = \{\}$
$\delta(q_1, 0) = \{q_2\}$
$\delta(q_1, 1) = \{\}$
$\delta(q_1, \epsilon) = \{\}$
$\delta(q_2, 0) = \{\}$
$\delta(q_2, 1) = \{q_3\}$
$\delta(q_2, \epsilon) = \{\}$
$\delta(q_3, 0) = \{q_3\}$
$\delta(q_3, 1) = \{q_3, q_4\}$
$\delta(q_3, \epsilon) = \{\}$
$\delta(q_4, 0) = \{q_5\}$
$\delta(q_4, 1) = \{\}$
$\delta(q_4, \epsilon) = \{\}$
$\delta(q_5, 0) = \{\}$
$\delta(q_5, 1) = \{q_6\}$
$\delta(q_5, \epsilon) = \{\}$
$\delta(q_6, 0) = \{\}$
$\delta(q_6, 1) = \{\}$
$\delta(q_6, \epsilon) = \{\}$

5.

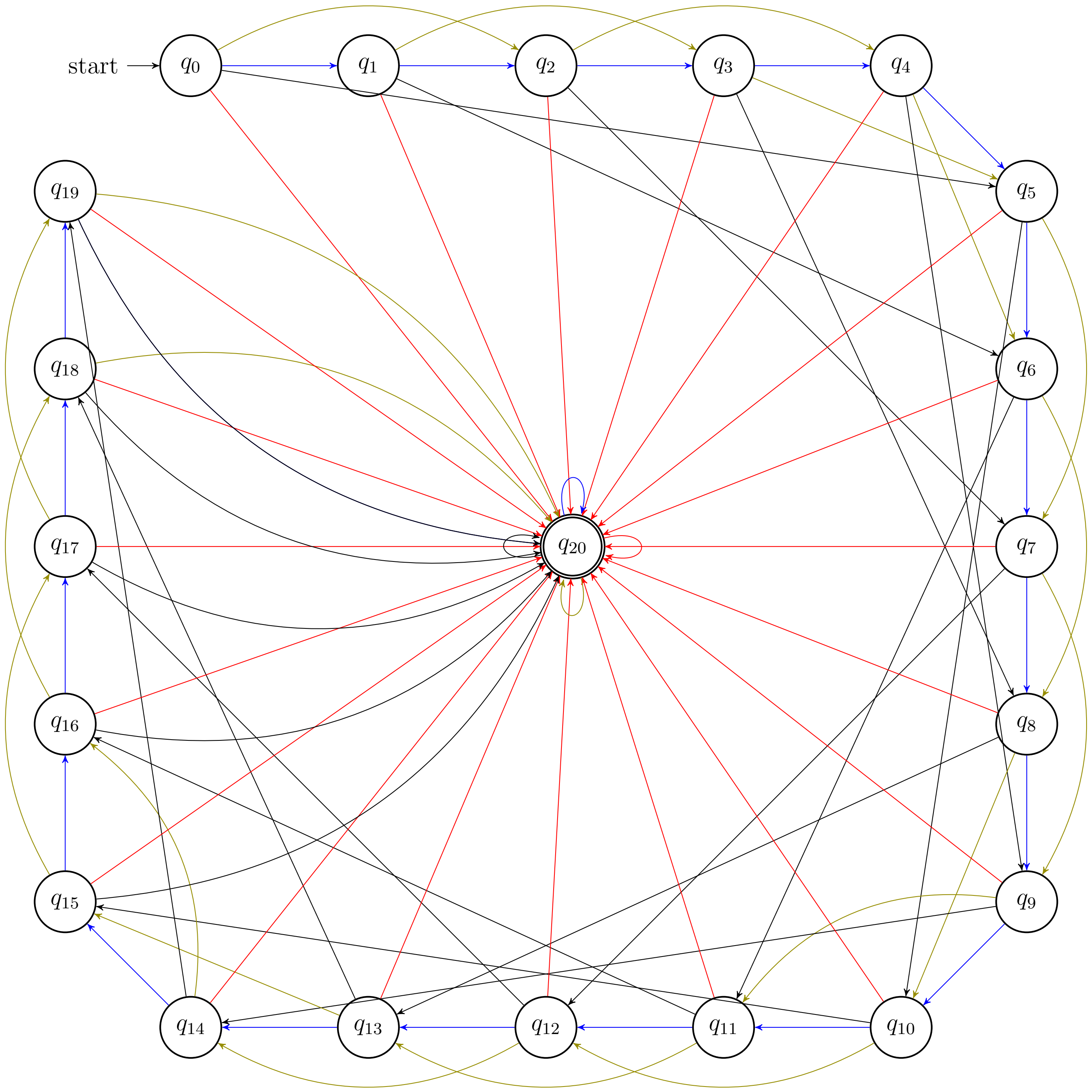
Key/Legend:

→ \$0.05

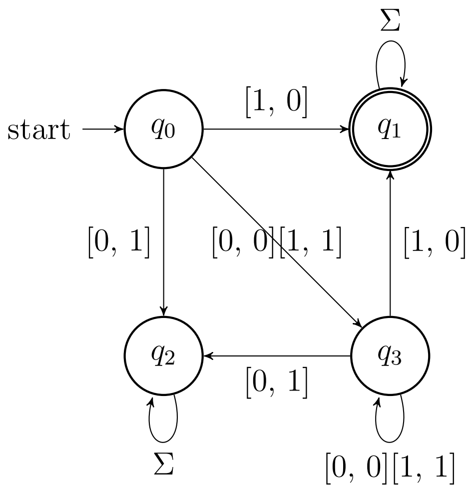
→ \$0.10

→ \$0.25

→ \$1.00



6.



7. $\mathscr{L}_1 - \mathscr{L}_2$ can be expressed as $(\mathscr{L}_1 \bigcup \mathscr{L}_2) \bigcap \overline{\mathscr{L}_2}$, assuming they are within the same universal set. This defines set difference in terms of intersection, union and complement, for which the regular languages are known to be regular. Thus it can be concluded that $\mathscr{L}_1 - \mathscr{L}_2$ is a regular language.

8. $\Sigma = \{0, 1, +, =\}$
ADD $= \{x = y + z \mid x, y, z$ are binary integers and x is the sum of y and $z\}$

Proof by contradiction

For this proof, we will assume that ADD is a regular language and it has some pumping length, p. Let the string $s = 1^p = 1^{p-1}0 + 1$ where $s \epsilon$ ADD and $p \leq |s|$.

There must exist some x, y, z such that $s = xyz$ with,

(a) $|y| > 0$
(b) $|xy| \leq p$
(c) for each $i \geq 0$, $xy^iz \ \epsilon$ ADD

From this string, s can only be broken down into three parts, xyz, in one way: y must be in the first sequence of $1's$. This must be done to obey rules (a) and (b).

We let:
$y \Rightarrow 1^m$ such that $m \geq 1$
$x \Rightarrow 1^{p-m}$
$z \Rightarrow = 1^{p-1}0 + 1$

if $i = 0$,
$xy^0z = (1^{p-m})(1^m)^0(= 1^{p-1}0 + 1)$
$xy^iz = 1^{p-m} = 1^{p-1}0 + 1$

Since $m \geq 1$, $p - m \neq p$ which means $1^{p-m} \neq 1^{p-1}0 + 1$ in terms of binary addition. Therefore, there exists some i such that $xy^iz \notin$ ADD. Since this is a contradiction of (c), it can be concluded that ADD is not regular.

9.

Regular Expression	**Recognized Strings**	**Non-Recognized Strings**
a^*b^*	ϵ	aba
	ab	baa
$a(ba)^*bb$	abb	ϵ
	ababb	bababb
$a^+ \cup b^*$	bb	ab
	a	bab
$(\epsilon \cup a)b$	b	ϵ
	ab	bb