CHAPTER 2
COMPLEX NUMBERS AND QUADRATIC EQUATIONS

1. The amplitude of $\frac{1+i\sqrt{3}}{\sqrt{3}+i}$ is

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$

(c) $\frac{2\pi}{3}$ (d) $\frac{\pi}{6}$ *(2005)*

2. If α, β, γ are the roots of the equation $2x^3 - 3x^2 + 6x + 1 = 0$, then $\alpha^2 + \beta^2 + \gamma^2$ is equal to

(a) $-\frac{15}{4}$ (b) $\frac{15}{4}$

(c) $\frac{9}{4}$ (d) 4 *(2005)*

3. The modulus and amplitude of $\frac{1+2i}{1-(1-i)^2}$ are

(a) $\sqrt{2}$ and $\frac{\pi}{6}$ (b) 1 and 0

(c) 1 and $\frac{\pi}{3}$ (d) 1 and $\frac{\pi}{4}$

(2005, 2013)

4. The real part of $\frac{1}{1+\cos\theta+i\sin\theta}$ is

(a) $-\frac{1}{2}$ (b) $\frac{1}{2}$

(c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$ *(2005)*

5. If α, β and γ are the roots of the equation $x^3 - 8x + 8 = 0$, then $\Sigma\alpha^2$ and $\Sigma\frac{1}{\alpha\beta}$ are respectively

(a) 0 and −16 (b) 16 and 8

(c) −16 and 0 (d) 16 and 0 *(2006)*

6. The complex number $\frac{(-\sqrt{3}+3i)(1-i)}{(3+\sqrt{3}i)(i)(\sqrt{3}+\sqrt{3}i)}$ when represented in the Argand diagram is

(a) in the second quadrant

(b) in the first quadrant

(c) on the Y-axis (imaginary axis)

(d) on the X-axis (real axis) *(2006)*

7. If $2x = -1+\sqrt{3}i$, then the value of $(1 - x^2 + x)^6 - (1 - x + x^2)^6 =$

(a) 32 (b) −64

(c) 64 (d) 0 *(2006, 2013)*

8. The modulus of amplitude of $(1+i\sqrt{3})^8$ are respectively

(a) 256 and $\pi/3$ (b) 256 and $2\pi/3$

(c) 2 and $2\pi/3$ (d) 256 and $8\pi/3$

(2006)

9. The conjugate of the complex number $\frac{(1+i)^2}{1-i}$ is

(a) $1 - i$ (b) $1 + i$

(c) $-1 + i$ (d) $-1 - i$ *(2007)*

10. The imaginary part of i^i is

(a) 0 (b) 1

(c) 2 (d) −1 *(2007)*

11. The amplitude of $(1 + i)^5$ is

(a) $\frac{3\pi}{4}$ (b) $\frac{-3\pi}{4}$

(c) $\frac{-5\pi}{4}$ (d) $\frac{5\pi}{4}$ *(2007)*

12. If 1, ω, ω^2 are the cube roots of unity then $(1 + \omega)(1 + \omega^2)(1 + \omega^4)(1 + \omega^8)$ is equal to

(a) 1 (b) 0

(c) ω^2 (d) ω *(2007)*

13. If α is a complex number satisfying the equation $\alpha^2 + \alpha + 1 = 0$ then α^{31} is equal to

(a) 1 (b) i (c) α (d) α^2

(2008)

14. If Z is a complex number such that $Z = -\bar{Z}$, then
    (a) Z is any complex number
    (b) Real part of Z is the same as its imaginary part
    (c) Z is purely real
    (d) Z is purely imaginary *(2008)*

15. The value of $\sum_{K=1}^{6}\left[\sin\frac{2K\pi}{7} - i\cos\frac{2K\pi}{7}\right]$ is
    (a) $-i$ (b) -1 (c) i (d) 0 *(2008)*

16. The real root of the equation $x^3 - 6x + 9 = 0$ is
    (a) 6 (b) -3 (c) -6 (d) -9 *(2008)*

17. If α and β are the roots of $x^2 + x + 1 = 0$, then $\alpha^{16} + \beta^{16} =$
    (a) 2 (b) 0
    (c) 1 (d) -1 *(2009)*

18. The complex number $\frac{1+2i}{1-i}$ lies in
    (a) fourth quadrant (b) first quadrant
    (c) second quadrant (d) third quadrant *(2009)*

19. If P is the point in the Argand diagram corresponding to the complex number $\sqrt{3}+i$ and if OPQ is an isosceles right angled triangle, right angled at 'O', then Q represents the complex number
    (a) $\sqrt{3}-i$ or $1-i\sqrt{3}$ (b) $-1 \pm i\sqrt{3}$
    (c) $-1+i\sqrt{3}$ or $1-i\sqrt{3}$
    (d) $1 \pm i\sqrt{3}$ *(2009)*

20. The smallest positive integral value of 'n' such that $\left[\frac{1+\sin\frac{\pi}{8}+i\cos\frac{\pi}{8}}{1+\sin\frac{\pi}{8}-i\cos\frac{\pi}{8}}\right]^n$ is purely imaginary is,
    (a) 2 (b) 8
    (c) 4 (d) 3 *(2009)*

21. If $a, -a, b$ are the roots of $x^3 - 5x^2 - x + 5 = 0$, then b is a root of
    (a) $x^2 - 5x + 10 = 0$ (b) $x^2 + 3x - 20 = 0$
    (c) $x^2 + 5x - 30 = 0$ (d) $x^2 - 3x - 10 = 0$ *(2010)*

22. The least positive integer n, for which $\frac{(1+i)^n}{(1-i)^{n-2}}$ is positive is
    (a) 4 (b) 3 (c) 2 (d) 1 *(2010)*

23. If $x + iy = (-1+i\sqrt{3})^{2010}$, then $x =$
    (a) 2^{2010} (b) -2^{2010} (c) -1 (d) 1 *(2010)*

24. If α, β and γ are roots of $x^3 - 2x + 1 = 0$, then the value of $\sum\left(\frac{1}{\alpha+\beta-\gamma}\right)$ is
    (a) $\frac{-1}{2}$ (b) -1 (c) 0 (d) $\frac{1}{2}$ *(2011)*

25. The value of $\left|\frac{1+i\sqrt{3}}{\left(1+\frac{1}{i+1}\right)^2}\right|$ is
    (a) 20 (b) 9 (c) $\frac{5}{4}$ (d) $\frac{4}{5}$ *(2011)*

26. If ω is an imaginary cube root of unity, then the value of $(1-\omega+\omega^2)\cdot(1-\omega^2+\omega^4)\cdot(1-\omega^4+\omega^8)\cdot$ .... ($2n$ factors) is
    (a) 2^{2n} (b) 2^n (c) 1 (d) 0 *(2011)*

27. If $P(x, y)$ denotes $z = x + iy$ in argand's plane and $\left|\frac{z-1}{z+2i}\right| = 1$, then the locus of P is a/an
    (a) hyperbola (b) ellipse
    (c) circle (d) straight line *(2011)*

28. In Argand's plane, the point corresponding to $\frac{(1-i\sqrt{3})(1+i)}{(\sqrt{3}+i)}$ lies in
    (a) quadrant I (b) quadrant II
    (c) quadrant III (d) quadrant IV *(2011)*

29. If $\log_2(9^{x-1} + 7) - \log_2(3^{x-1} + 1) = 2$, then values of x are
    (a) 1, 2 (b) 0, 2 (c) 0, 1 (d) 1, 4 *(2012)*

30. The number of solutions of equation $z^2 + \bar{z} = 0$, where $z \in C$ are
    (a) 6 (b) 1 (c) 4 (d) 5 *(2012)*

31. If α is a complex number such that $\alpha^2 - \alpha + 1 = 0$, then $\alpha^{2011} =$

(a) 1 (b) $-\alpha$ (c) α^2 (d) α

(2012)

32. If the conjugate of $(x + iy)(1 - 2i)$ is $1 + i$, then

(a) $x = -\frac{1}{5}$ (b) $x - iy = \frac{1+i}{1-2i}$

(c) $x + iy = \frac{1-i}{1-2i}$ (d) $x = \frac{1}{5}$ *(2012)*

33. If α, β, γ are the roots of the equation $x^3 + 4x + 2 = 0$, then $\alpha^3 + \beta^3 + \gamma^3 =$

(a) -6 (b) 2 (c) 6 (d) -2

(2012)

34. If α and β are different complex numbers with $|\beta| = 1$, then $\left|\frac{\beta - \alpha}{1 - \bar{\alpha}\beta}\right|$ is equal to

(a) 2 (b) $\frac{1}{2}$ (c) 1 (d) $\frac{1}{3}$

(2012, 2014)

35. If the roots of the equation $x^3 + ax^2 + bx + c = 0$ are in A.P., then $2a^3 - 9ab =$

(a) $9c$ (b) $18c$

(c) $27c$ (d) $-27c$

(2013)

36. If 1, ω, ω^2 are three cube roots of unity, then $(1 - \omega + \omega^2)(1 + \omega - \omega^2)$ is ______

(a) 2 (b) 4 (c) 1 (d) 3

(2015)

37. If $z = \frac{(\sqrt{3} + i)^3 (3i + 4)^2}{(8 + 6i)^2}$, then $|z|$ is equal to

(a) 1 (b) 3 (c) 0 (d) 2

(2015)

38. If α and β are the roots of $x^2 - ax + b^2 = 0$, then $\alpha^2 + \beta^2$ is equal to ______

(a) $2a^2 - b^2$ (b) $a^2 + b^2$

(c) $a^2 - 2b^2$ (d) $a^2 - b^2$

(2015)

39. The real part of $(1 - \cos\theta + i\sin\theta)^{-1}$ is

(a) $\frac{1}{2}$ (b) $\frac{1}{1+\cos\theta}$

(c) $\tan\frac{\theta}{2}$ (d) $\cot\frac{\theta}{2}$

(2016)

40. The simplified form of $i^n + i^{n+1} + i^{n+2} + i^{n+3}$ is

(a) 0 (b) 1 (c) -1 (d) i

(2016)

41. If $\left(\frac{1+i}{1-i}\right)^m = 1$, then the least positive integral value of m is

(a) 4 (b) 1

(c) 2 (d) 3 *(2017)*

42. If $\left(\frac{1-i}{1+i}\right)^{96} = a + ib$, then (a, b) is

(a) (1, 1) (b) (1, 0)

(c) (0, 1) (d) (0, -1) *(2018)*

43. If α and β are roots of the equation $x^2 + x + 1 = 0$, then $\alpha^2 + \beta^2$ is

(a) 1 (b) $\frac{-1 - i\sqrt{3}}{2}$

(c) $\frac{-1 + i\sqrt{3}}{2}$ (d) -1

(2019)

ANSWER KEY

1.	(d)	**2.**	(a)	**3.**	(b)	**4.**	(b)	**5.**	(d)	**6.**	(c)	**7.**	(d)	**8.**	(b)
9.	(d)	**10.**	(a)	**11.**	(d)	**12.**	(a)	**13.**	(c)	**14.**	(b)	**15.**	(c)	**16.**	(b)
17.	(d)	**18.**	(c)	**19.**	(c)	**20.**	(c)	**21.**	(d)	**22.**	(b)	**23.**	(a)	**24.**	(b)
25.	(d)	**26.**	(a)	**27.**	(d)	**28.**	(d)	**29.**	(a)	**30.**	(c)	**31.**	(d)	**32.**	(c)
33.	(a)	**34.**	(c)	**35.**	(d)	**36.**	(b)	**37.**	(d)	**38.**	(c)	**39.**	(a)	**40.**	(a)
41.	(a)	**42.**	(b)	**43.**	(d)										

1. **(d)** : Let $z=\dfrac{1+i\sqrt{3}}{\sqrt{3}+i}\times\dfrac{\sqrt{3}-i}{\sqrt{3}-i}=\dfrac{(1+i\sqrt{3})(\sqrt{3}-i)}{3+1}$

$=\dfrac{(\sqrt{3}+3i-i+\sqrt{3})}{4}=\dfrac{(2\sqrt{3}+2i)}{4}$

i.e., $z=\dfrac{(\sqrt{3}+i)}{2}=\dfrac{\sqrt{3}}{2}+i\left(\dfrac{1}{2}\right)$

amplitude $=\tan^{-1}\dfrac{y}{x}=\tan^{-1}\left(\dfrac{1}{2}\cdot\dfrac{2}{\sqrt{3}}\right)$

$=\tan^{-1}\left(\dfrac{1}{\sqrt{3}}\right)=\dfrac{\pi}{6}.$

2. **(a)** : Given equation $2x^3-3x^2+6x+1=0$

$\alpha+\beta+\gamma=-3/2,\ \alpha\beta\gamma=1/2,\ \Sigma\alpha\beta=3$

$\alpha^2+\beta^2+\gamma^2=(\alpha+\beta+\gamma)^2-2(\Sigma\alpha\beta)$

$=(-3/2)^2-2\times3=\dfrac{9}{4}-6=\dfrac{9-24}{4}=-\dfrac{15}{4}$

3. **(b)** : $z=\dfrac{1+2i}{1-(1-i)^2}=\dfrac{1+2i}{1-(1-1-2i)}=\dfrac{1+2i}{1+2i}=1$

$\therefore$ Modulus = 1; Argument = 0

4. **(b)** : $\dfrac{1}{1+\cos\theta+i\sin\theta}\times\dfrac{(1+\cos\theta)-i\sin\theta}{(1+\cos\theta)-i\sin\theta}$

$=\dfrac{1+\cos\theta-i\sin\theta}{(1+\cos\theta)^2+\sin^2\theta}$

$=\dfrac{1+\cos\theta-i\sin\theta}{1+\cos^2\theta+2\cos\theta+\sin^2\theta}=\dfrac{1+\cos\theta-i\sin\theta}{1+1+2\cos\theta}$

$=\dfrac{1}{2}\left[\dfrac{1+\cos\theta-i\sin\theta}{1+\cos\theta}\right]$ $\therefore$ real part is $\dfrac{1}{2}$

5. **(d)** : Given equation $x^3-8x+8=0$

$\therefore$ $\Sigma\alpha=0,\ \Sigma\alpha\beta=-8,\ \alpha\beta\gamma=-8$

$\Rightarrow$ $(\alpha+\beta+\gamma)^2=\alpha^2+\beta^2+\gamma^2+2(\alpha\beta+\beta\gamma+\gamma\alpha)$

$\Rightarrow$ $0=\alpha^2+\beta^2+\gamma^2+2(-8)\Rightarrow\Sigma\alpha^2=16.$

Now,

$\dfrac{1}{\alpha\beta}+\dfrac{1}{\beta\gamma}+\dfrac{1}{\alpha\gamma}=\dfrac{\alpha\beta\gamma(\alpha+\beta+\gamma)}{\alpha^2\beta^2\gamma^2}=\dfrac{\alpha+\beta+\gamma}{\alpha\beta\gamma}=0$

$\therefore$ $\Sigma\alpha^2=16,\ \Sigma\dfrac{1}{\alpha\beta}=0.$

6. **(c)** : $\dfrac{(-\sqrt{3}+3i)(1-i)}{(3+\sqrt{3}i)(i)(\sqrt{3}+\sqrt{3}i)}$

$=\dfrac{(-\sqrt{3}+3i)(1-i)}{(3i-\sqrt{3})(\sqrt{3})(1+i)}$

$=\dfrac{1}{\sqrt{3}}\left[\dfrac{1-i}{1+i}\times\dfrac{1-i}{1-i}\right]=\dfrac{1}{\sqrt{3}}\left[\dfrac{1+(i)^2-2i}{2}\right]=\dfrac{-i}{\sqrt{3}}.$

$\therefore$ Given complex number in Argand plane lies on y-axis (imaginary axis).

7. **(d)** : Given $2x=-1+\sqrt{3}i$

$\Rightarrow$ $x=\dfrac{-1+\sqrt{3}i}{2}=\omega$

Now, $(1-\omega^2+\omega)^6-(1-\omega+\omega^2)^6$

$=(-\omega^2-\omega^2)^6-(-\omega-\omega)^6$ $(\because\ 1+\omega+\omega^2=0)$

$=(-2\omega^2)^6-(-2\omega)^6=(-2)^6(\omega^3)^4-(-2)^6(\omega^3)^2$

$=(-2)^6-(-2)^6=0$ $(\because\ \omega^3=1).$

8. **(b)** : $(1+i\sqrt{3})^8=(2)^8\left[\dfrac{-1-i\sqrt{3}}{2}\right]^8=(2)^8(\omega^2)^8$

$=2^8(\omega^3)^5\omega=2^8\omega=2^8\left(-\dfrac{1}{2}+i\dfrac{\sqrt{3}}{2}\right)$

$=2^8\left(\cos\dfrac{2\pi}{3}+i\sin\dfrac{2\pi}{3}\right)$

$\therefore$ Modulus $=2^8=256$ and argument $=2\pi/3$.

9. **(d)** : $\dfrac{(1+i)^2}{1-i}=\dfrac{1+i^2+2i}{1-i}=\dfrac{1-1+2i}{1-i}$

$=\dfrac{2i}{1-i}\times\dfrac{1+i}{1+i}=\dfrac{2i(1+i)}{1-(i)^2}=\dfrac{2i(1+i)}{1-(-1)}$

$=\dfrac{2i(1+i)}{2}=i+i^2=i-1.$

$\therefore$ Required conjugate is $-i-1$.

10. **(a)** : $A=i^i$

$\log A=\log i^i\Rightarrow\log A=i\log i$

$\Rightarrow\log A=i\log(0+i)$

$\Rightarrow\log A=i[0+i\pi/2]$

$\Rightarrow\log A=-\pi/2$

$\Rightarrow A=e^{-\pi/2}$

Therefore imaginary part is 0.

11. **(d)** : $(1+i)^5=\left(\sqrt{2}\right)^5\left(\dfrac{1}{\sqrt{2}}+i\dfrac{1}{\sqrt{2}}\right)^5$

$=\left(\sqrt{2}\right)^5\left(\cos\dfrac{\pi}{4}+i\sin\dfrac{\pi}{4}\right)^5$

$= \left(\sqrt{2}\right)^5\left(\cos\frac{5\pi}{4}+i\sin\frac{5\pi}{4}\right)$

$\therefore$ Amplitude $= \frac{5\pi}{4}$

12. (a) : If 1, ω, ω^2 are the cube root of unity. Then, $1+\omega+\omega^2=0$ and $\omega^3=1$.

$\therefore (1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8)$

$=(1+\omega)(-\omega)(1+\omega)(-\omega)\ -\omega^2(-\omega)(-\omega^2)(-\omega)$

$=\omega^3\times\omega^3=1\times 1=1.$

13. (c) : α is a complex number satisfying the equation $\alpha^2+\alpha+1=0$

$\therefore\quad \alpha^3=1 \qquad [\because\ \omega^3=1]$

Now, $\alpha^{31}=(\alpha^3)^{10}\cdot\alpha=1\cdot\alpha=\alpha.$

14. (b) : Let $z=x+iy$ be any complex number.

$z=-\bar{z}$ (given)

$\Rightarrow x+iy=-(x-iy)\Rightarrow 2x=0\Rightarrow x=0$

$\therefore z=0+iy=iy$ is purely imaginary.

15. (c) : Let $z=\cos\frac{2\pi}{7}+i\sin\frac{2\pi}{7}$

$\therefore z^k=\cos\frac{2k\pi}{7}+i\sin\frac{2k\pi}{7}$

(Using De Moivre's theorem)

Taking $\sum_{k=1}^{6}\left[\sin\frac{2k\pi}{7}-i\cos\frac{2k\pi}{7}\right]$

$=(-i)\sum_{k=1}^{6}\left(\cos\frac{2k\pi}{7}+i\sin\frac{2k\pi}{7}\right)=(-i)\sum_{k=1}^{6}z^k$

(using (i))

$=(-i)[z+z^2+\ldots+z^6]$ is a G.P. series

$=(-i)\frac{z(1-z^6)}{1-z}=(-i)\left(\frac{z-z^7}{1-z}\right)$

$=(-i)\left(\frac{z-\left(\cos\frac{2\pi}{7}+i\sin\frac{2\pi}{7}\right)^7}{1-z}\right)$

$=(-i)\left(\frac{z-\left(\cos 7\cdot\frac{2\pi}{7}+i\sin 7\cdot\frac{2\pi}{7}\right)}{1-z}\right)$

$=(-i)\left(\frac{z-1}{1-z}\right)=i.$

16. (b) : Given equation is $x^3-6x+9=0$

Since $x=-3$ satisfy above equation

$\therefore$ real root is $x=-3$.

17. (d) : $x^2+x+1=0$

$\Rightarrow x=\frac{-1\pm\sqrt{1^2-4\cdot1\cdot1}}{2}=\frac{-1\pm\sqrt{3}i}{2}$

or $\alpha=-\frac{1}{2}+\frac{\sqrt{3}i}{2},\ \beta=-\frac{1}{2}-\frac{\sqrt{3}i}{2}$

$\beta^2=\left(-\frac{1}{2}-\frac{\sqrt{3}i}{2}\right)^2=-\frac{1}{2}+\frac{\sqrt{3}}{2}i=\alpha$

Also $1+\beta+\beta^2=0,\ \beta^3=1$

$\therefore \alpha^{16}+\beta^{16}=(\beta^2)^{16}+\beta^{16}=\beta^{16}(1+\beta^{16})$

$=(\beta^3)^5.\beta\,(1+(\beta^3)^5\beta)=\beta\,(1+\beta)$

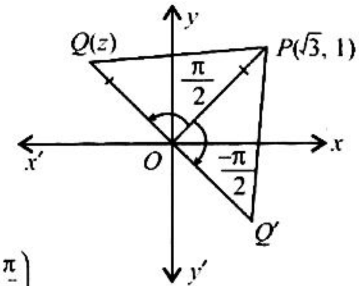
$=\beta\,(-\beta^2)=-\beta^3=-1$

18. (c) : $\frac{1+2i}{1-i}=\frac{1+2i}{1-i}\cdot\frac{1+i}{1+i}=\frac{1+2i+i+2i^2}{1-i^2}$

$=\frac{-1+3i}{2}=-\frac{1}{2}+\frac{3}{2}i=\left(-\frac{1}{2},\frac{3}{2}\right)$

$\therefore$ Complex number lies in 2nd quadrant.

19. (c) : In the Argand diagram point Q may be either in 2nd or 4th quadrant which is given by



$\frac{z-0}{(\sqrt{3}+i)-0}=\frac{OQ}{OP}e^{i\left(\pm\frac{\pi}{2}\right)}$

$\Rightarrow z=(\sqrt{3}+i)\left(\cos\left(\pm\frac{\pi}{2}\right)+i\sin\left(\pm\frac{\pi}{2}\right)\right)$

$=(\sqrt{3}+i)(\pm i) \qquad (\because\ OP=OQ)$

$=-1+\sqrt{3}i$ or $1-i\sqrt{3}$

20. (c) : $\left[\frac{1+\sin\frac{\pi}{8}+i\cos\frac{\pi}{8}}{1+\sin\frac{\pi}{8}-i\cos\frac{\pi}{8}}\right]^n$

$=\left[\frac{1+\cos\left(\frac{\pi}{2}-\frac{\pi}{8}\right)+i\sin\left(\frac{\pi}{2}-\frac{\pi}{8}\right)}{1+\cos\left(\frac{\pi}{2}-\frac{\pi}{8}\right)-i\sin\left(\frac{\pi}{2}-\frac{\pi}{8}\right)}\right]^n$

$=\left[\frac{1+\cos\frac{3\pi}{8}+i\sin\frac{3\pi}{8}}{1+\cos\frac{3\pi}{8}-i\sin\frac{3\pi}{8}}\right]^n$

$$= \left[\frac{1+2\cos^2\left(\frac{3\pi}{16}\right)-1+2i\sin\frac{3\pi}{16}\cos\frac{3\pi}{16}}{1+2\cos^2\left(\frac{3\pi}{16}\right)-1-2i\sin\frac{3\pi}{16}\cos\frac{3\pi}{16}}\right]^n$$

$$= \left[\frac{\cos\frac{3\pi}{16}+i\sin\frac{3\pi}{16}}{\cos\frac{3\pi}{16}-i\sin\frac{3\pi}{16}}\right]^n$$

$$= \left(\cos^2\frac{3\pi}{16}-\sin^2\frac{3\pi}{16}+2i\sin\frac{3\pi}{16}\cos\frac{3\pi}{16}\right)^n$$

$$= \left(\cos\frac{3\pi}{8}+i\sin^2\frac{3\pi}{16}\right)^n = \cos\frac{3\pi n}{8}+i\sin\frac{3\pi n}{8}$$

which is purely imaginary then

$$\cos\frac{3\pi n}{8} = 0 = \cos\frac{\pi}{2}$$

$$\Rightarrow \frac{3\pi n}{8} = (2k+1)\frac{\pi}{2} \Rightarrow n = \frac{4}{3}(2k+1), k \in I$$

keep $k = 1, 4, 7, 10$ ....

we get $n = 4, 12, 20, 28,$ ....

$\therefore$ Smallest positive integral value of n is 4

21. (d) : Sum of the roots $a, -a, b$ is

$a + (-a) + b = 5 \Rightarrow b = 5$

Roots of eq^n $x^2 - 5x + 10 = 0$ are $\frac{5 \pm \sqrt{25-40}}{2} \neq 5$

Roots of eq^n $x^2 + 3x - 20 = 0$ are $\frac{-3 \pm \sqrt{9-80}}{2} \neq 5$

Roots of eq^n $x^2 + 5x - 30 = 0$ are $\frac{-5 \pm \sqrt{25+120}}{2} \neq 5$

Roots of eq^n $x^2 - 3x - 10 = 0$ are $\frac{3 \pm \sqrt{9+40}}{2}$

$$= \frac{3\pm 7}{2} = \frac{3+7}{2}, \frac{3-7}{2} = 5, -2$$

22. (b) : $\frac{(1+i)^n}{(1-i)^{n-2}} = \frac{(1+i)^2(1+i)^{n-2}}{(1-i)^{n-2}}$

$$= (1+i)^2\left(\frac{1+i}{1-i}\right)^{n-2} = (1+i)^2\left(\frac{(1+i)(1+i)}{1^2+1^2}\right)^{n-2}$$

$$= (1+i)^2\left(\frac{1-1+2i}{2}\right)^{n-2} = (2i)(i)^{n-2}$$

$= 2(i)^{n-1} = 2(-1)^{n-1}$

[$2(-1)^{n-1} > 0$ if n is positive odd number greater than 1]

23. (a) : $x + iy = (-1+i\sqrt{3})^{2010}$ ...(1)

Let ω be the cube root of unity

$\omega = \frac{-1+i\sqrt{3}}{2} \Rightarrow 2\omega = -1+i\sqrt{3}$ (put in (1))

$x + iy = (2\omega)^{2010} = 2^{2010} \times (\omega^3)^{670} = 2^{2010}$ [$\because \omega^3 = 1$]

24. (b) : $\sum\left(\frac{1}{\alpha+\beta-\gamma}\right)$

$$= \frac{1}{\alpha+\beta-\gamma}+\frac{1}{\alpha-\beta+\gamma}+\frac{1}{-\alpha+\beta+\gamma}$$

$$= \frac{(\alpha-\beta+\gamma)(-\alpha+\beta+\gamma)+(\alpha+\beta-\gamma)(-\alpha+\beta+\gamma)+(\alpha+\beta-\gamma)(\alpha-\beta+\gamma)}{(\alpha+\beta-\gamma)(\alpha-\beta+\gamma)(-\alpha+\beta+\gamma)}$$

$$= \frac{(-2\beta)(-2\alpha)+(-2\gamma)(-2\alpha)+(-2\gamma)(-2\beta)}{(-2\gamma)(-2\beta)(-2\alpha)}$$

(It is given that $\alpha + \beta + \gamma = 0, \alpha\beta + \beta\gamma + \alpha\gamma = -2, \alpha\beta\gamma = 1$)

$$= \frac{4(-2)}{8(1)} = -1$$

25. (d) : Now, $|1+i\sqrt{3}| = \sqrt{1+3} = 2$ ...(i)

Also, $1+\frac{1}{i+1} = 1+\frac{i-1}{i^2+1} = 1+\frac{(i-1)}{-2} = \frac{3}{2}-\frac{i}{2}$

So, $\left|1+\frac{1}{i+1}\right| = \sqrt{\left(\frac{3}{2}\right)^2+\left(-\frac{1}{2}\right)^2} = \sqrt{\frac{9}{4}+\frac{1}{4}} = \frac{\sqrt{10}}{2}$ ...(ii)

Thus, $\left|\frac{1+i\sqrt{3}}{\left(1+\frac{1}{i+1}\right)^2}\right| = \frac{2}{10/4} = \frac{4}{5}$ (from (i) &(ii))

26. (a) : First factor $= -\omega - \omega = -2\omega$

Second factor $= 1 - \omega^2 + \omega = -\omega^2 - \omega^2 = -2\omega^2$

3rd factor, 5th factor, .. are equal to first factor which is -2ω

4th factor, 6th factor, ... are equal to the second factor which is $-2\omega^2$

Required product $= (-2\omega)(-2\omega^2)(-2\omega)(-2\omega^2)$ .... $2n$ factors $= (4\omega^3)(4\omega^3)$ ...... n factors

$= 4 \cdot 4$ .... n factors $= 4^n = 2^{2n}$.

27. (d) : Given, $\left|\frac{z-1}{z+2i}\right| = 1$

$\Rightarrow$ $|z-1| = |z+2i| \Rightarrow |z-1| = |z-(-2i)|$

So, z forms that set of points is equidistant from 1 and $(-2i)$ in the Argand plane.

So, z lies on the perpendicular bisector of 1 and $(-2i)$ in the Argand plane. Option (d) is correct.

28. **(d)** : Now, $\dfrac{(1-i\sqrt{3})(1+i)}{(\sqrt{3}+i)}$

$= \dfrac{-i(i+\sqrt{3})(1+i)}{(\sqrt{3}+i)} = -i(1+i) = 1-i$

Clearly, above point lies in quadrant IV.

29. **(a)** : By verification $x = 1, 2$

30. **(c)** : Let $z = x + iy$,

$z^2 = -\bar{z} \Rightarrow (x+iy)^2 = -(x-iy)$.

By simplifying we will get

$z_1 = 0 + i0, z_2 = -1 + i0, z_3 = \dfrac{1}{2} + i\dfrac{\sqrt{3}}{2}, z_4 = \dfrac{1}{2} - i\dfrac{\sqrt{3}}{2}$

$\therefore$ Number of solutions are 4

31. **(d)** : $\alpha^2 - \alpha + 1 = 0 \Rightarrow \alpha = \dfrac{1 \pm \sqrt{1-4}}{2} = \dfrac{1 \pm \sqrt{3}i}{2}$

$\Rightarrow \alpha = -\omega \therefore \alpha^{2011} = (-\omega)^{2011} = -\omega = \alpha$

32. **(c)** : $\overline{(x+iy)(1-2i)} = 1+i$

$\Rightarrow (x-iy)(1+2i) = 1+i$

$\Rightarrow x - iy = \dfrac{1+i}{1+2i} \Rightarrow x + iy = \dfrac{1-i}{1-2i}$

33. **(a)** : $x^3 + 0x^2 + 4x + 2 = 0, \alpha + \beta + \gamma = 0$

$\therefore \alpha^3 + \beta^3 + \gamma^3 = 3\alpha\beta\gamma = 3(-2) = -6$

34. **(c)** : $\left|\dfrac{\beta-\alpha}{1-\bar{\alpha}\beta}\right| = \dfrac{\sqrt{(\beta-\alpha)\overline{(\beta-\alpha)}}}{\sqrt{(1-\bar{\alpha}\beta)\overline{(1-\bar{\alpha}\beta)}}}$

$= \dfrac{\sqrt{(\beta-\alpha)(\bar{\beta}-\bar{\alpha})}}{\sqrt{(1-\bar{\alpha}\beta)(1-\alpha\bar{\beta})}} = \sqrt{\dfrac{\beta\bar{\beta}-\beta\bar{\alpha}-\alpha\bar{\beta}+\alpha\bar{\alpha}}{1-\alpha\bar{\beta}-\bar{\alpha}\beta+\alpha\bar{\alpha}\beta\bar{\beta}}}$

$= \sqrt{\dfrac{|\beta|^2-\beta\bar{\alpha}-\alpha\bar{\beta}+|\alpha|^2}{1-\alpha\bar{\beta}-\bar{\alpha}\beta+|\alpha|^2|\beta|^2}}$

$= \sqrt{\dfrac{1-\beta\bar{\alpha}-\alpha\bar{\beta}+|\alpha|^2}{1-\beta\bar{\alpha}-\alpha\bar{\beta}+|\alpha|^2}} = \sqrt{1} = 1 \quad (\because |\beta| = 1)$

35. **(d)** : We know that the roots of the equation $x^3 + ax^2 + bx + c = 0$ are in A.P.

Let $\alpha = -1, \beta = 1, \gamma = 3$

So, $(x+1)(x-1)(x-3) = 0$

$\Rightarrow x^3 - 3x^2 - x + 3 = 0$

$\Rightarrow a = -3, b = -1$ and $c = 3$

Now, $2a^3 - 9ab = 2(-3)^3 - 9(-3)(-1)$

$= -54 - 27 = -81 = -27c$

36. **(b)** : Since $1, \omega, \omega^2$ are cube roots of unity

$\therefore 1 + \omega + \omega^2 = 0$

Now, $(1 - \omega + \omega^2)(1 + \omega - \omega^2)$

$= (-\omega - \omega)(-\omega^2 - \omega^2) = (-2\omega)(-2\omega^2) = 4\omega^3 = 4$

37. **(d)** : $z = \dfrac{(\sqrt{3}+i)^3(3i+4)^2}{(8+6i)^2}$

Taking Modulus on both sides, we get

$|z| = \dfrac{|\sqrt{3}+i|^3|3i+4|^2}{|8+6i|^2}$

$= \dfrac{(\sqrt{3+1})^3(\sqrt{9+16})^2}{(\sqrt{64+36})^2} = \dfrac{8\times 25}{100} = 2$

38. **(c)** : Given equation is

$x^2 - ax + b^2 = 0$

Since α and β are its roots

$\therefore \alpha + \beta = a, \alpha\beta = b^2$

Now, $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = a^2 - 2b^2$

39. **(a)** : $(1 - \cos\theta + i\sin\theta)^{-1} = \dfrac{1}{1-\cos\theta+i\sin\theta}$

$= \dfrac{1}{1-\cos\theta+i\sin\theta} \times \dfrac{1-\cos\theta-i\sin\theta}{1-\cos\theta-i\sin\theta}$

$= \dfrac{1-\cos\theta-i\sin\theta}{1+\cos^2\theta-2\cos\theta+\sin^2\theta} = \dfrac{(1-\cos\theta)-i\sin\theta}{2(1-\cos\theta)}$

$\therefore$ Real part of given expression is $\dfrac{1}{2}$

40. **(a)** : $i^n + i^{n+1} + i^{n+2} + i^{n+3}$

$= i^n + i^{n+1} + i^n \cdot i^2 + i^{n+1} \cdot i^2$

$= i^n + i^{n+1} - i^n - i^{n+1} \quad [\because i^2 = -1]$

$= 0$

41. **(a)** : Given, $\left(\dfrac{1+i}{1-i}\right)^m = 1 \Rightarrow i^m = i^4 \Rightarrow m = 4$

42. **(b)** : Given, $\dfrac{1-i}{1+i} = \dfrac{1-i}{1+i} \times \dfrac{1-i}{1-i} = \dfrac{(1-i)^2}{1-i^2} = -i$

$\therefore \left(\dfrac{1-i}{1+i}\right)^{96} = (-i)^{96} = 1 = 1 + 0i \quad \text{...(i)}$

Comparing (i) with $a + ib$, we get $a = 1, b = 0$

So, the value of $(a, b) = (1, 0)$

43. **(d)** : We have, $x^2 + x + 1 = 0 \quad \text{... (i)}$

Since α, β are roots of equation (i).

$\therefore \alpha + \beta = -1, \alpha\beta = 1$

Now, $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = 1 - 2 = -1$