✎ Evolution of commitment and level of participation in public goods games

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ABSTRACT

This study explores how cooperation and commitment develop in public goods games (PGGs) using stochastic evolutionary game theory. We examine how factors like prior commitments, costs, and participation levels affect group behavior and cooperation.

Our methodology involves extensive use of the EGTtools Python library to simulate and analyze a diverse set of strategies, including unconditional contributors, defectors, and commitment proposers operating under different thresholds. These simulations evaluate payoffs, compute stationary distributions, and reveal the impact of structured commitments on group cooperation levels.

Key findings from this study demonstrate that moderate penalties for non-compliance and adaptable participation requirements significantly enhance the stability of cooperative arrangements. These mechanisms address the classic "tragedy of the commons," offering practical solutions for overcoming challenges in resource-sharing scenarios.

This work enhances our understanding of cooperation dynamics and provides practical insights for creating better cooperative frameworks.

keywords : Public Goods Games, Commitment, Moderate Penalties, Resource-sharing Scenarios 1103 characters

1 INTRODUCTION

The paper *"Evolution of commitment and level of participation in public goods games"* [1] investigates how prior commitments among agents influence cooperation in public goods games (PGGs). Using stochastic evolutionary game theory, the authors model the dynamics of cooperation when agents propose and accept commitments, taking into account costs, participation levels, and repeated interactions.

The study demonstrates that prior commitments, if properly structured, can lead to a high level of cooperation in public goods games, especially when the cost of arranging commitments is relatively low compared to the benefits of cooperation. The authors identify an optimal level of participation that ensures the success of cooperative ventures, dependent on the severity of the social dilemma and the cost of arranging the commitment. In multi-round interactions, agents need to adapt their strictness in enforcing commitments, with leniency proving beneficial in short-term agreements and stricter enforcement necessary for long-term interactions.

Additionally, the paper analyzes the influence of costs associated with commitment (e.g., setup costs and penalties for defection) and reveals that moderate compensation levels are sufficient for sustaining cooperation, eliminating the need for excessively punitive measures. Insights from the study can be applied to the design of self-organizing multi-agent systems and the formulation of policies in human cooperative scenarios, especially where cooperation is crucial but non-compliance needs regulation.

The findings address the classical issue of the "tragedy of the commons," offering a pathway to resolve this dilemma. By connecting theoretical results to real-world applications, such as environmental treaties and coalition formation in multi-agent systems, the work provides actionable insights for policymakers and system designers. Furthermore, it builds on earlier research in two-player games and extends it to complex group dynamics, incorporating novel perspectives on fostering cooperation under varying conditions.

2 RELATED WORK

. Evolution of Commitment in the Spatial Public Goods Game through Institutional Incentives.This paper investigates how institutionalincentives, such as punishment and rewards, influence the evolution of commitment and cooperation in spatial public goods games[2].
The authors introduce a commitment threshold that group members must meet for a commitment to be formed and explore the effects of punishing non-compliant players and rewarding compliant ones. The study finds that conditional behavior based on commitment can enhance cooperation, with punishment mechanisms being particularly effective at intermediate commitment thresholds and reward mechanisms promoting high levels of cooperation across various thresholds.
The main difference is introduction of institutional incentives, such as rewards and punishments, and studies their impact in spatial public goods games. The selected paper, however, focuses on non-spatial games and investigates how multi-round commitments evolve with participation thresholds.

. Good agreements make good friends[3].
This article explores the use of commitment strategies to study the evolution of cooperation and compares them to costly punishment strategies in a one-shot Public Goods Game (PGG). It examines five distinct strategies: C (cooperator), D (defector), COMP (commitment proposer), FREE (commitment free-rider), and FAKE (fake committer). The study demonstrates that commitment is a more effective solution for promoting cooperation compared to costly punishment strategies.
Unlike our study, this article does not incorporate the threshold parameters F and F', which are critical to determining the levels of participation in the game. The article does not deeply address the concept of cooperation cost, which plays a significant role in our

analysis. Our work introduces the notion of multiple rounds, where interactions take place over several iterations, enabling a dynamic analysis of strategies. In contrast, the second article focuses on a one-shot game, where the interactions are simpler and do not evolve over time.

. Avoiding or restricting defectors in public goods games[4]
This study examines strategies to mitigate the impact of defectors in Public Goods Games (PGGs). The authors explore two primary mechanisms: **Avoidance Strategy**: This mechanism involves refusing to play the game when there are non-committers in the group. **Restriction Strategy**: This approach introduces an additional cost, denoted as ϵ_r, to restrict the access of non-committers to the PGG. These strategies are analyzed alongside well-known strategies such as C (cooperation), D (defection), FREE (commitment free-riders), and FAKE (fake committers). The study demonstrates that both mechanisms enhance cooperation, with the restriction strategy being particularly effective in larger group interactions.
While both studies aim to promote cooperation in PGGs, they propose different approaches: The first study focuses on establishing prior commitments and sharing the associated costs to foster cooperation. This study emphasizes implementing restriction mechanisms to deter defectors during or after the game.

3 PRACTICAL ASPECTS OF THE REPRODUCTION

Existing Resources

There was no code directly provided with the article. To reproduce the results and generate the graphs, we used Python code, the $\mathrm{egttools}$ library, and some notebooks such as Homework 3 and other materials available from the course. These resources, derived from the article [4], were instrumental. The article itself offered detailed descriptions of the payoff equations and strategy implementations, supported by well-designed decision trees that clarified the scenarios and guided the implementation process.

Implementation Process

The primary objective of this project was to reproduce the figures essential for analyzing the results. The implementation process commenced with the definition of the PGG class, which inherits from $\mathrm{AbstractNPlayerGame}$ in the $\mathrm{egttools}$ library. Following the structure of the provided notebooks, we implemented three key methods:

- $\mathrm{play}()$ – Executes the game.
- $\mathrm{calculate\_payoffs}()$ – Computes the payoffs for all strategies.
- $\mathrm{\_\_init\_\_}()$ – Initializes the class with the following parameters:
  - $\mathrm{group\_size}$: int – Number of participants in the PGG.
  - c: float – Cost of cooperation.
  - r: float – Enhancement factor for the public good.
  - eps: float – Shared cost to propose a commitment.
  - delta: float – Cost incurred by those who commit but do not contribute.

Additionally, we defined a game with nine strategies: C, D, $\mathrm{COMPf(1\text{-}5)}$, FREE, and FAKE. The definitions for these strategies are as follows, show also on the figure 1:

Definitions of Strategies.

- $\mathrm{COMPF(1\text{-}5)}$: Commitment proposing strategies. Each COMPF contributes to the public good if at least F participants (including itself) commit to contributing. Otherwise, it refuses to play, resulting in zero payoff for all players. The five variants, $\mathrm{COMPF1}$, $\mathrm{COMPF2}$, ..., $\mathrm{COMPF5}$, represent increasing levels of required participation ($F = 1, 2, ..., 5$).
- FAKE: Fake committers. These players accept a commitment proposal but fail to contribute when the game is played. They attempt to exploit the proposers and other contributors, assuming they can avoid compensation penalties.
- FREE: Commitment-free riders. These players defect unless they are proposed a commitment. Upon receiving a proposal, they accept and cooperate during the game, benefiting from the arrangement without bearing the initial cost of proposing it.
- C: Unconditional contributors. These players always cooperate and commit when proposed a commitment deal, regardless of the game dynamics.
- D: Unconditional defectors. These players never commit to a proposal and always defect during the game, maximizing their individual payoff at the expense of the group.

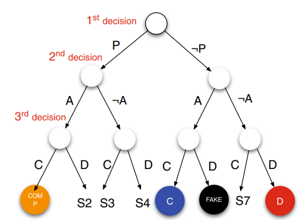

Figure 1: Differents strategies

The main challenge in implementing this class lies in the play function, as it involves significant complexity compared to the relatively simple or unchanged logic in the other methods. The initial part of this function defines the various strategies and calculates the individual rewards for the group. The total contribution is computed as:

$$\text{total\_contribution} = c \times \text{nb\_contributors},$$

where c is the cost of cooperation. The total reward is derived from the enhancement factor r applied to the total contribution:

$$\text{total\_reward} = r \times \text{total\_contribution}.$$

Finally, the individual reward is calculated by distributing the total reward equally among all participants:

$$\text{individual\_reward} = \frac{\text{total\_reward}}{\text{group\_size}}.$$

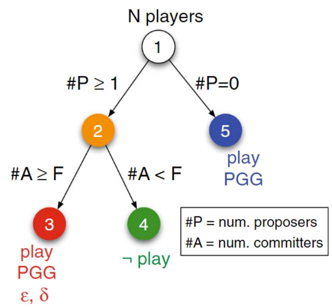

Figure 2: Scenario Tree

The implementation adheres to the process outlined in Figure 1 ([**?** , Figure 1] to ensure consistency with the theoretical framework.

Determining the Participation Threshold (F). To determine the required participation threshold (F), we evaluate all the COMPF strategies present in the group. From these, the highest F is selected, representing the strictest level of commitment required for the public goods game to proceed.

Method Structure: Three Key Parts. The play function is logically divided into three main parts:

(1) **Initial Check for Commitments:**
   - If there is at least one commitment (nb_commitment $\geq$ 1), the method moves to the next decision point, corresponding to Node 2 in Figure 2.

(2) **Evaluating the Participation Threshold (F):**
   - If the number of acceptances (nb_accept) meets or exceeds the required threshold (F), the public goods game (PGG) proceeds. This corresponds to Node 3 in the figure 2.
   - Otherwise, the game does not proceed, and this corresponds to Node 4 in fig.2, where no payoff is distributed.

(3) **Payoff Adjustments:**
   - Each strategy's payoff is adjusted based on its behavior:
     – **Cooperators (C)** and **Free Riders (FREE)** incur the cost of cooperation ($-c$).
     – **Fake Committers (FAKE)** incur the penalty for failing to contribute ($-\delta$).
     – **Commitment Proposers (COMPF):** Their payoffs are adjusted based on the balance of penalties collected from defectors, shared costs for proposing commitments (eps), and the cost of cooperation. The reward for a proposer is calculated as:

$$\text{Reward} = \frac{(\text{nb\_fake} \times \delta - \text{eps})}{\text{nb\_commitment}} - c,$$

where nb_fake represents the number of fake committers in the group.

This structure ensures a robust implementation of the public goods game dynamics while adhering closely to the theoretical model.

Next, we computed the stationary_distribution for a specific game using parameters provided in the article, leveraging the egttools library. We also generated the invasion diagram. To plot additional points, we implemented a loop over varying parameters, creating specific game objects for each parameter set. Using these objects, we initialized an evolver with PairwiseComparison from egttools [5]. to compute the stationary distribution. The strategy frequencies were stored by summing the relevant strategies, facilitated by the numpy library.

Plotting was primarily achieved with the axes.contour() method, which required four parameters. Most other figures followed a similar methodology with adjustments for specific parameters, which will be detailed in the next section.

For Figures 5 and 6, we introduced new parameters such as R (number of rounds) and F'. To manage this, we adjusted the PGG class by redefining it appropriately.

Encountered Difficulties

Several challenges arose during reproduction:

- The parameter c was not explicitly provided in the article. We derived its value based on the invasion graph figure. We compute its value by finding the smallest distance between, the value on the invasion graph given in the paper and the one that we compute with multiple values of c in a loop.
- Computing average payoffs for strategies posed difficulties. While the article provides pairwise payoff equations, our implementation required averaging payoffs for each strategy.
- Implementing F' proved challenging. The article mentions 29 strategies, but using the egttools library, computing the transition matrix became infeasible due to excessive computation time, even with SML or estimate_stationary_distribution() with PairwiseComparisonNumerical.

Additional Tests/Analysis

To connect this study with the course concepts, we compared the effectiveness of **commitment** strategies versus **punishment** strategies in promoting cooperation within Public Goods Games (PGGs).

1. PGG with *COMP, C, D, FREE,* and *FAKE* Strategies

In this scenario3, strategies invade more and more in the *contributers* when gamma (punition cost) is growing. This result is logical since cooperative strategies were added, inherently raising overall cooperation levels. However, it is notable that the *COMP* (commitment) strategy was more effective than punishment strategies in resisting invasion. This suggests that commitment is preferred over punishment in stabilizing cooperation.

3. PGG with *COMP, C,* and *D* Strategies

In this configuration 4, *commitment* strategies dominated with approximately 50% presence, while defectors occupied around 30%. Compared to the second invasion diagram, commitment visibly enhanced cooperation where punishment failed. This demonstrates

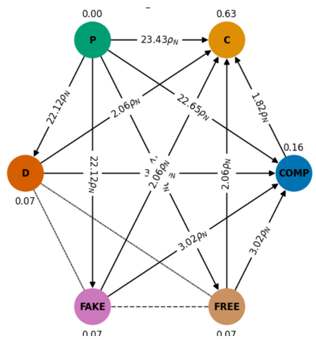

Figure 3: COMP and Punish invasion diagram

that commitment strategies can effectively suppress defection and foster higher cooperation levels.

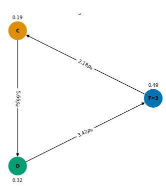

Figure 4: Ony COMPf diagram invasion

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The comparative analysis indicates that the *COMP* (commitment) strategy is more effective than punishment in fostering cooperation within the studied PGG models. While punishment may have contextual benefits , it does not consistently sustain cooperation like show in this article[6]. Commitment strategies, however, provide a proactive and reliable framework for encouraging cooperative behavior.like say the conclusion of this article [3]

4 METHODS

This section provides an overview of the methods used to produce the results presented in this work. The implementation process leverages stochastic evolutionary game theory to analyze the dynamics of cooperation under varying strategies in public goods games (PGGs) with commitments. Below, we detail the key components of the implementation.

Game Implementation: PGGWithCommitment

The public goods game with commitments (PGGWithCommitment) was implemented as a subclass of AbstractNPlayerGame [7], which defines the framework for N-player games in the egttools library. The primary objectives were to define the game dynamics, calculate payoffs for all strategies, and analyze the effects of commitments on cooperation.

Initialization. The PGGWithCommitment class takes the following parameters:

- group_size (N): The number of participants in the PGG.
- c: The cost of cooperation.
- r: The enhancement factor (multiplier) applied to the public good.
- eps (ϵ): The shared cost for proposing a commitment.
- delta (δ): The cost imposed on players who commit but fail to contribute.

The class also defines nine strategies (COMP1–COMP5, C, D, FAKE, FREE), representing different behaviors in the game, as described in the literature [1].

Game Dynamics. The play method implements the dynamics of the game:

- **Classical PGG:** If no commitments are proposed, the game proceeds as a standard PGG, with only cooperators (C) contributing.
- **PGG with Commitments:** If commitments are proposed, contributors include cooperators (C), free riders (FREE), and commitment proposers (COMP1–COMP5). The minimum participation threshold (F) is determined based on the strictest commitment requirement in the group.
- If the number of players accepting the commitment (nb_accept) meets or exceeds F, the PGG proceeds, and payoffs are distributed accordingly. Otherwise, the game is not played, and no payoffs are distributed.

Payoffs for each strategy are adjusted based on their behavior, including costs of cooperation (c), penalties for fake committers (δ), and shared costs of proposing commitments (ϵ).

Evolutionary Analysis

The PairwiseComparison class was used to simulate evolutionary dynamics, employing the Moran process to model strategy selection. The transition matrix and fixation probabilities were calculated for a population of size Z, considering the intensity of selection (β). The following analyses were conducted:

- **Gradient of Selection:** The gradient of selection quantifies the evolutionary advantage of each strategy under given conditions.
- **Stationary Distribution:** The stationary distribution of strategies was calculated from the transition matrix, providing insights into long-term population behavior.

Visualization and Results

Figures were generated to visualize the results:

- **Invasion Diagrams:** The invasion diagram (e.g., Figure 3a) illustrates the fixation probabilities and stationary distribution of strategies.
- **Avoidance Frequencies:** Contour plots show the avoidance frequency of cooperation and commitment under varying values of ϵ and r, providing insights into parameter sensitivity.

All visualizations were produced using Matplotlib, ensuring clarity and consistency.

References to Literature

The implementation and methods follow the theoretical framework described in [1], which analyzes commitment strategies in PGGs using stochastic evolutionary game theory. Additional details on the dynamics of commitment strategies and their role in promoting cooperation are outlined in the same study.

We utilized Large Language Models to enhance our understanding of Python libraries, such as find more faster and easier documentation of NumPy,Latex methods and obtaining practical examples for their usage. Additionally, LLMs were employed to improve the structure of our report„especially what to put in practical part and method as well as in refining phrasal structure and translations. Copilot auto completion was enabled but do not write use relevant code, only repetition code.

5 RESULTS

In this section, we will analyze the figures one by one, comparing the original results presented in the paper with our experimental findings. This approach allows us to identify consistencies and discrepancies, offering a deeper understanding of how parameter variations and methodological choices impact the dynamics of public goods games.

5.1 Constraints on the evolutionary viability of COMPF

The first set of results focuses on the comparison between the original paper's findings Figure 5(a) and our simulations under varying values of the parameter *c*. In the original paper, the distribution of strategies across participants showed a clear differentiation, with 16% adopting the unconditional cooperation strategy (C) and consistent differences of approximately 2% between the commitment strategies *F3*, *F4*, and *F5*. Our reproduced results demonstrate notable differences depending on the chosen value of *c*, as outlined below.

- For $c = 0.65$, the unconditional cooperation strategy (C) achieves a proportion of 13%, which is relatively close to the 16% reported in the original paper. However, the differentiation between *F3*, *F4*, and *F5* is minimal, with their proportions being almost equal. This suggests that while lower values of *c* can approximate the level of cooperation observed, they fail to replicate the structured separation between these strategies.
- For $c = 1.5$, the proportion of participants adopting C drops to 7%, significantly below the expected 16%. Nevertheless, the separation between *F3*, *F4*, and *F5* is more pronounced, closely aligning with the original findings. This indicates that higher values of *c* improve the differentiation among commitment strategies at the cost of reduced overall cooperation.
- Additionally, for c <0.61, we observe the emergence of the *FREE* strategy, where participants defect unless proposed a commitment. This behavior is absent in the original paper and highlights the influence of low cooperation costs in promoting opportunistic strategies.

The diagram of invasion shows that the population predominantly migrates toward COMPF strategies, with a particular focus on higher participation levels, though F=3 is the most frequent. It also illustrates that FAKE and FREE strategies are reduced to

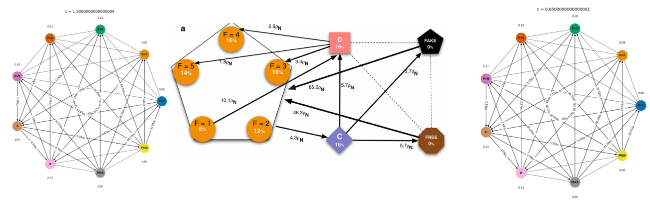

Figure 5: Invasion diagram with different c and the one from the article place in the middle

nearly zero. However, Defectors and Contributors remain at approximately equal levels, each representing about one-fifth of the population. The COMPF strategies play a crucial role in promoting cooperation by suppressing FAKE strategies, thereby improving overall cooperative behavior. Yet, the persistence of Defectors and Contributors highlights potential areas for further refinement.

5.2 Cooperation and Commitment level

The second set of results are from Figure 3b and Figure 3c 7. It highlight the impact of the arrangement cost (ε) and the multiplication factor (r) on the avoidance frequency of cooperation and commitment. These frequencies are derived from the combined strategies observed in the simulations. A notable aspect is the strong correlation between lower arrangement costs and higher levels of cooperation and commitment. When ε is minimal, both figures demonstrate peak levels of cooperation and commitment. Similarly, an increase in the multiplication factor r leads to a steady rise in these levels, emphasizing the role of resource enhancement in fostering cooperative behavior. However, discrepancies emerge when comparing the original paper's figures7 to our simulations6. These differences can be attributed to the following factors:

- **Propagation of Initial Errors:** As observed in Figure 3a, minor discrepancies in the initial strategy distributions can propagate through subsequent simulations, influencing the overall trends.
- **Stochastic Nature of the Model:** The stochastic framework introduces variability, amplifying minor inconsistencies during simulations. This effect is particularly evident in regions with intermediate ε and r values, where transitions appear smoother in our results.
- **Fixed Parameters:** While our parameters ($N = 5$, $Z = 100$, $\delta = 2$, $\beta = 0.25$) align with those of the original paper, minor differences in initial conditions, such as the distribution of strategies, may account for the observed gradients.

Despite these differences, the general trends remain consistent: reducing ε and increasing r enhances cooperation and commitment levels. These findings reinforce the importance of minimizing the costs of arranging commitments to sustain collective action

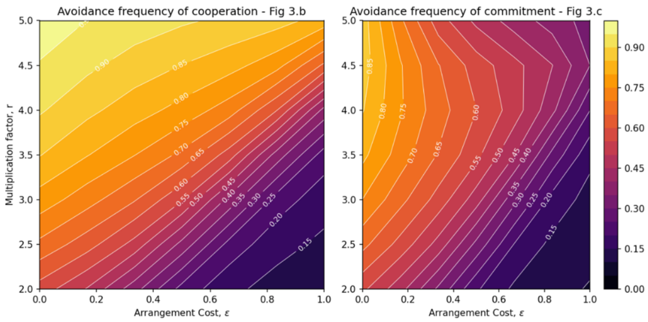

Figure 6: Our figures 3b and 3c

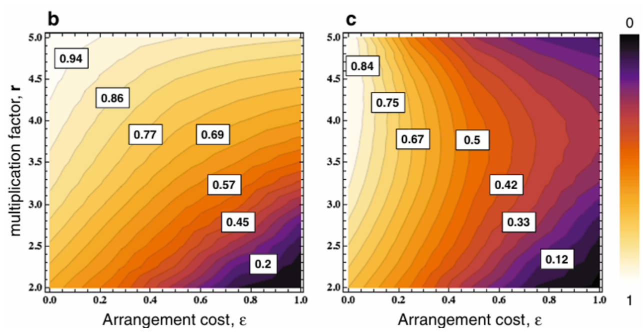

Figure 7: Figures 3b and 3c from the paper

5.3 Emergence of cooperation and sufficient participation levels

Figure 4a and Figure 4b 8 explore the average individual payoff in the population under varying arrangement costs (ε) and multiplication factors (r). These results provide insights into how cooperation dynamics influence individual and collective welfare. In both the simulated and original figures, the trends are consistent: decreasing ε and increasing result in higher average payoffs. This highlights the importance of reducing commitment costs to enhance individual and group welfare. The simulated Figure 4a 8captures these trends well, with maximum payoffs close to the original values (approximately 3.63 vs. 3.4). However, the simulated contours display smoother transitions, likely due to stochastic variability in the model. Figure 4b 8 compares the average individual payoff in the population of nine strategies to that of the simpler C-D strategies. While the overall trends align, discrepancies are observed for ε =3, where the payoff remains below 1.5 in the simulations compared to 2 in the original. These differences may result from:

- **Error Propagation:** Discrepancies in earlier figures, such as Figure 3a, likely influence the dynamics observed here.
- **Stochastic Effects:** Variability in the simulation could amplify differences, particularly for intermediate values of

ε

Despite these differences, general findings remain robust: reducing ε and increasing r enhance individual welfare, underscoring the value of minimizing commitment costs to sustain cooperative dynamics.

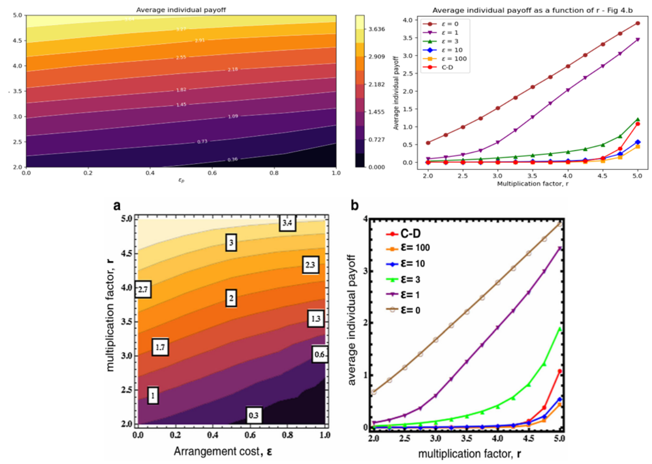

Figure 8: Above our figures, under figures from the paper

5.4 Otpimality of Participation Level

Figure 5a and Figure 5b 10analyse the average level of commitment (COMPF) and the optimal commitment thresholds (F*) under varying arrangement costs (ε) and multiplication factors (r). In Figure 5a 9, the trends align well with the original results, showing an increase in commitment levels as r rises and ε decreases. However, a noticeable discrepancy is observed for r=2.8, where our simulations reach a commitment level of 3.6, compared to r=2.3 in the original figure. This difference likely stems from initial distribution errors propagated from Figure 3a, as well as the stochastic nature of the model, which can amplify differences in intermediate ranges of r. Despite this, the alignment at r=4.4 (commitment level of 3.2) suggests that the overall dynamics are well captured. In Figure 5b 9, the discrepancies are more pronounced. The $F^* = 2$ region is nearly absent in our results, and the $F^* = 4$ region occupies a larger range (2.3 to 3.6) compared to the original (2.5 to 3.4). This misalignment impacts the interpretation of optimal thresholds, as the boundaries between F* regions are shifted. These differences can be attributed to:

- **Initial Strategy Distributions:** Errors in Figure 3a propagate through the model, affecting the representation of F* thresholds.
- **Choice of c=1.4:** While this parameter highlights differences between F*, it diverges from the initial configuration of the original paper, influencing the results.
- **Stochastic Effects:** Variability in the model amplifies discrepancies, particularly for intermediate ranges of r.

Despite these differences, the general trends remain consistent: higher r and lower ε result in increased commitment levels and higher optimal thresholds.

Figure 5c and Figure 5d 12 present the optimal commitment thresholds (F*) for fixed multiplication factors (r=2.5 and r=4.0), as functions of the arrangement cost (ε) and compensation cost (δ). These results highlight the robustness of the model in capturing the relationships between key parameters and F*. In Figure 5c 14, the general structure is preserved, with $F^* = 4$ and $F^* = 5$ regions clearly delineated. However, a notable discrepancy arises in the transition

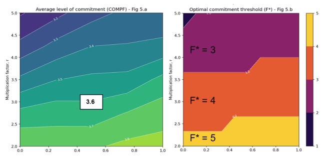

Figure 9: Our Figure 5 a and b

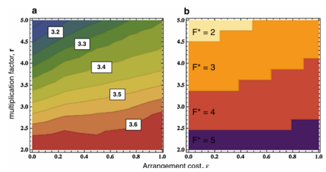

Figure 10: Figure 5 a and b from the paper

point: our simulation shows the shift at $\varepsilon = 0.75$, compared to $\varepsilon = 1.0$ in the original paper. Additionally, a minor irregularity in the matrix produces an outlier (F* = 4) in an incorrect region. Despite this, the key property is retained: the compensation cost (δ) does not influence the optimal F*. Similarly, in Figure 5d 14, the regions F* = 3 and F* = 4 are consistent with the original results, but the transition point is again shifted to $\varepsilon = 0.75$. This systematic shift suggests that the parameter configuration (c=1.4, $\delta = 6$) or initial strategy distributions might have introduced slight deviations. Overall, the simulations confirm the primary insights:

- The optimal F* increases with ε and decreases with r, consistent with the harsher public goods game dilemma.
- The compensation cost (δ) does not affect the value of F*, validating the robustness of the model.

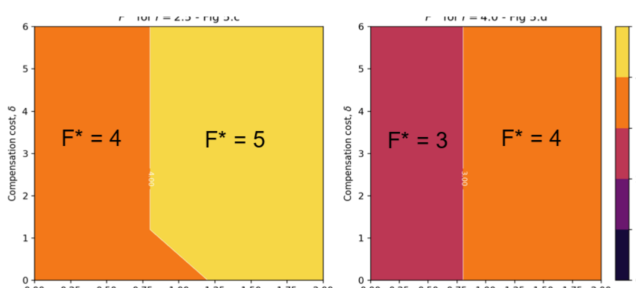

Figure 11: Our Figure 5 c and d

5.5 Lenience in long-term commitments

Figure 6a and Figure 6b analyse the average of F' in the population among all commitment strategies and the F' with the highest frequency. The parameters that vary are R and r. R is the number of rounds during which the commitment lasts. The results show that one of the most dominant F' are strategies with an F' = 1. Furthermore, as the number of rounds increases, a slower strategy is favoured. This contradicts what the original paper. There must have been an error when implementing the code. Unlike the other codes for the figures, the stochdynamics class was used to calculate the transition matrix. This choice was made because the calculation time for the transition matrix when using an abstract class became far too long in view of its matrix, which had a size of 29x237336.

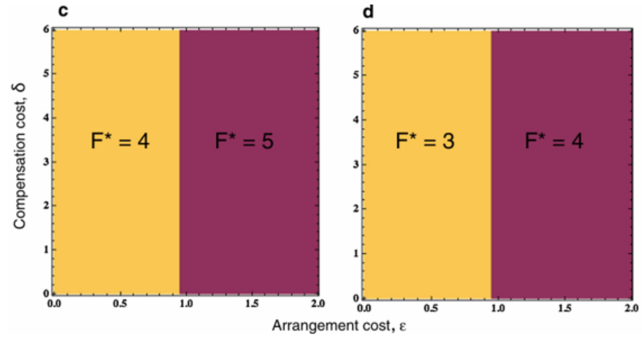

Figure 12: Figure 5 a and b from the paper

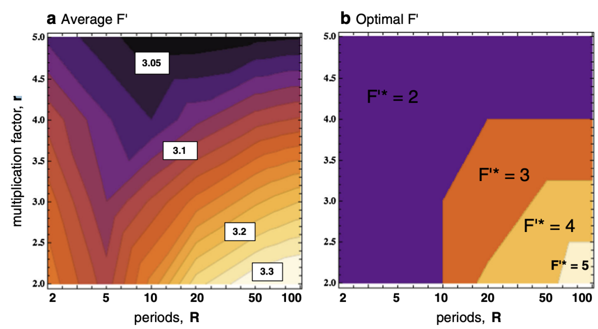

Figure 13: Paper Figure 6a and 6b

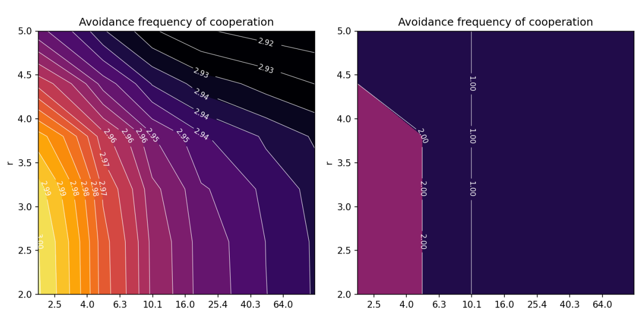

Figure 14: Our Figure 6 a and b

6 DISCUSSION

The study conducted by The Anh Han, Luís Moniz Pereira, and Tom Lenaerts offers significant insights into the role of commitment strategies and participation thresholds in fostering cooperative behavior in Public Goods Games (PGGs). Using evolutionary game theory models, the authors demonstrate how prior commitments and the cost structures surrounding these commitments impact cooperation dynamics.

The study's findings suggest that low costs for arranging commitments relative to the cost of cooperation encourage widespread cooperative behavior. An optimal participation threshold naturally emerges, influenced by the dilemma's severity and commitment costs. Interestingly, while short-term agreements allow leniency, long-term interactions demand stricter adherence to commitments. However, increasing compensation costs for breaking commitments only enhances cooperation up to a certain limit, beyond which additional penalties become ineffective.

While these findings align with theoretical predictions, several points merit further consideration:

- **Model Sensitivity to Parameters:** The results show significant sensitivity to parameters such as the cost of cooperation (c) and the cost of arranging commitments (ε). For instance, using $c = 1.4$ highlighted strategy differences but also introduced deviations from the original setup.
- **Impact of Initial Conditions:** Discrepancies in Figure 3a suggest that errors in initial distributions propagate through simulations. Precise calibration of initial conditions is essential for consistency.
- **Stochastic Variability:** The stochastic nature of the model introduces variability, particularly in intermediate parameter ranges. Addressing this may require increasing simulation iterations or averaging over multiple runs.
- **Compensation Cost Dynamics:** Analysis of Figures 5c and 5d shows that while compensation costs (δ) do not alter the optimal commitment threshold (F^*), transitions between thresholds differ slightly from the original study. This implies a need for more precise modeling of the relationship between ε and F^*.

In terms of future implementations, the development of adaptive algorithms that dynamically adjust parameters based on real-time feedback could advance the study of public goods games. We could also adding more strategies to approximate better th real world.

7 CONCLUSIONS

This study successfully reproduces and extends key findings from the original research on public goods games, shedding light on the interplay between cooperation, commitment, and strategy thresholds. Our simulations reaffirm the core dynamics of the original model while highlighting areas for further refinement.

A significant contribution of our work lies in identifying the sensitivity of the model to initial conditions and parameter choices, such as c. This factor influenced the alignment of our results with the original paper, emphasizing the need for precise calibration to minimize discrepancies. Furthermore, the stochastic nature of the simulations introduced variability, particularly in intermediate parameter ranges, which warrants further exploration to enhance robustness.

Our findings underscore the importance of reducing the costs of arranging commitments (ε) and optimizing multiplication factors (r) to sustain cooperation in public goods games. Additionally, the invariance of the optimal commitment threshold (F^*) to the compensation cost (δ) demonstrates the robustness of the model across varying conditions.

In conclusion, our work builds on the theoretical foundation laid by the original paper, providing both validation and new perspectives on the dynamics of public goods games. By addressing the challenges identified, we pave the way for further advancements in the modeling and application of cooperative systems.

7.1 Acknowledge

The code is in the following github link : https://github.com/Teytey2002/Projet_Learning.git

The file Project.ipynb in were the figure 3-5 are made and

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