DSP Exam Questions

An Open-Source Summary

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Contents

Part I.
First Document

1. January 30 2024

1.1. Question 1

1. Chapter-2 p.40 ("Polyphase decomposition: Example…"): Explain how the presented equations are exploited in general oversampled filter banks (=Chapters 12-13).

2. Chapter-3 p.19 ("3. Least squares estimation…"). Explain the meaning of the parameters K and L, and how these have to be set. What procedure does Matlab use to solve such a least squares estimation problem?

3. Chapter-4 p.10 ("This is a 'Quadratic Optimization'…"): Provide similar formulas for Type-2 linear phase FIR filter design (p.8), and explain all the symbols used in these formulas.

4. Chapter-5 p.16 ("From (*) (p.12), it follows that…"): Generalize the given formulas for the case with 3 power complementary filters (as on p.20).

5. Chapter-6 p.33: Provide a justification for the 'lumping' of noise sources (as illustrated in subsequent pages) that is explicitly based on the given formulas (with 'DC-gain' and 'noise-gain').

1.2. Question 2

1. Chapter-7 p.24 ("MMSE cost function can be expanded as…"): How does the Wiener filter formula ($w_{WF} = (X_{uu})^{-1} X_{du}$) and/or its components (X_{uu} and X_{du}) change in the case of a 'linear combiner' problem (as on p.20)? What is the computational complexity (to solve the resulting set of equations) in this case (as on p.27)?

2. Chapter-8: Explain in your own words how the characteristics of the filter input signal (u_k) influence the behavior of the LMS algorithm? What is then the 'ideal' filter input signal in this respect?

3. Chapter-9 p.35 ("4-by-4 example…"): If the adaptive filter input signal and the desired output signal have different 'dynamics' (for instance if the characteristics of one are very stationary, while the characteristics of the other are very non-stationary), would it be possible/useful to apply two different exponential weighting factors (a first one in the part corresponding to the input signal and a second one in the part corresponding to the desired output signal of the signal flow graph)?

4. Chapter-10 p.19 ("Theorem…"): The graph also shows an equality for the accumulated product of the cosines of the rotation angles. Provide an explanation for this equality.

5. In Chapter-11 p.24 ("relevant sub-problem is…"): Explain why the framed triangular matrix and corresponding right-hand side vector in the first equation correspond to the formulas that are added in the second equation.

1.3. Question 3

1. Chapter-12 p.36 ("A solution is as follows…"): Prove that the conditions Fo(z)=H1(-z) and F1(z)=-Ho(-z) indeed lead to alias-free operation. How is the frequency response of Fo(z) and F1(z) then related to the frequency response of Ho(z) and H1(z)?

2. Chapter-13 p.12 ("(ii) Necessary & sufficient condition…"): Explain in your own words the statement "hence $pr(z) = $ pure delay $= z - \delta$, and all other $pn(z) = 0$". Explain how this then leads to the next formula.

3. Chapter-14 p.13 ("Conclusion: economy in…"): Explain in your own words the statement "N filters for the price of 1".

4. Chapter-14 p.27 ("Example-1: Define B(z4)…"):

a) Specify B(z) for the case where N=5 and D=2.

b) Derive the conditions for perfect reconstruction and specify when the resulting set of equations can be solved.
   cfr. slide 31 for an example

2. January 11 2024

2.1. Question 1

1. Chapter-2 p36 ("Example: HP anti-aliasing…"): Explain the relevance of the presented operations in the filter bank context (=Chapters 12-13-14).

2. Chapter-3 p.29 ("At the Rx, throw away L samples corresponding…"). Rewrite the formula for the case where the order of the FIR channel is larger than the cyclic prefix length.

3. Chapter-4 ("Filter Design"): Explain how the filter phase response is controlled in IIR filter design, and compare this to linear-phase FIR filter design.

4. Chapter-5 p.30 ("Derivation similar to p.22…"): Explain why the highlighted element in the first formula has to be a zero (unlike in p.21 and p.22).

5. Chapter-6: Draw a "parallel realization" of $H(z) = (1 + az - 1) - 1 + (1 + bz - 1) - 1$. Insert all relevant quantization noise sources and compute the corresponding noise transfer functions. Can some of these noise sources be lumped into an equivalent noise source? Why (not)?

2.2. Question 2

1. Chapter-7 p.24 ("MMSE cost function can be expanded as…"): How does the Wiener filter formula ($wWF = (Xuu) - 1Xdu$) and/or its components (Xuu and Xdu) change in the case of a multi-channel FIR problem (as on p.19)?

2. Chapter-8: Explain how the LMS algorithm can be viewed as an RLS algorithm with a specific substitution for the input signal correlation matrix. Based on this link with RLS, provide an intuitive explanation for the statement that the convergence of the LMS depends on the correlation matrix eigenvalue spread.

3. Chapter-9 p.38 ("Residual extraction…"): Consider the case where uk is an all-zero vector and dk is non-zero (and R[k] is full-rank). What would be the corresponding rotation angles and epsilon, and hence the a posteriori and a priori residual.

4. Chapter-10 p.20 ("The main trick…"): Redraw the signal flow graph when the "main trick" is used to remove the column with R15, R25, … Define the relevant epsilon-signals in the signal flow graph (with subscripts & superscripts).

5. In Chapter-11 p.25 ("Recursive Square-Root…"): Could residual extraction (cfr. Chapter 9) be added to this algorithm (would it also require only the "lower-right/lower part" as stated on p.23)? What exactly would be the meaning of the extracted residuals?

2.3. Question 3

1. Chapter-12 p.12 ("Noise reduction…"): If D=4 (instead of D=3) and if the Gi's are not all equal to 1, what would be a condition for alias-free operation (a general formula is sufficient here) and what would be the resulting (linear) "distortion function"?

2. Chapter-13 p.11 ("This can be verified…"). Provide an equivalent verification where in the first step R(z)E(z) is swapped with the upsampling (instead of the downsampling).

3. Chapter-13 p.31 ("Given E(z)…"). Consider the case with N=4, D=1 and LE=3. Construct a (simple) example with transfer functions Hi(z) and Fi(z) that provide perfect reconstruction.

4. Chapter-14 p.27 ("Example-1: Define B(z4)…"):

a) Specify B(z) for the case where N=7 and D=4.

b) Provide the corresponding proof (similar to the proof on p.27) (with explanation in words for non-trivial steps) that a 7-channel DFT-modulated filter bank is indeed obtained with this B(z).

3. January 31 2023 (example exam)

3.1. Question 1

1. **Chapter-3**: Specify the computational complexity (number of multiplications per second) of the DMT receiver operations (only those discussed in Chapter-3), provide (approximate) formulas that contain the main DMT parameters (N, L, symbol rate, ...).

2. **Chapter-4 p.8** ("Linear Phase FIR Filters – Type 1...Type 2..."). Explain why 'modulating' a type-3 filter results in a Type-3 filter. Illustrate the statement "BP<->BP" by sketching the filter frequency responses. Are the two 'BPs' exactly the same?

3. **Chapter-5 p.18** ("Repeated application results in 'lossless lattice' ..."): Explain the appearance of the multiplication by 'cosq4' and 'sinq4' in this lossless lattice realization.

4. **Chapter-5 p.30** ("Derivation similar to p.22 (overlap-save, similar for overlap-add) ..."): Explain why the highlighted element in the first formula has to be a zero (unlike in p.21 and p.22).

5. **Chapter-6 p.32**

3.2. Question 2

1. **Chapter-7 p.8** ("example: channel identification") and p.14 ("example: channel equalization (training mode)"): Explain and compare these two example applications.

2. **Chapter-8 p.21** ("LMS analysis in a nutshell: Noisy gradients...."): Explain the 'noisy gradients' effect in an acoustic echo cancellation set-up, with a near-end speaker that is sometimes active and sometimes not active.

3. **Chapter-9 p.35** ("4-by-4 example"): Give an intuitive explanation for the fact that exponential weighting can be implemented here only by adding multiplications with l after the delay operations.

4. **Chapter-10 p.13** ("Preliminaries / LS residuals are not changed..."): Explain in your own words how this property is exploited in p.19 ("Theorem").

5. In **Chapter-11 p.24** ("Relevant sub-problem is..."): If the Kalman Filter would also have to compute xk-1|k (next to xk|k and xk+1|k) how would the formulas on the slide have to be adapted?

3.3. Question 3

1. **Chapter-12 p.4 ("What we have in mind is this...")**: In the given block scheme (without any upsampling/down-sampling), what would be the condition for perfect reconstruction? Would a set of power complementary filters provide perfect reconstruction?

> Does someone have an idea for this question? Especially the last part if a set of power complementary functions provides perfect reconstruction, my intuition says yes, but I don't have a good explanation for it. Doesn't seem logical to me, we have to invert the damage done by the H(z)'s, only possible by filter with transfer function equal to 1/H(z)

(which is not power complementary)

2. **Chapter-13 p.18** ("Design Procedure:..."): Explain in your own words how the stability issue mentioned in the last sentence is overcome by having D< N instead of D=N ?

3. **Chapter-13 p.33** ("Example N=4..."): Indicate how the formula changes (number of unknowns versus number of equations) if the order of all the synthesis filters is increased by one (=copy the formula and then highlight where in the matrices additional entries appear).

4. **Chapter-14 p.27** ("Example-1: Define B(z4)..."): Specify B(z) for the case where N=4 and D=3 and provide the corresponding proof that a 4-channel DFT-modulated filter bank is obtained with this B(z) (similar to the proof on p.27 for N=8 and D=4).

5. **Chapter-15 p.12** ("If maximally decimated..."). Provide an interpretation of the last formula by comparison with the DTFT on p.4. (What are the basis functions now? How many basis functions?)

4. January 12 2023 (example exam)

4.1. Question 1

1. **Chapter-4 ("Filter Design"):** Explain how the filter phase response is controlled in IIR filter design, and compare this to FIR filter design.

Design of analog filter -> digital, difficult to control phase response. IIR can never have linear phase response, compared to FIR, easy to control and have certain linear phase response.

For a linear phase filter H(w) should be symmetric or antisimetric. From other course Bessel thomson approximation has a maximally flat group delay.

The phase characteristic is taken into account by including it in the optimisation problem. This is not the case for FIR filters where only the magnute is included in the optimization function.

2. **Chapter-5 p.10 ("Repeated application results in 'lattice form' ... "):** Explain the appearance of the multiplication by 'b0' in the lattice realization.

B0 is preserved through the transformations to lattice structure

3. **Chapter-5 p.28 ("Derivation similar to p.22 (overlap-save, similar for overlap-add) ... "):** Explain why the highlighted element in the first formula has to be a zero (unlike in p.21 and p.22).

Does anyone have an idea? Is it because the coefficient b4 is in the polyphase > components? Why don't we leave out the entire row of 0's here and the u[k-7] > component, seems like it is not contributing?
Maybe something to do with that FFT is used...

Thanks, seems logical!

The extra zeros are required to make sure the resulting circulant matrix is square (FFT structure only valid for circulant matrices).

4. **Chapter-6 p.13 ("Coefficient quantization effect on pole locations / Higher-order systems..."):** Explain in your own words the meaning of the (approximate) equation, and how it leads to the given conclusions.

Gives relation between difference in coefficient value and the quantized coefficient and the difference of the root value and root value after quantization. Closely spaced roots result in an amplification of the error made on the coefficient to the error made on the location of the root, resulting in poor behavior.

5. **Chapter-6 p.35 ("PS: In a direct form realization. . . "):** Explain in detail why it is that all quantization noises can be lumped into e1 and e2. What are the corresponding noise transfer functions?

Transfer functions of separate noise sources are equivalent. Transfer functions are both 1? The upper part is referred to the input and the lower part to the output.

Linearity makes it possible to just sum the noise sources (only + operations in path where noise is generated). Transfer function of e2[k] = 1 and transfer function of e1[k] = IIR filter itself?

4.2. Question 2

1. **Chapter-7 p.19 ("PS: Can generalize FIR filter to 'multi-channel FIR filter'. . . "):** For this specific (3-input) example, provide formulas for the (input auto-)correlation matrix, the cross-correlation vector, and for the Wiener filter.

Combine 3 input vectors into u and three coefficient vectors into w, reduces to the same problem as before. What happens to input correlation matrix and cross correlation vector? Stay also the same?

2. **Chapter-8 p.37 ("compare to p.32-33. . . ."):** Explain in your own words the appearance of the summation, the first formula, and the statement (in p.36) that "D takes the place of L+1".

Replacing the original order of the filter L (thus L+1) coefficients by D polyphase components, which will all have L_D = L+1/D coefficients per polyphase component. D takes the place of L+1 means that originally we would have a block length of L+1, but by writing in polyphase components this is replaced by a block length of D, which can be lower, limiting the introduced delay.

3. **Chapter-9 p.26 ("QRD for LS estimation. . . "):** Explain why the ' * ' in the second formula does not play a role (in the third formula).

These rows in the vector are independent of w and do not contribute to minimizing the w norm of the vector w.r.t. w.

4. **Chapter-10 p.20 ("The main trick. . . "):** Redraw (sketch) the signal flow graph when the "main trick" is used to remove the column with R12. Define the relevant epsilon-signals in the signal flow graph (with subscripts & superscripts).

5. **Chapter-11 p.10 ("PPS: Note that if in p.9 matrix U has only 1 row. . . "):** Explain in your own words, based on the given formulas, how the standard RLS algorithm formulas can be related to a specific linear regression parameter estimation problem with a specific 'initial estimate'.

For a regressor and an error that has a unit covariance matrix, the least squares parameter estimation of a linear MMSE estimator with an initial estimate can be seen as the same estimate of the standard RLS algorithm where the correlation matrix is replaced with the error covariance matrix of the initial estimate.

4.3. Question 3

1. **Chapter-12 p.29 ("Now insert DFT-matrix...")**: Matrix F is said to be a DFT matrix, but more generally can be any square invertible matrix. In general, would it also be possible and meaningful to have a non-square matrix F (with some alternative for the inversion)?

> Yes, this is what we are doing in the case of an oversampled filter bank, with E(z) and R(z) rectangular matrices

2. **Chapter-13 p.12 ("Necessary & sufficient condition for PR is then...")**: Based on the first)formula, provide an expression for the distortion function (function of delta and r).

   $T(z) = z^{-r*z}$-delta ???

   I think T(z) = zˆ-r * pr(zˆ4), so T(z) = zˆ-r * zˆ-4*delta

   Red is correct <3

   But isnt R and E after downsample? How is it zˆ-4? -> R(z)E(z) = zˆ-delta I_N after upsampling to fourth power. This is for r = 0 here, so extra zˆ-r required for general case.

3. **Chapter-14 p.13 ("Conclusion: economy in...")**: Explain in your own words the statement "N filters for the price of 1".

> Filter is implemented only once, then modulated to higher frequencies by taking the IFFT.

4. **Chapter-14 p.27 ("Example-1: Define B(z4)...")**: Specify B(z) for the case where N=8 and D=6 and provide the corresponding proof that an 8-channel DFT-modulated filter bank is obtained with this B(z) (similar to the proof on p.27 for N=8 and D=4).

5. **Chapter-15 p.15 ("Example: N=4, d=2...'")**: Explain/derive the "minimum norm solution" formulas. What is the relevance of such a "minimum norm solution"?

5. 26 January 2021

Question 1

1. Chapter-3 p.26 ('To ensure the time-domain signal is real-valued, have to choose...'): Why is a real-valued time-domain signal needed? How does this choice modify the receiver structure?

$(W_{WF} = (X_{uu})^{-1}.X_{du}$

At receiver: only half of values used, redundancy

2. Chapter-4 p.12: Explain by means of a few formulas how the minimization problem can be solved using QR-decomposition, instead computing Qˆ-1.p.
   See 7 january 2021

3. Chapter-5: Consider a linear phase FIR filter. Is it possible to use a (LPC) lattice realization for this filter? A lossless lattice realization?
   No, 26 jan 2017 Q1.4
   What is the reason that lossless lattice realization is not possible?

4. Chapter-5 p.13: Explain the relevance of the Schur-Cohn stability test for the derivation of the lattice-ladder realization.
   IIR Lattice ladder realization: Ki's are computed from the coefficients of the denominator$\rightarrow$ this time the stability test can be used to see if the transfer function is stable

5. Chapter-6 p.35: Explain why it is that all quantization noises can be lumped into e1 and e2. What are the corresponding noise transfer functions?
   11 january 2019, Q1.2

Question 2

1. Chapter-7 p.40: Provide an intuitive explanation for the unrealizable Wiener filter formula for the two considered cases.

2. Chapter-8 p.19: Explain the 'noisy gradients' effect in an acoustic echo cancellation set-up, with a near-end speaker that is sometimes active and sometimes not active.
   F

3. Chapter-9 p.26: In this structure with 'residual extraction', where are the actual FIR filter coefficients, i.e. the estimated model for the echo path?

4. Chapter-9 p.35: Redraw (sketch!) the (relevant parts of the) signal flow graph when the 'main trick' is used to remove the column with R15,…R45. Define the relevant epsilon-signals to the signal flow graph (with subscripts & superscripts).

5. Chapter-10 p.19: Explain the statement "is seen to require only the lower right/lower part".

Question 3

1. Chapter-11 p.39: Explain why the R(zD) is a D-by-N matrix (and not an N-by-N or D-by-D or N-by-D matrix).
   Otherwise dimensions not compatible and PR criterium not satisfied

2. Chapter-12 p.7: Explain (in +/- half a page) the design procedure mentioned at the bottom of the slide.

3. Chapter-12 p.31: Specify B(z) for the case where N=5 and D=4.

4. Chapter-13 p.18: Explain the statement 'let B(z) take the place of distortion function T(z)'. Will there be any distortion in this case?

5. In Chapter-14 p.12, explain the meaning of the reconstruction formula (at the bottom of the slide) and compared to the reconstruction formula of p.4.
   Compare with 7 january 2021, Q3.5

6. 7 January 2021 (Written exam due to Covid-19)

"

Question 1

1. Chapter-3 slide 19: What is the meaning of this formula? Provide a suitable solution strategy to solve the minimization problem. Determine a good approximation by minimizing the distance between the exact and approximate solution.

Strategy? Iterative?

In this slide we see the least squares estimation that we can use for the channel estimation. We can for example send a training sequence every once in a while and thus we know what the expected signal would be. This expected signal, the training sequence that has been transmitted, form the matrix with the x-components, further referred to as the X-matrix

The signal that is actually received is represented by the y-vector, containing the y-components. We then want to find the h-vector that minimizes the difference y-Xh. This h-vector will contain the estimated channel coefficients. We find the best estimation for the actual channel coefficients by calculating the square of the two-norm of the difference y-Xh and then find the values in the h-vector for which this is minimal. To solve this we can write this in a matrix form of the given values and convert it into the following least squares problem: Xh=y. We can now use the least squares method to find a solution:

1) Compute the matrix X^TX and the vector X^Ty

2) Form the augmented matrix for the matrix equation $X^{TXh=X}Ty$ and row reduce

3)This equation is always consistent and any solution $\hat{h}$ is a least squares solution. In Matlab there are built in functions that can calculate this least squares problem for us.

I think in practice done with the QR decomposition and backsubstitution.

2. Chapter-4 slide 12: Give an alternative strategy to $Q^{-1}*p$ to solve the minimization problem. Provide formulas. I was thinking maybe if you take $c^T = A$, d = x and Gd=y than you have an similar problem as question 1? See chapter 9 QRD for LS

   Based on the explanation above (do notice the matlab notation): $Gd = [Gd(\omega_1), Gd(\omega_2), ...], A = [c^t(\omega_1); c^t(\omega_2); ...]$, x remains x. Now: Ax = Gd can be solved using leased .squares. I don't know how to add the weighted function tho. Note that this can only be done with a 'discretized' quadratic optimization problem. This is not compatible with the problem of slide 10!

3. Compare the complexity of the FIR realizations based on the multiplications that we have seen in the course.

   1. Direct -> L+1 multiplications and L additions

   2. Transposed direct -> L+1 multiplications and L additions

   3. Lattice (lpc lattice) -> 2L+1 multiplications and 2L additions

   4. Lossless lattice (2 outputs) -> 4L+2 multiplications and 2L additions

   5. Lossless lattice (N outputs) -> 2NL+N multiplications and 2(N-1)L additions

   6. Frequency domain implementation O(log(L))

   7. Frequency domain implementation using D-fold polyphase decomposition O(log(D))

4. Chapter-5 slide 14: Why do we need scaling? Is the scaling unique? ? $\rightarrow$ has to do with the sum being one on the next slide and both number will be positive.

   Scaling needed if the magnitude of the H(z) itself is already larger than 1, to make this power complementarity possible?

I don't think the scaling is unique as there are many scaling factors to let the magnitude of H(z) be below 1.

5. Chapter-6 slide 34: Explain why the noise sources are equivalent up to a delay. Why is this delay not a problem to combine them together? Every component (multiplier/adder) has a noise source, the difference between a noise source at adder 1 (most left) and adder 2 is 1 delay element. This will also be visible in the noise transfer functions. You can rewrite/adapt the transfer functions to make it like the source is located at another place. If you do this for all the noise sources to 1 specific point, you can add the transfer functions. (I would add that a delay is a linear operation, allowing each linearly shifted noise signal to be added to the others in the leftmost node and keeping and LTI filter) ← the delay identity has the extra requirement for the noise sources to be *ergodic*. Linearity is required for the additions and multiplications, though.

   Quantization noise is assumed to be truly random, so a delay will not influence this noise contribution? There is no correlation between successive errors, so delay won't influence these error contributions. Correct??

   I think you also need to mention that the calculation of the effect on the output signal of a noise source does not change by lumping them together if the mean and covariance are correctly adapted.

Question 2

1. Chapter-7 slide 38: Explain the statement and the condition for the AEC. (Irr. error = variance of near signal) Even the unrealizable Wiener coefficients/transfer function cannot make the desired signal-input signal cross correlation equal to the desired signal autocorrelation due to the noise being added after the compensated transfer function in the model. Thus the noise autocorrelation is not accounted for in the model and cannot be compensated => forms the irreducible error.

2. Chapter-8 slide 34: What is a rank-1 update? How does it define the complexity?

   All vectors can be written as a scaling of the first one => matrix product in the second term of the weight update equation simplified to a concatenation of scaled vectors = $O(L^2)$ instead of a standard matrix product which would be $O(L^3)$

3. Chapter-9 slide 26: What is the relevance of error residuals for AEC(acoustic error correction)? If there is no error remaining, this means that the filter properly represents the "channel" (acoustics, echo's, …). So how lower the error, how better. Isn't the residual the voice of the speaker on the other side, instead of your echo (in an AEC scenario)? (i.e. the part you actually want to *keep*) -> yes, I agree -> no, I don't agree I agree up to a certain part. I think it will be your voice of the speaker on the other side so the part you want to keep + the error which you can't reduce down. (course 2023: chapter 9: slide 40 it says it's the near-end signal + residual echo) i agree with this

4. Chapter-9 slide 32: Explain the property and explain why it's useful for the derivation. Changing the input does not change the residual output. Useful? Check slide 35 and the way the notations to the epsilon signals are done.

5. Chapter-10 slide 20: How are the indicated parts of the first formula identified?
   Course text on adaptive filters, page 136:
   $$x_{LS} = (A^T A)^{-1} \cdot A^T b.$$

Question 3

1. Chapter-11 slide 34: Why is the polyphase decomposition of E(z) and R(z) per-row and per-column?

2. Chapter-12 slide 11: Explain DD(L_E + L_R + 1) and show equations.
   Every entry in R(z)*E(z) is of order LE+LR, thus has LE+LR+1 coefficients. The dimension of the identity matrix is D, so there D*D equations for the polynomials with each LE+LR+1 coefficients, thus there are in total D*D(LE+LR+1) equations.
   Equations are: each element in $R(z) * E(z) = z - \delta$ if on the main diagonal, and 0 otherwise.

3. Chapter-12 slide 25: Why does every En(z) need to be unimodular and FIR to have E(z) modular and FIR?
   $RN - 1 - n(z) = z - \delta/En(z)$ (from slide 23). In order for RN-1-n(z) to be FIR, En(z) can only have 1 term in z, thus every En(z) has to be unimodular

4. Chapter-13 slide 7: Rewrite the selection matrix [0_4x4 I_4x4] into [I_4x4 0_4x4]. What does this mean for the Bi's?
   The bottom 4 rows and the top 4 rows in the circulant matrix on slide 7 will be swapped. The Bi's are then calculated as [B0, B1, ..., B7]T = F [b0, 0, 0, 0, 0, b3, b2, b1]T (from the property on slide 5)

5. Chapter-14 slide 24: Explain the reconstruction formula and compare it to the formula on p4. Why is x0 a separate term?
   Slide 25: Reconstruction formula may be viewed as an expansion of u[k], using a set of basis functions (infinitely many) (they are weighted + shifted versions of IR synthesis filters)
   x0 is a different term because f0 has the same upsampling factor as f1
   Additionally: if the x0 part wasn't separate, the summation of the separate subbands wouldn't cover the entire spectrum.

7. 10 August 2020

Question 1

1. Explain how WLS for FIR filter design reduces the degrees of freedom when imposing a linear phase requirement.
   Due to the linear phase requirement we know the H(w) is symmetric. The d-coefficients are related to the IR and thanks to the symmetry, there are roughly half as many d-coefficients as H's (see also slide 6). When we take a look at the optimization criterion, we see it's an overdetermined system. Is Symmetry a requirement for linear phase? Either symmetry or antisymmetric, but both can be used to reduce the degrees of freedom
   <f

2. Can QRD be used in WLS FIR filter design? How?
   Chapter 9, QRD for LS estimation? The solution to the discretised optimisation problem (ch4, slide 12) is xopt=A^-1 * p (xopt = Q^-1 * p on the slide, here replaced with A to avoid confusion), where xopt contains the coefficients of the IR such that the distance to the desired IR is minimised. The set of equations to this problem can be numerically solved more efficiently with QRD on matrix A: A=Q*R, where Q is orthogonal so Q^T*Q=I and Q^{-1=Q}T and R is upper triangle so R^-1 is easy to compute. The relevant problem can be written as min_x ||A*x-p||^2. Since multiplying with Q^T is an orthogonal transformation (which preserves norm) min_x ||Q^T *(A*x-p)||^2 is an equivalent problem (same as ch9, slide 12). So Q^{T*A*xopt=Q}T*.Q*R*xopt=R*xopt=phat -> xopt=R^-1*phat (Q^T*p=phat), which is very easy to compute What is the point? Q*x_opt = p is a determined system. There is no error because the error is left in mu.

3. When WLS FIR filter design is used for filters with an arbitrary required phase response, how does the design

procedure change? Give relevant formulas.
Is this a question related to question 2 or the first one?

Fir filters can also have arbitrary phase responses, if they are not symmetric, thus it is not necessary to change it into an IIR filter. When designing the filter, we can no longer use the symmetric property, thus all coefficients have to be determined by the LS calculations, rather than only half of them. The phase response will have to be included in the objective function, as on slide 4-9. The procedure does not change by a lot, except that the transfer functions within the squared operation will now be complex.

If the impulse response is not symmetric, the phase response will not be linear. Causality only means that h[k<0] = 0.

Alternatively, use windowed design

Question 2

1. For overdetermined Ax=b, LS is x_LS=$(A^{T*A})(-1) A^{T}*b$, manipulate this to justify back substitution in QRD RLS.
   The given formula is the original equation of the LS solution. To achieve better numerical results another representation is used to calculate the solution.
   Algebraic substitution: A = Q_tilde*R => AR^{-1} = Q_tilde
   $(A^{T*A})(-1) A^{T}*b$
   = $(A^{T*A})(-1) (Q\_tilde*R)^{T}*b = (A^{T*A})(-1) R^{T Q\_tilde}T*b$
   Bring the $R^T = (R^T)(-1)^{(-1)} = (R^{(-1)})T^{(-1)}$ in the inverse
   = $(R^{(-1)})T*A^{T*A}(-1) *Q\_tilde^{T}*b$
   Merge the two transposes together
   = $((A*R^{(-1)})T*A)^{(-1)*Q\_tilde}T*b$
   Knowing AR^{-1} = Q_tilde
   = $(Q\_tilde^{T*A})(-1)*Q\_tilde^{T}*b$
   This is the same form as with the triangular backsubstitution on slide 12.

2. How is Chapter-9, slide 32 used in derivation of QRD LSL? See january 24 2020: Q2.1

3. Chapter-9, slide 33, the set of input signals [u[k] u[k-1] u[k-2] u[k-3] u[k-4]]
   is extended to [u[k] u[k-1] u[k-2] u[k-3] u[k-4] v[k]] with v[k] independent of u[k], is QRD LSL still possible? Sketch a block scheme, omit details.
   I think this is possible. Why the 'independent'? How can it be added? As the epsilons are a prediction based on the input signals
   What do you think? In chapter 10, slide 22 he says that "The square root RLS algorithm form before is a special case of the square root Kalman filter algorithm (for Vk=0), so I guess if Vk isn't equal to zero like in this question, you'd end up with the square-root-Kalman filter? Does anyone think that's indeed what he's referring to, I'm really not sure…
   I think is is more a case like on the screenshots added under "24 januari 2020", Q2.3

Question 3

1. For maximally decimated filter banks, provide an intuitive explanation for perfect reconstruction despite aliasing in each channel.
   In a maximally decimated filter bank is D=N. The input is split into several frequency bands using the analysis filters. These are all separated and decimated which results in a stretched spectrum. When subband processing

is done, these channels are compressed using upsampling. The copies are also present and these can be filtered out afterwards using a synthesis filter (interpolation filter, same band as the original). When the separate bands are combined this results in the original signal. PR is possible because of the synthesis bank, which is designed to remove aliasing effects.

Is this what you expect from this question or should we explain the polyphase decomposition and the alias transfer function + distortion function?

I think just explain chapter 11, slide 23 to 28?

```
1          I think that the explanation is correct and you can complete it with the
              slides 23 to 28
```

2. Compare maximally decimated filter banks with oversampled filter banks. What are the advantages/disadvantages of oversampling?

   Maximally decimated (D=N):

   - Not much design freedom

   - R(z) has to be IIR (FIR no ?) (No it is IIR see slide 11 of Ch 12, L_R would need to be infinite aka IIR, this is also stated on slide 12 at the bottom in case D = N)

   Oversampled (D<N):

   + Easier to design

   + FIR R(z) can always be found

3. Design a DFT modulated filter bank with 4 channels and 3 fold decimation. Give relevant formulas.

   This is an oversampled DFT-modulated FB. Modifying the example of the slides should suffice.

```
1        B(z) \= \[E0(z^4)     z^-1\*E4(Z^4)       z^-2 E8\]
2
3                 \[z^-3\*E9        E1                      z^-1\*E5\]
4
5                 \[z^-2\* E6       z-3\*E10              E3      \]
6
7                 \[z^-1\*E3        z^-2 E7        z^-3\*E11\]
```

Question 4 (about acoustic modem)

1. What is the relevance of channel equalisation in the acoustic modem?

   The signal Xk will be modified by a channel H(z) and if you do the FFT at the receiver you won't have the same signals as at the transmitter due to this channel H(z). To recover the perfect signal (with the noise of course), one has to do the Freq-domain EQualization (FEQ) after having removed the CP and having done the FFT. slide 25 Chapter-3.

   The channel equalization is easy due to the fact that you use a prefix -> makes the matrix H diagonal.

2. What is the relevance of the prefix in OFDM? What are the requirements for this prefix?

The OFDM frame together with the prefix allows for a circular convolution. In the receiver, the L samples corresponding with the prefix are thrown away which results in a circulant matrix. This circulant matrix can then be factorized using IDFT/DFT(H= F^-1 * diag(H0,..,Hn-1) * F). With this, a diagonal matrix is obtained. The resulting output Y is then simply the input X multiplied with some scalar H(all in frequency domain), so the X can be estimated using component wise division. The prefix length needs to be longer than the impulse response of the channel. (slide 31-35')

3. How does one derive a channel model for the acoustic modem? What if one wants to equalise without modelling the channel?

You can send training frames that are known at both sides. You know X and Y, by using the OFDM modulation with CP, it is very easy to derive a channel model. Y_k^(n) = Hn * X_k ^(n) (slide 35,ch 3) which leads to H = Y_k * X_k^(-1) -> Least Square estimation

For the second part of the question, I understand it like this. In order to avoid losing channel resources for training frames, a adaptive filter can be used and you find a channel for each time.

8. 24 januari 2020

Question 1

1. The derivation of the FIR Lossless Lattice Realization (from one given filter transfer function) may involve 'scaling' (Chapter-5, s14). Why is this needed? is the scaling unique?

   In one of the questions from previous years someone mentioned that it is bad to have a big/small H(z). Personally I think this is more the interpretation of the formula. The equation must hold so the sum of both magnitudes should equal one. A square is always positive (unless i^2, but shouldn't matter here as we're taking the magnitude?) and when this value exceeds 1 the equation cannot hold anymore. So we should scale it to make sure it holds (normalize)?

   ^ This sounds logical indeed. I also think the scaling is unique: assume that we have all the coefficients (b0, … , bL) given. We are looking for a scaling factor. The only equation to solve for is the one on ch5, s14:

   1 equation, 1 unknown -> no degrees of freedom

   I don't think the scaling in unique. If you scale H smaller then H tildetilde will just become bigger and vice versa no? (CH5 s15)

   I think that for ch5, s14, the magnitude of H and Htildetilde should be complementary over the whole ω-axis after scaling (so here "ω" is a variable and not a constant)

   On second thought I think the scaling is not unique: As long as $H(z) * H(z^{-1}) < 1$, the formula (*) (ch5, s14) will ensure the coefficients of Htildetilde to be complementary.

So if H(z)*H(z^-1) < 1, scaling suffices

1. Explain how an FIR lossless lattice realization is derived for 3 given FIR filters that are power complimentary (Chapter-5 s22). Give formulas and signal flow graphs and explain how the required sequence of orthogonal rotations is defined.

2. If the resulting realization (Chapter-5 s22) represents an analysis filter bank, how can a synthesis filter bank be constructed such that perfect reconstruction is achieved for (decimation) D=1?

   Should we execute the rotations in reverse? Again no power loss

   I think they want you to refer to CH12 s9 here, but there maximum decimation is used so i'm not sure

   That's more likely indeed.

   PR: R(z)*E(z)=z^-delta*I_D = 1 if D=1 and delta = 0. So if R(z)=[H_tilde_tilde(z) H_tilde(z) H(z)] (a row vector with all TFs), then E(z) = [H_tilde_tilde(z^-1) H_tilde(z^-1) H(z^-1)]T (a column vector). Then the PR criterion is the same as the lossless criterion. (=indeed slide 9 ch12, paraunitary FBs)

   Should not be this??:

1. Formulate in your own words the Schur-Cohn stability criterion to test the 'stability' of a polynomial.
2. Where does the Schur-Cohn test appear in the context of FIR filter realisation, and what is its relevance there?
3. If the Schur-Cohn test is applied to a linear-phase FIR filter, what will be the outcome? What does this mean in terms of linear-phase FIR realization?
4. Where does the Schur-Cohn test appear in the context of IIR filter realisation, and what is its relevance there?
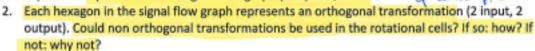
Question 2

1. Explain how the QRD-LSL algorithm is derived from the QRD-RLS algorithm. Explain the meaning of the epsilon notation and the 'main-trick' in the derivation. What is the relevance of the property illustrated on Chapter-9 s32? See previous years

   On this slide, you can observe the invariance of the residuals. It does not change under permutations. This applies to the a posteri and a priori error. This is the key for the derivation.

2. If the adaptive input signal and the desired output signal have different 'dynamics' (for instance if the characteristics of one are very stationary, while the characteristics of the other are very non-stationary), would it be possible/useful to apply two different exponential weighting factors in the desired output signal of the (signal flow graph of) QRD-LSL?

   Two different weighting factors for the desired output signal? That sounds weird. In the SFG the exponential weighting factor is included in the small black box next to the rotation. How would it be possible to apply two different factors? Does this question suggest to adjust the weights for the input AND the output?

   The weighting factor is factored out, so I think this shouldn't be different.

   Cfr 16 augustus 2017

   I think they mean different weights for input (so in the rotation with R) and desired output (rotation with z), but no idea if that would work

3. How would you use the QRD-LSL algorithm for acoustic echo cancellation in a scenario with 1 loudspeaker and 2 microphones (i.e. stereo recording)? Define all input/output signals.

   Cfr 13 augustus 2018 Q3.3 and have a look at fig 7.13 (equal channel length) and 7.17 (unequal) in the pdf on his website about adaptive filters (also available on burgiclan i think) **In the images below there is only 1 desired signal... But below they mentioned there should be two desired signals. However I think it should be the same. Because you want to calculate the filter coefficients and you'd like to have an estimation for your channel. What do you think?**

   But if you look at CH9 slide 26 you see the u comes from the loudspeaker and d from the microphone so isn't it logic that you have 2 desired signals here? Then you have 2 different errors which are the near end signal of the two microphones (with residual echo)

   I agree. How are the two residual errors combined to have one output at the end?

   I don't think they are combined together, they are both just the near end signal of one of the microphones

   Oh, so it's more like one mic in one corner of the room and another at the opposite corner? Both providing a good signal for that location? Not to have a better result globally?

   Yes I think this is what they mean. It's not that the residual is in this case an error but with echo cancellation it is the near end signal without echo

   Alright Thanks!

   (ftp://ftp.esat.kuleuven.ac.be/pub/SISTA/moonen/reports/chapter7.ps)

Here are <u>two</u> screenshots:

1. Explain the operation of the signal flow graph (SFG) of QRD-RLS. → *slide 24, ch. 8* *of cursus p. 85*
2. Each hexagon in the signal flow graph represents an orthogonal transformation (2 input, 2 output). Could non orthogonal transformations be used in the rotational cells? If so: how? If not: why not?
3. Consider the linear system y[k] = a*u[k] + b*u[k-1]. Could QRD-RLS be used in a 'system identification' application, where 'a' and 'b' are determined from observations of y[k] and u[k]? If so: how? If not: why not?
4. Consider the non linear system y[k] = a*u[k]*u[k-1] + b*u[k-2]*u[k-2]. Could QRD-RLS be used in a 'system identification' application, where 'a' and 'b' are determined from observations of y[k] and u[k]? If so: how? If not: why not?

Question 3

1. Explain why & how polyphase decompositions are used to derive conditions for perfect reconstruction for oversampled analysis/synthesis filter banks.
   The polyphase decomposition is used to make things beautifully simple. Oversampled: D<N; Explain slides of ch11, s39 but with a bit of background from the previous slides.

2. Explain (briefly) how the conditions for alias-free and perfect reconstruction are derived for oversampled analysis/synthesis filter banks (using polyphase decomposition). Ch11, s40, with again background of previous slides

3. Explain (in detail) how these conditions for perfect reconstruction can be used to design oversampled filter banks (with FIR analysis/synthesis filters).
   Slides about the general PR-FB design? Ch12 s10-12

9. 22 januari 2020 AM

9.1. Question 1

1. Explain how 'weighted least squares' optimization is used for FIR and IIR filter design. You have a set of correct values and you have a function to approximate these values. You want to minimize the distance between the correct and approximate values. This can be done by taking the best values. These best values are determined by a least square problem. The distances of certain correct values with his approximation are more important than at other values. These more important values need to be taken into account more by giving them a higher weight. The less important values will get a smaller weight.

2. When WLS optimization is used for IIR filter design, how can the filter's phase response be controlled/designed?

3. When WLS optimization is used for FIR filter design and the desired FIR filter has a desired phase response different from a linear phase response, how would the design procedure have to be adjusted? Provide formula's. (answer result is almost the same as with linear phase, in the end the cosines of c(w) are replaced by phase shifts.)

9.2. Question 2

1. Explain how QRD-LSL (least-squares-lattice) algorithm is derived from the QRD-RLS algorithm. Explain the meaning of the epsilon notation and the 'main trick' in the derivation. What is the relevance of the property illustrated in Chapter-9 p 32? (the fact that epsilon doesn't change when permutations)

2. A hexagon in the signal flow graph (SFG) represents a transformation with a 2-by-2 orthogonal matrix. Would it be possible (in QRD-RLS or QRD-LSL) to use non-orthogonal transformations to introduce the zero's in specific positions of the input vector (for example Chapter-9 p 17 replace [cos(theta) sin(theta); -sin(theta) cos(theta)] by [1 tan(theta); -tan(theta) 1])? If ot why not? (answer chapter 9 p 12: orthogonal transformation preserves the norm)

No because if you use non orthogonal transformations you would introduce non-zero values in the positions where you previously created zeros and thus it would be impossible to create the triangular matrix and won't be able to use the QRD decomposition.

What do you think about that answer ?
Sounds good for me. . .

I don't agree with previous answers. It is entirely possible to construct non-orthogonal transformations that introduce zeros in the same way as a Givens rotation would. For example left-multiplication with [1 0; 0 0] would do this.

However, we are after all still trying to calculate a sosition (starting from an already nicely structured matrix) so the left-multiplicant in the triangularization needs to be orthogonal. In general, this happens if the transformations done on the matrix are orthogonal transformations since the composition of orthogonal transformations/matrices is again an orthogonal transformation/matrix.

3. How can the SFG for QRD_LSL be extended to allow for 'decision-directedchannel equalization' (as in Chapter-7, p15). (answer: choose first zero as target output). In the pdf on page 123 there's a formula of W_LS with decision directed mode and also a note about the performance: "However, one can verify that decision-directed operation introduces an undesirable (inefficient) feedback path in the SFG." But I have no idea what should change in the SFG

9.3. Question 3

1. What are DFT-modulated filter banks? What are particular advantages of such filter banks.

2. What are oversampled filter banks? What are particular advantages of such filter banks?

3. Consider a 5-channel DFT-modulated filter bank with 4-fold downsampling. How can the analysis bank and synthesis bank be realized efficiently? And then how can perfect reconstruction be obtained in this case? Provide formulas.

10. 20 januari 2020 AM

Question 1

1. Explain how an FIR (LPC) lattice realization is derived for a given FIR filter. Derivation of FIR 3 Lattice Realization slide 8-13

2. What is the relevance of the Schur-Cohn stability test appearing in this derivation?
   This is the procedure to compute the Ki's (reflection coefficients)

3. For a given FIR (LPC) lattice realization, how can the effect of quantization errors in the arithmetic operations be analyzed?
   Quantization noise is usually analyzed in a statistical manner. This is based on assumptions:
   - Each quantization error is random
   - Successive errors at output of given multiplier/adder are uncorrelated
   - Quantization errors at output of different multipliers/adders are uncorrelated
   A noise source is added after each multiplier/adder and since the filter is a linear filter this is added to the output.

   Independent sources can be summed and I think this results at the end in an equivalent noise added at y and y_tilde.
   If you look at one block, there's a sum at the end with:
   - the error resulting from the b0 multiplication (mutual)
   - error from k3 multiplication (different/out)
   - error from sum at end (different/out)

   Although this is used a lot, it would be better to do an analysis in a deterministic manner as quantization is a nonlinear effect.

4. What are linear phase filters? What is the relevance of the linear phase characteristic in practice?
   Linear phase filters are those filters whose phase response varies linearly with frequency. Each of the frequency components of the signal is delayed by a constant amount. Hence such filters preserve the shape of the signal and introduce no distortions. This is an important characteristic required for some applications which cannot tolerate phase distortions at all.

5. Can linear phase filters be realized as an LPC lattice?
   No, 26 jan 2017 Q1.4

Question 2

1. Explain how the LMS algorithm is derived as a 'stochastic gradient' algorithm. Explain all formulas used in the derivation.

2. In an acoustic echo cancellation (AEC) scenario with one loudspeaker and one microphone, if LMS is used for the adaptive filtering, how do the characteristics of the loudspeaker signal influence the behavior of the LMS algorithm? What is then the ideal loudspeaker signal in this respect?

I'm not sure but with AEC, I think it is a fast time-varying system so if we want to track it properly, we should choose a sufficiently large \mu because if it is too small the LMS solution will lag behind. And instead of only having the noise due to the environment that stays as in the MSE, you will also have some other noise because the filter won't be perfect. And it leads to the formula of the pdf pg 72 that you can compare with slide 38 in chapter 7

The choice of \mu must be based on a compromise between fast tracking (large \mu) and small J_noise (small \mu). That is because we have stochastic parameters with LMS and not statistical as with MSE.

3. In an AEC scenario with one loudspeaker and one microphone, if LMS is used for the adaptive filtering, how do the characteristics (or presence) of the near-end speaker signal influence the behavior of the LMS algorithm? What is

then the ideal near-end signal in this respect?

4. In a stereo AEC scenario with two loudspeakers (playing two different signals) and one microphone, can LMS still be applied? if yes, what would the LMS update formula look like?

   If one would like to have both signals separated afterwards, he needs two filters. One to filter out the first and another one for the second signal. If however both signals contribute to the same output then only the desired signals should be combined.

   In the first situation there are two FIR filters and in the second only one.

   I don't know how the desired signals can be combined...

   In AEC you don't want the output from the loudspeakers, you need to filter this out of the received signal (see CH7 s 9). So in this case you have one desired response, the near end signal with echo, and 2 inputs/ far end signals from the two loudspeakers. Since the 2 loudspeakers are independent, wouldn't it be possible to just substract both of them? (something like on CH7 s19)

Question 3

1. Explain the context and general operation of the Short-time Fourier Transform.

```
1              STFT is for example used in a short lived sine wave, where the
               frequencies mustn't be viewed from an infinity time perspective
               but from a finite one. For example when making a spectogram, it'
               s only the instantaneous frequencies that are important so you
               get a different frequency spectrogram for each time bin.
```

In practice, the procedure for computing STFTs is to divide a longer time signal into shorter segments of equal length and then compute the Fourier transform separately on each shorter segment.

2. Explain how the STFT is realized by means of a (maximally decimated or oversampled) DFT-modulated filter bank.

3. Explain how the WOLA design and perfect reconstruction (PR) conditions relate to the filter bank design procedure and PR conditions in Chapter 12.

11. 10 januari 2020

11.1. Question 1

1. What are noise transfer functions? Explain how such noise transfer functions are used in the analysis of quantisation effects in filter implementation.

   See previous years

2. What would change in the NTF analysis if there would be a MIMO system instead of a SISO system as described in Chapter-6?

3. Are there other/better ways of analysing the effect of quantisation errors in filter implementations and realisations? If yes, why? If yes, what other ways?

See previous years + explain what deterministic analysis is (input is not uncorrelated with error).

We suppose that the errors are uncorrelated, but that is not always the case. If they were uncorrelated, we could just add it up but is is a bit more complicated than that. If we do a deterministic simulation (analysis is to complicated) then unwanted effects occur as for instance zero input limit cycle oscillations up.

11.2. Question 2

1. In which sense is the Kalman Filter a generalisation of the Least Squares parameter estimation.

   KF is dynamic, parameter estimation is static.

2. Explain how a square-root Kalman Filter algorithm is derived.

   See previous years (just explain what is going on in the slides of SRKF, why you can throw away the not relevant part to the problem (because the lhs is already triangular and you don't need the most part for zeroing the lhs using Givens rotations))

3. What can you say about the computational complexity of the QRD-RLS and the Square Root KF?

   Same in big O notation, but since you have to zero less (since Vk=0), the QRD-RLS is a bit faster than SRKF. O(L)?

11.3. Question 3

1. What are DFT-modulated filter banks? What are the advantages of such FB's?

   See previous years

2. Most analysis/synthesis filter banks have a complex impulse response (time domain). Why is that? How can this be avoided?

Most filter banks are DFT modulated, so this means using a prototype filter (easier to design). Prototype filter is shifted in the frequency domain, this translated to a complex time domain impulse response (see slide 12.14). This can be avoided by using cosine-modulated FB's (slide 12.27), since they have a symmetrical frequency response and thus real in time domain. Or you could just design every Hn(z) separately.

3. Explain how a 6-channel oversampled (D=5) DFT-modulated filter bank can be realised.

Explain examples at the end of chapter 12.

12. 10 januari 2020

Question 1

1. Explain windowed FIR filters Take a design and convert the design to a predetermined range/ window.

2. Explain the relation between windowed FIR filters and WLS It will generate the same result.

3. Explain whether the windowed FIR filter will always have a linear phase or how we can enforce a linear phase Add extra delay to turn an non-linear, non-causal filter into an linear causal filter.

Question 2

1. Explain the SFG of QRD-LSR

2. Explain the main trick that is used in the derivation of the QRD-LSL algorithm.

3. The epsilon-signals are referred to as forward and backward prediction errors (up to a scaling with the 'cosines product'). Specify the forward and backward prediction problems that are referred to here.

Question 3

1. Explain the difference in freedom of design between the maximally decimated and oversampled case in the **general analysis/synthesis case**.

2. Explain the difference in freedom of design between the maximally decimated and oversampled case in the **DFT-modulated FB**.

3. If E(z) contains only zero order polynomials, what order polynomials can R(z) contain?

13. 11 januari 2019

13.1. Question 1

1. What are noise transfer functions? Explain how such noise transfer functions are used in the analysis of quantization effects in filter implementation.

A noise transfer function describes how the noise acts on the output : $H\_n0 = Y(z)/E(z)$ where $E(z)$ is the noise signal. It gives more information about the influence of each noise source to the output. Each adder and multiplier introduce noise. This means that given for a filter realization and assuming a linear model we can lump all the noise sources together in one equivalent source. We suppose that the quantization errors at the output of given multipliers/adder are uncorrelated.

As the quantizer introduces noise (after a multiplier or adder), if you study the noise, you are studying the influence of the quantization on the complete system.

1. Why is it that in such analysis different quantization noise sources can be lumped into one equivalent noise source, for specific filter realizations? (use formulas)

   Assume a direct form implementation of a FIR filter: $y(k) = b0*u(k) + b1*u(k-1) + b2*u(k-2)$. Now assume that each addition and multiplication has a noise source (m = multiplier, a is adder): $y(k) = b0*u(k) + em(k) + ea(k) + b1*u(k-1) + em(k-1) + ea(k-1) + b2*u(k-2) + em(k-2) + ea(k-2)$. All of these noise sources can be added because they are the same, upto a certain delay. This can be one equivalent source in a direct form representation (can be verified graphically) or two in the case of a transposed direct form representation.

2. Are there other/better ways of analyzing the effect of quantization errors in filter implementations and realizations? If yes, why? If yes, what other ways?

   ```
   1  Yes, quantization is actually a deterministic effect. However, analysing it as
        such is very difficult and is only helpful for finding limit cycles in the
        filter (which only occur in feedback systems).
   ```

13.2. Question 2

1. Explain how general 'overlap-add' and 'overlap-save' based FIR filter realizations are derived.

Overlap-add and overlap-save filters are so called block filters, in these type of filters, a 'block' of output samples is calculated at once. The coefficients of a normal FIR filter can be represented in matrix form with these filters (see slide 5 of chapter 13). This matrix can be extended and formed into a circulant matrix. These types of matrices can be decomposed using IDFT and DFT matrices. The resulting matrix will be a diagonal matrix with the entries being the frequency domain representation of the filter coefficients. This can then be implemented in a filterbank where now the FIR coefficients are 2replaced with the distortion function(T(z)) coefficients. In an overlap-save implementation, the current block of samples is DFT modulated together with the previous samples in the analysis bank. This is then IDFT modulated where the first half is thrown away, while the second half is saved in the synthesis bank. In an overlap-add, the analysis bank does a DFT with the current block and the synthesis bank does the IDFT and adds the result to the previous IDFT block.

2. Explain how the process delay (latency) and computational complexity of such realizations can be controlled.

```
1  In overlap-save, the order of efficiency is O(log(L)), for large L, with L being
       the filter order. The latency however, is of order O(L). This can only be
       controlled by changing the filter order. In the overlap-add the efficiency
       and latency are different based on the amount of channels N and the filter
       order. If N\~L, then efficiency is O(log(L)) and latency nois O(L), but if N
       \<\< L, then efficiency is O(L) and the latency is O(N).
```

3. Consider a 'overlap-save' based realization based on a 8-channel analysis and synthesis prototype filter. Derive an expression for the analysis and synthesis filter and

sketch the frequency response of this filters.

See slide 11-12 of chapter 13 for derivation. Frequency responses are All-pass (I think) since the entries are only ones, zeroes, or delays.

13.3. Question 3

1. Explain how the QRD-LSL algorithm is derived from QDR-RLS. Also explain the epsilon-notation and the main trick that is used in this derivation.

In the QRD-RLS, there is an asterisk(*) which was not being used. However, that star can be used to calculate least squares residuals. When a small part of the SFG (slide 33 in chapter 9) is taken, it can be viewed as a smaller subsystem. This subsystem also has its own residual terms. These epsilons are used as: $\epsilon^{k-2}_{k-1:k}$, where the subscript k-1:k refers to the subsystem, in other words, this residual is calculated with the signals going from time k-1 to time k. Or the subsystem is fed with the signals u(k) and u(k-1). The superscript refers to the the time index, so in words: "$\epsilon^{k-2}_{k-1:k}$ is the residual at time k-2 of the subsystem which is fed with the input signals u(k) and u(k-1)." Now it can be shown that, two residuals at a different time, can create another residual which is equal to another residual in the system. This means that the calculation for the latter residual can be removed and as such some rotations and multiplications are removed. Take for example slide 35 of chapter 9. In this SFG it is shown that the residual fed with k-1:k at time k+1, together with the residual fed with k-1:k, at time k-2, can create a residual which is the same (upto a delay) as the residual fed with k-2:k at time k-3. So that circuitry can be removed (see slide 36).

2. The epsilon-signals are referred to as forward and backward prediction errors (up to a scaling with the 'cosines product'). Specify the forward and backward prediction problems that are referred to here.

3. In an acoustic echo cancellation scenario with one loudspeaker and two microphones (ie. where an echo contribution is to be cancelled in each of the two microphone signals), is it possible to apply a QRD-LSL-algorithm? Sketch an efficient realization (clearly defining input and output signals).

14. 13 augustus 2018

14.1. Question 1

1. What are noise transfer functions? Explain how such noise transfer functions are used in the analysis of quantization effects in filter implementation.

   Chapter 6: each multiplier and adder introduces noise. The transfer function from e[k] to the output is the noise transfer function. It gives more info about the influence of each noise source to the output. This means that given a filter realisation and assuming a linear model, we can lump all the noise-sources together in one equivalent source.

As the quantisation introduces noise (after a mult or add), if you study the noise you are studying the influence of that quantisation on the complete system.

1. Do the properties of the quantization mechanism play a role in such analysis?

   (slide 20) Different quantizations gives different errors => different noise. This gives different noise mean and variation -> important for the analysis

2. The analysis relies on linearity (explain) whereas quantization is a non-linear operation. Could this be a contradiction?

The analysis we use is statistical it assumes all noise sources are independent (white) and the quantisation error is random. Quantization may be non- linear, the error is linear?????

Quantization is non-linear and therefore produces some errors which are known as quantization errors. These errors result in what is known as quantization noise. The noise itself is viewed as additive white gaussian noise so it is equal for all frequencies.

```
1      The filter system which is being studied is assumed to be linear time
          invariant (LTI), it only uses additions, multiplications, and delays. What
          is assumed in the noise analysis is that the noise sources from
          quantization have a linear relation to the output (so are added). Therefore
           the noise can come from a non linear source, but the analysis relies on
          the circuit being linear.
```

m In the noise analysis we assume quantisation noise to be additive, but this is not actually the case. You can not just add it after a multiplication since it is dependent on which numbers are added, this makes it very unpredictable (see cycle errors)

14.2. Question 2

1. In a maximally decimated (MD) filter bank aliasing usually occurs because of the down-sampling operation. Provide an intuitive explanation why perfect reconstruction (PR) is still possible, in spite of the downsampling and aliasing.

If we choose our synthesis filters in a way that they restore the aliasing distortion, PR is possible -> slides 24-28. If we look at all filters in total we still have all the information available despite the aliassing.

2. What are DFT modulated filter banks? What are the advantages of such FB's?

DFT modulated filter banks are uniform filter banks that are based on a polyphase decomposition of a prototype filter H0 and followed by an inverse DFT, hence all the other Hi filters are derived from only one prototype filter. The main advantage is that this kind of filter banks is that are easier to design: yodo : you only design one!!! The rest of the filters are shifted versions of the prototype filter. Low computational cost also i think...

Low computational cost because only N polyphase components (= 1 filter) and DFT versus N^2 polyphase components (= N filters).

3. How can DFT-modulated PR filter banks be designed?

Maximally decimated:

4) Design prototype H_0(z)

5) Calculate E_n(z) = polyphase component

6) Assuming all E can be inverted -> choose R

```
1  \!\!\! not every E stable \-\> choose E unimodular or paraunitary\!\!\!
2  Not a lot of freedom
3  Oversampled \-\> the same but more E and R so more freedom in design.
```

14.3. Question 3

1. Explain the main trick that is used in the derivation of the QRD-LSL algorithm.

   LSL = RLS?? No, LSL = H9 part 2, fast algorithm for FIR RLS LSL = Least square lattice, RLS = Recursive least squares
   As most of the input vector is reused in for the next input vector (h[k] -> h[k-1]; h[k-1] -> h[k-2];.....
   On slide 32 is shown that when a random permutation is applied to the input, the residues remain the same.
   In slide 35 it is shown that the residual for a given problem is also the residual for other, related problems; for example $\epsilon_{k-1:k+1}^{k-2}$ may be viewed as the epsilon-output of a 1-dimensional RLS problem, with left-hand side $\epsilon_{k-1:k}^{k+1}$ and right-hand side $\epsilon_{k-1:k}^{k-2}$. After a delay element this becomes $\epsilon_{k-2:k}^{k-3}$. Since we now have a way to generate this epsilon, the other way is redundant and thus can be removed. This process is then repeated until the SFG on slide 37 is reached.

Can someone explain how we can calculate the filtercoefficeint with this? (i found a formula but it only uses the R matrix, and with this there is no R...

Filter coeff are not calculated (slide 22)

1. Compare the complexity of the QRD-LSL algorithm with the complexity of LMS.

LMS $\rightarrow$ O(6L)--> moet dit niet 2L multiplications zijn? (slide 16) en kan big O-notatie hier gebruikt worden?
QRD-LSL $\rightarrow$ O(24L) $\rightarrow$ en dit moet 24*L multiplications zijn, niet O(24L)

2. In an acoustic echo cancellation scenario with one loudspeaker and two microphones (ie. where an echo contribution is to be cancelled in each of the two microphone signals), is it possible to apply a QRD-LSL-algorithm? Sketch an efficient realisation (clearly defining input and output signals).

The input is equal for both microphone signals, but the desired output is different. By simply adding an extra desired signal we can reuse the same procedure.

Do we need to calculate an epsilon for both microphone signals? And can we use the same R matrix for both signals?

Yes different epsilons are needed, remember that epsilon is: e = d - w*x. So if there is another desired signal d, then there is also another epsilon.

14.4. Question 4: Acoustic modem project

1. Explain how the OFDM modulation format (including a cyclic prefix) provides an easy equalization of the transmission channel. What are conditions that the transmission channel has to satisfy for this 'cyclic prefix trick' to work properly?

The OFDM frame together with the prefix allows for a circular convolution. In the receiver, the L samples corresponding with the prefix are thrown away which results in a circulant matrix. This circulant matrix can then be factorized using IDFT/DFT(H= F^-1 * diag(H0,..,Hn-1) * F). With this, a diagonal matrix is obtained. The resulting output Y is then simply the input X multiplied with some scalar H(all in frequency domain), so the X can be estimated using component wise division. The prefix length needs to be longer than the impulse response of the channel. (slide 31-35')

1. If a channel model is required for the equalization, explain how it can be derived. Is it possible to run the equalization without first deriving a channel model?

    A channel model can be acquired by using trainingframes?? The data in these frames is known at the receiver, so the channel model can be derived with a least squares estimation.
    By measurement, and ise slide 19

15. 26 januari 2018

15.1. Question 1

1. Explain the lossless lattice realization for 4 FIR filters. Explain the orthogonal matrix transformations.

P18: we want it lossless so we want power-complementary:
1= H(z)H(z$^{-1)+\hat{H}(z)\hat{H}(z}$-1) => we only want rotations to conserve the energy!!!

2. Sometimes scaling is necessary, explain.

Stability: if H(z) is very big/ small -> H(z^-1) very small/big -> not good

3. Are there any … matrices present (matrices with determinant = cte*z^x)

… = unimodulary:
The orthogonal transformation matrix and the order reduction matrix.

15.2. Question 2

1. Explain the context and need of the short time fourier transform (STFT).

STFT is for example used in a short lived sine wave, where the frequencies mustn't be viewed from an infinite time perspective but from a finite one. For example when making a spectrogram, it's only the instantaneous frequencies that are important, so you get a different frequency spectrogram for each time bin.

2. Explain the link to DFT modulated filter banks

In DFT modulated FBs we shift one LPF over the frequency axis and obtain a set of N filters the spans all frequencies. In this context, I think we design a short time prototype filter and the inverse fourier transform operation shifts it over all time samples?? Anyone know if this is correct?

15.3. Question 3

1. Explain the main trick in the fast RLS algorithm

2. You have 2 microphones and one speaker, can the algorithm be used? How?

16. 15 januari 2018

16.1. Question 1

1. Formulate in your own words the Schur-Cohn stability criterion to test the 'stability' of a polynomial.

For a polynomial to be stable, all the zeros have to be within the unity circle. Instead of calculating every single root of a polynomial (which takes a lot of effort), you can simply use the Schur-Cohn criterion. This dictates that the polynomial is stable if your reflection coëfficients $|K_i| < 1$. The reflection coëfficients can be calculated based on your polynomial coëfficients: $K_0 = b_L/b_0$ etc...

26 jan 2017

1. Where does the Schur-Cohn test appear in the context of FIR filter realisation, and what is its relevance there?

Appears in the lattice realization. Used for computing kappais from bis so we can be sure that all zeros lie inside the unit circle and are therefore stable.
The zeros of a FIR filter do not need to lie inside the unit circle, a FIR filter is always guaranteed to be stable, so I don't think there is any relevance of this test for FIR filters (only for IIR filters as in question 1.4).

No, different structures can be unstable even it's FIR

26 jan 2017

1. If the Schur-Cohn test is applied to a linear-phase FIR filter, what will be the outcome? What does this mean in terms of linear-phase FIR realization?

abs(ki) is altijd 1, dus lattice realization van FIR linear fase filter is onmogelijk?
Yes

26 jan 2017

4. Where does the Schur-Cohn test appear in the context of IIR filter realisation, and what is its relevance there?

IIR Lattice ladder realization: Ki's are computed from the coefficients of the denominator$\rightarrow$ this time the stability test can be used to see if the transfer function is stable

26 jan 2017

16.2. Question 2

1. What are DFT modulated filter banks? What are the advantages of such FB's?

2. How can oversampled DFT filter banks be designed? What are the advantages?

3. Explain how an 8-channel oversampled (D=4) filter bank can be realised.

Example 1 in slides 29-34, chapter 12

4. How can the analysis and synthesis filter of such a filter bank be derived?

    Analysis: $[H\_0 \ H\_1 \ H\_2 \ldots H\_{D-1}]\hat{}T = IFFT * B(z\hat{}D) * [1 \ z\hat{}-1 \ z\hat{}-2 \ldots z^{-(D-1)]}T$
    Synthesis: $[F\_0 \ F\_1 \ F\_2 \ldots F\_{D-1}] = [z\hat{}-(D-1) \ldots z\hat{}-2 \ z\hat{}-1 \ 1] * C(z\hat{}D) * FFT$

16.3. Question 3

1. Explain the operation of the signal flow graph (SFG) of QRD-RLS.

2. Each hexagon in the signal flow graph represents an orthogonal transformation (2 input, 2 output). Could non orthogonal transformations be used in the rotational cells? If so: how? If not: why not?

No, because if you use non orthogonal transformations you would introduce non-zero values in the positions where you previously created zeros and thus it would be impossible to create the triangular matrix.

3. Consider the linear system $y[k] = a*u[k] + b*u[k-1]$. Could QRD-RLS be used in a 'system identification' application, where 'a' and 'b' are determined from observations of $y[k]$ and $u[k]$? If so: how? If not: why not?

4. Consider the non linear system $y[k] = a*u[k]*u[k-1] + b*u[k-2]*u[k-2]$. Could QRD-RLS be used in a 'system identification' application, where 'a' and 'b' are determined from observations of $y[k]$ and $u[k]$? If so: how? If not: why not?

17. 16 augustus 2017

17.1. Question 1

1. What are noise transfer functions? Explain how such noise transfer functions are used in the analysis of quantization effects in filter implementation.

2. Do the properties of the quantization mechanism play a role in such analysis?

3. The analysis relies on linearity (explain) whereas quantization is a non-linear operation. Could this be a contradiction?

17.2. Question 2

1. What are DFT modulated filter banks? What are the advantages of such FB's?

2. How can maximally decimated DFT filter banks be designed?

3. How can oversampled DFT filter banks be designed?

17.3. Question 3

1. What is a rank-1 update? How does it define the complexity?

2. Residuals: explain a priori and a posteriori residuals of SFG

The a priori residual is the difference between the desired signal and the filtered input, using the filter coefficients of the previous iteration.
The a posteriori residual is the difference between the desired signal and the filtered input, using the filter coefficients of the current iteration.

3. Is it useful to use two exponential weighting factors if the output and input are very different: static input and non static output (for example)? (something like this)

I don't think so: you're supposed to use the same weighting factor for both the desired (input) and the received (output) signals (ch.8 slide 37) else the LS solution won't calculate the filter coëfficient correctly

17.4. Question 4 (Acoustic modem project)

1. Explain how the OFDM modulation format (include cyclic prefix) provides easy equalization of the transmission channel. What conditions are to be met for the cyclic prefix?

2. Explain how RLS is used for channel equalization in the acoustic modem project. What input for the adaptive filter? What is the desired output signal? What is the order of the filter? (awnser: 1, for every carrier/subband)

Does anyone have an idea?
I think we used NLMS in week 7 (and not RLS) . And the input of the adaptive filter is the signal received at te microphone ($x[k]*h[k]+n[k]$). The desired output is the output of the decision device which takes the output of the adaptive filter as input.
The order of the filter is 1, you try to find an estimation for every subband carrier channel.

18. 16 jan 2017

Question 1

1. Formulate in your own words the Schur-Cohn stability criterion to test the 'stability' of a polynomial.

We have a polynomial B(z) and want to know whether all zeros of B(z) lie inside the unit circle. If $B(z) = \text{sum}\{i=0..L\}(b_i*z^{-i})$, then calculate kappa_0 as b_L/b_0. Calculate all new coefficients b'_0..b'_{L-1}. Now you are ready to calculate kappa_1 as b'_{L-1}/b'0. If all L Kappa_i satisfy $|K_i| < 1$, the zeros lie inside the unit circle, and thus, the polynomial is stable. (+ see p87 sl 173)

2. Where does the Schur-Cohn test appear in the context of FIR filter realisation, and what is its relevance there?

It appears in lattice ladder realisation. Its relevance is mainly to calculate the different kappa_i to realise the filter. (should be lattice realization instead of lattice ladder I think)

3. Where does the x test appear in the context of IIR filter realisation, and what is its relevance there?
    Schur-cohn is used to compute the reflection coefficients of a lattice-ladder realization

- the reflection coefficients are calculated from the ai of the IIR filter (the denominator), because all Ki < 1 (sine of theta), this ensures the stability of the IIR filter

-> correct?? I think so Yes, this is why you cannot make a lattice realisation of an unstable IIR

4. If the Schur-Cohn test is applied to a linear-phase FIR filter, what will be the outcome? What does this mean in terms of linear-phase FIR realization?

Linear phase filters have symmetrical impulse responses, so the first kappa will be bL/b0, which will be 1 or -1, depending on even or odd symmetry. K_0 = +-1. Then, when calculating the "next-generation" coefficients b1' to bL',
A zero will appear in the denominator, because of kappa being 1.
Is it possible to build it in lattice realization if kappa = 1? No
This means it is - at least with this design procedure - impossible to design a linear phase lattice FIR filter. $\rightarrow$ better? And the next generation coefficients are infinity I think? Divided by 0 $\rightarrow$it will be 0/0 for even symmetry and inf for odd symmetry I think. Indeed, but what does that mean? Just you can't realize with this realization? Marcske remained vague in his slides, but I guess we don't know how to or if it is realisible. I think other realizations are possible, but not the lattice, like direct form. Exactly, no Lattice, yes direct form, transposed form or lossless lattice!

5. Use the Schur-Cohn stability test to derive the stability conditions for a 2nd order polynomial A(z) = (1 + a1*z^-1 + a2*z^-2)

Kappa_0 = a2/a0 = a2
A0' = a0;
a1' = (a1-kappa0*a1)/(1-kappa0^2) = (a1-a1a2)/(1-a2^2)
Kappa_1 = a'1/a'0 = a'1 = (a1-a1a2)/(1-a2)^2
For the system to be stable, all kappa should have a modulus smaller than 1:
|k0| < 1 $\Leftrightarrow$ |a2| < 1
|k1| < 1 $\Leftrightarrow$ |(a1-a1a2)/(1-a2)^2| < 1 $\Leftrightarrow$ |a1(1-a2)| < |(1-a2)^2| $\Leftrightarrow$ |a1| < |1-a2|
If those conditions are met, every zero of the polynomial will lie inside the unit circle.
Note that there is no criterion for a0, as this is already set to 1 in the given polynomial.
Also note that this is a strict criterion in that when it's not met, the poles will not lie strictly inside the unit circle!
I think there was a mistake in the denominator above: $(1-a2^2)$ changed to $(1-a2)^2$. If I keep the first option then my result is |a1| < |1+a2|
See also chapter 6 slide 12

Question 2
Consider an oversampled (analysis + synthesis) filter bank.

1. Explain how a condition for alias-free operation is derived, based on polyphase representations of the analysis and synthesis filters.

Chapter-11 slides 34-36

2. By using this condition, derive an expression for the 'distortion' function as a function of the analysis filters H(z) and synthesis filters F(z).

3. Explain how a condition for perfect reconstruction is derived, again based on polyphase representations of the analysis and synthesis filters.

Question 3

1. Explain the operation of the signal flow graph (SFG) of QRD-RLS.

2. Explain how the QRD-LSL (least-squares-lattice) algorithm is derived. What is the meaning of the epsilon-notation? Meaning of the epsilon-notation: The SFG can be seen as a collection of nested RLS problems. (e.g. when you remove the last 3 rows on the figure of the SFG from the slides, you can view the remaining 2 rows as a SFG that solves 4 RLS problems simultaneously. They all have the same U-matrix as left hand side (u[k] & u[k-1]). The right hand side for each RLS problem is u[k-2], u[k-3] and u[k-4]. The signals between row 2 and row 3 can thus be seen as residues of an RLS problem (= epsilons). The same is true for all the other rows.

   On slide 32 is shown that when a random permutation is applied to the input, the residues remain the same.
   In slide 35 it is shown that the residual for a given problem is also the residual for other, related problems; for example $\epsilon^{k-2}_{k-1:k+1}$ may be viewed as the epsilon-output of a 1-dimensional RLS problem, with left-hand side $\epsilon^{k+1}_{k-1:k}$ and right-hand side $\epsilon^{k-2}_{k-1:k}$. After a delay element this becomes $\epsilon^{k-3}_{k-2:k}$. Since we now have a way to generate this epsilon, the other way is redundant and thus can be removed. This process is then repeated until the SFG on slide 37 is reached.

3. The epsilon-signals are referred to as forward and backward prediction errors (up to a scaling with the 'cosines product'). Specify the forward and backward prediction problems that are referred to here.

   A posteriori residual = d_k - (u_k)^T*w_LS[k]
   = epsilon*prod_{i=1}^{L+1}(cos(teta_i)) t

   A priori residual = d_k - (u_k)^T*w_LS[k-1]
   = epsilon/prod_{i=1}^{L+1}(cos(teta_i))

19. 28 jan 2016

Question 1

1. how is the IIR lattice related to direct form FIR
   A lattice-ladder with all theta's 0 (this means H(z) is FIR) becomes a direct-form FIR

2. why do you need to scale the TF for Lossless Lattice realisation?
   If H(z)*H(z^-1) > 1 the magic equation isn't possible I think?
   Anyone a further explanation on this? Do we have to prove that the second term of the magic equation can never be negative?

3. how is the schur-kohn stability test related to the implementation of lattice FIR filters?\

Question 2

1. explain how PR is achieved given the AA condition of a DFT modulated FB

The requirements for perfect reconstruction are:

1) Alias-free → alias transfer function A(z) = 0

2) DIstortion-free → T(z) = z^(-delta) i.e. T(z) is a pure delay

Based on the R(z)*E(z) product, a necessary and sufficient condition for perfect reconstruction is that R(z)*E(z) = z^(-delta)*I_N (see page 192).
Is AA standing for anti-aliasing? I think so :)

    2. what are unimodulary matrices? how are they used in DFT modulated FB?

A unimodulary matrix is a matrix of which the determinant = constant * z^(d). We use them in filter banks because we need to make sure that if we design a FIR analysis filter, that the synthesis filter (which is ~ the inverse of the analysis filter) is also a FIR filter.

    3. what are paraunitary matrices? how are they used in DFT modulated FB?

Paraunitary matrices are unimodulary matrices that are built from unitary E_l matrices. The definition according to google is that if U is paraunitary, then U(z)*U(z^-1) = I. They are cool because they have some properties listed at p. 197 (sl. 394).

Question 3

    1. what is the difference between optimal filters and RLS filters?

Optimal filtering is based on given statistical information. In RLS filtering this information is not given and is estimated based on real samples.

    2. explain how standard RLS is a special case of standard KF

Slide 336

    3. explain how the algorithm of RLS can be seen in the algorithm of KF

20. 12 Jan 2016

Question 1

    1. What are 'zero-input limit cycle oscillations'? Where and how do these appear?
       Oscillations in the absence of input, they are unwanted and only appear in IIR filters. Solution: MAGNITUDE truncation + good choice of filter realization

    2. Explain how from a given frequency domain specification (e.g. low-pass filter characteristic) a filter can be designed and realized that is guaranteed to be free of limit cycle oscillations. Explain (briefly) different steps and options in the design process.

First choice: FIR/IIR → FIR is always free of limit cycle oscillations.
Second choice: filter realization, FIR choice doesn't matter for limit cycle oscillations
IIR choice of lossless lattice realization or latice-ladder realization with ¶¶¶magnitude truncation guarantees no limit cycle oscillations. With other realizations/quantization limit cycle oscillations are possible.

Question 2

    1. What are oversampled DFT-modulated filter banks? What are particular advantages of such filter banks?

Oversampled DFT-modulated filter banks are DFT-modulated filter banks in which the #channels > decimation. This leads to more design freedom in the polyphase components. In maximally decimated there is 0 design freedom in the polyphase components.

2. Consider an 8-channel DFT-modulated filter bank with 4-fold down-sampling. How can the analysis bank and synthesis bank be realized efficiently?

Explain slide 414-417

3. …And then how can PR be obtained in this case (N=8,D=4)?

Explain slide 418-421

Question 3

1. Explain the operation of the SFG (signal-flow graph) of QRD-RLS.

Does anyone know a good video/explanation on the internet? You can find some explanation on dropbox, or on the DSP site (in ghostview format), I think it was used in the past years, some kind of course written by Marcy

2. A hexagon in the SFG represents a transformation with a 2-by-2 orthogonal matrix. Would it be possible to use non-orthogonal transformations (to introduce the zeros in specific positions of the input vector)? How/why not? No, because if you use non orthogonal transformations you would introduce non-zero values in the positions where you previously created zeros and thus it would be impossible to create the triangular matrix.

3. What is residual extraction? How is it realized in the SFG of QRD-RLS? For which application(s) is it relevant? Sometimes the actual filter coefficients aren't important, but only the residual is.

A posteriori residual = d_k - (u_k)^T*w_LS[k]
= epsilon*prod_{i=1}^{L+1}(cos(teta_i))
By simply multiplying all the cos(teta_i)'s and the final epsilon at the end of the RLS problem we can easily extract the error signal. This is mainly used in applications such as acoustic echo cancellation where only the error is important. Because the actual filter coefficients aren't important, we can skip the final step of calculating them through back substitution.

21. 11 Jan 2016

Question 1e

1. Formulate in your own words the Shur-Cohn stability criterion to test the 'stability' of a polynomial.

2. Where does the Shur-Cohn test appear in the context of FIR filter realisation, and what is its relevance there?

3. If the Schur-Cohn test is applied to a linear-phase FIR filter, what will be the outcome? What does this mean in terms of linear-phase FIR realization?

4. Where does the Shur-Cohn test appear in the context of IIR filter realization and what is its relevance there.

Question 2

1. What are oversampled DFT-modulated filter banks? What are particular advantages of such filter banks?

2. Consider an 8-channel DFT-modulated filter bank with 4-fold downsampling. How can the analysis bank and the synthesis bank be realized efficiently?

3. … And then how can perfect reconstruction be obtained in this case. (N=8, D=4)

Question 3

1. Explain how a square-root Kalman Filter algorithm is derived. Estimate (as 'O(xx)') the resulting computational complexity per sampling interval.

2. Explain how the Kalman Filter extends Recursive Least Squares parameter oversaFIRestimation.

22. 13 Jan 2015 (am)

1. Question 1:

    1. What are noise transfer functions? Explain how such noise transfer functions are used in the analysis of quantisation effects in filter implementations

    2. Do the properties of the quantisation mechanism play a role in such analysis?

    3. The analysis relies on linearity, whereas quantisation is a non-linear operation. Could this be a contradiction?

2. Question 2:

    1. Explain how polyphase decompositions are used for a convenient representation of maximally decimated analysis and synthesis banks. Explain in detail how the polyphase matrices are constructed.

    2. Explain how polyphase decompositions are used for a convenient representation of oversampled analysis and synthesis banks. Explain in detail how the polyphase matrices are constructed.

    3. Develop a similar polyphase decomposition based on representation for transmultiplexers, where the number of channels is equal to the up-/downsampling factor. We haven't seen transmultiplexers!

3. Question 3:

    1. Explain the main trick that is used in the derivation of the QRD-LSL algorithm.

    2. Compare the complexity of the QRD-LSL algorithm with the complexity of LMS.

    3. In an acoustic echo cancellation scenario with one loudspeaker and two microphones (ie. where an echo contribution is to be cancelled in each of the two microphone signals), is it possible to apply a QRD-LSL-algorithm? Sketch an efficient realisation (clearly defining input and output signals).

23. 13 Jan 2015 (pm)

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1  See wiki
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24. 12 januari 2015

1. Ruis

    1. Wat zijn noise transfer functions en waarvoor worden ze gebruikt?

    2. Heeft het quantisatieproces bij ruisanalyse effect op die analyse?

    3. quantisatie is een niet-lineair proces, de analyse veronderstelt lineariteit, dit lijkt een tegenspraak. Leg uit.

2. Filterbanken

    1. Leg in detail de vereisten uit voor de afwezigheid van aliasing bij polyfase gedecomposeerde filterbanken.

    2. Hetzelfde maar nu voor PR in plaats van AA.

    3. Leid equivalente eisen af voor een transmux.

3. Adaptieve filters (QRD updating)

    1. Leg de signal flow graph voor QRD RLS updating.

    2. We gebruiken hiervoor orthogonale transformaties, mag dit ook met niet-orthogonale transformaties? Waarom (niet)?

25. 14 januari 2014

1. We willen een filter ontwerpen zonder limit cycles. Hoe doen we dit? Overloop het ontwerpproces en licht de keuzes toe die gemaakt moeten worden.

    1. Wat is een oversampled DFT-modulated filter bank? Wat zijn de voordelen hiervan?

    2. Gegeven een DFT-modulated FB met 8 kanalen en 4-voudige decimatie. Hoe kunnen we op een efficiënte manier de analyse- en synthesebank realiseren? Hoe bekomen we een FIR unimodular PR FB? Hoe bekomen we een FIR paraunitary PR FB?

2. Acoustic echo cancellation met (N)LMS

    1. Wat is de invloed van het far-endsignaal op de convergentie? Wat zou een optimale keuze voor het far-endsignaal zijn?

    2. Wat is de invloed van het near-endsignaal op de convergentie? Is een on-offsignaal te verkiezen, of een continu signaal (bv. muziek)?

    3. Vergelijk de complexiteit van LMS en NLMS.

26. Januari 2013

1. Question 1

    1. What are noise transfer functions and how are these used to analyse the effect of quantization errors in filter realizations? Why is it that in such analysis filters, quantization noise sources can sometimes be lumped into a single noise source, for specific filter realizations? (use formulas)

    2. Are the assumptions under which such analysis is performed fully justified? (provide (counter-) examples)

    3. What are noise transfer functions like in a FIR lossless lattice realization?

2. Question 2

    1. Explain why and how polyphase decompositions are used to derive conditions for perfect reconstruction in maximally decimated filter banks.

    2. Consider a 6-channel filter bank with 3-fold up- and downsampling?. How can polyphase decompositions be used to derive conditions for perfect reconstruction? What is the condition for alias free operation? What is the condition for perfect reconstruction?

    3. Consider a 6-channel filter bank with 6-fold up- and downsampling, which provides perfect reconstruction. Does the same filter bank also provide perfect reconstruction under 3-fold up- and downsampling?

3. Question 3

    1. What is exponential weighting and why is it used in adaptive filtering? Explain how the LS cost function is modified by the exponential weighting.

    2. What is practical and advantageous about such exponential weighting?

    3. Explain how exponential weighting is included/appears in QRD-RLS.

    4. In an acoustic echo cancellation application, if the statistics of the far end signal are highly time-varying, would this require a larger/smaller exponential weighting factor?

    5. In an acoustic echo cancellation application, if the microphone records (lots of) background noise, next to echo and near-end speech, would this require a larger/smaller weighting factor?

27. Januari 2013

1. derive lossless lattice FIR,

    1. what is paraunitic transfer function?

2. Is lossless lattice paraunitair?

3. What is unimodular? lossless lattice unimodular?

2. derivation of 'overlap-save'. and relate to DFT oversampled filterbank

3. Discuss SFG QRD-RLS. deduce complexity and compare it with NLMS. extraction of residue, (additional question: why important with adaptive filter?). QRD RLS with or without residue extraction useful or not with adaptive filter?

De vragen hieronder werden toegevoegd op het oude VTK vakwikisysteem en werden later overgezet naar het huidige systeem

28. Januari 2012

1. Leg het gebruik van minimum least square optimisation uit bij het ontwerp van een FIR filters.

    2. Leg het ontwerp op basis van window-functies uit. Welke window zijn 'goede' windows die hiervoor in aanmerking komen? Waarom?

    3. Leg uit hoe het ontwerp op basis van windows gelinkt is aan het ontwerp op basis van minimum least square optimisation. Welk window? Welke gewichtsfunctie?

    4. Leg uit hoe OFDM (met cyclische prefix) een handig mechanisme heeft voor channel equalisation. Opdat dit zou werken moet het kanaal aan verschillende voorwaarden voldoen. Dewelke?

    5. OFDM kan ook voorgesteld worden als een transmuliplexer. Wat zijn dan de analyse/synthese filters (formules)? Aantal kanalen? Wat zijn de upsample/downsample factoren en wat is hun betekenis?

    6. Hoe wordt het QRD-LSL algoritme afgeleid van het QRD-RSL algoritme?

    7. Wat is residu-extractie? Hoe wordt dit gebruikt in acoustic echo cancellation?

    8. Hoe zou je dit implementeren voor het geval van 2 micro's en 1 luidspreker?

29. Januari 2011

1. Lattice ladder van IIR (dus ook die prove it)

    2. hoe weet je zeker dat $|a4| < 1$

    3. wat als alle a-coeff gelijk zijn aan nul. Is H~ dan nog altijd een allpass?

    4. Leg uit SFTF

5. Leg de link met DFT-gemod filterbanken

   1. Wat is de betekenis van het aantal kanalen?

   2. Wat is de betekenis van de decimatiefactor?

   3. Wat als de vensterlengte groter is dan het aantal kanalen?

6. Welke invloed heeft perfecte reconstrueerbaarheid op het ontwerp van de vensterfunctie?

7. Wat is MMSE? Teken de kostenfunctie voor een 2-tapsfilter

8. Wat betekent die kostenfunctie bij acoustic echo cancellation?

   1. Hoe verandert de kostenfunctie als de statistische eigenschappen van het far-end signaal veranderen?

   2. Welke invloed heeft het near-end signaal?

9. Waarom zet men in de praktijk soms de adaptatie af als het near-end signaal actief wordt?

10. Leg uit: Lossless Lattice FIR filter realisatie

11. Hoe wordt hierbij H~~ bepaald?

12. Bepaal H~~ en teken de realisatie voor: $H(z) = 1/\sqrt{2}*\cos(\theta) + z^{-1} * \sin(\theta)$

13. Leg uit MD-PR filterbanken en wat is de vereiste voor Perfecte reconstructie

14. Bespreek in detail de voorwaarde voor anti - aliasing

15. Bespreek in detail de voorwaarde voor Perfecte reconstructie

16. Bespreek QRD-LSL

17. Hoe ziet de implementatie van QRD-LSL voor accoustische cancellatie eruit met 1 luidspreker en 1 microfoon

18. Hoe ziet de implementatie van QRD-LSL voor accoustische cancellatie eruit met 2 luidsprekers en 2 microfoons

2. Leg uit hoe men aan de lattice ladder realisatie van een IIR filter komt. Wat gebeurt er wanneer $|a4| > 1$? Wanneer a4 gelijk is aan nul is Theta0 ook gelijk aan nul, wat als alle a-coëfficiënten gelijk zijn aan nul, hoe ziet H~ er dan uit? Is H~ nog steeds een APF?

3. Bijvragen: Wat voor filter is H~? (APF) Hoe zie je dat? (coëfficiënten in omgekeerde volgorde)

1. Wanneer op het einde de hele procedure moet herhaald worden op het overgebleven stuk, gaat dit zomaar? (Structuur is hetzelfde maar wat ook belangrijk is dat het overgebleven deel ook een APF is)

2. Wat is de betekenis van H~? (Ik heb gezegd dat die niet meteen voor iets dient maar dat die gewoon gebruikt wordt om tot die uiteindelijke structuur te komen)

3. Wanneer alle a-coëfficiënten nul zijn, welke structuur staat er dan nog?

4. Wat moet je doen wanneer $|a4| > 1$?

4. Leg uit wat Short Time Fourier Transform (STFT) is. Wat is het verband tussen STFT en DFT gemoduleerde filterbanken? Wat is de betekenis van de decimatiefactor? Wat betekent het aantal kanalen? Wat gebeurt er wanneer de window length groter is dan het aantal kanalen? Legt de eis om PR te hebben beperkingen op aan de keuze van de vensterfunctie?

5. Bijvragen: Vertel eens wat meer over filterbanken. (Ik was maar begonnen over die polyfase decompositie)

    1. Wat betekent frequentieresolutie?

    2. Bij die beperking op de vensterfunctie, hoe zit het met de inverse die je nodig hebt?

6. Leg de MSE kostfunctie uit bij optimale/adaptieve filtering. Schets de kostfunctie voor een filter met twee coëfficiënten. Wat is de betekenis van het minimum van de kostfunctie? Voor een echo cancellatie filter, wat gebeurt er met de vorm van de kostfunctie wanneer de statistische eigenschappen van het far-end signaal veranderen? Wat gebeurt er met de vorm wanneer het near-end signaal actief wordt? Waarom wordt praktisch adaptieve LMS uitgeschakeld wanneer er geen near-end signaal aanwezig is?

7. Bijvragen: Waarom hangt het optimale punt van de kostfunctie op een bepaalde afstand van het xy-vlak?

    1. Wat heeft de naam MSE te maken met de kostfunctie?

    2. Wat is /Xuu?

    3. Wat weet je over excess MSE? Is dat van toepassing wanneer de adaptieve LMS wordt uitgeschakeld wanneer er geen near-end signaal aanwezig is?

8. Leg uit hoe men aan de lattice ladder realisatie van een IIR filter komt. Wat gebeurt er wanneer $|a4| > 1$? Wanneer a4 gelijk is aan nul is Theta0 ook gelijk aan nul, wat als alle a-coëfficiënten gelijk zijn aan nul, hoe ziet H~ er dan uit? Is H~ nog steeds een APF?

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11. Bijvragen: Vertel eens wat meer over filterbanken. (Ik was maar begonnen over die polyfase decompositie)

    1. Wat betekent frequentieresolutie?

    2. Bij die beperking op de vensterfunctie, hoe zit het met de inverse die je nodig hebt?

12. Leg de MSE kostfunctie uit bij optimale/adaptieve filtering. Schets de kostfunctie voor een filter met twee coëfficiënten. Wat is de betekenis van het minimum van de kostfunctie? Voor een echo cancellatie filter, wat gebeurt er met de vorm van de kostfunctie wanneer de statistische eigenschappen van het far-end signaal veranderen? Wat gebeurt er met de vorm wanneer het near-end signaal actief wordt? Waarom wordt praktisch adaptieve LMS uitgeschakeld wanneer er geen near-end signaal aanwezig is?

13. Bijvragen: Waarom hangt het optimale punt van de kostfunctie op een bepaalde afstand van het xy-vlak?

    1. Wat heeft de naam MSE te maken met de kostfunctie?

    2. Wat is /Xuu?

    3. Wat weet je over excess MSE? Is dat van toepassing wanneer de adaptieve LMS wordt uitgeschakeld wanneer er geen near-end signaal aanwezig is?

Part II.

Second Document

!!! Note: please correct or add more to the answers

Last update: August 2024

30. August 13 2024

Question 1

1. Chapter-4 p.26,27,28: Describe briefly how these frequency responses have been generated. Explain how the comparison of these different responses leads to conclusions.

Ans: h[k] = h_d[k]*w[k]. Side lobes vs. smearing of main lobe

2. Chapter-5 p. 27: This can be seen as a filter bank without subband processing. Give E(z) and R(z).

Ans: ~Ch. 14 slide 19: E(z) = F*diag(1,1,1,1,0,0,0,0), R(z) = diag(1,1,1,1,1,1,1,1)*F^-1 = F-1

3. Chapter-5: Counting computational complexity of the different filter realisations as the number of multiplications per sample, which FIR filter realisation offers the lowest cost?

Ans: Direct form and transposed direct form

4. Chapter-6 p.13: Explain in your own words the meaning of the last equation, and how it leads to the given conclusions.

Same as Jan 12 exam

Question 2

1. Chapter-8 p.18 : Draw the SFG for the case of a multi-channelinput (as on chapter-7 p.19), but when 2 input signals instead of 3 are used with filter coefficients w0,…,w3 and w4,…,w7. Also give the LMS formula for this case.

2. Chapter-9 p.16: What is a 'rank-1 update' and how does it define the computational complexity of the standard RLS algorithm?

3. Chapter-10 p.20 ("The main trick…"): Redraw (sketch) the signal flow graph when the "main trick" is used to remove the column with R13. Define the relevant epsilon-signals in the signal flow graph (with subscripts & superscripts). Also indicate A, B, C, D & E.

4. Chapter-11 p.25: In the last formula, the matrices appear to cancel each other, so it appears that this notation is redundant. Why is the formula given like this?

The formula was given like this because the two matrices are obtained as a result of a QR decomposition. Basically, at time step k, we have a left hand side vector and a right hand side vector to which we are applying a QR decomposition. It was proven on p.24 that the left hand side vector is P-½ k/k-1 with the state space parameters at time k and that the right hand side vector is P-½ k/k-1 xk/k-1. By applying the same QR decomposition to both sides we will see that we get P-½ k+1/k and P-½ k+1/k xk+1/k respectively, which are then used to get xk+1/k. So, the formula was written like this since its two components originate from a QR decomposition.

Question 3

1. Chapter-12 p.14 : Explain the statement "PR guarantees distortion-free desired near-end speech signal". Where does the near-end speech signal appear in the signal flow graph?

2. Chapter-13 p.8 : Explain why, in the formula for E(z), the vector containing the delay elements appears as a row vector, while in the formula for R(z), it appears as a column vector

This is because in the R(z), it is multiplied from the left while in E(z), tit is multiplied from the right, this becomes more clear when the filterbank is oversampled, and tha matrix dimensions need to be correct to allow multiplication.

3. Chapter-14 p.27 ("Example-1: Define B(z4)…"): Specify B(z) for the case where N=5 and D=3

4. Chapter-14 p.27 ('Proof is simple'): Modify this proof for N=5 and D=3 and explain the non-trivial steps.

Question 4 (Acoustic modem project)

1. Chapter-3: How is the cyclic prefix removed in practice?

2. Chapter-3 p.36: What is the computational complexity (in general) of the steps 'S/P', 'FFT' and 'FEQ'?

3. Chapter-3 p.27 ('So this will be…'): A higher-order QAM-constellation (for instance 16-QAM instead of 4-QAM) can be used to increase the number of transmitted bits per second. In a practical channel, is there a limit to the order of the QAM-constellation?

4. Chapter-3 p.35 ('Note that…'): The frequency domain channel equalization relies on the inverse $(H_n)^{-1}$. How should the channel equalization be performed when $H_n = 0$?

31. January 30 2024

Question 1

1. Chapter-2 p.40 ("Polyphase decomposition: Example…"): Explain how the presented equations are exploited in general oversampled filter banks (=Chapters 12-13).

The polyphase decomposition splits the filter into two polyphase components $E_0(z^2)$ and $E_1(z^2)$. Then, using the noble identities we can operate the filtering process at a lower sampling rate. This is very helpful especially when it comes to filter banks since it would allow us to reduce complexity and increase efficiency since this would require less computation time and fewer samples.

2. Chapter-3 p.19 ("3. Least squares estimation…"). Explain the meaning of the parameters K and L, and how these have to be set. What procedure does Matlab use to solve such a least squares estimation problem?

K: number of considered samples
L: number of filter coefficients (filter order)

How to set them: K>L since what we want to have is an overdetermined set of linear equations (K equations L unknowns) which we solve using least squares estimation. In MATLAB we just do y/x.
3. Chapter-4 p.10 ("This is a 'Quadratic Optimization'…"): Provide similar formulas for Type-2 linear phase FIR filter design (p.8), and explain all the symbols used in these formulas.
4. Chapter-5 p.16 ("From (*) (p.12), it follows that…"): Generalize the given formulas for the case with 3 power complementary filters (as on p.20).

5. Chapter-6 p.33: Provide a justification for the 'lumping' of noise sources (as illustrated in subsequent pages) that is explicitly based on the given formulas (with 'DC-gain' and 'noise-gain').

They have the same gain transfer function going to the output so they are lumped together into one noise source and their means are just added together. (same with their variances)

Question 2

1. Chapter-7 p.24 ("MMSE cost function can be expanded as…"): How does the Wiener filter formula ($wWF=(Xuu)^{-1}Xdu$) and/or its components (Xuu and Xdu) change in the case of a 'linear combiner' problem (as on p.20)? What is the computational complexity (to solve the resulting set of equations) in this case (as on p.27)?

Xuu is not necessarily Toeplitz anymore since the inputs are distinct instead of being delayed versions of each other. Xdu also won't have any special structure. The equation wWF = Xuu-1Xdu stays the same since it doesn't depend on the special structures and it just what we get when we set the gradient equal to zero.

2. Chapter-8: Explain in your own words how the characteristics of the filter input signal (uk) influence the behavior of the LMS algorithm? What is then the 'ideal' filter input signal in this respect?

3. Chapter-9 p.35 ("4-by-4 example…"): If the adaptive filter input signal and the desired output signal have different 'dynamics' (for instance if the characteristics of one are very stationary, while the characteristics of the other are very non-stationary), would it be possible/useful to apply two different exponential weighting factors (a first one in the part corresponding to the input signal and a second one in the part corresponding to the desired output signal of the signal flow graph)?

4. Chapter-10 p.19 ("Theorem…"): The graph also shows an equality for the accumulated product of the cosines of the rotation angles. Provide an explanation for this equality.

The rotation cells compute bcos(theta)-asin(theta). At first the b is 1 meaning that the first rotation cell outputs cos(theta1), which is the b for the second rotation cell, thus the output of the 2nd rotation cell is cos(theta1)cos(theta2), etc.

5. In Chapter-11 p.24 ("relevant sub-problem is…"): Explain why the framed triangular matrix and corresponding right-hand side vector in the first equation correspond to the formulas that are added in the second equation.

Question 3

6. Chapter-12 p.36 ("A solution is as follows…"): Prove that the conditions Fo(z)=H1(-z) and F1(z)=-Ho(-z) indeed lead to alias-free operation. How is the frequency response of Fo(z) and F1(z) then related to the frequency response of Ho(z) and H1(z)?

Frequency response of F0(z) becomes the frequency response of H1(z) but shifted by pi (same as F1 and H0 however the magnitude is inverted so F1 becomes a HP filter if Ho is LP).

If we fill in these F1 and F0 in the formula for A(z) on slide 34, we can see that A(z) = 0. The alias transfer function is zero so we have an alias-free filterbank

2. Chapter-13 p.12 ("ii) Necessary & sufficient condition…"): Explain in your own words the statement "hence pr(z)=pure $delay = z - \delta$, and all other pn(z)=0". Explain how this then leads to the next formula.

3. Chapter-14 p.13 ("Conclusion: economy in…"): Explain in your own words the statement

"N filters for the price of 1".

You only need to implement the polyphase components of the prototype filter H0, and these will automatically be shited to the right frequencies by the IDFT that comes after

4. Chapter-14 p.27 ("Example-1: Define B(z4)…"):

a) Specify B(z) for the case where N=5 and D=2.

b) Derive the conditions for perfect reconstruction and specify when the resulting set of equations can be solved.

32. January 11 2024

Question 1

```
1    Chapter-2 p36: Give the link with chapters 12, 13 and 14
```

```
1    Chapter-3 p 29: Rewrite the equation for the case that the filter order of the
         FIR filter is greater than the cyclic prefix length.
```

```
1    Chapter-4 ("Filter Design"): Explain how the filter phase response is controlled
         in IIR filter design, and compare this to FIR filter design.
```

For IIR filters it is difficult to control the phase response because of their non-linear behavior + the impulse response is infinitely long

For FIR filter linearity is easily achieved + impulse response is finite

```
1    Chapter-6: Draw a 'parallel' realization of H(z) \= (1+az\-1)\-1 \+ (1+bz\-1)\-1
         and compute the noise transfer functions. Can the noise sources be lumped
         into equivalent noise sources?
```

5. Chapter-5, p.30: Explain why the highlighted element has to be a zero (compared to p. 22)

Explanation on slide 31, the E interchange positions in the matrix

Question 2

```
1    Chapter-7 p24: Does the Wiener filter solution WWF \= Xuu\-1 \* Xdu\-1 or its
         components change in the case of multi-channel?
```

It doesn't change because it is a generalized version in cascade with the input signals to the filter being delayed versions of each other (slide19-20)

```
1    Chapter-8: How can LMS be viewed as RLS, For a certain substitution in
         correlation matrix (Not sure about last part)
```

From chapter9 slide 15-16: an LS solution at time k can be computed from solution at time k-1. Matrix update using rank-1 matrix -> computing the inverse -> computing the coefficients by vector update. Basically Kalman gain vector x a priori residual, you check the previous set of coefficients and apply that to the input to get the next output. The value of the a priori determines the direction of the gain vector + if it has to be updated (a=/0) or not (a=0)

```
1  Textbook p63. "The original LMS algorithm is a simple one line updating algorithm,
       which may be derived from the RLS algorithm by ignoring the covariance matrix
       update and setting Xuu(k-1)^-1 to I in the updating formula for w leading to ..."
```

```
3
  1    Chapter-9 p38: u\[k\] is an all zero vector and R is of full rank. Give the a
          posteriori and a priori residuals for this case.
```

A posteriori residual = multiplication with the product of the rotor angles

A priori residual = division with the product of the rotor angles

If u[k] is all zero $\Rightarrow$ a posteriori residual = a priori residual = d[k]? Which makes sense since the filter can't adapt if no input is provided

```
4
  1    Chapter-10 p.20 ("The main trick..."): Redraw (sketch) the signal flow graph
          when the "main trick" is used to remove the column with R15, R25, ....
          Define the relevant epsilon-signals in the signal flow graph (with
          subscripts & superscripts).
```

```
  1    Chapter-11 p25: Can residual extraction be added here? What do the residuals
          then mean?
```

Question 3

```
5
  1    Chapter-14 p.27 ("Example-1: Define B(z4)..."): Specify B(z) for the case where
          N=7 and D=4 and provide the corresponding proof that a 7-channel DFT-
          modulated filter bank is obtained with this B(z) (similar to the proof on p
          .27 for N=8 and D=4).
```

```
  1    ???
```

```
3
  1    ???
```

```
  1    ???
```

Unknown Qs:

chapter-5, p.30: Explain why the highlighted element has to be a zero (compared to p. 22)

chapter-12, p.12: If D=4 (instead of D=3) and if the Gi's are not equal to 1. What is the condition for alias-free operation (a general formula is sufficient here) and what would be the resulting (linear) "distortion function"?

Attenuation<1 => subtracting the signals? Change in Gi, SNR is decreased

chapter-13, p. 11: Verify for when R(z)E(z) is switched with the up sampling instead of the down sampling

chapter-13, p. 32: For D=1, N=4, Le=3, design a simple H(z) and F(z).

33. August 17 2023

Question 1

1. Chapter-4 p.12 ('This leads to an equivalent ('discretized')..'): Rewrite the quadratic optimization function as a least squares estimation problem for an overdetermined set of equations of the form minx ||A.x-b||2 (i.e. define A, x, and b). (Already in slides ?)

2. Chapter-5 p.8 ('FIR/3. Lattice Realization'): Explain why the special case |ki |=1 with a rank-deficient transformation matrix is problematic in the mathematical derivation.

3. Chapter-5: Explain in your own words how a direct-form FIR is generalized to an IIR lattice-ladder realization. Provide the expression for H^tilde (p.37) in the FIR-case.

4. Chapter-6 p.16 ('Example (continued)'): Explain in your own words the relevance of the 'coupled realization'.

Question 2

1. Chapter-8 p.27 ('Normalized LMS'): Explain in your own words the meaning of the two terms in the 'specific optimization problem' that corresponds to NLMS for the case where the step size is equal to 1.

The problem here is represented as a cost function that needs to be minimized. The main objective here is to minimize the a posteriori error, which is the second term. However, assume the cost function is only minimizing the a posteriori error with no regard to previous NLMS solutions (aka alpha = 0). Here, we could witness high fluctuations in NLMS solutions from one time step to the other, which results in a bad performance in time varying environments. This is why the first term exists: i tis here to penalize high fluctuations in NLMS solutions and adds a dependency of the currents weights on the previous weights, thus making sure we are getting smooth changes in weights while minimizing the a posteriori error.

2. Chapter-9 p.35 ('4x4 example'): Consider the cell that has 'R 24'. Specify (in words or by means of a formula for the 'epsilon-signals') all in- and outgoing arrows, i.e. all in- and outgoing signals/parameters.

3. Chapter-10 p.21 ('Result = QRD Lattice..'): Are the least squares residuals extracted with the fast QRD-LSL algorithm exactly the same (or an approximation of) the least squares residuals extracted with the original (non-fast) QRD-RLS algorithm?

4. Chapter-11 p.23 ('A RECURSIVE implementation…'): in your own words the statement "…to require only the lower-right/lower part".

To turn a triangular matrix with one extra row into a triangular matrix again, we only need the last 2 rows.

Question 3

1. Chapter-12 p.40 ('Similarly, for…'): For D<N (instead of D=N) the dimension reduction of the polyphase matrices (from NxN to DxN and NxD, as graphically illustrated in the slide) appears to reduce the design degrees of freedom (i.e. if D is made smaller than N, the number of polyphase components is reduced). Does this give a disadvantage in the perfect reconstruction design procedure? Does this conflict with the general observation that oversampled filter bank design is easier then maximally decimated filter bank design?

No, since we can adjust the orders of the FIR filters in R(z) and E(z) such that we get a number of unknowns higher than the number of equations. This will essentially mean that the set of equations will have more solutions hence making the design easier. Also, in the maximally decimated case, LR has to be very large, which corresponds to saying that every filter in R(z) has to be an IIR filter, which could introduce stability problems.

2. Chapter-13 p.19 ('PS: Inversion of…'): Explain in your own words the relevance of the observation that some FIR matrix transfer functions can have an FIR inverse.

3. Chapter-14 p.27 ('Example1…'): Specify B(z 3) for the case where N=5 and D=3.

4. Chapter-14 p.27: Modify the 'Proof is simple' part for the case N=5 and D=3.

Question 4 (Acoustic Modem Project – Only if this exam is a 2022-2023 retake exam)

1. By including the cyclic prefix, the OFDM modulation format provides an easy equalization of the transmission channel. What are conditions that the transmission channel has to satisfy for this 'cyclic prefix trick' to work properly? Are these conditions satisfied in a practical acoustic channel?

2. How does the distance between the transmitter (loudspeaker) and receiver (microphone) influence the transmission, or in other words what happens if transmitter and receiver are moved away from each other?

3. Chapter-3 p.27 ('So this will be…'): A higher-order QAM-constellation (for instance 16-

QAM instead of 4-QAM) can be used to increase the number of transmitted bits per

second. In a practical channel, is there a limit to the order of the QAM-constellation?

4. Chapter-3 p.35 ('Note that…'): The frequency domain channel equalization relies on the

inverse $(H n)^{-1}$. How should the channel equalization be performed when $Hn =0$?

34. January 31 2023

Question 1

1. Chapter-3: Specify the computational complexity (number of multiplications per second) of the DMT receiver operations

(only those discussed in Chapter-3), provide (approximate) formulas that contain the main DMT parameters (N, L, symbol

rate, …).

DMT = discrete multi-tone

Receiver: (see slide 36): FFT: N.log(N) calculations, Channel equalization: N/2 calculations (see slide 26 (symbols are

duplicated(complex conj) to create real signals)), and every N/(symbolrate) seconds this calculation needs to be done =>

number of multiplications per second = (N.log(N)+N/2).symbolrate/N

2. Chapter-4 p.8 ("Linear Phase FIR Filters – Type 1…Type 2…"). Explain why 'modulating' a Type-3 filter results in
   a Type-3 filter. Illustrate the statement "BP<->BP" by sketching the filter frequency responses. Are the two 'BPs'
   exactly the same?
   I think you interpret a modulation of -1,1,-1,1… as a shift of pi. So it makes sense that LP <-> HP but BP <-> BP.
   https://tomroelandts.com/articles/spectral-reversal-to-create-a-high-pass-filter

3. Chapter-5 p.18 ("Repeated application results in 'lossless lattice' …"): Explain the appearance of the multiplication
   by 'cosq4' and 'sinq4' in this lossless lattice realization.
   The rotation angles preserve the norm of the vectors and the power of the input signal becuase of their
   orthogonality$\Rightarrow$ lossless. The cos(theta4) and sin(theta4) appear because the system is a 4th order system (order =
   #delay elements)

4. Chapter-5 p.30 ("Derivation similar to p.22 (overlap-save, similar for overlap-add) …"): Explain why the highlighted
   element in the first formula has to be a zero (unlike in p.21 and p.22).
   Same as in Jan 12 exam

5. Chapter-6 p.32 ("Statistical analysis is based on the following assumptions…"): For the given example, compute
   the noise transfer functions for e1 and e2. Can e1 and e2 be lumped into an equivalent noise source? Why (not)?
   They can be i think, like the transposed form

Question 2

1. Chapter-7 p.8 ("example: channel identification") and p.14 ("example: channel equalization (training mode)"): Explain

and compare these two example applications.

Channel identification = to provide the mathematical model for signal propagation

Channel equalization = the same but multipath propagation?

Channel identification uses system identification (i.e. provides a model for the channel) whereas channel equalization is

a way of doing inverse modeling (the plant which is the mobile reciever outputs to the adaptive filter and the desired

response is the initial training sequence)

2. Chapter-8 p.21 ("LMS analysis in a nutshell: Noisy gradients.…"): Explain the 'noisy

gradients' effect in an acoustic echo cancellation set-up, with a near-end speaker that is

sometimes active and sometimes not active.

Expectation implies statistical information which in practice is not given. So you use estimations from the observed signal. These estimations of course are not perfect so what happens is that the gradient has an error and therefore will not be 0 at the WF solution. Wont be zero at the instantaneous value

3. Chapter-9 p.35 ("4-by-4 example"): Give an intuitive explanation for the fact that exponential weighting can be implemented here only by adding multiplications with l after the delay operations.
   Exponential weighting = giving less weight to older samples.
   I would assume the weighting has to come after the process is done so that it would not change the values?
   See slide 30: R[k-1] and z[k-1] is multiplied with lamba => every element of R[k-1] and z[k-1] is multiplied with lambda for the calculation of the new R[k] and z[k]

4. Chapter-10 p.13 ("Preliminaries / LS residuals are not changed…"): Explain in your own words how this property is exploited in p.19 ("Theorem").
   From slide 13, we know that after random fixed permutation the LS residual remains unchanged $\Rightarrow$ the order is not affected. Similarly, in slide 19, this is shown when the same signal is produced in 2 different ways, one at the addition of rotational elements and one at the source signal u. This means that the system is redundant and has high computational complexity

5. In Chapter-11 p.24 ("Relevant sub-problem is…"): If the Kalman Filter would also have to compute xk-1|k (next to xk|k and xk+1|k) how would the formulas on the slide have to be Adapted?
   I assume the matrix size wll have to change, adding one more row for sure. But for the other matrices idk
   Matrices are expanded to include the equations at time k-1 since this is where our relevant subproblem will come from (check p.20). The inherited upper triangular matrix (p.24) will be inherited from k-2 so the matrix is effectively Pk-1|k-2 since we are using samples up to k-2 to get an estimate of xk-1.
   Question 3

6. Chapter-12 p.4 ("What we have in mind is this…"): In the given block scheme (without any upsampling/downsampling), what would be the condition for perfect reconstruction? Would a set of power complementary filters provide perfect reconstruction?
   Perfect reconstruction $\Rightarrow$ IN = OUT $\Rightarrow$ y[k] = u[k-d] (general condition)
   Based on intuition, power complementary filters should do perfect reconstruction since we don't have downsampling/upsampling, and in that case we would have zero aliasing whereas if we downsample/upsample, we will have some unwanted response which is cancelled out using the synthesis filters usually.

7. Chapter-13 p.18 ("Design Procedure:…"): Explain in your own words how the stability issue mentioned in the last sentence is overcome by having D<N instead of D=N?
   At the D = N case, the order of R(z) Lr has to be infinitely large so it leads to an R(z) IIR where stability is not ensured.
   At the D < N case, the Lr has to be sufficiently large for an underdetermined set of equations can be solved so R(z) is always FIR therefore, there are no stability issues, as we know FIR filters ensure stability.
   Regard this answer, makes more sense; in the D=N case we are using the inverse of E matrix which always leads to IIR slide 19

8. Chapter-13 p.33 ("Example N=4…"): Indicate how the formula changes (number of

unknowns versus number of equations) if the order of all the synthesis filters is increased
by one (=copy the formula and then highlight where in the matrices additional entries
appear).
We know that the number of coefficients is equal to Lr + Le -1 and the number of equations is equal to Lr + Le. From
the inequality in slide 31, we can deduce that there cant be perfect reconstruction if the #unknowns >= #equations.
+++

9. Chapter-14 p.27 ("Example-1: Define B(z4)…"): Specify B(z) for the case where N=4 and
   D=3 and provide the corresponding proof that a 4-channel DFT-modulated filter bank is
   obtained with this B(z) (similar to the proof on p.27 for N=8 and D=4).
   N=4, D=3 $\Rightarrow$ N' = NxD / gcd(N,D) = 4x3 / 1 = 12
   Adapt the equation on slide 26 for H0(z) and En'(z)
   Adapt B(z^12) matrix on slide 27, matrix size DxN
   Adapt the equation on slide 27 for proof of B matrix

10. Chapter-15 p.12 ("If maximally decimated…"). Provide an interpretation of the last formula
    by comparison with the DTFT on p.4. (What are the basis functions now? How many basis
    Functions?

The STFT windows certain sections of the signal at time k while DTFT analyses the whole signal with no regards to time
localization. Here the basis functions are the impulse responses of the synthesis filters and their shifted versions over all
possible values of kbar.

35. January 12 2023

Question 1

1. Chapter-4 ("Filter Design"): Explain how the filter phase response is controlled in IIR filter design, and compare this to
FIR filter design.
In IIR, the phase response is influenced by the poles. In the z-plane, the angle of the complex pole determines the phase
contribution to the overall phase response. The pole angle determines how phase changes as f increases. Poles closer to
the unit circle and smaller angles induce smaller phase changes across f while poles near the centre of the unit circle and
large angles produce abrupt changes in phase => this is from chatGPT, so use at your own risk.

2. Chapter-5 p.10 ("Repeated application results in 'lattice form' …"): Explain the appearance of the multiplication by
   'b0' in the lattice realization.
   My guess is that you can't get rid of the b0 because there is a limit to the number of order reductions you can
   accomplish. In p.8 it wouldn't make sense to get rid of the z^-0 term because you would be multiplying by 0 in that
   case.
   If you look at the bottom of slide 8, there are equations provided for the new coefficients you get from doing a single
   iteration of the lattice 'application'. b4 is substituted for a coefficient k0 that is common to both y[k] and y~[k] . It
   states that b0' = b0, i.e. b0 is unchanged. So it stands that after repeated application, b0 is still unchanged, and
   thus can be applied right at the start of the realization. (This is really just something I noticed, my explanation is
   probably gibberish though)

3. Chapter-5 p.28 ("Derivation similar to p.22 (overlap-save, similar for overlap-add) …"): Explain why the highlighted element in the first formula has to be a zero (unlike in p.21 and p.22).
    My guess is that since you have only 4 b terms (because for 4-fold decomposition there are 4 expressions) compared to 5 in p.22 it makes sense that there is a 0 left over.

4. Chapter-6 p.13 ("Coefficient quantization effect on pole locations / Higher-order systems…"): Explain in your own words the meaning of the (approximate) equation, and how it leads to the given conclusions.
    For the Lth root, the quantisation error for the root is the summation of L with, in the denominator, a product of the distances of the Lth root that you are analysing and all other roots that you have. So, in a high order polynomial you are going to have many roots (and of course they all have to fit inside the unit circle) so the distance between some of these poles is going to be very small. So large error, and therefore high-order is bad. They will be closer together since there will be less space => more sensitive
    Also, the (ai~ - ai) component implies that if the summed difference between the intended coefficients and the quantised coefficients (the total error in the coefficients due to quantisation) is large, the difference between the intended root and the quantised root you get is larger. This makes sense, because if your quantization does a poor job of representing the filter coefficients, it's reasonable to assume your root approximations will also suffer.

5. Chapter-6 p.35 ("PS: In a direct form realization…"): Explain in detail why it is that all quantization noises can be lumped into e1 and e2. What are the corresponding noise transfer functions?
    The quantization noises can be lumped into equivalent sources because of the linearity of the direct form realization. It is 2 equivalent sources because of the structure of the input/output and the position of the adders/multiplies. We know that after each adder/multiplier there is a noise source.
    Not really sure what on earth he means by TF of the noise sources, but heres my honest guess. I think the TF of ei[k] is just the filter TF since its introduced at the same point as u[k]. By this logic e2[k] has a TF of 1 (e.g. nothing happens to it at the ouput) because it's introduced at the final adder of the realization (and isn't fed back into the filter).

Question 2

1. Chapter-7 p.19 ("PS: Can generalize FIR filter to 'multi-channel FIR filter'…"): For this specific (3-input) example, provide formulas for the (input auto-)correlation matrix, the cross-correlation vector, and for the Wiener filter.
Matrix for Xuu matrix slide 23. I assume the vector ukT will become a 3x3 matric because of the 3inputs as in slide 19 ⇒ the Xuu will triple in size? The matrix will still be symmetrical, non negative and have the toeplitz form
I agree that the u_k becomes a 3x3 matrix: u = [u_0[k] u_1[k] u_1[k] ; u_0[k-1] u_1[k-1] u_1[k-1] ; u_0[k-2] u_1[k-2] u_1[k-2]]. But that means that X_uu = E{u u} still stays a 3x3-matrix.

Maybe the 3x3-matrix isn't the right option for u, but make u a 9x1-vector. Then, the X_uu matrix will become a 9x9-matrix that is symmetric and Toeplitz.

2. Chapter-8 p.37 ("compare to p.32-33…."): Explain in your own words the appearance of the summation in the first formula, and the statement (in p.36) that "D takes the place of L+1".

3. Chapter-9 p.26 ("QRD for LS estimation…"): Explain why the ' * ' in the second formula does not play a role (in the third formula).
    It does not play a role because when you look at the right-hand triangular matrix, on the bottom of that is 0, since this multiplies w, that portion that the minus sign multiplies will be 0. There is no w for the bottom row that will minimise the cost function because there is no w for that part.

4. Chapter-10 p.20 ("The main trick…"): Redraw (sketch) the signal flow graph when the "main trick" is used to remove the column with R12. Define the relevant epsilon-signals in the signal flow graph (with subscripts & superscripts).

We always keep the diagonal elements! In this case, from column 2 we only remove element R12. The epsilon notation shows were the triangular matrix starts and ends in a sense. Element D ϵ k+1 k-1:k, element E ϵ k k-1:k, element before the delay ϵ k k-1:k+1, element aftr the delay ϵ k-1 k:k

5. In Chapter-11 p.10 ("PPS: Note that if in p.9 matrix U has only 1 row..."): Explain in your own words, based on the given formulas, how the standard RLS algorithm formulas can be related to a specific linear regression parameter estimation problem with a specific 'initial estimate' .

Question 3

1. Chapter-12 p.29 ("Now insert DFT-matrix..."): Matrix F is said to be a DFT matrix, but more generally can be any square invertible matrix. In general, would it also be possible and meaningful to have a non-square matrix F (with some alternative for the inversion)?

In theory no, since you can't compute the inverse of a non-square matrix but the IDFT is different. If you do the inverse of a DFT you simply need to do F^H where H is conjugate and transposed, so by using this you can use rectangular DFT matrices instead of just square. Look at slide 26 of chapter 3, some formula to do with this.

2. Chapter-13 p.12 ("Necessary & sufficient condition for PR is then..."): Based on the first formula, provide an expression for the distortion function (function of delta and r).

3. Chapter-14 p.13 ("Conclusion: economy in..."): Explain in your own words the statement "N filters for the price of 1". I guess this is about only having to design a prototype filter that serves for multiple subbands. In slide 11 you just cycle through the n values to get a new filter every time.
   See slide 13: H_0, H_1, ... all use the results of the same E_0, E_1, E_2 and E_3. In other words, you don't have to implement all the polyphases of each H separately.

4. Chapter-14 p.27 ("Example-1: Define B(z4)..."): Specify B(z) for the case where N=8 and D=6 and provide the corresponding proof that an 8-channel DFT-modulated filter bank is obtained with this B(z) (similar to the proof on p.27 for N=8 and D=4).
   Not specific to this question but in general N defines the number of rows and D defines the number of columns. The number of elements within the matrix is given by N' (formula shown in some previous slide). You always start at the top left and always go diagonally down. Whenever you reach the rightmost column, you drop 1 row down and go to the leftmost column. Whenever you reach the bottommost row, you shift 1 column to the right and go to the topmost row. Whenever you do the shift from right to left you multiply the element by z^-D. This results in the matrix B(z^D). (as in slide 27). This is then converted to B(z) because you use the nobel identities to switch the structure round (downsampling before). You divide the exponent of z by the downsampling factor (Note: after a lot of consulting reports I found online and a bit of trial and error, I found that the exponent of z becomes N'/D = N/gcd(N,D) rather than dividing it by D. I know this seems weird for this example since the exponent becomes 4, but it's consistent with the other examples i've seen, and just dividing it by D gives you a fraction = 4/3, which makes even less sense).=> This is just what I observed, not any formal rules, Moonen at one point in the audio says "do not ask questions, this is how we do it" (for the general construction of matrix).

5. Chapter-15 p.15 ("Example: N=4, d=2...'"). Explain/derive the "minimum norm solution" formulas. What is the relevance of such a "minimum norm solution"

36. January 13 2022

Question 1

1. Chapter-4 p.10 ("This is a 'Quadratic Optimization'..."): Provide similar formulas for Type-2

linear phase FIR filter design, and then explain all the symbols used in these formulas.

Maybe something like this: H()=e-jL/2cos(/2)Gd()

And in the long equation add cos(/2)Gd() where Gd is.

Gd is the amplitude factor of the desired signal

2. Chapter-5 p.6 (p7) ("Starting point is direct form…"):

   a. For the second realization (i.e. after retiming) specify the noise transfer function
      $(G(z)=\ldots)$ for every individual arithmetic operation.

   b. Which of these noise transfer functions can be lumped, to simplify further analysis?
      Explain why such lumping would indeed be allowed.

3. Chapter-5 p.18 (slide16 i think) ("From (*) (page 14), it follows that…"):

   a. Provide similar formulas for the next order reduction (i.e. the second order
      reduction, following the first order reduction as specified on this page).
      To reduce the 3rd order, b0xb3+ ~~b0 x ~~b3 = 0 continue with the orthogonal vectors. From the matrix with the b
      coefficients we add the zero on the first line in order to shift the bs and the zero on the second line of the matrix to
      remove the coefficient b3 in this case. Continue with the same process on the slide 16

   b. After the last order reduction, explain why a specific 0-order system was obtained (see 'Explain' on p.20).
      B0 remains after every order reduction but idk why specifically (slide 10)

Question 2

1. Chapter-7 p.24 ("MMSE cost function can be expanded as…"): How does the final Wiener
filter formula $W_{WF}=(X_{uu})^{-1}.X_{du}$ change in the case of a multi-channel FIR filter problem (as
on p.19)?

The uk will be a vector? (slide20)

As in the explanation of exercise 2.1 of jan 12 2023, I think u_k becomes a 3x3-matrix and that the w_wf changes from a
vector to a 3x3-matrix because X_du is now a 3x3-matrix instead of a vector (not sure of this anymore)

2. Chapter-9 p.37&38 (slide19) ("2.3 Exponentially weighted RLS…"): Explain (intuitively) how the inclusion of an
   exponential weighting factor "lambda" in the LS cost function leads to the appearance of factors 1/(lambda)2 in the
   RLS formulas.
   1/lambdaˆ2 appears because we are computing an inverse matrix
   Intuitively: lambda is multiplied with the input of time k-1 to have less emphasis on the older inputs. X_uu en X_du
   ((in slides N_uu and N_du is used)) are by definition ~ transpose(u).u so the X_uu[k-1] and X_du[k-1] are multiplied
   by lambdaˆ2. Now, in the RLS formulas the inverse of X_uu and X_du are used => 1/ lambdaˆ2

3. Chapter-9 p.21 (slide 35?)("4-by-4 example…"): If the adaptive filter input signal and the desired output signal have
   different 'dynamics' (for instance if the characteristics of one are very stationary, while the characteristics of the
   other are very non-stationary), would it be
   possible/useful to apply two different exponential weighting factors (a first one in the part
   corresponding to the input signal and a second one in the part corresponding to the
   desired output signal of the signal flow graph)?
   Slide 30, there can be the same exponential weight factor

4. Chapter-9 p.35 (chapter-10 slide20) ("The main trick..."): Redraw (sketch) the (relevant parts of the) signal flow graph when the "main trick" is used to remove the column with R15, R25, etc. Define the relevant epsilon-signals in the signal flow graph (with subscripts & superscripts).

   Remove elements R15, R21, R35, R45. Dont remove R55 because it belongs in the diagonal of the triangular matrix A (slide19). I think the epsilon connect where we want the triangular matrix to end. It will connect to u[k-4] and u[k+1] for the inputs and the output to what remains, at the R55 element. So they will be defined as ϵ k+1 k-1:k (D matrix element), ϵ k-3 k-1:k (E matrix element), ϵ k-3:k-1:k+1 (before the delay element), ϵ k-4 k-3:k (after the delay element)

5. In Chapter-10 p.20(chapter-11 slide 24) ("relevant sub-problem is..."): Explain in detail why the Kalman filter problem can indeed be reduced to the specified sub-problem.

In a Kalman filter, we can reduce the complexity by not computing the smoothed values (x0|k, x1|k, ...) since the set of equations at time k can be reduced to a triangular set which allows triangular backsubstitution, meaning that xk+1|k will be computed first, hence solving the main problem of computing xk+1|k without using the smoothed values.

Question 3

1. Chapter-11 p.12 (chapter-12 slide14) ("Filter Bank Applications"): Explain the statement "PR guarantees distortion-free desired near-end speech signal".

Perfect reconstruction aims for IN = OUT signal by using appropriate filter banks and adding gain +++

2. Chapter-12 p.10 (chapter-13 slide30) ("Design Procedure:..."):

   a. Explain in detail the last sentence ("It will turn out..."). Why is D=N excluded?
      For D=N there is no FIR for R(z), it will be IIR

   b. Provide a "contrived exception" for D<N.
      +++

3. Chapter-13 p.20 ("A filter bank representation..."): i cant find which slide it is referring to

   a. If H(z)=I, then perfect reconstruction is achieved in that y[k] will be a delayed version of u[k]. In the general case where H(z) is N-by-N, how large is the delay? Provide an intuitive explanation for this.

   b. Explain in your own words how polyphase decompositions are exploited to trade off delay (latency) against complexity in frequency domain filter realizations (chapter-5 slide26)
      For large sample blocks $\Rightarrow$ large complexity reduction but large latency
      Have to derive intermediate realizations with smaller latency and the expense of smaller complexity reduction
      TRADE OFF

37. February 1 2022

Question 1

1. Chapter-4 p.10 ("This is a 'Quadratic Optimization'..."): Provide similar formulas for general (i.e. not restricted to linear phase) FIR filter design, and explain all the symbols

used in these formulas.

Arent all FIR filters linear by default? I dont understand the q

(See slide 6 and 7) Phase is non-linear if impulse respons is non-symmetric => not possible to simplify to freq response with cosines

=> c does not exist anymore of cosines but of exp()

=> x doesn't consist of d_ks but h[k]s

2. Chapter-5:

   a. Explain in your own words the relevance of the Schur-Cohn stability test appearing
      in the derivation of the FIR lattice realization.
      We use the schur-cohn stability test to see if all the zeros of B are withing the unit circle. We compute all b_i's and from those all the k_i's. To be stable the condition $|k_i| < 1$ has to be satisfied.

   b. Explain in your own words the relevance of the Schur-Cohn stability test appearing
      in the derivation of the IIR lattice-ladder realization.
      For the same reason but this time we compute the a_i's and from those all the k_i's which correspond to the rotation angles $\sin(\theta)$

3. Chapter-5 p.12 ("Repeated application results in…"):

   a. For the special case with $k_0=k_1=k_2=0$, specify the noise transfer functions ($G(z)=\ldots$) for every individual arithmetic operation.

   b. Which of these noise transfer functions can be lumped, to simplify further analysis? Explain why such lumping would indeed be allowed.

Question 2

1. Chapter-7 p.24 ("MMSE cost function can be expanded as…"): How does the Wiener filter formula ($w_{WF}=(X_{uu})^{-1}X_{du}$) and/or its components (X_{uu} and X_{du}) change in the case of a 'linear combiner' problem (as on p.20)?
Same as 13/1/22 exam

2. Chapter-8: Explain in your own words how the characteristics of the filter input signal (u_k) influence the behavior of the LMS algorithm? What is then the 'ideal' filter input signal in this respect?
   I have written down that the observant signal is related somehow to auto-correlation, cross-correlation and that it has to be computed over a time window, maybe something about stochastic characteristics?
   See slide 12

3. Chapter-9 p.16 ("QR-updating for RLS estimation…"): If the adaptive filter is used in an acoustic echo cancellation application, should the tuning of the exponential weighting factor (i.e. setting the lambda to a large or small value) be based on the characteristics (dynamics) of the input signal, or on the characteristics (dynamics) of the path, or both?
   Same as previous exam

4. Chapter-9 p.31 ("Preliminaries / LS residuals are not changed…"): Explain in your own words how this property is exploited in p.36 ("Theorem").
   Same as previous exam

5. In Chapter-10 p.6 (chapter-11 slide7) ("The Mean Squared Error (MSE) of the estimation is.."): In which sense is the MSE defined here different from the MSE used in the MMSE problem formulation in Chapter-7?
   Does not take into account the desired signal?

Question 3

1. Chapter-11 p.10(chapter-12 slide 13) ("Filter Bank Applications/Subband Coding"): Explain the relevance of perfect reconstruction in (lossless) subband audio coding.
The full band signals are split into subbands and then are downsamples. The subband signals are separately encoded

2. Chapter-12 p.24(chapter-13 p.30) ("Design Procedure:…"): Explain in your own words how the stability issue mentioned in the last sentence is overcome by having D<N instead of D=N?
   Same as previous exam

3. Chapter-12 p.30 (chapter-14 slide 27) ("Example-1: Define B(z).."):

a. Provide a formula for the B-matrix in the case where N=6, D=5.
   N' = NxD / gcd(N,D) = 6x5 / 1 = 30
   Same as previous exam

b. What exactly is demonstrated by the proof at the bottom of the page?
   6-channel DFT-modulated filter bank is obtained with this B(z)

4. Chapter-13 (chapter-5 slide 26): Explain in your own words how polyphase decompositions are exploited to trade off delay (latency) against complexity in frequency domain filter realizations
   same as previous exam

38. January 7 2021

Question 1

1. In Chapter-3 p.19, explain the meaning of the formula. What would be a suitable solution strategy to solve the minimization problem? Idk which slide it is referring to

2. In Chapter-4 p.12, what could be an alternative strategy to solve the minimization problem, alternative to computing $Q^{-1} \cdot p$? Illustrate by means of one or a few formulas.
   Use the minmax design?

3. Counting computational complexity of a filter realization as the number of multiplications per sample, which FIR filter realization offers the lowest computational complexity? Provide concise complexity indications for the FR filter realizations that have been discussed.
   The direct form has the lowest computational complexity because it uses less adders and multipliers and u[k], u[k-1] are produced at the same time

4. In Chapter-5 p.14,(chapter-6 p20) why is a 'scaling' needed? Is the scaling unique? Illustrate by means of a figure.
   Scaling is needed to ensure that the roots of R(z) is inside the unit circle

5. In Chapter-6 p.34, explain in detail why it is that all noise transfer functions are the same up to a delay. And then why is it that the delay does not play a role, such that the noise sources can be lumped?
   Noise transfer functions are equal all together because of the fact that the DC gain and the noise gain are equal thats why the noise TF can be lumped into one equivalent noise source.
   Idk about the "up to a delay" part

Question 2

1. Provide an intuitive explanation for the statement (and the condition under which the statement holds) that in an acoustic echo cancellation set-up, the irreducible error is equal to the variance of the near-end signal, Chapter-7 p.38.

2. What is a 'rank-1 update', as mentioned in Chapter-8 p.34 (chapter-9 slide 16), and how does it define the computational complexity of the standard RLS algorithm?
   Rank-1 update = inverse common correlation matrix
   The complexity is reduced

3. Explain the relevance of 'residual extraction' in an acoustic echo cancellation setup, Chapter-9 p.26 (slide 40).
   In general residual extraction can be used to compute least squares residuals without explicitly using the least squares filter vector. It models the acoustic path from the loudspeaker to the microphone ⇒models the echo contribution in the microphone's signal which is the least squares residual. The a priori and posteriori are being extracted sufficiently and then subtracted ⇒ only thing that matters are the residuals and not the filter coefficients in this case.

4. How is the property illustrated in Chapter-9 p.32 (slide 26) exactly used in the derivation of the QRD-LSL (least-squares-lattice) algorithm?
   Orthogonal factorization and triangular substitution?

5. In Chapter-10 p.20(chapter-11, p.24), explain why the quantities propagated from time k-1 to time k in the first formula can be identified as indicated in the second formula.

Question 3

1. In Chapter-11 p.34 (chapter-12 slide38), explain why the E(zN) has polyphase components ordered in a per-row fashion, whereas the R(zN) has polyphase components ordered in a percolumn fashion.
Is it because of downsampling and upsampling?

2. In Chapter-12 p.11(chapter-13 slide 31), explain the statement that there are D.D.(LE+LR+1) equations and indicate what these equations look like.
   (Le+Lr+1) = #coefficients DxD = dimension of the identity matrix = #equations

3. In Chapter-12 p.25 (chapter-14 p.21), explain the statement that E(z) is FIR & unimodular if and only each En(z) is FIR & unimodular.

4. In Chapter-13 p.7, rewrite the formulas such that the 'selection matrix' becomes [I4x4 04x4] instead of [04x4 I4x4]. How does this change the definition of the Bi's? Explain.

5. In Chapter-14 p.24, explain the meaning of the reconstruction formula (compared to the reconstruction formula of p.4). Why is there a separate term with the xo's?

39. January 26 2021

Question 1

1. Chapter-3 p.26 ('To ensure the time-domain signal is real-valued, have to choose…'): Why is a real-valued time-domain signal needed? How does this choice modify the receiver structure?

   2. Chapter-4 p.12: Explain by means of a few formulas how the minimization problem can be solved using QR-decomposition, instead computing Q-1 .p.

   3. Chapter-5: Consider a linear phase FIR filter. Is it possible to use a (LPC) lattice realization for this filter? A lossless lattice realization?

   4. Chapter-5 p.13: Explain the relevance of the Schur-Cohn stability test for the derivation of the lattice-ladder realization.

   5. Chapter-6 p.35: Explain why it is that all quantization noises can be lumped into e1 and e2. What are the corresponding noise transfer functions?

Question 2

1. Chapter-7 p.40: Provide an intuitive explanation for the unrealizable Wiener filter formula for the two considered cases.

   2. Chapter-8 p.21: Explain the 'noisy gradients' effect in an acoustic echo cancellation set-up, with a near-end speaker that is sometimes active and sometimes not active.

   3. Chapter-9 p.26 (38): In this structure with 'residual extraction', where are the actual FIR filter coefficients, i.e. the estimated model for the echo path?
   In the delay line
   Nope, it is not present in this system, that is the beauty, because we can calculate the residual without calculating explicitly the model

   4. Chapter-9 p.35 (chapter-10): Redraw (sketch!) the (relevant parts of the) signal flow graph when the 'main trick' is used to remove the column with R15,…R45. Define the relevant epsilon-signals to the signal flow graph (with subscripts & superscripts).
   Same as previous exam. Removing all the elements of those columns except the diagonal elements

   5. Chapter-10 p.19 (chapter-11 p.23): Explain the statement "is seen to require only the lowerright/lower part".

Question 3

1. Chapter-11 p.39 (chapter-12 p.40): Explain why the R(zD) is a D-by-N matrix (and not an N-by-N or D-by-D or N-by-D matrix).
Due to the perfect reconstruction condition for the case of D<N

   2. Chapter-12 p.7(chapter-13 p.20): Explain (in +/- half a page) the design procedure mentioned at the bottom of the slide.

   3. Chapter-12 p.31(chapter-14?): Specify B(z) for the case where N=5 and D=4.
   Same as previous exam

   4. Chapter-13 p.18: Explain the statement 'let B(z) take the place of distortion function T(z)'. Will there be any distortion in this case?

   5. In Chapter-14 p.12, explain the meaning of the reconstruction formula (at the bottom of the slide) and compared to the reconstruction formula of p.4.

40. August 10 2020

Question 1

1. Explain (max 10 lines) how in 'weighted least squares' (WLS) based FIR filter design, imposing a linear phase response reduces the design degrees of freedom?

    2. Can QR-decomposition be used anywhere in the WLS based FIR filter design process? If yes, where/how?

    3. When in WLS based FIR filter design the desired FIR filter has (next to a desired amplitude response) a desired phase response that is not linear, how would the design procedure have to be adjusted? Provide formulas.

Question 2

1. For an overdetermined set of linear equations A.x=b, the least squares solution is given as $x_{LS}=(A^TA)^{-1} . A^Tb$. Manipulate this last formula to derive an equivalent formula that justifies the 'backsubstitution' step in the QRD-RLS algorithm.

    2. How is the property illustrated in Chapter-9 page-32 exactly used in the derivation of the QRD-LSL (least-squares-lattice) algorithm (max 1 page)?

    3. If in Chapter-9 page-33 the set of filter input signals
    [u[k] u[k-1] u[k-2] u[k-3] u[k-4]]
    is extended to [u[k] u[k-1] u[k-2] u[k-3] u[k-4] v[k]]
    (where v[k] is a second signal, independent from u[k]) would it still be possible to derive/use a QRD-LSL algorithm? If yes, sketch a block scheme (omit details).

Question 3

1. In a maximally decimated (MD) filter bank aliasing usually occurs because of the downsampling operation. Provide an intuitive explanation (max 10 lines) why perfect reconstruction (PR) is still possible in an analysis/synthesis filter bank, in spite of the downsampling and aliasing.
The analysis filters are anti-aliasing filters to prevent aliasing in the subband signals after decimation

    2. Compare maximal decimation with oversampling in an analysis/synthesis filter bank. What are advantages/disadvantages of one versus the other (max 1 page)?
    Maximal decimation D=N
    Disadvantages: stability concern + large
    Advantages: power complementary property

Nothing in the lectures for oversampling

    3. Consider a 4-channel DFT-modulated filter bank with 3-fold downsampling. How can the analysis bank and synthesis bank be realized efficiently? And then how can perfect reconstruction be obtained in this case? Provide formulas

41. January 15 2018

1. Formulate in your own words the Schur-Cohn stability criterion to test the 'stability' of a polynomial.
2. Where does the Schur-Cohn test appear in the context of FIR filter realisation, and what is its relevance there?
3. If the Schur-Cohn test is applied to a linear-phase FIR filter, what will be the outcome? What does this mean in terms of linear-phase FIR realization?
4. Where does the Schur-Cohn test appear in the context of IIR filter realisation, and what is its relevance there?

1. Same as previous exam

2. »

3. »

4. »

1. Explain the operation of the signal flow graph (SFG) of QRD-RLS.
2. Each hexagon in the signal flow graph represents an orthogonal transformation (2 input, 2 output). Could non orthogonal transformations be used in the rotational cells? If so: how? If not: why not?
3. Consider the linear system y[k] = a*u[k] + b*u[k-1]. Could QRD-RLS be used in a 'system identification' application, where 'a' and 'b' are determined from observations of y[k] and u[k]? If so: how? If not: why not?
4. Consider the non linear system y[k] = a*u[k]*u[k-1] + b*u[k-2]*u[k-2]. Could QRD-RLS be used in a 'system identification' application, where 'a' and 'b' are determined from observations of y[k] and u[k]? If so: how? If not: why not?

2. Only orthogonal transform because it defined the rotational angles in order to reduce the power order. The orhogonality preserves the norm of the vectors and the power of the input signals $\Rightarrow$ lossless

42. January 17 2017

1. Formulate in your own words the Schur-Cohn stability criterion to test the 'stability' of a polynomial.
2. Where does the Schur-Cohn test appear in the context of FIR filter realisation, and what is its relevance there?
3. Where does the Schur-Cohn test appear in the context of IIR filter realisation, and what is its relevance there?
4. If the Schur-Cohn test is applied to a linear-phase FIR filter, what will be the outcome? What does this mean in terms of linear-phase FIR realization?
5. Use the Schur-Cohn stability test to derive the stability conditions for a 2nd order polynomial A(z) = (1 + a1*z^-1 + a2*z^-2)

All in previous exam

43. August 16 2017

± 13 janv: 2015

1. What are noise transfer functions? Explain how such noise transfer functions are used in the analysis of quantization effects in filter implementation.
2. Do the properties of the quantization mechanism play a role in such analysis?
3. The analysis relies on linearity (explain) whereas quantization is a non-linear operation. Could this be a contradiction?

1. Noise is apparent in every realization after an adder/multiplier. Quantization requires removing least significant bits which introduces quntization noise.

2. The quantization noise is analyzed in a statistical manner, it is non linear and deterministic which introduces oscillations

3. The oscillations can be apparent only if the filter has feedback $\Rightarrow$ linear filters FIR cannot have this non-linearity

1. Explain the signal flow graph (SFG) of QRD-RLS
2. Residuals: explain a priori and a postiori residuals of SFG
3. Is it useful to use two exponential weighting factors if the output and input are very different: static input and non static output (for example)? (something like this)

1. Same as previous exam

2. A priori = the difference of the desired output at time k and the filter output of the previous time k-1 using its previous design coefficients. A posteriori = same but using the updated design coefficients

1. Explain the operation of the signal flow graph (SFG) of QRD-RLS.
2. Explain how the QRD-LSL (least-squares-lattice) algorithm is derived. What is the meaning of the epsilon-notation?
3. The epsilon-signals are referred to as forward and backward prediction errors (up to a scaling with the 'cosines product'). Specify the forward and backward prediction problems that are referred to here.

1. Same as previous exam

2. The epsilon superscript = time index of the right hand side signal. The epsilon subscript = time indices of the set of the left hand side signals

3. Forward prediction = the next input sample is predicted using the previous sample. The opposite for backward prediction

44. January 11 2016

1. Explain how a square-root Kalman Filter algorithem is derived. Estimate (as 'O(xx)') the resulting computational complexity per sampling interval.
2. Explain how the Kalman Filter extends Recursive Least Squares parameter estimation.

45. January 12 2016

1. What are 'zero-input limit cycle oscillations? Where and how do these appear?
2. a) Explain how from a given frequency domain specification (e.g. low-pass filter characteristic) a filter can be designed and realized that is guaranteed to be free of limit cycle oscillations.
3. b) Explain (briefly) different steps and options in the design process.

1. In the quantization of arithmetic operations. When there is zero input it is expected for the output to also be zero but because of the quantization process, a quantization noise is introduced that created non linearities = zero input limit cycle oscillations (due to non stochastic signals)

2. When the filter is linear = does not have feedback then there are no oscillations

1. Explain the operation of the SFG (signal-flow graph) of QRD-RLS.
2. A hexagon in the SFG represents a transformation with a 2-by-2 orthogonal matrix. Would it be possible to use non-orthogonal transformations (to introduce the zeros in specific positions of the input vector)? How/why not?
3. What is residual extraction? How is it realized in the SFG of QRD-RLS? For which application(s) is it relevant?

1. Same as previous exam

2. »

3. Residual extraction refers to the low matrix elements of the QRD algorithm. The residual extraction process creates a priori and posteriori residuals that are relevant with the product of the rotation angles in the SFG. The residuals can be used to compute least squares and in the filter design process instead of using the coefficients

46. January 28 2016

1. how is the IIR lattice related to direct form FIR
2. why do you need to scale the TF for Lossless Lattice realisation?
3. how is the schur-kohn stability test related to the implementation of lattice FIR filters?\

1. See chap4 slide 4

2. The two TF have to have the same order so scaling is needed

3. Same as previous exam

1. what is the difference between optimal filters and RLS filters?
2. explain how standard RLS is a special case of standard KF
3. explain how the algoritm of RLS can be seen in the algoritm of KF

1. Optimal filters are based on the steepest descent iteration method = using the previous sample to compute the next one. The quantities used for the wiener solution are statistical. In contrast with the RLS where is uses expected quantities. The common thing they have is that the result is the same = LS solution

2. With the kalman gain vector?

47. January 13 2015

1. Explain the main trick that is used in the derivation of the QRD-LSL algorithm.
2. Compare the complexity of the QRD-LSL algorithm with the complexity of LMS.
3. In an acoustic echo cancellation scenario with one loudspeaker and two microphones (ie. where an echo contribution is to be cancelled in each of the two microphone signals), is it possible to apply a QRD-LSL-algorithm? Sketch an efficient realisation (clearly defining input and output signals).

2. QRD is numerically better method to compute the LS solution

48. Various questions

48.1. Exam 1

1. Derive lossless lattice for a 3-port. Show the mathematical derivation as well as SFGs.

2. Does this realization have a pareunitary transfer function?

3. Does this realization have a unimodular transfer function?

4. Explain DFT modulated filter bank

5. What are the advantages/disadvantages

6. Compare it to another modulated filter bank

7. Does the complexity compare with those other modulated filter bank compare to a 1-input 4-output lossless ladder FIR filter (in other words what is the complexity of cosine modulated FB and 1-input 4-output lossless ladder)

8. Explain how QR-RLS works

9. Explain the SFG for QRLS

10. How is the 'a posteri' residuals computed a posteriori = division of the residual extraction with the rotational angle product = the difference between the desired output at time k with the filter output at time k-1 using the updated coefficients

48.2. Exam 2

1. Lattice ladder of IIR. How can you be sure that |a4| < 1?

2. What if all a-coeff are equal to zero. Is H~ still an allpass? If all a=0 then all theta=0 $\Rightarrow$ the all pass part of the realization becomes only a delay line

3. Explain SFTF

4. Make the link with DFT modded filter banks What is the meaning of the number of channels?

5. What if the window length is greater than the number of channels? What does the decimation factor mean? D>N

6. How does perfect reconstructability affect window function design?

7. What is MMSE? Draw the cost function for a 2-step filter

8. What does the cost function mean for acoustic echo cancellation?

9. How does the cost function change as the statistical properties of the far end signal change?

10. What influence does the near end signal have

11. In practice why is the adaptation sometimes turned off when the near end signal becomes active?

48.3. Exam 3

1. Explain the use of minimum least square optimization in the design of FIR filters.

2. Explain the design based on window functions. Which windows are 'good' windows that qualify for this? Why?

3. Explain how the windows-based design is linked to the design based on minimum least square optimization. Which windows? Which weight function?

4. Explain how OFDM (with cyclic prefix) has a convenient mechanism for channel equalization. For this to works the channel must meet several conditions. Which ones?

5. OFDM can also be represented as a transmiltiplexer. What are the analysis filters (formulas)? Number of channels? What are the upsample/downsample factors and what is their meaning?

6. How is the QRD-LSL alsogorhtm derived from the QRD-RLS algorithm?

7. What is residual extraction? How is this used in acoustic echo cancellation

8. How would you implement this for the case of 2 microphones and 1 speaker?

48.4. Exam 4

1. Explain SFG for QRD-RLS, and how is the QRD-LSL derived from this.

2. Explain on p 12.9 the equality with regard to the cos-products (the ones that are therefore not used ecplicitly).

3. Can QRD-LSL be used for the "linear combiner" p 9.29?

4. What is DFT-modulated FB, explain how to obtain an efficient realizaton for analysis and synthesis bank

5. How does this lead to a design procedure for PR? Possibilities and limitations of that procedure? How can FIR lattice filters (p3.13) be used here?

6. Explain how lattice ladder realization for IIR is derived

7. p.340: sin = a4, what if |a4|>1? Same as previous exam

8. if a4=0 then theta0=0. what if all theta = 0? is H~ still all-pass? No its only a delay line

48.5. Exam 5

1. Explain lossless lattice for FIR

2. Calculate thetas for H(z)=1/sqrt(2)+1/sqrt(2)*z^-1

3. Explain: maximally decimated DFT filter bank with perfect reconstruction

4. $E_i(z) = 1+c*z^{-1}+d*z^{-2}$ for i=0..3 determine c and d for an optimal filter (+ what is optimal) and what does the spectrum look like

5. Explain QRD-RLS with residue extraction (+ signal flow in diagram).

6. Draw an acoustic echo cancellation schematic with 2 micros and 2 loudspeakers

48.6. Exam 6

1. Discuss WLS in FIR and how to solve via quadratic equation

2. Explain how obtained linear phase filter can be implemented via lattice/loseless lattice and what H~(z)/ H~~(z) looks like (also linear phase or not)

3. Can you also obtain linear phase for IIR filter, if A(z) and B(z) are both linear phase filters? can this also be calculated as sum of quadratic equation?

4. Describe how polyphase decomposition can be used to achieve perfect reconstruction.

5. then describe the conditions for alias-free, and for perfect reconstruction

6. now also derives this for transmultiplexers

7. Discuss steepest descent method, and how LMS follows from this

8. discuss influence of loudspeaker signal spectrum in echo cancellation on convergence of LMS

9. discuss influence of spectrum of near-end signal in echo cancellation on convergence of LMS

48.7. Exam 7

1. Explain QRD-RLS with extraction residue, and explain what a priori and postpriori residue are. Same in previous exam

2. explain SFG for QRD-RLs

3. how can one use qrd-rls in echocancelation where sometimes the filter has to adjust adaptively but sometimes not adjust filter coef but still keep working.(about the question, basically Rls just works if normal but the new values of R and z are not adjusted, I think anyway)

4. what about qrd-lsl when it comes to point b and c

5. What is the importance of 'perfect reconstruction' (PR) in relation to filter bank design? Show with an application and explain

6. In a maximum-decimated (MD) filterbank, aliasing usually occurs due to the downsampling. operation. Explain why perfect reconstruction is still possible (intuition)? Previous exam

7. What is a general procedure for the design of MD-PR filter banks

8. What are 'modulated' MD-PR filter banks.

9. -Explain the design procedure for modulated MD-PR filter banks

10. Explain how lossless lattice ladder realization for FIR is derived.

11. derive lossless lattice from above but with three outputs, so H forms a power complementary thing together with 2 other functions. Instead of 1 as above.

48.8. Exam 8

1.Explain the principle of 'weighted least squares' FIR filter design

1. Explain how a quadratic optimization problem is obtained when the filter design is limited to a set of 'sample' frequencies.

2. How can the procedure be adapted if the linear phase behavior is not structurally imposed? (and where the desired phase behavior may or may not be linear

3. Specify the quadratic optimization problem (formulas) in this case (again limiting the filter design to a set of 'sample' frequencies

2.What are oversampled DFT-modulated perfect-reconstructing filter banks?What are the benefits of this?

1. Consider a 6 channel DFT modulated filterbank with 3-way downsampling. Describe how such a filter bank is designed, and how a para-unit structure can be obtained. Draw the resulting block diagram and sketch a possible frequency response of the 6 analysis filters

3.What is Residue Extraction? Explain how residue extraction is organized in QRD-based RLS. Are a priori residuals larger or smaller than a priori residuals?

1. Explain how the QRD-LSL (least-square-lattice) algorithm is derived. Also explain the epsilon notation used

48.9. Exam 9

1. Explain WLS to IIR. Do you also get a quadratic optimization problem here?

2. Explain Steigliz-McBride. Where appropriate? Formulas? What about sample frequencies?

3. Overlap-save method as representation of an oversampled filter bank? Explain + draw frequency characteristics of analysis and synthesis filter.

4. Find an alternative transformation of T(z) and show how this leads to overlap-add.

5. What is Residue Extraction? How is this organized in QRD based RLS? Is the a priori larger/ smaller than the a posteriori? Same as previous exam

6. How is QRD-LSL derived? Does epsilon notation mean? Is residue extraction possible here too? Same as previous exam

48.10. Exam 10

1. How do you derive lossless lattice realization for FIR filters?

2. Suppose you have two transfer functions that you want to realize together (1-input/2-output). Can you embed this in a lossless 1-input/3-output system? How do you arrive at a lossless lattice realization? Also work out how to calculate the orthogonal transformations.

3. What are unimodular and paraunitary matrices and what is their importance in perfect reconstruction filter banks?

4.) What are DFT modulated filter banks? What are the pros and cons? Additional question: how can you remedy this disadvantage (not paraunitary unless for trivial choices)? c) How would you design a DFT modulated transmultiplexer?

5. What is QRD-RLS with Residue Extraction? What are a priori and a posteriori residues? Same as previous exam Explain the SFG (signal flow graph) b) How can you use the above for acoustic echo cancellation with 1 loudspeaker and 2 microphones?

6. What if you have two speakers and 1 microphone

48.11. Exam 11

1. Explain: Lossless Lattice FIR filter realization

2. How is H~~ determined? c) Determine H~~ and draw the realization for: $H(z) = 1/sqrt(2)*cos(theta) + z^{(-1)} * sin(theta)$

3. Explain MD-PR filter banks and what is the requirement for Perfect reconstruction

4. b) Discuss in detail the condition for anti - aliasing Same as previous exam

5. Discuss in detail the condition for Perfect reconstruction Same as previous exam

6. Discuss QRD-LSL b) What does the implementation of QRD-LSL for acoustic cancellation look like with 1 loudspeaker and 1 microphone

7. What does the implementation of QRD-LSL for acoustic consolation look like with 2 loudspeakers and 2 microphones

48.12. Exam 12

1. Explain lossless lattice in FIR.

2. Given are 4 FIR filters that are power complementary. Explain how you could turn this into a lossless lattice structure. (In other words, multiple output lossless lattice, but for 4 instead of 3 as in the course. Just explain conceptually, don't deduce in detail. He asked which exact elements in that matrix are nullified by the rotations)

3. What are maximally decimated DFT modulated filter banks? What are the benefits of this? (Additional question: in the general case those R_i's are not stable if E_i's are FIR. Can you formulate conditions on the E_i's so that R_i's are still stable?)

4. A maximally decimated DFT filter bank has polyphase components $E_i(z) = 1 + c_i z^{-1} + d_i z^{-2}$. for $i = 0..3$. Describe a method to determine optimal c_i and d_i (and explain what is optimal).

5. How can you interpret wiener adaptive filters as steepest slope filters? Explain how you arrive at conditions for the step size μ from this. How do you make the transition to LMS?

6. b. What is the effect of the eigenvalue spectrum of the input signal on the operation of the filter? (Tip that was not given on the exam, but that he asked about: if all lambdas are equal, that would apparently correspond to an input signal that is white noise…)

48.13. Exam 13

1. Explain how one arrives at the lattice ladder realization of an IIR filter. What happens when $|a4| > 1$? When a4 is equal to zero, Theta0 is also equal to zero, what if all a coefficients are equal to zero, what does H~ look like? Is H~ still an APF? Same as previous exam

2. Additional questions: What kind of filter is H~? (APF) How do you see that? (coefficients in reverse order).When at the end the whole procedure has to be repeated on the remaining piece, is this just an option? (Structure is the same but what is also important is that the remaining part is also an APF)

3. What does H~ mean?

4. When all a-coefficients are zero, what structure is left? Same as previous exam

5. What to do when $|a4| > 1$? Same as previous exam

6. Explain what Short Time Fourier Transform (STFT) is.

7. What is the relationship between STFT and DFT modulated filter banks?

8. What does the decimation factor mean?

9. What does the number of channels mean? What happens when the window length is greater than the number of channels?

10. Does the requirement to have PR impose restrictions on the choice of window function?

11. Explain the MSE cost function for optimal/adaptive filtering. Sketch the cost function for a filter with two coefficients.

12. What is the significance of the minimum of the cost function?

13. For an echocancellation filter, what happens to the shape of the cost function when the statistical properties of the far-end signal change?

14. What happens to the shape when the near end signal becomes active?

15. Why is practically adaptive LMS disabled when no near-end signal is present?

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