ME44206 Quantitative Methods for Logistics

Faculty of 3mE Department Maritime & Transport Technology

Assignment 2

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1 Question A

Sets, parameters and decision variables

The following indices have been used in the mathematical model:

- i, j index for nodes, where $i, j \in C$ for customer nodes and $i, j \in D$ for depots
- v index for vehicles, where $v \in V$

The set N is the set of all nodes, such that $N = C \cup D$ and uses the indices i and j.
The different parameters, that are used in the mathematical model, are given in Table 1.

Name	Description
xc_i	x-coordinate of node i
yc_i	y-coordinate of node i
c_{ij}	Distance from i to j where: $c_{ij} = \sqrt{(xc_j - xc_i)^2 + (yc_j - yc_i)^2}$
b_v	Capacity of vehicle v
a_i	Demand of customer i
K	Maximum number of vehicles to be used
rt_i	Ready time of node i
dt_i	Due time of node i
pt_i	Processing time at node i
M	Large positive integer

Table 1: Definitions of parameters and constants

The decision variables, which are used in the mathematical model, are given in Table 2.

Name	Description
x_{ijv}	$\begin{cases} 1, & \text{if vehicle } v \text{ is travelling from node } i \text{ to node } j \\ 0, & \text{otherwise} \end{cases}$
z_{iv}	$\begin{cases} 1, & \text{if node } i \text{ is visited by vehicle } v \\ 0, & \text{otherwise} \end{cases}$

Table 2: Definitions of decision variables

Mathematical model

The defined sets, parameters and decision variables can be used to formulate the mathematical model. In equations (OF1) - (C9) this mathematical model is defined. Each equation will be explained in detail.
Equation (OF1) is the objective function, which minimizes the total distance travelled by all the vehicles. Next, there are 8 constraints and a constraint on the domain of the decision variables. The first two constraints, that is formulated in equations (C1A) and (C1B), defines that every customer with a pick-up is visited only once and that every customer with a delivery cannot be visited. The second constraint is stated in two parts, namely in equation (C2A) and (C2B). Equation (C2A) states that the number of vehicles that enter the depot should be equal to K, where K is the number of available vehicles. Equation (C2B) states that the number of vehicles that leave the depot should equal to K as well. The third constraint ensures that the load capacity of the vehicle is not higher than the capacity of the vehicle, which is stated in equation

(C3). The fourth constraint defines the flow continuity of each customer node in equation (C4). In equation (C5), the fifth constraint is given, which states the arrival time of the vehicle at the next node. The sixth constraint, given in equation (C6), defines the starting time at the depot of a vehicle. The seventh constraint, stated in equation (C7), ensures that the arrival time at node i is before the closing time of node i. Finally, the eight constraint ensures that the arrival time at node i is after the opening time of node i. The objective function, the eight functional constraints and the final domain constraint of the decision variables define the mathematical model of this Vehicle Routing Problem.

$$\min \sum_{i \in N} \sum_{j \in N} \sum_{v \in V} c_{ij} x_{ijv} \tag{OF1}$$

$$\sum_{v \in V} z_{iv} = 1 \quad \forall i \in C \text{ with } a_i > 0 \tag{C1A}$$

$$\sum_{v \in V} z_{iv} = 0 \quad \forall i \in C \text{ with } a_i < 0 \tag{C1B}$$

$$\sum_{i \in C} \sum_{j \in D} \sum_{v \in V} x_{ijv} = K \tag{C2A}$$

$$\sum_{i \in D} \sum_{j \in C} \sum_{v \in V} x_{ijv} = K \tag{C2B}$$

$$\sum_{i \in N} a_i z_{iv} \leq b_v \quad \forall v \in V \tag{C3}$$

$$\sum_{j \in N} x_{ijv} = \sum_{j \in N} x_{jiv} = z_{iv} \quad \forall v \in V, i \in C \tag{C4}$$

$$t_{iv} + c_{ij} + pt_i - M(1 - x_{ijv}) \leq t_{jv} \quad \forall i \in C, j \in N, v \in V \tag{C5}$$

$$rt_i + c_{ij} - M(1 - x_{ijv}) \leq t_{jv} \quad \forall i \in D, j \in C, v \in V \tag{C6}$$

$$t_{iv} \leq dt_i \quad \forall i \in N, v \in V \tag{C7}$$

$$t_{iv} \geq rt_i \quad \forall i \in N, v \in V \tag{C8}$$

$$\begin{aligned} &t_{iv} \geq 0, && \forall i \in N, \forall v \in V \\ &x_{ijv} \in \{0,1\} && \forall i,j \in N, v \in V \\ &z_{iv} \in \{0,1\} && \forall i \in N, v \in V \end{aligned} \tag{C9}$$

2 Question B

The optimal solution that has been found is 82.9473 as the total distance travelled. In Figure 1 the nodes that have been visited by the vehicle with a capacity of 200 are mapped. The red dots are the depots in the problem. As can be seen in the figure the vehicle starts at depot 1 and follows the following route: customer 7, customer 12, customer 13, customer 10, customer 5, customer 4 and finally depot 0.

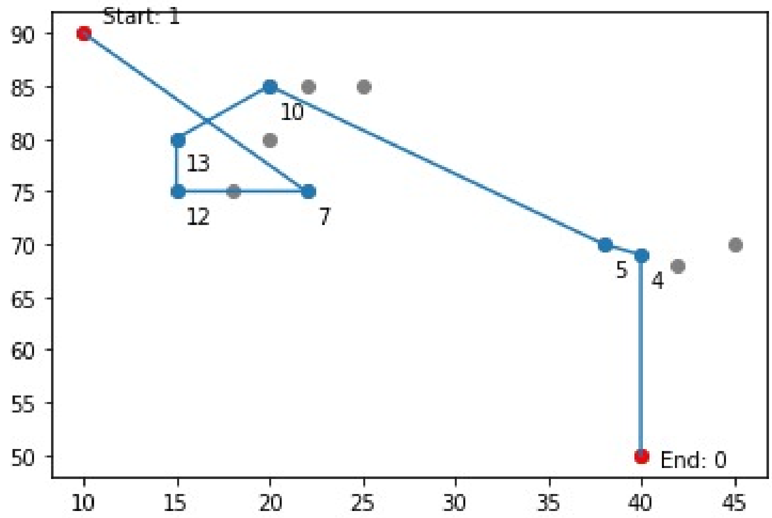

Figure 1: Map of the route of the vehicle with capacity 200

In Table 3 the time values at the origin and destination are given, just like the load values at the origin and destination of the visited nodes.

Origin	Destination	Time O	Time D	Load O	Load D
1	7	0.0	92.0	0	30
7	12	92.0	250.0	30	50
12	13	250.0	345.0	50	60
13	10	345.0	475.0	60	100
10	5	475.0	605.0	100	110
5	4	605.0	702.0	110	130
4	0	702.0	1236.0	130	130

Table 3: Time and load values of the vehicle at each customer

3 Question C

The flow continuity constraint (C4) is changed in order to include the depots as well, so it is applicable to all nodes. Now (C4) ensures that all nodes visited by a vehicle have an in- and outgoing link, so all vehicles have to return to the same depot. Since now we are also considering having multiple vehicles at our disposal but we do not necessarily use them all, we consider the depot-visiting constraints C2A and C2B to be inequalities. This yields the following equations:

$$\sum_{i \in C} \sum_{j \in D} \sum_{v \in V} x_{ijv} \leq K \tag{C2A*}$$

$$\sum_{i \in D} \sum_{j \in C} \sum_{v \in V} x_{ijv} \leq K \tag{C2B*}$$

$$\sum_{j \in N} x_{ijv} = \sum_{j \in N} x_{jiv} = z_{iv} \quad \forall v \in V, i \in N \tag{C4*}$$

4 Question D

The mathematical model that has been provided in Question A is implemented with the adaptations made in Question C. Three cases are implemented and the results are given in this chapter.

4.1 1 vehicle with a capacity of 200

When implementing 1 vehicle with a capacity of 200 the total distance travelled is 94.5437. In Figure 2 the map of the vehicle is shown. It can be seen that the vehicle starts and ends in depot 0. Furthermore, the nodes are visited in the following order: depot 0, customer 7, customer 12, customer 13, customer 10, customer 5 and finally, customer 4.

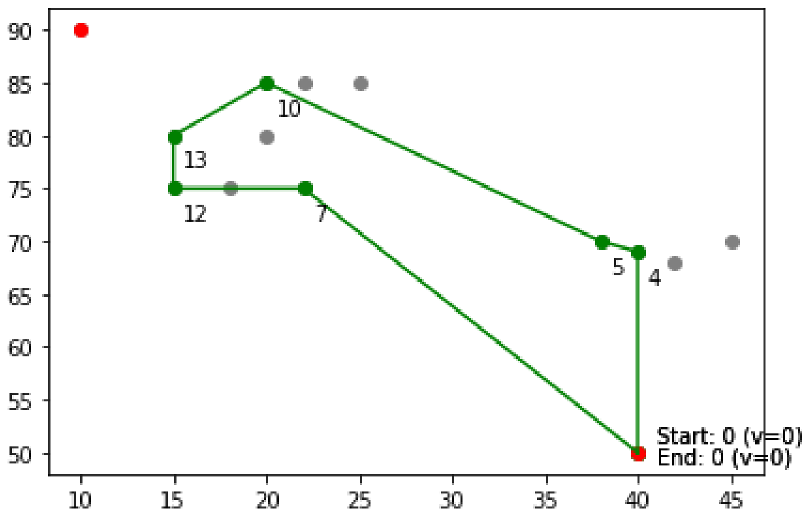

Figure 2: Map of route of the vehicle with capacity 200

In Table 4 the time and load values at the origin and destination are given of the visited customers.

Origin	Destination	Time O	Time D	Load O	Load D
0	7	0.0	92.0	0	30
7	12	92.0	250.0	30	50
12	13	250.0	345.0	50	60
13	10	345.0	475.0	60	100
10	5	475.0	605.0	100	110
5	4	605.0	702.0	110	130
4	0	702.0	1236.0	130	130

Table 4: Time and load values of the vehicle at each customer

4.2 2 vehicles with a capacity of 100 each

The total distance travelled by 2 vehicles with a capacity of 100 each gives a result of 90.7966. In Figure 3 the routes of both vehicles are given. The first vehicle starts and ends at depot 1 and visits the following customers in the following order: customer 7, customer 12, customer 13, customer 10 and finally, it ends in depot 1. For the second vehicle, which starts in depot 0, customer 4 is visited first and then customer 5, after it returns to depot 0 again.

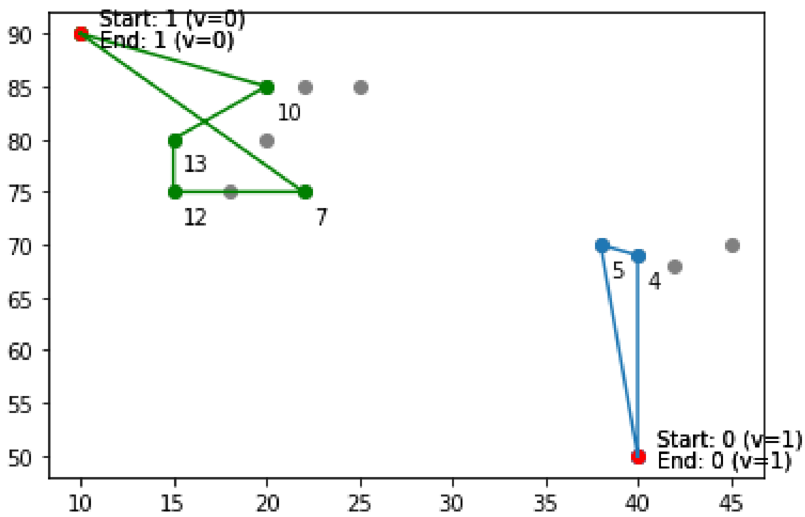

Figure 3: Map of route of both vehicles with capacity 100

In Table 5 and 6 the time and load values of vehicle 1 and vehicle 2, respectively, can be found.

Origin	Destination	Time O	Time D	Load O	Load D
1	7	0.0	30.0	0	30
7	12	30.0	250.0	30	50
12	13	250.0	345.0	50	60
13	10	345.0	475.0	60	100
10	1	475.0	1236.0	100	100

Table 5: Time and load values of vehicle 1 at each customer

Origin	Destination	Time O	Time D	Load O	Load D
0	5	0.0	605.0	0	10
5	4	605.0	702.0	10	30
4	0	702.0	1236.0	30	30

Table 6: Time and load values of vehicle 2 at each customer

4.3 2 vehicles with a capacity of 75 each

When 2 vehicles are considered with a capacity of 75, the total distance travelled is 107.8678. In Figure 4 the routes of both vehicles can be found. Now, the first vehicle starts again at depot 1. It starts at depot 1, goes to customer 13, customer 12, customer 10 and finally, returns to depot 1. For the second vehicle the route stays the same, which is starting at depot 0, customer 7, customer 5, customer 4 and ending at depot 0.

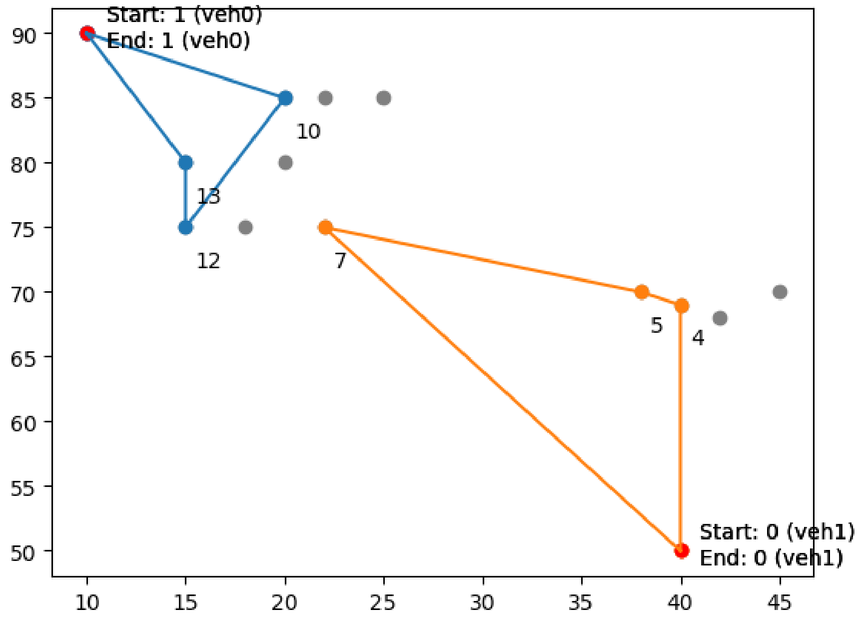

Figure 4: Map of route of both vehicles with capacity 75

In Table 7 and 8 the time and load vehicles of vehicle 1 and vehicle 2, respectively, can be found.

Origin	Destination	Time O	Time D	Load O	Load D
1	12	0.0	250.0	0	20
12	13	250.0	345.0	20	30
13	10	345.0	475.0	30	70
10	1	475.0	1236.0	70	70

Table 7: Time and load values of vehicle 1 at each customer

Origin	Destination	Time O	Time D	Load O	Load D
0	7	0.0	30.8	0	30
7	5	30.8	605.0	30	40
5	4	605.0	702.0	40	60
4	0	702.0	1236.0	60	60

Table 8: Time and load values of vehicle 2 at each customer

4.4 Comparison

The results of the total distance travelled in Question D are all larger compared to the result of total distance travelled in Question B. This is an expected result, since the vehicle always needs to travel back to the starting depot in Question D, while in Question B a depot that is located closest to the final visited customer can be chosen.

When looking at the three different cases of Question D it can be seen that the case where 2 vehicles can be used have a lower total distance travelled by these vehicles together, compared to the case where 1 vehicle is available. This makes sense, since the case with 1 vehicle needs to return to the starting depot, which results in a higher total distance travelled. In addition, smaller loops can be created because several vehicles can be used with their own routes.

Another comparison that can be made is the two cases with two vehicles. In 4.2, two vehicles are used with a capacity limit of 100. In 4.3, two vehicles can be used with a capacity limit of 75. As the first vehicle in 4.2 used the maximum load capacity, the vehicles with a capacity of 75 need to be rearranged. This results in a shift of customer 7 from the first to the second vehicle in section 4.3.

5 Question E

To allow for multiple objectives, the weighted sum method is introduced. This means that an extra set M for the number of objective functions (index m), a parameter w_m and a parameter for the deport costs dc_i are introduced. Here, the w_m parameter represents the individual weights of the objective and relates the value of a time/distance unit to the value of a monetary unit (value-of-time). The first objective remains the same as before and represents the total distance traveled. The second objective function, that will represent the depots cost per vehicle, is computed as the sum over all vehicles that leave a depot times the costs for the depot (dc_c).

Name	Description
dc_i	Depot costs of depot i
w_m	Relative weight of objective function w

Table 9: Additional parameters and constants

$$\min \sum_{m \in M} w_m f_m(x_{ijv}) \tag{OF2}$$

with

$$f_1(x_{ijv}) = \sum_{i \in N} \sum_{j \in N} \sum_{v \in V} c_{ij} x_{ijv}$$

and

$$f_2(x_{ijv}) = \sum_{i\in D}\sum_{j\in C}\sum_{v\in V} dc_i x_{ijv}$$

6 Question F

6.1 10 vehicles and no depot costs

The objective function is changed to OF2. When 10 vehicles are used to visit the customers with a capacity limit of 200, the objective function value is 696.0263. For this calculation, an optimality gap of 0% is used. In figure 5 the routing of the different vehicles can be seen. The routes and loads are shown in table 10. The third vehicle will not be used.

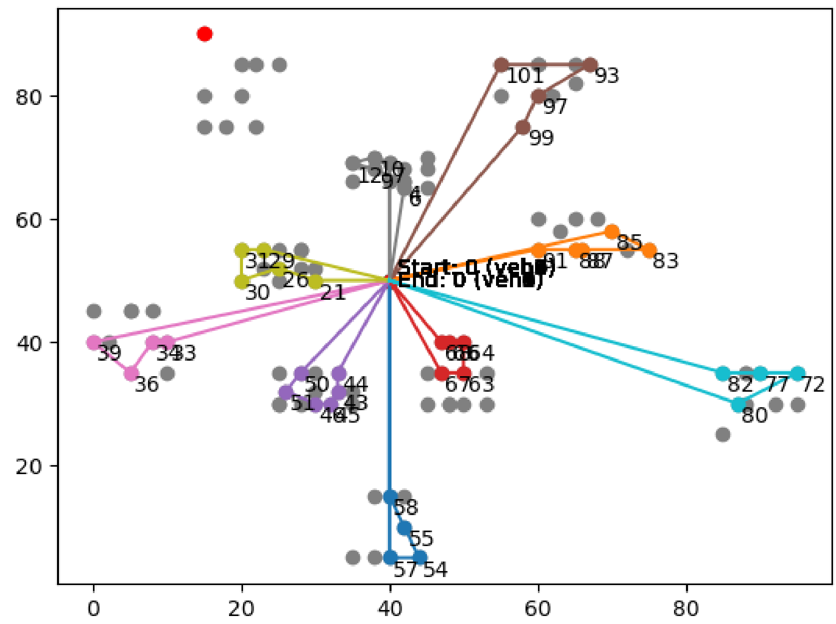

Figure 5: Route map (10 vehicles, 200 capacity, no depot costs)

Vehicle	Route	Vehicle load
1	0, 58, 55, 54, 57, 0	0, 40, 80, 100, 130, 130
2	0, 91, 88, 87, 83, 85, 0	0, 10, 30, 40, 60, 80, 80
3	unused	unused
4	0, 68, 66, 64, 63, 67, 0	0, 10, 20, 70, 90, 100, 100
5	0, 44, 43, 45, 46, 51, 50, 0	0, 10, 30, 40, 50, 60, 70, 70
6	0, 99, 97, 93, 101, 0	0, 20, 30, 50, 70, 70
7	0, 33, 34, 36, 39, 0	0, 30, 70, 80, 110, 110
8	0, 6, 4, 9, 12, 10, 7, 0	0, 10, 20, 40, 50, 60, 80, 80
9	0, 21, 26, 30, 31, 29, 0	0, 10, 50, 60, 70, 90, 90
10	0, 82, 77, 72, 80, 0	0, 30, 40, 60, 70, 70

Table 10: Routes and Loads (10 vehicles, 200 capacity, no costs)

6.2 10 vehicles with depot costs

For this route planning, 10 vehicles will be used with a maximum capacity of 200 each. There are costs associated with the deposit of 150 (for depot 0) and 100 (for depot 1). In this case, the third and fifth vehicle will not be used. The routing of the 10 vehicles is shown. The value of the objective function is 3280.7241 (with an optimality gap of 0%). From figure 7 can be seen that the two different depots both will be used compared to subsection 6.1 where only depot 0 was used. In this case, 3 vehicles make use of depot 0, and 5 vehicles make use of depot 1.

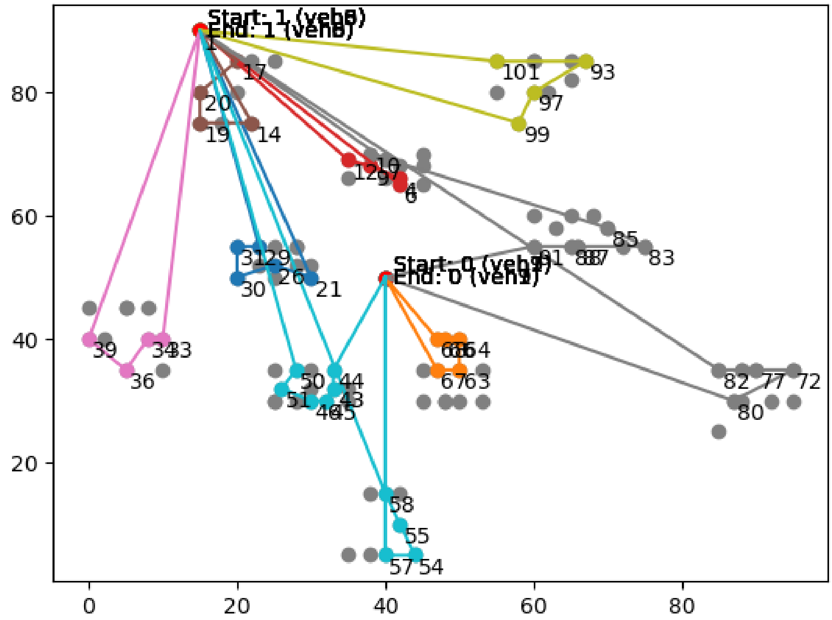

Figure 6: Route map (10 vehicles, 200 capacity, 150,100 costs)

Vehicle	Route	Vehicle load
1	1, 21, 26, 30, 31, 29, 1	0, 10, 50, 60, 70, 90, 90
2	0, 68, 66, 64, 63, 67, 0	0, 10, 20, 70, 90, 100, 100
3	unused	unused
4	1, 6, 4, 9, 12, 1	0, 10, 20, 40, 50, 50
5	unused	unused
6	1, 14, 19, 20, 17, 1	0, 30, 50, 60, 100, 100
7	1, 33, 34, 36, 39, 1	0, 30, 70, 80, 110, 110
8	0, 91, 88, 87, 83, 85, 10, 7, 1, 82, 77, 72, 80, 0	0, 10, 30, 40, 60, 80, 90, 110, 110, 140, 150, 170, 180, 180
9	1, 99, 97, 93, 101, 1	0, 20, 30, 50, 70, 70
10	0, 44, 43, 45, 46, 51, 50, 1, 58, 55, 54, 57, 0	0, 10, 30, 40, 50, 60, 70, 70, 110, 150, 170, 200, 200

Table 11: Routes and Loads (10 vehicles, 200 capacity, 150,100 costs)

6.3 Experiment with different depot costs

In this subsection experiments are performed with different depot costs. In table 12, the costs for depot 0 and 1 are shown with the resulting value of the OF. The goal is to minimize the OF value. In case of depot 0 with a costs of 0 and depot 1 with cost of 500, the OF has the lowest value. The reason that the costs of that OF are low is that the depot is centrally located and is has no costs. From the table 12 becomes clear that there is a preference for depot 0 which is more centrally located. Once the cost of that depot is higher, the overall cost also becomes higher or other routes are chosen to reduce the costs of the OF. Therefore, in order to make this comparison properly, several counterfactual costs have been chosen.

Depot 0	Depot 1	Result OF
150	100	3280.7241
100	150	2743.1234
500	0	3980.7241
0	500	743.123

Table 12: Different depot costs with results of the objective function

6.4 15 vehicles and depot costs

In this subsection, 15 vehicles can be used with a maximum capacity of 120 each and depot costs of 150 (for depot 0) and 100 (for depot 1). Furthermore, experiments are done with different capacities (90, 120 and 150). As expected, higher capacity (150) gives lower cost and lowest capacity (90) will give higher costs. The computation times are also shown in table 13. The route map of the vehicles with a capacity of 120 is shown in figure 7 and the belonging routes are shown in table 14. From the 15 available vehicles, 5 vehicles will not be used.

Capacity	Number of vehicles	Result OF	Computation Time
90 (5% opt. gap)	15	3520.139	3214.76 s
120 (base)	15	3327.551	2239.24 s
150	15	3287.202	29.28 s

Table 13: Different capacity values of the vehicles with results of the objective function

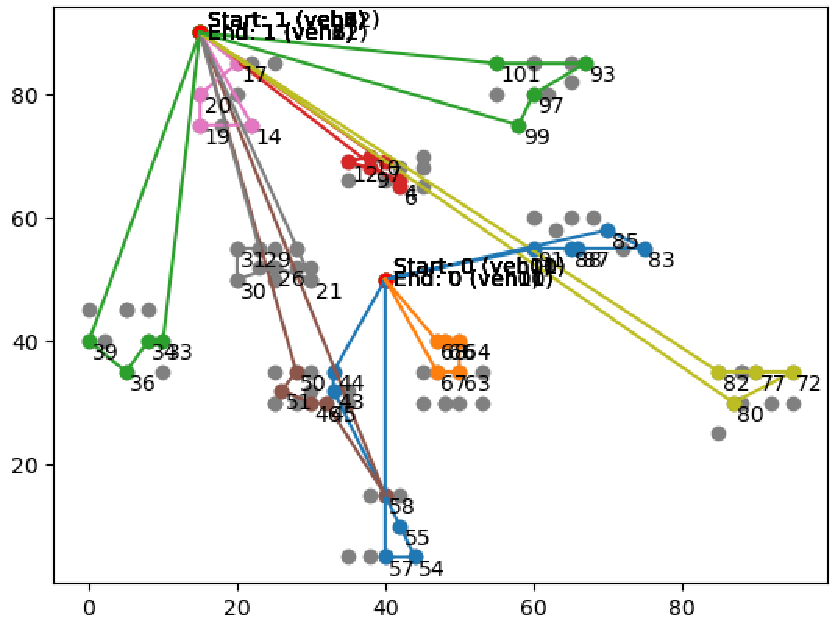

Figure 7: Route map (15 vehicles, 120 capacity, 150,100 costs)

Vehicle	Route	Vehicle load
1	0, 44, 43, 55, 54, 57, 0	0, 10, 30, 70, 90, 120, 120
2	unused	unused
3	1, 33, 34, 36, 39, 1	0, 30, 70, 80, 110, 110
4	1, 6, 4, 9, 12, 10, 7, 1	0, 10, 20, 40, 50, 60, 80, 80
5	unused	unused
6	1, 58, 45, 46, 51, 50, 1	0, 40, 50, 60, 70, 80, 80
7	1, 14, 19, 20, 17, 1	0, 30, 50, 60, 100, 100
8	1, 21, 26, 30, 31, 29, 1	0, 10, 50, 60, 70, 90, 90
9	1, 82, 77, 72, 80, 1	0, 30, 40, 60, 70, 70
10	unused	unused
11	0, 91, 88, 87, 83, 85, 0	0, 10, 30, 40, 60, 80, 80
12	0, 68, 66, 64, 63, 67, 0	0, 10, 20, 70, 90, 100, 100
13	1, 99, 97, 93, 101, 1	0, 20, 30, 50, 70, 70
14	unused	unused
15	unused	unused

Table 14: Routes and Loads (15 vehicles, 120 capacity, 150,100 depot costs)

7 Question G

To complete the model, the deliveries should also be included. This is achieved by omitting constraint C1B and the restriction on the customer nodes in C1A and adding a constraint to ensure the pick-ups are being visited before the corresponding deliveries. To couple the corresponding pick-ups and deliveries, a list (PID) of tuples (a, b) is created in the pre-processing of the data where a is taken to be the pick-up ID of a parcel and b that of the corresponding delivery location. Next, the time-variable is used to define the arrival time at the pick-up node

a to be before the arrival time at node b, the delivery node (C10).

PID_d = tuple (a, b) of parcel d with its pick-up - delivery location where:
a = pick-up node of parcel d
b = delivery node of parcel d

$$t_{d(0)v} \leq t_{d(1)v} \quad \forall d \in PID, v \in V \tag{C10}$$

Because now the load on a vehicle can also diminish after a delivery, constraint C3 no longer suffices. To accurately track the load of the vehicles, a new decision variable Q_{iv} is introduced. Q_{iv} is defined as the load of vehicle v at arrival at node i and thus covers the load at all nodes except at the start depot. The decision variable works quite similar to t_{iv} and so most constraints will show similarities. Constraint C11, similar to C5, computes the load of the vehicle at the next node to be the load at the current node together with the demand quantity q_i at that node. The third part on the LHS, which uses the large integer M, checks whether the vehicle actually travels from i to j and makes sure the constraint is satisfied regardless the value of x_{ijv}. Constraint C12 defines the load at the starting node of each vehicle to be empty, just like C6 for the time variable. Finally, since the load cannot exceed a vehicles capacity and cannot have a load lower than 0, a load capacity constraint is added (C13).

$$Q_{iv} + a_i - M(1 - x_{ijv}) \leq Q_{jv} \quad \forall i \in C, j \in N, v \in V \tag{C11}$$

$$Q_{iv} - M(1 - x_{ijv}) \leq 0 \quad \forall i \in C, j \in N, v \in V \tag{C12}$$

$$0 \leq Q_{iv} \leq b_v \quad \forall i \in N, v \in V \tag{C13}$$

8 Question H

In this subsection 10 vehicles with a capacity of 200 each are used. There are 4 depots with a cost of (150, 100, 100, 125) respectively. The results are shown in Table 15 and the routing map is shown in Figure 8. Almost all vehicles make use of depot 0, except for vehicle 2. The resulting value of the OF is 728.0686 with a computational time of 58.24 seconds.

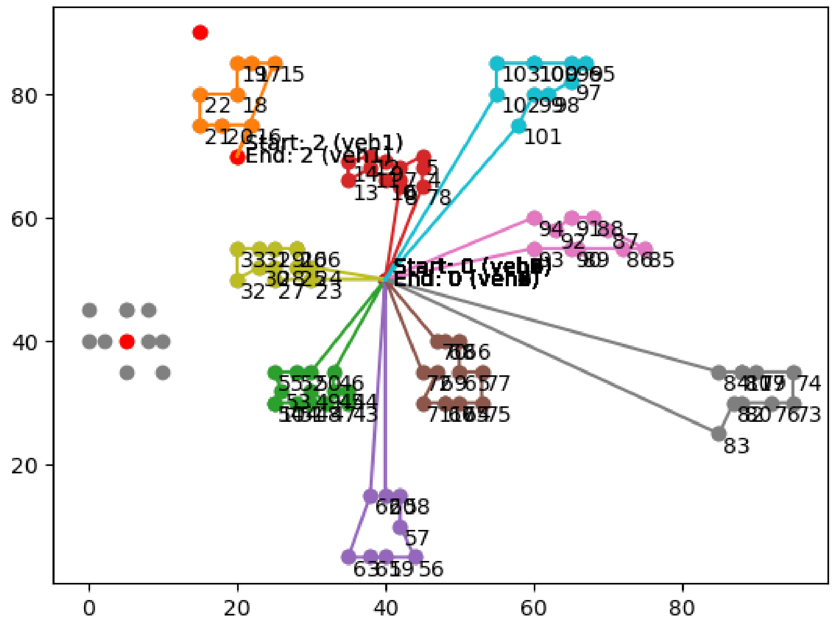

Figure 8: Route map (10 vehicles, 200 capacity)

Vehicle	Route	Vehicle load
1	unused	unused
2	2, 16, 20, 21, 22, 18, 19, 17, 15, 2	0, 30, 0, 20, 30, 20, 60, 20, 0, 0
3	0, 46, 45, 44, 43, 47, 49, 48, 51, 54, 104, 53, 55, 52, 50, 0	0, 10, 30, 20, 0, 10, 0, 10, 0, 10, 0, 10, 0, 10, 0, 0
4	0, 8, 6, 10, 11, 13, 14, 12, 9, 7, 5, 4, 78, 0	0, 10, 20, 10, 30, 10, 20, 30, 50, 40, 20, 10, 0, 0
5	0, 60, 58, 57, 56, 59, 61, 63, 62, 0	0, 40, 0, 40, 60, 90, 70, 30, 0, 0
6	0, 70, 68, 66, 65, 77, 75, 64, 67, 105, 71, 69, 72, 0	0, 10, 20, 70, 90, 40, 30, 20, 30, 20, 0, 10, 0, 0
7	0, 93, 90, 89, 86, 85, 87, 88, 91, 92, 94, 0	0, 10, 30, 40, 20, 40, 60, 40, 30, 10, 0, 0
8	0, 84, 81, 107, 79, 74, 73, 76, 80, 82, 83, 0	0, 30, 50, 30, 40, 60, 30, 20, 0, 10, 0, 0
9	0, 23, 27, 28, 30, 32, 33, 31, 29, 26, 106, 25, 24, 0	0, 10, 0, 40, 0, 10, 20, 40, 30, 40, 30, 10, 0, 0
10	0, 101, 99, 98, 97, 95, 96, 100, 109, 103, 102, 0	0, 20, 30, 10, 0, 20, 0, 30, 0, 20, 0, 0

Table 15: Routes and Loads (10 vehicles, 200 capacity)