Assignment 2

Quantitative Methods for Logistics

ME44206

Group 30
Thom Gietelink
Daan van der Klooster
Max Crebolder
Erik Scherpenhuijsen Rom: 4422104

TUDelft

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Subquestions A and B

1.1. Subquestion A

Table 1.1 shows the notation used for this mathematical model. In this case there are 7 nodes with the first node being the depot and the rest being customer locations. The parameter defining the distance from one node to another is calculated using the x and y coordinates of nodes i and j in equation 1.1.

$$d_{i,j} = \sqrt{(xco_j - xco_i)^2 + (yco_j - yco_i)^2} \tag{1.1}$$

Sets and Indices		
N	Set of nodes	$i,j \in N$
L	Set of customer locations	$i,j \in L$
S	Subset of nodes	$S \subset N$
Parameters		
xco_i	x-coordinate of node i	$[distanceunit]$
xco_j	x-coordinate of node j	$[distanceunit]$
yco_i	y-coordinate of node i	$[distanceunit]$
yco_j	y-coordinate of node j	$[distanceunit]$
$d_{i,j}$	Distance from node i to node j	$[distanceunit]$
rt_i	Ready time at node i	$[timeunit]$
dt_i	Due time at node i	$[timeunit]$
st_i	Service time at node i	$[timeunit]$
M	Large positive integer	$[unit]$
Variables		
$t_{i,j}$	Time variable continuous and linear	$[timeunit]$
Decision Variables		
$x_{i,j}$	Binary indicator if vehicle is travelling from node i to node j: (1 if true, 0 if false)	$[unit]$

Table 1.1: Description of all notation used in mathematical model

The objective function is to minimize the total distance travelled by the vehicle where the vehicle travels through each node during its assigned time window. In this case the capacity of the vehicle is not limited and only one vehicle is used. The objective function is mathematically described in equation 1.2.

$$min \sum_{i \in N} \sum_{j \in N} d_{i,j} x_{i,j} \tag{1.2}$$

The following constraints are set regarding this objective function in order to define its TSP characteristics. To ensure that every node in i or j has two links to other nodes, constraints 1.3 and 1.4 are added. To ensure symmetry, constraint 1.5 is added to make sure that if a path from i to j is made, it cannot be made in the other direction. To eliminate any sub-tours, constraint 1.6 is added to ensure one route passes through all nodes. The starting time of the vehicle at the depot is stated in constraint 1.7. The arrival time of the vehicle at node i is stated in constraint 1.8, taking the service time at that node into account. To ensure that the arrival time of the vehicle at node i is after the opening time and before closing time of that node, constraints 1.9 and 1.10 are added respectively. To ensure that the vehicle closes the loop and that the depot is both the start position as well as the end position, constraint 1.11 is added. Constraints 1.12 and 1.13 are added to ensure the variable $x_{i,j}$ is in fact binary and the arrival time at node i is a positive time value.

$$\sum_{j \in N} x_{i,j} = 2 \qquad \forall i \in N \tag{1.3}$$

$$\sum_{i \in N} x_{i,j} = 2 \qquad \forall j \in N \tag{1.4}$$

$$x_{i,j} = x_{j,i} \qquad \forall i \in N, \forall j \in N \tag{1.5}$$

$$\sum_{i \in S} \sum_{j \in S} x_{i,j} \leq |S| - 1 \qquad \forall S \subset N \tag{1.6}$$

$$rt_i + d_{i,j} - M(1 - x_{i,j}) \leq t_j \qquad i = 0, j \in L \tag{1.7}$$

$$t_i + d_{i,j} + st_i - M(1 - x_{i,j}) \leq t_j \qquad \forall i \in N, j \in N \tag{1.8}$$

$$t_i \geq rt_i \qquad \forall i \in N \tag{1.9}$$

$$t_i \leq dt_i \qquad \forall i \in N \tag{1.10}$$

$$x_{0,j} = x_{i,0} = 1 \tag{1.11}$$

$$x_{i,j} \in 0, 1 \qquad \forall i \in N, j \in N \tag{1.12}$$

$$t_i \geq 0 \qquad \forall i \in N \tag{1.13}$$

1.2. Subquestion B

2 Subquestions C and D

2.1. Subquestion C

2.2. Subquestion D

3

Subquestion E to H

3.1. Subquestion E
3.2. Subquestion F
3.3. Subquestion G
3.4. Subquestion H

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Subquestions I and J

4.1. Subquestion I

4.2. Subquestion J