Exponential function

Wikipedia

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Introduction

The exponential funtion is a mathematical function denoted by $f(x) = exp(x)$ or e^x. The real exponential function $exp : \mathbb{R} \to \mathbb{R}$ can be characterized in a variety of equivalent ways. It is commonly defined by the following power series:

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!} \tag{1}$$

$$= 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \dots \tag{2}$$

Implementation

The exponential function has been implemented with the following code:

```
static double ex(double x) {
  if(x<0) return 1/ex(-x);
  if(x>1.0/8) return Pow(ex(x/2),2);
  return 1+x*(1+x/2*(1+x/3*(1+x/4*(1+x
      /5*(1+x/6*(1+x/7*(1+x/8*(1+x/9*(1+x
      /10)))))))));
  }
```

This implementation exhibits a recursive behavior where the last statement is the power series from (1) with $k = 10$ and this is the base case. The two lines above the base case are cases that successively reduces toward the base case. The first line returns 1 divided by the function itself with argument -x, such when a negative x value is passed to the function it gives $\frac{1}{e^x}$. The second line basically says return $\left(e^{x/2}\right)^2 = e^x$, which is the function itself, if x is a bit above zero. If the second line just would be written as *return ex(x);* then we would get a stack overflow exception, which means that the recursion would go on infinitely.
The above implementation is tested by making a plot and also by plotting some tabulated values as a reference. This can be seen in figure .