Homework 1

Problem 1:

$$P(\text{open}|u) = \sum_s P(\text{open}|u,s)\cdot P(s)$$

$$= P(\text{open}|u,\text{open})P(\text{open}) + P(\text{open}|u,\text{close})\cdot P(\text{close})$$

$$= 1\times 0.5 + 0.8\times 0.5 = 0.9$$

Problem 2:

At the beginning : $P_0(\text{open}) = 0.5$, $P_0(\text{closed}) = 0.5$

After do_nothing : $P_1(\text{open}) = 0.5$, $P_1(\text{closed}) = 0.5$

After first measurment : (open)

$$P_2(\text{open}) = \eta\cdot P(Z=\text{open}|X=\text{open})\cdot P_1(\text{open}) = \eta\times 0.4\eta = \frac{8}{11}$$

$$P_2(\text{closed}) = \eta\cdot P(Z=\text{open}|X=\text{closed})\cdot P_1(\text{closed}) = 0.1\eta = \frac{3}{11}$$

After push :

$$\begin{bmatrix} P_3(\text{open}) \\ P_3(\text{closed}) \end{bmatrix} = \begin{bmatrix} 1 & 0.9 \\ 0 & 0.1 \end{bmatrix} \begin{bmatrix} P_2(\text{open}) \\ P_2(\text{closed}) \end{bmatrix} = \begin{bmatrix} \frac{10}{11} \\ \frac{0.3}{11} \end{bmatrix}$$

After second measurment : (open)

$$P_4(\text{open}) = \eta'\cdot P(Z=\text{open}|X=\text{open})\cdot P_3(\text{open}) = \frac{10}{11}\times 0.8\cdot\eta' =$$

$$P_4(\text{closed}) = \eta'\cdot P(Z=\text{open}|X=\text{closed})\cdot P_3(\text{closed}) = \frac{0.3}{11}\times 0.3\cdot\eta' =$$

Problem 3

(1) transition matrix : $\begin{bmatrix} 0.9 & 0 & 0 & 0.1 \\ 0 & 0.8 & 0.1 & 0.1 \\ 0 & 0.1 & 0.8 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.7 \end{bmatrix}$

(2) Let A_i denotes event { stay at room i }

$$\begin{bmatrix} 0.9 & 0 & 0 & 0.1 \\ 0 & 0.8 & 0.1 & 0.1 \\ 0 & 0.1 & 0.8 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.7 \end{bmatrix} \begin{bmatrix} P(A_1) \\ P(A_2) \\ P(A_3) \\ P(A_4) \end{bmatrix} = \begin{bmatrix} P(A_1) \\ P(A_2) \\ P(A_3) \\ P(A_4) \end{bmatrix}$$

And $P(A_1)+P(A_2)+P(A_3)+P(A_4)=1$.

$\Rightarrow$ Solve the equations : $\begin{cases} P(A_1) = 0.1 \\ P(A_2) = 0.1 \\ P(A_3) = 0.1 \\ P(A_4) = 0.7 \end{cases}$

(3) Let B = { The robot is going through door }

C = { The robot is going through the door between room 1 and 4 }

$\Rightarrow P(B) = 0.1P(A_1) + 0.2P(A_2) + 0.2P(A_3) + 0.3P(A_4) = 0.2$

$P(C) = 0.1P(A_1) + 0.1P(A_4) = 0.02$

$\therefore$ The result is $\frac{P(C)}{P(B)} = \frac{0.02}{0.2} = \frac{1}{8}$.

Problem 4:

$$bel(x_t) = p(x_t \mid u_{1:t}, z_{1:t})$$

$$= p(x_t \mid u_{1:t}, z_{1:t-1}, z_t)$$

$$= \eta\, p(z_t \mid u_{1:t}, z_{1:t-1}, x_t) \cdot p(x_t \mid u_{1:t}, z_{1:t-1})$$

$$= \eta\, p(z_t \mid x_t) \left[\sum_{x_{t-1}} p(x_t \mid x_{t-1}, u_t) \cdot p(x_{t-1} \mid u_{1:t}, z_{1:t-1}) \right]$$

$$= \eta\, p(z_t \mid x_t) \left[\sum_{x_{t-1}} p(x_t \mid x_{t-1}, u_t) \cdot p(x_{t-1} \mid u_{1:t-1}, z_{1:t-1}) \right]$$

$$= \eta\, p(z_t \mid x_t) \left[\sum_{x_{t-1}} p(x_t \mid x_{t-1}, u_t) \cdot bel(x_{t-1}) \right]$$

Problem 5:

$$bel(x_{0:t}) = p(x_{0:t} \mid u_{1:t}, z_{1:t})$$

$$= p(x_{0:t-1}, x_t \mid u_{1:t}, z_{1:t})$$

$$= \eta\, p(x_t \mid u_{1:t}, z_{1:t}, x_{0:t-1}) \cdot p(x_{0:t-1} \mid u_{1:t}, z_{1:t})$$

$$= \eta\, p(x_t \mid u_{1:t}, z_{1:t-1}, z_t, x_{0:t-1}) \cdot p(x_{0:t-1} \mid u_{1:t-1}, z_{1:t-1})$$

$$= \eta \cdot p(z_t \mid u_{1:t}, z_{1:t-1}, x_{0:t}) \cdot p(x_t \mid u_{1:t}, z_{1:t-1}, x_{0:t-1}) \cdot bel(x_{0:t-1})$$

$$= \eta \cdot p(z_t \mid x_t) \cdot p(x_t \mid x_{t-1}, u_t) \cdot bel(x_{0:t-1})$$