TITLE:

FUNDAMENTAL CONTROL OF LEO NANO SATELLITE

Abstract:

The approach in solving dynamics and control systems of satellite missions is usually complex. The complexity increases the farther away you are from the Earth's atmosphere. Hence create challenges for freshers who want to interpret and innovate the systems used for such missions. I myself, who is an Aerospace graduate had some difficulty understanding the dynamics and control system of spacecrafts. For us to rapidly advance our space mission technologies I believe this barrier must be greatly reduced.

Therefore, the idea for this report is built around simplifying some of this complex system. We must note that space dynamics and control is so vast, depending on what orbit or mission you are about to perform, some assumed constraints can be applied to numerically simulate the mission. More emphasis will be given to Low Earth Orbit, spherical nano satellite in this report. A spherical nano satellite with radius about 5cm, and of has uniform mass distribution for its payload and casing about its centre of mass (i.e., Body fixed frame). We can easily assume a point mass approach for the space mechanics of this satellite system.

Let's consider some few characteristic associated with LEO satellites (e.g., ISS). We know that low amount of energy is required to place a satellite into LEO orbit. And their long life-cycle due to the less impact of radiation, when compared to the higher orbits. Its proximity to the Earth's atmosphere, makes communication at a higher Bandwidth possible. We should also understand that this proximity to the earth has few draw backs, such as the significant atmospheric drag (below 300km from Earth's surface), and perturbation due to Earth's oblateness. This draw backs, causes Orbital decay (i.e., Reduction in the orbit size, by pulling the satellite closer and closer towards the earth). Hence, if re-boosting/thrusting of the satellite is not routinely done, the satellite may re-enter the earth's atmosphere. Hence, LEO is defined by an orbital period of 128minutes or less, which corresponds to an attitude of about 2000km or less, above Earth's sea level.

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1. Introduction

Recent study has shown that micro and nano satellite are more cost effective. We are going to be focusing our attention in the control of a spherical nano satellite, which is about 10kg in weight. Let's assume a Ball Lens In The Space (BLITS) satellite, with attitude determination as its mission requirements.

2. Guidance, Navigation and Control (GNC)

Let's consider a general configuration of a satellite's GNC as shown below:

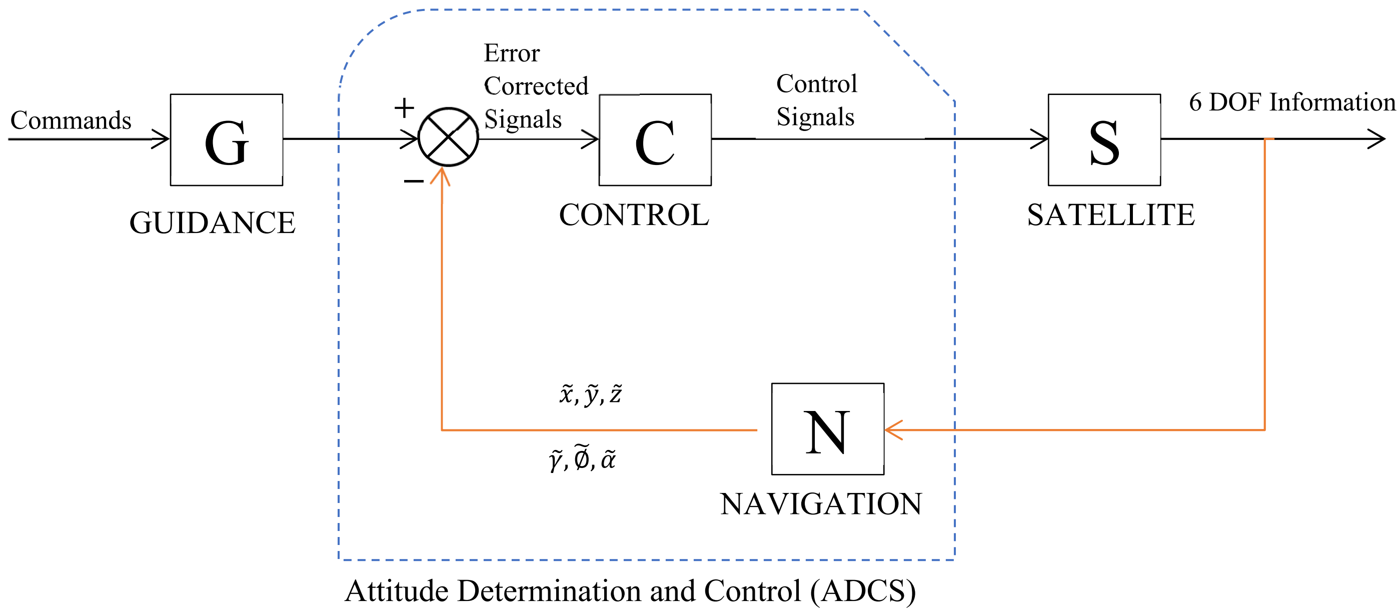


Figure 1.1 General Guidance, Navigation and Control Block Diagram

Fundamentally this is in form of a close loop system, where the Guidance block contains the commands which the satellite must perform, such as rotational motion of yaw, pitch or roll, and or translational motion. Satellite block is the physical attitude position of the satellite, which has about six degree of freedom (6 DOF for translational and rotational motion). The determination of this attitude position (or the state of the satellite), is done in the Navigation block, using sensors (such as rate gyro), global positioning system (GPS), etc. The estimated measurement for the transnational motion can be in spherical or polar coordinate system or in cartesian coordinate system, $\tilde{x}, \tilde{y}, \tilde{z}$, and for the rotational motion we use the Euler-angles, pitch ($\tilde{\alpha}$), yaw ($\tilde{\emptyset}$) and roll ($\tilde{\gamma}$). This estimation will also include their rates. There is also a Summing point or an Error computation block, which automatically computes the difference between the desired attitude position command signals from the Guidance Block and estimated attitude position signals from the Navigation block. And hence produces and error free or corrected signals for the Control Block, which in turn sends controls signals to the Satellite block again.

Having a wholistic view of the above figure, can say that the Satellite Block represents the physics of the satellite, whose orbital position can be described in:

- Orbital elements; semi-major axis (a), orbital inclination (i), and Eccentricity (e), or
- Longitude, Latitude and Altitude, or
- North, East, Down (NeD)

For LEO satellite we assume the satellite as point when estimating it orbital position and motion. The commands in the Guidance block are sent to the Command and Data Handling (CDH) board of the satellite, for a nano LEO satellite. The blocks enclosed in the blue dash lines are called Attitude Determination and Control.

2.1 Attitude Determination and Control (ADCS):

The ADCS board, which is located on the satellite, performs two main tasks, these are:

- Attitude Estimation: this can be done using,
  - Global positioning system (GPS): measure position of the satellite relative to Earth.
  - Horizon sensor; measures the horizon of the satellite with respect to the Earth.
  - Magnetometer; measures the magnetic field acting on the satellite.
  - Sun sensors; measures the vector (direction) of the sun.
  - Star Trackers; takes pictures of the stars that can used for measurement.
  - Rate Gyros; measures how fast the satellite is spinning in different directions.

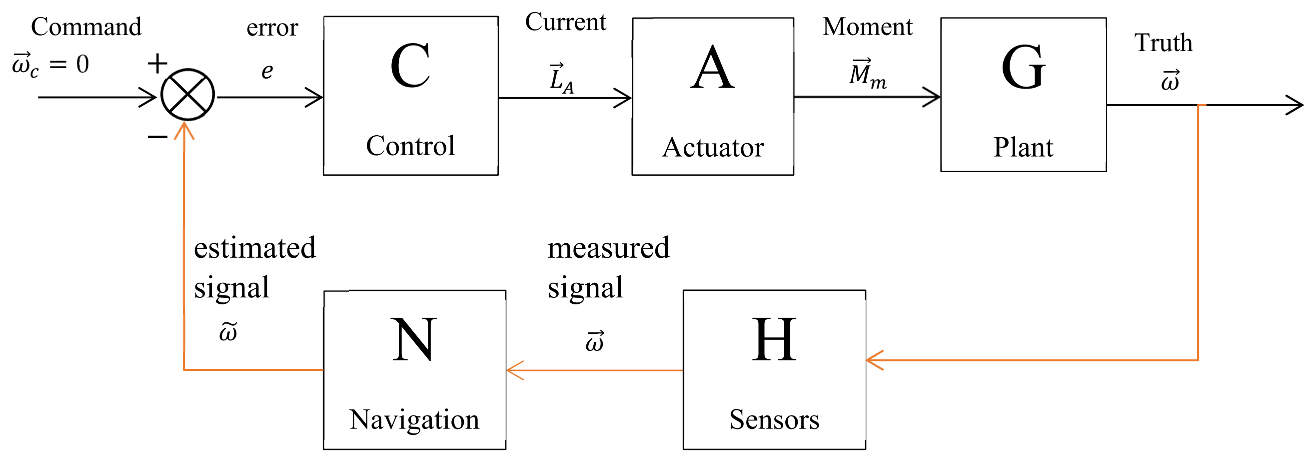
  These will give us the result from our Navigation Block, which is our Nav-state, that tells us where the satellite is, and angular rate of spin.

- Attitude Control: this can be done using,
  - Reaction Wheels; controls through the angular momentum of a spinning rotor.
  - Control Moment Gyros (CMG); controls through the torque created by rotating a gimble which carries a reaction wheel.
  - Magnetorquers (torque rods); controls by passing a current through a coil, which creates magnetic field that can oppose Earth's magnetic field.
  - Thrusters; controls through pulse jets.

Some trade-offs will be made during the selection of attitude estimation and control devices for our project, because we don't necessarily require all of them on-board our small LEO nano satellite.

3. State Estimation for Magnetorquers Control Using Complimentary Filters

Performing control using magnetorquers we must try to estimate the state of the satellite, i.e., we want to measure the angular rate and magnetic field of the system. Let's assume the below block diagram for our control system:



From the above we can see that we send a guess angular velocity command of $\vec{\omega}_c = 0$, This is then added or subtracted with the estimated angular velocity signal $\tilde{\omega}$, from the navigation block, to yield the error signal in the error block. This signal is then passed to control block, which generates current signal $\vec{L}_A$, in amperes, that is required by the actuators (magnetorquers) to generate magnetic field. This magnetic fields will oppose the earth's magnetic field acting around the satellite and thus producing moment force used in the plant block.

An onboard sensor, typically rate gyro and magnetometer is used to measure the angular velocity, and thus passes a measured angular velocity signal $\vec{\omega}$, to the navigation block, which then tries to filter this noisy measured signal using complimentary or other types, such as the Extended Kalman Filter (EKF). But for this project we will be making use of the complimentary filter which converts the measured noisy signal into an estimated signal $\tilde{\omega}$.

3.1 The General Estimation

The discrete measurement from the sensor is given as,

$$\vec{y}_k = \vec{h}(\vec{X}_k) + \vec{v}_k + \vec{b}_k \qquad (3.1)$$

Where, $\vec{X}_k$ is the state vector of the system

$\vec{h}$ is the non-linear vector function of X

$\vec{v}_k$ is the random noise vector

$\vec{b}_k$, is the bias or offset vector.

For the noise vector let's assume it's covariant as $cov(\vec{v}_k) = E[\vec{v}_k \vec{v}^T] = \Upsilon$,

with E as the expectation operator and some matrix Υ, which tells us how much noise is in our system. For instance, if Υ is really big, then we can say that our sensors are bad, but if this matrix is small then our sensors are said to be good. Hence Υ is like a tuning parameter for us to play with and see what happens to the measurement.

Also, we can linearize $\vec{h}(\vec{X}_k)$ by taking the Jacobi of it, we will get $h_k = H$, in matrix form.

3.2 First method using Least Square Regression

in solving the above will be using the trendline fitting which considers multiple measurement that is greater than our state vector (i.e., $\vec{\omega}$ for pitching, rolling and yawing). This method is known as linear Least Square Regression. So that we have the measurement as,

$\vec{Y} = \begin{bmatrix} \vec{y}_1 \\ \vdots \\ \vec{y}_N \end{bmatrix} - \begin{bmatrix} \vec{b} \\ \vdots \\ \vec{b} \end{bmatrix}$, in this case we assume that we know our bias vector and that it is constant.

From this, eqn. (3.1) an be simplified and written as,

$$\vec{Y} = H\vec{X}_k + \vec{v}$$

Now with $\vec{X}_k$ as our truth signal, it is difficult to solve due to the noise vector $\vec{v}$, Hence we utilize the least square regression developed by Carl Friedrich, so that estimation of our state at the k^{th} measurement as,

$$\tilde{X}_k = (H^T H)^{-1} H^T \vec{Y} \qquad (3.2)$$

3.3 Another method will be using a complimentary filter

This ensures that er use our previous k^{th} measurement for every new $(k+1)^{th}$ measurement. This is known as the Sequential Linear Least Square Estimator (Filter). So let's assume that the previous measurement, $\tilde{X}_{k-1}$ is known, obtained as,

$$\tilde{X}_{k-1} = \vec{X}_k + \vec{W}_k$$

With $\vec{X}_k$ as the truth signal and $\vec{W}_k$ as noise model. Taking the covariant of this noise model yields,

$$cov(\vec{W}_k) = E[\vec{W}_k \vec{W}^T] = \mathrm{q}$$

Where the noise expectation matrix q, tells us how good or bad the previous measurements are.

After complex simplification of the above we get,

$$\tilde{X}_k = \Omega(\vec{y}_k - \vec{b}) + \Gamma \tilde{X}_{k-1} \qquad (3.3)$$

Where Ω and Γ are some complex matrices, which are factors for the current and previous measurement respectively.

3.4 Example of General Estimation for Angular Velocity Measurement

Let's assume that for every k^{th} time step that we have a measurement,

$$\tilde{\omega}_k = \vec{\omega}_k + \vec{v}_k + \vec{b} \qquad (3.4)$$

Where the truth $\vec{\omega}_k$ is direct linear value measurement from the onboard rate gyro sensor. This means that the non-linear terms in eqn. (3.1) becomes identity, $h_k = H = I$.

Also, from eqn. (3.2) we can now say that $H^T H = N$, which is the total number of measurement as a scalar quantity. And that $H^T \vec{Y}$, is the summation of all the measurement, so that we can rewrite eqn. (3.2) as $\tilde{X}_k = \frac{\sum \vec{Y}}{N}$, we can see that this is like taking the average of the signal.

Hence, we can say that eqn. (3.4) will become, $\widetilde{\omega}_k = average\ of \left\{ \begin{bmatrix} \vec{\omega}_1 \\ \vdots \\ \vec{\omega}_N \end{bmatrix} - \begin{bmatrix} \vec{b} \\ \vdots \\ \vec{b} \end{bmatrix} \right\}$

Note that this method can only estimate some number of measurements at one given instance of time. But because our satellite is always on motion, which is dependent on time. So, this method will make the measured signal data messier and more unusable, hence we will be considering the below method for our case study.

This second method uses complimentary filter,

Let's consider the following assumption and substitute into eqn. (3.3) for the complex matrices,

$$\mathrm{q}^{-1} = s$$

$$\Upsilon^{-1} = 1 - s$$

we will get the measurement for the k^{th} time step as,

$$\widetilde{\omega}_k = (1 - s)(\vec{\omega}_k - \vec{b}) + s\widetilde{\omega}_{k-1}$$

The above is an important equation we will utilize for our numeric integration.

3.5 Mathematical Analysis of the Complimentary Filter

Analysing the above equation as follows,

If $s = 1$,

So that, $\widetilde{\omega}_k = \widetilde{\omega}_{k-1}$, in other words this ignores the new sensor values and uses only the previous values.

And our assumption $\Upsilon^{-1} = 0$, which means that the noise expectation matrix for the current measurement $\Upsilon \rightarrow \infty$, as we know, this says that our sensors are really bad.

If $s = 0$,

So that, $\widetilde{\omega}_k = \vec{\omega}_k - \vec{b}$, in other words this ignores the previous measured values and uses only the new values directly from the sensors.

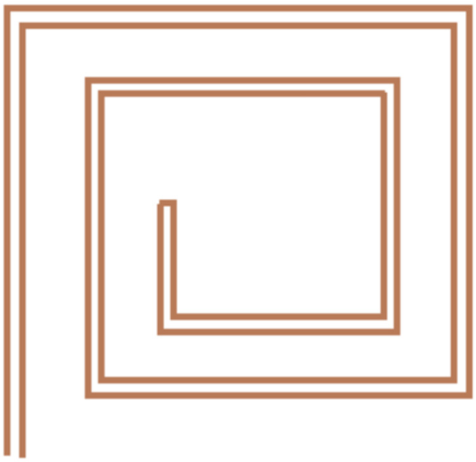
And our assumption $\mathrm{q}^{-1} = 0$, which means that the noise expectation matrix of the previous measurement, $\mathrm{q} \rightarrow \infty$, this shows that the previous measurement is really bad.

In conclusion we can play around with the s or q and Υ parameters to observe how the system will behave. And also, we can consider similar equations for the magnetometer measurement, by using complimentary filters to filter out the noise vector.

4. B-dot Control Using Magnetorquers

4.1 Magnetorquers Operation Principle.

Let's assume we have a planar coil of wire as shown below.



Where;

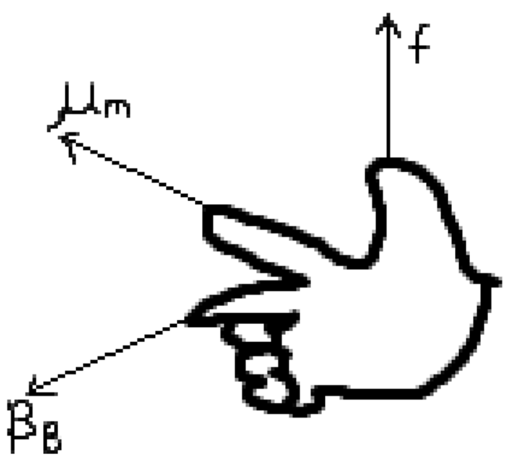
A is the area of coils

n is the number of turns

i is the current passing through the coils

We know that if some current i is made to flow through the above coil setup, it will induce a magnetic field, which is perpendicular to it (i.e., using the familiar right-hand rule). We can also have that the magnetic-moment generated by this induced magnetic field which is perpendicular to the coil plane as,

$$\vec{\mu}_M = nA\left(i_x\hat{x}_B + i_y\hat{y}_B + i_z\hat{z}_B\right)$$



Using the right-hand rule as shown we can say that taking the cross product of the magnetic-moment vector $\vec{\mu}_M$ and the magnetic field vector β_B (at the body reference frame) will give the Magnetic-Torque, that will eventually move the satellite as,

$$\vec{M}_M = \vec{\mu}_M \times \vec{\beta}_B \qquad (4.1)$$

4.2 B-dot Controller Model

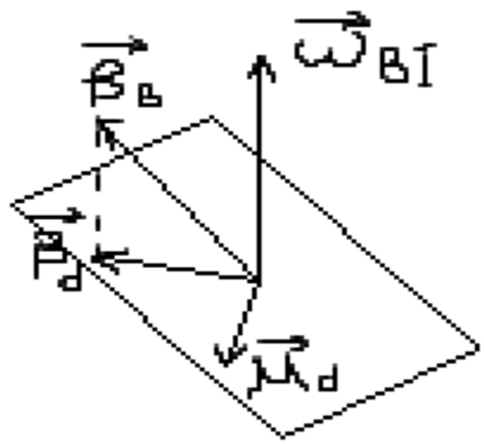
Given a satellite that is rotating/tumbling about one axis, and then we have a desire to oppose that given motion. Thus, to control the satellite we will need a torque of the form,

$$\vec{M}_M = -k\vec{\omega}_{BI}$$

Where k is some constant and $\vec{\omega}_{BI}$ is the angular velocity vector observed from the inertial frame, on the body fixed frame. Hence if we are to have a condition where the above is equal to eqn. (4.1) as,

$$\vec{M}_M = -k\vec{\omega}_{BI} = \vec{\mu}_M \times \vec{\beta}_B$$

But this might be difficult to archive, because magnetic field vector $\vec{\beta}_B$, can be in any orientation at given position and inclination of the satellite. Thus, we know that ideal orientation of $\vec{\beta}_B$, is it being orthogonal to the angular velocity vector, which is given as $\vec{\beta}_d$, in the figure below;



What this implies is that the desire moment, will be given by,

$$\vec{\mu}_d = k\vec{\omega}_{BI} \times \vec{\beta}_B$$

Substituting this into eqn. (4.1) gives,

$$\vec{M}_M = (k\vec{\omega}_{BI} \times \vec{\beta}_B) \times \vec{\beta}_B$$

What this means is that if $\vec{\omega}_{BI} \times \vec{\beta}_B = 0$, (i.e., both vectors are co-linear) then this control equation will not work. Although for most scenarios such co-linear occurrence are not common, but it is still important to note.

5. Reference trajectory and Satellite Dynamics

For this section I will be citing my project report for AE 8133 – Space Mechanics (2021) on "Fundamental Space Dynamics of a Near Frictionless Contact Between two surfaces of a Spherical Nano Satellite". But for this project we will be focusing only on the control of the satellite. From the cited paper we had a spherical LEO satellite with the following parameters obtained from CAD modelling:

Inner ball radius and shape	80mm radius, semi spherical
Overall dimension and shape	90 mm radius, spherical
Number of ball bearing	5
Origin	At main sphere concentric center [0, 0, 0] of body fixed frame [x, y, z] (shown. in fig. 1.3)
Payload's center of mass	At [0, 0, -30.01] mm from origin
Satellite's center of mass	At [0, 0, -15.11] mm from origin
Mass of the nanosatellite	8kg
Moment of inertial [Ixx, Iyy, Izz]	[0.023, 0.023, 0.027] kg m^2

And a BLITS satellite reference trajectory given as,

orbit	Sun-synchronous near-circular
mean altitude	832 km
inclination	98.85°
period	101.3 minutes
local equatorial crossing time	12:00 hours

6. Numeric Integration.

For the sensor dynamics we are going to use a fixed step integrator using Runge-Kutta-4 (RK4), method. RK4 algorithm converges faster than the Euler's method and the equation is given below as;

$$\vec{k}_1 = \dot{\vec{x}}(t_k, \vec{x}_k)$$

$$\vec{k}_2 = \dot{\vec{x}}\left(t_k + \frac{\Delta t}{2}, \vec{x}_k + \vec{k}_1 \frac{\Delta t}{2}\right)$$

$$\vec{k}_3 = \dot{\vec{x}}\left(t_k + \frac{\Delta t}{2}, \vec{x}_k + \vec{k}_2 \frac{\Delta t}{2}\right)$$

$$\vec{k}_4 = \dot{\vec{x}}\left(t_k + \Delta t, \vec{x}_k + \vec{k}_3 \Delta t\right)$$

$$\vec{k} = \frac{1}{6}\left(\vec{k}_1 + 2\vec{k}_2 + 2\vec{k}_3 + \vec{k}_4\right)$$

And the next state is given by,

$$\vec{x}_{k+1} = \vec{x}_k + \vec{k}\Delta t$$

6.1 Initial Conditions

N/B: That the space dynamics of our satellite are already discussed in the cited paper, with the initial conditions, given as,

Angular rate, $\vec{\omega}_{BI} = \begin{bmatrix} p \\ q \\ r \end{bmatrix}_B = \begin{bmatrix} 10 \\ 5 \\ 3 \end{bmatrix}$ deg/sec

$$\gamma = \emptyset = \alpha = 0$$

$$x(0) = 6.357e3 + mean\ altitude, \qquad y(0) = z(0) = \dot{x}(0) = 0$$

$\vec{r} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$, Semi-major axis, $a = norm(\vec{r})$, inclination, $i = 98.85°$

for near circular orbit we have that circular velocity, $v_c = \sqrt{(\mu/a)}$

$$\dot{y}(0) = v_c cos(i), \qquad \dot{z}(0) = -v_c sin(i);$$
$$timestep = 1$$

We will perform orbital simulation for two orbital period, $timespan = [0: timestep: 2T]$
Also we have to update the magnetic field for every 10sec.

Please note the satellite is assumed to have a positive damping constant $C = 0.00003$, due to the coefficient of friction.

For the sensor block we can generate some random values with;
Magnetic field scale bias = $4e^{-7}$ tesla
Magnetic field scale noise = $1e^{-5}$ tesla
Angular velocity scale bias ≅ 0.57 degree
Angular velocity scale noise ≅ 0.057 degree
These bias and noise values will be added to the corresponding magnetic field and angular velocity vector, in order to create some artificial noise and bias to the system model.

For the navigation block,

We have the previous navigation values for the magnetic field, $\vec{\beta}_{Bprev} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$,

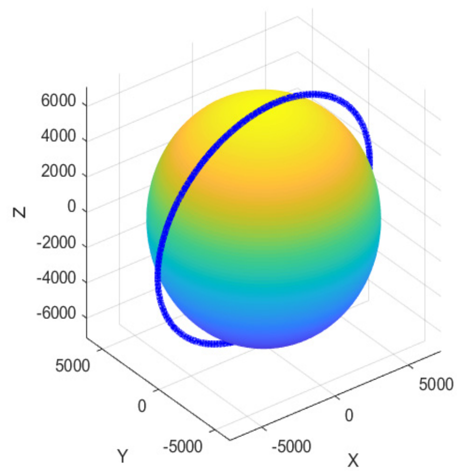
and for angular velocity as $\vec{\omega}_{BIprev} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.

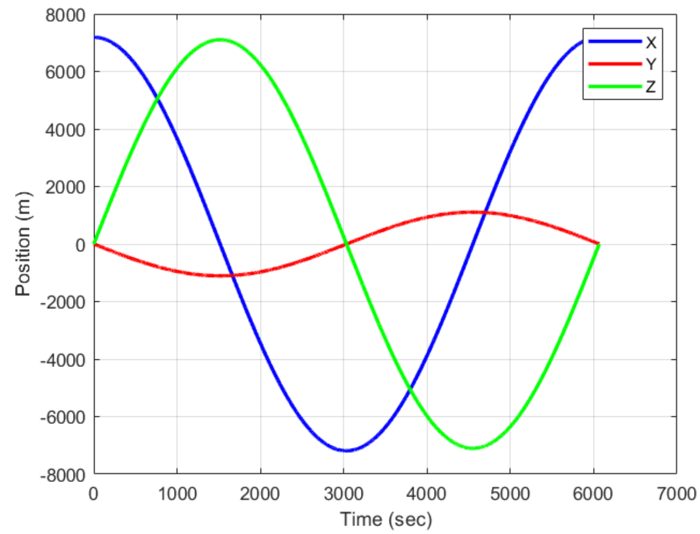
Also, we set our filter, $s = 70\%$ and later change it to $s = 10\%$ to see the behavior of the system

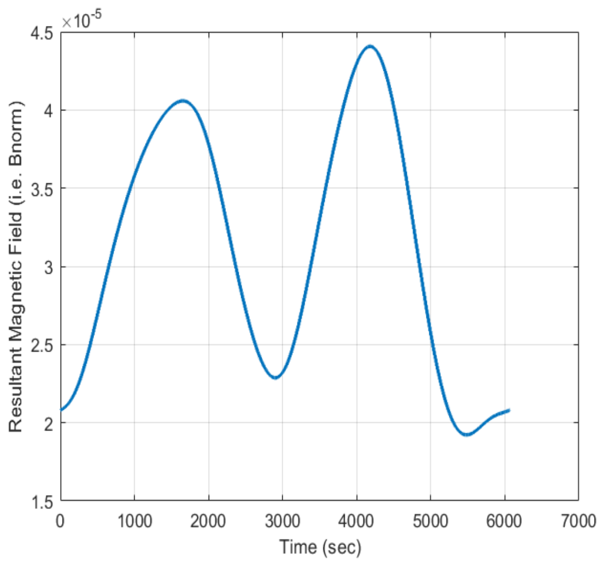
For the control block we put,
$k = 67200$,
and magnetorquers parameters as,
number of turns, $n = 84$,
Area occupied by the coils, $A = 0.02\ m^2$,
So that we have the current i as the output parameter in amperes, which is then use to compute the magnetic torque using magnetic moment of $\vec{\mu}_M = nAi$ and taking cross product of it with the resolved magnetic field in the inertial frame $\vec{\beta}_{BI}$.

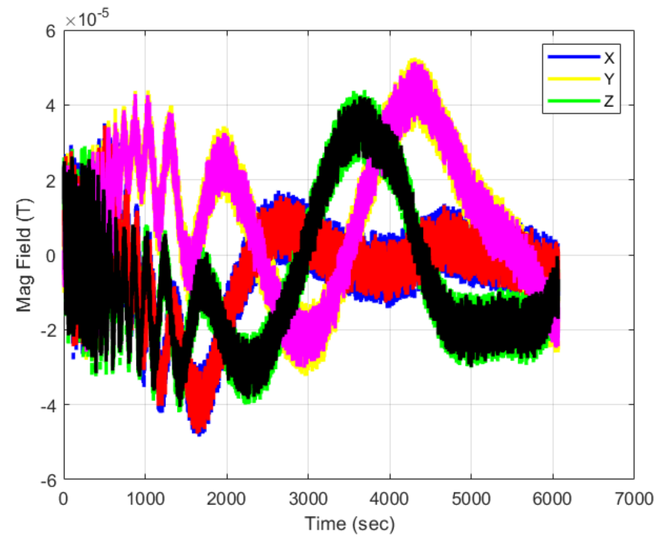
7. Results and Discussion

Running the simulation in MATLAB yields the following results;

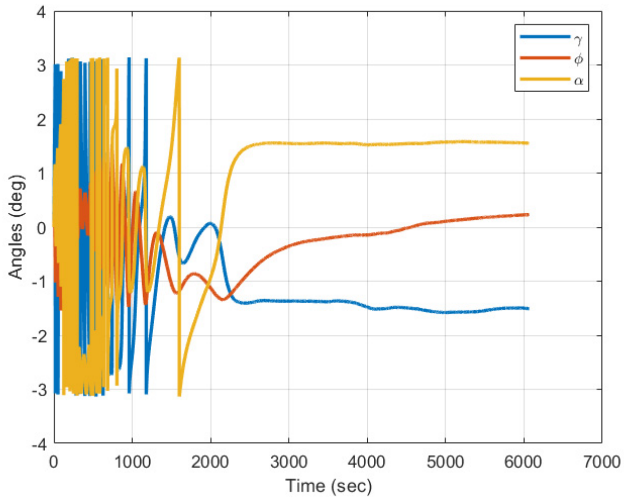


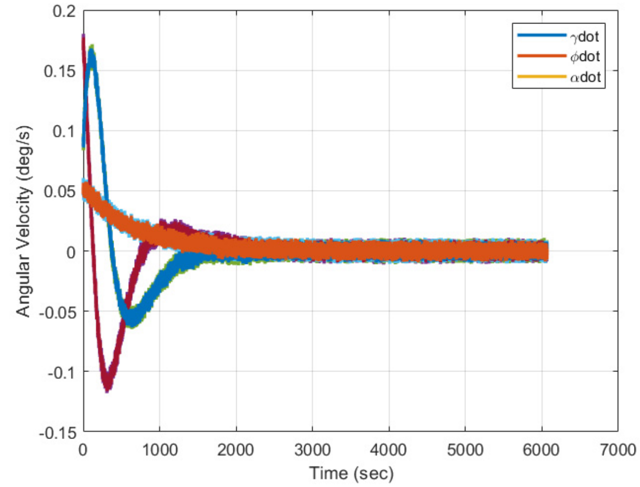




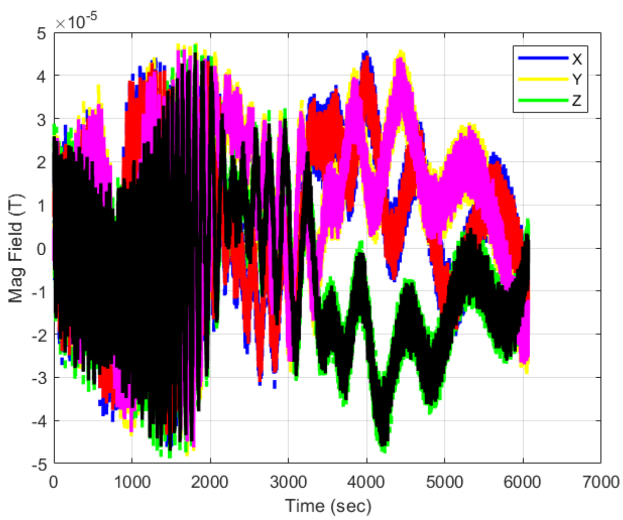


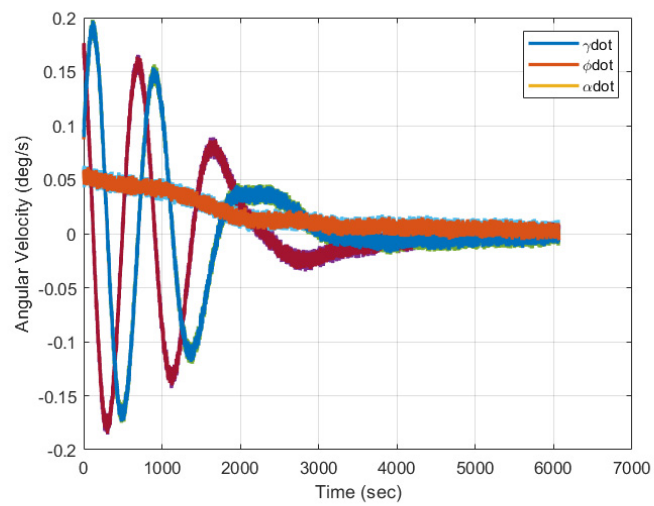
The frequency of the magnetic field is observed to greatly reduce as it gets in-tuned with the orbital motion. Likewise for the angular velocity shown below.





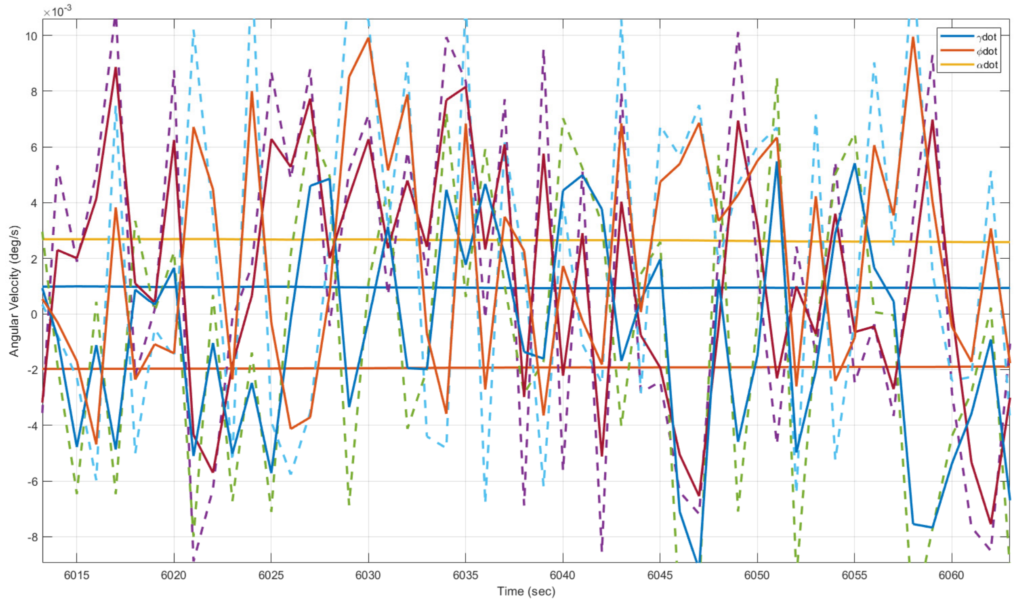
From the above attitude representation. We can say that the de-tumbling effect of the controller is not very visible in this case, due to the energy dissipation effect caused by the coefficient of friction between the two contacts of our satellite from the cited paper. Therefore, for us to fully see the effect of the controller on the satellite, let's assume that the positive damping constant $C = 0$. Hence, we will get the following plots for the magnetic field and angular rate;



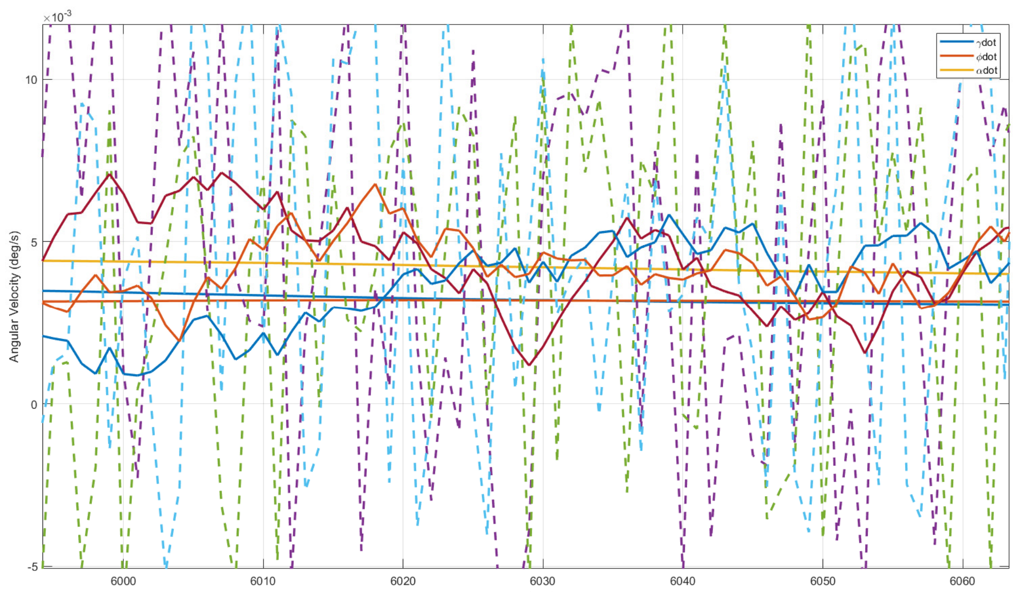


From the above we can clearly see the difference between our previous plot and these plots, as the controller tries to de-tumble the satellite gradually with time. Thus the frequency reduces but not as fast as before when we had energy dissipation effect.

Also zooming-in on the angular velocity plot for our filter parameter $s = 70\%$, we can see the truth signals as straight lines, the noisy signals in dash lines and then the filtered signals as continuous curvy lines as shown below;



If we change the filter parameter $s = 10\%$ in the navigation block we will see that the complimentary filter becomes aggressive at filtering out noise and the curves gets a lot closer to the truth signals and become less spiky as shown below;



8. Conclusion

In conclusion we've been able to take a more fundamental approach of performing control simulation and analysis for a LEO nano satellite. This should help facilitate the understanding basic spacecraft dynamics and control system and how they operate. For future work we can validate these results with a more practical approach.

9. References

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