ASSIGNMENT 4

1. The following MATLAB code will solve the given problem, I have also included the MATLAB filename.m file to my submission:

```
clear
%%EQUATIONS OF MOTION

syms M R L1 L2 Mt1 Mt2 Mt3 mu beta1 beta2
U=-
M*mu/R+(mu/(2*R^3))*(Mt1*L1^2+Mt2*L2^2+2*Mt3*L1*L2*cos(beta
1-beta2))-
(3*mu/(2*R^3))*(Mt1*L1^2*cos(beta1)^2+Mt2*L2^2*cos(beta2)^2
+2*Mt3*L1*L2*cos(beta1)*cos(beta2));
syms Rdot thetadot w1(t) w2(t) theta
T=(1/2)*M*(Rdot^2+R^2*thetadot^2)+(1/2)*Mt1*w1^2*L1^2+(1/2)
*Mt2*w2^2*L2^2+Mt3*w1*w2*L1*L2*cos(beta1-beta2);
L=T-U;
% R-Equation
DL_Rdot=diff(L,Rdot);
syms Rdott(t) beta1t(t) beta2t(t) Rt(t)
DL_Rdott=subs(DL_Rdot,Rdot,Rdott);
eqmt=diff(DL_Rdott,t)-
subs(diff(L,R),{R,beta1,beta2},{Rt,beta1t,beta2t});
syms Rdot(t) beta1(t) beta2(t)
eqmR=subs(eqmt,{Rdott,Rt,beta1t,beta2t},{Rdot,R,beta1,beta2
})
% Theta-Equation
syms M R L1 L2 Mt1 Mt2 Mt3 mu beta1 beta2
syms Rdot thetadot w1(t) w2(t) theta
syms beta1dot beta2dot
L_new=subs(L,{w1,w2},{thetadot+beta1dot,thetadot+beta2dot})
;
DL_thetadot=diff(L_new,thetadot);
syms thetadott(t) Rt(t) beta1t(t) beta2t(t) beta1dott(t)
beta2dott(t)
DL_thetadott=subs(DL_thetadot,{thetadot R beta1 beta2
beta1dot beta2dot},...
    {thetadott Rt beta1t beta2t beta1dott beta2dott});
eqmt=diff(DL_thetadott,t)-
subs(diff(L_new,theta),{R,beta1,beta2},{Rt,beta1t,beta2t});
syms thetadot(t) R(t) beta1(t) beta2(t) beta1dot(t)
beta2dot(t)
```

```
eqmtheta_1=subs(eqmt,{thetadott Rt beta1t beta2t beta1dott
beta2dott},...
    {thetadot R beta1 beta2 beta1dot beta2dot});
syms w1dot w2dot
eqmtheta=subs(eqmtheta_1,{thetadot+beta1dot,thetadot+beta2d
ot,diff(thetadot,t)+diff(beta1dot,t)...

,diff(thetadot,t)+diff(beta2dot,t)},{w1,w2,w1dot,w2dot})
% Beta1-Equation
syms beta1dot beta2dot beta1 beta2 thetadot
DL_beta1dot=diff(L_new,beta1dot);
syms thetadott(t) beta1t(t) beta2t(t) beta1dott(t)
beta2dott(t) Rt(t)
DL_beta1dott=subs(DL_beta1dot,{thetadot beta1 beta2
beta1dot beta2dot},{thetadott beta1t beta2t beta1dott
beta2dott});
eqmt=diff(DL_beta1dott,t)-
subs(diff(L_new,beta1),{R,beta1,beta2},{Rt,beta1t,beta2t});
syms thetadot(t) beta1(t) beta2(t) beta1dot(t) beta2dot(t)
R(t)
eqmbeta1_1=subs(eqmt,{thetadott Rt beta1t beta2t beta1dott
beta2dott},{thetadot R beta1 beta2 beta1dot beta2dot});
eqmbeta1_2=subs(eqmbeta1_1,{thetadot+beta1dot,thetadot+beta
2dot,diff(thetadot,t)+diff(beta1dot,t)...

,diff(thetadot,t)+diff(beta2dot,t)},{w1,w2,w1dot,w2dot});
syms thetadot beta1dot beta2dot
eqmbeta1=subs(eqmbeta1_2,{thetadot+beta1dot,thetadot+beta2d
ot},{w1,w2})
% Beta2-Equation
syms beta1dot beta2dot beta1 beta2 thetadot
DL_beta2dot=diff(L_new,beta2dot);
syms thetadott(t) beta1t(t) beta2t(t) beta1dott(t)
beta2dott(t) Rt(t)
DL_beta2dott=subs(DL_beta2dot,{thetadot beta1 beta2
beta1dot beta2dot},{thetadott beta1t beta2t beta1dott
beta2dott});
eqmt=diff(DL_beta2dott,t)-
subs(diff(L_new,beta2),{R,beta1,beta2},{Rt,beta1t,beta2t});
syms thetadot(t) beta1(t) beta2(t) beta1dot(t) beta2dot(t)
R(t)
eqmbeta2_1=subs(eqmt,{thetadott Rt beta1t beta2t beta1dott
beta2dott},{thetadot R beta1 beta2 beta1dot beta2dot});
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eqmbeta2_2=subs(eqmbeta2_1,{thetadot+beta1dot,thetadot+beta
2dot,diff(thetadot,t)+diff(beta1dot,t)...

,diff(thetadot,t)+diff(beta2dot,t)},{w1,w2,w1dot,w2dot});
syms thetadot beta1dot beta2dot
eqmbeta2=subs(eqmbeta2_2,{thetadot+beta1dot,thetadot+beta2d
ot},{w1,w2})


%%NUMERIC INTEGRATION

syms M R L1 L2 Mt1 Mt2 Mt3 mu beta1 beta2
m1=10; m2=1; m3=1; L1=1; L2=1 ;
M=m1+m2+m3;
gamma1=(m2+m3)/M; gamma2=m3/M ;
Mt1=m1*gamma1^2+m2*(1-gamma1)^2+m3*(1-gamma1)^2;
Mt2=m1*gamma2^2+m2*gamma2^2+m3*(1-gamma2)^2;
Mt3=m1*gamma1*gamma2-m2*(1-gamma1)*gamma2+m3*(1-gamma1)*(1-
gamma2);
mu=3.986*10^5;

% Converting Equation of Motion into first order form
syms Rdot(t) thetadot(t) beta1dot(t) beta2dot(t) beta1
beta2
eqmR=subs(eqmR)==0;
eqmtheta=subs(eqmtheta_1)==0;
eqmbeta1=subs(eqmbeta1_1)==0;
eqmbeta2=subs(eqmbeta2_1)==0;
syms Rddot thetaddot beta1ddot beta2ddot
eqmR=subs(eqmR,diff(Rdot,t),Rddot);
eqmtheta=subs(eqmtheta,diff(thetadot,t),thetaddot);
eqmbeta1=subs(eqmbeta1,diff(beta1dot,t),beta1ddot);
eqmbeta2=subs(eqmbeta2,diff(beta2dot,t),beta2ddot);
syms beta1(t) beta2(t) R(t) theta(t)
eqmR=subs(eqmR,{Rdot,thetadot,beta1dot,beta2dot},...
    {diff(R),diff(theta),diff(beta1),diff(beta2)});
eqmtheta=subs(eqmtheta,{Rdot,thetadot,beta1dot,beta2dot},..
.
    {diff(R),diff(theta),diff(beta1),diff(beta2)});
eqmbeta1=subs(eqmbeta1,{Rdot,thetadot,beta1dot,beta2dot},..
.
    {diff(R),diff(theta),diff(beta1),diff(beta2)});
eqmbeta2=subs(eqmbeta2,{Rdot,thetadot,beta1dot,beta2dot},..
.
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```
    {diff(R),diff(theta),diff(beta1),diff(beta2)});

EqR=diff(R,t,t)==solve(eqmR,Rddot);
Eqtheta=diff(theta,t,t)==solve(eqmtheta,thetaddot);
Eqbeta1=diff(beta1,t,t)==solve(eqmbeta1,beta1ddot);
Eqbeta2=diff(beta2,t,t)==solve(eqmbeta2,beta2ddot);
Eqs=[EqR;Eqtheta;Eqbeta1;Eqbeta2];
[V,Y]=odeToVectorField(Eqs);
F=matlabFunction(V,'vars',{'t','Y'});
% Initial Conditions
R_0=6878; Rdot_0=0;
theta_0=0; thetadot_0=sqrt(mu/R_0^3); % Orbital Angular
Velocity (rad/s)
beta1_0=deg2rad(10); beta1dot_0=0.01*thetadot_0; %(rad/s)
beta2_0=deg2rad(-15); beta2dot_0=0.01*thetadot_0; %(rad/s)
% Simulation time
tstart=0;                % Simulation start time (in
seconds)
nor=2;                   % Simulation end time (in orbits)
tperiod=2*pi/thetadot_0;  % Orbital time period (sec)
tend=tperiod*nor; % Simulation end time (in seconds)
tstep=0.5;                 % Time step (in seconds)
t=tstart:tstep:tend;     % Simulation time
init_con=[theta_0;thetadot_0;R_0;Rdot_0;beta1_0;beta1dot_0;
beta2_0;beta2dot_0];
options=odeset('abstol',1e-9,'reltol',1e-9);
[t,x]=ode45(F,t,init_con);

theta=thetadot_0*t;
deg2rad=pi/180;
%Plotting
figure(1)
zoom on
subplot(10,1,1:5),plot(theta/(2*pi),x(:,1)/deg2rad);
set(findobj(gca,'Type','line','Color',[0 0
1]),'LineWidth',1)
set(findobj(gca,'Type','axes'),'Fontsize',12)
xlabel('Orbits',"FontSize",16);
ylabel('\theta (deg)',"FontSize",16);
legend('Numerical');
subplot(10,1,6:10),plot(theta/(2*pi),x(:,2)/deg2rad);
set(findobj(gca,'Type','line','Color',[0 0
1]),'LineWidth',1)
set(findobj(gca,'Type','axes'),'Fontsize',12)
```

```
xlabel('Orbits',"FontSize",16);
ylabel('\thetadot (deg/sec)',"FontSize",16);
legend('Numerical');

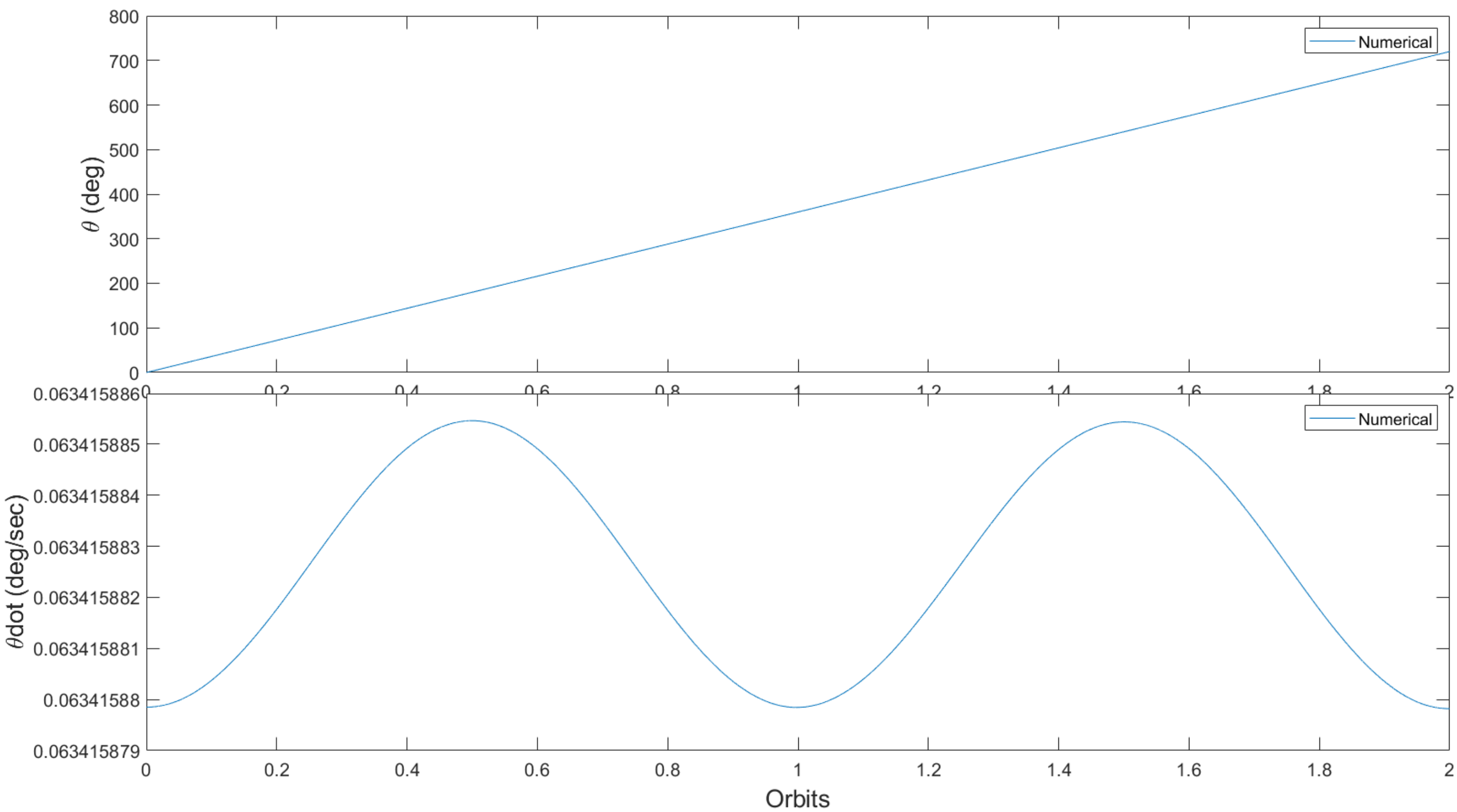
figure(2)
zoom on
subplot(10,1,1:5),plot(theta/(2*pi),x(:,3));
set(findobj(gca,'Type','line','Color',[0 0
1]),'LineWidth',1)
set(findobj(gca,'Type','axes'),'Fontsize',12)
xlabel('Orbits',"FontSize",16);
ylabel('R (Km)',"FontSize",16);
legend('Numerical');
subplot(10,1,6:10),plot(theta/(2*pi),x(:,4)/deg2rad);
set(findobj(gca,'Type','line','Color',[0 0
1]),'LineWidth',1)
set(findobj(gca,'Type','axes'),'Fontsize',12)
xlabel('Orbits',"FontSize",16);
ylabel('Rdot (Km/s)',"FontSize",16);
legend('Numerical');

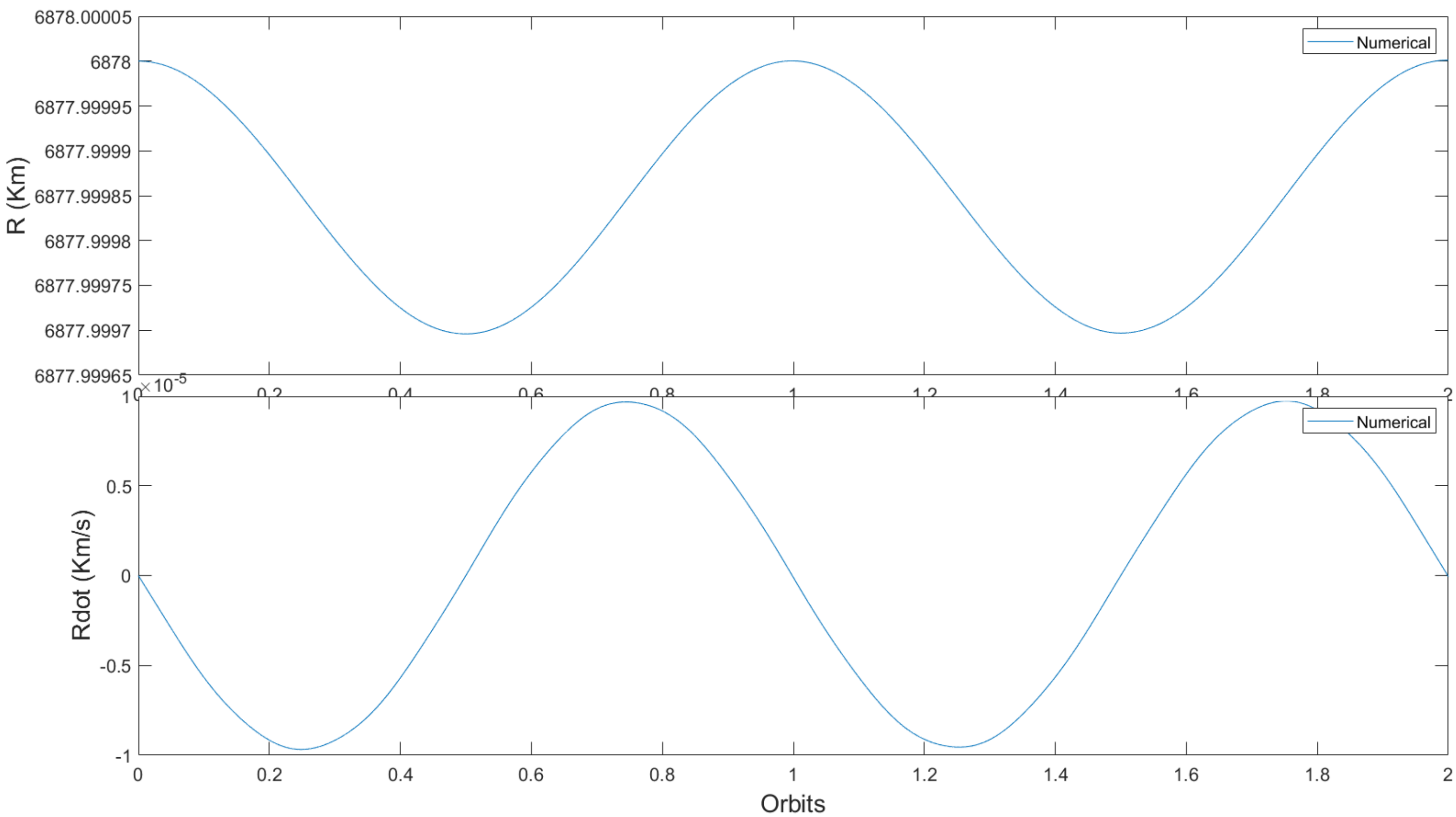
figure(3)
zoom on
subplot(10,1,1:5),plot(theta/(2*pi),x(:,5)/deg2rad);
set(findobj(gca,'Type','line','Color',[0 0
1]),'LineWidth',1)
set(findobj(gca,'Type','axes'),'Fontsize',12)
xlabel('Orbits',"FontSize",16);
ylabel('\beta1 (deg)',"FontSize",16);
legend('Numerical');
subplot(10,1,6:10),plot(theta/(2*pi),x(:,6)/deg2rad);
set(findobj(gca,'Type','line','Color',[0 0
1]),'LineWidth',1)
set(findobj(gca,'Type','axes'),'Fontsize',12)
xlabel('Orbits',"FontSize",16);
ylabel('\beta1dot (deg/sec)',"FontSize",16);
legend('Numerical');

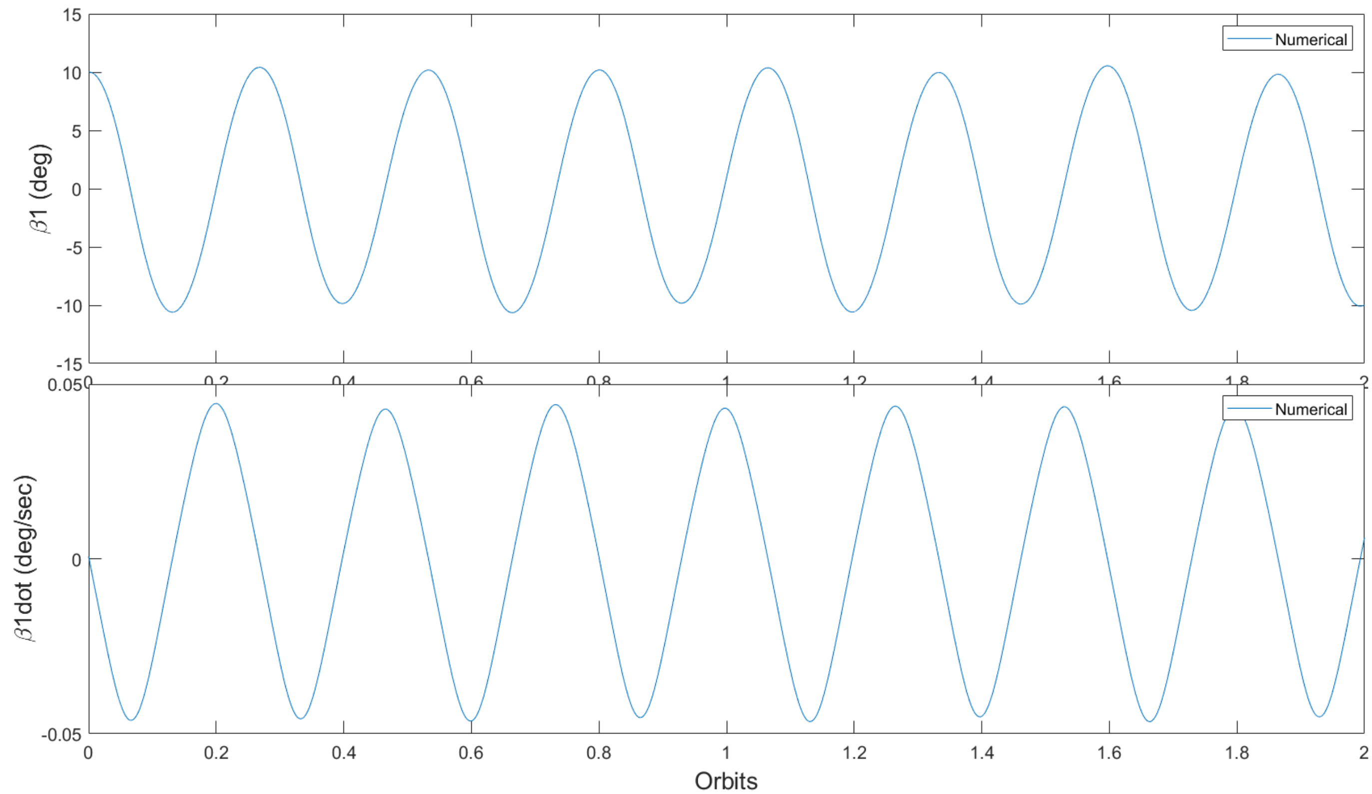
figure(4)
zoom on
subplot(10,1,1:5),plot(theta/(2*pi),x(:,7)/deg2rad);
set(findobj(gca,'Type','line','Color',[0 0
1]),'LineWidth',1)
set(findobj(gca,'Type','axes'),'Fontsize',12)
```

```
xlabel('Orbits',"FontSize",16);
ylabel('\beta2 (deg)',"FontSize",16);
legend('Numerical');
subplot(10,1,6:10),plot(theta/(2*pi),x(:,8)/deg2rad);
set(findobj(gca,'Type','line','Color',[0 0
1]),'LineWidth',1)
set(findobj(gca,'Type','axes'),'Fontsize',12)
xlabel('Orbits',"FontSize",16);
ylabel('\beta2dot (deg/sec)',"FontSize",16);
legend('Numerical');
```

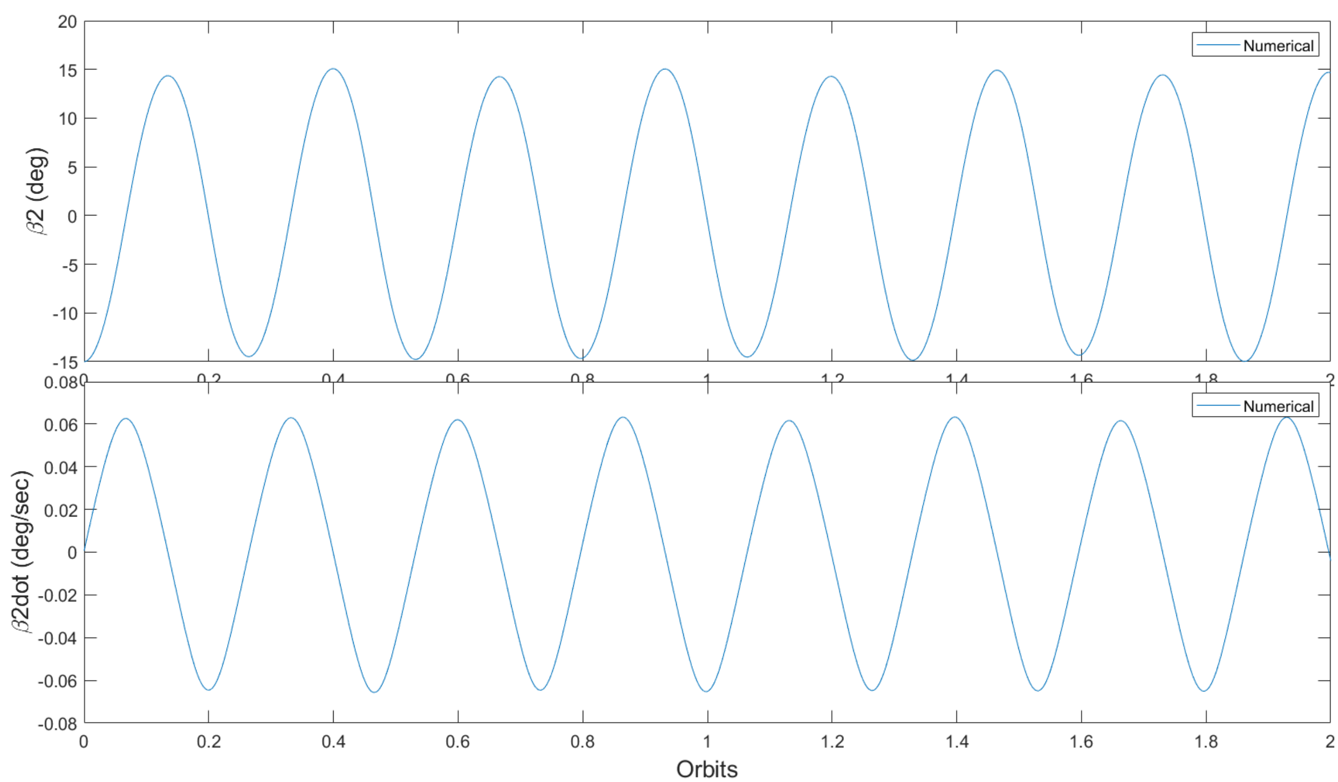
The Following Graphs are obtained from running the above code:




6878.00005
6878
6877.99995
6877.9999
6877.99985
6877.9998
6877.99975
6877.9997
6877.99965
R (Km)
Numerical
×10^-5
1
0.5
0
-0.5
-1
Rdot (Km/s)
Numerical
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
Orbits



Numerical
β1 (deg)
15
10
5
0
-5
-10
-15
Numerical
β1dot (deg/sec)
0.05
0
-0.05
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
Orbits



Numerical
β2 (deg)
20
15
10
5
0
-5
-10
-15
Numerical
β2dot (deg/sec)
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
Orbits

2) Given the following Euler equations for an attitude motion of a spacecraft as,

$$I_x \dot{\omega}_x - (I_y - I_z)\omega_y \omega_z = -C\omega_x$$
$$I_y \dot{\omega}_y - (I_z - I_x)\omega_z \omega_x = -C\omega_y$$
$$I_z \dot{\omega}_z - (I_x - I_y)\omega_x \omega_y = -C\omega_z \quad \text{—— (2)}$$

Lets reduce the form of this equations, by collecting the principal moment inertial as,

$$K_1 = \frac{(I_y - I_z)}{I_x}, \quad K_2 = \frac{I_z - I_x}{I_y}, \quad K_3 = \frac{I_x - I_y}{I_z}$$

So that eqns. (2) becomes,

$$\dot{\omega}_x = -\frac{C\omega_x}{I_x} + K_1 \omega_y \omega_z$$

$$\dot{\omega}_y = -\frac{C\omega_y}{I_y} + K_2 \omega_z \omega_x$$

$$\dot{\omega}_z = -\frac{C}{I_z}\omega_z + K_3 \omega_x \omega_y$$

Lets assume that ~~our~~ the origin of this system is one of its equilibruim point, so that The Lyapunov function, must satisfy

$$\cancel{V(\omega) = \int_0^\omega g(\omega)} \qquad V(0) = 0$$
$$V(\omega) > 0$$

$$V(\omega) = \int_0^\omega g(\omega)\, d\omega$$

So that we can have ~~where $g(\omega) = \frac{C\omega}{I} + K_1\omega_y$~~ the following Lyapunov function Candidate

$$V = C_1 \omega_x^2 + C_2 \omega_y^2 + C_3 \omega_z^2 \quad \text{—— (3)}$$

where, C_1, C_2 and C_3 are constant that are to be determined, and that they are positive - Constant. These constants are assumed to be different from the positive damping constant. ~~for now~~.

Now lets consider the derivative of V along it trajectories.

we know that

$$\dot{V} = \frac{dV}{d\omega}\dot{\omega} \quad \text{where } V \text{ is a function of } \omega \text{ as in eqn (3)}$$

so that in matrix form this will be,

$$\dot{V} = \begin{bmatrix} 2c_1\omega_x & 2c_2\omega_y & 2c_3\omega_z \end{bmatrix} \begin{bmatrix} -\frac{c}{I_x}\omega_x + k_1\omega_y\omega_z \\ -\frac{c}{I_y}\omega_y + k_2\omega_x\omega_z \\ -\frac{c}{I_z}\omega_z + k_3\omega_x\omega_y \end{bmatrix}$$

Expanding the above yields;

$$\dot{V} = -2c_1\frac{c}{I_x}\omega_x^2 + 2k_1c_1\omega_x\omega_y\omega_z - 2c_2\frac{c}{I_y}\omega_y^2 + 2k_2c_2\omega_x\omega_y\omega_z - 2c_3\frac{c}{I_z}\omega_z^2 + 2k_3c_3\omega_x\omega_y\omega_z$$

if we eliminate the cross terms $\omega_x\omega_y\omega_z = 0$ we get,

$$\dot{V} = -2\left[c_1 \frac{c}{I_x}\omega_x^2 + c_2\frac{c}{I_y}\omega_y^2 + c_3\frac{c}{I_z}\omega_z^2\right]$$

we know that $c>0$ and $I>0$, let assume $c_i = \frac{I_j}{c}$, $i=1,2,3$ and $j=x,y,z$

So that we have

$$\dot{V} = -2\left[\omega_x^2 + \omega_y^2 + \omega_z^2\right]$$

∴ we have $\dot{V} < 0$ which encompasses the entire State space. We can say that the System is globally Asymptotically Stable