AE 8133 – Space Mechanics
Project Phase III: Final Report

Submission Due date: Wednesday April 14 (by 3 pm)

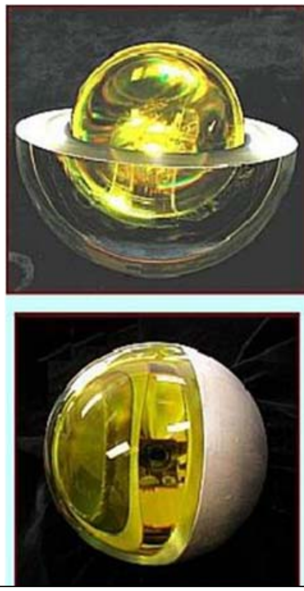
Title

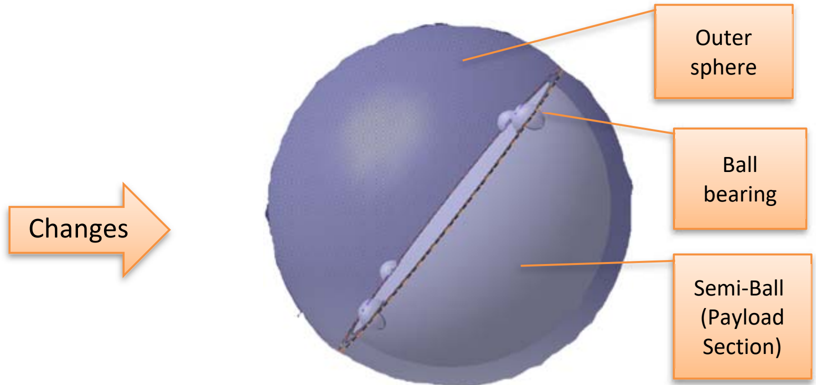
Fundamental Space Dynamics of a Near Frictionless Contact Between two surfaces of a Spherical Nano Satellite

1. Introduction

My proposed space mission is to describe the fundamental mechanics of a new nano-satellite concept/idea, that I came up with. This fundamental description will involve calculating its translational orbit dynamics, which is for a Low Earth Orbit Satellite. And its attitude rotational dynamics using quaternions and Euler angles. The satellite idea is similar to the BLITS (Ball Lens in the Space) satellite, but with some major changes.

This conceptual satellite is a spherically shaped satellite, consisting two outer hemisphere (5mm thick glass) and an inner semi-ball, which houses the payload. The contact surface between the inner semi-ball and the outer glass are 5 ball bearings which allow for free movement of the semi-ball inside the glass. Note that all spheres are concentric except for the ball bearings.





Rendered view of new Concept with bearings between the inner semi-ball and

Current BLITS satellite with glued contact

A key assumption taken, is that the satellite will be made to spin about the pitch axis immediately it is put into orbit, with an rpm greater than 10. This creates a centrifugal force on the semi-ball, causing the payload to face any direction about the spin axis.

2. Mission Objectives & Performance Measures

2.1 Mission Objectives

Orbital and attitude characteristics determination for a near frictionless contact of a nano-spherical LEO satellite.

2.2 Mission Performance Measures

Considering our reference satellite (i.e., BLITS), which is a passive nano satellite used for Satellite Laser Ranging (SLR) measurements, with zero signal signature due to the used of high-refraction-index glass (TF105 type). We can also assume similar case for our satellite but with some major changes, which are that our satellite will be an active satellite with magnetorquers for control and that we want to ensure that the inner semi-ball (i.e., the payload), always faces the earth at $\pm 5°$ in all direction.

3. System Model

As discussed earlier, for our attitude model we are going to convert from Euler-angles to Quaternions due to its advantages over the other for numeric integration as well as removing the problem of singularity,

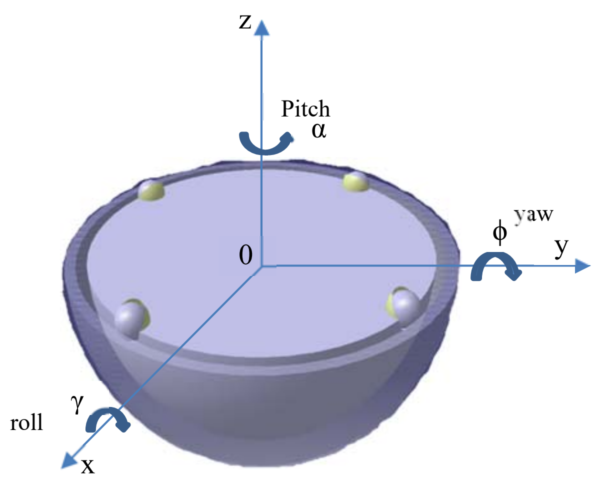
$$q_0 = \cos\left(\frac{\phi}{2}\right)\cos\left(\frac{\gamma}{2}\right)\cos\left(\frac{\alpha}{2}\right) + \sin\left(\frac{\phi}{2}\right)\sin\left(\frac{\gamma}{2}\right)\sin\left(\frac{\alpha}{2}\right)$$
$$q_1 = \sin\left(\frac{\phi}{2}\right)\cos\left(\frac{\gamma}{2}\right)\cos\left(\frac{\alpha}{2}\right) - \cos\left(\frac{\phi}{2}\right)\sin\left(\frac{\gamma}{2}\right)\sin\left(\frac{\alpha}{2}\right)$$
$$q_2 = \cos\left(\frac{\phi}{2}\right)\sin\left(\frac{\gamma}{2}\right)\cos\left(\frac{\alpha}{2}\right) + \sin\left(\frac{\phi}{2}\right)\cos\left(\frac{\gamma}{2}\right)\sin\left(\frac{\alpha}{2}\right)$$
$$q_3 = \cos\left(\frac{\phi}{2}\right)\cos\left(\frac{\gamma}{2}\right)\sin\left(\frac{\alpha}{2}\right) - \sin\left(\frac{\phi}{2}\right)\sin\left(\frac{\gamma}{2}\right)\cos\left(\frac{\alpha}{2}\right)$$

We will need to convert back from Quaternions to Euler-angles, during our simulation graphical representation, due to its advantage over the other in terms of practical interpretation of simulation results.

$$\phi = tan^{-1}\left(\frac{2(q_0q_1 + q_2q_3)}{1 - 2({q_1}^2 + {q_2}^2)}\right)$$
$$\gamma = sin^{-1}\big(2(q_0q_2 - q_3q_1)\big)$$
$$\alpha = tan^{-1}\left(\frac{2(q_0q_3 + q_1q_2)}{1 - 2({q_2}^2 + {q_3}^2)}\right)$$

3.1 System Description

For the translational model we will be performing our orbit computation and determination on the inertial reference frame which is on the ECI. While for our attitude determination will be estimated on the body fixed frame. As shown below:



With Z-axis acting as its local vertical.

3.2 Assumptions

We assume a point mass system for the translational dynamics of our satellite with the following parameters obtained from CAD modelling:

Inner ball radius and shape	80mm radius, semi spherical

Overall dimension and shape	90 mm radius, spherical
Number of ball bearing	5
Origin	At main sphere concentric center [0, 0, 0] of body fixed frame [x, y, z] (shown. in fig. 1.3)
Payload's center of mass	At [0, 0, -30.01] mm from origin
Satellite's center of mass	At [0, 0, -15.11] mm from origin
Mass of the nanosatellite	8kg
Moment of inertial [Ixx, Iyy, Izz]	[0.023, 0.023, 0.027] kg m^2

And because of the complexity and time constraint for this project we will assume the satellite as a completely rigid system for the rotational dynamics. And that the body frame is at its principal axis, $I = \begin{bmatrix} Ixx & 0 & 0 \\ 0 & Iyy & 0 \\ 0 & 0 & Izz \end{bmatrix} = [0.023, 0.023, 0.027]^T$

3.3 System Equations of Motion

Translational Kinematics and Dynamics: We can use the Gravity Model for our orbital motion as,

Inertial velocity vector, $\vec{v} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix}$

Inertial acceleration, $\vec{a} = \begin{bmatrix} \ddot{x} \\ y \\ \ddot{z} \end{bmatrix} = \left(-\frac{Gm_{\oplus} m_s \hat{r}}{r^2}\right)/m_s$

Mass of the nanosatellite, $m_s = 8\ kg$

For virtualization we can consider one orbital period, $T = 2\pi/\sqrt{(\frac{\mu}{a^3})}$, during numeric integration. We can therefore use ode45 on the state vector $[\vec{v}; \vec{a}\ ]$ in MATLAB to solve this system as describe later in this paper.

Rotational Kinematics and Dynamics: We assume our system as a rigid body, which we have already obtained the following,

$\vec{\omega}_{bI} = \begin{bmatrix} p \\ q \\ r \end{bmatrix}_B = H^{-1} \begin{bmatrix} \dot{\gamma} \\ \dot{\emptyset} \\ \dot{\alpha} \end{bmatrix}$, we can assume different initial angular rate for the system here.

quaternions, $q = \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix}$,

The derivative of the quaternions, yields, $\begin{bmatrix} \dot{q}_0 \\ \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 0 & -p & -q & -r \\ p & 0 & r & -q \\ q & -r & 0 & p \\ r & q & -p & 0 \end{bmatrix} \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix}$

For the attitude dynamics we have the Euler equation of motion,

That the Total torque acting on the satellite, $\sum \vec{T} = \left(\frac{d\vec{H}}{dt}\right)_I$, subscript I stands for the Inertial frame.

The RHS is given by, $\sum \vec{T} = \vec{T}_{SRP} + \vec{T}_{Aero} + \vec{T}_{Mag}$

The LHS is given by, $\left(\frac{d\vec{H}}{dt}\right)_I = \left(\frac{d\vec{H}}{dt}\right)_B + \vec{\omega}_{BI} \times \vec{H}$,

expanding the above using Derivative Transport Theorem and chain rule,

will give, $\left(\frac{d\vec{H}}{dt}\right)_I = \dot{I}\vec{\omega}_{BI} + I\dot{\vec{\omega}}_{BI} + \vec{\omega}_{BI} \times \vec{H}$, Although our principal moment of inertia could change with time due to the freedom of rotation of the semi-ball with the hemisphere, but let's assume that the coefficient of friction between both contact surfaces, $C_f = 0.5$, so that we can say that $\dot{I}\vec{\omega}_{BI} = 0$, for now. In future work we will try to treat this.

Hence if we are to consider a control system using magnetorquers. Let's assume that our magnetorquers, $\tau_m = [0,0,0]^T$, so that we will have our rotational dynamics as,

$$\tau_m = I\dot{\vec{\omega}}_{BI} + \vec{\omega}_{BI} \times \vec{H}, \tag{3}$$

making $\dot{\vec{\omega}}_{BI}$ subject of formular and writing it terms of $\begin{bmatrix} p \\ q \\ r \end{bmatrix}_B$

We get, $\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix}_B = I^{-1}\left(\tau_m - \begin{bmatrix} p \\ q \\ r \end{bmatrix}_B \times \vec{H}\right)$, where, $\vec{H} = I\begin{bmatrix} p \\ q \\ r \end{bmatrix}_B$, we will use this equation doing our numeric integration in MATLAB.

3.4 External disturbance model

Magnetic Field Model:

Although Earth's magnetic disturbance will be very small (in nanoteslas), but if we are going to control our satellite with magnetorquers then this becomes a very important models for our system. We are going to utilize the International Geomagnetic Reference Field (IGRF) Model, to generate the earth's magnetic field. This IGRF model utilizes time, latitude, longitude, altitude, and coordinate system. For our case we can have the time set to 01-Jan-2020, which is the most updated date in its database as of now. Since we assume the Z-axis of our ECI to be pointing to the north, we can say that our coordinate system is geocentric. Hence converting from Convert Cartesian, we first need to convert it to spherical coordinate using, $\rho = \sqrt{x^2 + y^2 + z^2}$

$\emptyset_E = 0, \quad \gamma_E = \cos^{-1}(\frac{z}{\rho}), \quad \alpha_E = \tan^{-1}(\frac{y}{x}),$

Using above we can now obtain the latitude, longitude coordinates with the following equation,

$$\lambda_{LAT} = 90 - \gamma_E \frac{180}{\pi}$$

$$\lambda_{LON} = \alpha_E \frac{180}{\pi}$$

And for attitude, $h = \rho$, since we are using geocentric system.

Note that the magnetic field output $\vec{\beta}_{NED}$, from the IGRF model is in North, East, Down (NED) coordinate. To transform this to the inertial frame we use,

$$\vec{\beta}_I = T_{IB}(0, \gamma_E + \pi, \alpha_E)\vec{\beta}_{NED}$$

Where T_{IB}, is the transformation matrix from the spherical frame to the inertial reference frame. Given as,

$$T_{IB} = \begin{bmatrix} \cos\gamma_E cos\alpha_E & sin\emptyset_E sin\gamma_E cos\alpha_E - cos\emptyset_E sin\alpha_E & cos\emptyset_E sin\gamma_E cos\alpha_E + sin\emptyset_E sin\alpha_E \\ cos\gamma_E sin\alpha_E & sin\emptyset_E sin\gamma_E sin\alpha_E + cos\emptyset_E cos\alpha_E & cos\emptyset_E sin\gamma_E sin\alpha_E - sin\emptyset_E cos\alpha_E \\ -sin\gamma_E & sin\emptyset_E cos\gamma_E & cos\emptyset_E cos\gamma_E \end{bmatrix}$$

3.5 Reference or Desired Trajectory

The reference trajectory is same as that of the BLITS satellite, which is a LEO satellite, with the following parameters:

orbit	Sun-synchronous near-circular
mean altitude	832 km
inclination	98.85°
period	101.3 minutes
local equatorial crossing time	12:00 hours

4. **Mathematical Analysis**

Although the satellite has the payload as a free moving part which should change the behavior of the system, but for this scope of our project, lets, assume a positive damping constant $C = 0.00003$, which is dependent on the coefficient of friction of the five bearings in contact with the two surfaces. So that fom eqn. (3) we can rewrite the attitude equation of motion in 3D form,

Where the skew matrix, $\vec{\omega}_{BI} \times = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}$.

So that we have our assumed equation of motion for our initial condition of $\tau_m = [0,0,0]^T$ as,

$$Ixx\dot{p} - (Iyy - Izz)qr = -0.5p$$
$$Iyy\dot{q} - (Izz - Ixx)rp = -0.5q$$
$$Izz\dot{r} - (Ixx - Iyy)pq = -0.5r$$

We can simplify this by, $Ixx = I_x, Iyy = I_y, Izz = I_z$ and $k_1 = \frac{I_y - I_z}{I_x}$, $k_2 = \frac{I_z - I_x}{I_y}$, $k_3 = \frac{I_x - I_y}{I_z}$,

This becomes,

$$\dot{p} = -\frac{0.5}{I_x}p + k_1 qr$$
$$\dot{q} = -\frac{0.5}{I_y}q + k_2 rp$$
$$\dot{r} = -\frac{0.5}{I_z}r + k_3 pq$$

We can have the following Lyapunov function candidate,

$$V = C_1 p^2 + C_2 q^2 + C_3 r^2$$

Where C_1, C_2 *and* C_3 are some constant that will be determined.

Now consider the derivative of V, along its trajectory.

We know that $\dot{V} = \frac{dV}{d(pqr)}(\dot{pqr})$

In matrix form this will be written as,

$$\dot{V} = [2C_1p \quad 2C_2q \quad 2C_3r]\begin{bmatrix} -\dfrac{p}{2I_x} + k_1qr \\ -\dfrac{q}{2I_y} + k_2rp \\ -\dfrac{r}{2I_z} + k_3pq \end{bmatrix}$$

Expanding the above will yield,

$$\dot{V} = -\frac{C_1p^2}{I_x} + 2C_1k_1pqr - \frac{C_2q^2}{I_y} + 2C_2k_2pqr - \frac{C_3r^2}{I_z} + 2C_3k_3pqr$$

Solving the above we will get,

$$\dot{V} = -\left(\frac{C_1p^2}{I_x} + \frac{C_2q^2}{I_y} + \frac{C_3r^2}{I_z}\right)$$

We know that $I > 0$, we can say that $C_i = I_j$ for $i = 1,2,3$ and $j = x, y, z$

We will now have,

$$\dot{V} = -(p^2 + q^2 + r^2)$$

Therefore, we have that $\dot{V} < 0$, which encompasses the entire state variables. We can say that the system is globally asymptotically stable.

5. Numerical Simulation Results and Discussion

Initial Condition and Assumed parameters: Some of our initial conditions are.

Angular rate, $\begin{bmatrix} p \\ q \\ r \end{bmatrix}_B = \begin{bmatrix} 10 \\ 5 \\ 3 \end{bmatrix}$ deg/sec

$$\gamma = \emptyset = \alpha = 0$$

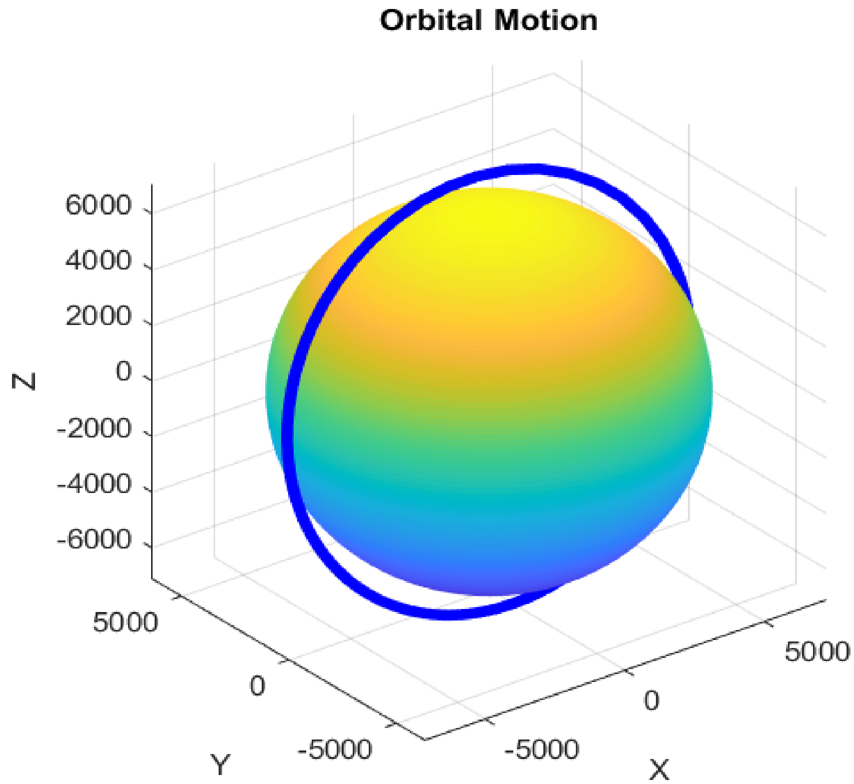
$$x(0) = 6.357e3 + mean\ altitude, \qquad y(0) = z(0) = \dot{x}(0) = 0$$

$\vec{r} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$, Semi-major axis, $a = norm(\vec{r})$, inclination, $i = 98.85°$

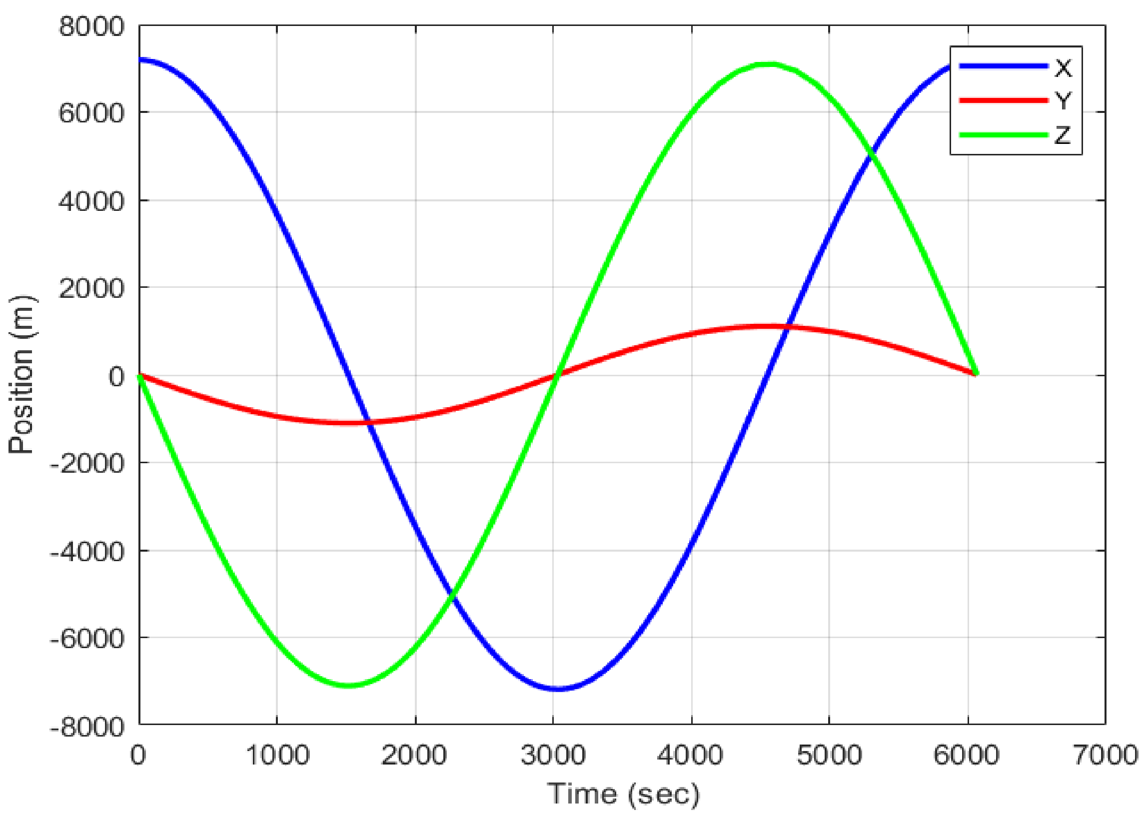
for near circular orbit we have that circular velocity, $v_c = \sqrt{(\mu/a)}$

$$\dot{y}(0) = v_c cos(i), \qquad \dot{z}(0) = -v_c sin(i);$$

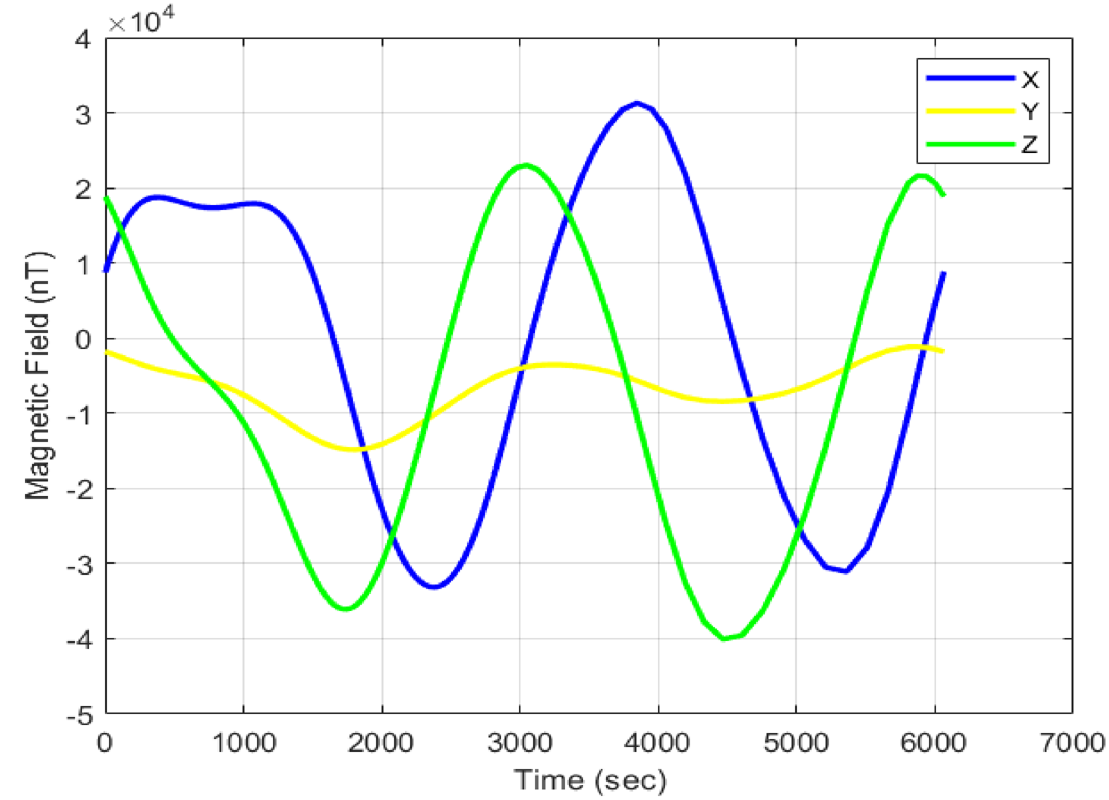
We will perform orbital simulation for one orbital period, $timespan = [0\ \ T]$

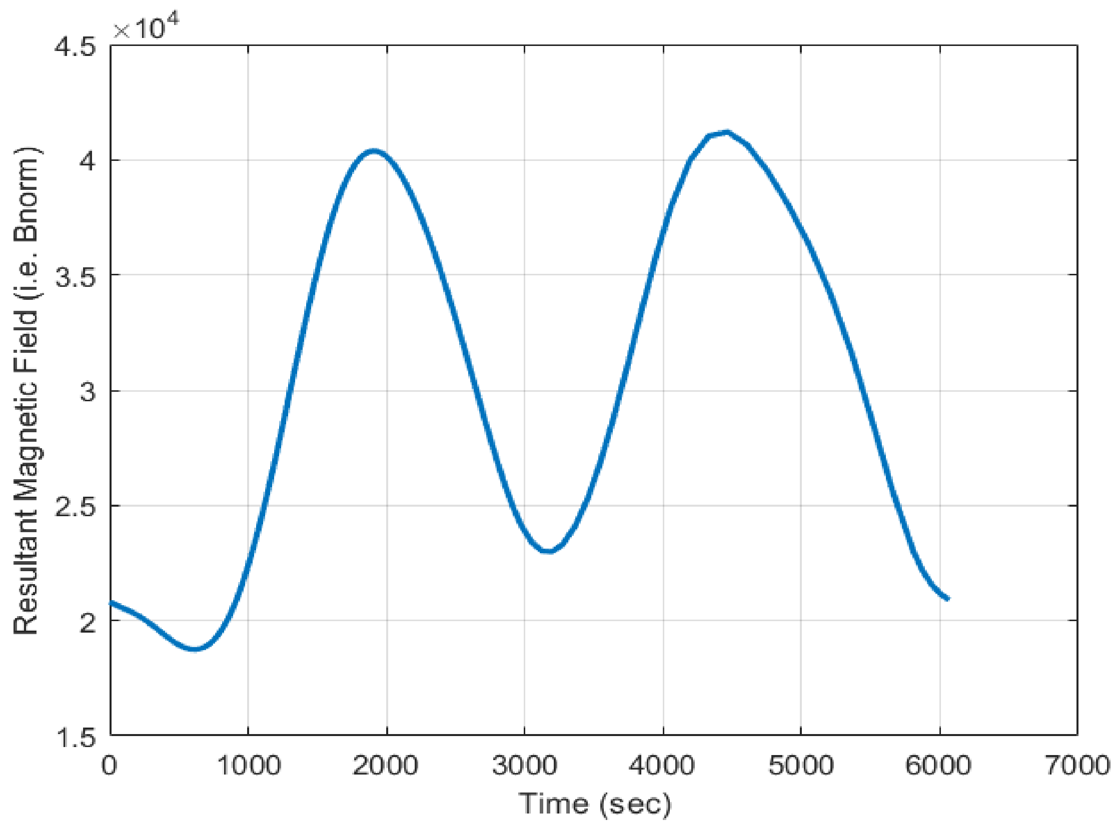


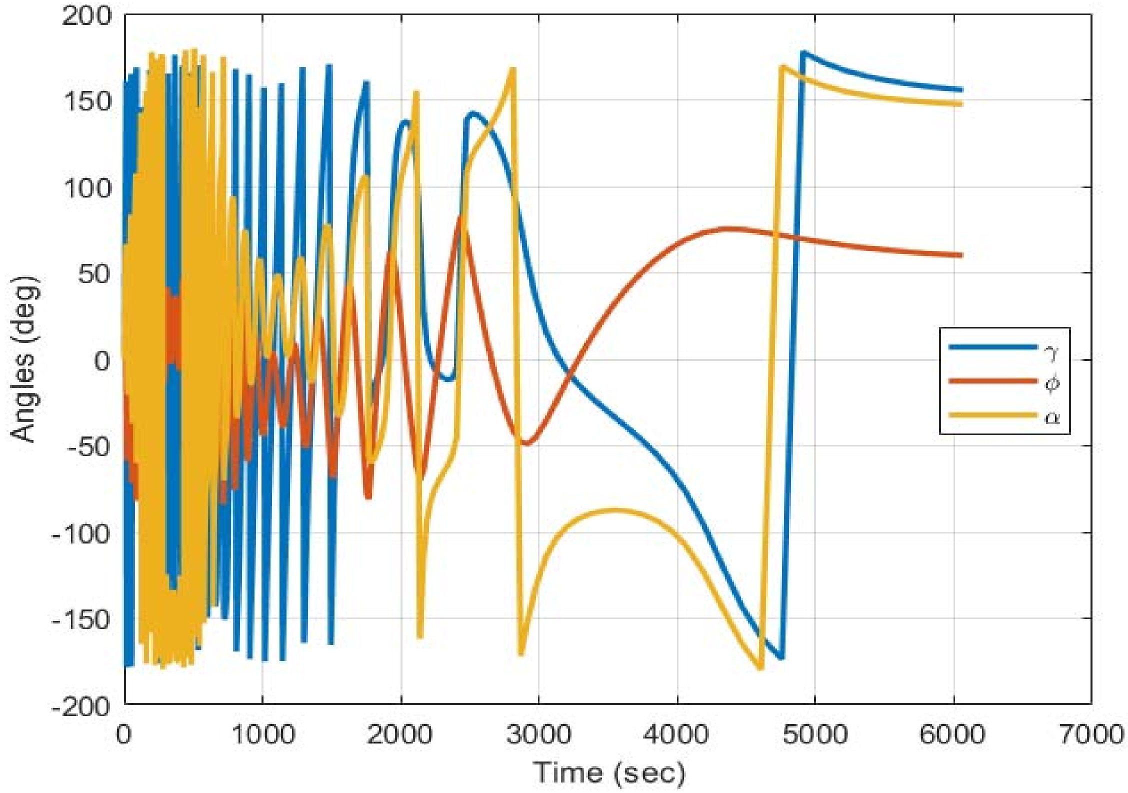
Orbital Motion
6000
4000
2000
0
-2000
-4000
-6000
Z
5000
0
-5000
Y
5000
0
-5000
X



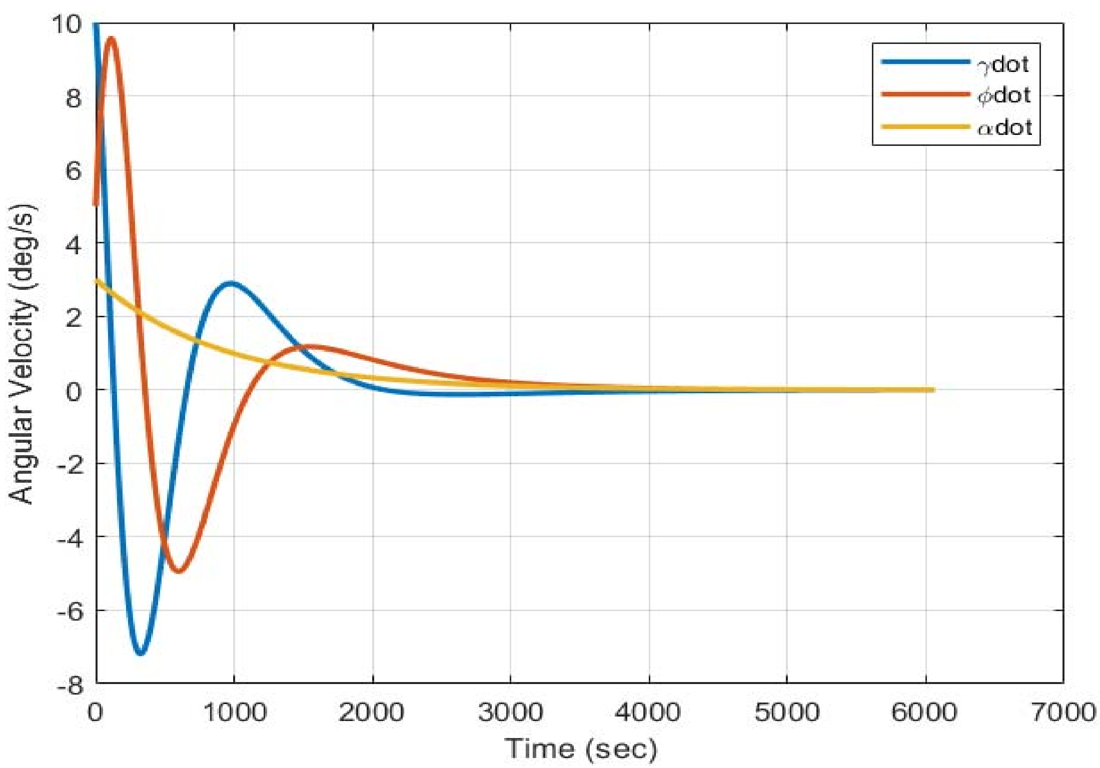
8000
6000
4000
2000
0
-2000
-4000
-6000
-8000
Position (m)
0
1000
2000
3000
4000
5000
6000
7000
Time (sec)
X
Y
Z


×10^4
X
Y
Z
Magnetic Field (nT)
Time (sec)
4
3
2
1
0
-1
-2
-3
-4
-5
0
1000
2000
3000
4000
5000
6000
7000


×10^4
Resultant Magnetic Field (i.e. Bnorm)
Time (sec)
4.5
4
3.5
3
2.5
2
1.5
0
1000
2000
3000
4000
5000
6000
7000



Angles (deg)
Time (sec)
γ
ϕ
α



Angular Velocity (deg/s)
Time (sec)
γdot
ϕdot
αdot

From the above figure we can see the orbital representation of our satellite with its position from the ECI in X, Y and Z axis. The We can also observe the estimated magnetic field around the satellite. The attitude representation is finally shown in the form of the behavior of the pitch, roll and yaw angles with time about the orbit. Both the angles and the angular rate are observed to gradually become stable. This is true as verified using Lyapunov method, which analytically states that the system is asymptotically stable.

6. Conclusions

We have been able to establish the wholistic behavior of the system, which is asymptotically stable. And we also obtained the magnetic field vector values, which will be used with B-dot Control system using Magnetorquers to eventually control the attitude of the satellite, to meet its mission requirement of SLR (in order words, a precise attitude determination measurement system).

Note that this project is very complex due to the freedom of the inner semi-ball. Hence to fully account for the performance of the payload we need to change some assumptions to properly estimate the behavior of the inner semi-ball. Because if we are to use magnetorquers to control the inner semi-ball, this will also affect the outer hemisphere motion due to the coefficient of friction between their contacts, which is not zero. This can be done in future work of this project.

References

- Kumar, K. D. "Fundamentals of Dynamics and Control of Space Systems," Journal of Guidance, CreateSpace Publisher, 2012.
- Bate, R., Mueller, D. and White, J., 1971. ”Fundamentals of astrodynamics”. New York: Dover Publications.
- Deakin, R. E., 2007. ”SATELLITE ORBITS”. [online] Available at: <https://www.researchgate.net/publication/228860752> [Accessed 25 January 2021].
- Carlos, J. M., 2020. ”Spaceflight Mechanics and Control”. [paper] Available at: https://github.com/cmontalvo251/MATLAB/tree/master/Aircraft_Flight_Mechanics.
- FSUE-IPIE, 2021. ”BLITS - eoPortal Directory - Satellite Missions”. [online] Directory.eoportal.org. Available at: <https://directory.eoportal.org/web/eoportal/satellite-missions/b/blits> [Accessed 31 January 2021].