Assignment 4

1) To generate a free transfer from the Lunar DPO to the L2 Lyapunov Oribit, First we have to generate the unstable and stable manifold for a periodic orbit of the Lunar DPO and L2 respectively. Using the given initial conditions in the form: ~~of a monodromy matrix~~

$$X = [x, y, x', y', 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, \cdots 0, 0, 0, 1]^T$$

The monodromy matrix of X is the computed and it's eigen values and vectors. For the DPO the eigen value, $\lambda_u > 1$, the eigen vector corresponding to this is:

$$u_u = \begin{bmatrix} 0.1335 \\ -0.1053 \\ 0.7036 \\ -0.6900 \end{bmatrix}.$$

Also for the L2, the eigen value, $\lambda_s < 1$ is selected, and the corresponding eigen vector is:-

$$u_s = \begin{bmatrix} -0.1828 \\ -0.2823 \\ 0.8114 \\ 0.4780 \end{bmatrix}$$

Using the initial condition

$$\phi(t', x_0) \pm \epsilon \frac{\Phi(t', x_0)\, u}{\|\Phi(t', x_0)\, u\|}$$

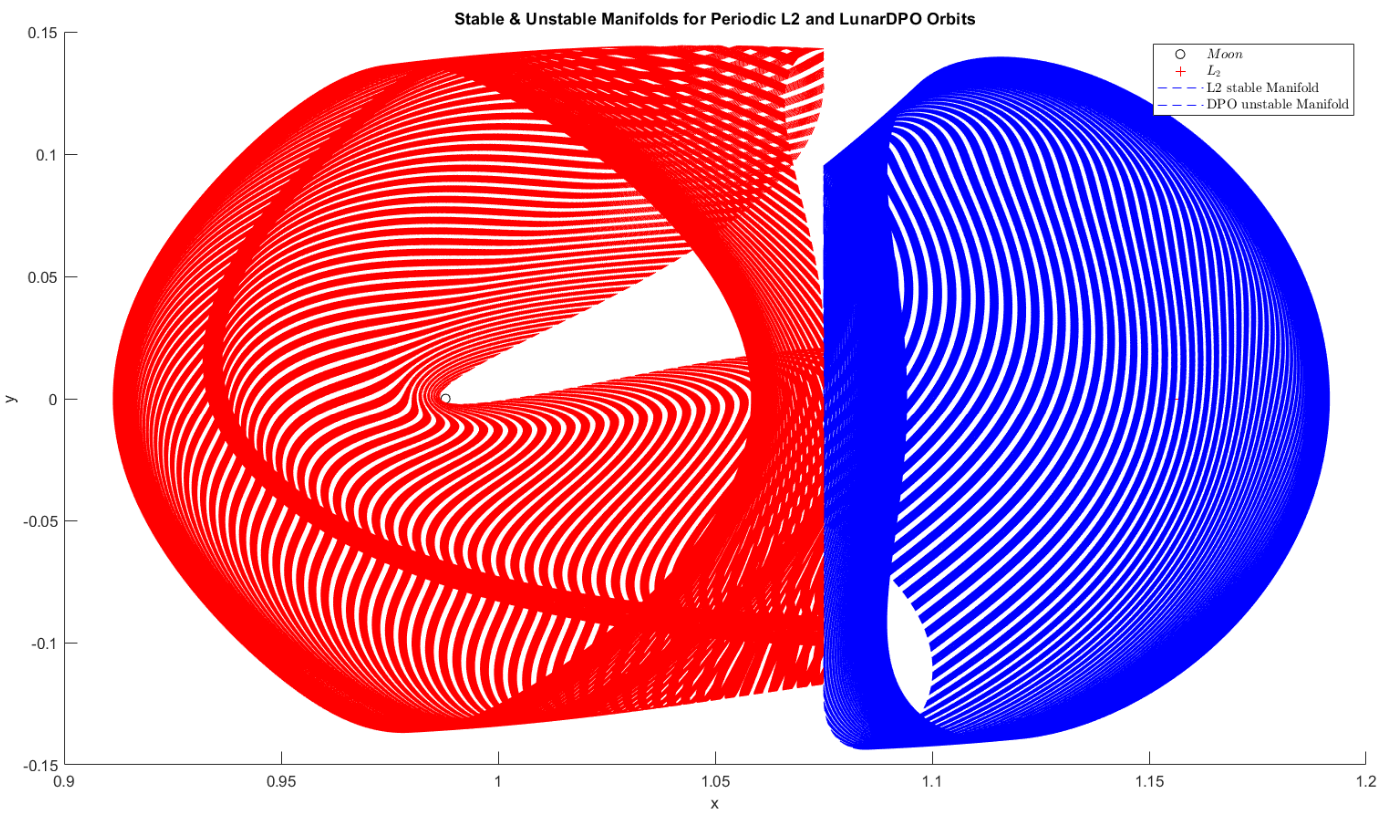
where $\Phi(t', x_0)$, is the State transition matrix

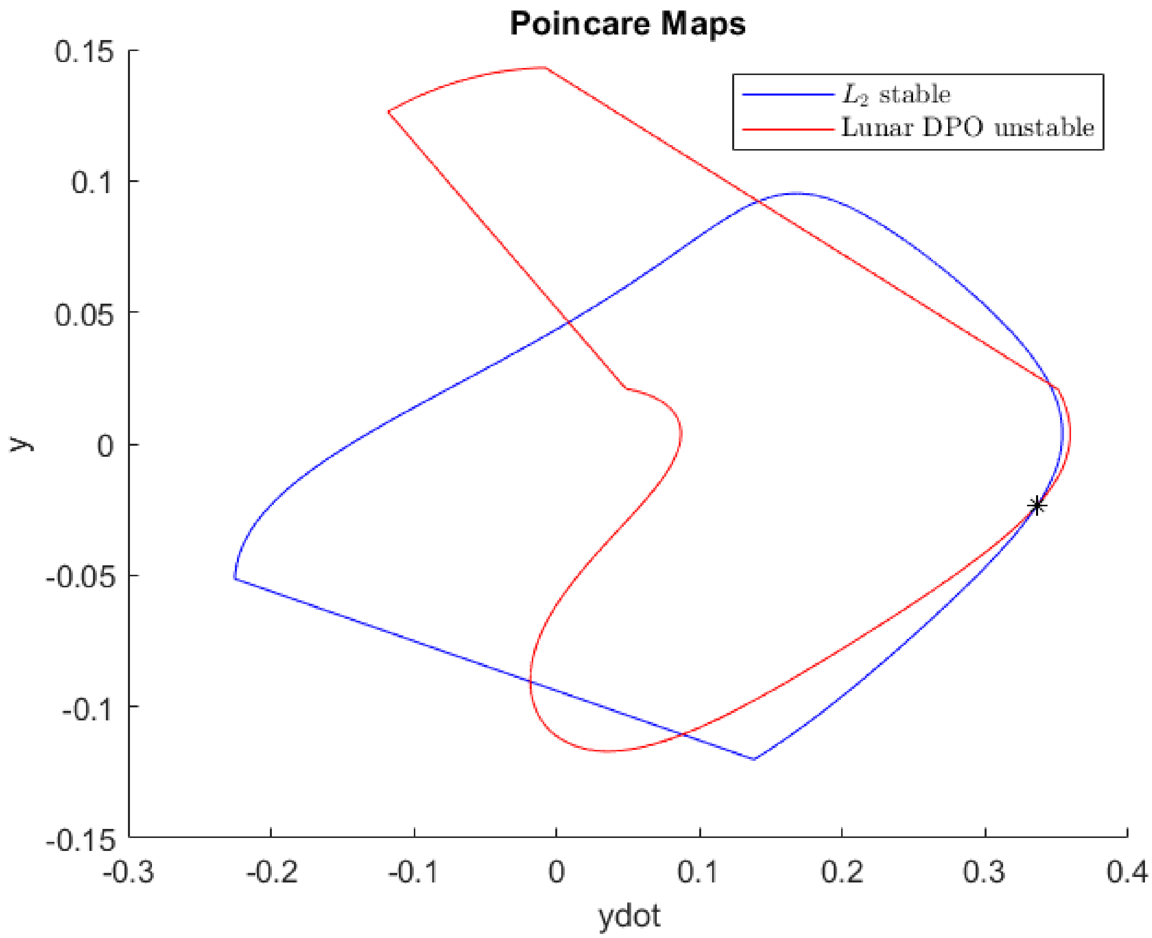
Using ODE event function we can terminate the propagation of the manifolds at the Surface of section, of, $x = 1.075$.
At the end of this computation, the y and y' values are extracted, which is used to generate the poincare map. From this poincare map the intersection point between the two manifolds can be used to compute x' in eqn. (78) of the notes. Hence we now have the initial condition

$$x_0 = [x, y, x', y']^T \cancel{= [1.075, -0.0236, 0.0903, 0.35]}$$

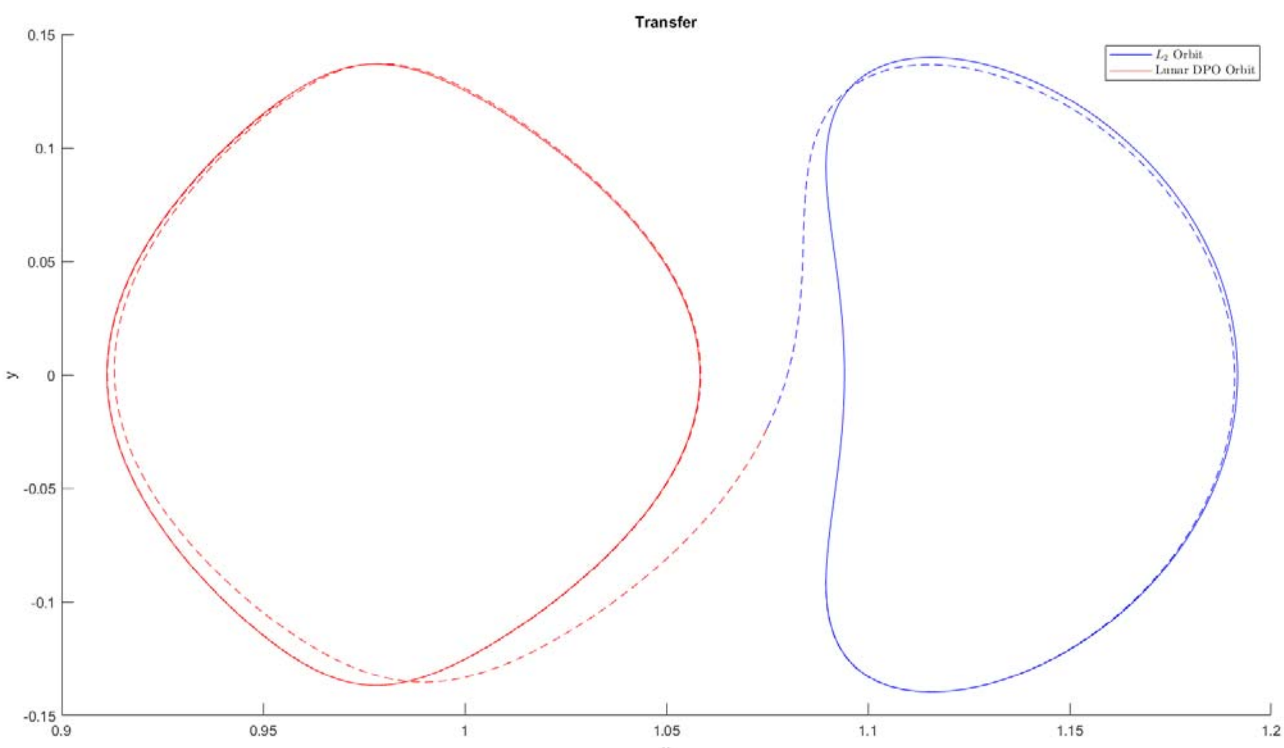
Which when integrated forward in time ~~will~~ and ~~give~~ backward in time will give the free transfer from Lunar DPO to L2 orbit.
The resulting plots are shown below:-

$$x_0 = [1.075, -0.0236, 0.0903, 0.3359]^T$$





Poincare Maps
L_2 stable
Lunar DPO unstable
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
y
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
ydot



Transfer
L_2 Orbit
Lunar DPO Orbit
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
y
0.9
0.95
1
1.05
1.1
1.15
1.2
x

2) Using similar procedure as in question 1, but generating the stable manifold for the Lunar DPO using, the monodromy eigen vector :

$$u_s = \begin{bmatrix} -0.1335 \\ -0.1053 \\ 0.7036 \\ 0.6900 \end{bmatrix}$$

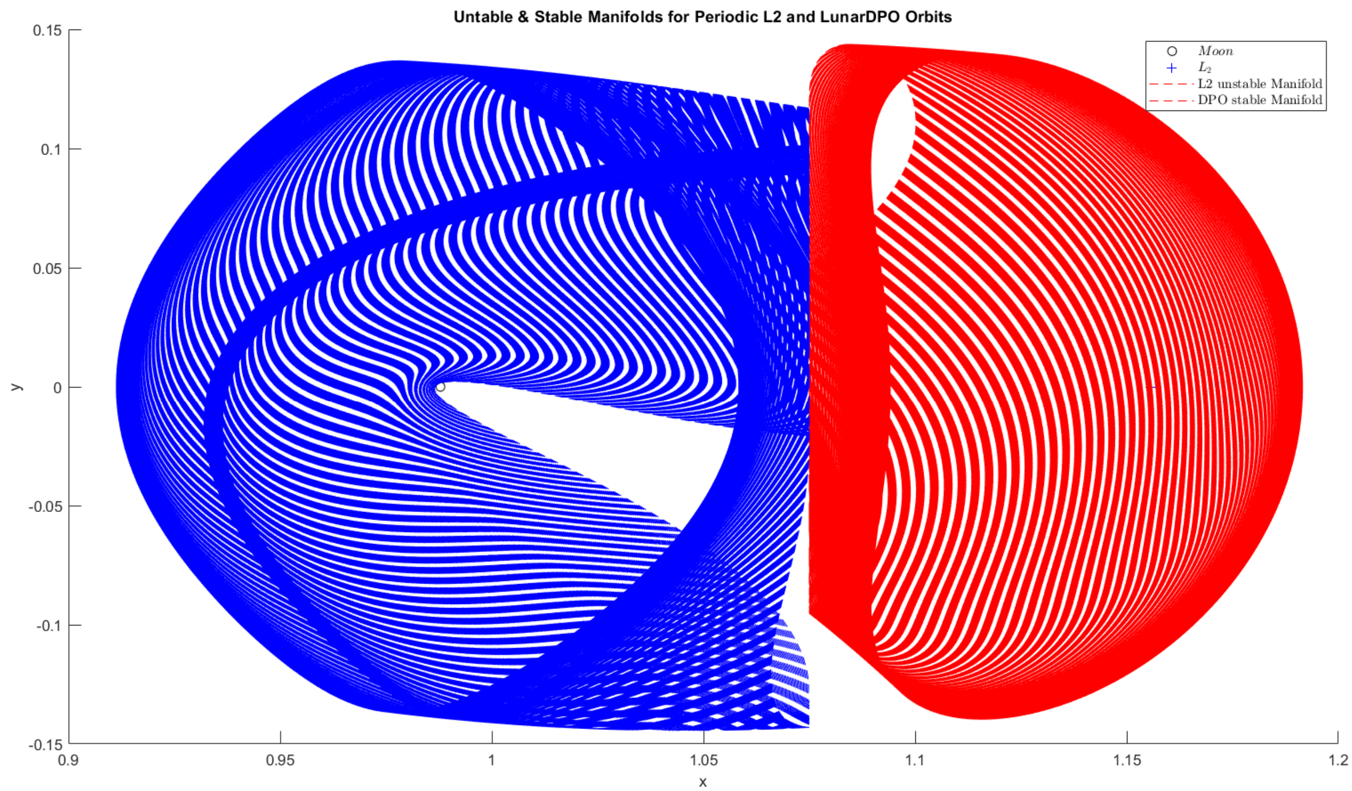
and an unstable manifold for L_2, using the monodromy eigen vector :-

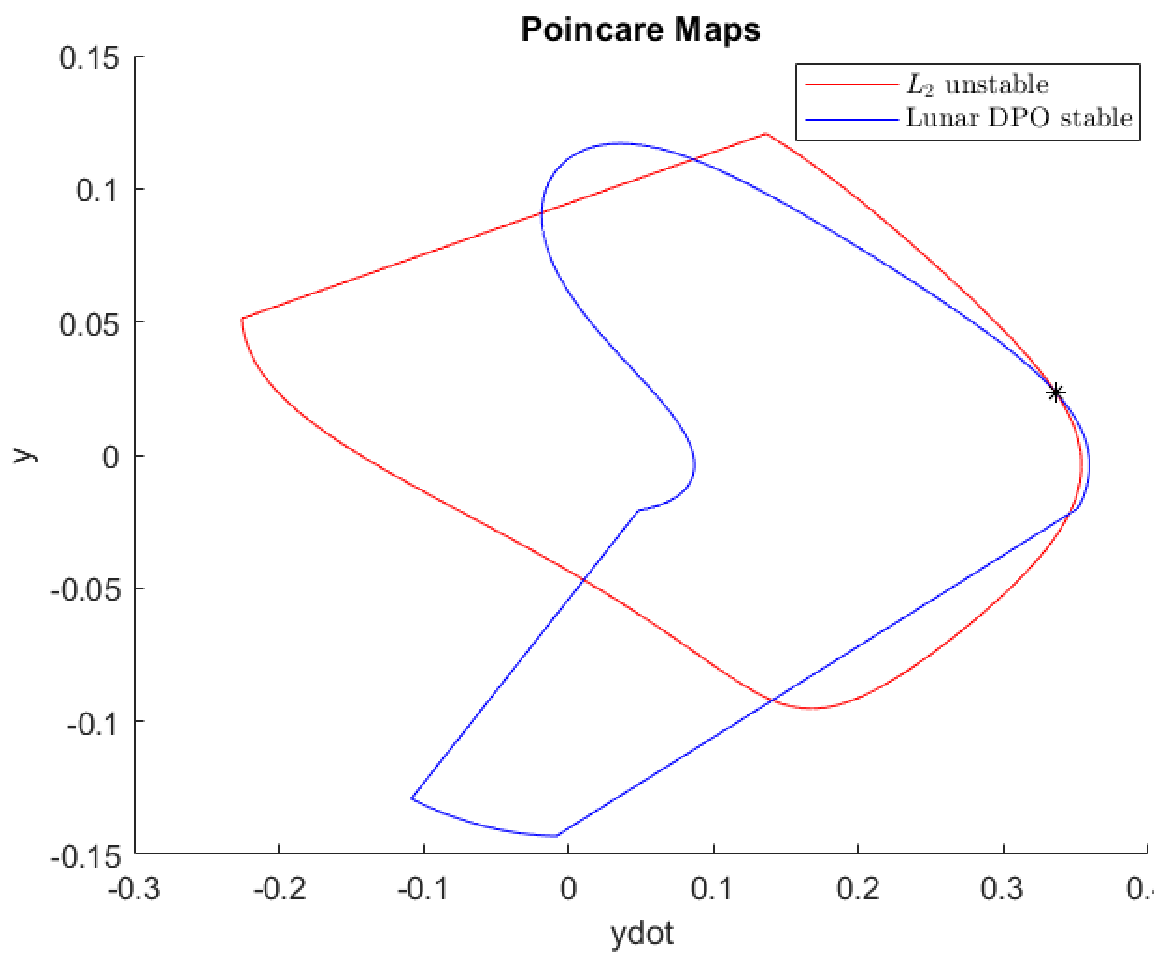
$$u_u = \begin{bmatrix} -0.1828 \\ 0.2823 \\ -0.8114 \\ 0.4780 \end{bmatrix}$$

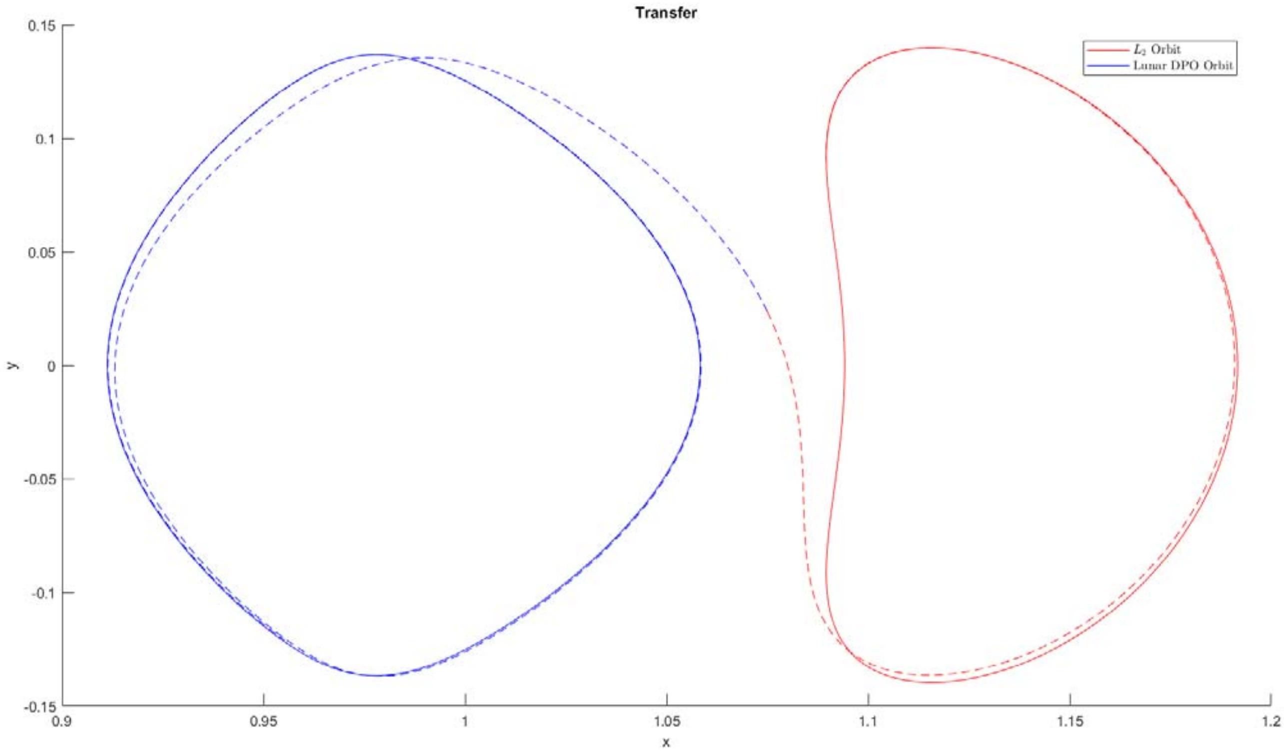
Similarly the poincare map is generated, at the intersection point between both manifolds y, y' is used to also calculate x' but we must put a negative sign for the calculated x' value, since the tranfer is L_2 to DPO ($x'<0$). Similarly with this initial condition ~~the~~ transfer can be generated, when integrated forward and backward in time using it equation of motion. ~~$x_0 = [$~~ $]^T$

The results for this simulation is shown below:

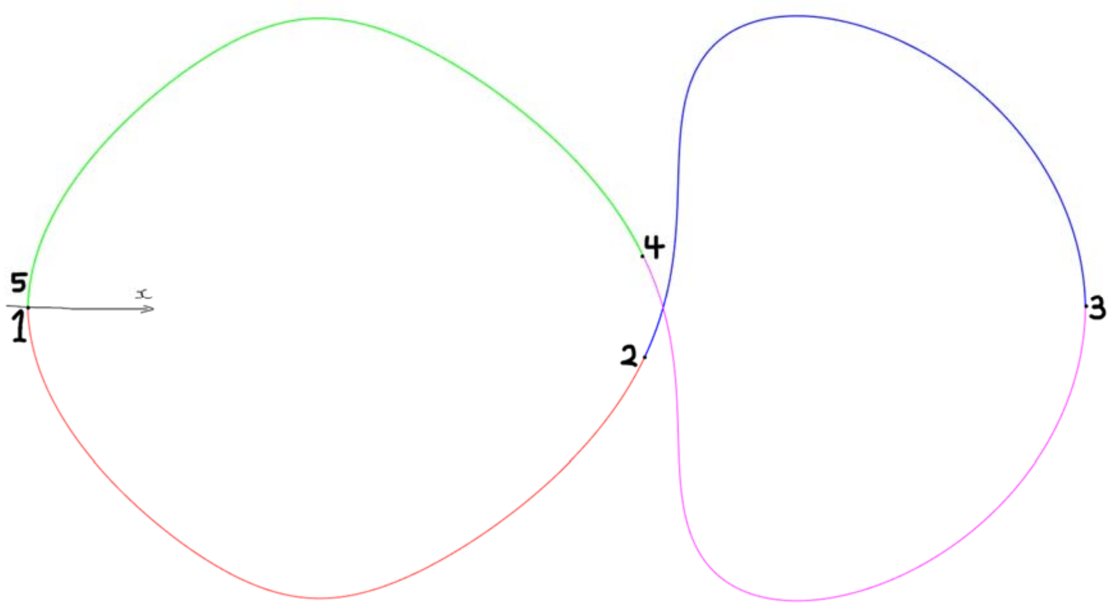
$x_0 = [1.075, 0.0236, -0.0903, 0.3359]^T$


Untable & Stable Manifolds for Periodic L2 and LunarDPO Orbits
Moon
L_2
L2 unstable Manifold
DPO stable Manifold
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
y
0.9
0.95
1
1.05
1.1
1.15
1.2
x


Poincare Maps
L_2 unstable
Lunar DPO stable
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
y
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
ydot



3) From the Obtain transfers from the above two questions, ~~the~~ a new orbit about Lunar DPO and L_2 can be generated using the multi shooting differential corrector. From the transfer segments shown below;



set patch point 1 and 5 to be at the ~~y~~ x-axis crossing (ie., $y=0$). By doing this we now have four patch points with their corresponding time and velocity, which used as input for the multi-shooting method.

The obtained ~~new~~ orbit about the Lunar DPO and L_2 orbits ~~are~~ is shown below:-