AE 8133: Space Mechanics

Problem Set1

Due Date: March 31, 2021 (Wednesday, 3 p.m.)

1. A three-body space system is orbiting around a spherical Earth (Fig. 1). It is comprised of three spacecraft m_1, m_2 and m_3 connected through rigid massless cables of length L_1 and L_2. The kinetic and potential energies of the system are obtained as

$$T = \frac{1}{2}M\left(\dot{R}^2 + \dot{\theta}^2 R^2\right) + \frac{1}{2}M_{t1}\omega_1^2 L_1^2 + \frac{1}{2}M_{t2}\omega_2^2 L_2^2 + M_{t3}\omega_1\omega_2 L_1 L_2 cos(\beta_1 - \beta_2) \tag{1}$$

$$\begin{aligned} U = &-\frac{\mu M}{R} + \frac{\mu}{2R^3}\left\{M_{t1}L_1^2 + M_{t2}L_2^2 + 2M_{t3}L_1L_2cos(\beta_1 - \beta_2)\right\} \\ &- \frac{3\mu}{2R^3}\left\{M_{t1}L_1^2cos^2\beta_1 + M_{t2}L_2^2cos^2\beta_2 + 2M_{t3}L_1L_2cos\beta_1cos\beta_2\right\} \end{aligned} \tag{2}$$

where $M = m_1 + m_2 + m_3$, $\omega_1 = \dot{\theta} + \dot{\beta}_1$, $\omega_2 = \dot{\theta} + \dot{\beta}_2$, and

$$\begin{aligned} M_{t1} &= m_1\gamma_1^2 + m_2(1-\gamma_1)^2 + m_3(1-\gamma_1)^2 \\ M_{t2} &= m_1\gamma_2^2 + m_2\gamma_2^2 + m_3(1-\gamma_2)^2 \\ M_{t3} &= m_1\gamma_1\gamma_2 - m_2(1-\gamma_1)\gamma_2 + m_3(1-\gamma_1)(1-\gamma_2) \end{aligned}$$

with $\gamma_1 = (m_2 + m_3)/M$ and $\gamma_2 = m_3/M$. Note that the system center of mass does not coincide with the center of mass of the spacecraft m_1.

For this given system,

(i) derive the equations of motion of the system corresponding to R, θ, β_1 and β_2 degrees of freedom.

(ii) perform numerical simulation of the given system for a time period of two orbits with the following initial state conditions and system parameters:

$$R(0) = 6878\text{km}, \dot{R}(0) = \theta(0) = 0, \dot{\theta}(0) = \frac{\mu}{R(0)^3}, \beta_1 = 10deg,$$

$$\dot{\beta}_1 = 0.01\dot{\theta}(0) \text{ rad/s}, \beta_2 = -15 \text{ deg}, \dot{\beta}_2 = 0.01\dot{\theta}(0) \text{ rad/s}$$

$$m_1 = 10 \text{ kg},\ m_2 = m_3 = 1 \text{ kg},\ L_1 = 1 \text{ km},\ L_2 = 1 \text{ km}$$

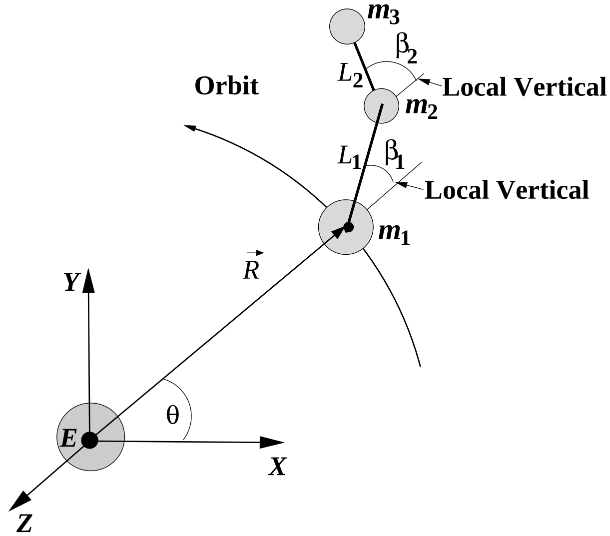

Figure 1: System undergoing in-plane libration.

Show the numerical simulation results for R, θ, β_1 and β_2.

It is recommended to use Matlab Symbolic Toolbox to answer the above questions. Submit your Matlab code with the results.

2. The Euler equations of attitude motion of a spacecraft are given by

$$I_x\dot{\omega}_x - (I_y - I_z)\omega_y\omega_z = -C\omega_x$$
$$I_y\dot{\omega}_y - (I_z - I_x)\omega_z\omega_x = -C\omega_y$$
$$I_z\dot{\omega}_x - (I_x - I_y)\omega_x\omega_y = -C\omega_z$$

where I_j and ω_j are the principal moment of inertia and angular of the spacecraft about the j-axis, $j = x, y, z$; C is a positive damping constant; and ω_j is the angular velocity of the spacecraft about the j-axis, $j = x, y, z$. Determine the stability of the given system (whether it is asymptotic stable or unstable) using Lyapunov stability theory.