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The Spread of Disease

Math 114 Mathematical Modeling

St. Mary's College

February 22, 2019

Problem Statement

An isolated town has a population of 100,000 residents. Last week there were 18 new cases of people infected by a mild disease that lasts three weeks and leaves the person immune from further disease. Direct contact with an infected person leads to an infection of a previously uninfected person. This week there are 40 new cases. It is estimated that 30% of the existing population is immune because of previous exposure.

1. Make a list of assumptions that you need to make in order to develop a dynamical system model using difference equations.

2. Develop a model that describes how the number of new cases each week develops.

3. What is the eventual number of people who will become infected?

4. Vary the assumptions you make in this model to develop a feel for how sensitive the model is to your assumptions.

List of Assumptions

- Every person in the population is equally exposed to any peoples that are infected.
- Every person that is exposed to an individual that is infected has the same chance of becoming infected themselves.
- During the 0th week the infection is introduced into the population, 18 people develop cases of the sickness.
- During the first week the infection is introduced into the population, 40 people develop cases of the sickness.
- People recover on their third week on getting the disease, not after their third week of getting the disease.
- That the 30% of people that are immune in week 2 represent the only people of the population that are immune.

The Model

Based upon an equation given by the book, (Section 1.2—Example 3—p.13) we can assume that the spread of disease can be modeled with the following equation: $\Delta i_n = i_{n+1} - i_n = k i_n (N - i_n)$, where i_n is the number of people that developed the disease on week n, k is a constant, and N is the total population. In this equation, $i_n(N - i_n)$ represents the number of times an infected person will come into contact with someone who can become sick. k is the probability that someone will become sick if they encounter someone that is infected. Unlike the equation in the book, the people in the problem statement will get better after 3 weeks of having the disease. Also, we don't only care about the spread of disease, but rather the total number of people that are infected. This is reflected in the following equation: $i_{n+1} = i_n + k i_n (N - i_n - s_n) - \Delta i_{n-2}$. This reads as the following: the number of people that have the disease in week n+1 is equal to the number of people that have the disease in week n plus the number of people that contracted the disease in week n+1, minus the number of people that have become immune to the disease. The number of people that have contracted the disease is calculated by multiplying the constant k, the number of people that had the disease on week n, and the number of people that are succeptible to the disease. The number of people succeptible to the disease is found by subtracting the number of sick people on week n and the number of immune people on week n. We can determine our constant k by calculating the number of people that become infected on week 1. We do so with the following formula, using the data that we were given in the problem statement:

$$40 = (100,000 - 100,000 * 0.3 - 18)$$

$$k = 3.1754197111003146E - 5$$

Each week, we calculate the number of people that developed the disease, the number of people that became immune to the disease, the total number of infected, the total number of those that are immune, and the total number of those that are susceptible.

We calculate the total number of people that developed a new case of the disease by using a formula that is similar to the one we used to solve for K:

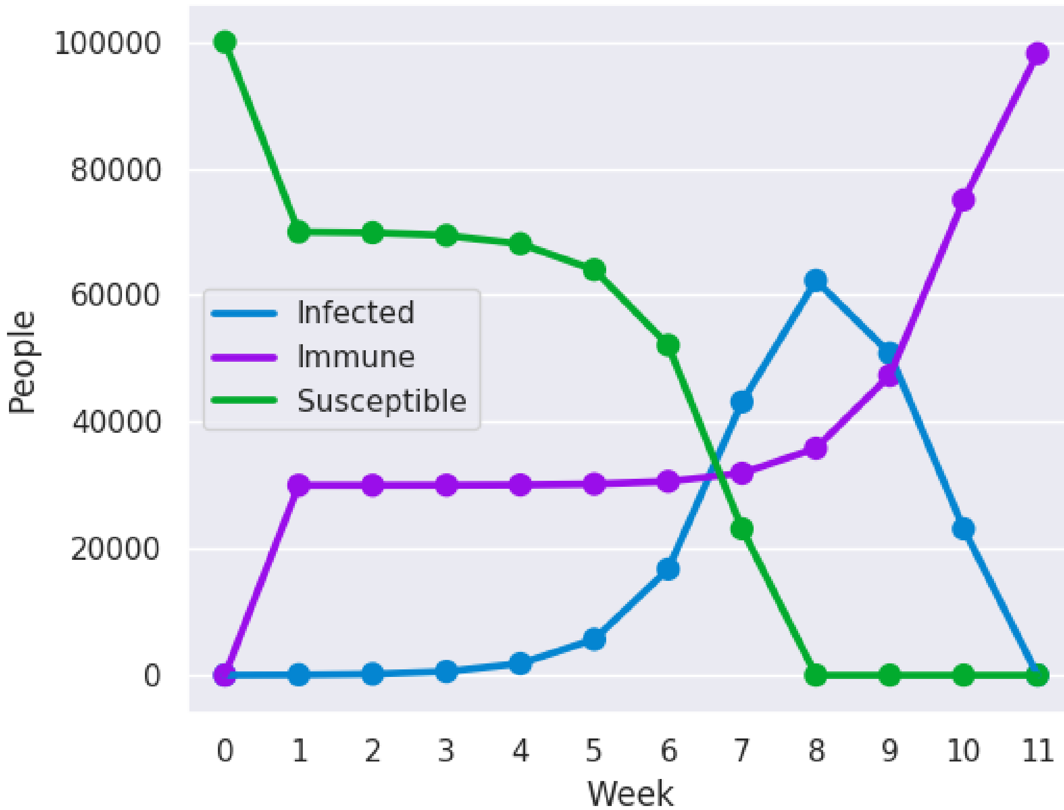
$$\Delta I_n = S_n * I_n * k - \Delta P_n$$

Where ΔI_n represents the change in the infected population, S_n represents the number of people that are susceptible at week n, I_n represents the number of people that are infected for week n, k is our constant, and ΔP_n represents the number of people that became immune and are now protected from the disease at week n, which is the same as ΔI_{n-3}. We record how many people are immune at week n by adding ΔI_{n-3} to After we calculate how many people developed the disease, we then calculate how many people developed immunity. This can be done by calculating ΔI_{n-3}. Using these two numbers, we calculate how many people have the disease on week n. Finally, we use the new data to calculate the number of people that are now susceptible to the disease, which is equivalent to: $S_n = S_{n-1} + \Delta I_n - \Delta I_{n-3}$ Using these data points, we are now able to watch the disease take its course through the population.

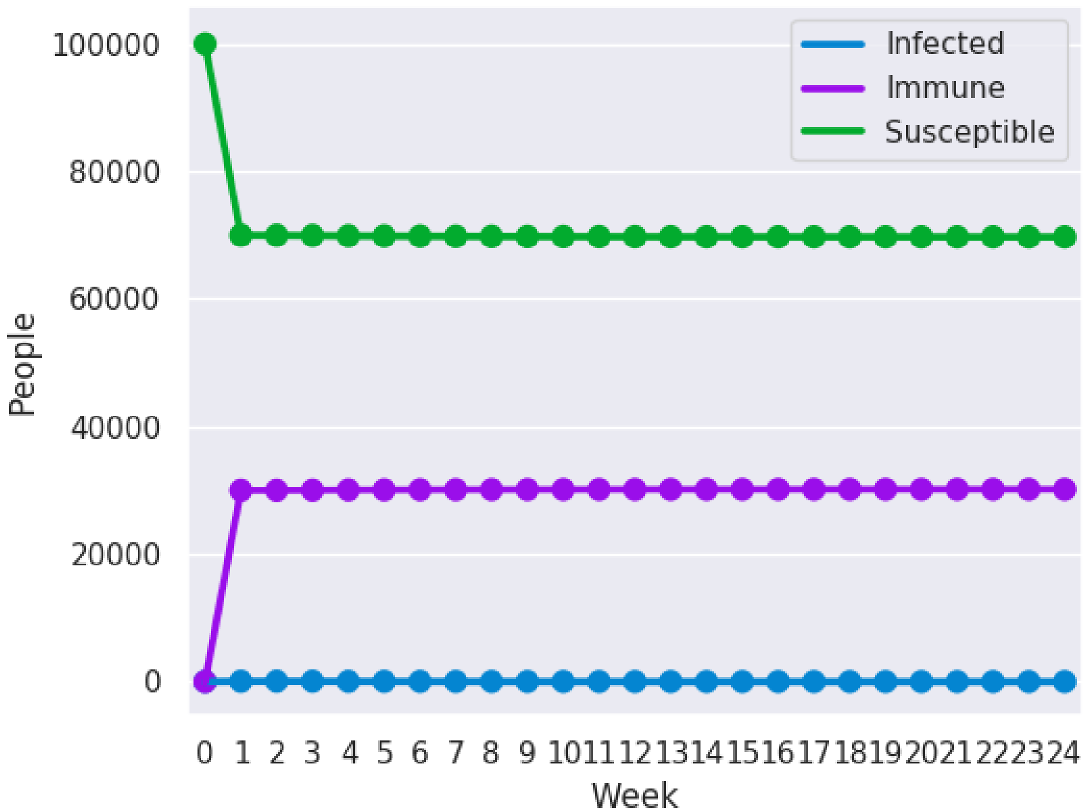
Findings

When running the model, we get the following data:

Week Number	Sick People	New Cases	Immune People	Became Immune	Susceptible People
0	18.0	18.0	0.0	0.0	99982.0
1	58.0	40.0	29994.6	29994.6	69987.4
2	186.899	128.899	29994.6	0.0	69858.501
3	583.497	414.598	30012.6	18.0	69425.903
4	1829.852	1286.356	30052.6	40.0	68099.548
5	5657.911	3956.957	30181.499	128.899	64013.692
6	16744.167	11500.854	30596.097	414.598	52098.239
7	43158.32	27700.508	31882.452	1286.356	23111.375
8	62312.738	23111.375	35839.41	3956.957	0.0
9	50811.884	0.0	47340.264	11500.854	0.0
10	23111.375	0.0	75040.772	27700.508	0.0
11	0.0	0.0	98152.148	23111.375	0.0



The graph is built from the data table above. As we can see, the number of sick people peaks at 5,657.911, which we could round up to 5,657 or 5,658. The only way for the virus to not infect everyone is if we decrease k by a magnitude of 10, which results in the chart below:



If we continue to decrease the k-value, the infection spreads slower, which leads to it doing less and less damage to the given population. It would be interesting to make the model more complicated by having a certain percent of those that are infected die, or a certain percent of those that are immune loosing their immunity.