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The Spread of Disease

Math 114 Mathematical Modeling

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Problem Statement

An isolated town has a population of 100,000 residents. Last week there were 18 new cases of people infected by a mild disease that lasts three weeks and leaves the person immune from further disease. Direct contact with an infected person leads to an infection of a previously uninfected person. This week there are 40 new cases. It is estimated that 30% of the existing population is immune because of previous exposure.

1. Make a list of assumptions that you need to make in order to develop a dynamical system model using difference equations.

2. Develop a model that describes how the number of new cases each week develops.

3. What is the eventual number of people who will become infected?

4. Vary the assumptions you make in this model to develop a feel for how sensitive the model is to your assumptions.

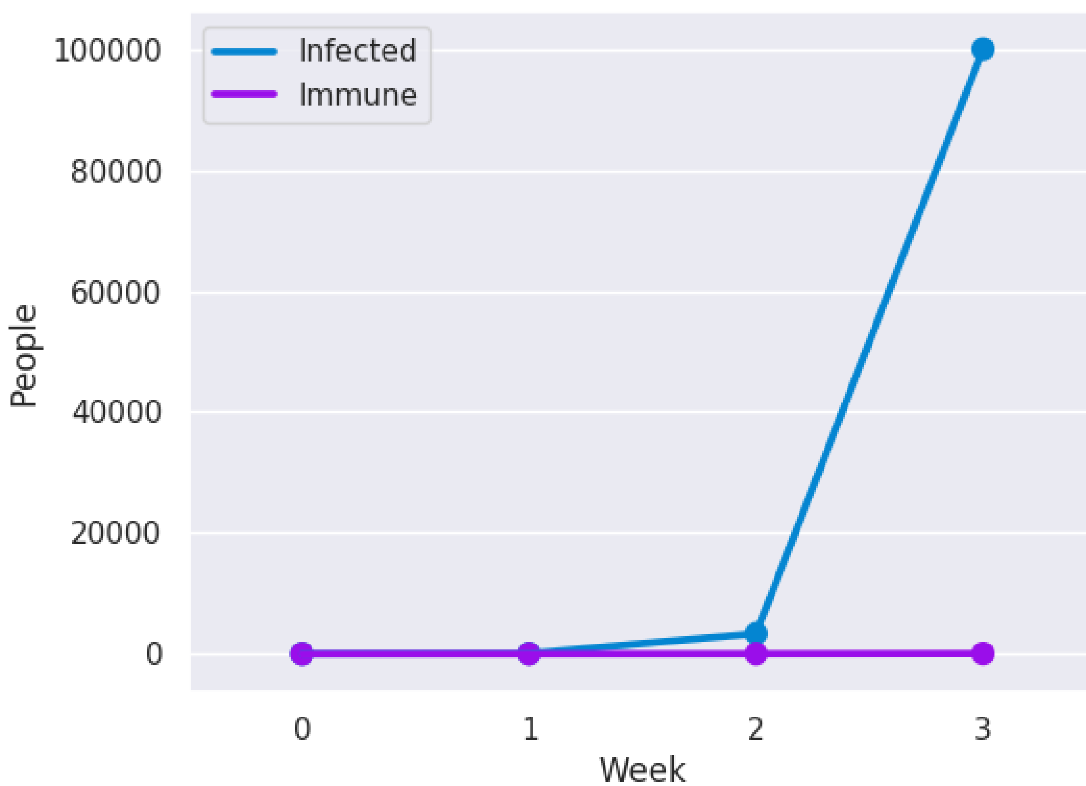
List of Assumptions

- Every person in the population is equally exposed to any peoples that are infected.
- Every person that is exposed to an individual that is infected has the same chance of becoming infected themselves.
- That the first week the infection is introduced into the population, 18 people develop cases of the sickness.
- That the second week the infection is introduced into the population, 40 people develop cases of the sickness.
- That the 30% of people that are immune in week 2 are immune via genetic trait.

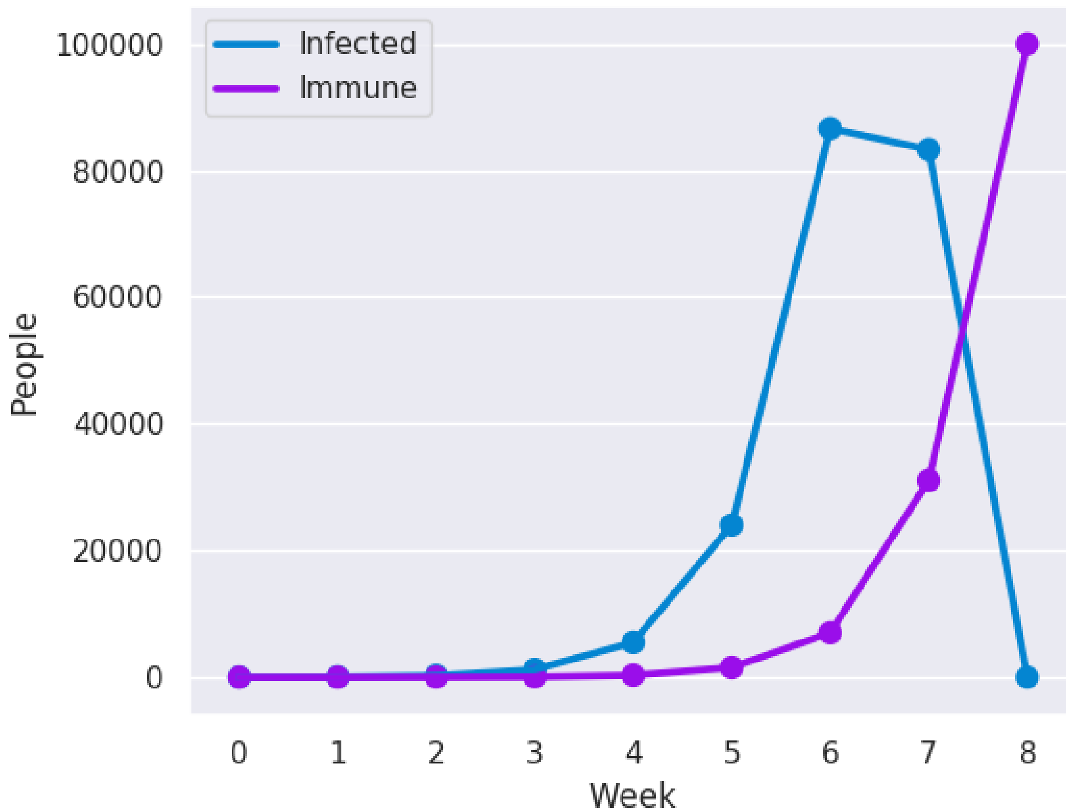
The Model

Based upon an equation given by the book, (Section 1.2—Example 3—p.13) we can assume that the spread of disease can be modeled with the following equation: $\Delta i_n = i_{n+1} - i_n = k i_n (N - i_n)$, where i_n is the number of people that developed the disease on week n, k is a constant, and N is the total population. In this equation, $i_n(N - i_n)$ represents the number of times an infected person will come into contact with someone who can become sick. k is the probability that someone will become sick if they encounter someone that is infected. Unlike the equation in the book, the people in the problem statement will get better after 3 weeks of having the disease. Also, we don't only care about the spread of disease, but rather the total number of people that are infected. This is reflected in the following equation: $i_{n+1} = i_n + k i_n (N - i_n - s_n) - \Delta i_{n-2}$. This reads as the following: the number of people that have the disease in week n+1 is equal to the number of people that have the disease in week n plus the number of people that contracted the disease in week n+1, minus the number of people that have become immune to the disease. The number of people that have contracted the disease is calculated by multiplying the constant k, the number of people that had the disease on week n, and the number of people that are succeptible to the disease. The number of people succeptible to the disease is found by subtracting the number of sick people on week n and the number of immune people on week n. The constant, k is found by calculating the number of infected individuals for week one. Given that week zero had 18 infected, and week one had 58 infected, we are able to solve for succeptible population, which eventually allows us to find out k-value (k=0.00057157).

People Who Become Infected



As shown by the figure above, the disease infects everyone by week three. If we decrease our k-value by a factor of fifteen, everyone is immune to the disease by week 8.



If we continue to decrease the k-value, the infection spreads slower, which leads to it doing less and less damage to the given population. It would be interesting to make the model more complicated by having a certain percent of those that are infected die, or a certain percent of those that are immune loosing their immunity.