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Written HW

Section 3.2 #'s 2a, 2b, 2c

Section 3.3 #'s 4, 9, 10

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3.2 #'s 2a, 2b, 2c

For these three problems, we are tasked with writing a mathematical model that minimizes the largest deviation between the data and the line $\hat{y} = ax + b$. We do this by solving for the estimates of a and b via the following definitions:

$$a = \frac{n\sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{n\sum_{i=1}^{n} x_i^2 - (\sum_{i=1}^{n} x_i)^2}$$

$$b = \frac{\sum_{i=1}^{n} x_i^2 \sum_{i=1}^{n} y_i - \sum_{i=1}^{n} x_i y_i \sum_{i=1}^{n} x_i}{n\sum_{i=1}^{n} x_i^2 - (\sum_{i=1}^{n} x_i)^2}$$

We judge how good our line is by using least squares:

$$\sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

Data for 2a:

x	y
1.0	3.6
2.3	3.0
3.7	3.2
4.2	5.1
6.1	5.3
7.0	6.8

Using our definitions for a and b, we get the following equation (rounded after 3 digits):

$$\hat{y} = 0.564x + 2.215$$

This has a least squares score of: 12508.995.

Data for 2b:

x	y
29.1	0.0493
48.2	0.0821
72.7	0.123
92.0	0.154
118	0.197
140	0.234
165	0.274
199	0.328

Using our definitions for a and b, we get the following equation (rounded after 3 digits):

$$\hat{y} = 0.00164x + 0.00293$$

This has a least squares score of: 118346.107

Data for 2c:

x	y
2.5	4.32
3.0	4.83
3.5	5.27
4.0	5.74
4.5	6.26
5.0	6.79
5.5	7.23

Using our definitions for a and b, we get the following equation (rounded after 3 digits):

$$\hat{y} = 0.969x + 1.893$$

This has a least squares score of: 7287.636

3.3 #'s 4,9,10

4) Make an appropriate transformation to fit the model $P = ae^{bt}$ using Equation (3.4). Estimate a and b.

In order to do this, we use equation 3.8 and the definitions that accompany it.

$$ln(y) = ln(\alpha) + mln(x)$$

$$y = \alpha + xe^{n}$$

$$m = \frac{n\sum_{i=1}^{n} ln(x_i)ln(y_i) - (\sum_{i=1}^{n} ln(x_i))(\sum_{i=1}^{n} ln(y_i))}{n\sum_{i=1}^{n}(ln(x_i)^2) - (\sum_{i=1}^{n} ln(x_i))^2}$$

$$ln\alpha = \frac{\sum_{i=1}^{n}(ln(x_i)^2)(ln(y_i)) - \sum_{i=1}^{n}(ln(x_i)ln(y_i))\sum_{i=1}^{n}(ln(x_i))}{n\sum_{i=1}^{n}(ln(x_i)^2) - (\sum_{i=1}^{n} ln(x_i)^2)}$$

Data for 4:

t	P
7	8
14	41
21	133
28	250
35	280
42	297

Using our definitions for m and ln(α), we get the following equation (rounded after 3 digits):

$$\hat{y} = 0.09727 + xe^{0.010207}$$

This has a least squares score of: 326010.755

9) Given the data for the ponderosa pine, make models of the following form:

- $y = ax + b$
- $y = ax^2$
- $y = ax^3$
- $y = ax^3 + bx^2 + c$

Data for 9:

x	y
17	19
19	25
20	32
22	51
23	57
22	51
25	71
28	113
31	140
32	153
33	187
36	192
37	205
39	250
42	260

y=ax+b :

Using the previous methods, we get the following equation:

$$y = 10.433x - 175.704$$

This has a least squares score of: 277541.645

y=ax2 :** // There was a latex bug that prevented me from doing ax^2

We use the following equation to get our a:

$$a = \frac{\sum_{i=0}^{n} x_i^n y_i}{\sum_{i=0}^{n} x_i^{2n}}$$

According to our implementation, a = 0.147. This results in a least squares of: 5185.290.

y=ax3 :**

According to our implementation, a = 0.00412. This results in a least squares of 298132.235

y=ax3+bx**2+c:**

Since this equation has degree of three, we use 4 points when constructing our lagrange multiplier:

$$19\frac{(x-19)(x-20)(x-22)}{(17-19)(17-20)(17-22)}+25\frac{(x-17)(x-20)(x-22)}{(19-17)(19-20)(19-22)}+32\frac{(x-17)(x-19)(x-22)}{(20-17)(20-19)(20-22)}$$

$$+51\frac{(x-17)(x-19)(x-20)}{(22-17)(22-19)(22-20)}$$

Using this method we get a least squares of 738883.844.

10) Using the data below, fit it to the model y=ax(3/2)**

Period	Distance$_{From_Sun}$
7.60*10**6	5.79*10**10
1.94*10**7	1.08*10**11
3.16*10**7	1.5*10**11
5.94*10**7	2.28*10**11
3.74*10**8	7.79*10**11
9.35*10**8	1.43*10**12
2.64*10**9	2.87*10**12
5.22*10**9	4.5*10**12

Using this methods we get $0.667x^{14.219}$

The least squares for this is 1.672148106802607e+38