

Men with no Hats Problem

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Ross, Intro to Probability Models Chapter 1. Problem 32

Theorem 1. Suppose that all n men at a party throw their hats in the center of the room. Each man then randomly selects a hat. Show that the probability that none of the n men selects his own hat is

$$\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \dots + \frac{(-1)^n}{n!}.$$

Proof. 1. $n = 1$, the probability that the one person doesn't select his own hat is 0

2. $n = 2$, $P = \frac{1}{2}$

3. $n = 3$, $P = \frac{3! \cdot 2 \cdot 1}{3! \cdot 3!} = \frac{1}{3}$, there are $3!$ ways to choose and $3!$ ways to order a group of 3 but only $2!$ ways to not select your own hat for any group of 3.

4. n_4 , $P =$

□