Machine Learning and Statistical Learning Theory II
Midterm Proposal

Daniel Yu

September 20, 2024

1 Team

Daniel Yu

2 Presentation Proposal

2.1 Topic

Reproducing Kernel Hilbert Spaces (RKHS) and Specific Application in Gaussian Processing

2.2 Summary

The aim of this presentation is to introduce the concept of **Reproducing Kernel Hilbert Spaces (RKHS)** and explain their significance in mathematical theory, as well as their practical applications in diverse fields such as machine learning, signal processing, statistics, and finance. By the end of the presentation, students will understand:

- What RKHS is and how it is constructed.
- How the kernel trick simplifies complex problems by working in high-dimensional spaces.
- Application of RKHS to Gaussian Processing with example in option volatility pricing

3 Outline

Part 1: Introduction to Hilbert Spaces and Kernels

- **What is a Hilbert Space?**
  - Generalization of Euclidean space to infinite dimensions.
  - Inner product, norm, and completeness.
- **The Role of Kernels**
  - Defining similarity functions (kernel functions).
  - Examples of common kernels (linear, polynomial, Gaussian/RBF).
- **Motivation for RKHS**
  - How kernels can implicitly map data into higher dimensions.
  - The idea of non-linear problems being easier to solve in transformed spaces.

Part 2: Reproducing Kernel Hilbert Spaces

- **Defining RKHS**
  - What makes RKHS special?
  - Reproducing property: $f(x) = \langle f, K_x \rangle$.
- **The Kernel Trick**
  - Explanation of the kernel trick and how it avoids explicit computation of high-dimensional transformations.
  - Real-world benefits of using kernel functions.
- **Examples of Kernels in RKHS**
  - Gaussian (RBF), polynomial, and other examples that are widely used in practice.

Part 3: Applications of RKHS

- **Machine Learning: Support Vector Machines (SVM) and Kernel Methods**
  - How RKHS is used to classify non-linear data in SVMs.
  - Benefits of the kernel trick in machine learning.
- **Finance: Volatility Modeling**
  - Using RKHS with Gaussian/RBF kernel and associated Gaussian Process to estimate volatility surfaces for options pricing.

Part 4: Conclusion and Discussion

- Summary of the key points from the presentation.
- Q&A.

4 Resources

- *Elements of Statistical Learning* by Hastie
- *Probabilistic Machine Learning* by Murphy
- TODO - Will add papers, other textbooks, and any online resources I find!