MECH230 - Fall 2024
Recommended Problems - Set 08

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Power The power of a force $\mathbf{F}$ acting on a particle with velocity $\mathbf{v}$ is defined to be

$$\mathcal{P} = \mathbf{F} \cdot \mathbf{v} \tag{1}$$

which has units of N$\cdot$ m/s = Watt in SI.

Work The work of a force $\mathbf{F}$ over the interval $[t_A, t_B]$ is the integral of the power over that interval

$$W_{\mathbf{F},AB} = \int_{t_A}^{t_B} \mathbf{F} \cdot \mathbf{v} dt = \int_{\mathbf{r}_A}^{\mathbf{r}_B} \mathbf{F} \cdot d\mathbf{r} \tag{2}$$

where $\mathbf{r}_A = \mathbf{r}(t_A)$ and $\mathbf{r}_B = \mathbf{r}(t_B)$. Notice that is the the angle between the force and the displacement vector is acute, then the work of the force is positive. A force is workless if it is perpendicular to the velocity/displacement of the particle.

Kinetic Energy Define the kinetic energy of a particle to be

$$T = \frac{1}{2} m \mathbf{v} \cdot \mathbf{v}. \tag{3}$$

The work energy theorem for a particle The work energy theorem for a particle is obtained by dotting the balance of linear momentum with $d\mathbf{r}$ and integrating it to obtain

$$T_B - T_A = W_{\mathbf{F},AB}. \tag{4}$$

Conservative Forces A conservative force $\mathbf{F}_c$ is such that

$$W_{\mathbf{F}_c,AB} = -(U_B - U_A) \tag{5}$$

where U is called the potential energy function.

Examples of conservative forces include

- any constant force $\mathbf{C}$ with $U = -\mathbf{c} \cdot \mathbf{r}$,

- the gravitational force $\mathbf{F}_G = G\frac{M_e m}{(R_e+h)^2}(-\mathbf{e}_r)$ with $U = -\frac{GM_e m}{r}$, and
- the spring force $\mathbf{F}_s = -K\varepsilon\frac{\mathbf{r}-\mathbf{r}_A}{||\mathbf{r}-\mathbf{r}_A||}$ with $U = \frac{1}{2}K\varepsilon^2$. The spring stretch $\varepsilon = ||\mathbf{r}-\mathbf{r}_A|| - \ell_0$.

The tension, friction, and normal are examples of nonconservative forces, $\mathbf{F}_{nc}$.

We can rewrite the work energy theorem as

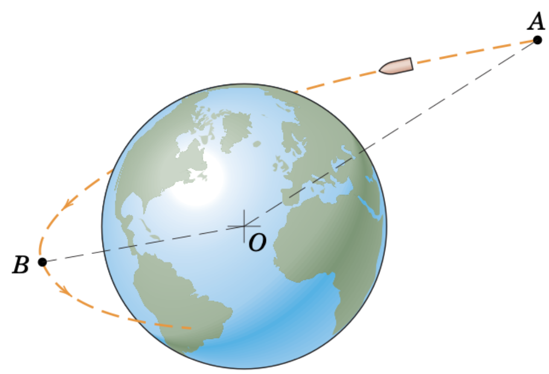
$$(\mathbf{T}_B + U_B) - (\mathbf{T}_A + U_A) = W_{\mathbf{F}_{nc},AB}. \tag{6}$$

Conservation of energy Thus, if all the forces doing work on the system are conservative, then the energy of the system $E = T + U$ is conserved. Here, U is the total potential energy of the system.

These problems are taken from J. L. Meriam, L. G. Kraige, and J. N. Bolton (MKB), Engineering Mechanics: Dynamics, Ninth Edition, Wiley, New York, 2018.

1. [MKB 03-028] During its motion, the only force acting on the spacecraft is the gravitational force. Hence, all the forces doing work on the satellite are conservative, so the energy of the spacecraft is conserved. Use the conservation of energy to solve this problem. Refer to table D/2 copied at the end of this booklet for the necessary numerical values.

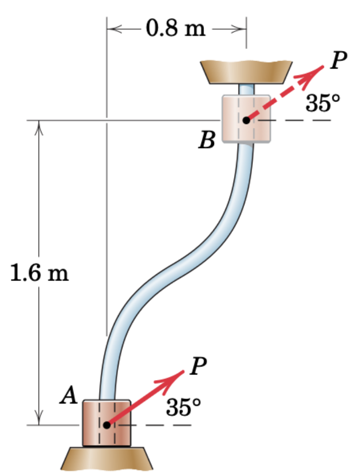
3/128 Upon its return voyage from a space mission, the spacecraft has a velocity of 24 000 km/h at point A, which is 7000 km from the center of the earth. Determine the velocity of the spacecraft when it reaches point B, which is 6500 km from the center of the earth. The trajectory between these two points is outside the effect of the earth's atmosphere.



PROBLEM 3/128

2. [MKB 03-080]

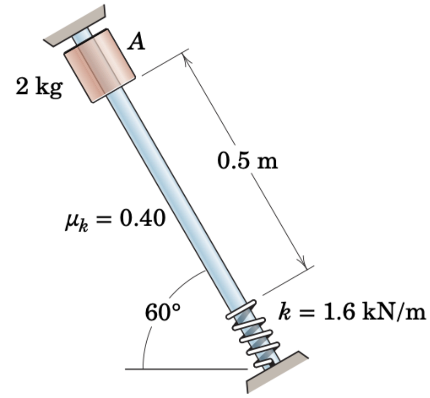
3/80 The 2-kg collar is at rest in position A when the constant force P is applied as shown. Determine the speed of the collar as it passes position B if (a) $P = 25$ N and (b) $P = 40$ N. The curved rod lies in a vertical plane, and friction is negligible.



PROBLEM 3/80

3. [MKB 03-084]

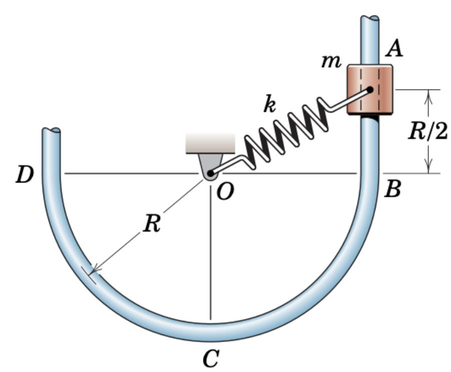
3/84 The 2-kg collar is released from rest at A and slides down the inclined fixed rod in the vertical plane. The coefficient of kinetic friction is 0.40. Calculate (a) the velocity v of the collar as it strikes the spring and (b) the maximum deflection x of the spring.



PROBLEM 3/84

4. [03-094]

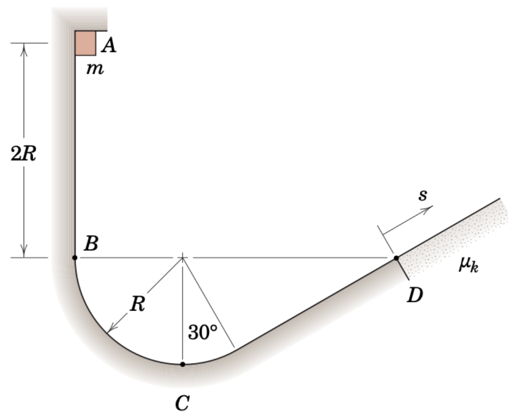
3/94 The collar of mass m is released from rest while in position A and subsequently travels with negligible friction along the vertical-plane circular guide. Determine the normal force (magnitude and direction) exerted by the guide on the collar (a) just before the collar passes point B, (b) just after the collar passes point B (i.e., the collar is now on the curved portion of the guide), (c) as the collar passes point C, and (d) just before the collar passes point D. Use the values $m = 0.4$ kg, $R = 1.2$ m, and $k = 200$ N/m. The unstretched length of the spring is $0.8R$.



PROBLEM 3/94

5. [03-101]

3/101 The small slider of mass m is released from rest while in position A and then slides along the vertical-plane track. The track is smooth from A to D and rough (coefficient of kinetic friction μ_k) from point D on. Determine (a) the normal force N_B exerted by the track on the slider just after it passes point B, (b) the normal force N_C exerted by the track on the slider as it passes the bottom point C, and (c) the distance s traveled along the incline past point D before the slider stops.



PROBLEM 3/101

6. [03-263]

3/263 The simple 2-kg pendulum is released from rest in the horizontal position. As it reaches the bottom position, the cord wraps around the smooth fixed pin at B and continues in the smaller arc in the vertical plane. Calculate the magnitude of the force R supported by the pin at B when the pendulum passes the position $\theta = 30°$.

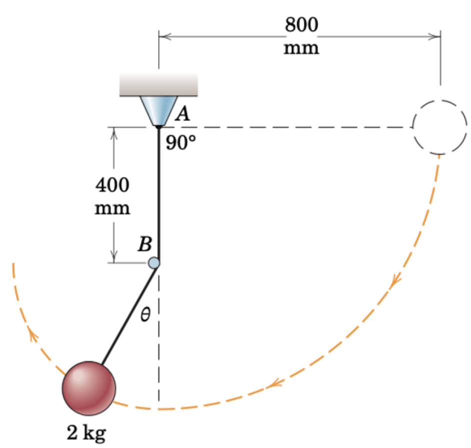


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TABLE D/2 Solar System Constants

Universal gravitational constant	$G = 6.673(10^{-11})\ \text{m}^3/(\text{kg}\cdot\text{s}^2)$
	$= 3.439(10^{-8})\ \text{ft}^4/(\text{lb-sec}^4)$
Mass of Earth	$m_e = 5.976(10^{24})\ \text{kg}$
	$= 4.095(10^{23})\ \text{lb-sec}^2/\text{ft}$
Period of Earth's rotation (1 sidereal day)	= 23 h 56 min 4 s
	= 23.9344 h
Angular velocity of Earth	$\omega = 0.7292(10^{-4})$ rad/s
Mean angular velocity of Earth–Sun line	$\omega' = 0.1991(10^{-6})$ rad/s
Mean velocity of Earth's center about Sun	= 107 200 km/h
	= 66,610 mi/hr

Body	Mean Distance to Sun km (mi)	Eccentricity of Orbit e	Period of Orbit solar days	Mean Diameter km (mi)	Mass Relative to Earth	Surface Gravitational Acceleration m/s^2 (ft/sec^2)	Escape Velocity km/s (mi/sec)
Sun	—	—	—	1 392 000 (865 000)	333 000	274 (898)	616 (383)
Moon	384 398[1] (238 854)[1]	0.055	27.32	3 476 (2 160)	0.0123	1.62 (5.32)	2.37 (1.47)
Mercury	57.3×10^6 (35.6×10^6)	0.206	87.97	5 000 (3 100)	0.054	3.47 (11.4)	4.17 (2.59)
Venus	108×10^6 (67.2×10^6)	0.0068	224.70	12 400 (7 700)	0.815	8.44 (27.7)	10.24 (6.36)
Earth	149.6×10^6 (92.96×10^6)	0.0167	365.26	12 742[2] (7 918)[2]	1.000	9.821[3] (32.22)[3]	11.18 (6.95)
Mars	227.9×10^6 (141.6×10^6)	0.093	686.98	6 788 (4 218)	0.107	3.73 (12.3)	5.03 (3.13)
Jupiter[4]	778×10^6 (483×10^6)	0.0489	4333	139 822 (86 884)	317.8	24.79 (81.3)	59.5 (36.8)

[1]Mean distance to Earth (center-to-center)
[2]Diameter of sphere of equal volume, based on a spheroidal Earth with a polar diameter of 12 714 km (7900 mi) and an equatorial diameter of 12 756 km (7926 mi)
[3]For nonrotating spherical Earth, equivalent to absolute value at sea level and latitude 37.5°
[4]Note that Jupiter is not a solid body.