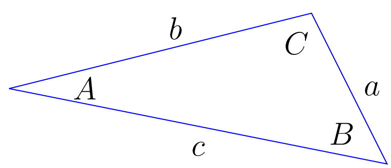
# MECH230 - Fall 2024
# Recommended Problems - Set 13

Theresa Honein

November 6, 2024

Sine law and cosine law




Referring to the above triangle with interior angles $A$, $B$, and $C$ and with side lengths $a$, $b$, and $c$, the sine law and cosine law are respectively

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}, \tag{1}$$

$$c^2 = a^2 + b^2 - 2ab\cos(C). \tag{2}$$

Instantaneous center of rotation If a rigid body is rotating about a fixed point $O$, the the velocity vector $\mathbf{v}_P$ of any point $P$ on the rigid body is perpendicular to the position vector from $O$ to $P$

$$\mathbf{v}_P \perp (\mathbf{v}_P - \mathbf{v}_O). \tag{3}$$

A body in general plane motion at an instant to has an instantaneous center of rotation (IC) such that $\mathbf{v}_{IC} = 0$ but $\mathbf{a}_{IC} \neq 0$ because the center of rotation moves in space. If the velocities of two points on a rigid body are known, IC lies on the intersection point of the two perpendiculars to the velocity vectors at these points.

A particle in a noninertial frame Considered two particles $A$ and $B$ with $A$ being fixed to the rigid body and $B$ moving with respect to the rigid body, then

$$\mathbf{r}_B - \mathbf{r}_A = x\mathbf{e}_x + y\mathbf{e}_y, \tag{4}$$

$$\mathbf{v}_B - \mathbf{v}_A = \boldsymbol{\omega} \times (\mathbf{r}_B - \mathbf{r}_A) + \mathbf{v}_{rel}, \tag{5}$$

$$\mathbf{a}_B - \mathbf{a}_A = \boldsymbol{\alpha} \times (\mathbf{r}_B - \mathbf{r}_A) + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times (\mathbf{r}_B - \mathbf{r}_A)) + 2\boldsymbol{\omega} \times \mathbf{v}_{rel} + \mathbf{a}_{rel}, \tag{6}$$

where

$$\mathbf{v}_{rel} = \dot{x}\mathbf{e}_x + \dot{y}\mathbf{e}_y, \quad \text{and} \quad \mathbf{a}_{rel} = \ddot{x}\mathbf{e}_x + \ddot{y}\mathbf{e}_y. \tag{7}$$

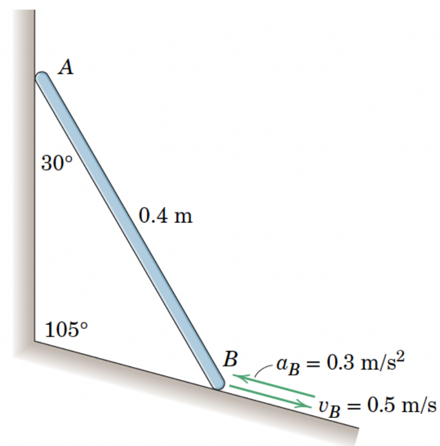
The Coriolis acceleration is $2\boldsymbol{\omega} \times \mathbf{v}_{rel}$ and the centripetal acceleration is $\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{v}_{rel})$.

These problems are taken from J. L. Meriam, L. G. Kraige, and J. N. Bolton (MKB), Engineering Mechanics: Dynamics, Ninth Edition, Wiley, New York, 2018.

In the following problems, feel free to introduce corrotational basis vectors as you see fit.

1. [MKB 05-102]

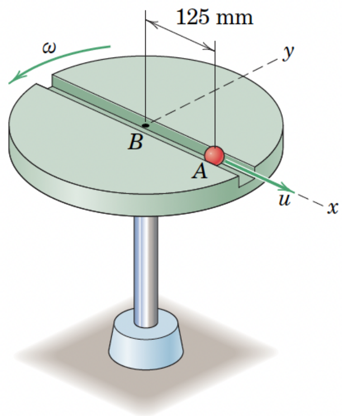
**5/102** The bar of Prob. 5/66 is repeated here. The ends of the 0.4-m bar remain in contact with their respective support surfaces. End $B$ has a velocity of 0.5 m/s and an acceleration of 0.3 m/s$^2$ in the directions shown. Determine the angular acceleration of the bar and the acceleration of end $A$.




**PROBLEM 5/102**

2. [MKB 05-125]

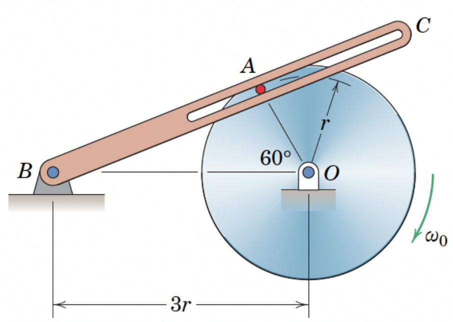
**5/125** The disk rotates with angular speed $\omega = 2$ rad/s. The small ball $A$ is moving along the radial slot with speed $u = 100$ mm/s relative to the disk. Determine the absolute velocity of the ball and state the angle $\beta$ between this velocity vector and the positive $x$-axis.




PROBLEM 5/125

3. [05-143]

**5/143** The disk rotates about a fixed axis through point $O$ with a clockwise angular velocity $\omega_0 = 20$ rad/s and a counterclockwise angular acceleration $\alpha_0 = 5$ rad/s$^2$ at the instant under consideration. The value of $r$ is 200 mm. Pin $A$ is fixed to the disk but slides freely within the slotted member $BC$. Determine the velocity and acceleration of $A$ relative to slotted member $BC$ and the angular velocity and angular acceleration of $BC$.




PROBLEM 5/143