

MECH 230 Dynamics - Section 1

Homework 1

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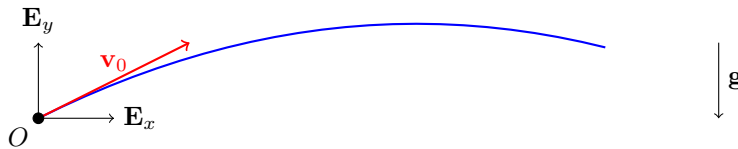
Due Wednesday September 4, 2024

This homework will walk you through reproducing Figure 1.1 and 1.6 of

O. M. O'Reilly, *Engineering Dynamics: A Primer*, Third Edition
Springer-Verlag, New York, 2019.

The electronic version of this text is available for free [here](#).

1. Read [Matlab's ode45 tutorial](#).
2. Consider a particle of mass m that is launched into the air from a point with initial velocity \mathbf{v}_0 at $t = 0$. At this instant, $\mathbf{r} = \mathbf{r}_0 = x_0\mathbf{E}_x + y_0\mathbf{E}_y + z_0\mathbf{E}_z$. During the subsequent motion of the particle it is under the influence of a vertical gravitational force $-mg\mathbf{E}_y$. In SI units, g is approximately 9.81 meter per second per second (m s^{-2}). The particle is also under the influence of the drag force $\mathbf{F}_D = -\frac{1}{2}\rho C_d A v \mathbf{v}$.



- (a) Write expressions of the position, velocity, and acceleration vectors for the particle in Cartesian coordinates.
- (b) Draw a free body diagram of the particle and express the forces acting on it in vector form.
- (c) Write the expression for the balance of linear momentum (also known as Newton's second law or Euler's first law) for the particle in vector form.
- (d) Project the balance of linear momentum equation along the \mathbf{E}_x , \mathbf{E}_y , \mathbf{E}_z directions to obtain three scalar differential equations for the motion of the particle.
- (e) To solve these differential equations numerically, we start by defining the column vector

$$y = [y(1); y(2); y(3); y(4); y(5); y(6)]$$

where

```
y1 = x
y2 = y
y3 = z
y4 = dxdt
y5 = dydt
y6 = dzdt
```

Then,

```
dy1dt = y4;
dy2dt = ... (1);
dy3dt = ... (2);
dy4dt = -k*sqrt(y4^2+y5^2+y6^2)*y4;
dy5dt = ... (3);
dy6dt = ... (4);
```

where k replaces the expression $\frac{\rho C_d A}{2m}$. Using your answers in part (d), provide the missing expressions (1), (2), (3), and (4).

- (f) In Matlab, create a script, in which you define the function the function `eom` as follows:

```
% defining the equation of motion
function dydt = eom(t,y,k,g)
    dydt = zeros(6,1);
    dydt(1) = y(4);
    dydt(2) = ...;
    dydt(3) = ...;
    dydt(4) = -k*sqrt(y(4)^2+y(5)^2+y(6)^2)*y(4);
    dydt(5) = ...;
    dydt(6) = ...;
end
```

where, again, you need to provide the missing expressions.

- (g) To solve these equations of motion for several values of k , we use Matlab's `ode45` inside a for loop. At the beginning of your script, insert the following code snippet.

```
k = [0, 0.01, 0.05];
g = 9.81; % gravitational constant
tspan = [0 15]; % simulation time span
```

```

y0 = ...;          % vector of initial conditions

for i=1:3
    % solving the differential eom
    [t,y] = ode45(@(t,y) eom(t,y,k(i),g), tspan, y0);
end

```

Define the vector of initial conditions y_0 knowing that

$$\mathbf{r}_0 = \mathbf{0},$$

$$\mathbf{v}_0 = 5\mathbf{E}_x + 10\mathbf{E}_y.$$

- (h) To reproduce Fig 1.6, add the following code snippet inside the for loop after solving the differential equations of motion.

```

% Fig 1.6
v = sqrt(y(:,4).^2+y(:,5).^2+y(:,6).^2); % calculating speed
figure(1) % initializing figure 2
hold on % plot new curves without erasing old ones
box on % box figure
plot(t,v) % plot speed as a function of time
axis([0 15 0 40]) % axis limits axis([xmin xmax ymin ymax])
xlabel('t (seconds)') % x-axis label
ylabel('v (m/s)') % y-axis label
quiver(2,5,-1,4,'k') % arrow towards increasing k, see documentation

```

Add code to reproduce Fig 1.1 that includes the three projectile trajectories and two unit vectors \mathbf{E}_x and \mathbf{E}_y . You will need to use the command `axis equal`.

- (i) To calculate the terminal velocity of the particle, we set the accelerations to zero in the equations of motion and solve for the velocity vector:

$$\mathbf{v}_{\text{term}} = -v_{\text{term}}\mathbf{E}_y = -\left(\frac{2mg}{\rho C_d A}\right)^{1/2} \mathbf{E}_y.$$

Verify that this result agrees with your replica of Fig. 1.6.

Instructions Your homework submissions should consist of

1. A hard copy of your answers to questions (2a-e) and (2i).
2. A hard copy of your complete final code containing your implementation of questions (2f-h).
3. Hard copy of your reproductions of figures 1.1 and 1.6.