MECH 230 Dynamics
Homework 2

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Due Wednesday September 18, 2024

1. Consider a particle tracing a circular helix. Its position vector is expressed in Cartesian coordinates as

$$\mathbf{r} = x\mathbf{E}_x + y\mathbf{E}_y + z\mathbf{E}_z = R\cos(\omega t)\mathbf{E}_x + R\sin(\omega t)\mathbf{E}_y + \alpha R\omega t\mathbf{E}_z.$$

   (a) Create a Matlab script `helix.m` in which you write the following code to plot the helix for $R = 1$, $\omega = 2$, and $\alpha = 0.5$. Note that you can change your view angle manually or using the command `view`.

```
% Running this script builds an animation of a particle tracing a circular
% helix

R = 1;  % radius of helix
omega = 2;  % angular velocity parameter
alpha = 0.5;

n = 30;
t = linspace(0,5,n);    % plotting trajectory for 0<t<5s with n time steps

x = R*cos(omega*t);
y = R*sin(omega*t);
z = R*alpha*omega*t;

figure()
hold on

plot3(0,0,0,'*','linewidth',2,'color','k')  % plotting the orgin

plot3(x,y,z,'color','k','linewidth',2)    % plotting the helix

quiver3(0,0,0,1,0,0,'linewidth',2,'color','b')  % plot Ex
quiver3(0,0,0,0,1,0,'linewidth',2,'color','r')  % plot Ey
quiver3(0,0,0,0,0,1,'linewidth',2,'color','g')  % plot Ez
```

```
view(30,45) % set view angle, can be also changed manually
axis equal

% plotting er in blue
er = [x./(x.^2+y.^2);       % the x-coordinate of er
    y./(x.^2+y.^2);         % the y-coordinate of er
    zeros(1,n)];            % the z-coordinate of er
quiver3(x,y,z,er(1,:),er(2,:),er(3,:),'color','b')
% plotting etheta in red
etheta = [-y./(x.^2+y.^2);  % the x-coordinate of etheta
    x./(x.^2+y.^2);         % the y-coordinate of etheta
    zeros(1,n)];            % the z-coordinate of etheta
quiver3(x,y,z,etheta(1,:),etheta(2,:),etheta(3,:),'color','r')
```

(b) In cylindrical polar coordinates, the position vector can be expressed as

$$\mathbf{r} = r\mathbf{e}_r + z\mathbf{E}_z.$$

Identify the expressions for r, θ, $\mathbf{e}_r$ and $\mathbf{e}_\theta$ in terms of R and t.

(c) Read the Matlab documentation on `quiver3`.

(d) Copy the following code to the end of your script to plot the $\{\mathbf{e}_r, \mathbf{e}_\theta, \mathbf{E}_z\}$ basis on the helix.

```
% plotting er in blue
quiver3(x,y,z,x./(x.^2+y.^2),y./(x.^2+y.^2),zeros(1,n),'color','b')
% plotting etheta in red
quiver3(x,y,z,-y./(x.^2+y.^2),x./(x.^2+y.^2),zeros(1,n),'color','r')
```

(e) Calculate $\mathbf{v} = \frac{d\mathbf{r}}{dt}$.

(f) Knowing that $\mathbf{v} = ||\mathbf{v}||\mathbf{e}_t$ and $\frac{ds}{dt} = v = ||\mathbf{v}||$, verify that

$$\mathbf{e}_t = \frac{1}{\sqrt{1+\alpha^2}}\left(\mathbf{e}_\theta + \alpha\mathbf{E}_z\right),$$
$$\frac{ds}{dt} = R\dot{\theta}\sqrt{1+\alpha^2}.$$

(g) The chain rule implies that

$$\frac{d\mathbf{e}_t}{dt} = \frac{d\mathbf{e}_t}{ds}\frac{ds}{dt}.$$

Use this result to calculate $\frac{d\mathbf{e}_t}{ds}$.

(h) Recall that

$$\frac{d\mathbf{e}_t}{ds} = \kappa\mathbf{e}_n$$

where $\kappa \geq 0$ and $\mathbf{e}_n \cdot \mathbf{e}_n = 1$. Vertify that

$$\mathbf{e}_n = -\mathbf{e}_r$$
$$\kappa = \frac{1}{R(1+\alpha^2)}$$

(i) Recall that $\mathbf{e}_b = \mathbf{e}_t \times \mathbf{e}_n$. Calculate $\mathbf{e}_b$.

(j) Recall the Serret-Frenet relations and the definition of the torsion τ:

$$\frac{d\mathbf{e}_b}{ds} = -\tau \mathbf{e}_n.$$

Verify that

$$\tau = \frac{\alpha}{R(1+\alpha^2)}.$$

(k) On a new figure, plot the helix, and the basis $\{\mathbf{e}_t, \mathbf{e}_n, \mathbf{e}_b\}$ along its length. Here's one way to plot $\mathbf{e}_t$:

```
figure()
hold on

plot3(0,0,0,'*','linewidth',2,'color','k')  % plotting the orgin

plot3(x,y,z,'color','k','linewidth',2)    % plotting the helix

quiver3(0,0,0,1,0,0,'linewidth',2,'color','b')  % plot Ex
quiver3(0,0,0,0,1,0,'linewidth',2,'color','r')  % plot Ey
quiver3(0,0,0,0,0,1,'linewidth',2,'color','g')  % plot Ez

view(30,45) % set view angle, can be also changed manually
axis equal

Ez = [0;0;1];
% plotting et in blue
et = 1/sqrt(1+alpha^2)*(etheta+alpha*Ez);
quiver3(x,y,z,et(1,:),et(2,:),et(3,:),'color','b')
% plotting en in red
% ???
% ???
% plotting eb in green
% ???
% ???
```

(l) How do κ and τ change as you increase α?

Deliverables: Your submission should include the following.

- A hard copy of your answers to parts b, e, g, i and l.
- A hard copy of the plot you obtained in part k.
- A hard copy of the code.