MECH230 - Fall 2024
Recommended Problems - Set 11

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Kinematics of a system of particles Consider a system of n particles each with mass m_i and with position vector $\mathbf{r}_i$ from the origin.

The total mass of the system of particles is $m = \sum_{i=1}^{n} m_i$.

The center of mass of the system of particles C is defined to be located at

$$\mathbf{r}_C = \frac{\sum_{i=1}^{n} m_i \mathbf{r}_i}{m}. \tag{1}$$

The velocity and acceleration of the center of mass are then obtained by differentiation $\mathbf{r}_C$.

$$\mathbf{v}_C = \frac{\sum_{i=1}^{n} m_i \mathbf{v}_i}{m}, \quad \text{and} \quad \mathbf{a}_C = \frac{\sum_{i=1}^{n} m_i \mathbf{a}_i}{m}. \tag{2}$$

The linear momentum of a system of particles is the sum of the linear momenta of its constituents which is also the linear momentum of the center of mass

$$\mathbf{G} = \sum_{i=1}^{n} \mathbf{G}_i = m\mathbf{v}_C. \tag{3}$$

The angular momentum of a system of particles about point P is the sum of the angular momenta of its constituents about P

$$\mathbf{H}^P = \sum_{i=1}^{n} (\mathbf{r}_i - \mathbf{r}_P) \times m_i \mathbf{v}_i. \tag{4}$$

This expression can be rewritten as

$$\mathbf{H}^P = \mathbf{H} + (\mathbf{r} - \mathbf{r}_P) \times \mathbf{G}. \tag{5}$$

where the angular momentum $\mathbf{H}^C$ of the system about the center of mass can be written equivalently as

$$\mathbf{H} = \sum_{i=1}^{n} (\mathbf{r}_i - \mathbf{r}_C) \times m_k \mathbf{v}_i = \sum_{i=1}^{n} (\mathbf{r}_i - \mathbf{r}_C) \times m_i (\mathbf{v}_i - \mathbf{v}_C). \tag{6}$$

The kinetic energy of the system of particles is the sum of the kinetic energies of its constituents. This is not equal to the $\frac{1}{2}m\mathbf{v}_C \cdot \mathbf{v}_C$.

$$T = \sum_{i=1}^{n} T_i = \sum_{i=1}^{n} \frac{1}{2} m_i \mathbf{v}_i \cdot \mathbf{v}_i = \frac{1}{2} m \mathbf{v}_C \cdot \mathbf{v}_C + \frac{1}{2} \sum_{i=1}^{n} m_k (\mathbf{v}_i - \mathbf{v}_C) \cdot (\mathbf{v}_i - \mathbf{v}_C). \tag{7}$$

Kinetics of a system of particles

The balance of linear momentum for a system of particles is obtained by adding the balance of linear momenta of the individual particles $\mathbf{F}_i = m_i \mathbf{a}_i$

$$\mathbf{F} = \sum \mathbf{F}_i = \sum m_i \mathbf{a}_i = m \mathbf{a}_c. \tag{8}$$

Here, $\mathbf{F}$ is the net external force acting on the system.

The balance of angular momentum of a system of particles about any point P is

$$\dot{\mathbf{H}}^P = \mathbf{M}^P - \mathbf{v}^P \times \mathbf{G}. \tag{9}$$

If P is a fixed point O, then this equation simplifies to $\mathbf{M}^O = \dot{\mathbf{H}}^O$.

If P is the center of mass C, then this equation simplifies to $\mathbf{M}^C = \dot{\mathbf{H}}^C$.

The work-energy theorem for a system of particles is the sum of the work-energy theorems of the individual particles

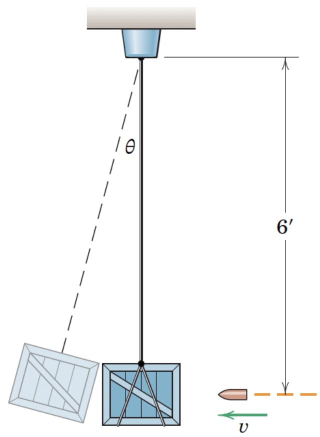
$$\sum \dot{T}_i = \sum \mathbf{F}_i \cdot \mathbf{v}_i. \tag{10}$$

Now that different points in the system have different displacements/velocities, it is important to note that the work of a force i is obtained by dotting $\mathbf{F}_i$ with the velocity/displacement of its point of application.

These problems are taken from J. L. Meriam, L. G. Kraige, and J. N. Bolton (MKB), Engineering Mechanics: Dynamics, Ninth Edition, Wiley, New York, 2018.

1. [MKB 03-168] Take the origin to be at the fixed point where the pendulum is suspended. In this problem, show that during collision (which takes up a very small time interval), the angular momentum of combined system of the pendulum and the bullet are conserved.

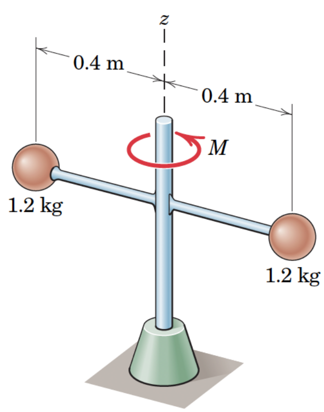
3/168 The ballistic pendulum is a simple device to measure projectile velocity v by observing the maximum angle θ to which the box of sand with embedded projectile swings. Calculate the angle θ if the 2-oz projectile is fired horizontally into the suspended 50-lb box of sand with a velocity $v = 2000$ ft/sec. Also find the percentage of energy lost during impact.



PROBLEM 3/168

2. [MKB 03-178] In this problem, you will use the integral form of the balance of angular momentum. Choose your system appropriately and take the origin to be at the green base.

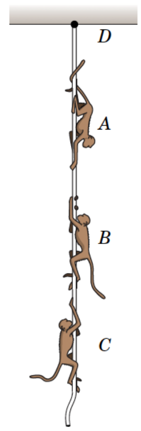
3/178 The rigid assembly which consists of light rods and two 1.2-kg spheres rotates freely about a vertical axis. The assembly is initially at rest and then a constant couple $M = 2$ N·m is applied for 5 s. Determine the final angular velocity of the assembly. Treat the small spheres as particles.



PROBLEM 3/178

3. [04-008] Take an appropriate cut in the rope to find the desired quantity.

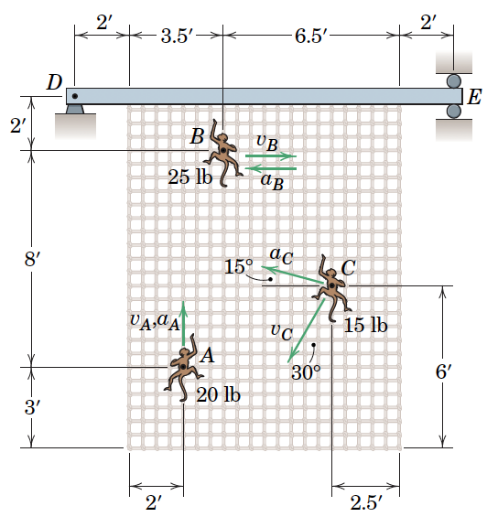
4/8 Three monkeys A, B, and C weighing 20, 25, and 15 lb, respectively, are climbing up and down the rope suspended from D. At the instant represented, A is descending the rope with an acceleration of 5 ft/sec^2, and C is pulling himself up with an acceleration of 3 ft/sec^2. Monkey B is climbing up with a constant speed of 2 ft/sec. Treat the rope and monkeys as a complete system and calculate the tension T in the rope at D.



PROBLEM 4/8

4. [04-009]

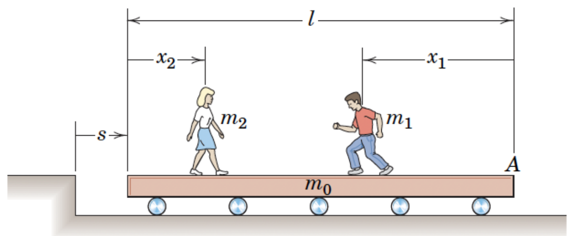
4/9 The monkeys of Prob. 4/8 are now climbing along the heavy rope wall suspended from the uniform beam. If monkeys A, B, and C have velocities of 5, 3, and 2 ft/sec, and accelerations of 1.5, 0.5, and 2 ft/sec^2, respectively, determine the changes in the reactions at D and E caused by the motion and weight of the monkeys. The support at E makes contact with only one side of the beam at a time. Assume for this analysis that the rope wall remains rigid.



PROBLEM 4/9

5. [04-019] In this problem, identify a direction of conservation of linear momentum. On your way to the answer, you will notice that the velocity of the center of mass of the system will remain zero, so the center of mass will remain stationary.

4/19 The man of mass m_1 and the woman of mass m_2 are standing on opposite ends of the platform of mass m_0 which moves with negligible friction and is initially at rest with $s = 0$. The man and woman begin to approach each other. Derive an expression for the displacement s of the platform when the two meet in terms of the displacement x_1 of the man relative to the platform.



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