MECH230 - Fall 2024
Recommended Problems - Set 12

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Angular velocity and angular acceleration Consider the reference and current configuration of a rigid body in general plane motion. At any time t, any material line on the rigid body in the current configuration makes the same angle θ with its precursor in the reference configuration. Define the angular velocity of the rigid body to be $\boldsymbol{\omega} = \dot{\theta}\mathbf{E}_z$ where $\mathbf{E}_z$ is the fixed axes of rotation. $\boldsymbol{\alpha} = \dot{\boldsymbol{\omega}}$ is the angular acceleration of the rigid body.

Corotational basis Let the orthonormal basis $\{\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z = \mathbf{E}_z\}$ be a corotational basis that is fixed on and corotates with the rigid body such that

$$\mathbf{e}_x = \cos(\theta)\mathbf{E}_x + \sin(\theta)\mathbf{E}_y, \tag{1}$$
$$\mathbf{e}_y = -\sin(\theta)\mathbf{E}_x + \cos(\theta)\mathbf{E}_y. \tag{2}$$

Then, $\dot{\mathbf{e}}_x = \boldsymbol{\omega} \times \mathbf{e}_x$ and $\dot{\mathbf{e}}_y = \boldsymbol{\omega} \times \mathbf{e}_y$.

Velocity and acceleration analysis for material points on a RB Consider two material points A and B with

$$\mathbf{r}_B - \mathbf{r}_A = x\mathbf{e}_x + y\mathbf{e}_y. \tag{3}$$

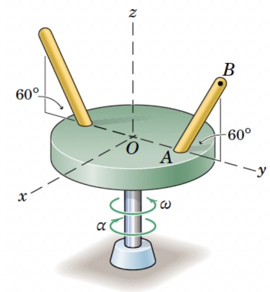
As the rigid body moves, x and y remain constants, hence

$$\mathbf{v}_B - \mathbf{v}_A = \boldsymbol{\omega} \times \mathbf{r}_B - \mathbf{r}_A, \tag{4}$$
$$\mathbf{a}_B - \mathbf{a}_A = \boldsymbol{\alpha} \times \mathbf{r}_B - \mathbf{r}_A + \boldsymbol{\omega} \times \mathbf{v}_B - \mathbf{v}_A. \tag{5}$$

These problems are taken from J. L. Meriam, L. G. Kraige, and J. N. Bolton (MKB), Engineering Mechanics: Dynamics, Ninth Edition, Wiley, New York, 2018.

1. [MKB 05-010]

5/10 The device shown rotates about the fixed z-axis with angular velocity $\omega = 20$ rad/s and angular acceleration $\alpha = 40$ rad/s^2 in the directions indicated. Determine the instantaneous velocity and acceleration of point B.

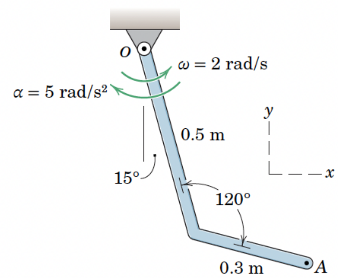


$\overline{OA} = 300$ mm, $\overline{AB} = 500$ mm

PROBLEM 5/10

2. [MKB 05-013]

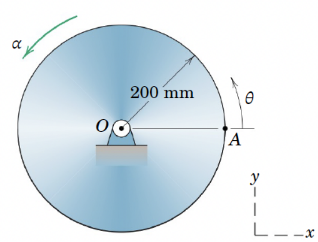
5/13 The bent flat bar rotates about a fixed axis through point O with the instantaneous angular properties indicated in the figure. Determine the velocity and acceleration of point A.



PROBLEM 5/13

3. [05-018]

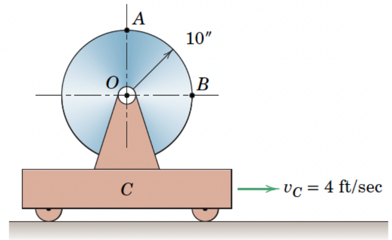
5/18 Point A of the circular disk is at the angular position $\theta = 0$ at time $t = 0$. The disk has angular velocity $\omega_0 = 0.1$ rad/s at $t = 0$ and subsequently experiences a constant angular acceleration $\alpha = 2$ rad/s^2. Determine the velocity and acceleration of point A in terms of fixed $\mathbf{i}$ and $\mathbf{j}$ unit vectors at time $t = 1$ s.



PROBLEM 5/18

4. [05-049]

5/49 The cart has a velocity of 4 ft/sec to the right. Determine the angular speed N of the wheel so that point A on the top of the rim has a velocity (*a*) equal to 4 ft/sec to the left, (*b*) equal to zero, and (*c*) equal to 8 ft/sec to the right.



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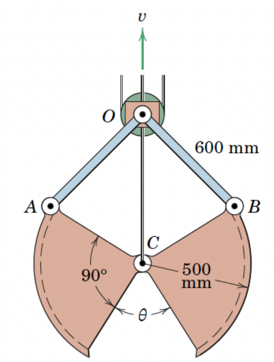
5. [05-065]

5/65 The elements of a simplified clam-shell bucket for a dredge are shown. The cable which opens and closes the bucket passes through the block at O. With O as a fixed point, determine the angular velocity ω of the bucket jaws when $\theta = 45°$ as they are closing. The upward velocity of the control cable is 0.5 m/s as it passes through the block.



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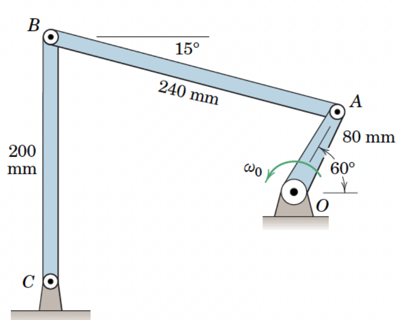
6. [05-069]

5/69 SS A four-bar linkage is shown in the figure (the ground "link" OC is considered the fourth bar). If the drive link OA has a counterclockwise angular velocity $\omega_0 = 10$ rad/s, determine the angular velocities of links AB and BC.



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