MECH230 - Fall 2024
Recommended Problems - Set 19

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The Koenig decomposition for the kinetic energy of a rigid body is

$$T = \frac{1}{2} m \mathbf{v}_C \cdot \mathbf{v}_C + \frac{1}{2} \mathbf{H}^C \cdot \boldsymbol{\omega}. \tag{1}$$

If the body has a point O with zero velocity, $\mathbf{v}_O = \mathbf{0}$, then

$$T = \frac{1}{2} m \mathbf{v}_O \cdot \mathbf{v}_O. \tag{2}$$

The work-energy theorem for a rigid body is

$$\frac{dT}{dt} = \mathbf{F} \cdot \mathbf{v}_C + \mathbf{M} \cdot \boldsymbol{\omega} = \sum_{i=1}^{K} \mathbf{F}_i \cdot \mathbf{v}_i + \mathbf{M}_e \cdot \boldsymbol{\omega}. \tag{3}$$

When integrated between $[t_A, t_B]$, the above expression becomes

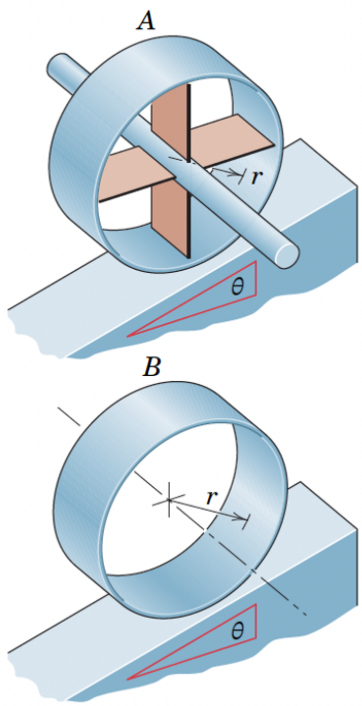
$$T_B - T_A = \sum_{i=1}^{K} W_{\mathbf{F}_i, AB} + W_{\mathbf{M}_e, AB}. \tag{4}$$

To establish energy conservation results, this theorem is used in a similar manner to the one employed with particles and systems of particles.

These problems are taken from J. L. Meriam, L. G. Kraige, and J. N. Bolton (MKB), Engineering Mechanics: Dynamics, Ninth Edition, Wiley, New York, 2018.

1. [MKB 06-096]

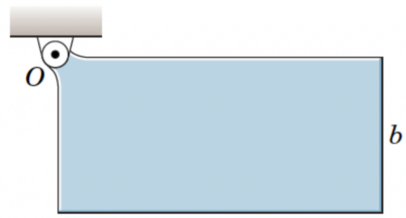
6/96 The two wheels of Prob. 6/62, shown again here, represent two extreme conditions of distribution of mass. For case A all of the mass m is assumed to be concentrated in the center of the hoop in the axial bar of negligible diameter. For case B all of the mass m is assumed to be concentrated in the rim. Determine the speed of the center of each hoop after it has traveled a distance x down the incline from rest. The hoops roll without slipping.



PROBLEM 6/96

2. [MKB 06-099]

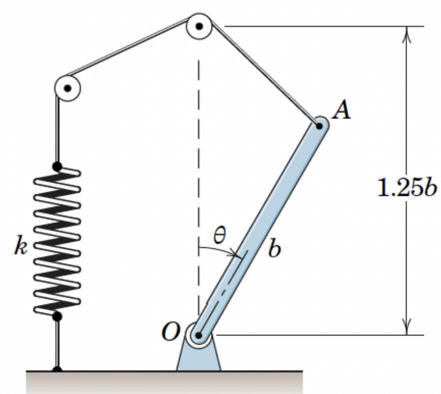
6/99 The uniform rectangular plate is released from rest in the position shown. Determine the maximum angular velocity ω during the ensuing motion. Friction at the pivot is negligible.



PROBLEM 6/99

3. [06-108]

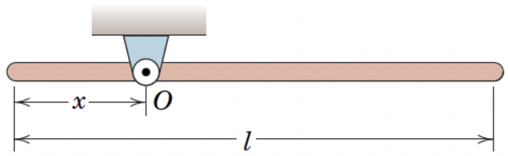
6/108 The system is at rest with the spring unstretched when $\theta = 0$. The 5-kg uniform slender bar is then given a slight clockwise nudge. The value of b is 0.4 m. (a) If the bar comes to momentary rest when $\theta = 40°$, determine the spring constant k. (b) For the value $k = 90$ N/m, find the angular velocity of the bar when $\theta = 25°$.



PROBLEM 6/108

4. [06-112]

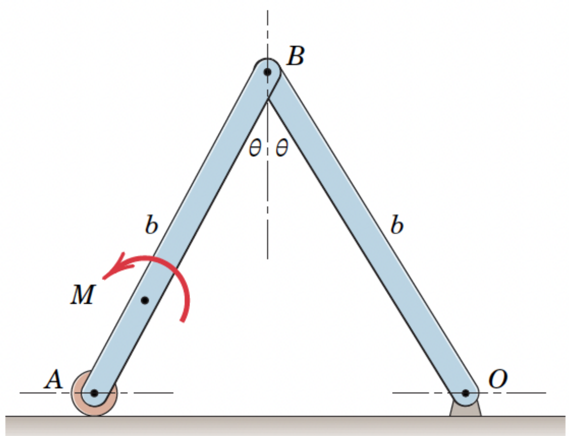
6/112 For the pivoted slender rod of length l, determine the distance x for which the angular velocity will be a maximum as the bar passes the vertical position after being released in the horizontal position shown. State the corresponding angular velocity.



PROBLEM 6/112

5. [06-118]

6/118 The two slender bars each of mass m and length b are pinned together and move in the vertical plane. If the bars are released from rest in the position shown and move together under the action of a couple M of constant magnitude applied to AB, determine the velocity of A as it strikes O.



PROBLEM 6/118