MECH230 - Fall 2024
Recommended Problems - Set 15

Theresa Honein

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Center of Mass The center of mass C of a body $\mathcal{B}$ has position vector

$$\mathbf{r}_C = \frac{\int_{\mathcal{B}} \mathbf{r} dm}{\int_{\mathcal{B}} dm} \tag{1}$$

where $\mathbf{r}$ is the position vector to a typical differential mass dm on the rigid body. The center of mass of a a rigid body acts as if it is a material point of the rigid body.

Linear Momentum The linear momentum of a rigid body $\mathcal{B}$ is

$$\mathbf{G} = \int_{\mathcal{B}} \mathbf{v} dm = m\mathbf{v}_C. \tag{2}$$

Angular Momentum The angular momentum of a rigid body $\mathcal{B}$ relative to any material point P on the body is

$$\mathbf{H}^P = \int_{\mathcal{B}} (\mathbf{r} - \mathbf{r}_P) \times \mathbf{v} dm. \tag{3}$$

In terms of the angular momentum $\mathbf{H}^C$ about the mass center

$$\mathbf{H}^P = \mathbf{H}^C + (\mathbf{r}_C - \mathbf{r}_P) \times \mathbf{G}, \quad \text{where} \quad \mathbf{H}^C = \int_B (\mathbf{r} - \mathbf{r}_C) \times \mathbf{v} dm.$$

Letting $\mathbf{r} - \mathbf{r}_C = x\mathbf{e}_x + y\mathbf{e}_y + z\mathbf{e}_Z$ can calculate

$$\begin{bmatrix} \mathbf{H}^C \cdot \mathbf{e}_x \\ \mathbf{H}^C \cdot \mathbf{e}_y \\ \mathbf{H}^C \cdot \mathbf{e}_z \end{bmatrix} = \begin{bmatrix} I_{xx}^C & I_{xy}^C & I_{xz}^C \\ I_{xy}^C & I_{yy}^C & I_{yz}^C \\ I_{xz} & I_{yz} & I_{zz} \end{bmatrix} \begin{bmatrix} \boldsymbol{\omega} \cdot \mathbf{e}_x \\ \boldsymbol{\omega} \cdot \mathbf{e}_y \\ \boldsymbol{\omega} \cdot \mathbf{e}_z \end{bmatrix} \tag{4}$$

where

$$\begin{array}{ccc} I_{xx}^C = \int_B (y^2 + z^2) dm, & I_{yy}^C = \int_B (x^2 + z^2) dm, & I_{zz}^C = \int_B (x^2 + y^2) dm, \\ I_{xy}^C = -\int_B xy dm, & I_{yz}^C = -\int_B yz dm, & I_{xz}^C = -\int_B xz dm \end{array} \tag{5}$$

The moments of inertia of typical shapes about regular axes can be found in tables or online.

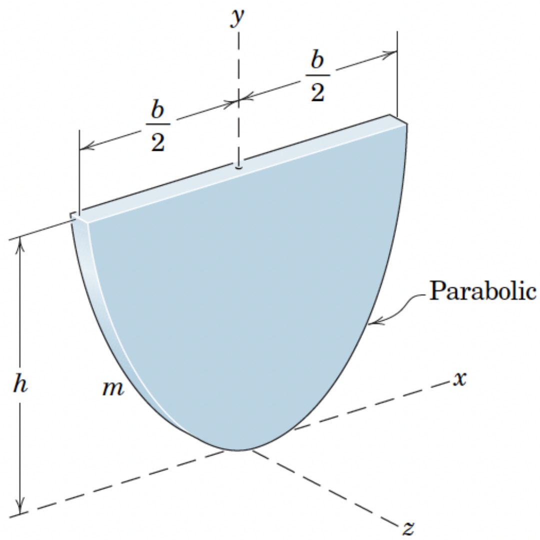
Parallel axis theorem Consider a material point A on the rigid body such that $\mathbf{r}_A - \mathbf{r}_C = A_x\mathbf{e}_x + A_z\mathbf{e}_y + A_z\mathbf{e}_z$, then according to the parallel axis theorem

$$I_{xx}^A = I_{xx}^C + m(A_y^2 + A_z^2), \quad I_{xy}^A = I_{xy}^C - mA_xA_y, \quad etc. \tag{6}$$

These problems are taken from J. L. Meriam, L. G. Kraige, and J. N. Bolton (MKB), Engineering Mechanics: Dynamics, Ninth Edition, Wiley, New York, 2018.

1. [MKB B-004]

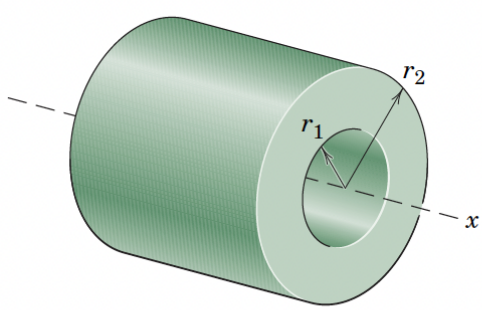
B/4 Determine the mass moment of inertia of the uniform thin parabolic plate of mass m about the x-axis. State the corresponding radius of gyration.



PROBLEM B/4

2. [MKB B-029]

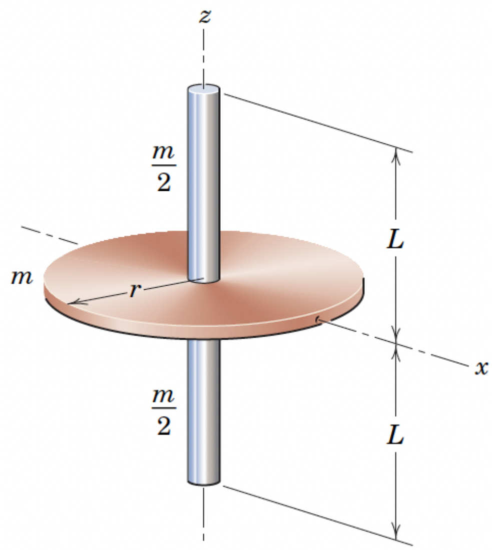
B/29 Determine I_{xx} for the cylinder with a centered circular hole. The mass of the body is m.



PROBLEM B/29

3. [B-032]

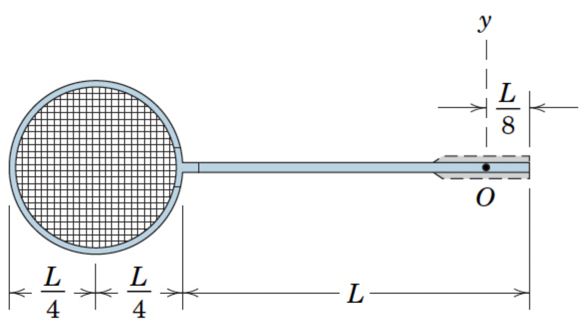
B/32 Determine the length L of each of the slender rods of mass $m/2$ which must be centrally attached to the faces of the thin homogeneous disk of mass m in order to make the mass moments of inertia of the unit about the x- and z-axes equal.



PROBLEM B/32

4. [B-034]

B/34 A badminton racket is constructed of uniform slender rods bent into the shape shown. Neglect the strings and the built-up wooden grip and estimate the mass moment of inertia about the y-axis through O, which is the location of the player's hand. The mass per unit length of the rod material is ρ.



PROBLEM B/34