MECH230 - Section 2
Midterm 1 Formula Sheet

Kinematics in Cartesian Coordinates

$$\begin{aligned} \mathbf{r} &= x\mathbf{E}_x + y\mathbf{E}_y + z\mathbf{E}_z, \\ \mathbf{v} &= v_x\mathbf{E}_x + v_y\mathbf{E}_y + v_z\mathbf{E}_z = \dot{x}\mathbf{E}_x + \dot{y}\mathbf{E}_y + \dot{z}\mathbf{E}_z, \\ \mathbf{a} &= a_x\mathbf{E}_x + a_y\mathbf{E}_y + a_z\mathbf{E}_z = \ddot{x}\mathbf{E}_x + \ddot{y}\mathbf{E}_y + \ddot{z}\mathbf{E}_z. \end{aligned} \tag{1}$$

Rectilinear Motion Consider a rectilinear motion of a particle in the direction of $\mathbf{E}_x$.

$$\begin{aligned} \mathbf{r} &= x\mathbf{E}_x, \\ \mathbf{v} &= v\mathbf{E}_x = \dot{x}\mathbf{E}_x, \\ \mathbf{a} &= a\mathbf{E}_x = \ddot{x}\mathbf{E}_x. \end{aligned} \tag{2}$$

$$a = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx}. \tag{3}$$

Kinematics in Cylindrical Polar Coordinates

$$\begin{aligned} \mathbf{r} &= r\mathbf{e}_r + z\mathbf{E}_z, \\ \mathbf{v} &= \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta + \dot{z}\mathbf{E}_z, \\ \mathbf{a} &= \left(\ddot{r} - r\dot{\theta}^2\right)\mathbf{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\mathbf{e}_\theta + \ddot{z}\mathbf{E}_z, \end{aligned} \tag{4}$$

where

$$\mathbf{e}_r = \cos(\theta)\mathbf{E}_x + \sin(\theta)\mathbf{E}_y, \qquad \mathbf{e}_\theta = -\sin(\theta)\mathbf{E}_x + \cos(\theta)\mathbf{E}_y. \tag{5}$$

Kinematics in the Serret-Frenet Basis

$$v = ||\mathbf{v}|| = \frac{ds}{dt}, \quad \mathbf{e}_t = \frac{\mathbf{v}}{v}, \quad \frac{d\mathbf{e}_t}{ds} = \kappa\mathbf{e}_n, \quad \mathbf{e}_b = \mathbf{e}_t \times \mathbf{e}_n, \quad \frac{d\mathbf{e}_b}{ds} = -\tau\mathbf{e}_n, \quad \rho = \frac{1}{\kappa}. \tag{6}$$

$$\begin{aligned} \mathbf{v} &= v\mathbf{e}_t. \\ \mathbf{a} &= \dot{v}\mathbf{e}_t + \kappa v^2\mathbf{e}_n. \end{aligned} \tag{7}$$

The Balance of Linear Momentum for a particle $\quad \mathbf{F} = \dot{\mathbf{G}}$ where $\mathbf{G} = m\mathbf{v}$.

Spring Forces A spring of stiffness K with unstretched length ℓ_0 whose base is at point A and whose free end is attached to a mass m with position vector $\mathbf{r}$ applies a force on m that is

$$\mathbf{F}_s = -K(||\mathbf{r} - \mathbf{r}_A|| - \ell_0)\frac{\mathbf{r} - \mathbf{r}_A}{||\mathbf{r} - \mathbf{r}_A||}. \tag{8}$$

Friction Forces

- Static friction is unknown but satisfies that static friction criterion $||\mathbf{F}_f|| \leq \mu_s||\mathbf{N}||$.
- Kinetic friction is prescribed according to Coulomb's friction model to be $\mathbf{F}_f = -\mu_k||\mathbf{N}||\frac{\mathbf{v}_{rel}}{||\mathbf{v}_{rel}||}$.