MECH 230 Dynamics
Homework 5

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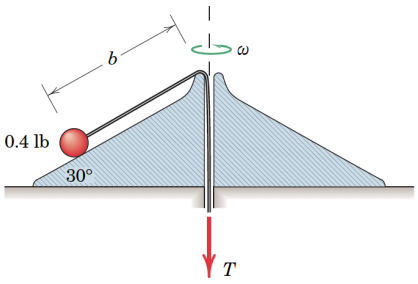
Due Wednesday October 16, 2024

1. Read the problem statement of MKB 3/193.

3/193 **SS** The 0.4-lb ball and its supporting cord are revolving about the vertical axis on the fixed smooth conical surface with an angular velocity of 4 rad/sec. The ball is held in the position $b = 14$ in. by the tension T in the cord. If the distance b is reduced to the constant value of 9 in. by increasing the tension T in the cord, compute the new angular velocity ω and the work $U'_{1\text{-}2}$ done on the system by T.



PROBLEM 3/193

2. Set up a cylindrical-polar coordinate system whose origin is taken to be at the vertex of the cone.
3. The position vector to the object of mass m can be written as

$$\mathbf{r} = r\mathbf{e}_r + z\mathbf{E}_z.$$

   Express r and z in terms of b and the 30° angle.
4. Differentiate the position vector to find the velocity vector of the particle of mass m in terms of b. At the instants considered, before and after the chord is shortened, b is constant.
5. For notational convenience, we also define the following unit vectors in the

$\{\mathbf{e}_r, \mathbf{E}_z\}$ plane.

$$\mathbf{t} = -\cos(30^\circ)\mathbf{e}_r + \sin(30^\circ)\mathbf{E}_z$$
$$\mathbf{n} = \sin(30^\circ)\mathbf{e}_r + \cos(30^\circ)\mathbf{E}_z.$$

Add these vectors to your sketch of the cone.

6. Draw the free body diagram of the particle m and write the expressions of the forces acting on the particle where applicable.
7. Write the Work-Energy theorem for the particle. Is the total mechanical energy of the system conserved?
8. By looking at the moments of the forces, show that the angular momentum of the system in the fixed $\mathbf{E}_z$ direction, $\mathbf{H}_O \cdot \mathbf{E}_z$, is conserved.
9. Calculate $\mathbf{H}_O \cdot \mathbf{E}_z$ and use the previously found conservation to find the angular velocity of the mass after b is shortened.
10. Compute the work done by the tension in the chord $U_{\mathbf{T},1-2}$ on the mass as the chord was shortened.