Assignment 4

Submission through Moodle is due by Sunday, November 5th at 23:55

1. [10 Points] For each CFG G below, describe $L(G)$:

    (a) $S \rightarrow EaS \mid E$
       $E \rightarrow aEb \mid bEa \mid EE \mid \lambda$

    (b) $S \rightarrow aSd \mid A \mid B \mid C$
       $A \rightarrow aAc \mid C$
       $B \rightarrow bBd \mid C$
       $C \rightarrow bCc \mid \lambda$

2. [15 Points] For each of the languages below, prove that the language is context-free:

    (a) $\{vwv^R : v, w \in \{a, b\}^+ \text{ and } |w| = 3\}$.

    (b) $\{w \in \{a, b, c\}^* : |w| = 5n_a(w) + 2n_b(w)\}$.

    (c) $\{r : r \text{ is valid regular expression over the alphabet } \Sigma = \{a, b, c\}\}$.

3. [15 Points] For each pair (G, w) below, draw a derivation tree to show that $w \in L(G)$:

    (a) $G : S \rightarrow aSb \mid bSa \mid SS \mid \lambda$, and $w = aabbbabbaa$.

    (b) $G :$
       $S \rightarrow aSb \mid A \mid B$
       $A \rightarrow aAa \mid C$
       $B \rightarrow bBb \mid C$
       $C \rightarrow bCa \mid \lambda$
       and $w = aaabaabb$.

    (c) $G :$ your answer to Q2(c), and $w = (aa^* + b)^*(c + \lambda)$.

4. [12 Points] Convert the following CFG G into a Chomsky Normal Form (CNF) by following the steps below:

$$S \rightarrow ASBB \mid a \mid b$$
$$A \rightarrow AaA \mid AB \mid \lambda$$
$$B \rightarrow BbB \mid BA \mid \lambda$$
$$C \rightarrow ABB$$

    (a) Remove λ-productions from G.

(b) Remove unit-productions from the grammar in your answer to (a).

(c) Remove useless-productions from the grammar in your answer to (b).

(d) Finish the conversion of the grammar in your answer to (c) to the CNF.