COMP 335 Worksheet
Languages, DFAs and NFAs

1. Prove that if $L_1 \subseteq L_2$, then $L_1^* \subseteq L_2^*$.
2. Let $L = \{w \in \{0,1\}^* \mid n_0(w) \neq n_1(w)\}$ What is L^*?
3. What are the strings in the language $L\emptyset$?
4. Under what circumstances is $L^+ = L^* - \{\lambda\}$?
5. Let $\Sigma = \{a, b\}$. Give some examples of strings that are *in* and *not in* the following languages.
   (a) $\{w \mid \text{for some } u \in \Sigma\Sigma, w = uu^R u\}$
   (b) $\{w \mid ww = www\}$
   (c) $\{w \mid \text{for some } u, v \in \Sigma^*, uvw = wvu\}$
   (d) $\{w \mid \text{for some } u \in \Sigma^*, www = uu\}$
6. Let $\Sigma = \{a, b\}$. Give DFAs for the following languages over Σ. (Note: A *run* in a string is a substring of length at least two, as long as possible, and consisting entirely of the same symbol. For instance, the string *abbbaab* contains a run of b's of length 3 and a run of a's of length 2.)
   (a) $L_1 = \{w \mid w \text{ ends with the string } ab\}$
   (b) $L_2 = \{w \mid w \text{ contains the string } aba\}$
   (c) $L_3 = \{w \mid w \text{ contains exactly one } a\}$
   (d) $L_4 = \{a^n b^m \mid n + m \text{ is even}\}$
   (e) $L_4 = \{a^n b^m \mid n \geq 3, m \text{ is even}\}$
   (f) $L_5 = \{w \mid |w| \bmod 3 = 1\}$
   (g) $L_6 = \{w \mid n_a(w) \bmod 3 > 1\}$
   (h) $L_7 = \{w \mid n_a(w) \bmod 3 > n_b(w) \bmod 3\}$
   (i) $L_8 = \{w \mid w \text{ contains no runs of length} < 4\}$
   (j) $L_9 = \{w \mid w \text{ contains exactly two runs of } a\text{'s of length} < 3\}$
   (k) $L_{10} = \{w \mid w \text{ the second symbol from the end of } w \text{ is a } b\}$
7. Design an NFA with no more than five states for the language $\{abab^n \mid n \geq 0\} \cup \{aba^n \mid n \geq 0\}$.
8. Design an NFA with no more than three states for the language $\{ab, abc\}^*\}$.
9. Design an NFA for the language $\{(ab)^i \mid i \geq 0\} \cup \{(ba)^i \mid i \geq 0\}$.
10. Convert the NFA obtained in the previous question to a DFA.