

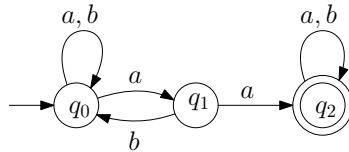
CONCORDIA UNIVERSITY
Dept. of Computer Science and Software Engineering
COMP 335 – Introduction to Theoretical Computer Science
Fall 2023

Assignment 3

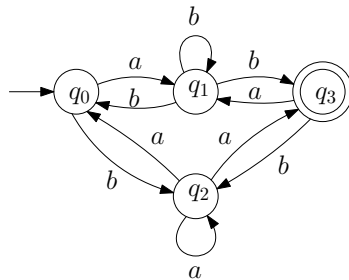
Submission through Moodle is due by Monday, October 16th at 23:55

1. [20 Points] Let $\Sigma = \{a, b\}$. For each of the following languages over Σ , first give a regular expression for it, then give a regular grammar for it.
 - (a) $\{w \in \Sigma^* : n_a(w) \text{ is divisible by 3 and every } a \text{ in } w \text{ is immediately followed by at least one } b\}$
 - (b) $\{w \in \Sigma^* : n_a(w) \text{ is odd and } n_b(w) \text{ is also odd}\}$
2. [20 Points] Let $\Sigma = \{a, b\}$. For each of the following languages over Σ , prove that the language is regular
 - (a) $L_3 = \{w \in \Sigma^* : n_a(w) \text{ is divisible by 4 and } w \text{ ends in } aab\}$.
 - (b) $L_4 = \{w \in \Sigma^* : w \text{ does NOT contain a substring } y \in L_3\}$.
3. [20 Points] Use generalized transition graphs (GTG) to convert the following NFAs (alphabet $\Sigma = \{a, b\}$) into regular expressions. Show your work, i.e., intermediate GTGs.

(a)



(b)



4. [10 Points] Let $\Sigma = \{a, b\}$. Consider the function f defined as follows:

$$\text{For } w \in \Sigma^* \text{ we have } f(w) = a^{n_a(w)}b^{n_b(w)},$$

or, in words, f rearranges input w so that all a 's precede all b 's.

For a language L , we define $f(L) = \{f(w) : w \in L\}$, that is we form $f(L)$ by applying f to every string in L . Consider the following statement:

If L is regular then $f(L)$ is regular.

Clearly, state whether the statement is TRUE or FALSE, and prove your answer.

5. [20 Points] For each of the following languages, prove that it is not regular using the pumping lemma.

(a) $L = \{w \in \{a, b, c\}^* : n_a(w) = n_b(w) + n_c(w)\}.$

(b) $L = \{a^n b^k : n \geq k^3\}$