

COMP 335 Worksheet

Context-free pumping lemma and closure properties

1. Let $\Sigma = \{a, b\}$. Determine if the following languages are context-free, and prove your answers.

- (a) $\{ww \mid w \in (a+b)^*\}$
- (b) $\{a^n b^j \mid n \leq j^2\}$
- (c) $\{a^n b^m c^\ell \mid \ell = nm\}$
- (d) $\{a^n b^j c^k \mid k > n, k > j\}$
- (e) $\{w \in (a+b+c)^* \mid n_a(w) < n_b(w) < n_c(w)\}$
- (f) $\{a^n w w^R a^n \mid n \geq 0, w \in (a+b)^*\}$
- (g) $\{a^n b^m a^\ell b^k \mid n+m = \ell+k\}$
- (h) $\{a^n b^m a^\ell b^k \mid n+\ell = m+k\}$
- (i) $\{a^n b^n c^k \mid n \leq k \leq 2n\}$
- (j) $\{a^n b^n \mid n \text{ is not a multiple of } 5\}$
- (k) $\{a^{n!} \mid n \geq 0\}$

2. Let L_1 be a cfl, L_2 a dcfl and let R be regular. Can we conclude that

- (a) $L_1 - R$ is a cfl?
- (b) $R - L_1$ is a cfl?
- (c) $L_2 \cup R$ is a dcfl?
- (d) $L_2 \cap R$ is a dcfl?