Concordia University

Dept. of Computer Science and Software Engineering

COMP 335 – Introduction to Theoretical Computer Science

Fall 2023

Assignment 6

Submission through moodle is due by Sunday, December 3rd at 23:55

1. [10 points] Let L be a Deterministic CFL over an alphabet Σ. Let $f_1(L) = L_1 = \{w : wa \in L$, for some $a \in \Sigma\}$ and $f_2(L) = L_2 = \{w : aw \in L$ for some $a \in \Sigma\}$. Only one of L_1 and L_2 is guaranteed to be a DCFL. Which one? Justify your answer.

2. [40 points] For each of the following 4 languages L, determine to which of the following categories L belongs: (i) regular (ii) context-free but not regular, and (iii) not context-free. To show that L belongs to category (ii), you need to show two things: (1) L is CF and (2) L is not regular.

   (a) $L_1 = \{wuw^R : \ w, u \in \{a, b\}^*\}$

   (b) $L_2 = \{wuw^R : \ w, u \in \{a, b\}^\star, \ |w| = |u|\}$

   (c) $L_3 = \{w_1 c w_2 : \ w_1 \neq w_2^R, \ w_1, w_2 \in \{a, b\}^*\}$

   (d) $L_4 = \{a^n b^j c^k d^l : \ n \leq k, \ j \leq l\}$

3. [20 points] Design standard Turing machines that *accept* the following languages. In each case, draw the transition diagram of your TM and give a brief description in English of your design strategy.

   (a) $L_5 = \{ww : \ w \in \{a, b\}^\star\}$

   (b) $L_6 = \{wuw^R : \ w, u \in \{a, b\}^\star, \ |w| = |u|\}$

4. [20 points] Draw transition diagrams for standard Turing machines that *compute* the following functions. In each case, briefly describe in English your design strategy.

   (a) $f_1(1^n 0 1^m) = 1^{|m-n|}$

   (b) $f_2(0^n) = 0^k$, where $k \equiv n (\text{mod} 3)$