Exercise 6.1

In the lecture, we saw that the correlation-based methods can be interpreted as a feedforward comb filter. This filter is given by

$$e(n) = x(n) - ax(n-\tau) \tag{1}$$

where $x(n)$ is the input signal and $e(n)$ is the output signal.

(a) Compute the frequency response and the amplitude response. Sketch the latter for a delay of τ, 2τ, and 3τ. What is the consequence of this for pitch estimation?

Solution: If we set $x(n) = \mathrm{e}^{\mathrm{j}\omega n}$, we obtain

$$e(n) = \mathrm{e}^{\mathrm{j}\omega n} - a\mathrm{e}^{\mathrm{j}\omega(n-\tau)} = \left[1 - a\mathrm{e}^{-\mathrm{j}\omega\tau}\right]\mathrm{e}^{\mathrm{j}\omega n} \ . \tag{2}$$

Thus,

$$H(\omega) = 1 - a\mathrm{e}^{-\mathrm{j}\omega\tau} \tag{3}$$

is the frequency response of the feedforward comb filter. If we use Euler's formula given by

$$\mathrm{e}^{\mathrm{j}\theta} = \cos(\theta) + \mathrm{j}\sin(\theta) \ , \tag{4}$$

we can also write the frequency response as

$$H(\omega) = 1 - a\cos(\omega\tau) + \mathrm{j}a\sin(\omega\tau) \ . \tag{5}$$

The amplitude response is the magnitude of this complex number and given by

$$|H(\omega)| = \sqrt{(1 - a\cos(\omega\tau))^2 + (a\sin(\omega\tau))^2} = \sqrt{1 + a^2 - 2a\cos(\omega\tau)} \ . \tag{6}$$

From the amplitude response, we see that it bounded by

$$1 - a \leq |H(\omega)| \leq 1 + a \tag{7}$$

when $a > 0$. The amplitude response attains the upper bound when

$$\cos(\omega\tau) = -1 \quad \Leftrightarrow \quad \omega = (2k+1)\pi/\tau \qquad \text{for } k = 0, \pm 1, \pm 2 \cdots \tag{8}$$

Conversely, the amplitude response attains the upper bound when

$$\cos(\omega\tau) = 1 \quad \Leftrightarrow \quad \omega = 2k\pi/\tau \qquad \text{for } k = 0, \pm 1, \pm 2 \cdots \tag{9}$$

The amplitude spectra for a delay of τ (red) and 2τ (blue) are sketched in Fig. 1.

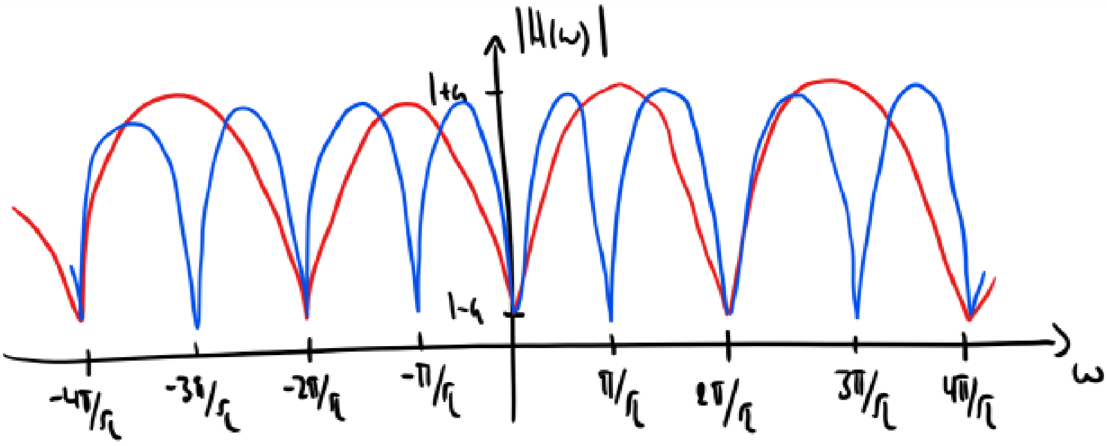

Figure 1: Amplitude spectra of a feedforward comb filter for a delay of τ (red) and 2τ (blue).

Exercise 6.2

On Moodle, you can download the viola signal `09viola.flac` and the speech signal `roy.wav`. We would like to estimate the fundamental frequency/pitch of these signals.

(a) Implement the comb filtering pitch estimation method as a function in, e.g., MATLAB. The function should have the following input:

- a segment of data and
- the lower and upper limits for the fundamental frequency in cycles/sample.

The output of the function should be the estimated fundamental frequency.

Solution: See the MATLAB file `combFilterPitchEstimator.m`.

The above function can estimate the fundamental frequency for a segment of data. We now wish to analyse entire audio files.

(b) Write a function that takes in an audio file and displays the estimated fundamental frequency track in cycles/second (Hz). The function should have the following input

- Filename of the audio file,
- the segment length in seconds,
- the overlap between segments as a percentage,
- the lower and upper limits for the fundamental frequency in cycles/sample

The output of the function should be the estimated fundamental frequencies as a function of time.

Solution: See the MATLAB file `extractPitchTrack.m`.