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Subject: Signal and Systems Laboratory

Subject Code: CSS453

Lab 5: Laplace Transforms

Common import statements and declarations for all code snippets

```
import sympy
sympy.init_printing()

import sympy.plotting as plt
%matplotlib inline

from IPython.display import display, Math

import numpy as np

t, s = sympy.symbols('t, s', real = True)
a = sympy.symbols('a', real=True, positive=True)
```

1. Find the Laplace transform of the following:

(a) $y = t^2$

```
f = t * t
display(sympy.laplace_transform(f, t, s)[0])
```

Output:

> ans=
>
> $\frac{2}{s^3}$

(b) $y = e^{-at} + e^{-3at}$

```
f = sympy.exp(-a*t)+sympy.exp(-3*a*t)
display(sympy.laplace_transform(f, t, s)[0])
```

Output:

> ans =
>
> $\frac{1}{(a+s)} + \frac{1}{(3a+s)}$

(c) $y = e^{2at}\sin{(2t)}$

```
f = sympy.exp(2*t)*sympy.sin(2*t)
display(sympy.laplace_transform(f, t, s)[0])
```

Output:

> ans =
>
> $\frac{2}{((s-2)^2+4)}$

(d) $y = e^{3t} + \cos 6t - e^{-3t}\cos 6t$

```
f = sympy.exp(3*t) + sympy.cos(6*t) - sympy.exp(-3*t)*sympy.cos(6*t);
display(sympy.laplace_transform(f, t, s)[0])
```

Output:

> ans =
>
> $\frac{1}{(s-3)} - \frac{(s+3)}{((s+3)^2+36)} + \frac{s}{(s^2+36)}$

(e) $y = u(t-2) + 2u(t-3) - 2r(t-2)$

```
f = sympy.Heaviside(t-2) + 2*sympy.Heaviside(t-3) - 2*(t-3)*sympy.Heaviside(t-3)
display(sympy.laplace_transform(f, t, s)[0])
```

Output:

> ans =
>
> $\frac{(se^s+2s-2)e^{-3s}}{s}$

2. Consider the two functions $f(t) = u(t)u(3-t)$ **and** $g(t) = u(t) - u(t-3)$

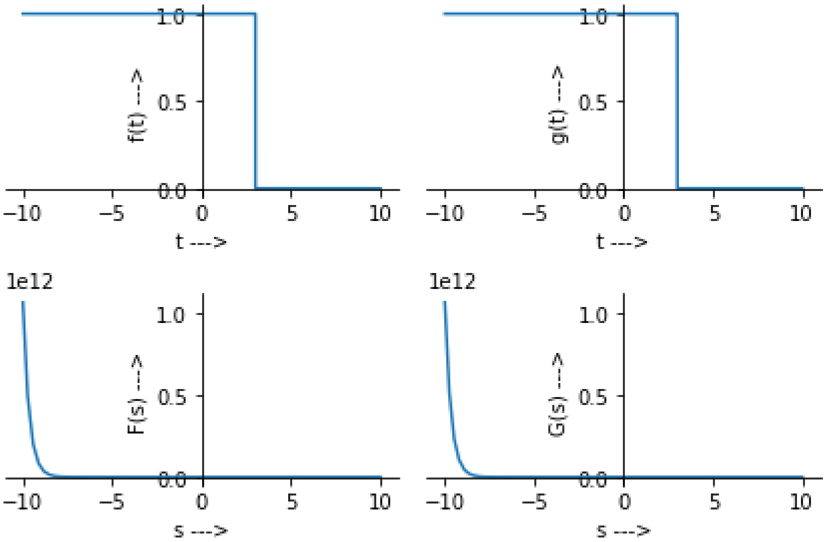
(a) Are the two functions identical

(b) Show that $L[f(t)] = L[g(t)]$

```
f = sympy.Heaviside(t)*sympy.Heaviside(3-t)
p1 = plt.plot(f, xlabel='t --->', ylabel='f(t) --->', show=False)
g = sympy.Heaviside(t) - sympy.Heaviside(t-3)
p2 = plt.plot(g, xlabel='t --->', ylabel='g(t) --->', show=False)
F = sympy.laplace_transform(f, t, s)[0]
G = sympy.laplace_transform(g, t, s)[0]
p3 = plt.plot(F, xlabel='s --->', ylabel='F(s) --->', show=False)
p4 = plt.plot(G, xlabel='s --->', ylabel='G(s) --->', show=False)
plt.PlotGrid(2, 2, p1, p2, p3, p4)
display(F)
display(G)
```

Output:

> $\frac{1-e^{-3s}}{s}$
>
> $\frac{1}{s} - \frac{e^{-3s}}{s}$



From plots 1 and 2 it is evident that the two functions are identical

From plots 3 and 4 it can be proved that Laplace transforms of both f and g are equal

3. Find the Laplace transform of:

$$f(t) = \begin{cases} 0 & 0 \leq t < \ 1 \\ t-1 & 1 \leq t < 2 \\ 0 & 2 \leq t \end{cases}$$

```
f = (t-1)*sympy.Heaviside(t-1) + (1-t)*sympy.Heaviside(t-2)
display(sympy.laplace_transform(f, t, s)[0])
```

Output:

> $\frac{(-s+e^{s}-1)e^{-2s}}{s^2}$

4. Find the inverse laplace of:

(a) $F(s) = \frac{1}{s}$

```
F = 1/s
display(sympy.inverse_laplace_transform(F, s, t))
```

Output:

> ans=
>
> 1

(b) $F(s) = \frac{10}{s^2+25} + \frac{4}{s-3}$

```
F = 10/(s**2 + 25) + 4/(s-3)
display(sympy.inverse_laplace_transform(F, s, t))
```

Output:

> ans=
>
> $4e^{3t} + 2sin(5t)$

(c) $F = \frac{e^{-3s}(2s+7)}{s^2+16}$

```
F = sympy.exp(-3*s)*(2*s + 7)/(s**2 + 16)
display(sympy.inverse_laplace_transform(F, s, t))
```

Output:

> ans =
>
> $2sin(12)sin(4t) + \frac{7sin(4t)cos(12)}{4} - \frac{7sin(12)cos(4t)}{4} + 2cos(12)cos(4t)$

(d) $F = \frac{s^2+5s-3}{(s^2+16)(s-2)}$

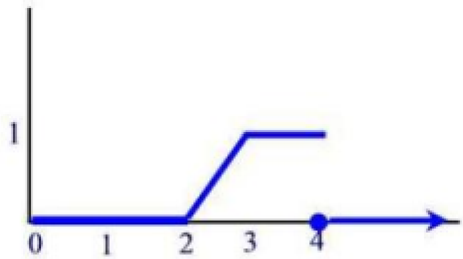
```
F = (s**2 + 5*s -3)/((s**2 +16)*(s-2))
display(sympy.inverse_laplace_transform(F, s, t))
```

Output:

> ans =
>
> $\frac{9cos(4t)}{20} + \frac{11e^{2t}}{20} + \frac{59sin(4t)}{40}$

5. The graph of $f(t)$ is given below. Represent $f(t)$ as a combination of Heaviside step functions

calculate the Laplace transform of $f(t)$.




```
f = (t-2)*sympy.Heaviside(t-2) + (2-t)*sympy.Heaviside(t-4)
display(sympy.laplace_transform(f, t, s)[0])
```

Output:

> ans =
>
> $\frac{(-2s+e^{2s}-1)e^{-4s}}{s^2}$

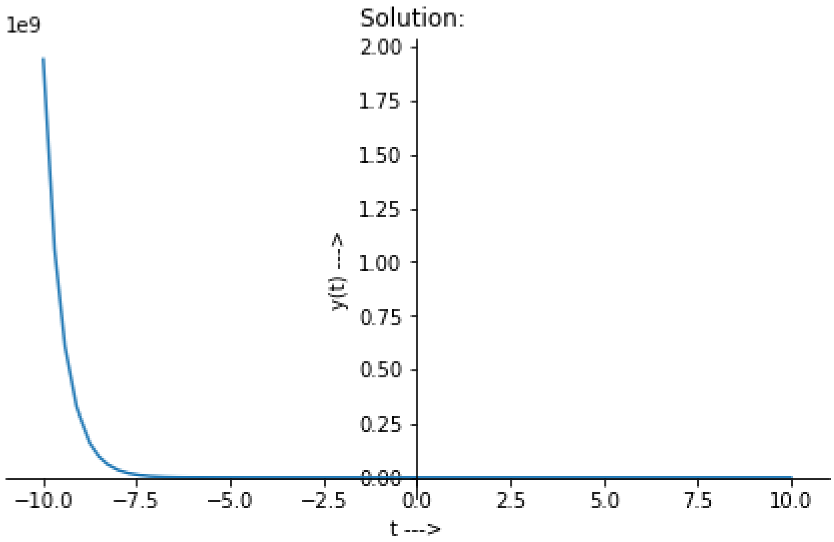
6. Solve the initial value problem using Laplace transform:

(a) $y' + 2y = 4t \quad , \quad y(0) = 3$

```
t, s, Y = sympy.symbols('t, s, Y', positive=True)
f = 4*t
F = sympy.laplace_transform(f, t, s)[0]
Y1 = s*Y - 3
Sol = sympy.solvers.solve(Y1 + 2*Y - F, Y)
y = sympy.inverse_laplace_transform(Sol[0], s, t)
display(y)
plt.plot(y, xlabel='t --->', ylabel='y(t) --->', title='Solution: ')
```

Output:

> y =
>
> $2t + 4e^{-2t} - 1$

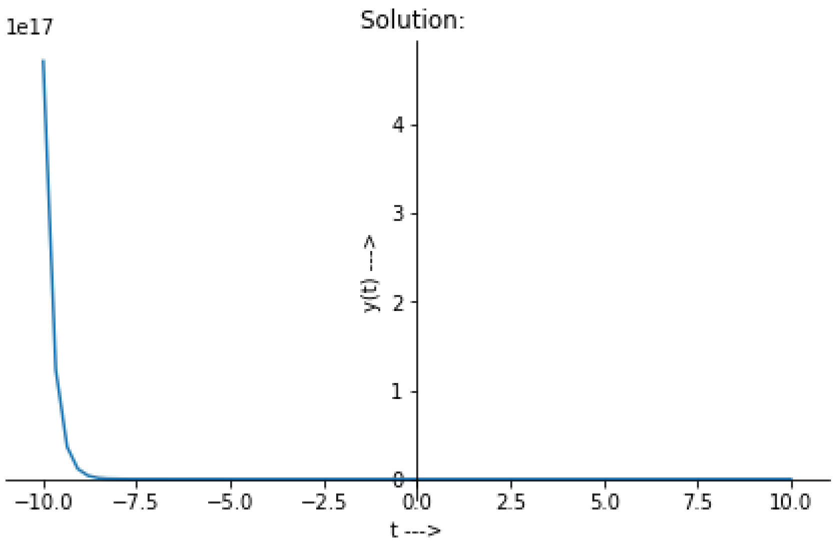


(b) $y'' + 3y' + 2y = 6e^{-}t, \quad y(0) = 1, \quad y'(0) = 2$

```
t, s, Y = sympy.symbols('t, s, Y', positive=True)
f = (12*sympy.Heaviside(t-1) - 12*sympy.Heaviside(t-3))
F = sympy.laplace_transform(f, t, s)[0]
Y1 = s*Y -2
sol = sympy.solvers.solve(Y1 + 4*Y - F, Y)[0]
y = sympy.inverse_laplace_transform(sol, s, t)
display(y)
plt.plot(y, xlabel='t --->', ylabel='y(t) --->', title='Solution: ')
```

Output:

> y =
>
> $3e^{-2t} - 2e^{-t} + 6te^{-t}$



(c)

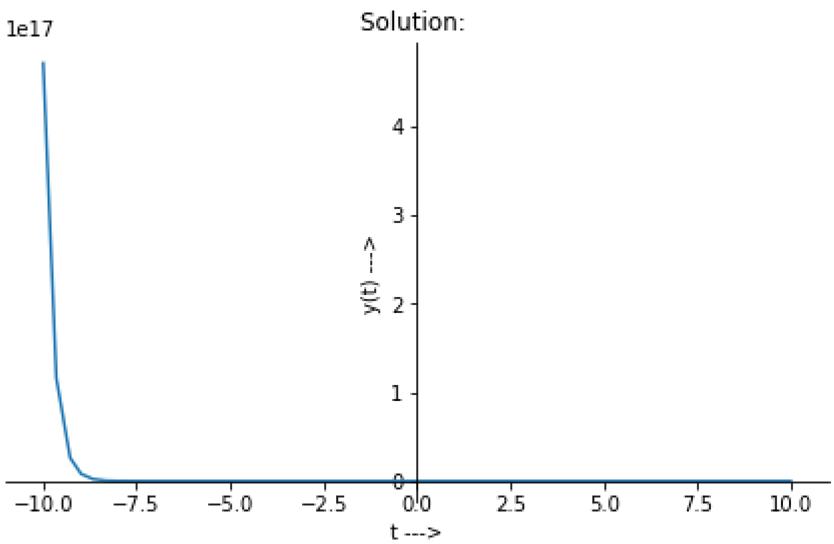
$$y' + 4y + 2y = g(t), \quad y(0) = 2, \text{ where } g(t) = 12 \quad \text{if , } (1 < t < 3) \quad \text{else , } 0$$

```
t, s, Y = sympy.symbols('t, s, Y', positive=True)
f = (12*sympy.Heaviside(t-1) - 12*sympy.Heaviside(t-3))
F = sympy.laplace_transform(f, t, s, noconds=True)
Y1 = s*Y - 2
sol = sympy.solvers.solve(Y1 + 4*Y - F, Y)
y = sympy.inverse_laplace_transform(sol[0], s, t)
display(y)
plt.plot(y, xlabel='t --->', ylabel='y(t) --->', title='Solution: ')
```

Output:

> y =
>
> $$\left(-3e^{4t}\theta(t-3) + 3e^{4t}\theta(t-1) + 3e^{12}\theta(t-3) - 3e^{4}\theta(t-1) + 2\right)e^{-4t}$$

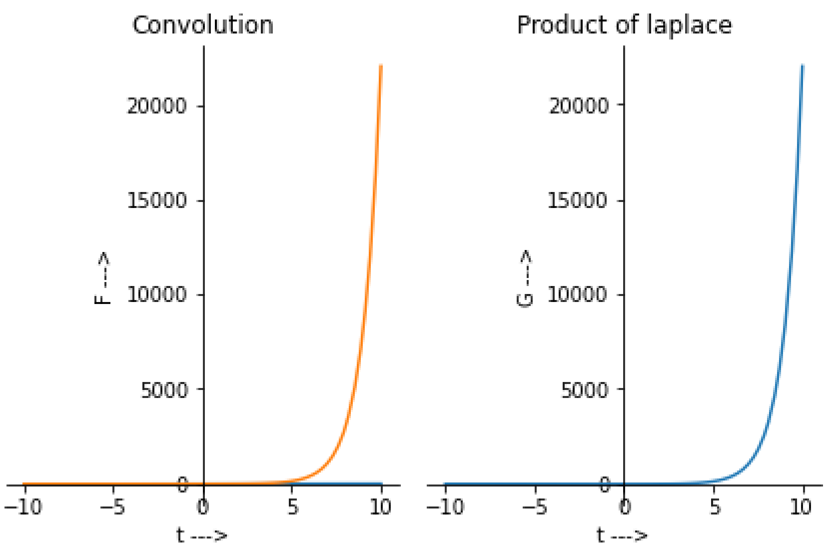
Here $\theta(t)$ is `sympy.Heaviside(t)`



7. Verify that multiplication in s domain is equivalent to convolution in time domain.

```
t, s, a = sympy.symbols('t, s, a', positive=True)
x = sympy.symbols('x')
f = t
f1 = sympy.exp(t)
F = sympy.integrate(f.subs(t, x)*f1.subs(t, t-x),(x, 0, oo))
G = sympy.inverse_laplace_transform(sympy.laplace_transform(f, t, s,
noconds=True)*sympy.laplace_transform(f1, t, s, noconds=True), s, t,
noconds=True)
p1 = plt.plot(t, F.subs({a:4}), xlabel='t --->', ylabel='F --->',
title='Convolution', show=False)
p2 = plt.plot(G.subs({a:4}), xlabel='t --->', ylabel='G --->', title='Product of
laplace', show=False)
plt.PlotGrid(1, 2, p1, p2)
```

Output:



We can see that the graphs are same, hence they are equal.