Exercise Sheet Tutorial V.3

1. Consider a pendulum of length l with mass m, as shown in Figure 1. A gravitational field of uniform acceleration g is acting on the mass.

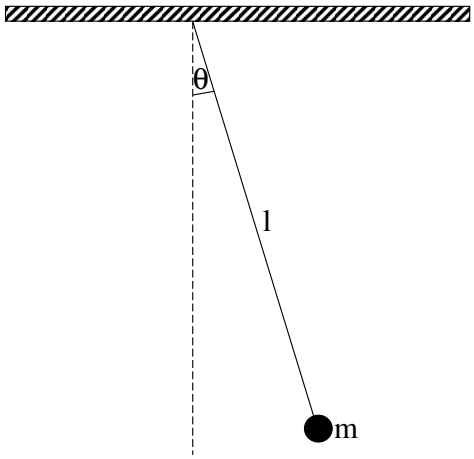

Figure 1: A simple pendulum of length l and mass m. θ denotes the angle between the pendulum and the vertical.

   - Verify that the potential energy of the pendulum is given by $E_{pot} = mgl(1 - \cos\theta)$, where θ denotes the angle between the pendulum and the vertical as shown in Figure 1.
   - Based on the generalized coordinate θ, write down the Lagrangian of the system and the equation of motion using the Lagrangian formalism.
   - Show that the conjugate momentum of θ is $p_\theta = ml^2\dot{\theta}$, write down the Hamiltonian of the system and the equations of motion using the Hamiltonian formalism.

2. A C-program `velocityVerletSho.c` is provided which integrates the equation of motion for the simple harmonic oscillator according to the Velocity-Verlet algorithm. For simplicity, the mass and force constant are defined as `MASS=1` and `K=1`, respectively. The initial conditions are defined as `X0=1` (coordinate) and `V0=0` (velocity). The number of integration steps is `NSTEPS` and the time step size is given by `deltat = Tper / TFAC`, where `Tper` is the period of oscillation

and `TFAC` is the number of time steps per period of oscillation. The program can be compiled with

```
gcc -o velocityVerletSho velocityVerletSho.c
```

and run with

```
./velocityVerletSho
```

For each time step, it prints the time in units of period, coordinate, momentum, total energy

$$E_{tot} = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 \tag{1}$$

The output can be written to a file `out.dat` with

```
./velocityVerletSho > out.dat
```

That is, in this file, there are five columns which report the time in units of period, coordinate, momentum, total energy and the energy of the shadow Hamiltonian, respectively.

- Look at the program and verify the equation of motion of the simple harmonic oscillator, as well as the equations of the Velocity-Verlet algorithm.
- Edit, compile and run the program for variables `TFAC=10`, `50` and `100` and write corresponding output data to files `out10.dat`, `out50.dat` and `out100.dat`, respectively.
- Make phase space plots for the three above runs, *i.e.* plot the momentum *vs.* the coordinate. Interpret.
- Plot the energy *vs.* time. Interpret.