Time Integration property. [Fourier Transform]

When a periodic signal $x(t)$ is integrated, Then the Fourier Transform representation becomes,

$$\int_{-\infty}^{t} x(t)\,dt \xleftrightarrow{FT} \frac{1}{jw} X(jw)$$

Note: Integration has finite values and is periodic only if $a_0 = 0$

Proof:

The inverse FT is given by.

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(jw)\, e^{jwt}\, dw$$

Integrating the signal,

$$\int_{-\infty}^{t} x(t)\,dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(jw)\, \frac{e^{jwt}}{jw}\, dw$$

$$= \frac{1}{jw} \frac{1}{2\pi} \int_{-\infty}^{\infty} X(jw)\, e^{jwt}\, dw$$

$$= \frac{1}{jw} X(jw).$$