

## Assignment HW9 due 10/08/2022 at 01:00pm SAST

**Problem 1. (1 point)**

Eliminate the parameter in  $x = -t^2 + 4t$  and  $y = t - 3$  and then identify the parametric curve and sketch its image in the  $xy$ -plane on a piece of paper.

Equation: \_\_\_\_\_

Image in the  $xy$ -plane:

- Choose
- Circle
- Semicircle opening up
- Semicircle opening down
- Semicircle opening right
- Semicircle opening left
- Ellipse
- Ellipse opening up
- Ellipse opening down
- Ellipse opening right
- Ellipse opening left
- Hyperbola opening up
- Hyperbola opening down
- Hyperbola opening right
- Hyperbola opening left
- Parabola opening up
- Parabola opening down
- Parabola opening right
- Parabola opening left

**Problem 2. (1 point)**

Find the length of the curve defined by the parametric equations

$$x = \frac{5}{3}t, \quad y = 5 \ln((t/3)^2 - 1)$$

from  $t = 6$  to  $t = 8$ .

**Problem 3. (1 point)**

Find a Cartesian equation relating  $x$  and  $y$  corresponding to the parametric equations

$$x = 5 \sin(5t) \quad y = 9 \cos(5t)$$

Write your answer in the form

$$P(x, y) = 0$$

where  $P(x, y)$  is a polynomial in  $x$  and  $y$  such that the coefficient of  $y^2$  is 25.

Answer: \_\_\_\_\_ = 0

Find the equation of the tangent line to the curve at the point corresponding to  $t = \pi/15$ .

Answer:  $y =$  \_\_\_\_\_

**Problem 4. (1 point)**

Consider the curve given by the parametric equations

$$x = t(t^2 - 48), \quad y = 2(t^2 - 48)$$

a.) Determine the point on the curve where the tangent is horizontal.

$t =$  \_\_\_\_\_

b.) Determine the points  $t_1, t_2$  where the tangent is vertical and  $t_1 < t_2$ .

$t_1 =$  \_\_\_\_\_

$t_2 =$  \_\_\_\_\_

**Problem 5. (1 point)**

Consider the following parametric equation.

$$x = 15(\cos \theta + \theta \sin \theta)$$

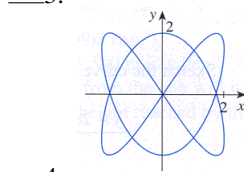
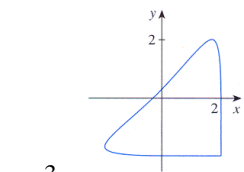
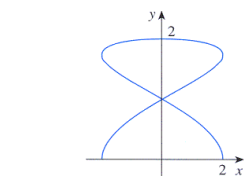
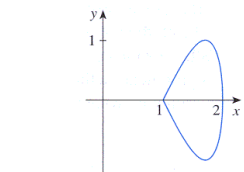
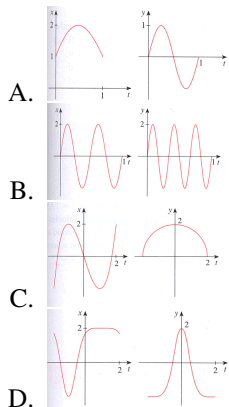
$$y = 15(\sin \theta - \theta \cos \theta)$$

What is the length of the curve for  $\theta = 0$  to  $\theta = \frac{9}{8}\pi$ ?

Answer: \_\_\_\_\_

**Problem 6. (1 point)**

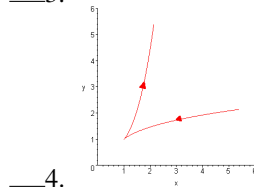
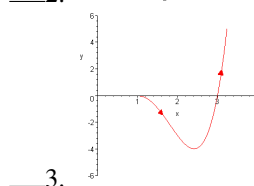
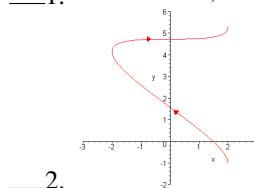
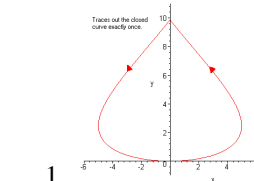
Match the graphs of the parametric equations  $x = f(t)$  and  $y = g(t)$  in A-D with the parametric curves in 1-4.



**Problem 7. (1 point)**

Below you are given four parametric equations and their plots. Match each plot to the correct set of parametric equations. (Note: Values along the axes are given in Cartesian coordinates.)

- A.  $x = 1 + \sqrt{t}$ ,  $y = t^2 - 4t$ ,  $0 \leq t \leq 5$   
 B.  $x = 2\cos(t)$ ,  $y = t - \cos(t)$ ,  $0 \leq t \leq 2\pi$   
 C.  $x = 5\sin(t)$ ,  $y = t^2$ ,  $-\pi \leq t \leq \pi$   
 D.  $x = e^{-t} + t$ ,  $y = e^t - t$ ,  $-2 \leq t \leq 2$



**Problem 8. (1 point)**

Use the parametric equations of an ellipse

$$x = a\cos(\theta), \quad y = b\sin(\theta), \quad 0 \leq \theta \leq 2\pi,$$

to find the area that it encloses.

Area = \_\_\_\_\_

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**Problem 9. (1 point)**

Consider the parametric curve given by

$$x = t - e^t, \quad y = 4t + 4e^{-t}$$

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(a) Find  $dy/dx$  and  $d^2y/dx^2$  in terms of  $t$ .

$$dy/dx = \underline{\hspace{2cm}}$$

$$d^2y/dx^2 = \underline{\hspace{2cm}}$$

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(b) Using "less than" and "greater than" notation, list the  $t$ -interval where the curve is concave upward.

Use upper-case "INF" for positive infinity and upper-case "NINF" for negative infinity. If the curve is never concave upward, type an upper-case "N" in the answer field.

$t$ -interval:  $\underline{\hspace{1cm}} < t < \underline{\hspace{1cm}}$

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**Problem 10. (1 point)**

Which of the following integrals represents the area of the surface obtained by rotating the parametric curve  $x = t - t^2$ ,  $y = \frac{4}{3}t^{3/2}$ ,  $1 \leq t \leq 2$ , about the  $x$ -axis?

- A.  $\int_1^2 2\pi(t - t^2) \sqrt{1 - 4t + 4t^2} dt$
- B.  $\int_1^2 \frac{8\pi}{3} t^{3/2} \sqrt{1 - 2t + 2t^{1/2}} dt$
- C.  $\int_1^2 2\pi(t - t^2) \sqrt{1 + 4t^2} dt$
- D.  $\int_1^2 \frac{8\pi}{3} t^{3/2} \sqrt{1 + 4t^2} dt$
- E.  $\int_1^2 \frac{8\pi}{3} t^{3/2} \sqrt{1 - 4t + 4t^2} dt$
- F.  $\int_1^2 2\pi(t - t^2) \sqrt{1 - 2t + 2t^{1/2}} dt$

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**Problem 11. (1 point)**

Find the area of the surface obtained by rotating the curve of parametric equations

$$x = 10\cos^3 \theta, \quad y = 10\sin^3 \theta, \quad 0 \leq \theta \leq \pi/2$$

about the  $y$  axis.

Surface area =  $\underline{\hspace{2cm}}$