Transient Modeling of Flywheel-Based Pitchers

Thaddeus Hughes
hughes.thad@gmail.com
thaddeus-maximus.github.io

May 23, 2020

Abstract

Flywheel based pitchers are very common ways to propel projectiles. They are well-suited to many such cases in that they are reasonably light, do not require keeping track of states, provide inherent strength-of-shot adjustment, and can be used in continous applications.

Unfortunately, they are somewhat of a mystery, with design essentially relegated to trial-and-error rather than intuitive understanding or numerical simulation. This paper aims to change that.

Hooded Pitcher

Consider the following *hooded* pitcher. A wheel (red) is spun up to an initial RPM, and then a ball (blue) is introduced between it and an arced hood (black), which is concentric with the wheel.

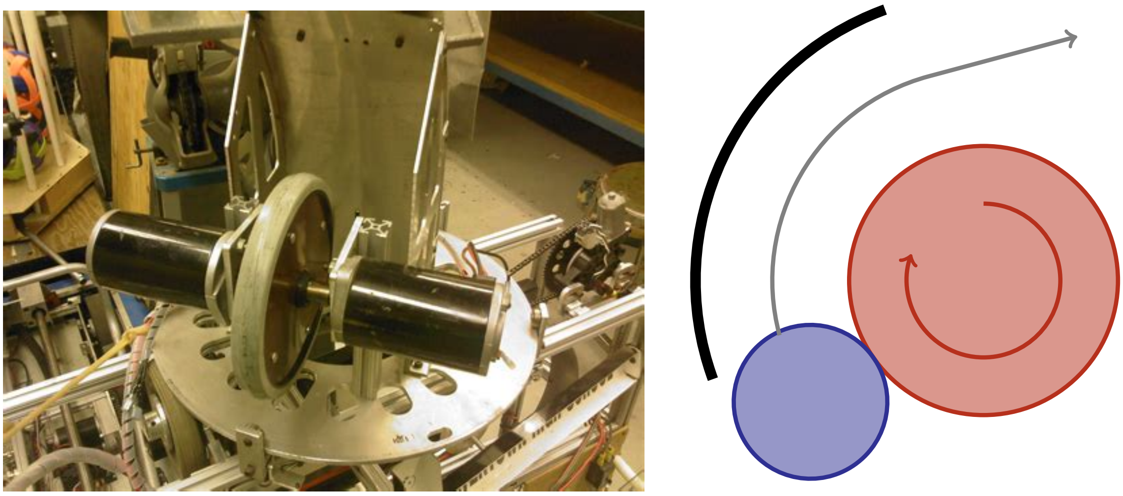

Figure 1: Hooded pitcher: example and schematic.

There are already many parameters of interest in this system.

d_w, the wheel diameter

I_w, the wheel's moment of inertia

$\omega_{w,0}$, the wheel's initial velocity

d_b, the ball's initial diameter

I_b, the ball's moment of inertia

w, the gap between the hood and the surface of the wheel

θ, the angle between inlet and outlet

What makes this work? The ball is accelerated as the wheel makes contact with the ball. The hood provides constraint, keeping the ball from simply spinning. Free-body diagrams may make this more clear.

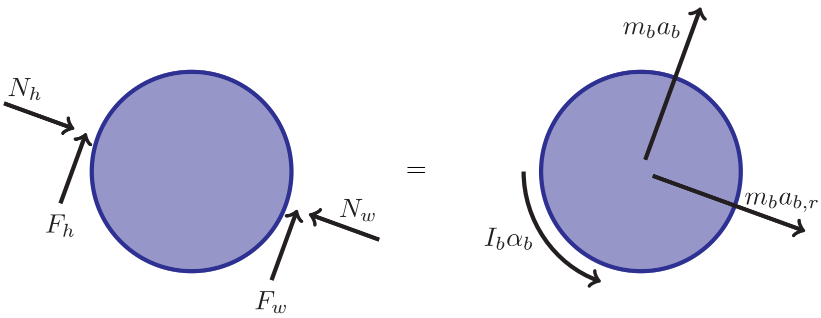

Figure 2: Free-Body and Kinetic Diagrams for Ball

For the sake of simplicity, gravity is neglected during this split-second interaction. This leads to force balances

$$F_h + F_w = m_b a_b \tag{1}$$

$$N_h - N_w = m_b a_{b,r} = m_b \frac{v_b^2}{R} \tag{2}$$

$$(F_w - F_h)\frac{w}{2} = I_b \alpha_b, \tag{3}$$

where R is the radius of the path travelled by the ball ($R = \frac{d_w + w}{2}$).

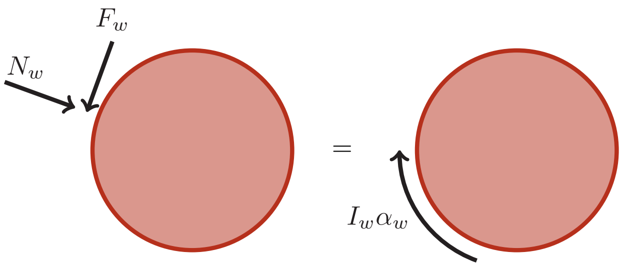

Figure 3: Free-Body and Kinetic Diagrams for Wheel

This leads to the torque balance

$$-N_w \frac{d_w}{2} = I_w \alpha_w \tag{4}$$

At this point we have four governing equations, four unknown forces, and four unknown kinematic properties. Let's recap and solve what we have so far to get expressions for the accelerations.

$$\alpha_b = (F_w - F_h)\frac{w}{2I_b} \tag{5}$$

$$a_b = \frac{F_h + F_w}{m_b} \tag{6}$$

$$\alpha_w = -F_w \frac{d_w}{2I_w} \tag{7}$$

Where does the normal force come from? Well, it comes from the ball's compression! But the ball is also accelerating along the axis of the compression, making this not quite straightforward.

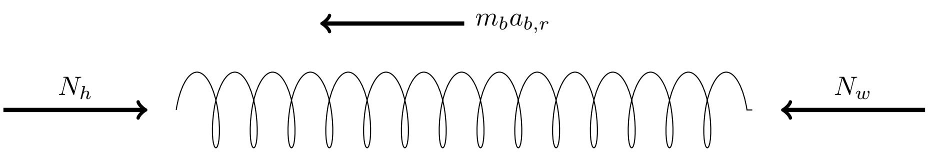

Figure 4: Massed spring model of the ball.

The inertial forces can be resolved in such a spring of stiffness k_b squeezed by δ as so:

$$N_h = k_b\delta + \frac{m_b a_{b,r}}{2} = k_b\delta + \frac{m_b v_b^2}{2R} \tag{8}$$

$$N_w = k_b\delta - \frac{m_b a_{b,r}}{2} = k_b\delta - \frac{m_b v_b^2}{2R} \tag{9}$$

Side takeaway: As the ball accelerates through the hood, it lifts off the wheel. This effect is magnified with a smaller arc radius.

We can relate the normal forces to the frictional or tangential forces with a basic coloumb friction model. More sophisticated models are probably more accurate, but the intent of this model is to provide intuitively understandable results and general guidelines which should be followed up by real-world testing, so this is an appropriate model.

$$F_h \leq \mu_h N_h \; sign(0 - v_{b,top}) \tag{10}$$
$$F_w \leq \mu_w N_w \; sign(v_{w,surf} - v_{b,bottom}) \tag{11}$$

Anyone who's simulated anything can immediately see the glaring problem here- sign functions have glaring discontinuities. There are many strategies to solve this. Here's what I've come up with.

First, recognize that the sign associated with the wheel should either be positive or zero. The ball isn't going to start over-spinning with the wheel. Second, recognize that the spin of the ball should never exceed the speed of the ball such that the top of the ball travels backwards, so its sign should be negative or zero. Third, the sign becomes zero, the condition to calculate is no longer one of two surfaces at arbitrary velocities exchanging forces, but of two surfaces that have *coupled*. This brings the driving physics away from force balance and towards pure kinematics. That is to say,

$$\begin{aligned} v_{w,surf} &= v_{b,bottom} \\ a_{w,surf} &= a_{b,bottom} \\ \alpha_w \frac{d_w}{2} &= a_b + \alpha_b \frac{w}{2} \end{aligned} \tag{12}$$

and

$$\begin{aligned} 0 &= v_{b,top} \\ 0 &= a_{b,top} \\ 0 &= a_b - \alpha_b \frac{w}{2}. \end{aligned} \tag{13}$$

Substituting what we found earlier about the accelerations gives us

$$\begin{aligned} -F_w \frac{d_w}{2I_w} \frac{d_w}{2} &= \frac{F_h + F_w}{m_b} + (F_w - F_h) \frac{w}{2I_b} \frac{w}{2} \\ F_w \frac{d_w}{2I_w} + \frac{F_w}{m_b} + F_w \frac{w}{2I_b} \frac{w}{2} &= -\frac{F_h}{m_b} + F_h \frac{w}{2I_b} \frac{w}{2} \\ F_w &= F_h \frac{\frac{w^2}{4I_b} - \frac{1}{m_b}}{\frac{d_w}{2I_w} + \frac{1}{m_b} + \frac{w^2}{4I_b}} \end{aligned} \tag{14}$$

in the case of no wheel slip, and

$$\begin{aligned} 0 &= \frac{F_h + F_w}{m_b} - (F_w - F_h) \frac{w}{2I_b} \frac{w}{2} \\ \frac{F_h}{m_b} + F_h \frac{w}{2I_b} \frac{w}{2} &= -\frac{F_w}{m_b} + F_h \frac{w}{2I_b} \frac{w}{2} \\ F_h &= F_w \frac{\frac{w^2}{4I_b} - \frac{1}{m_b}}{\frac{w^2}{4I_b} + \frac{1}{m_b}} \end{aligned} \tag{15}$$

in the case of no hood slip. In these cases, the force of one is directly proportional to the other. This means that when both hood and wheel stop slipping,

$$F_h = F_w = 0 \tag{16}$$

But how is state determined? We simply go back to the sign equations.

$$\textit{top attached when } 0 \leq v_b - \omega_b \frac{w}{2} \tag{17}$$

$$\textit{bottom attached when } \omega_w \frac{d_w}{2} \leq v_b + \omega_b \frac{w}{2} \tag{18}$$

This developed friction model can be summarized with this pseudocode, which would be ran every iteration of the simulation loop.

$Fw \leftarrow mw\ Nw$
$Fh \leftarrow mh\ Nh$
if *hood attached* and *wheel attached* **then**
  $Fw \leftarrow 0$
  $Fh \leftarrow 0$
else if *wheel attached* **then**
  $Fw \leftarrow$ *Fw in attached state*
else if *hood attached* **then**
  $Fh \leftarrow$ *Fh in attached state*

... insert interesting simulation code ...

if *ball top speed goes positive* **then**
  hood attached $\leftarrow$ true
if *ball bottom speed exceeds wheel speed* **then**
  wheel attached $\leftarrow$ true

All that's left is to set up the state equations and initial conditions.

$$\frac{d}{dt}\omega_b = \alpha_b \tag{19}$$

$$\frac{d}{dt}v_b = a_b \tag{20}$$

$$\frac{d}{dt}u_b = v_b \tag{21}$$

$$\frac{d}{dt}\omega_w = \alpha_w \tag{22}$$

$$v_b(0) = 0 \tag{23}$$

$$u_b(0) = 0 \tag{24}$$

$$\omega_w(0) = \omega_{w,0} \tag{25}$$

$$\textit{when } u_b \geq \theta R \textit{ terminate} \tag{26}$$

Dual-Wheel Pitchers

Dual wheel pitchers are a little more complex as the effective gap is not constant during the ball's motion, and the wheel's normal forces contribute to ball acceleration. However, the basic strategy remains the same.

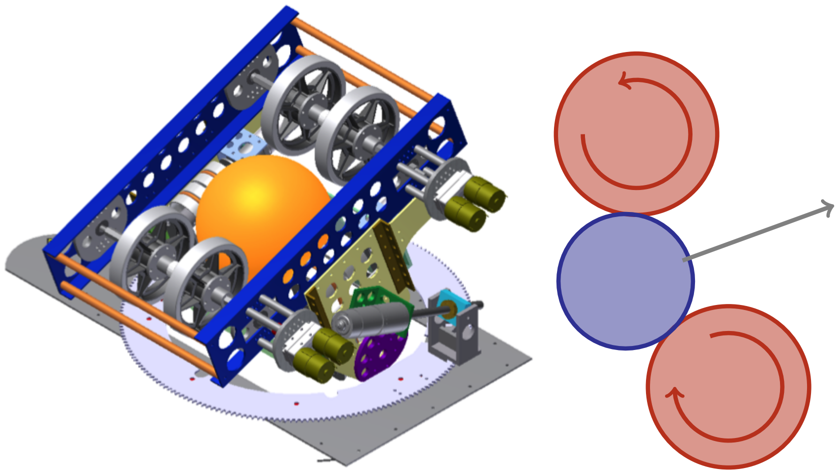

Figure 5: Dual wheel pitcher: example and schematic.

Some critical dimensions are:

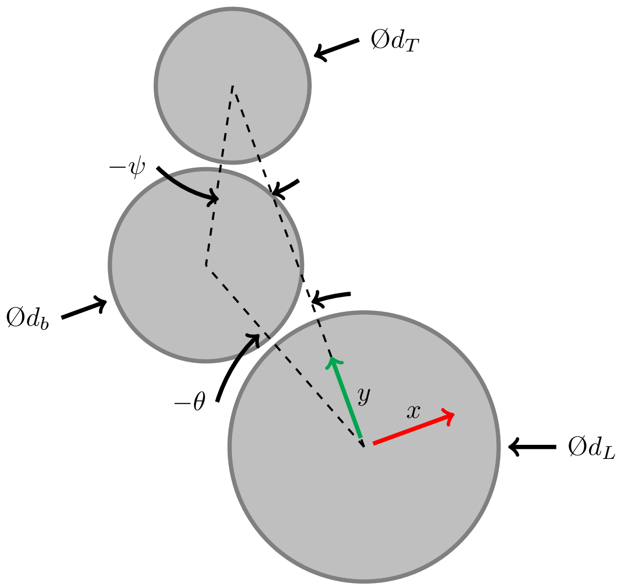

Figure 6: Dual wheel pitcher key dimensions and coordinates. Note sign on angles.

Geometry allows us to relate these as so:

$$tan\theta = \frac{x}{y} \tag{27}$$

$$tan\psi = \frac{x}{\frac{d_L+d_T}{2} + w - y} \tag{28}$$

The instantaneous radii of the ball (distance from center of ball to top and bottom wheels) can be readily computed.

$$R_B = \sqrt{x^2+y^2} - \frac{d_B}{2} \tag{29}$$

$$R_T = \sqrt{x^2 + (\frac{d_T+d_B}{2} + w - y)^2} - \frac{d_T}{2} \tag{30}$$

To analyze the ball, begin by drawing free-body and kinetic diagrams.

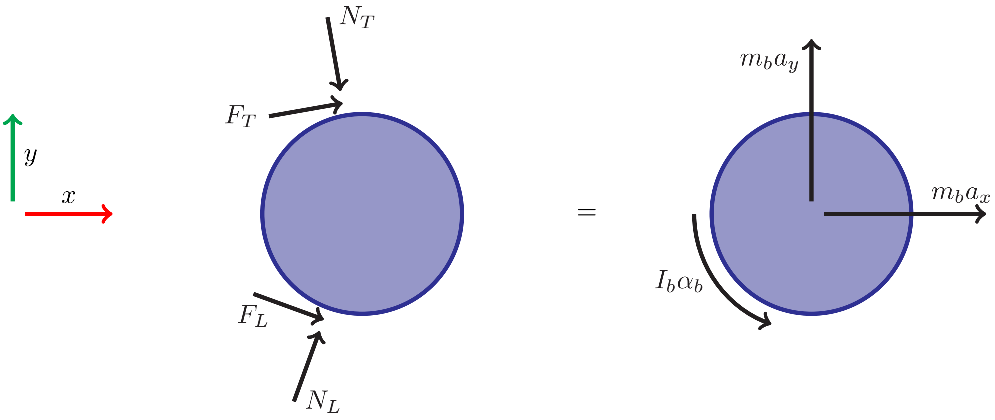

Figure 7: Free-Body and Kinetic Diagrams for Ball

$$F_T cos\psi + F_L cos\theta + N_T sin\psi + N_L sin\theta = m_b a_x \tag{31}$$
$$F_T sin\psi - F_L sin\theta - N_T cos\psi + N_L cos\theta = m_b a_y \tag{32}$$
$$F_L R_L - F_T R_T = I_b \alpha_b \tag{33}$$

We will assume negligible vertical acceleration, so $a_y = 0$. We can then solve and find

$$a_x = \frac{F_T cos\psi + F_L cos\theta + N_T sin\psi + N_L sin\theta}{m_b} \tag{34}$$
$$\alpha_b = \frac{F_L R_L - F_T R_T}{I_b} \tag{35}$$

As for the flywheel, there are two cases: the flywheels are separately powered and not linked, and the flywheels are centrally powered and linked. The indepdendent case is the same as that of the hooded pitcher, that is to say,

$$\alpha_{T,free} = -F_T \frac{d_T}{2I_T} \tag{36}$$
$$\alpha_{L,free} = -F_L \frac{d_L}{2I_L} \tag{37}$$

In the linked case, a torque constrains the two wheels.

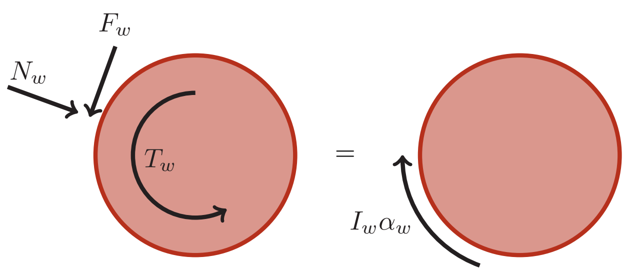

Figure 8: Free-Body and Kinetic Diagrams for Wheel

This leads to the torque balance

$$-N_w\frac{d_w}{2} - T_w = I_w\alpha_w$$
$$-F_T\frac{d_T}{2} - T_T = I_T\alpha_T \tag{38}$$
$$-F_B\frac{d_B}{2} - T_B = I_B\alpha_B \tag{39}$$

The two wheels are coupled by a gear reduction G.

$$\alpha_L = \alpha_T * G \tag{40}$$
$$-T_L * G = T_T \tag{41}$$

Solving these four equations yields

$$\alpha_{L,coupled} = \frac{-F_T\frac{d_T}{2G} - F_L\frac{d_L}{2}}{I_L + I_T/G^2} \tag{42}$$
$$\alpha_{T,coupled} = \alpha_L/G = \frac{-F_T\frac{d_T}{2G} - F_L\frac{d_L}{2}}{GI_L + I_T/G} \tag{43}$$

The ball can, again, be modeled as a spring.

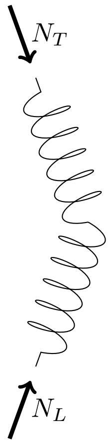

Figure 9: 2-D spring model for ball

The friction model that follows is the same idea as that of the hooded pitcher, though it contains many more terms.

When the lower wheel grips:

$$v_{L,surf} = v_{b,bottom}$$
$$a_{L,surf} = a_{b,bottom}$$
$$\alpha_L\frac{d_L}{2} = a_b cos\theta + \alpha_b R_L \tag{44}$$
$$\tag{45}$$

When the top wheel grips:

$$v_{T,surf} = v_{b,top}$$
$$a_{T,surf} = a_{b,top}$$
$$\alpha_T \frac{d_T}{2} = a_b cos\psi - \alpha_b R_T \tag{46}$$
$$\tag{47}$$

First let's handle the simpler case of free, uncoupled wheels. For the lower side, we need to find F_L.

$$-F_L \frac{d_L}{2I_L}\frac{d_L}{2} = (\frac{F_T cos\psi + F_L cos\theta + N_T sin\psi + N_L sin\theta}{m_b})cos\theta + \frac{F_L R_L - F_T R_T}{I_b} R_L \tag{48}$$
$$(\frac{N_T sin\psi + N_L sin\theta}{m_b})cos\theta + F_T \frac{R_T}{I_b} R_L - \frac{F_T}{m_b} cos\psi cos\theta - F_L \frac{d_L}{2I_L}\frac{d_L}{2} - \frac{F_L}{m_b} cos^2\theta - F_L \frac{R_L^2}{I_b} = 0 \tag{49}$$
$$\textit{recognizing that this can be written in the form of } A + BF_T + CF_L = 0 \tag{50}$$
$$A = (N_T sin\psi + N_L sin\theta)\frac{cos\theta}{m_b} \tag{51}$$
$$B = \frac{R_T R_L}{I_b} - \frac{cos\psi cos\theta}{m_b} \tag{52}$$
$$C = -\frac{d_L^2}{4I_L} - \frac{cos^2\theta}{m_b} - \frac{R_L^2}{I_b} \tag{53}$$
$$F_{L,\ bottom\ gripped,\ free\ wheels} = \frac{A + BF_T}{-C} \tag{54}$$

For the top side, we need F_T.

$$-F_T \frac{d_T}{2I_T}\frac{d_T}{2} = (\frac{F_T cos\psi + F_L cos\theta + N_T sin\psi + N_L sin\theta}{m_b})cos\psi - \frac{F_L R_L - F_T R_T}{I_b} R_T \tag{55}$$
$$(\frac{N_T sin\psi + N_L sin\theta}{m_b})cos\psi - F_T \frac{R_T}{I_b} R_T - \frac{F_T}{m_b} cos^2\psi - F_T \frac{d_T}{2I_T}\frac{d_T}{2} + \frac{F_L}{m_b} cos\theta cos\psi - F_L \frac{R_L R_T}{I_b} = 0 \tag{56}$$
$$\textit{recognizing that this can be written in the form of } D + EF_L + FF_T = 0 \tag{57}$$
$$D = (N_T sin\psi + N_L sin\theta)\frac{cos\psi}{m_b} \tag{58}$$
$$E = \frac{R_L R_T}{I_b} - \frac{cos\theta cos\psi}{m_b} \tag{59}$$
$$F = -\frac{d_T^2}{4I_T} - \frac{cos^2\psi}{m_b} - \frac{R_T^2}{I_b} \tag{60}$$
$$F_{T,\ top\ gripped,\ free\ wheels} = \frac{D + EF_L}{-F} \tag{61}$$

The same analysis is repeated for the coupled flywheel case. When the lower wheel grips, we need to find F_L.

$$\frac{-F_T\frac{d_T}{2G}-F_L\frac{d_L}{2}}{I_L+I_T/G^2}\frac{d_L}{2}=$$

$$\left(\frac{F_T cos\psi+F_L cos\theta+N_T sin\psi+N_L sin\theta}{m_b}\right)cos\theta+\frac{F_L R_L-F_T R_T}{I_b}R_L \tag{62}$$

$$\left(\frac{N_T sin\psi+N_L sin\theta}{m_b}\right)cos\theta+F_T\frac{R_T}{I_b}R_L-\frac{F_T}{m_b}cos\psi cos\theta-F_T\frac{\frac{d_T}{2G}}{I_L+I_T/G^2}\frac{d_L}{2}$$

$$-F_L\frac{\frac{d_L}{2}}{I_L+I_T/G^2}\frac{d_L}{2}-\frac{F_L}{m_b}cos^2\theta-F_L\frac{R_L^2}{I_b}=0 \tag{63}$$

$$\textit{recognizing that this can be written in the form of } A+BF_T+CF_L=0 \tag{64}$$

$$A=(N_T sin\psi+N_L sin\theta)\frac{cos\theta}{m_b} \tag{65}$$

$$B=\frac{R_T R_L}{I_b}-\frac{cos\psi cos\theta}{m_b}-\frac{\frac{d_T d_L}{4G}}{I_L+I_T/G^2} \tag{66}$$

$$C=-\frac{\frac{d_L^2}{4}}{I_L+I_T/G^2}-\frac{cos^2\theta}{m_b}-\frac{R_L^2}{I_b} \tag{67}$$

$$F_{L,\ bottom\ gripped,\ free\ wheels}=\frac{A+BF_T}{-C} \tag{68}$$

For the top side, we need F_T.

$$\frac{-F_T\frac{d_T}{2G}-F_L\frac{d_L}{2}}{GI_L+I_T/G}\frac{d_T}{2}=$$

$$\left(\frac{F_T cos\psi+F_L cos\theta+N_T sin\psi+N_L sin\theta}{m_b}\right)cos\psi-\frac{F_L R_L-F_T R_T}{I_b}R_T \tag{69}$$

$$\left(\frac{N_T sin\psi+N_L sin\theta}{m_b}\right)cos\psi-F_T\frac{R_T}{I_b}R_T-\frac{F_T}{m_b}cos^2\psi-F_T\frac{\frac{d_T}{2G}}{GI_L+I_T/G}\frac{d_T}{2}$$

$$-F_L\frac{\frac{d_L}{2}}{GI_L+I_T/G}\frac{d_T}{2}-\frac{F_L}{m_b}cos\theta cos\psi+F_L\frac{R_L R_T}{I_b}=0 \tag{70}$$

$$\textit{recognizing that this can be written in the form of } D+EF_L+FF_T=0 \tag{71}$$

$$D=(N_T sin\psi+N_L sin\theta)\frac{cos\psi}{m_b} \tag{72}$$

$$E=\frac{R_L R_T}{I_b}-\frac{\frac{d_L d_T}{4}}{GI_L+I_T/G}-\frac{cos\theta cos\psi}{m_b} \tag{73}$$

$$F=-\frac{\frac{d_T^2}{4G}}{GI_L+I_T/G}-\frac{cos^2\psi}{m_b}-\frac{R_T^2}{I_b} \tag{74}$$

$$F_{T,\ top\ gripped,\ free\ wheels}=\frac{D+EF_L}{-F} \tag{75}$$

For the case of both being gripped, we simply solve simultaneously.

$$A+BF_T+CF_L=0 \text{ and } D+EF_L+FF_T=0$$

$$F_{L,\ both\ gripped}=\frac{AF-BD}{BE-CF} \tag{76}$$

$$F_{T,\ both\ gripped}=\frac{CD-AE}{BE-CF} \tag{77}$$

State is determined as

$$top\ attached\ when\ \omega_T \frac{d_T}{2} \leq v_b - \omega_b R_T \tag{78}$$

$$lower\ attached\ when\ \omega_L \frac{d_L}{2} \leq v_b + \omega_b R_L. \tag{79}$$

All that's left is to set up the state equations and initial conditions.

$$\frac{d}{dt}\omega_b = \alpha_b \tag{80}$$

$$\frac{d}{dt}v_b = a_b \tag{81}$$

$$\frac{d}{dt}x = v_b \tag{82}$$

$$\frac{d}{dt}\omega_T = \alpha_T \tag{83}$$

$$\frac{d}{dt}\omega_L = \alpha_L \tag{84}$$

$$v_b(0) = 0 \tag{85}$$

$$x(0) = 0 \tag{86}$$

$$\omega_T(0) = \omega_{T,0} \tag{87}$$

$$\omega_L(0) = \omega_{L,0} \tag{88}$$

$$when\ x > x_end\ terminate \tag{89}$$

Hybrid Pitchers

An interesting design proliferated during the FRC 2020 season, consisting of a hooded pitcher with an extra set of 'afterburner' wheels where the hood would terminate. This could be modeled fairly easily (I simply haven't gotten to it yet). I may transfer the calculator into a single one consisting of only the hybrid pitcher, as hooded and dual-wheel pitchers could be considered just special cases.

Sanity Checks

These are new analysis techniques so there is no prior art to compare to. However, we can still perform some sanity checks.

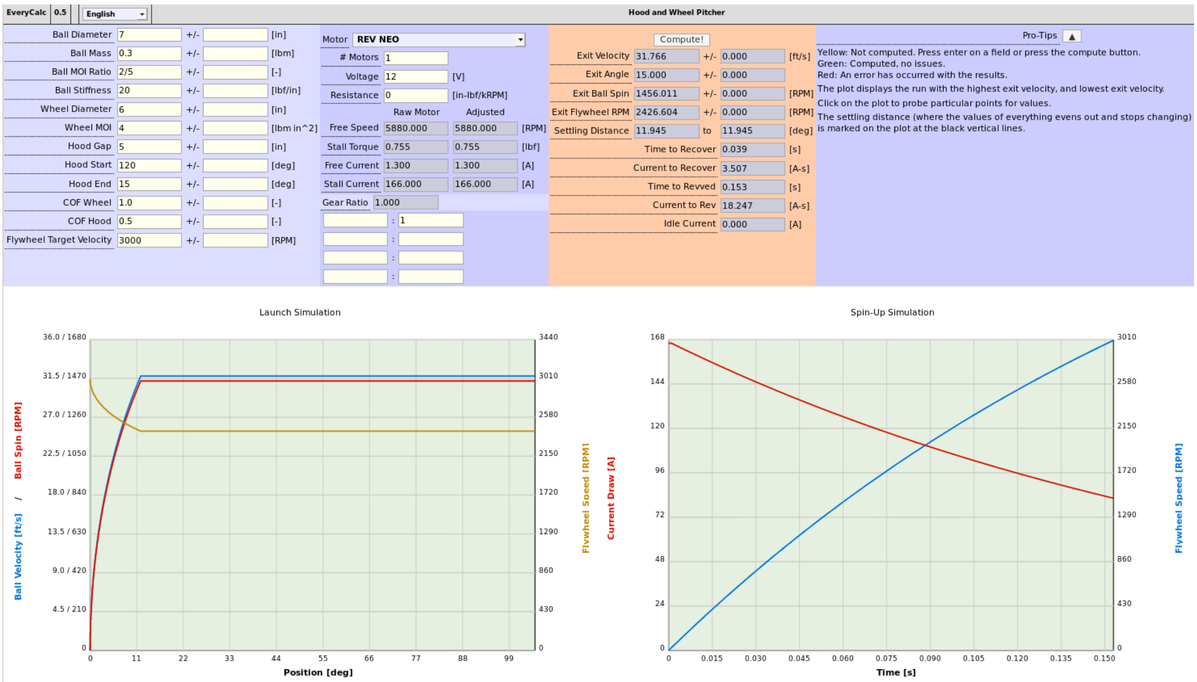

Figure 10: Hooded Flywheel Calculator - Exemplary Case

We can quickly check the relationship between ball velocity, ball spin, and wheel speed at exit. If the ball has finally latched onto the wheel, we'd see that the surface speeds of the wheel and ball should match. That is to say,

$$v_b + \omega_b \frac{w}{2} = \omega_w \frac{d_w}{2}. \tag{90}$$

Numerically in this case,

$$31.766\frac{\text{ft}}{\text{s}} + 1456 \text{ RPM} \times \frac{2\pi/60\text{rad/s}}{\text{RPM}} \times \frac{5}{2}\text{in} \times \frac{\text{ft}}{12\text{in}} = 2426.6 \text{ RPM} \times \frac{2\pi/60\text{rad/s}}{\text{RPM}} \times \frac{6}{2}\text{in} \times \frac{\text{ft}}{12\text{in}} \tag{91}$$

$$63.5310\frac{\text{ft}}{\text{s}} \approx 63.5282\frac{\text{ft}}{\text{s}}. \tag{92}$$

Is the wheel drop realistic? Let's look at the lost energy. There's definitely some losses as the ball interaction has friction, but let's look at it.

$$E = m_b v_b^2 + I_b \omega_b^2 + I_w \omega_w^2 \tag{93}$$

$$E_{init} = 4lbm\ in^2 \times (3000\text{RPM})^2 = 115.5\text{J} \tag{94}$$

$$E_{final} = 0.3\text{lbm} \times (31.766\text{ft/s})^2 + \frac{2}{5}0.3\text{lbm}(\frac{5\text{in}}{2})^2 \times (1456\text{RPM})^2 + 4lbm\ in^2 \times (2426\text{RPM})^2 = 93.45\text{J} \tag{95}$$

$$\eta = \frac{E_{final}}{E_{init}} = \frac{93.45\text{J}}{115.5\text{J}} = 80.9\% \tag{96}$$

That's an efficiency less than 100% (good, physics didn't break), and it isn't dreadfully low (signalling that there isn't too much slippage).

The rev-up simulation does have some prior art to compare to, luckily. We can run basically the exact same simulation in the simple mechanism calculator.

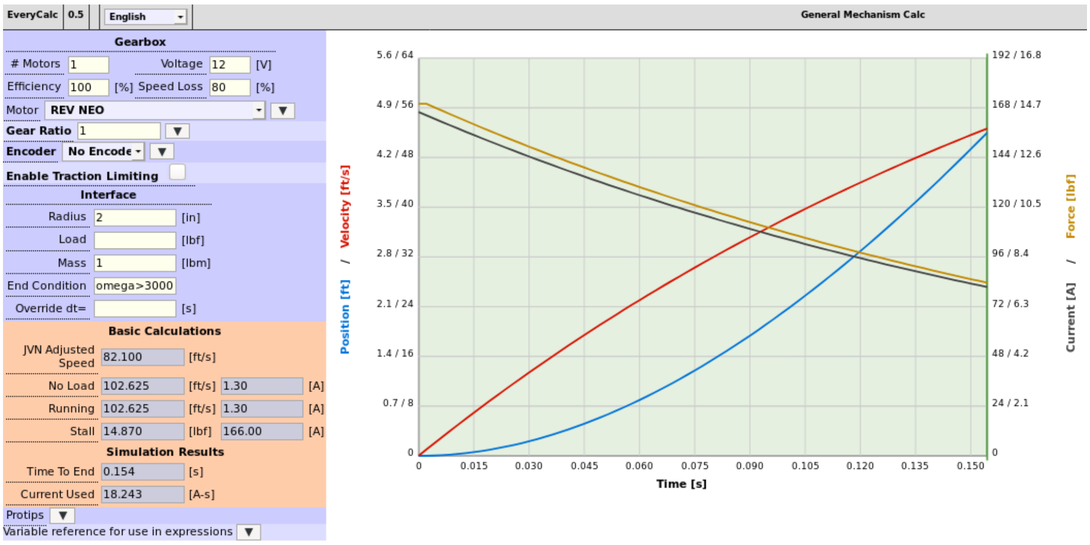

Figure 11: Revving up a 4 $lbm\ in^2$ flywheel with a single NEO motor.

This shows it takes 0.154 seconds to rev up- nearly the same as the 0.153 in the previous example. Note that the efficiency is specified as 100 percent- this is because the rev-up calculator instead uses a bearing friction value that the general mechanism calculator doesn't have.