Center-to-center calculation for power transmission belts

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Abstract

Belts are pretty easy to use and calculate the appropriate distances for. When this center distance is calculated and manufactured properly, they should not require adjustment.

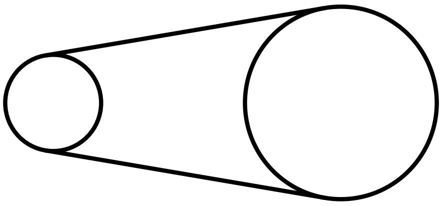

Figure 1: Belt and Sprockets

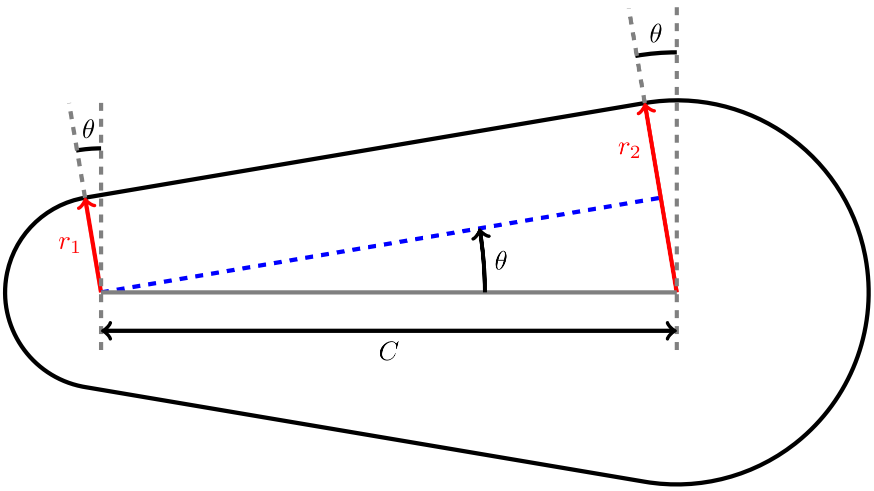

Figure 2: Belt Dimensions, Labeled

Quickly, the pitch radii and diameters of the pulleys are:

$$d_1 = 2r_1 \tag{1}$$

$$d_2 = 2r_2 \tag{2}$$

$$sin(\theta) = \frac{r_2 - r_1}{C} \tag{3}$$

The total length of the pulley L can be expressed as:

$$L = 2 < \text{straight segment} > + < \text{arc for pulley 1} > + < \text{arc for pulley 2} >$$

$$L = 2\frac{C}{cos(\theta)} + r_1(\pi - 2\theta) + r_2(\pi + 2\theta) \tag{4}$$

The trig identity for the cosine of an arcsine will be helpful:

$$cos(asin(x)) = \sqrt{1 - x^2} \tag{5}$$

Putting this all together lets us determine the total belt length in terms of pitch diameters d_1, d_2, and the center-center distance C:

$$L = \frac{2C}{\sqrt{1 - (\frac{d_2 - d_1}{2C})^2}} + \frac{d_1}{2}(\pi - 2\theta) + \frac{d_2}{2}(\pi + 2\theta) \tag{6}$$

This equation isn't easy to analytically solve for C in terms of d_1, d_2, and L. WolframAlpha yields a solution, though it is quite atrocious. I found that it's best to use a numeric algorithm (such as bisection, which my calculator uses).

The same approach can be taken with a crossed drive belt (which is used in order to reverse direction of rotation).

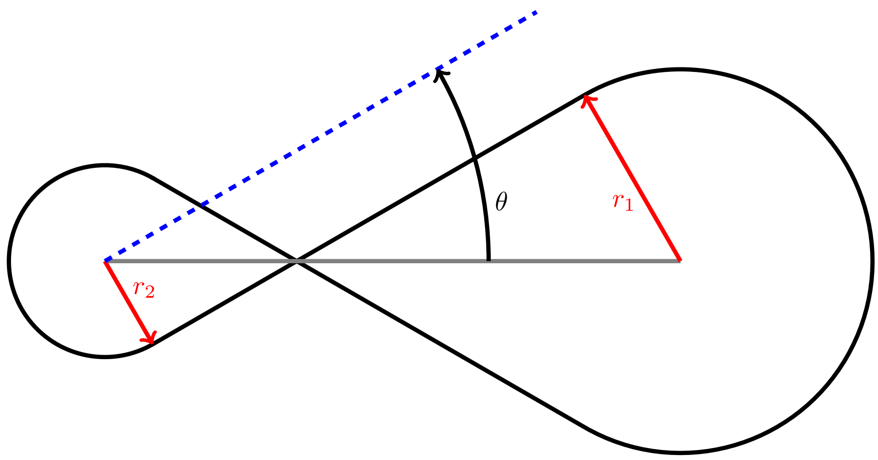

Figure 3: Belt Dimensions, Labeled

The belt angle now is

$$sin(theta) = \frac{r_2 + r_1}{C} \tag{7}$$

$$L = 2\frac{C}{cos(\theta)} + r_1(\pi + 2\theta) + r_2(\pi + 2\theta) \tag{8}$$

Resulting in:

$$L = \frac{2C}{\sqrt{1 - (\frac{d_2 + d_1}{2C})^2}} + \frac{d_1 + d_2}{2}(\pi + 2\theta) \tag{9}$$