

Transient Modeling of Flywheel-Based Pitchers

Thaddeus Hughes
hughes.thad@gmail.com
thaddeus-maximus.github.io

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Abstract

Flywheel based pitchers are very common ways to propel projectiles. They are well-suited to many such cases in that they are reasonably light, do not require keeping track of states, provide inherent strength-of-shot adjustment, and can be used in continuous applications.

Unfortunately, they are somewhat of a mystery, with design essentially relegated to trial-and-error rather than intuitive understanding or numerical simulation. This paper aims to change that.

Hooded Pitcher

Consider the following *hooded* pitcher. A wheel (red) is spun up to an initial RPM, and then a ball (blue) is introduced between it and an arced hood (black), which is concentric with the wheel.

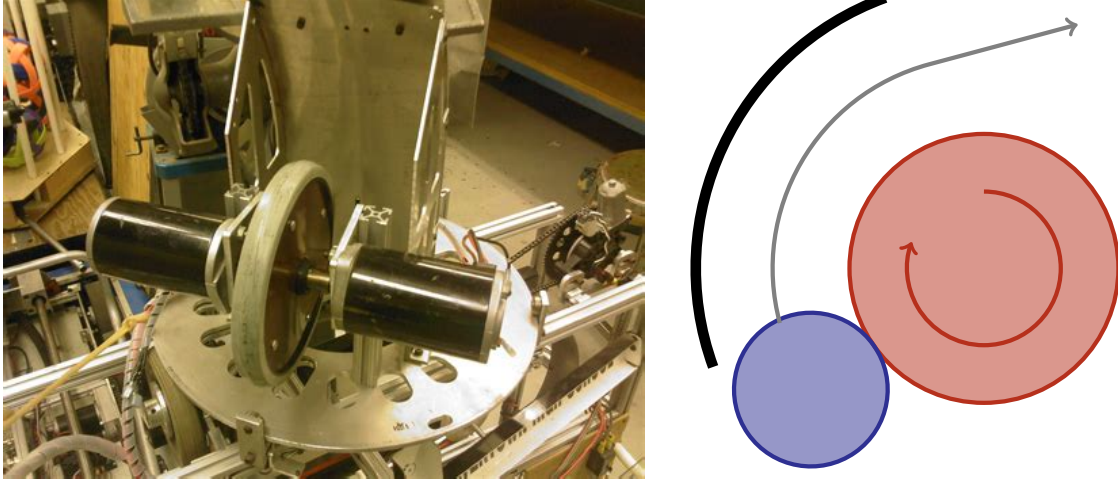


Figure 1: Hooded pitcher: example and schematic.

There are already many parameters of interest in this system.

d_w , the wheel diameter

I_w , the wheel's moment of inertia

$\omega_{w,0}$, the wheel's initial velocity

d_b , the ball's initial diameter

I_b , the ball's moment of inertia

w , the gap between the hood and the surface of the wheel

θ , the angle between inlet and outlet

What makes this work? The ball is accelerated as the wheel makes contact with the ball. The hood provides constraint, keeping the ball from simply spinning. Free-body diagrams may make this more clear.

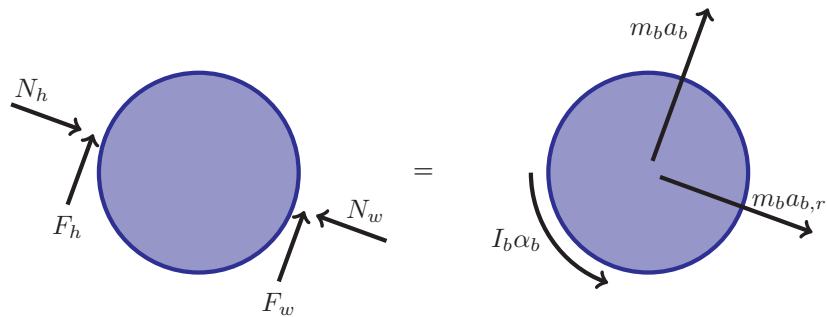


Figure 2: Free-Body and Kinetic Diagrams for Ball

For the sake of simplicity, gravity is neglected during this split-second interaction. This leads to force balances

$$F_h + F_w = m_b a_b \quad (1)$$

$$N_h - N_w = m_b a_{b,r} = m_b \frac{v_b^2}{R} \quad (2)$$

$$(F_w - F_h) \frac{w}{2} = I_b \alpha_b, \quad (3)$$

where R is the radius of the path travelled by the ball ($R = \frac{d_w + w}{2}$).

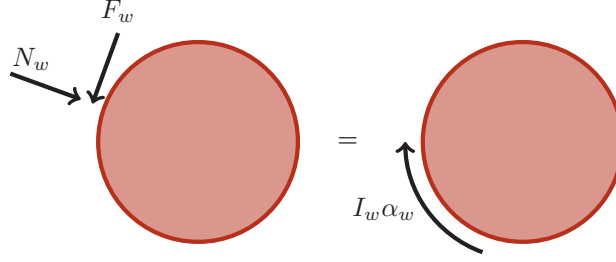


Figure 3: Free-Body and Kinetic Diagrams for Wheel

This leads to the torque balance

$$-N_w \frac{d_w}{2} = I_w \alpha_w \quad (4)$$

At this point we have four governing equations, four unknown forces, and four unknown kinematic properties. Let's recap and solve what we have so far to get expressions for the accelerations.

$$\alpha_b = (F_w - F_h) \frac{w}{2I_b} \quad (5)$$

$$a_b = \frac{F_h + F_w}{m_b} \quad (6)$$

$$\alpha_w = -F_w \frac{d_w}{2I_w} \quad (7)$$

Where does the normal force come from? Well, it comes from the ball's compression! But the ball is also accelerating along the axis of the compression, making this not quite straightforward.

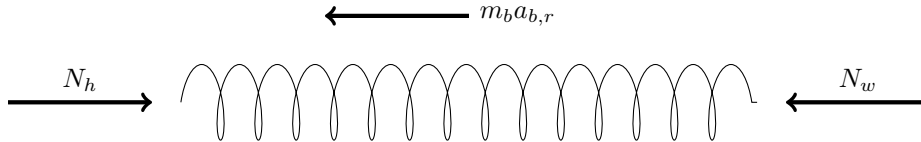


Figure 4: Massed spring model of the ball.

The inertial forces can be resolved in such a spring of stiffness k_b squeezed by δ as so:

$$N_h = k_b \delta + \frac{m_b a_{b,r}}{2} = k_b \delta + \frac{m_b v_b^2}{2R} \quad (8)$$

$$N_w = k_b \delta - \frac{m_b a_{b,r}}{2} = k_b \delta - \frac{m_b v_b^2}{2R} \quad (9)$$

Side takeaway: As the ball accelerates through the hood, it lifts off the wheel. This effect is magnified with a smaller arc radius.

We can relate the normal forces to the frictional or tangential forces with a basic coulomb friction model. More sophisticated models are probably more accurate, but the intent of this model is to provide intuitively understandable results and general guidelines which should be followed up by real-world testing, so this is an appropriate model.

$$F_h \leq \mu_h N_h \text{ sign}(0 - v_{b,top}) \quad (10)$$

$$F_w \leq \mu_w N_w \text{ sign}(v_{w,surf} - v_{b,bottom}) \quad (11)$$

Anyone who's simulated anything can immediately see the glaring problem here- sign functions have glaring discontinuities. There are many strategies to solve this. Here's what I've come up with.

First, recognize that the sign associated with the wheel should either be positive or zero. The ball isn't going to start over-spinning with the wheel. Second, recognize that the spin of the ball should never exceed the speed of the ball such that the top of the ball travels backwards, so its sign should be negative or zero. Third, the sign becomes zero, the condition to calculate is no longer one of two surfaces at arbitrary velocities exchanging forces, but of two surfaces that have *coupled*. This brings the driving physics away from force balance and towards pure kinematics. That is to say,

$$\begin{aligned} v_{w,surf} &= v_{b,bottom} \\ a_{w,surf} &= a_{b,bottom} \\ \alpha_w \frac{d_w}{2} &= a_b + \alpha_b \frac{w}{2} \end{aligned} \quad (12)$$

and

$$\begin{aligned} 0 &= v_{b,top} \\ 0 &= a_{b,top} \\ 0 &= a_b - \alpha_b \frac{w}{2}. \end{aligned} \quad (13)$$

Substituting what we found earlier about the accelerations gives us

$$\begin{aligned} -F_w \frac{d_w}{2I_w} \frac{d_w}{2} &= \frac{F_h + F_w}{m_b} + (F_w - F_h) \frac{w}{2I_b} \frac{w}{2} \\ F_w \frac{d_w}{2I_w} + \frac{F_w}{m_b} + F_w \frac{w}{2I_b} \frac{w}{2} &= -\frac{F_h}{m_b} + F_h \frac{w}{2I_b} \frac{w}{2} \\ F_w &= F_h \frac{\frac{w^2}{4I_b} - \frac{1}{m_b}}{\frac{d_w}{2I_w} + \frac{1}{m_b} + \frac{w^2}{4I_b}} \end{aligned} \quad (14)$$

in the case of no wheel slip, and

$$\begin{aligned} 0 &= \frac{F_h + F_w}{m_b} - (F_w - F_h) \frac{w}{2I_b} \frac{w}{2} \\ \frac{F_h}{m_b} + F_h \frac{w}{2I_b} \frac{w}{2} &= -\frac{F_w}{m_b} + F_h \frac{w}{2I_b} \frac{w}{2} \\ F_h &= F_w \frac{\frac{w^2}{4I_b} - \frac{1}{m_b}}{\frac{w^2}{4I_b} + \frac{1}{m_b}} \end{aligned} \quad (15)$$

in the case of no hood slip. In these cases, the force of one is directly proportional to the other. This means that when both hood and wheel stop slipping,

$$F_h = F_w = 0 \quad (16)$$

But how is state determined? We simply go back to the sign equations.

$$\text{top attached when } 0 \leq v_b - \omega_b \frac{w}{2} \quad (17)$$

$$\text{bottom attached when } \omega_w \frac{d_w}{2} \leq v_b + \omega_b \frac{w}{2} \quad (18)$$

This developed friction model can be summarized with this pseudocode, which would be ran every iteration of the simulation loop.

```

Fw ← mw Nw
Fh ← mh Nh
if hood attached and wheel attached then
    Fw ← 0
    Fh ← 0
else if wheel attached then
    Fw ← Fw in attached state
else if hood attached then
    Fh ← Fh in attached state

... insert interesting simulation code ...

if ball top speed goes positive then
    hood attached ← true
if ball bottom speed exceeds wheel speed then
    wheel attached ← true

```

All that's left is to set up the state equations and initial conditions.

$$\frac{d}{dt} \omega_b = \alpha_b \quad (19)$$

$$\frac{d}{dt} v_b = a_b \quad (20)$$

$$\frac{d}{dt} u_b = v_b \quad (21)$$

$$\frac{d}{dt} \omega_w = \alpha_w \quad (22)$$

$$v_b(0) = 0 \quad (23)$$

$$u_b(0) = 0 \quad (24)$$

$$\omega_w(0) = \omega_{w,0} \quad (25)$$

$$\text{when } u_b \geq \theta R \text{ terminate} \quad (26)$$

Dual-Wheel Pitchers

Hybrid Pitchers

An interesting design proliferated during the FRC 2020 season, consisting of a hooded pitcher with an extra set of 'afterburner' wheels where the hood would terminate. This could be modeled fairly easily (I simply haven't gotten to it yet). I may transfer the calculator into a single one consisting of only the hybrid pitcher, as hooded and dual-wheel pitchers could be considered just special cases.

Evaluation and Comparison to Empirical Data