Adiabatic Model of a Pneumatic Cylinder

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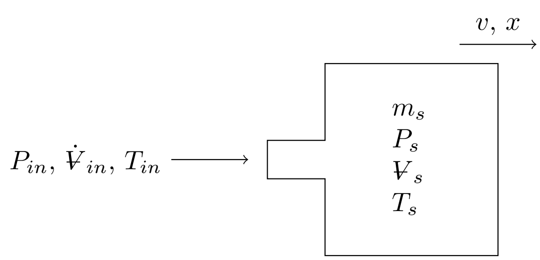
March 24, 2020

Abstract

There are some basic models for pneumatic cylinders that result in modeling cylinders as constant-force devices, or nearly so. This is a rough first pass at creating a model that is a little more sophisticated. This model assumes that the air in the cylinder is adiabatic; that is, no heat transfer takes place. This model will be a differential equation intended to be used in conjunction with a numerical DE solver due to the non-linear nature of many conditions.

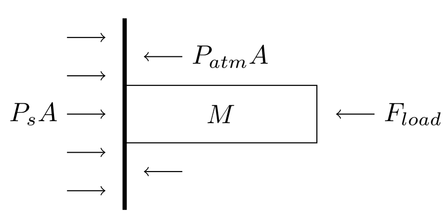
System Definition

We will analyze the open system that is the air contained within a pneumatic cylinder, plus the air in the hose leading up to it (inclusion of the hose will become of crucial importance and will be evident later on).



Air at pressure P_{in} and temperature T_{in} enters the cylinder at rate $\dot{V}_{in}$. It mixes with air in the cylinder (m_s, V_s) and is assumed to become of uniform pressure and temperature P_s and T_s.

The piston ram is moving at velocity v and with position x. x is zero when the cylinder is bottomed out (no air in the cylinder).



The cylinder itself is considered to have a mass M and sees the cylinder pressure, a resistive force F_{load}, and atmospheric pressure.

Conservation Principles In The Cylinder

First, let's apply conservation of mass to the air.

$$\frac{d}{dt}m_{sys} = \sum_{+in}\dot{m}$$
$$\frac{d}{dt}m_s = \dot{m}_{in}$$

We can use the ideal gas law $P\forall = mRT$ to write this in terms of knowns.

$$\frac{d}{dt}m_s = \dot{m}_{in} = \frac{P_{in}\dot{\forall}_{in}}{RT_{in}} \tag{1}$$

Now, let's apply conservation of energy.

$$\frac{d}{dt}E_{sys} = \sum \dot{Q}_{in} + \sum \dot{W}_{in} + \sum_{+in}\dot{m}(v^2/2 + gz + h)$$

We will assume:

- The energy of the system is entirely thermal; $E_sys = m_s u(T)$
- No heat transfer; $\dot{Q}_{in} = 0$
- Work is out of the system, defined by the pressure of the system and velocity of the ram; $\dot{W}_{in} = -\vec{F}\cdot\vec{v} = -P_sAv$
- Mass transfer into the system has negligible velocity (v) and head height (gz), leaving only enthalpy $h(T)$.

$$\frac{d}{dt}[m_s u(T)] = -P_sAv + \dot{m}_{in}h(T_{in})$$

The derivative here is quite ugly, since both the mass and temperature of the system are changing. However, it can be broken up with the product rule ($\frac{d}{dt}[xy] = y\frac{dx}{dt} + x\frac{dy}{dt}$).

$$m_s\frac{du(T_s)}{dt} + u(T_s)\frac{dm_s}{dt} = -P_sAv + \dot{m}_{in}h(T_{in})$$
$$m_s\frac{du(T_s)}{dt} = \dot{m}_{in}h(T_{in}) - P_sAv - u(T_s)\frac{dm_s}{dt}$$

Recognizing that $u(T)$ and $h(T)$ can be approximated as $u(T) = c_p T$ and $h(T) = c_v T$, we can then solve the equation for $\frac{dT_s}{dt}$.

$$\frac{dT_s}{dt} = \frac{\dot{m}_{in} c_v T_{in} - P_s A v - c_p T_s \dot{m}_{in}}{c_p m_s} \tag{2}$$

All of these are knowns or states aside from P_s, which can be determined with the ideal gas law, and the modeling of the system volume as based on cylinder extension, crossectional area, and initial (i.e. hose) dead volume.

$$\begin{aligned} P_s &= \frac{m_s R T_s}{\forall_s} \\ P_s &= \frac{m_s R T_s}{\forall_{dead} + xA} \end{aligned} \tag{3}$$

Conservation Principles on the Piston

We can simply apply conservation of linear momentum to the piston.

$$\frac{d}{dt} P_{sys} = \sum F + \sum \dot{m} v$$

We will assume:

- No mass transfer in this system, so $\frac{d}{dt} P_{sys} = M \frac{dv}{dt}$ and $\dot{m} = 0$
- The forces acting on the piston are the cylinder pressure, atmospheric pressure, and the external load.

$$\begin{aligned} M \frac{dv}{dt} &= P_s A - P_{atm} A - F_{load} \\ \frac{dv}{dt} &= \frac{(P_s - P_{atm}) A - F_{load}}{M} \end{aligned} \tag{4}$$

We are also interested in the position of the piston.

$$\frac{dx}{dt} = v \tag{5}$$

Limiting the Model

In review we have:

- A model for rate of change of m_s

- A model for rate of change of T_s
- Supporting equation for cylinder pressure P_s
- A model for velocity v
- A model for position x

We simply now need initial conditions, "bumper" conditions, and termination criteria.

We will assume the initial temperature T_s will be set to that of the input gas T_{in}. (This may seem like it would negate all the point of this model, but recall that when gases expand/contract, they change pressure)

$$T_s(t=0) = T_{in} \tag{6}$$

The initial mass of cylinder air can be found with the ideal gas law, assuming we start at atmospheric pressure (wholly unpressurized)

$$m_s(t = 0 = \frac{P_{atm} \forall_{dead}}{RT_s(t=0)} \tag{7}$$

Assume the piston starts from rest.

$$x(t=0) = 0 \tag{8}$$
$$v(t=0) = 0 \tag{9}$$